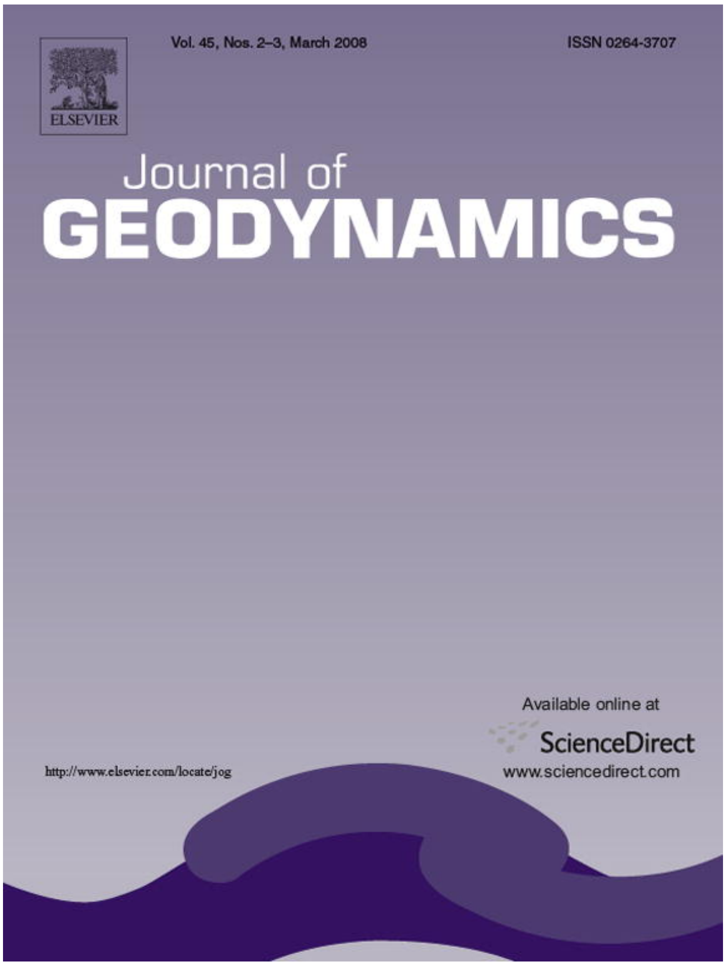
Gravitational and magnetic models of the core–mantle boundary and their correlation

Article *in* Journal of Geodynamics · March 2008

DOI: 10.1016/j.jog.2007.09.001

Ilya Prutkin
Friedrich Schiller University Jena


Vol. 45, Nos. 2–3, March 2008
ISSN 0264-3707
ELSEVIER
Journal of
GEODYNAMICS
Available online at
ScienceDirect
www.sciencedirect.com
http://www.elsevier.com/locate/jog

Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
Journal of Geodynamics 45 (2008) 146–153
JOURNAL OF GEODYNAMICS
http://www.elsevier.com/locate/jog

Gravitational and magnetic models of the core–mantle boundary and their correlation

I. Prutkin *

Delft Institute of Earth Observation and Space Systems (DEOS), Delft University of Technology, 2629 HS Delft, Kluyverweg 1, The Netherlands

Received 10 January 2007; received in revised form 12 September 2007; accepted 12 September 2007

Abstract

On the basis of a technique for 3D potential field data inversion worked out by the author earlier, gravitational and magnetic models of the core–mantle boundary (CMB) have been developed. The gravitational model agrees quite well with seismological data. The magnetic model represents a homogeneously magnetized body with the same external magnetic field as the Earth's core; the uplifts of its surface are related to regions with increased values of the magnetic field. The comparison of the models has revealed the correlation that regions exhibiting high magnetic field values in the Earth's core correspond to depressions in the core–mantle boundary, as reconstructed from gravity data. This correlation has led us to an hypothesis about core material flow. To examine the hypothesis in an indirect way, CMB magnetic field models at various epochs inferred by assuming a non-magnetic mantle and a homogeneously magnetized core have been studied. Changes of their shape appear to confirm our hypothesis.

Keywords: Core–mantle boundary; Gravitational and magnetic models; Core material flow; Potential field data inversion

1. Introduction

Bowin (2000) noted that, despite dynamic topography solutions, the problem to explain the main geoid anomalies (degrees 2–10) still exists. Due to the overwhelming degree 2 and 3 contributions, the global pattern of geoid anomalies has no consistent correlation with the topography, plate tectonic patterns or the magnetic field. At the same time, according to Bowin (2000), bands of positive anomalies in the degree 4–10 geoid pattern match well with locations of subduction zones. Very detailed considerations led Bowin (2000) to his suggestion to separate the sources into two parts: deep ones, responsible for the large energy of degrees 2 and 3, and shallower sources, responsible for degree 4–10 contributions. From his viewpoint, the first source should be core–mantle boundary topography, which might be produced by processes within the core rather than within the mantle.

The hypothesis that the CMB may be rough was advanced by Garland (1957). Later concepts of the existence of CMB relief irregularities were used systematically in the studies of Hide (1969, 1970). In particular, he suggested (Hide and Malin, 1970) that large-scale gravity and magnetic anomalies could be a result of the very same irregularities of CMB topography. Thereafter, the interest to the relief of this boundary has increased considerably thanks to the papers (Gudmundsson et al., 1986; Morelli and Dziewonski, 1987; Doornbos and Hilton, 1989), in which details of core geometry were investigated for the first time based on seismological data. Therefore, it seems quite topical to construct gravitational and magnetic models of the relief of the CMB and to compare them with seismological models and with each other.

