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Unveiling Geomagnetic Reversals: Insights From Tipping Points Theory

T. Alberti[1,2], F. Florindo[1], P. De Michelis[1], and G. Consolini[2]

[1]Istituto Nazionale di Geofisica e Vulcanologia, Rome, Italy, [2]INAF-Istituto di Astrofisica e Planetologia Spaziali, Rome, Italy

Key Points:

- A recently-proposed method based on tipping point theory detects and characterizes geomagnetic reversals from 25 to 36 million years ago
- A critical threshold is identified as an early warning indicator of polarity reversal occurrence
- A simple non-autonomous model is proposed to describe polarity reversals

Correspondence to:

T. Alberti,
tommaso.alberti@ingv.it

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Author Contributions:

Conceptualization: T. Alberti, F. Florindo
Data curation: F. Florindo
Formal analysis: T. Alberti
Investigation: T. Alberti, F. Florindo, P. De Michelis, G. Consolini
Methodology: T. Alberti
Resources: F. Florindo
Software: T. Alberti
Supervision: F. Florindo
Writing – original draft: T. Alberti, F. Florindo
Writing – review & editing: T. Alberti, P. De Michelis, G. Consolini

Abstract The geomagnetic field shows aperiodic reversals and excursions separated by stable polarity periods. Although the exact mechanisms responsible of reversals are still debated, several models of different complexity have been proposed. Here we use, for the first time, a different and novel approach based on the theory of tipping points to detect and characterize geomagnetic reversals occurred during the period 25–36 millions years ago by using a high-resolution magnetostratigraphic study conducted on a sedimentary section located on Maud Rise in the Southern Ocean. We detect a critical threshold for an early warning indicator of the occurrence of a polarity reversal below which a polarity reversal starts. Through the proper use of this early warning indicator we build up a simple non-autonomous stochastic model to describe the main features of polarity reversals. This approach could be helpful for building up a novel framework for paleomagnetic studies.

Plain Language Summary Geomagnetic polarity reversals involve the complete flip of Earth's magnetic field, and they provide valuable insights into geological and biological processes. Geomagnetic reversals have been extensively studied since they were first proposed in 1906, and paleomagnetic studies indicate that the Earth's magnetic field has undergone multiple polarity reversals over millions of years. The causes of these reversals are still being investigated, but one hypothesis suggests that a complex dynamics of the Earth's liquid outer core plays a key role. We use the concept of *tipping points* and their theory to identify an early warning signal for the occurrence of polarity reversals and provide a simple model to improve our capabilities of predicting them, also revealing that the timing for a complete reversal is of the order of 5,000 years, which is consistent with previous estimations.

1. Introduction

Geomagnetic polarity reversals have been a subject of intense scientific investigation for decades. These fascinating phenomena involve the complete reversal of Earth's magnetic field, with the magnetic North and South poles swapping places (Laj, 2021; Valet & Fournier, 2016). The study of geomagnetic polarity reversals provides valuable insights into the dynamic behavior of Earth's magnetic field and its impact on various geological and biological processes (Channell & Vigliotti, 2019; Pan & Li, 2023). The concept of geomagnetic reversals was first proposed by Bernard Brunhes in 1906 (B. Brunhes, 1906), who discovered evidence of reversed magnetization in baked sedimentary rocks that were aligned with reverse magnetization directions in overlying lavas from central France (Puy de Dome) (B. Brunhes, 1906). Since then, extensive research has been conducted to unravel the mechanisms and implications of these events. One of the seminal works in this field is the pioneering study by Matuyama (1929), which identified a major polarity reversal in the sedimentary records of Japan. Over the years, through new studies and advancements in paleomagnetic techniques, scientists have been able to deepen our understanding of geomagnetic polarity reversals and identify increasingly subtle fluctuations in the magnetic dipole. These developments have ultimately resulted in the establishment of Geomagnetic Polarity Time Scales (Cande & Kent, 1995). The causes of geomagnetic reversals remain an active area of research. One prevailing hypothesis suggests that they arise from the complex dynamics of the Earth's liquid outer core, where the planet's magnetic field is generated. It is believed that changes in the convective flow of molten iron within the outer core can lead to the generation of new magnetic fields with reversed polarity (Glatzmaiers & Roberts, 1995; Landeau et al., 2022). However, the exact mechanisms behind these processes and the factors that trigger reversals are still subjects of investigation. Nevertheless, putting geomagnetic reversals into a dynamical system framework they can be seen as bifurcations between two different stable states of opposite polarity which has been discussed as a stochastic resonance phenomenon (Carbone et al., 2020; Consolini & De Michelis, 2003; Lorito et al., 2005;

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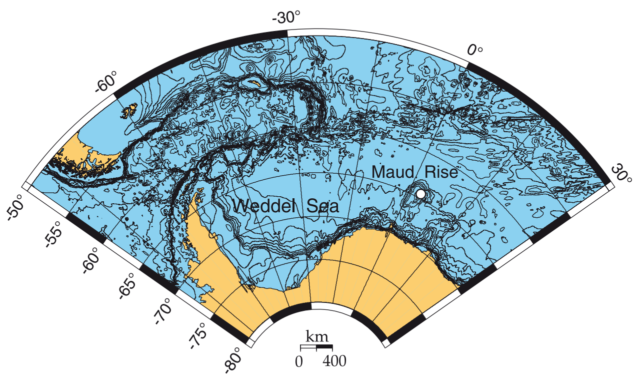

Figure 1. The location of the Hole 689D used in this study.

