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A definitive model of the geomagnetic field and its secular variation for 1965 – I. Derivation of model and comparison with the IGRF

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Summary. Using a very large body of post-1955 data, a spherical harmonic model of the geomagnetic field and its secular variation is derived for 1965.0. This model is compared with the original International Geomagnetic Reference Field (IGRF) and with individual models used, or proposed for use, in producing the IGRF. Positions of the dip-poles, the geomagnetic poles and the eccentric dipole are derived from the model, together with their rates of change, and comparisons are made with other estimates of these positions.

1 Introduction

Spherical harmonic models of the geomagnetic field are derived for a number of purposes. They are used in the preparation of magnetic charts for navigators and surveyors, for calculating the trajectories of charged particles outside the Earth and for removing the regional trend from survey data to reveal the anomalies. For these purposes, most users require the most up to date model possible, valid for the date at which the model was produced and, preferably, predicting the field for a few years after the date of production. Inevitably, such predictive models contain inaccuracies, since the mean date of observation of the data is some years before the required date, and extrapolation is involved. Spherical harmonic models may also be used to investigate processes in the Earth's deep interior; for example, the dynamics of the core, where both main field and secular change are believed to originate. For this purpose, the most accurate possible model is required.

The International Geomagnetic Reference Field (IGRF) (Zmuda 1971) was an attempt to satisfy as many users as possible by providing an internationally agreed model that would be a good representation of the main field for 1965.0, with secular change coefficients that would extend its usefulness for at least a decade. Individual models were submitted for assessment before 1968 March 15, and the final model, which was a weighted mean of several of the candidate models, was adopted on 1968 October 25. Despite the inherent weaknesses of such a model, the IGRF has proved remarkably successful, particularly for the removal of trend from surveys, which had previously been done by means of local polynomial fits, or the arbitrary adoption of one of the many global models available, which made the matching of anomaly charts for adjacent regions well-nigh impossible. However, as time passed, the imperfections of the model increased due to inaccuracies in the secular variation coefficients, resulting in unacceptable differences between the IGRF and the true field. In an attempt to extend its useful life without introducing discontinuities into the main field, a revised set of IGRF secular change coefficients was adopted for use from 1975.0. While this has reduced the rate of growth of errors in the IGRF, it is recognized that this is only a temporary measure, and that a new model must be adopted at an early date. Thus it is appropriate to assess the methods used for selecting the IGRF, and to see how closely they achieved their objective. For this purpose, it is convenient to compare the models submitted for the IGRF, and the IGRF itself, with the 'truth' for 1965.0.

For these two reasons (the provision of an accurate model for the study of core processes and for assessing the IGRF) a new model has been produced for 1965, based on all available survey, satellite and observatory data obtained in the interval 1955 to 1975. Since the model refers to a date near the centre of the period covered by the data, errors due to extrapolation are minimized. We refer to this as a 'definitive' model, partly to contrast it with the more usual predictive models, and partly because it seems unlikely that any significant improvement will be made in the representation of the field for 1965 by the acquisition of further data. The choice of 1965 is particularly appropriate, not only because it coincides with the date of the IGRF, but also because a large quantity of good quality data is available, with a suitable temporal and spatial distribution, near this date, and an adequate set of secular variation models exists which can be used to reduce the data to 1965.

In this paper, we present the model together with details of its derivation and the data upon which it is based, and examine the differences between it and the various IGRF candidates. In a later paper we will consider the problems of extrapolating the model down to the core–mantle interface and examine the physical implications at this level.

2 Data

The main field coefficients are based on a data set which is essentially the same as that described by Barraclough *et al.* (1975). We used all available post-1955 data in the following categories: (a) data from land, sea and aeromagnetic surveys, together with observatory annual mean values; (b) total intensity data from oceanographic surveys; (c) observations of declination made in the remoter oceanic regions by merchant vessels during routine compass checks; (d) observations of total intensity from the *POGO* series of satellites. The numbers of observations of the various magnetic elements – declination (D), inclination (I), horizontal intensity (H), north component (X), east component (Y), vertically downward component (Z) and total intensity (F) – in the first three categories are given in Table 1. Fig. 1 summarizes the time distribution of the data (all elements) in categories (a), (b) and (c); the mean date of observation is 1965.9. The spatial distribution of the data is reasonably good, as shown in figs 1 and 2 of Barraclough *et al.* (1975). The satellite data were included by using a spherical harmonic model based on the *POGO* observations.