In Bowin (2000), four point masses were used to simulate CMB topography. In this paper a more complex 3D geometry of the Earth's core is determined by means of new algorithms of potential field data inversion for 3D objects of arbitrary shape, developed by the author earlier (Prutkin, 1989). A short description of our approach is presented in Section 2. Section 3 is devoted to the main features of the gravitational model and its comparison with seismological data. The characteristics of the magnetic model and its relation with physics are discussed in Section 4. A correlation which has been observed between the models, is the subject of Section 5. In this section an hypothesis about core material flow as a possible explanation of

* Tel.: +31 1527 87394; fax: +31 1527 82348.
E-mail address: I.Prutkin@tudelft.nl.

0264-3707/$ – see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.jog.2007.09.001

the correlation is suggested. An attempt to prove the hypothesis is also included in Section 5.

Our idea to obtain an independent confirmation of the hypothesis is to study solutions, corresponding to various geomagnetic epochs. It should be emphasized that new gravity and magnetic data has become available owing to recent satellite missions. Earth gravity field model from GRACE (Tapley et al., 2005) is used while finding our gravitational model of the CMB. But the real challenge for our investigation is to compare magnetic models, corresponding to International Geomagnetic Reference Field for the epoch 2000, derived from the Ørsted and CHAMP missions (Maus et al., 2005), to IGRF1980, based on MAGSAT data, and to IGRF1960. The results of this comparison are included in Section 5. Section 6 contains the main conclusions of this study.

2. Mathematical theory

In this section the approach to 3D potential field data inversion is briefly described. For more details we refer to Prutkin (1989). We seek geometry of an unknown homogeneous body; the only requirement is that the object sought is assumed to be star convex relative to some point of the body. This means that there exists a point of the body such that the line segment from this point to any other point of the body is contained in the body. In this case we can introduce spherical coordinates r, θ, ϕ relative to the point, the body boundary can be determined by the equation $r = r(\theta, \phi)$, where $r(\theta, \phi)$ is a single-valued function. In the case of the core–mantle boundary, the unknown function $r(\theta, \phi)$ determines the geometry of the Earth's core, so the star convexity is not a substantial limitation. We introduce a Cartesian coordinate system, whose origin coincides with the center of the spherical coordinate system. Assume that $\mathbf{r}$ is the radius-vector of the observation point. If $V(\mathbf{r})$ and $W(\mathbf{r})$ denote gravitational and magnetic potentials of the body, respectively, we introduce the following functions into consideration (Prutkin, 1989):

$$\begin{aligned} \Psi(\mathbf{r}) &= V(\mathbf{r}) - \frac{1}{2}\,\mathbf{r}\cdot\nabla V(\mathbf{r}) = V - \frac{1}{2}\,r\frac{\partial V}{\partial r}, \\ \Psi_M(\mathbf{r}) &= W(\mathbf{r}) - \mathbf{r}\cdot\nabla W(\mathbf{r}) = W - r\frac{\partial W}{\partial r}. \end{aligned} \quad (1)$$

It was demonstrated (Prutkin, 1989) that the following relations are valid:

$$\Psi(\mathbf{r}) = \frac{G\sigma}{2}\int_0^{2\pi}\int_0^{\pi}\frac{r^3(\theta,\phi)\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi}{\sqrt{r^2(\theta,\phi) - 2br(\theta,\phi) + a^2}}, \quad (2)$$

$$\Psi_M(\mathbf{r}) = \int_0^{2\pi}\int_0^{\pi}\frac{(d - cr(\theta,\phi))r^3(\theta,\phi)\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi}{(r^2(\theta,\phi) - 2br(\theta,\phi) + a^2)^{3/2}}, \quad (3)$$

where G is the gravitational constant, σ is density, $\mathbf{r} = (x, y, z)$, $b = x\sin\theta\cos\phi + y\sin\theta\sin\phi + z\cos\theta$, $a^2 = x^2 + y^2 + z^2$, $c = I_x\sin\theta\cos\phi + I_y\sin\theta\sin\phi + I_z\cos\theta$, $d = I_x x + I_y y + I_z z$, I_x, I_y, I_z are magnetization components.

We proposed that (2) and (3) be used as integral equations for inverse problems in gravimetry and magnetometry. If a potential or its derivatives are given in the form of a volume integral, then the kernel function in the corresponding equation is rather complicated and, in addition, is a transcendental function of $r(\theta, \phi)$. If we apply a representation in the form of a surface integral, then the integrand includes derivatives of the unknown function, but differentiation of the approximately computed function $r(\theta, \phi)$ is an ill-posed problem. Eqs. (2) and (3) are free from both disadvantages: their integrands are algebraic relative to the function sought and do not contain its derivatives. The use of modified potentials instead of the conventional gravitational and magnetic potentials does not introduce any additional difficulties in constructing global models, because in such cases the harmonic coefficients of potential are applied, which relations with the harmonic coefficients of $\Psi(\mathbf{r})$ and $\Psi_M(\mathbf{r})$ are described by very simple formulas, as it follows from (1).