Molina-Cardín et al., 2021) and can be also approached through the emergent theory of *tipping points* (Ditlevsen & Johnsen, 2010; Lenton et al., 2008).

In the general context of complex systems tipping points refer to critical thresholds beyond which a small change or disturbance can lead to significant and potentially irreversible shifts in the state of the system (Lenton, 2011; Lenton et al., 2008). Tipping points can occur in several systems, including climate and ecosystems to social and economic structures (Boers & Rypdal, 2021; Kuehn, 2011; Schaeffer et al., 2017; Scheffer et al., 2009). The concept of tipping points has gained prominence in recent years due to growing concerns about the potential for abrupt and cascading changes in the Earth system (Boers et al., 2022; Lenton, 2021; Lohmann et al., 2021; Medeiros et al., 2017). This is particularly relevant for investigating the role of climate change in producing irreversible changes in its subsystems as the melting of polar ice caps and/or the collapse of major ice sheets (Boers & Rypdal, 2021), or the release of vast amounts of greenhouse gases from permafrost which can trigger feedback loops that accelerate warming and amplify the impacts of climate change (Lenton, 2021; Lenton et al., 2012; Scheffer et al., 2009). However, tipping points are not limited to the climate system alone, and the general theoretical framework behind their characterization can be applied to any complex system showing bifurcations, that is, transitions, between multiple (stable) states. Nevertheless, identifying and understanding tipping points is a challenging task, mainly due to the nonlinear nature of the systems where they occur, making difficult to predict when a tipping point will occur or what its precise consequences will be (Boers et al., 2022; Lenton, 2021). Furthermore, tipping points often involve complex interactions and feedback loops between multiple components of a system, making their analysis and prediction even more complex (Kuehn, 2011). Nonetheless, significant advances in mathematical modeling, data analysis, and empirical observations have been made to reveal indicators associated with tipping points (Boers et al., 2021; Boettner & Boers, 2022; Clarke et al., 2023).

In this work we apply, for the first time, the theory of tipping points to detect and characterize geomagnetic field reversals that occurred between 25 and 36 millions years ago by using a high-resolution magnetostratigraphic study conducted on a sedimentary section located on Maud Rise in the Southern Ocean. We provide evidence of a simple one-dimensional (1-D) model that relies on a bifurcation parameter λ which is able to provide an early warning indicator of the occurrence of a polarity reversal and, under certain conditions, in also detecting excursions of the geomagnetic field that do not result in reversals. We detect a critical threshold λ_c below which a polarity reversal starts as well as an average reversal time of about 5 Kyr, in agreement with previous studies (Clement, 2004). Through the proper use of this early warning indicator λ we are able to detect the main features of polarity reversals using a non-autonomous stochastic model and reconcile our results in a similar recently-proposed framework (Molina-Cardín et al., 2021). This analysis can be the starting point to provide insight on future polarity reversals and we are confident that the proposed formalism can be helpful for building up a novel framework for paleomagnetic studies.

2. Data

During the Ocean Drilling Program (ODP) Leg 113, sedimentary sections were collected on Maud Rise in the Southern Ocean (Figure 1) (Barker et al., 1988).

This study is focused on Hole 689D, which is located near the crest of Maud Rise at Site 689 (64°31.01′S; 03°06.00′–03°06.30′E). A high-resolution magnetostratigraphic study was conducted on the sediments using u-channels and a cryogenic magnetometer in a magnetically shielded laboratory. After removing coring-induced magnetic overprints, the sediments from Hole 689D exhibited stable characteristic remanent magnetization (ChRM) directions with two polarity states with a distinct square-wave magnetostratigraphic signal (Florindo & Roberts, 2005). Correlating the magnetostratigraphic signal with the Geomagnetic Polarity Time Scale (GPTS) provided an age interpretation between the upper Eocene and the upper Oligocene, from Chron C16n.2n to C7n.2n, covering a time period from approximately 36 to 25 million years ago. Rock magnetic analyses revealed that magnetite with a narrow grain size range is the primary carrier of remanent magnetization in the studied sediments (Florindo & Roberts, 2005; Roberts et al., 2013). The geocentric axial dipole (GAD) field at the latitude