The data used to derive the secular variation coefficients are also as described by Barraclough *et al.* (1975). Briefly, they consist of observatory annual means, observations made at a selected number of repeat stations, survey data and satellite F data, the last two categories being included via spherical harmonic models.

3 Reduction and analysis

The main field data were reduced to 1965.0 as described by Barraclough *et al.* (1975), the only difference being that here the data for the period 1955–65 are given the same weight

Table 1. Numbers of observations used in the main field analysis.

Element	Category (a)	(b)	(c)	Total*	No. of tesseral means
D	92414		3989	92453.89	1448
I	82227			82227	1352
H	39221			39221	941
X	42			42	8
Y	42			42	8
Z	35786			35786	865
F	115099	302608		417707	1446
All	364831	302608	3989	667478.89	6068

* Giving weight 0.01 to category (c).

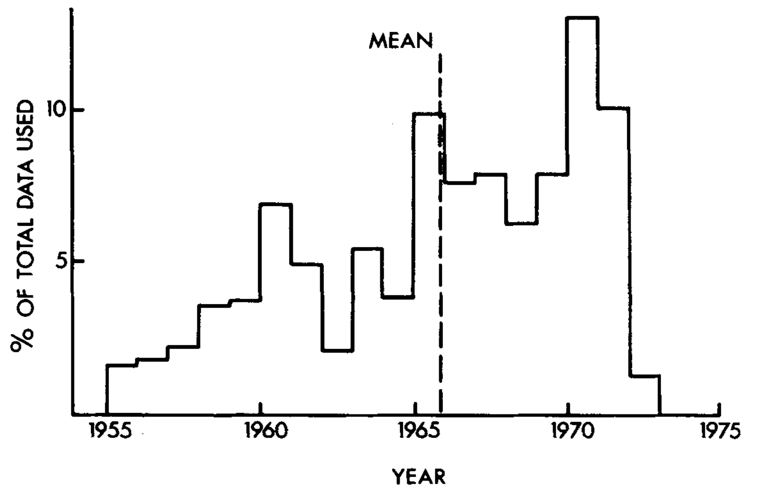

Figure 1. Distribution with time of data used in main field analysis.

as the post-1965 data in categories (a) and (b). This reference also gives details of the formation of tesseral means, rejection of anomalous data, treatment of the secular variation data and the techniques of spherical harmonic analysis used.

The results of this analysis are the sets of spherical harmonic coefficients given in Table 2, for the main field and for the secular variation up to degree (n) and order (m) of 8. The coefficients are in the Schmidt quasi-normalized form, and refer to a spheroidal earth of mean radius 6371.2 km and flattening 1/298.25.

4 Comparison with the IGRF

For each element, main field and secular variation differences, in the sense definitive model *minus* IGRF, were computed at intervals of 5° in latitude and longitude between 85° N and 85° S. Table 3 summarizes these comparisons, giving, for each element, the mean and rms values of the differences, the values of the maximum and minimum differences and the positions at which they occur. It is clear from these results that the IGRF represents the field at 1965.0 well. The secular variation components of the IGRF show larger relative

Table 2. Spherical harmonic coefficients of the main field and secular variation for 1965.0.

