Planar Graphs

- **Definition:** A graph is planar if it can be drawn without any edges crossing. When drawn this way, it divides the plane into regions called faces.
- **Key Question:** The central question is determining when it's possible to draw a graph without any edges intersecting.
- **Important Note:** The definition focuses on the *possibility* of drawing without crossings. A graph might appear non-planar but can be redrawn to show its planar nature.
- **Faces:**
  - When a planar graph is drawn without edge crossings, the edges and vertices create regions called faces.
  - The "outside" region is also considered a face.
  - The number of faces is an inherent property of the planar graph and doesn't change with different planar drawings.
  - Faces can only be counted when the graph is drawn in a planar representation.
- **Euler's Formula:** For any connected planar graph: $v - e + f = 2$, where v is the number of vertices, e is the number of edges, and f is the number of faces.
  - The document provides an explanation of why Euler's formula is true, using the concept of building up a graph step-by-step.
- **Non-Planar Graphs:**
  - Not all graphs are planar. If there are too many edges relative to the number of vertices, edges must intersect.
  - The smallest graph that demonstrates this is K5 (the complete graph with 5 vertices).
  - K5 is proven to be non-planar using a proof by contradiction and Euler's formula.
  - The proofs for non-planarity involve Euler's formula and analyzing the minimum number of edges surrounding each face (girth).
- **Connection to Polyhedra:**
  - Convex polyhedra can be represented as planar graphs.
  - Euler's formula for planar graphs also applies to convex polyhedra.
  - The document uses graph theory to prove that there are only five regular polyhedra