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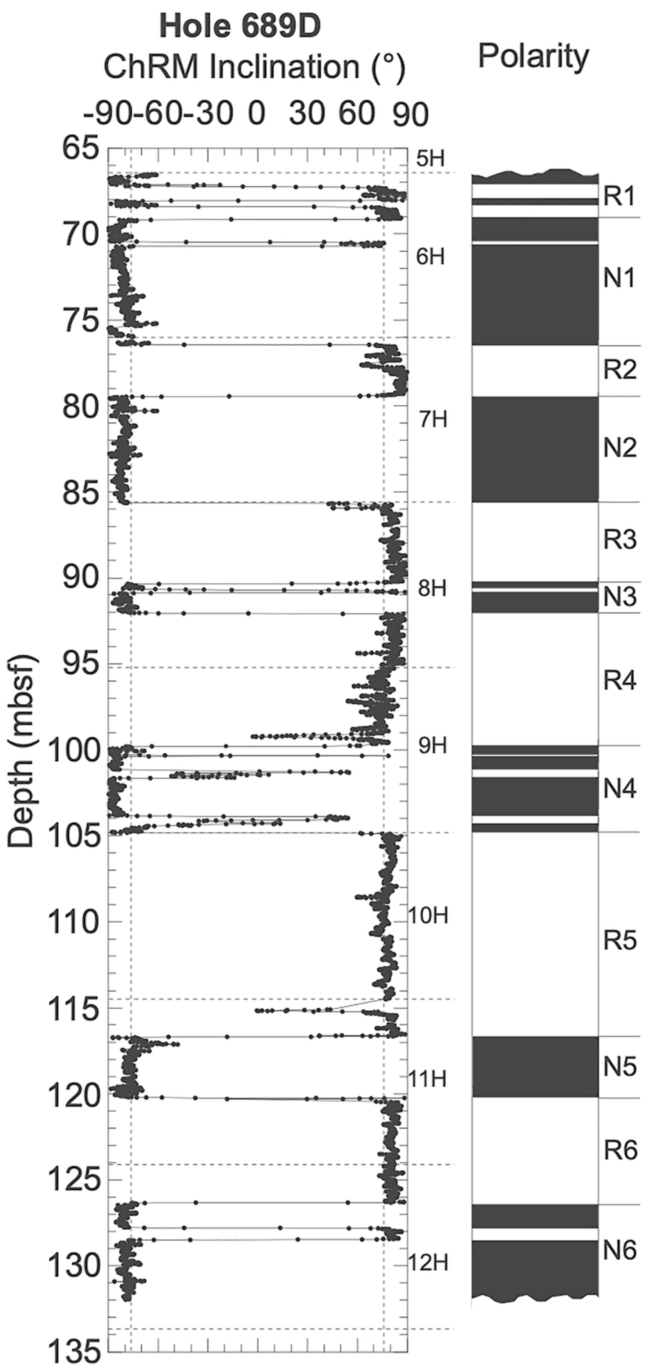

Figure 2. Down-core variations in the ChRM inclination for Hole 689D (left) as a function of the depth reported in meters below the seafloor (mbsf). The magnetic polarity zonation is shown on the right. Black (white) represents normal (reversed) polarity intervals.

of site 689D (about 64°S) has an inclination of ±76°. ChRM inclinations are therefore sufficient to determine polarity despite the fact that the cores lack azimuthal orientation. After alternating field demagnetization the ChRM inclinations have a clear bimodal distribution that demonstrates the presence of two stable polarity states (Figure 2).

The normal and reversed polarity ChRM inclinations are indistinguishable from expected GAD field values for the site latitude. The ChRM inclinations clearly define a series of magnetozones and the polarity intervals are bounded by polarity transitions only a few centimeters in thickness. In two cases, corresponding to core breaks, the boundary between the magnetozones is indicated by sharp shifts in inclination (i.e., 85.6 and 104.8 mbsf). This indicates that an unspecified amount of section is missing at these core breaks. Correlation of the paleomagnetic polarity pattern with the GPTS provides a direct age interpretation from Chron C16n.2n to C7n.2n. On the basis of the correlation to the GPTS, the average sedimentation rate for the studied interval of Hole 689D is ~0.6 cm/Kyr.