The author proposed (Prutkin, 1983) the local corrections method for solving nonlinear potential inverse problems like (2) and (3). Its idea is that, in computing the integrals in (2) and (3), and also in evaluating the field values on some closed surface, containing the object sought within it, the very same nodes (θ_i, ϕ_j) are used; in each iteration an attempt is made to decrease the difference between the given and approximate field values at a fixed node only by means of a change in the value of the function sought at the same node. These considerations lead to decomposition of the inverse problem and reduction of time expenditures to solve it approximately by an order of magnitude.

To improve the solution, we are proceeding from the local corrections method to a solving of the integral equation for the inverse problem by the Newton method. This method is applied rather rarely, since it ensures convergence (but extremely rapid) only in the presence of an initial approximation close to the solution sought. In our case, such an approximation is provided by the local corrections method.

As is known, the Newton method involves solving of the linearized problem in each iteration. Assume that variation of the object boundary is described by the function $\delta r(\theta, \phi)$ and $\delta V(\mathbf{r})$ is the corresponding variation of the gravity potential. It is found that the following relation holds:

$$\delta V(\mathbf{r}) = G\sigma\int_0^{2\pi}\int_0^{\pi}\frac{r^2(\theta,\phi)\sin\theta\,\delta r(\theta,\phi)}{\sqrt{r^2(\theta,\phi) - 2br(\theta,\phi) + a^2}}\,\mathrm{d}\theta\,\mathrm{d}\phi. \quad (4)$$

Applying the operator

$$I_x\frac{\partial}{\partial x} + I_y\frac{\partial}{\partial y} + I_z\frac{\partial}{\partial z}$$

to both sides of the Eq. (4) and using the Poisson formula, for the variation of magnetic potential, we obtain

$$\delta W(\mathbf{r}) = \int_0^{2\pi}\int_0^{\pi}\frac{(d - cr(\theta,\phi))r^2(\theta,\phi)\sin\theta}{(r^2(\theta,\phi) - 2br(\theta,\phi) + a^2)^{3/2}}\,\delta r(\theta,\phi)\,\mathrm{d}\theta\,\mathrm{d}\phi. \quad (5)$$

The same notations are used in (4) and (5) as in (2) and (3). Formulas (4) and (5) are proposed as integral equations for the linearized inverse problems in gravimetry and magnetometry. In their properties, they are entirely similar to Eqs. (2) and (3). Their integrands are algebraic relative to the function $r(\theta, \phi)$ and do not contain its derivatives. This circumstance is extremely important because the kernel of the integral equation of the linearized problem in the Newton method must be computed again in each iteration.

A study is made for the problem of parameterization of the solution. It is suggested that a solution be sought in the form of a truncated spherical harmonic expansion. In such a case, linear systems of small size are derived and the solution of regularization problem is very simple.

As the regularizing functionals, we use analogs of the Tikhonov first-order smoothing functional. A complexity is that the derivatives of the spherical functions, in contrast, for example, to the terms of a double Fourier series, no longer form an orthogonal system. The following functional is applied:

$$\Omega(r) = \int_0^{2\pi} \int_0^{\pi} \left(\left(\frac{\partial}{\partial\theta} r(\theta,\phi) \right)^2 \sin\theta + \left(\frac{\partial}{\partial\phi} r(\theta,\phi) \right)^2 \frac{1}{\sin\theta} \right) \mathrm{d}\theta\,\mathrm{d}\phi. \quad (6)$$

The computations indicated that if $r(\theta, \phi)$ is substituted into (6) in the form of a truncated spherical harmonic expansion, a quadratic form is obtained relative to expansion coefficients, whose matrix is extremely close to a diagonal one. Accordingly, when one uses the necessary condition of extremum, it is sufficient to add to the matrix of the linear system of the least squares method a diagonal matrix multiplied by the regularizing parameter. We note that the sequence of diagonal elements is ascending, which makes it possible to suppress undesirable high-frequency oscillations of the boundary of the object sought.

3. Gravitational model

Now we apply the described method of 3D potential field data inversion for constructing a two-layer model of the Earth. The gravitational model is developed using Eq. (2). The values of the function $\Psi(\mathbf{r})$ have been calculated by means of the harmonic coefficients of gravity field model GGM02C (Tapley et al., 2005), obtained from GRACE data. Coefficients up to degree 10 have been taken into account. The model of the Earth consists of a mantle and a core, which are assumed to have a homogeneous density. The field of a rotated ellipsoid with the mean Earth's density has been subtracted from the gravity field values. The CMB topography is the source of the residual field. The CMB density contrast value $\Delta\sigma = 4.34\,\mathrm{g/cm}^3$ has been taken from the PREM density model of the Earth (Dziewonski and Anderson, 1981). The CMB geometry is determined by the function $r(\theta, \phi)$. The approach described above has been applied to find it.

