The homogeneous and inhomogeneous ECE current, Part IV

Horst Eckardt[1], Anil Goel[2]
A.I.A.S. and UPITEC
(www.aias.us, www.upitec.org)
Paper 457, Copyright © by AIAS, July 19, 2024, revised:

June 3, 2025

Abstract

This paper is the fourth in sequence that describes effects of the homogeneous current, which was shown to be a voltage flow rather than a flow of magnetic monopoles. Important aspects for practical use of this potential flux current are developed. Kirchhoff's laws for electronic circuits are extended to this current, which was previously called "cold current" by practitioners. An additional set of circuit laws is derived which is dual to the known laws for the electronic current. Impedances in AC circuits are also derived for the dual laws.

Keywords: ECE theory, ECE2 theory, electrodynamics, potential area flux density, circuit laws, Kirchhoff's laws, impedance.

1 Introduction

In ECE theory [1–3], a symmetric form of field equations has been developed. These comprise all areas of the unified field, like electrodynamics and mechanics. In the latest papers [4–6], the dual form of electrodynamics has been worked out in further detail. In the first of these papers [4], the ECE2 field equations were shown to contain current terms that consist of a conductivity part and a Lorentz force part. This holds for both the inhomogeneous and the homogeneous currents. In the second paper [5], polarization effects have been studied. The homogeneous current has been identified as a vacuum current, and it has been shown that energy transfer from the vacuum is possible.

The third paper [6] brought some remarkable new insights. The homogeneous current is a current of flowing potentials, and it has the characteristics of electricity rather than of a flow of magnetic monopoles. Therefore, we renamed it the potential flux current. The wave equations have been extended by conductivity terms for both types of currents, and the Heaviside flow (the energy flow outside of a conductor) is derived from the extended Poynting theorem. Moreover, the conductivity terms in the wave equations lead to expansion

---

[1]email: mail@horst-eckardt.de
[2]email: anil@anilgoel.com

velocities of electromagnetic waves that can be smaller or even larger than the speed of light in vacuo.

In this paper, we explain the consequences of the preceding papers with respect to practical electrical engineering. Our approach here is a simplified version of the original theory [7] which we have translated into standard vector algebra. The original theory is based on Clifford algebra and contains even more information. The potential flux current leads to a second set of circuit laws (Kirchhoff's laws) that are dual to the original ones. For AC currents, the impedances of standard elements, like coils and capacitors, take different forms when the second type of voltages/currents derived from the potential flux current are present. This opens new possibilities for the construction of electromagnetic devices.

2 Extended field equations

In Parts I-III of this article series [4–6], we have described the two currents of ECE theory [1–3], the homogeneous and inhomogeneous current. They are the right sides of the extended Maxwell-Heaviside equations:

$$\boldsymbol{\nabla} \cdot \mathbf{B} = \rho_p, \tag{1}$$

$$\frac{\partial \mathbf{B}}{\partial t} + \boldsymbol{\nabla} \times \mathbf{E} = \mathbf{V}, \tag{2}$$

$$\boldsymbol{\nabla} \cdot \mathbf{D} = \rho_e, \tag{3}$$

$$-\frac{\partial \mathbf{D}}{\partial t} + \boldsymbol{\nabla} \times \mathbf{H} = \mathbf{J}. \tag{4}$$

These equations are the Gauss law, the Faraday law, the Coulomb law, and the Ampère-Maxwell law. Therein, $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic induction, $\mathbf{J}$ is the electronic current density vector, and ρ_e is the volume charge density. The new terms on the right side are the potential density ρ_p and the potential area flux density $\mathbf{V}$. The equations hold for a spacetime with curvature and torsion, in contrast to the original Maxwell-Heaviside equations, which are valid only for special relativity.

3 Kirchhoff's circuit laws

3.1 Standard form

Kirchhoff's laws are widely used for the computation of electric circuits, and form the basis for circuit simulation software, such as SPICE. These laws handle electric devices as "point devices", i.e., no spatial extensions are considered, only their logical behavior is relevant. This allows circuits to be described by ordinary differential equations. If geometric dimensions of devices play a role, one has to use partial differential equations, which require finite element methods for their solution.

We will now take a closer look at the two Kirchhoff laws: the current law and the voltage law [8].