m	n	g_n^m (nT)	h_n^m (nT)	$\dot{g}_n^m$ (nT yr^{-1})	$\dot{h}_n^m$ (nT yr^{-1})
0	1	-30337.1		20.8	
1	1	-2119.4	5777.2	9.4	-4.4
0	2	-1660.9		-22.4	
1	2	2999.4	-2016.4	0.3	-9.2
2	2	1595.8	115.7	3.8	-18.0
0	3	1298.9		-1.1	
1	3	-2036.7	-406.5	-10.1	6.2
2	3	1291.0	242.1	-1.7	2.5
3	3	853.9	-163.6	-5.2	-6.9
0	4	957.7		-1.3	
1	4	802.8	147.2	-0.2	2.8
2	4	479.5	-267.8	-4.3	0.7
3	4	-390.1	14.8	-0.7	2.6
4	4	253.8	-267.9	-3.5	-2.1
0	5	-218.7		1.3	
1	5	357.4	17.6	0.2	2.1
2	5	252.9	125.7	2.7	2.1
3	5	-29.9	-128.3	-0.7	-2.7
4	5	-156.5	-95.9	-0.7	1.5
5	5	-59.0	83.1	1.2	0.0
0	6	45.2		-0.3	
1	6	61.9	-9.5	0.3	-0.5
2	6	9.0	99.6	1.2	0.1
3	6	-228.3	67.9	2.8	1.3
4	6	5.1	-31.0	0.0	-1.6
5	6	2.5	-9.0	0.1	0.3
6	6	-110.0	-4.7	-0.8	2.1
0	7	78.4		-0.3	
1	7	-58.4	-58.8	-0.4	-1.4
2	7	6.2	-26.7	-0.7	0.0
3	7	12.8	0.8	0.1	0.1
4	7	-23.5	3.9	0.7	0.2
5	7	-5.6	26.7	0.4	-0.6
6	7	14.0	-22.2	0.4	-0.2
7	7	2.6	-9.3	-0.7	1.1
0	8	12.9		0.5	
1	8	3.8	5.3	0.3	-0.2
2	8	-5.1	-11.5	0.2	-0.4
3	8	-16.5	12.4	0.1	-0.4
4	8	-0.2	-16.9	-0.3	-0.1
5	8	4.9	5.3	-0.5	0.6
6	8	0.8	22.8	0.7	-0.5
7	8	11.2	-3.2	-0.4	-0.6
8	8	4.9	-15.9	-0.1	0.2

Table 3. Global comparisons between the definitive model for 1965.0 and IGRF 1965 (definitive *minus* IGRF) each based on 2520 values computed at 5° intervals of latitude and longitude between 85° N and 85° S.

			Extreme values of difference					
Element	Mean difference	RMS difference	Minimum	Position Latitude	Position Longitude	Maximum	Position Latitude	Position Longitude
a) Main field (units:- minutes of arc for D and I, nT for other elements)								
D	-1.9	22.5	-243.6	75°N	100°W	297.6	65°S	140°E
I	-0.0	7.7	-31.6	5°N	15°E	39.7	15°S	75°W
H	1.2	48.1	-145.0	20°N	130°W	144.0	15°N	155°E
X	-10.1	53.9	-153.4	20°N	130°W	143.7	15°N	155°E
Y	-0.0	54.1	-220.9	15°S	55°W	147.2	15°S	90°W
Z	4.5	86.7	-305.5	5°N	15°E	318.7	15°S	75°W
F	-1.5	59.6	-132.2	80°N	165°E	179.5	60°N	100°E
b) Secular variation (units:- minutes per year for $\dot{D}$ and $\dot{I}$, nT yr^{-1} for other elements)								
$\dot{D}$	0.1	4.3	-39.9	75°N	95°W	55.3	75°N	100°W
$\dot{I}$	-0.0	1.0	-3.4	5°S	20°E	3.2	15°S	70°W
$\dot{H}$	-3.2	9.0	-36.6	35°N	95°E	19.2	50°S	130°W
$\dot{X}$	-2.6	9.3	-36.6	35°N	95°E	20.4	55°S	120°W
$\dot{Y}$	-0.0	9.6	-21.3	10°S	55°W	27.3	10°S	35°E
$\dot{Z}$	-1.1	15.6	-45.5	60°N	95°E	44.7	35°S	125°W
$\dot{F}$	-8.1	14.1	-46.4	55°N	95°E	14.6	5°S	15°E

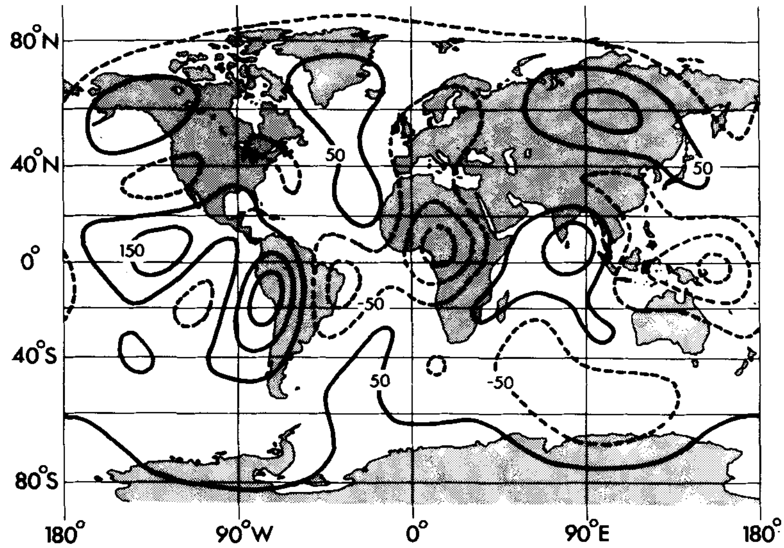

Figure 2. Chart of vertical component differences (definitive *minus* IGRF). Contour interval 100 nT. Solid lines – positive difference; dashed lines – negative difference.