3. Theory of Tipping Points

The geomagnetic field polarity variability is a typical real-world example of a two-state system in which large-scale (abrupt) changes are driven by unknown processes of both (possible) internal and/or external source mechanisms. The most simple theory at the basis of describing abrupt transitions in a bi-stable system is that of fast-slow dynamical systems. Let $y(t)$ be a time variable describing the temporal changes between the two states of the system (in our case $y_+ = +76°$ and $y_- = -76°$), depending on a slow-variable parameter $\mu(t)$, then the dynamics of the system can be described by the following equations

$$\frac{dy}{dt} = f(y, \mu) \tag{1}$$

$$\frac{d\mu}{dt} = \varepsilon\, g(y, \mu) \tag{2}$$

where $\varepsilon \ll 1$. The theory of fast-slow systems, first introduced by Hasselmann (1976), allows us to reduce the 2-D dynamical system represented by Equations 1 and 2 to a 1-D dynamical system (Kuehn, 2011)

$$\frac{dy}{dt} = f(y, \mu) + \eta(t). \tag{3}$$

This implies that in a one-dimensional dynamical system, the state variable y, which evolves over time t, is dependent on a slowly changing parameter $\mu(t)$ and an additional rapidly varying time-dependent perturbation $\eta(t)$. The simplest choice for $\eta(t)$ is a white noise process, however in the following we relax this assumption, as recently proposed by Clarke et al. (2023), only requiring that its Fourier Transform exists. If we denote $y_\pm$ the two stable states of the system and we linearize Equation 3 around them, that is, $y(t) = y_\pm + x(t)$, we have

$$\frac{dx}{dt} \approx -\lambda x + \eta(t), \tag{4}$$

with

$$\lambda \doteq \frac{df(y, \mu)}{dy}\Big|_{y=y_\pm} \tag{5}$$

representing the rate at which the system returns to the equilibrium states $y_{\pm}$. By means of this approximation we remove the dependence of f on y as well as to directly link λ to f, providing a simple estimation of tipping probability and occurrence. Indeed, in the theory of fast-slow dynamical systems a tipping occurs if $\mu \to \mu_c$ which means $\lambda \to 0$. This phenomenon, known as critical slowing down, is at the basis of approaching a tipping point which in turn can be identified by looking at the time-dependence of λ.

Classically, λ can be identified by looking at the behavior of the variance $\langle \Delta x \rangle^2$ and auto-correlation R_{xx} of the system (Boers et al., 2022; Lenton et al., 2008; Medeiros et al., 2017). Indeed, for near-equilibrium systems, close to a tipping point, $\langle \Delta x \rangle^2$ increases and $R_{xx} \to 1$. This is based on the assumption that $\eta(t)$ is standard Wiener process, that is, $\eta(t) = \sigma W(t)$, a stochastic process where $W(t)$ is a $\delta -$ correlated unit variance Gaussian noise and σ its root-mean-square amplitude. Thus Equation 4 represents an Ornstein-Uhlenbeck process for which it is known that $\langle \Delta x \rangle^2 = \frac{\sigma^2}{2\lambda}$ and $R_{xx}(\tau) = e^{-\lambda\tau}$. However, if we move to the Fourier space we can relax any hypothesis on the nature of the noise as recently introduced by Clarke et al. (2023) with the new method known as ratio of spectra (ROSA) method. Indeed, Equation 4 can be written as

$$-i\,\omega\,\hat{x} = -\lambda\,\hat{x} + \hat{\eta} \tag{6}$$

where $\hat{\cdot}$ stands for Fourier transform. By taking the squared modulus we have

$$\mathcal{R}(\omega) = \frac{\hat{x}^2}{\hat{\eta}^2} = \frac{1}{\omega^2 + \lambda^2} \tag{7}$$

such that by knowing the spectrum of the time series $x(t)$ and regardless of the nature of the noise, $\mathcal{R}(\omega)$ has a universal character. This has a profound implication for reducing a priori assumptions on the nature of the noise that could lead to misleading results (Clarke et al., 2023). Indeed, usually the noise is thought to be representative of the fast or of the unknown dynamics of a given system and, although a standard Wiener process has been widely employed in literature (Benzi et al., 1982; Boers & Rypdal, 2021; Boers et al., 2022; Consolini & De Michelis, 2003), it cannot always reproduced as a Brownian-like dynamics as in the case of turbulence (Alberti et al., 2023) or random dynamical systems with pullback attractors (Chekroun et al., 2011; Vannitsem et al., 2021).

According to Equation 7 to detect the time variations of λ we have to select a moving window of length τ_w and perform a best-fit estimation of the ratio $\mathcal{R}(\omega)$ such that $\lambda = \lambda(\tau_w)$. If $\lambda \to 0$ an early warning signal of approaching a tipping point ($\lambda = 0$) can be derived. Power spectra are evaluated by using the Welch's method (Clarke et al., 2023; Welch, 1967), while data gaps are linearly interpolated only if less than the 10% of data in each window.

Two issues are crucial to properly detect λ:

1. the choice of the time window τ_w: it must be smaller than the slow timescale τ_s;
2. the choice of $\eta(t)$ if not known a priori.