The obtained CMB relief is shown in Fig. 1. The main feature of the CMB topography, as revealed in Gudmundsson et al. (1986) using *PcP* waves and in Morelli and Dziewonski (1987) using both *PcP* and *PKP* waves, turned out to be an uplift in the northern part of the Greenwich meridian. The same uplift is the most essential feature of the relief detected by us proceeding from gravity data. Using data from *PKKP* waves, the authors of a later paper (Doornbos and Hilton, 1989) traced, along with the above-mentioned uplift, depressions of the core–mantle boundary. The depressions also agree well with our gravitational topography: the triangle depression in the top left corner, the depression stretched in horizontal direction in the middle of the bottom edge of the figure and the isometric depression in the top right corner are clearly recognisable. In Doornbos and Hilton (1989) the amplitude of the CMB variation was [−4, 4] km,

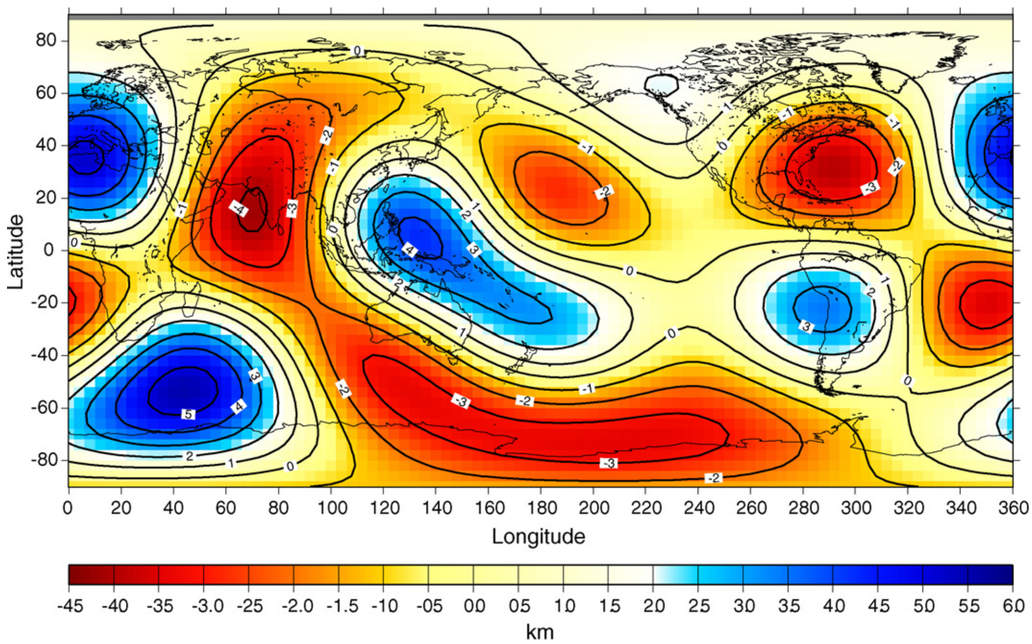


Fig. 1. CMB topography derived from gravity data. Harmonic coefficients of gravity field model GGM02C (Tapley et al., 2005) obtained from GRACE data are used, deviations of the boundary from the spheroid are shown.

which is quite close to our results. Similar estimates of CMB amplitude are obtained also in Garcia and Souriau (2000). Of course, it is possible to explain the Earth's gravity field without CMB topography, but the CMB geometry determined by us with the use of gravity data is adequately supported by seismological data, a fact which makes us quite confident that the topography actually exists.

4. Magnetic model

The magnetic model is developed using the harmonic coefficients of the 10th Generation International Geomagnetic Reference Field for the epoch 2000, derived from the Ørsted and CHAMP missions (Maus et al., 2005). The model of the Earth consists of a non-magnetic mantle and a homogeneously magnetized core. The relation of this model with reality is more complex than in the gravitational case and deserves a more detailed discussion. We should take into account three models of the sources of the Earth's main magnetic field. The first one corresponds to the geomagnetic dynamo theory and represents a system of electric currents in the core. As is known, each elementary electric current can be replaced by a magnetic dipole; this leaves the magnetic field unchanged. These considerations lead us to the second model of the core: an inhomogeneously magnetized sphere. Finally, a homogeneous equivalent can be constructed for such a sphere. The three models generate the same external magnetic field. We are concerned with the development of exactly the third model: a homogeneous magnetic equivalent for a system of electric currents in the core. Naturally, this model is the most unrealistic one. At the same time, the problem of constructing the first two models possesses a principal ambiguity: it is well known that numerous systems of electric currents and distributions of alternating magnetization exist, which generate a zero external magnetic field. While constructing a homogeneous magnetic equivalent, we find ourselves in the class of uniqueness. Besides, it is clear that high values of the internal magnetic field in the Earth's core lead to high values of magnetization in the second model and raising of the boundary for the homogeneous equivalent. Then, the shape of the model constructed by us represents a quite vivid illustration of the internal magnetic field in the core.

The homogeneous equivalent geometry is determined by the function $r(\theta, \phi)$. The approach described in Section 2 has been applied to find this function. Firstly, the $\Psi_M(\mathbf{r})$ values at the Earth's surface have been computed using Eq. (1) and the harmonic coefficients of the magnetic potential. Then the integral Eq. (3) is solved by the local corrections method. Newton's method is used to improve the initial solution. The linearized inverse problem is solved by means of the integral Eq. (5). In each iteration, the regularization is applied using the functional (6). Here, like in the gravitational case, a spherical harmonic expansion of $r(\theta, \phi)$ up to degree 4 is used according to Bowin's (2000) considerations and to the fact that such was the maximal order of the spherical harmonics used in constructing the CMB topography on the basis of seismological data. The shape of the model obtained is shown in Fig. 2.