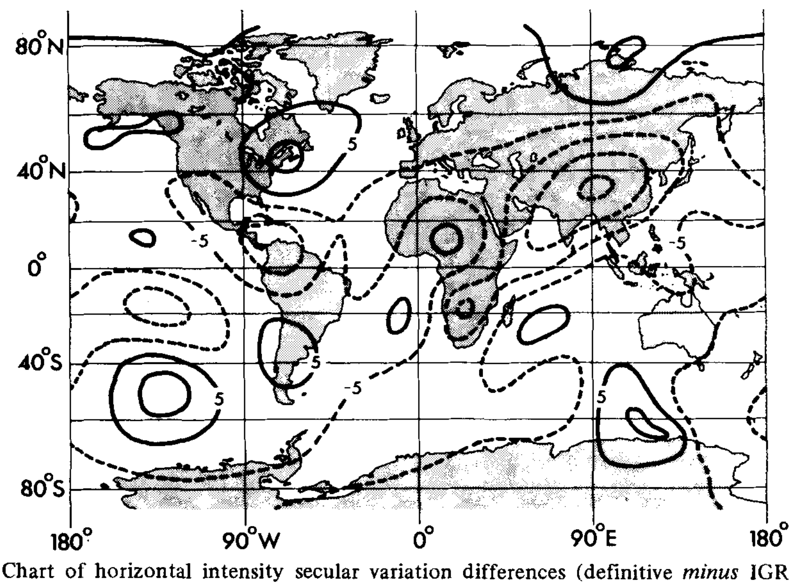

Figure 3. Chart of horizontal intensity secular variation differences (definitive *minus* IGRF). Contour interval 10 nT/yr. Solid lines – positive difference; dashed lines – negative difference.

differences from the definitive model, of the order of magnitude to be expected from regional studies of the discrepancies in the IGRF secular variation (see, e.g. Barraclough 1971).

The grid-point values of the differences between the definitive field and the IGRF were contoured. The resulting charts show, in the majority of cases, a patchwork of small foci, both positive and negative, with no signs of any systematic pattern. The exceptions are, for the main field, the chart of Z differences (Fig. 2) and, for the secular variation, the charts of differences in $\dot{H}$ and $\dot{Z}$ (Figs 3 and 4, respectively). Fig. 2 shows a series of large foci (up to ± 300 nT) extending round the Earth in a band which is approximately parallel to the dip-equator. They are equivalent to small differences in the positions which the two models give

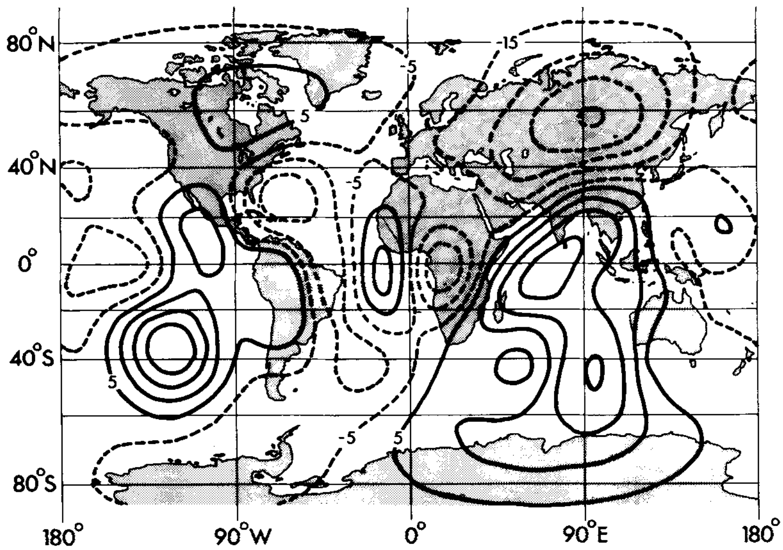

Figure 4. Chart of vertical component secular variation differences (definitive *minus* IGRF). Contour interval 10 nT/yr. Solid lines – positive difference; dashed lines – negative difference.