3.1.1 Current law

For any node (junction) in an electric circuit, the sum of the currents flowing into that node is equal to the sum of the currents flowing out of that node:

$$\sum_{i=1}^{n} I_{J,i} = 0. \tag{5}$$

This law is derived from the conservation of charge in the following way. The continuity equation connects the electric charge density ρ_e with the current density $\mathbf{J}$. It is derived from the Coulomb and Ampère-Maxwell laws [2], and reads

$$\frac{\partial \rho_e}{\partial t} + \mathbf{\nabla} \cdot \mathbf{J} = 0. \tag{6}$$

After taking the volume integral over a volume containing one junction, this equation gives

$$\int \frac{\partial \rho_e}{\partial t} \, dV + \int (\mathbf{\nabla} \cdot \mathbf{J}) \, dV = 0. \tag{7}$$

The first term is the electric current, by definition:

$$I_J = \int \frac{\partial \rho_e}{\partial t} \, dV. \tag{8}$$

If the current density is divergence-free, the inflow is equal to the outflow and the second term gives zero, thus resulting in

$$I_J = 0. \tag{9}$$

The current density in a junction consists of the sum of all current densities $\rho_{e,i}$ of the conductors connected to it:

$$I_J = \sum_i I_{J,i} = \sum_i \int \frac{\partial \rho_{e,i}}{\partial t} \, dV; \tag{10}$$

therefore,

$$\sum_i I_{J,i} = 0. \tag{11}$$

3.1.2 Voltage law

The second Kirchhoff law (voltage or loop law) states that the directed sum of the electric potential differences (voltages) around any closed loop is zero:

$$\sum_i U_{E,i} = 0. \tag{12}$$

For a circuit to be described by Kirchhoff's laws, it must be assumed that there is no external charge input or field interaction, for example, no antennas.

3

The second law is based on the Faraday law, in which both the electric and magnetic fields are zero on a surface far enough away from the circuit elements:

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = \mathbf{0}. \tag{13}$$

According to the Stokes theorem [9], we can apply the surface integral over the curl of a vector field where the surface S is that of the surrounding volume. This is then identical to the integral over any closed loop Γ on that surface:

$$\int_S (\boldsymbol{\nabla} \times \mathbf{E}) \cdot d\mathbf{S} = \oint_\Gamma \mathbf{E} \cdot d\mathbf{l}. \tag{14}$$
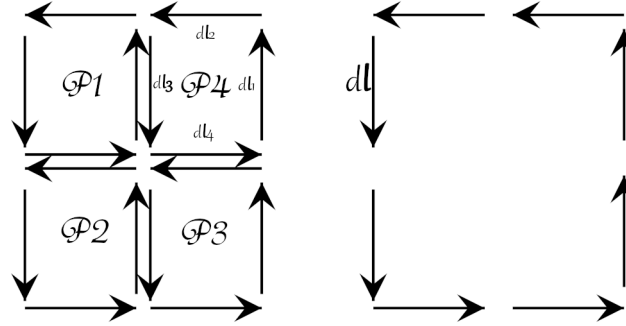

Figure 1: Integration paths for the Stokes theorem.

As depicted in Fig. 1, the surface can be subdivided into smaller parts P_i that cover each of the circuit elements. The line integrals can be taken over the parts P_i, but the integrations over the interior paths cancel out; consequently, their sum is equal to the integral over an outer path Γ. Because the line integral over an electric field is the voltage, it follows from Eq. (14) that

$$\sum_i U_{E,i} = \sum_i \oint_{P_i} \mathbf{E} \cdot d\mathbf{l} = \oint_\Gamma \mathbf{E} \cdot d\mathbf{l} = 0. \tag{15}$$

This is Kirchhoff's second law.

3.2 Kirchhoff's laws for the potential flux current

We will now derive Kirchhoff's laws for the potential density and potential flux current defined in Eqs. (1) and (2). We will see that these new laws are inverse to those of the electronic current. Since Eqs. (1) and (2) are dual to (3) and (4), we speak of the dual Kirchhoff laws.

3.2.1 Dual current law (node law for potential current)

Concerning the first law, conservation of this current must be guaranteed at node points. The "flux quantum" is a *volt-second* in this case. The number of these quanta must be conserved, as is the case for the electronic current.

