Discussion of Papers by P. F. Fougere, 'Spherical Harmonic Analysis, 1 and 2'

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In the conventional method of spherical harmonic analysis each observation at each position provides an equation containing a preselected number of unknowns, and the unknowns are evaluated by the method of least squares. A new method of spherical harmonic analysis has been proposed by *Fougere* [1963] and has been used in the analysis of annual mean data from 82 magnetic observatories [*Fougere*, 1965]. Another new method of spherical harmonic analysis has been proposed by *Roberts and Scott* [1963]. For randomly distributed data their method of exact fit and discard involves the solution of a number of simultaneous equations equal to the number of observations. Even for the small sample of data considered here, 246 unknowns need to be evaluated. Because of the excessive amount of computing time required, no solution was made using this method.

A comparison is made here between the results obtained by the method of Fougere and by the conventional method. To make the results of this analysis closely comparable with those of Fougere, the data used were identical with his and were taken to represent a potential field of purely internal origin. The degree and order of the coefficients were both taken as far as 8, yielding 80 parameters compared with Fougere's 77. The analysis was performed on an IBM 7094 computer in single precision (about 8 decimal digits).

The data consisted of observatory annual mean values of X, Y, and Z; the north, east, and vertically downward components, respectively, of the earth's magnetic field. It should be remarked that most of the data, which Fougere stated to be for the epoch 1960.0, are, in fact, centered on 1958.5, but this does not materially affect a comparison of methods. The X, Y, and Z solutions were coupled; the 82 observatories thus provided 246 observational equations. Table 1 presents the Schmidt coefficients that resulted

TABLE 1. Spherical Harmonic Coefficients, Epoch 1958.5 Unit 0.1 γ

g/h	n	m	Malin and Leaton	Fougere	Difference
g	1	0	−30574	−30509	−65
g	1	1	−2231	−2181	−50
h	1	1	+5716	+5841	−125
g	2	0	−1450	−1464	+14
g	2	1	+3018	+2971	+47
h	2	1	−1866	−1988	+122
g	2	2	+1529	+1673	−144
h	2	2	+224	+198	+46
g	3	0	+1216	+1147	+69
g	3	1	−1953	−2012	+59
h	3	1	−413	−390	−23
g	3	2	+1290	+1151	+139
h	3	2	+233	+291	−58
g	3	3	+934	+831	+103
h	3	3	−188	−174	−14
g	4	0	+943	+997	−54
g	4	1	+790	+863	−73
h	4	1	+105	+173	−68
g	4	2	+516	+583	−67
h	4	2	−311	−292	−19
g	4	3	−447	−312	−135
h	4	3	−26	+69	−95
g	4	4	+171	+237	−66
h	4	4	−282	−206	−76
g	5	0	−277	−267	−10
g	5	1	+305	+272	+33
h	5	1	+25	−39	+64
g	5	2	+228	+251	−23
h	5	2	+251	+161	+90
g	5	3	+2	−52	+54
h	5	3	−13	−127	+114
g	5	4	−106	−113	+7
h	5	4	−37	−147	+110
g	5	5	−47	−8	−39
h	5	5	+145	+88	+57
g	6	0	+80	+48	+32
g	6	1	+141	+131	+10
h	6	1	+10	+1	+9
g	6	2	+34	−15	+49
h	6	2	+13	+62	−49
g	6	3	−201	−224	+23
h	6	3	+31	+38	−7
g	6	4	+36	+13	+23

TABLE 1. (Continued)

g/h	n	m	Marlin and Leaton	Fougere	Difference
h	6	4	−89	+19	−108
g	6	5	+43	−71	+114
h	6	5	−84	+4	−88
g	6	6	−14	−67	+53
h	6	6	−37	−57	+20
g	7	0	+94	+104	−10
g	7	1	−105	−64	−41
h	7	1	−94	−69	−25
g	7	2	−25	+8	−33
h	7	2	−11	+7	−18
g	7	3	−36	−18	−18
h	7	3	+12	+37	−25
g	7	4	−108	−42	−66
h	7	4	+66	+22	+44
g	7	5	−44	+55	−99
h	7	5	+54	+8	+46
g	7	6	−42	−6	−36
h	7	6	+9	+62	−53
g	7	7	−66	−33	−33
h	7	7	−28	+3	−31
g	8	0	−35	−16	−19
g	8	1	+44	+6	+38
h	8	1	+6	−6	+12
g	8	2	+22	+13	+9
h	8	2	−3	−21	+18
g	8	3	−18	−5	−13
h	8	3	−10	−1	−9
g	8	4	+65	+33	+32
h	8	4	−70	−70	0
g	8	5	+47	−10	+57
h	8	5	+4	+5	−1
g	8	6	+26	−7	+33
h	8	6	+16	−11	+27
g	8	7	+81	+61	+20
h	8	7	+38	...	...
g	8	8	+34	...	...
h	8	8	+72	...	...
				rms =	±61.1

from this analysis and also the Schmidt coefficients of Fougere, for comparison.

To see which of the analyses gave the better representation of the data analyzed, the rms of the residuals of all the elements were calculated. From the present analysis the value was 297 γ, which should be compared with 320 γ for Fougere (deduced from data in his paper).

A second test of the quality of the analyses is the accuracy with which they fit data from other sources. It was not practicable to compare synthetic values from the two analyses with the very large amount of existing magnetic survey data, and it is difficult to select a small, unbiased sample. However two recent spherical harmonic analyses, those of *Leaton et al.* [1965] LME and *Cain et al.* [1965] GSFC(64) have been shown in the latter paper to represent the observed data obtained betwen 1955 and 1963 to a higher order of accuracy than any previous analysis; the rms residuals from 155,000 unweighted observations obtained in this period are 242 γ for LME and 260 γ for GSFC(64). Some indication of the fit of the present analysis and Fougere's analysis to these observations may be obtained by comparing the coefficients with those from LME and GSFC(64). Those coefficients for which secular variation information is available, i.e. the first 48 of LME and the first 35 of GSFC(64) were reduced to epoch 1958.5. The rms difference, LME reduced *minus* present analysis, is 56.5 γ, the rms difference, LME reduced *minus* Fougere, is 56.4 γ. The corresponding figures for GSFC(64) reduced *minus* present analysis and GSFC(64) reduced *minus* Fougere are 57.0 γ and 62.4 γ, respectively.

These figures confirm the view of *Winch* [1966] that there is little to choose between the two methods of analysis.

References

Cain, J. C., W. E. Daniels, S. J. Hendricks, and D. C. Jensen, An evaluation of the main geomagnetic field 1940–1962, *J. Geophys. Res., 70,* 3647–3674, 1965.

Fougere, P. F., Spherical harmonic analysis, 1, A new method and its verification, *J. Geophys. Res., 68,* 1131–1139, 1963.

Fougere, P. F., Spherical harmonic analysis, 2, A new model derived from magnetic observatory data for epoch 1960.0, *J. Geophys. Res., 70,* 2171–2179, 1965.

Leaton, B. R., S. R. C. Malin, and Margaret J. Evans, An analytical representation of the estimated geomagnetic field and its secular change for the epoch 1965.0, *J. Geomagnet. Geoelec., Kyoto, 17,* 187–194, 1965.

Roberts, P. H., and S. Scott, Truncation errors in the spherical harmonic analysis of the geomagnetic field and the problem of downward extrapolation, *J. Geomagnet. Geoelec., Kyoto, 15,* 148–160, 1963.

Winch, D. E., A criticism of the Gram-Schmidt orthogonalization process applied to spherical harmonic analysis, *J. Geophys. Res., 71,* to be published, 1966.

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