for the dip-equator. The secular change in the horizontal intensity $\dot{H}$ (Fig. 3) shows large differences in three regions: over China, in the eastern Pacific Ocean and over the South Pacific. These are all areas where secular variation data are very sparse. Fig. 4 shows large foci of the differences in $\dot{Z}$ near the dip-equator, over Siberia and in the Pacific and Indian Oceans. The latter three regions are again sparsely covered with data. Near the dip-equator, the IGRF relied heavily on satellite F data, and it has been shown (Lowes 1975; Barraclough & Nevitt 1976) that, in this region, such data provide a poor control of the values of Z and $\dot{Z}$ from a spherical harmonic model.

Having examined the global distribution of departures of the IGRF elements from the definitive model, we next consider the departures of each candidate model, as summarized by a single parameter derived directly from the coefficients. There were eight candidate models, whose characteristics are described by Zmuda (1971). Briefly, they are as follows: (1) Cain *et al.* (1967), GSFC (12/66)-1; (2) Fougere (1969), AFCRL 3/15/68, referred to here as Fougere 2; (3) Malin; (4) IZMIRAN (Tyurmina & Cherevko 1967); (5) Hurwitz *et al.* (1966), mean coefficients; (6) Leaton, Malin & Evans (1965), weighted mean of $(X + Y)$ and Z coefficients; (7) Cain & Cain (1968), *POGO* (3/68); (8) Fougere (1969), AFCRL 11/1/67, referred to here as Fougere 1. All the coefficients are listed by Cain & Cain (1968), although their version of model 4 differs slightly from that given by Tyurmina & Cherevko (1967). The main field coefficients are included in the catalogue of Barraclough (1978), designated 1960/16, 1965/15, 1965/04, 1965/21, 1965/09, 1965/03, 1965/13, 1965/14, respectively. Models 1, 2, 3 and 8 were based on surface and satellite data; models 5 and 6 on surface data only and models 4 and 7 on satellite data only. Where surface data were used, these were selected from the post-1900 compilation held in the World Digital Data Centres. All models included secular variation coefficients, those of models 1, 2, 7 and 8 derived simultaneously with those for the main field, those of the remaining models based almost exclusively on observatory annual means (though model 4 also included repeat station data and cross-over points from *Zarya* surveys). The only published source for the secular variation coefficients of models 3, 4 and 5 is Cain & Cain (1968).

The IGRF main field coefficients are a weighted mean of models 1 to 4, and the IGRF secular variation coefficients are the means of models 1 to 5.

The departures of these models, and the IGRF itself, from the definitive model are measured by a figure of merit derived as follows. If δg_n^m and δh_n^m denote the departures of the spherical harmonic coefficients of a model from those of the definitive model, then

$$\sigma_n^2 = (n + 1) \sum_{m=0}^{n} [(\delta g_n^m)^2 + (\delta h_n^m)^2]$$

is the mean square contribution to the vector field from the degree n terms (Lowes 1966). The total rms contribution (σ) to the vector field from all terms up to $n = 8$ is then given by

$$\sigma = \left\{ \sum_{n=1}^{8} \sigma_n^2 \right\}^{1/2}.$$

The results of these comparisons are presented in Table 4. Two points should be noted. Firstly, the coefficients of model 1 refer to 1960.0 and, before being used to derive the IGRF were converted to the appropriate 1965.0 values (which are given by Cain & Cain 1968) using the first- and second-order time derivative coefficients of the same model. The 1965.0 values were used in the present comparisons. Secondly, all but two of the main field models were produced using the assumption that the Earth is spheroidal in shape. The two exceptions, models 5 and 6, assumed a spherical shape. To make them more comparable with the other models we applied an approximate correction to them in the following way. Malin & Pocock (1969) produced, using the same data in every case, two sets of main field models, one set assuming the Earth to be spheroidal and the other using the spherical approximation. Taking the differences between the models with maximum degree and order of 8 from each set, we assumed that the application of these differences to the two spherical models would give an approximation to the coefficients which would have been derived had the spheroidal assumption been used.

Table 4 shows that the choice of constituents for the IGRF main field model was not the best that could have been made. Each of the rejected candidates compares better with the definitive model than does model 4. Accepting the reasons for excluding models 5 to 8

Table 4. RMS vector field differences (σ) between the definitive model for 1965.0 and IGRF and associated models.