We can use the same process that we used to reach Eq. (11). We start by deriving a continuity equation in the same way from the Gauss and Faraday laws, giving

$$\frac{\partial \rho_p}{\partial t} - \boldsymbol{\nabla} \cdot \mathbf{V} = 0. \tag{16}$$

This is the continuity equation for the potential flux density. Taking the volume integral over this equation gives

$$\int \frac{\partial \rho_p}{\partial t} \, dV - \int (\boldsymbol{\nabla} \cdot \mathbf{V}) \, dV = 0. \tag{17}$$

Because ρ_p is a voltage density with units of Vs/m^3, the first term gives the total voltage of the flow:

$$U_V = \int \frac{\partial \rho_p}{\partial t} \, dV. \tag{18}$$

The volume integral encompasses all potential flows in a node. Therefore, for the single contributions U_{Vi} of the node:

$$U_V = \sum_i U_{Vi} \tag{19}$$

and

$$\sum_i U_{Vi} = 0, \tag{20}$$

which is the dual node or "current" law that holds for the voltages of potential flow, in this case.

3.2.2 Dual voltage law (loop law for potential current)

The second law is based on the Ampère-Maxwell law (4):

$$-\frac{\partial \mathbf{D}}{\partial t} + \boldsymbol{\nabla} \times \mathbf{H} = \mathbf{J}. \tag{21}$$

Similarly to Eq. (13), we assume that both the electric and magnetic fields are zero on a surface far enough away from the circuit elements, and that there is no electric current:

$$\boldsymbol{\nabla} \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} = \mathbf{0}. \tag{22}$$

Proceeding analogously to Eq. (14), we find that

$$\int_S (\boldsymbol{\nabla} \times \mathbf{H}) \cdot d\mathbf{S} = \oint_\Gamma \mathbf{H} \cdot d\mathbf{l} \tag{23}$$

and, for the same reasons, the closed line integral over Γ vanishes. Since this integral represents the potential currents I_{Hi}, we get an equation that is the analogue of Eq. (15):

$$\sum_i I_{Hi} = \sum_i \oint_{P_i} \mathbf{H} \cdot d\mathbf{l} = \oint_\Gamma \mathbf{H} \cdot d\mathbf{l} = 0. \tag{24}$$

This is the dual of Kirchhoff's second law, which is valid for potential currents[1]:

$$\sum_i I_{Hi} = 0. \tag{25}$$

4 Examples for Kirchhoff's original and dual laws

4.1 Kirchhoff's original laws

For clarity, let us review the well-known laws for serial and parallel circuit elements.

4.1.1 Serial circuit

The current in a circuit path is always the same, and the voltages at the circuit path elements add up:

$$I_J = I_{J,1} = I_{J,2} = \cdots = I_{J,n}, \tag{26}$$
$$U_E = U_{E,1} + U_{E,2} + \cdots + U_{E,n}. \tag{27}$$

Ohm's law reads

$$U_E = I_J R, \tag{28}$$

from which we get the following:

$$U_E = U_{E,1} + U_{E,2} + \cdots + U_{E,n} = I_J \left(R_1 + R_2 + \cdots + R_n \right) \tag{29}$$
$$= I_J R_{\text{total}}.$$

4.1.2 Parallel circuit

In a parallel circuit, the voltages and currents behave inversely:

$$U_E = U_{E,1} = U_{E,2} = \cdots = U_{E,n}, \tag{30}$$
$$I_J = I_{J,1} + I_{J,2} + \cdots + I_{J,n}. \tag{31}$$

Using Ohm's law, we obtain

$$I_J = I_{J,1} + I_{J,2} + \cdots + I_{J,n} = U_E \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} \right) \tag{32}$$
$$= \frac{U_E}{R_{\text{total}}}.$$

---

[1] The index H for the current was chosen because it is computed as an integral over a closed loop of the H field.

4.2 Kirchhoff's dual laws

In the dual laws (see Section 3.2), I_J is replaced with U_V and U_E with I_H.