However, both issues can be tackled by looking at the behavior of the power spectral density (PSD) of the analyzed time series. Indeed, the slow timescale can be identified as the inverse of the frequency at which a spectral break occurs; while, the choice of $\eta(t)$ can be done by looking at the spectral slope of the highest-frequency part of the PSD. Last but not least, our analysis is based on the inclination of the geomagnetic field at different depths such that we do not deal with time series directly. However, since there is a close relation between the depth and the timing of polarity reversals the proposed formalism can be applied equivalently. This means that, in our case, if we define $I(z)$ the ChRM inclination as a function of the depth z Equations 4 and 7 become, respectively,

$$\frac{dI}{dz} \approx -\lambda\,I + \eta(z), \tag{8}$$

$$\mathcal{R}(k) = \frac{1}{k^2 + \lambda^2}, \tag{9}$$

and, accordingly, $\lambda(z)$ will become a function of the depth z.

4. Results and Discussions

Figure 3 reports the Fourier PSD of the ChRM inclination (blue line) and that corresponding to a red noise process (red line) with a correlation scale corresponding to slow scale $\ell_s = 1/k_s$ (Ditlevsen, 2022).

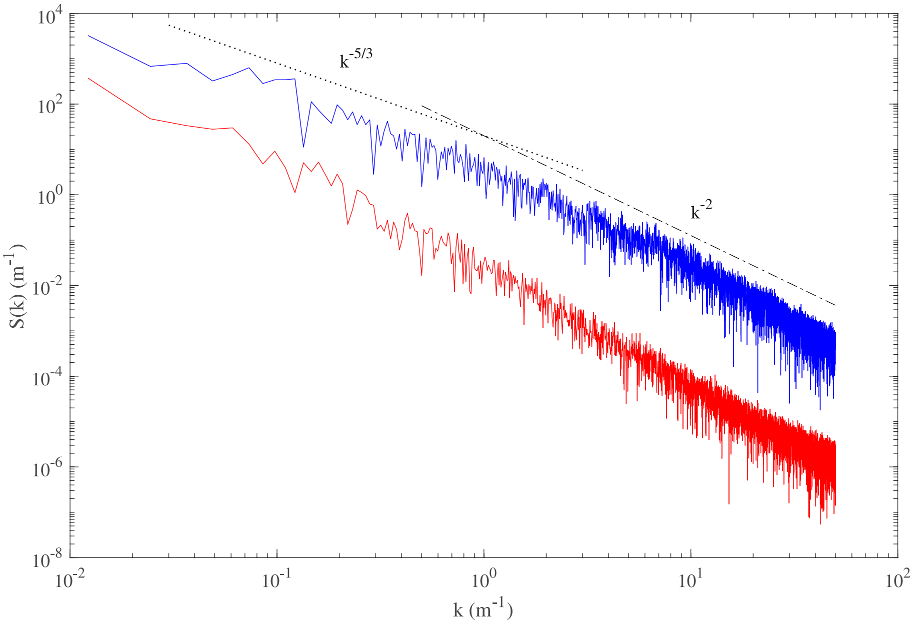

Figure 3. The Fourier PSD of the ChRM inclination (blue line) and that corresponding to a red noise process (red line) with a correlation scale corresponding to the slow scale $\ell_s = 1/k_s$. The dashed-dotted black line marks the k^{-2} slope of the high-wavevector range, while the dashed black line marks the $k^{-5/3}$ scaling of the low-wavevector range.

The PSD behavior evidences the existence of two different spectral slopes: a $k^{-5/3}$ scaling is found for $k < k_s \sim 1$ m^{-1}, and a steeper spectrum ($\sim k^{-2}$) for $k > k_s$. The low-wavevector range appears to be consistent, assuming that the amplitude and the direction of the dipole are plausibly related, with a turbulent dynamo regime (Moffatt, 1978; Sakuraba & Hamano, 2007; Schaeffer et al., 2017), while the high-wavevector can be representative of small-scale dissipation processes mainly driven by drift-wave turbulence (Berhanu et al., 2007; Moffatt, 1978). Based on the sedimentation rate of the site the scaling break $k_s \sim 1$ m^{-1} is in agreement with a frequency spectral break $f_s \sim 10$ Myr^{-1} previously reported in literature (Yamazaki & Oda, 2002). Based on Figure 3 we select as noise term $\eta(z)$ a red noise process, generated via an Ornstein-Uhlenbeck process with a relaxation scale $\ell_s \sim 1$ m, corresponding to the spectral break observed on the original data (k_s). For the moving window length, instead, we choice a set $\ell_w \in [\ell_s/50, \ell_s/10] = 0.02 - 0.1$ m, according to the condition based on the fast-slow theory of $\ell_w \ll \ell_s$, to take into account the uncertainty in the value of λ derived from Equation 9.