Model	σ Main field (nT)	Secular variation (nT/yr)
IGRF	124	21.5
1 Cain *et al.*	125	32.1
2 Fougere 2	181	38.8
3 Malin	182	19.6
4 IZMIRAN	491	23.0
5 Hurwitz *et al.*	283	45.6
6 Leaton *et al.*	254	24.6
7 Cain & Cain	182	55.8
8 Fougere 1	197	44.0
IGRF revised	115	19.4

(that only models assuming a spheroidal earth be used, and not more than one model from any institution) a more equitable weighting (proportional to $1/\sigma^2$) for those included would have been 50:24:23:3 instead of 58:19:14:9, as actually used. Nevertheless, the IGRF itself has a better figure of merit than any of the candidate models. Of the two models (4 and 7) based solely on satellite data, the former has the worst fit of all, whilst the latter compares well with the definitive model. This is probably due to the restricted coverage (50° N to 50° S) of the *Cosmos 49* data, on which model 4 was based, compared with the essentially global coverage given by the nearly polar orbits of *OGO 2* and *OGO 4*, whose data formed the basis of model 7. Indeed, it is interesting to note that the five best models all incorporated *OGO 2* data.

For the secular variation, the assumption of a spherical earth is acceptable and a model from each responsible organization was used for the IGRF, with equal weighting. Table 4 shows that a better choice of secular variation models could have been made, though the IGRF itself ranks higher than all but one of the candidate models. Considering only the five models used, the data of Table 4 suggest that the appropriate weighting should have been 15:10:39:29:7 for models 1 to 5, respectively. The secular variation coefficients of model 7 are the only ones based solely on satellite data and they give the worst result. In fact, none of the models where the secular variation coefficients were determined simultaneously with those for the main field (numbers 1, 2, 7 and 8) are very satisfactory (see also Barraclough 1976). The models based on observatory data are, with the exception of model 5, very much better. The poor quality of model 5 may result from the short time span of the data (1960–65) and the weighting scheme used, which was based on the isolation of a station rather than on the intrinsic merits of its data. The values of σ for a revised version of the IGRF, produced from the same models as were used for the original, but using the more appropriate weights discussed above, are also given in Table 4. A small but significant improvement over the original IGRF is shown in each case.

It is instructive to compare the rms differences in Table 4 with the rms vector fields themselves, derived by using the definitive model coefficients rather than the differences. The corresponding values are, for the main field, 44925 nT and, for the secular variation, 73 nT/yr. The rms difference for the IGRF is thus only 0.3 per cent of the total for the main field, but is 30 per cent of the total for the secular variation. By 1971 the errors arising from the uncertainties in secular variation were greater than those for the main field. This reflects the severely inadequate knowledge of the secular variation.

5 Positions of dipoles and dip-poles

The geomagnetic field can be represented, to a first approximation, by that of a magnetic dipole at the centre of the Earth and inclined to the axis of rotation. The geomagnetic poles are the intersections of the axis of the dipole with the Earth's surface. The latitude (ϕ_c) and longitude (λ_c) of the northern geomagnetic pole and the moment (M) of the dipole can be computed from the first three coefficients of a spherical harmonic model (Chapman & Bartels 1940). Expressions for the instantaneous rates of change of ϕ_c, λ_c and M in terms of these three coefficients and the corresponding secular variation coefficients can also be simply derived.

The parameters of the centred dipole and their rates of change are given in Table 5 for the present model and a selection of other recent models. As is usual, the strength of the dipole is measured by MR^{-3} where R denotes the mean radius of the Earth.

The values from the definitive model are consistent with those given by the other models. The 1965 position and dipole moment are closer to the Leaton *et al.* (1965) values than to

Table 5. Position and strength of centred dipole and rates of change for the period 1960 to 1975.

Model	Date	ϕ_c deg	$\dot{\phi}_c$ deg yr^{-1}	λ_c deg	$\dot{\lambda}_c$ deg yr^{-1}	MR^{-3} nT	$\dot{M}R^{-3}$ nT yr^{-1}
Malin (1969)	1960	78.64	0.00	290.66	-0.09	31152	-17
Leaton *et al.* (1965)	1965	78.56		289.81		30987	-16
Malin & Clark (1974)	1965	78.50	0.00	289.67	-0.08	30873	-22
Definitive	1965	78.53	0.01	290.15	-0.07	30955	-22
Barraclough *et al.* (1975)	1975	78.7	0.00	289.5	-0.06	30701	-29

Table 6. Position of the magnetic centre and rates of change for the period 1960 to 1975.