4.2.1 Serial circuit

In a serial circuit, we have:

$$U_V = U_{V,1} = U_{V,2} = \cdots = U_{V,n}, \tag{33}$$
$$I_H = I_{H,1} + I_{H,2} + \cdots + I_{H,n}. \tag{34}$$

To write the dual of Ohm's law, we have to use the conductance values $G = 1/R$:

$$I_H = U_V G, \tag{35}$$

from which we get the following:

$$I_H = I_{H,1} + I_{H,2} + \cdots + I_{H,n} = U_V \left(G_1 + G_2 + \cdots + G_n \right) \tag{36}$$
$$= U_V G_{\text{total}}.$$

4.2.2 Parallel circuit

In a parallel circuit, the inverse relations are

$$I_H = I_{H,1} = I_{H,2} = \cdots = I_{H,n}, \tag{37}$$
$$U_V = U_{V,1} + U_{V,2} + \cdots + U_{V,n}. \tag{38}$$

Using the conductance values from 4.2.1, we obtain

$$U_V = U_{V,1} + U_{V,2} + \cdots + U_{V,n} = I_H \left(\frac{1}{G_1} + \frac{1}{G_2} + \cdots + \frac{1}{G_n} \right) \tag{39}$$
$$= \frac{I_H}{G_{\text{total}}}.$$

4.3 Comparison with examples

The duality of Kirchhoff's laws is graphically presented in Figs. 2 and 3. The dual laws take some getting used to, since they are counter-intuitive for the electrical engineer. However, it should be helpful to remember that the potential current is a flow of potentials and behaves more like potentials than electric charge carriers.
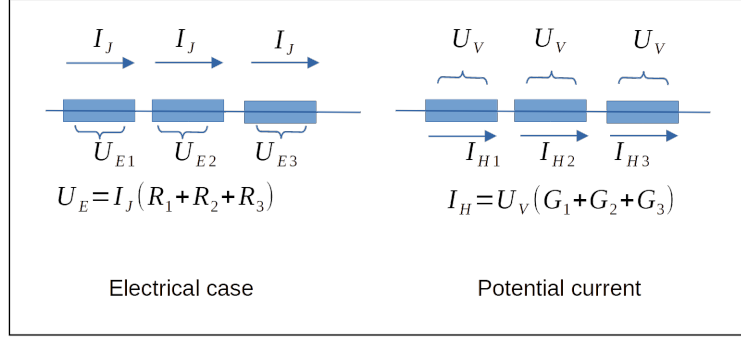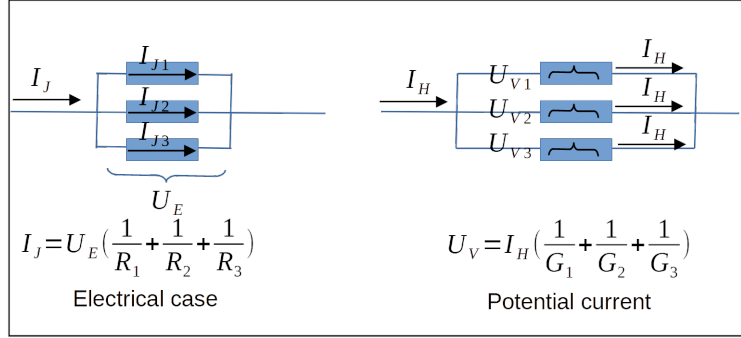
Figure 2: Comparison of serial circuits.

Figure 3: Comparison of parallel circuits.

5 AC resistances

For alternating currents, the impedances are frequency-dependent resistances. From standard electrical engineering, we know the induction law:

$$U_E = L\frac{dI_J}{dt}, \tag{40}$$

and the capacitance law:

$$I_J = C\frac{dU_E}{dt}, \tag{41}$$

where L and C are the inductance and capacitance values of the corresponding devices.

From the preceding sections, we see that the current-voltage dualities that appear through the potential current are

$$U_E \longleftrightarrow I_H$$

and

$$I_J \longleftrightarrow U_V.$$

By applying these dualities, we can directly formulate the equations dual to (40) and (41). The following are the new induction law:

$$U_V = L_H \frac{dI_H}{dt}. \tag{42}$$

and the new capacitance law:

$$I_H = C_V \frac{dU_V}{dt}, \tag{43}$$

In these equations, we have defined new constants C_V and L_H, which must be capacitance and inductance values to keep the units right. They could have values different from C and L.

To derive the AC properties of capacitance and inductance, one usually assumes harmonic behavior of the currents and voltages appearing on the right sides of Eqs. (40) and (41), for example,

$$I_J = I_{J0} \exp(i\omega t), \tag{44}$$

with a time frequency ω. We write Eqs. (40-43) with impedances for inductances and capacitances:

$$U_E = Z_L I_J, \text{ inductive,} \tag{45}$$
$$U_E = Z_C I_J, \text{ capacitive,} \tag{46}$$
$$I_H = G_L U_V, \text{ inductive,} \tag{47}$$
$$I_H = G_C U_V, \text{ capacitive.} \tag{48}$$

Evaluation of the time derivatives then gives

$$Z_L = i\omega L, \tag{49}$$
$$Z_C = \frac{1}{i\omega C}, \tag{50}$$
$$G_L = \frac{1}{i\omega L_H}, \tag{51}$$
$$G_C = i\omega C_V. \tag{52}$$

This is the complete dual structure of frequency-dependent impedance values.