Figure 4 reports the behavior of λ (red) as a function of the depth, compared with the behavior of the ChRM inclination (blue). The results are intriguing since it is clear that when approaching a geomagnetic polarity reversal the parameter λ tends to 0. This clearly suggests that the behavior of geomagnetic polarity reversals can be described in a simple framework based on Equation 8, strengthening the importance of using simple stochastic models in geomagnetism and paleomagnetism (Molina-Cardín et al., 2021; Raphaldini et al., 2021). Indeed, we are able to detect all geomagnetic polarity reversals occurring during the time interval covered by sedimentary Section 689D (i.e., approximately from 25 to 36 million years ago) by selecting a critical threshold $\lambda_c = 0.30^{0.41}_{0.20}$, where 0.20 and 0.41 are the 5th and the 95th percentile of the distribution of the λ values over the set of chosen windows. When $\lambda < \lambda_c$ a reversal occurs (see Figure 4, dashed-dotted black line). We are also able to observe that, when the system resides in one of the two states $y_\pm$ with small fluctuations, also λ presents some fluctuations and, sometimes, seems to decrease approaching the critical threshold λ_c but never crossing it. Furthermore, thanks to our estimation of a critical threshold λ_c and remembering that λ represents the rate at which the system comes back toward a stable state after a perturbation we can estimate which is the characteristic relaxation time τ_r for a reversal to be completed. Considering that for this site a sedimentation rate of $s_r \approx 0.6$ cm/Kyr is observed, then

$$\tau_r = \frac{\lambda_c}{s_r} \approx 5 \text{ kyr (95\% confidence level:}[3.3, 6.8] \text{ kyr).} \tag{10}$$

Current estimates suggest that the average duration for a complete reversal of the Earth's magnetic field spans thousands of years. However, it is important to note that this estimate is uncertain and lacks a universally

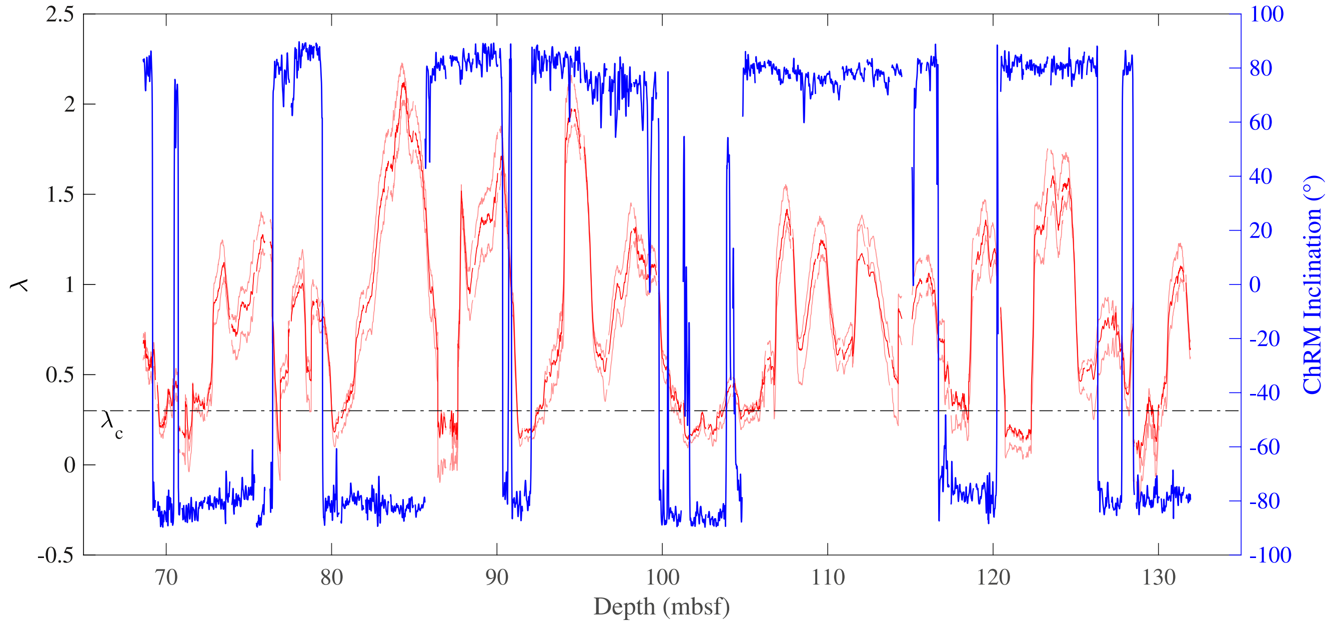

Figure 4. The behavior of λ (red) as a function of the depth in comparison with the behavior of the ChRM inclination (blue). The horizontal dashed-dotted black line marks a critical threshold $\lambda_c = 0.30^{0.41}_{0.20}$. The shaded red curves mark the 5th and the 95th percentiles of the distribution.