One should not treat the boundary of the homogeneous magnetic equivalent as a real physical boundary; it is only a representation of the internal magnetic field in the Earth's core. A rather considerable amplitude of the boundary variation could be caused by strong inhomogeneity of the magnetic field in the core. The isolines exhibit two characteristic trends: from the top left corner to the middle of the bottom edge of the figure and further to the top right corner. The same trends are present in all of the above-mentioned seismological models of the CMB

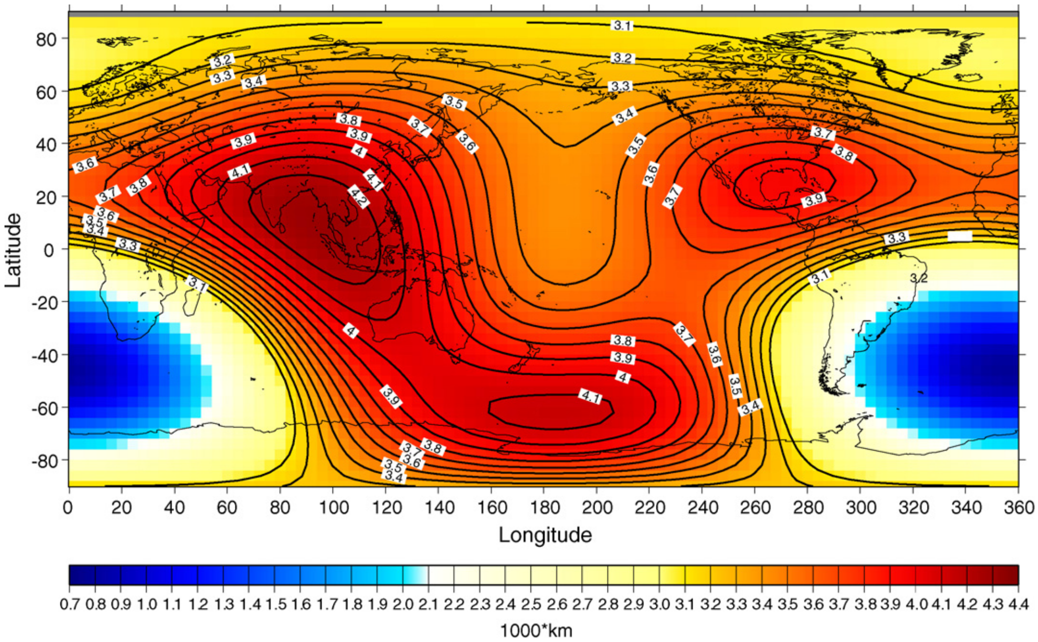


Fig. 2. Magnetic model of CMB. The model represents a homogeneous magnetic equivalent for the Earth's core, which is developed using the harmonic coefficients of IGRF2000 derived from Ørsted and CHAMP missions (Maus et al., 2005).

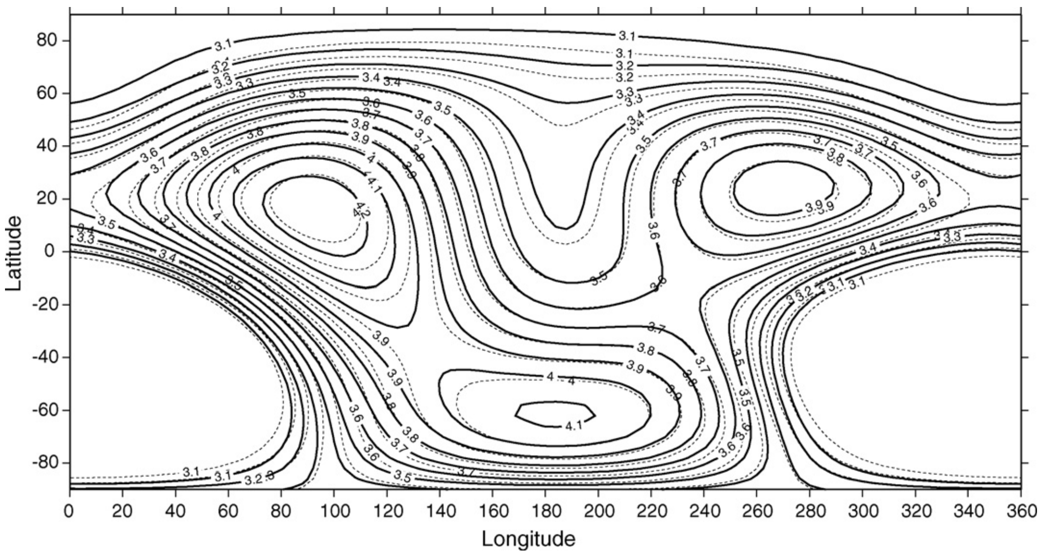


Fig. 3. Comparison of magnetic solutions for epochs 1960 and 1980. Geometry of homogeneous magnetic equivalents corresponding to IGRF1960 (dashed line) and IGRF1980 (solid line) are shown.

relief. The comparison of our gravitational and magnetic models with each other is the subject of the next section.