6 Discussion

We start by recalling that the voltage created by an electric field is

$$U = \int_\Gamma \mathbf{E} \cdot d\mathbf{l}, \tag{53}$$

where Γ is a curve in the electric field that has different starting and ending points, in general. Analogously, the current induced by a magnetic field is

$$I = \int_\Gamma \mathbf{H} \cdot d\mathbf{l}. \tag{54}$$

In classical physics, this current is often called a "magnetic voltage". In the case considered here, the **H** field flows outside of the conductor (or a ferromagnetic core); it represents the voltage area flux density **V** and, consequently, I is called I_H.

The dual versions of Kirchhoff's laws interchange the effects of a serial and a parallel connection. This could cause confusion: how can a serial connection, as in Fig. 2, have different currents in each resistance element? To answer this question, it can be helpful to visualize the potential current as consisting of voltages, internally. It then becomes easier be seen that multiple resistances in a series can produce an enhancement of current. An alternative interpretation is that we have negative resistances in such a circuit that are current sources instead of (ohmic) consumers. These two interpretations could explain the different voltages in the parallel circuit on the right side in Fig. 3.

In the case of AC impedances, we obtain from Eqs. (45-52) the following pairs:

$$U_E = i\omega L\, I_J, \tag{55}$$
$$U_V = i\omega L_H\, I_H, \tag{56}$$

and

$$U_E = \frac{1}{i\omega C}I_J, \tag{57}$$
$$U_V = \frac{1}{i\omega C_H}I_H. \tag{58}$$

We see that the impedance laws are maintained, for the most part: only the parameter values of the elements could differ for the standard and potential currents. We have to bear in mind, however, that the dual of a voltage is a current, and vice versa. Thus, when comparing Eqs. (41) and (42), for example, the appearance of the dual part means that a capacitance is the dual of an inductance, and vice versa.

The power consumption can be computed in the usual way for the electric current as well as for the potential current:

$$P_{EJ} = U_E I_J, \tag{59}$$
$$P_{VH} = U_V I_H. \tag{60}$$

From the preceding papers, we remember that a potential current gathers energy from the environment, while the electronic current dissipates energy. For practical use, instruments for measuring both types of voltages and currents are required. Unfortunately, no instruments exist for the potential current, and voltmeters show both types of voltage in sum. Additional development work would be required to produce suitable devices for research and practical employment.

Acknowledgment

References

[1] M. W. Evans, "Generally Covariant Unified Field Theory: The Geometrization of Physics", Vols. 1 to 7, Abramis Academic, Bury St Edmunds, 2005 ff.

[2] H. Eckardt, "ECE UFT - The Geometrical Basis of Physics, Vol. I - Classical Physics", textbook, epubli, Berlin, 2022, freely available as UFT Paper 438, Unified Field Theory (UFT) section of www.aias.us.

[3] H. Eckardt, "ECE UFT - The Geometrical Basis of Physics, Vol. II - Quantum Physics", textbook, in preparation, freely available as UFT Paper 448, Unified Field Theory (UFT) section of www.aias.us.

[4] H. Eckardt, "Properties of the homogeneous and inhomogeneous ECE current, Part I", UFT Paper 454, Unified Field Theory (UFT) section of www.aias.us.

[5] H. Eckardt, "Properties of the homogeneous and inhomogeneous ECE current, Part II", UFT Paper 455, Unified Field Theory (UFT) section of www.aias.us.

[6] H. Eckardt, "Properties of the homogeneous and inhomogeneous ECE current, Part III", UFT Paper 456, Unified Field Theory (UFT) section of www.aias.us.

[7] A. Goel, "RA-DIANT Next Generation Renewable Energy Device", Publications section of www.aias.us.

[8] https://en.wikipedia.org/wiki/Kirchhoff%27s_circuit_laws

[9] https://en.wikipedia.org/wiki/Generalized_Stokes_theorem