Model	Date	r_e km	$\dot{r}_e$ km yr^{-1}	θ_e deg	$\dot{\theta}_e$ deg yr^{-1}	λ_e deg	$\dot{\lambda}_e$ deg yr^{-1}
Malin (1969)	1960	432.3	2.2	75.04	-0.23	150.40	-0.23
Leaton *et al.* (1965)	1965	444.1		72.81		148.64	
Malin & Clark (1974)	1965	453	2.2	71.4	-0.24	149.0	-0.14
Definitive	1965	452	2.2	72.8	-0.22	148.8	-0.14
Barraclough *et al.* (1975)	1975	474	2.2	70.2	-0.26	148.0	-0.05

those of Malin & Clark (1974). This is not unexpected since the Malin & Clark model, whose main purpose was to give accurate secular variation values, was derived from observatory data only, whereas the other two models are based on more extensive data sets. The essentially westward drift of the geomagnetic pole and the accelerating decrease in the dipole moment (Harwood & Malin 1976) are confirmed.

A more accurate representation of the field can be obtained by shifting the dipole away from the centre of the Earth whilst retaining its strength and orientation. The position in spherical polar coordinates (r_e, θ_e, λ_e) of this eccentric dipole – the magnetic centre – can be computed from the first eight ($n = 1$, $n = 2$) coefficients of a spherical harmonic model using expressions given by Bartels (1936). In Table 6 the values of these coordinates, and of their annual rates of change $\dot{r}_e$, $\dot{\theta}_e$, $\dot{\lambda}_e$, are given for the definitive model and for the other models discussed above. The 1965 values from the definitive model are again consistent with those from the other models and the north-westward and outward movement of the magnetic centre is confirmed.

The dip-poles are those (idealized) points on the Earth's surface where the horizontal component (H) becomes zero. The dip-pole positions derived from the definitive model are: for the northern pole (75°.9 N, 101°.4 W) and for the southern pole (66°.4 S, 139°.6 E). These may be compared with the values adopted for the 1965 UK World Magnetic Charts of (75°.5 N, 100°.5 W) and (66°.5 S, 139°.5 E). Since the dip-poles are localized phenomena, more accurate positions are derivable from local surveys. Dawson & Dalgetty (1966) give a northern dip-pole position of (75°.6 N, 100°.3 W) on the 1965 charts for Canada. Although not based on a truly local survey of the region surrounding the dip-pole, these charts were derived from observations made in Canada and the pole position quoted is likely to be more accurate than those based on a global distribution of data.

The definitive model indicates the following rates of movement for the dip-poles: for the northern dip-pole 4.0 km/yr northwards, 1.9 km/yr eastwards; for the southern dip-pole 8.4 km/yr northwards, 4.1 km/yr westwards. The north polar values may be compared with the values quoted by Dawson & Loomer (1963), for the period 1948.0 to 1962.5, of 5.0 km/yr northwards, 0.1 km/yr eastwards.

6 Conclusions

A definitive model of the geomagnetic field and its secular variation, for 1965.0 has been described. Comparisons with the IGRF and with other associated models show that, although a better choice of constituent models could have been used to derive the IGRF, it still represents the field at 1965.0 very well. The representation of the secular variation by the IGRF is less good.

For the description of the main field, the importance of complete geographical data coverage, such as that provided by satellites in polar orbits, clearly emerges. On the other hand, it is equally apparent that satellite data (at least with the time span available in 1965) contribute little to the secular variation, which is best determined from independent analyses in which observatory data are given a high weight.

For future descriptions of the geomagnetic field, the problem will continue to be the secular variation. The main field for a given date can already be modelled to an accuracy of about 100 nT, and with the advent of MAGSAT, this should be improved still further, possibly to the point where the contributions from external (ionospheric and magnetospheric) sources need to be taken into account. There is no similar improvement to be expected in secular variation data, which will continue to be heavily dependent on observatories. Every effort should be made to ensure that the absolute standards at observatories are as high as possible, and to encourage the establishment of more observatories, particularly in the southern hemisphere. On the analytical side, it is important to make the optimum use of secular variation data by suitable weighting and by the judicious use of secular variation information from non-observatory sources (e.g. Barraclough *et al.* 1975). Malin & Clark (1974) have shown that the inclusion of some secular acceleration coefficients produces a significantly better fit to the observed field over at least a few years, so it is desirable that these should be included in any future IGRF.