5. Correlation between the models and its possible explanation

The following circumstance, which is revealed by a comparison of the constructed gravitational (Fig. 1) and magnetic (Fig. 2) models, seems to be the most interesting. It turned out that rising of the boundary of the homogeneous magnetic equivalent corresponds to depressions in the core–mantle boundary, reconstructed from gravitational data. Hide and Malin (1970) suggested that the correlation between the global features of the gravity and magnetic fields of the Earth is due to the fact that both fields are connected with the CMB topography. Now we can refine the hypothesis: regions exhibiting high values of the magnetic field in the Earth's core correspond to depressions in the core–mantle boundary recovered on the basis of gravity data.

With the aim to quantify the degree of correlation, a correlation coefficient is calculated between gravitational and magnetic solutions (Figs. 1 and 2). It should be noted that CMB topography derived from gravity is more complicated than CMB magnetic model; it includes not only depressions, but also uplifts, partly correlated with subduction zones. If we calculate a correlation coefficient for the whole globe, it is equal to −0.067 (i.e., nearly no correlation). But if we only take into consideration the area of uplifts of the magnetic model, the correlation coefficient is −0.797. This means that the absolute value of the correlation coefficient reaches approximately 80%; it is

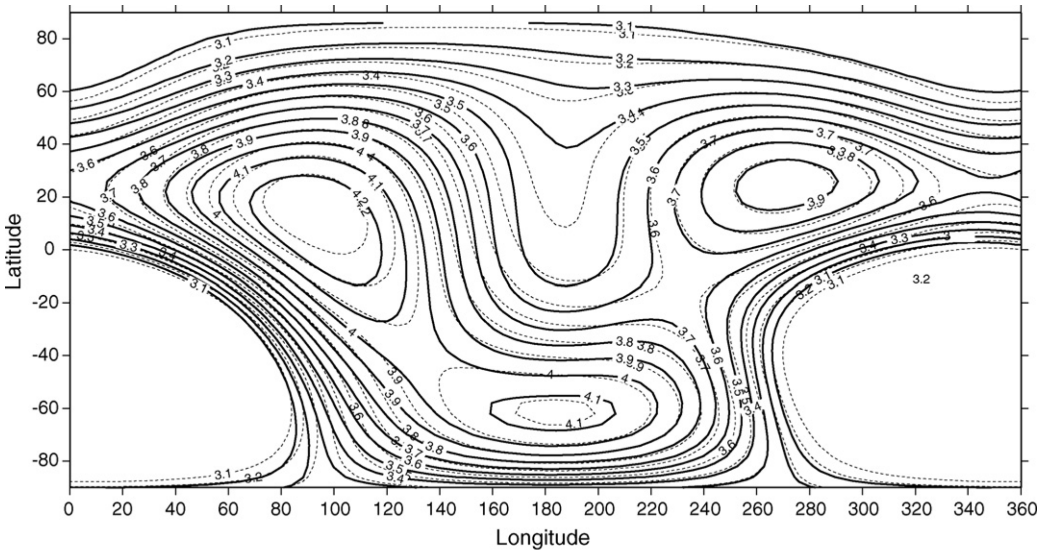


Fig. 4. Comparison of magnetic solutions for epochs 1980 and 2000. Geometry of homogeneous magnetic equivalents corresponding to IGRF1980 (dashed line) and IGRF2000 (solid line) are shown.

negative, because we compare depressions of the gravitational model and uplifts of the magnetic one.

A possible explanation of the detected correlation is, as the author believes, the flow of the core material from the core–mantle boundary to the Earth's centre. Such a flow could cause the formation of a "crater" on the surface of the core, and the crater shows up as a depression according to gravitational and seismological data. It is also possible that CMB thermal or topographic structure controls the Earth's core flow in such a way that the flow is downwelling under depressions of CMB and these downwellings act to concentrate magnetic flux. In both cases, the descending flow results in high values of the magnetic field and, consequently, in rising of the homogeneous equivalent boundary, which is well correlated with depressions of CMB according to gravity and seismological data.

Most important for us is to draw attention to this correlation. Of course, the author does not lay claim to any exhaustive physical interpretation of the results obtained. At the same time, there are some results that seem to be in favour of our hypothesis. For instance, Whaler (1990) reconstructed a velocity distribution on the core surface according to magnetic field variations data. A feature of a descending flow is that its velocity vector is nearly normal to the surface of the core; therefore, its projection on the surface is close to zero. In Whaler (1990), regions where