accepted precise value. The uncertainties arise due to the limitations and ambiguities inherent in paleomagnetic data and records, which serve as the basis for understanding geomagnetic reversals. Nevertheless, a study by Clement (2004) sheds further light on this subject. By analyzing 30 carefully selected records from the past four geomagnetic field reversals, Clement (2004) determined that the average transition time for the field to shift from one polarity to another is approximately 7,000 years. Interestingly, these reversals tended to occur faster at lower latitudes than at mid-to-high latitudes. The individual duration of these reversals exhibited a wide range, spanning from 2,000 to 12,000 years thus suggesting the significant influence of the non-dipole field in shaping the geomagnetic reversal phenomenon. Therefore, our analysis, resulting in an average reversal duration of approximately 5,000 years, is closely aligned with the findings reported by Clement (2004). However, it is important to underline that our analysis is based on the direction of the dipole and, thus, the duration of polarity reversal based on directions is bound to be different than a duration based on intensity. Particularly, the estimated transition time from inclination is shorter than that evaluated in terms of amplitude. Thus, our estimation of approximately 5,000 years is only specific to the definition based on directions.

5. Conclusions and Perspectives

In this study we presented for the first time an early warning signal analysis for detecting geomagnetic reversals based on the theory of tipping points widely employed in climate and paleoclimate (Boers & Rypdal, 2021; Boers et al., 2021, 2022; Boettner & Boers, 2022; Ditlevsen & Johnsen, 2010; Lenton, 2011; Lenton et al., 2008, 2012; Lohmann et al., 2021). This theory is well suited also for studying geomagnetic reversals as it captures the behavior of a system that remains in stable states or regimes for a prolonged period. However, once a tipping point is reached, the system undergoes a transition to a new state, often resulting in significant and enduring consequences. By using a long sedimentary section covering the period from 25 to 36 millions years ago we provide evidence of an early warning indicator λ of the occurrence of a polarity reversal and, under certain circumstances, to detect excursions (without reversals) of the geomagnetic field. We identified a critical threshold λ_c below which a polarity reversal starts as well as an average reversal time of about 5 Kyr, in agreement with previous independent studies (Clement, 2004). Through the proper use of this early warning indicator λ we can build up a simple 1-D model to reproduce the main feature of polarity reversals by rewriting Equation 3 in a commonly used for in the framework of tipping points as

$$\frac{dy}{dz} = -\frac{dU(y)}{dy} + \eta(z), \tag{11}$$

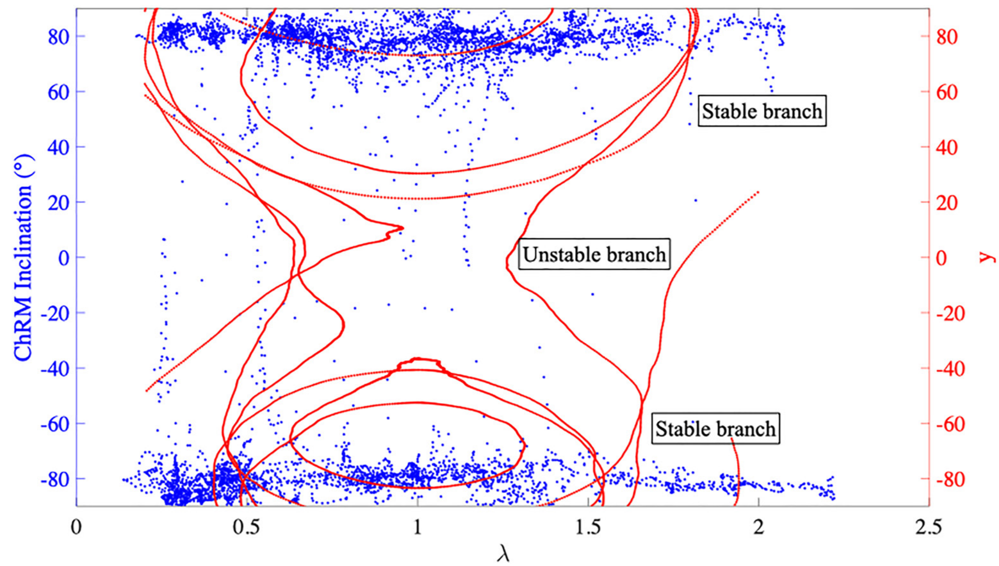

Figure 5. The phase-space dynamics of the observed λ versus the ChRM inclination $I(z)$ (blue dots) in comparison with those reconstructed from the model reported in Equation 11.

where the forcing term $f(y, \mu)$ is represented by the first derivative of a potential function $U(y, \lambda)$ defined as

$$U(y) = \frac{y^4}{4} - \frac{y^2}{2} - \lambda(z)\, y. \tag{12}$$

The model introduced in Equations 11 and 12 is a more general (non-autonomous) version of the widely employed Stochastic Resonance (SR) model (Benzi et al., 1982). Indeed, the equivalence can be obtained by substituting the periodic modulation function of the SR model with $\lambda(z)$. The condition $\lambda(z) \to 0$ is equivalent to have in-phase or out-of-phase resonances, that is, reversals of the state (from one well to the other). Clearly, when $\lambda \to 0$ there is the co-existence of the two symmetric (i.e., equi-probable) states, observed during a polarity reversal; while, when λ departs from zero, then the two wells are not symmetric, suggesting that the two states are not equi-probable, meaning that the system resides in one of the two stable states (i.e., $y = y_{\pm}$). Figure 5 reports the phase-space dynamics (i.e., the scatter-plot between the value of the ChRM inclination $I(z)$ and $\lambda(z)$) for the paleomagnetic measurements (blue line) and the modeled via Equation 11 (red line).