References

Barraclough, D. R., 1971. Secular variation in Scandinavia, *Nature Phys. Sci.*, **233**, 94–98.

Barraclough, D. R., 1976. Spherical harmonic analysis of the geomagnetic secular variation – a review of methods, *Phys. Earth planet. Int.*, **12**, 365–382.

Barraclough, D. R., 1978. Spherical harmonic models of the geomagnetic field, *Inst. Geol. Sci. Geomagn. Bull. No. 8.*

Barraclough, D. R. & Nevitt, Carol E., 1976. The effect of observational errors on geomagnetic field models based solely on total-intensity measurements, *Phys. Earth planet. Int.*, **13**, 123–131.

Barraclough, D. R., Harwood, J. M., Leaton, B. R. & Malin, S. R. C., 1975. A model of the geomagnetic field at epoch 1975, *Geophys. J. R. astr. Soc.*, **43**, 645–659.

Bartels, J., 1936. The eccentric dipole approximating the Earth's magnetic field, *Terr. Magn. atmos. Elect.*, **41**, 225–250.

Cain, J. C. & Cain, S. J., 1968. Derivation of the International Geomagnetic Reference Field [IGRF (10/68)], *NASA Preprint X-612-68-501.* Also published as *NASA Tech. Note D-6237* (1971).

Cain, J. C., Hendricks, S. J., Langel, R. A. & Hudson, W. V., 1967. A proposed model for the International Geomagnetic Reference Field – 1965. *J. Geomagn. Geoelectr., Kyoto*, **19**, 335–355.

Chapman, S. & Bartels, J., 1940. *Geomagnetism*, pp. 642–645, Clarendon Press, Oxford.

Dawson, E. & Dalgetty, L. C., 1966. Magnetic charts of Canada for epoch 1965.0, *Publ. Dom. Obs., Ottawa*, **31** (9), 379–405.

Dawson, E. & Loomer, E. I., 1963. The north magnetic dip pole, *Publ. Dom. Obs., Ottawa*, **28** (5), 195–208.

Fougere, P. F., 1969. Spherical harmonic analysis, 3. The Earth's magnetic field 1900–1965, *J. Geomagn. Geoelectr., Kyoto*, **21**, 685–695.

Harwood, J. M. & Malin, S. R. C., 1976. Present trends in the Earth's magnetic field, *Nature*, **259**, 469–471.

Hurwitz, L., Knapp, D. G., Nelson, J. H. & Watson, D. E., 1966. Mathematical model of the geomagnetic field for 1965, *J. geophys. Res.*, **71**, 2373–2383.

Leaton, B. R., Malin, S. R. C. & Evans, Margaret J., 1965. An analytical representation of the estimated geomagnetic field and its secular change for the epoch 1965.0, *J. Geomagn. Geoelectr., Kyoto*, **17**, 187–194.

Lowes, F. J., 1966. Mean-square values on sphere of spherical harmonic vector fields, *J. geophys. Res.*, **71**, 2179.

Lowes, F. J., 1975. Vector errors in spherical harmonic analysis of scalar data, *Geophys. J. R. astr. Soc.*, **42**, 637–651.

Malin, S. R. C., 1969. Geomagnetic secular variation and its changes, 1942.5 to 1962.5, *Geophys. J. R. astr. Soc.*, **17**, 415–441.

Malin, S. R. C. & Clark, Anne D., 1974. Geomagnetic secular variation 1962.5 to 1967.5, *Geophys. J. R. astr. Soc.*, **36**, 11–20.

Malin, S. R. C. & Pocock, Susan B., 1969. Geomagnetic spherical harmonic analysis, *Pure Appl. Geophys.*, **75**, 117–132.

Tyurmina, L. O. & Cherevko, T. N., 1967. Some results of a spherical harmonic analysis of the geomagnetic field from *Cosmos 49* data, *Geomagn. Aeron.*, **7**, 1126 (p. 917 in English translation).

Zmuda, A. J., 1971. World Magnetic Survey 1957–1969, *IAGA Bull. No. 28.*