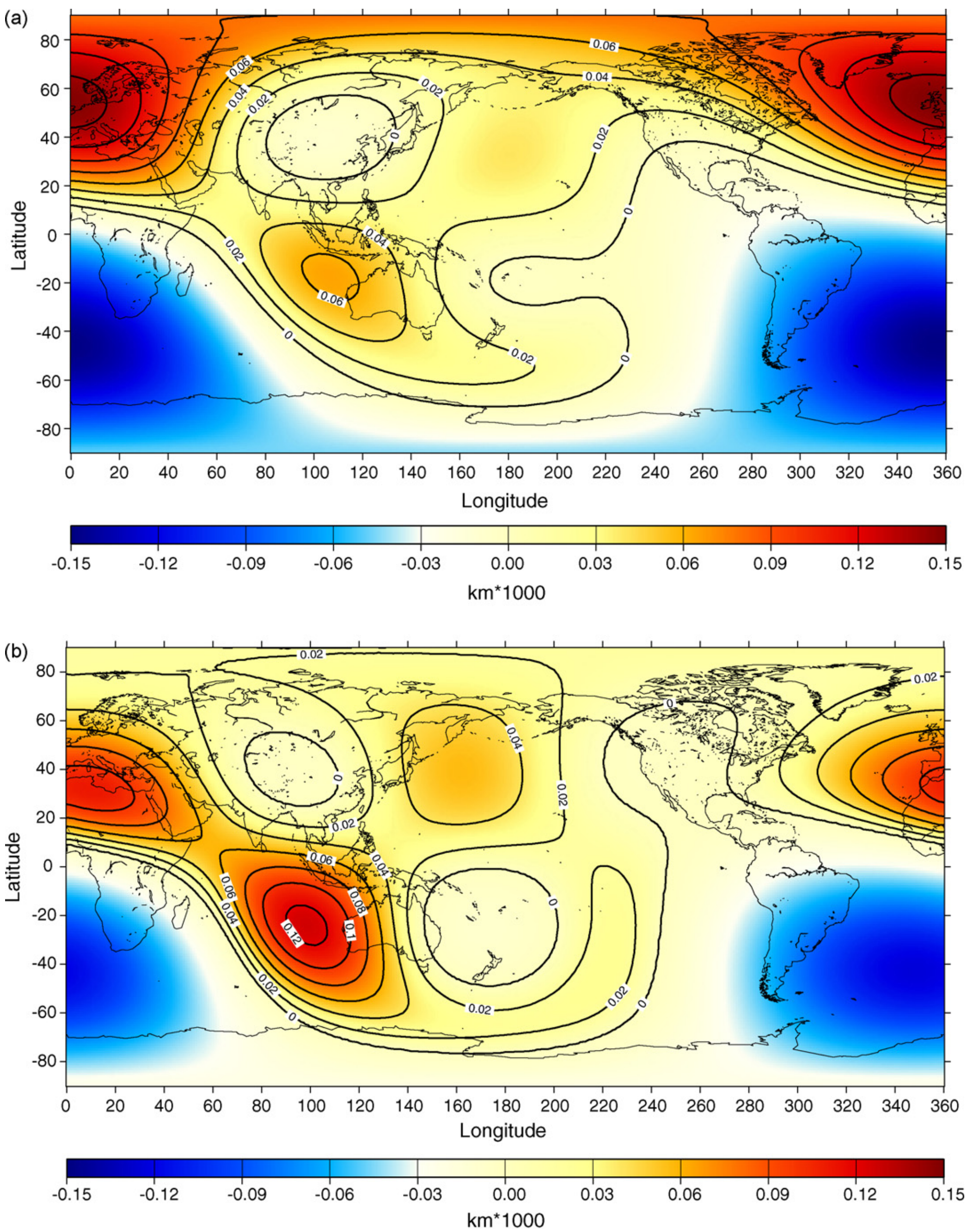


Fig. 5. Differences between magnetic solutions corresponding to various geomagnetic epochs. (a) Epoch 1980 – epoch 1960. (b) Epoch 2000 – epoch 1980.

the velocity is close to zero form a configuration quite similar to the one shown in Fig. 2. In Bloxham and Jackson (1990), temperature variations on the core–mantle boundary were investigated. The shape of the colder regions on the CMB also has substantial similarity with uplifts of the homogeneous equivalent boundary and depressions of the gravitational model that we found. Finally, in Golovkov et al. (1996), a triangular region of descending flow of core material is revealed according to geomagnetic jerks data, which is quite in agreement with Fig. 2.

Instead we have made an attempt to test our hypothesis in an indirect way. The idea is to find the geometry of homogeneous magnetic equivalents corresponding to various geomagnetic epochs. As mentioned above, the shape of such a model represents a quite vivid illustration of the internal magnetic field in the core; besides, by its recovery we find ourselves in the class of uniqueness. The comparison of homogeneous equivalents corresponding to various epochs provides an opportunity to observe the dynamics of the magnetic field. It should be noted that, if we accept the frozen flux hypothesis, the study of the behaviour of homogeneous magnetic equivalent relief isolines in time could be regarded as an independent approach for investigating core material flow.

Assuming that the hypothesis is valid and the flow still takes place, the internal magnetic field in regions of its increased value must grow further. As a result, the isolines of homogeneous magnetic equivalent relief should expand. Our idea to obtain an independent confirmation of the hypothesis is to study the geometry of homogeneous equivalents corresponding to various geomagnetic epochs in order to examine if the predicted expansion of their isolines actually takes place.

The harmonic coefficients of the 10th Generation International Geomagnetic Reference Field for the epochs 1960, 1980 and 2000 have been used to test our hypothesis. For each field model, the geometry of homogeneous magnetic equivalent has been found by means of the algorithm described in Section 2. Then the solutions corresponding to three different moments of time have been compared with each other. The results found according to IGRF1960 and IGRF1980 data are shown in Fig. 3.

For the uplift in the top right corner the expansion of isolines is visible only in the eastern part, but for uplifts in the top left corner and in the middle near to the bottom line of the figure it is quite evident; a new isoline with the value 4.1 has appeared here.

The next comparison is presented in Fig. 4. In this case we deal with two solutions found according to harmonic coefficients, both based on satellite data, being IGRF1980 (MAGSAT) and IGRF2000 (Ørsted and CHAMP).