We can clearly see the agreement between measurements and modeled values since the system oscillates between two different stable states $y_{\pm}$, with fluctuations around both states when $\lambda > \lambda_c$, while the jump between these two stable states occurs when the system passes through an unstable saddle point (an unstable branch or manifold of the dynamics, (Ditlevsen, 2022; Lohmann et al., 2021)). Our results can be reconciled with a recently proposed stochastic model for geomagnetic reversals reproducing the temporal asymmetry of the axial dipole moment (ADM) formed by two coupled Brownian particles moving in a double-well potential (Molina-Cardín et al., 2021). The authors found that the ADM oscillates around its equilibrium value with reversals, when both particles change sign and move in the opposite well, and excursions, when a single particle moves in the opposite well (see Figure 2 of Molina-Cardín et al. (2021)). The main novelty introduced by Molina-Cardín et al. (2021) is the use of two particle within the potential since, in this way, it is possible to reproduce the asymmetry of the ADM. However, our results demonstrate that two asymmetric states can be obtained using a single-particle approach (i.e., a 1-D model as in Equation 11) if a non-autonomous forcing is introduced in a simple 1-D stochastic model, that is, by considering a double-well potential function of the form as in Equation 12. By further inspecting Figure 4 we clearly observe that reversals occur when $\lambda \to 0$ and specifically when $\lambda < \lambda_c$, although we report the evidence of decreasing λ with no reversals when $\lambda > \lambda_c$. This suggest that our results can be reconciled with those reported by Molina-Cardín et al. (2021) if we define the occurrence of reversals when $\lambda < \lambda_c$ and excursions when $\lambda_c < \lambda < \lambda_e$, with $\lambda_e \sim 1$. Thus, our simple model can be used to characterize both reversals and excursions in a simple 1-D framework.

However, the main novelty we introduced with this work is not on the modeling point of view but instead on the improvement of possibly provide an early warning signal for future geomagnetic reversals. Indeed, our simple approach, widely applied in climate research for investigating future tipping points in several ecosystems (Boers & Rypdal, 2021; Boers et al., 2021, 2022; Lenton, 2021), allows us, by looking at the behavior of the parameter λ, in characterizing and providing an early warning signal of reversals when λ crosses the critical threshold $\lambda_c = 0.3$. This first study demonstrates the importance of the crossroad between dynamical systems and paleomagnetism and surely claim for future investigations on the same line. Indeed, studying tipping points and associated early warning signals is of utmost importance in understanding complex systems and their behavior. By identifying and comprehending these tipping points, we gain valuable insights into the potential trajectories and outcomes of the Earth system in terms of geomagnetic field variations on long timescales. Clearly our simple model cannot delve in discerning the causes and the processes behind geomagnetic reversals, which still remain a debate question and an open area of research, as well as, clearly distinguishing between reversals and excursions (Raphaldini et al., 2020). Further investigations, possibly on longer records and with a larger statistics of reversals and excursions, can help in finding a precise dynamical feature in the temporal behavior of model parameters (e.g., $\lambda(z)$) to detect and correctly classify both types of transitions. In this framework, machine learning approaches (Gwirtz et al., 2022), combined with the theory of tipping points, can help in improving the effectiveness of machine learning in pinpointing precursors of magnetic field reversals, possibly arising from the scarcity of data and its limited frequency resolution. Finally, additional analysis are needed to find a connection between stochastic model parameters and physical quantities which can be at the basis of polarity reversals, although a dynamo effect triggered by different mechanism can be observed in simple stochastic models (Carbone et al., 2020; Consolini & De Michelis, 2003; Molina-Cardín et al., 2021; Wicht & Meduri, 2016), deterministic dynamical systems based on the truncation of the magnetohydrodynamic (MHD) equations (Alberti et al., 2019), and numerical simulations (Aubert, 2019; Schaeffer et al., 2017). As a result, our research can serve as a starting point to provide insight on future polarity reversals, and we are confident that the proposed formalism can aid in the development of a novel framework for paleomagnetic research.

Conflict of Interest

The authors declare no conflicts of interest relevant to this study.

Data Availability Statement

Data is available at Alberti (2023).

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