The same isolines (IGRF1980 solution) are displayed with solid lines in Fig. 3 and with dashed lines in Fig. 4. Again the expansion of isolines is clearly recognizable, especially for the isoline corresponding to the value 4, which has broken up and bounds now not two separate peaks but one of greater area.

The main tendency of the isolines behaviour, as revealed by both comparisons, is their expansion. This fact seems to confirm our hypothesis.

We have also calculated differences between our magnetic solutions: epoch 2000 − epoch 1980 and epoch 1980 − epoch 1960 (see Fig. 5). At least two main depressions (in the top left corner and in the middle of the bottom edge) are definitely located in the area, where both differences are positive. Moreover, using these time difference maps, one could estimate the velocity of the boundary growth for the homogeneous magnetic equivalent.

6. Conclusions

The following conclusions are drawn from the conducted inversion of gravity and magnetic data into the CMB topography:

(1) Our algorithms of 3D potential field data inversion can be successfully applied to obtain global solutions according to harmonic coefficients of the latest satellite gravity and magnetic field models.
(2) The amplitude of variation and the main features of our gravitational model of CMB topography agree quite well with seismological data. This makes us quite confident that the relief actually exists.
(3) The magnetic model represents a homogeneous magnetic equivalent for the Earth's core; its shape provides a quite vivid illustration of the internal magnetic field in the core.
(4) A new correlation has been revealed by the comparison of the obtained gravitational and magnetic CMB models: regions exhibiting high values of the magnetic field in the Earth's core correspond to depressions of the CMB recovered on the basis of gravity data.
(5) The study of homogeneous magnetic equivalent geometry corresponding to various geomagnetic epochs is proposed as an independent approach to investigate the core material flow.
(6) The expansion of the isolines of the solutions, corresponding to epochs 1960, 1980 and 2000, tends to confirm our hypothesis about core material flow, which is suggested to explain the revealed correlation.

Acknowledgments

The author would like to thank D.Sc. Juergen Kusche and Prof. Dr. Roland Klees (DEOS) for useful remarks, which have helped to improve the quality of the paper. Their support is gratefully acknowledged.

References

Bloxham, J., Jackson, A., 1990. Lateral temperature variations at the core–mantle boundary deduced from the magnetic field. Geophys. Res. Lett. 17 (11), 1997–2000.

Bowin, C., 2000. Mass anomaly structure of the Earth. Rev. Geophys. 38 (3), 355–387.

Doornbos, D.J., Hilton, T., 1989. Models of the core–mantle boundary and the travel times of internally reflected core phases. J. Geophys. Res. 94 (B11), 15741–15751.

Dziewonski, A.M., Anderson, D.L., 1981. Preliminary reference Earth model. Phys. Earth Planet. Int. 25, 297–356.

Garcia, R., Souriau, A., 2000. Amplitude of the core–mantle boundary topography estimated by stochastic analysis of core phases. Phys. Earth Planet. Int. 117, 345–359.

Garland, G.D., 1957. The figure of the Earth's core and the nondipole field. J. Geophys. Res. 62, 486.
Golovkov, V.P., Simonyan, A.O., Yakovleva, S.V., 1996. Calculating the surface velocity field on the Earth's core from data on geomagnetic jerks. Geomagnet. Aeronomy 36 (1), 80–88.
Gudmundsson, O., Clayton, R.W., Anderson, D.L., 1986. Core mantle boundary topography inferred from ISC PcP travel times. Trans. Am. Geophys. Union 67 (44), 1100.
Hide, R., 1969. Interaction between Earth's liquid core and solid mantle. Nature 222, 1055–1056.
Hide, R., 1970. Earth's core–mantle interface. Q. J. R. Met. Soc. 96, 579–590.
Hide, R., Malin, S.R.C., 1970. Novel correlation between global features of Earth's gravitational and magnetic fields. Nature 225, 605–609.
Maus, S., Macmillan, S., Chernova, T., Choi, S., Dater, D., Golovkov, V., Lesur, V., Lowes, F., Luehr, H., Mai, W., McLean, S., Olsen, N., Rother, M., Sabaka, T., Thomson, A., Zvereva, T., 2005. The 10th Generation International Geomagnetic Reference Field. Geophys. J. Int. 161 (3), 561–565.
Morelli, A., Dziewonski, A.M., 1987. Topography of the core–mantle boundary and lateral homogeneity of the liquid core. Nature 325, 678–683.
Prutkin, I.L., 1983. Approximate solution of three-dimensional gravimetric and magnetometric inverse problems by the method of local corrections. Izvestiya. Phys. Solid Earth 19 (1), 38–41.
Prutkin, I.L., 1989. Nonlinear inverse potential problems and construction of two-layer models of Earth and Moon. Izvestiya. Phys. Solid Earth 25 (11), 913–918.
Tapley, B., Ries, J., Bettadpur, S., Chambers, D., Cheng, M., Condi, F., Gunter, B., Kang, Z., Nagel, P., Pastor, R., Pekker, T., Poole, S., Wang, F., 2005. GGM02—an improved Earth gravity field model from GRACE. J. Geod. 79 (8), 467–478.
Whaler, K.A., 1990. A steady velocity field at the top of the Earth's core in the frozen-flux approximation—errata and further comments. Geophys. J. Int. 102 (2), 507–509.