Power Electronics Reference

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Contents

Introduction

Just throwing together some concepts and examples for later reference

1 General Equations

1.1 Power and Energy

1.1.1 Instantaneous Power

This value is a positive real number if the element is supplying power, or is a negative real number if the element is consuming power.

$$\boxed{p(t) = v(t)i(t) \quad (W)}$$

- $\boldsymbol{p(t)}$:= Instantaneous Power (W)
- $\boldsymbol{v(t)}$:= Instantaneous Voltage (V)
- $\boldsymbol{i(t)}$:= Instantaneous Current (A)

1.1.2 Average Power

$$\boxed{P = \frac{1}{T}\int_{t_o}^{t_o+T} p(t)dt \quad (W)}$$

$$\boxed{P = \frac{1}{T}\int_{t_o}^{t_o+T} v(t)i(t)dt \quad (W)}$$

$$\boxed{P = \frac{W}{T} \quad (W)}$$

- $\boldsymbol{P}$:= Average Power (W)
- $\boldsymbol{p(t)}$:= Instantaneous Power (W)
- $\boldsymbol{v(t)}$:= Instantaneous Voltage (V)
- $\boldsymbol{i(t)}$:= Instantaneous Current (A)
- $\boldsymbol{T}$:= Time Period (Δs)
- $\boldsymbol{t_o}$:= Initial Time (s)
- $\boldsymbol{W}$:= Work (J)

1.1.3 Energy

$$\boxed{W = \int_{t_1}^{t_2} p(t)dt \quad (J)}$$

- $\boldsymbol{W}$:= Work (J)
- $\boldsymbol{p(t)}$:= Instantaneous Power (W)

1.2 Inductors

1.2.1 Current/Voltage Relationship

$$i_L(t_o + T) = \frac{1}{L}\int_{t_o}^{t_o+T} v_L(t)dt + i_L(t_o)$$

- $\boldsymbol{i_L(t)}$:= Instantaneous Current Through an Inductor (A)
- $\boldsymbol{v_L(t)}$:= Instantaneous Voltage Across an Inductor (V)
- $\boldsymbol{i_L(t_o)}$:= Initial Current Through an Inductor (A)
- $\boldsymbol{L}$:= Inductance (H)
- $\boldsymbol{t_o}$:= Initial Time (s)
- $\boldsymbol{T}$:= Time Period (Δs)

$$V_L = \frac{1}{T}\int_{t_o}^{t_o+T} v_L(t)dt$$

NOTE: When a sinusoidal current is applied through the inductor, the average voltage is zero ($V_L = 0$) for each period.

- $\boldsymbol{V_L}$:= Average Voltage Across and Inductor (V)
- $\boldsymbol{v_L(t)}$:= Instantaneous Voltage Across an Inductor (V)
- $\boldsymbol{t_o}$:= Initial Time (s)
- $\boldsymbol{T}$:= Time Period (Δs)

1.2.2 Power and Energy

$$w_L(t) = \frac{1}{2} L i_L^2(t)$$

- $\boldsymbol{w_L(t)}$:= Instantaneous Work an Inductor Does at Time t (J)
- $\boldsymbol{i_L(t)}$:= Instantaneous Current Through an Inductor (A)
- $\boldsymbol{L}$:= Inductance (H)

$$P_L = \frac{1}{T} \int_{t_o}^{t_o+T} p_L(t) dt$$

NOTE: The average power produced/consumed by the inductor is zero ($P_L = 0$) for each period when a periodic sinusoidal current is applied through the inductor.

- $\boldsymbol{P_L}$:= Average Power an Inductor Produces/Consumes (W)
- $\boldsymbol{p_L(t)}$:= Instantaneous Power an Inductor Produces/Consumes (W)
- $\boldsymbol{t_o}$:= Initial Time (s)
- $\boldsymbol{T}$:= Time Period (Δs)

1.3 Capacitors

1.3.1 Current/Voltage Relationship

$$v_C(t_o+T)=\frac{1}{C}\int_{t_o}^{t_o+T} i_C(t)dt+v_C(t_0)$$

- $\boldsymbol{v_C(t)}$:= Instantaneous Voltage Across a Capacitor (V)
- $\boldsymbol{i_C(t)}$:= Instantaneous Current Through a Capacitor (A)
- $\boldsymbol{v_C(t_o)}$:= Initial Voltage Across a Capacitor (V)
- $\boldsymbol{C}$:= Capacitance (F)
- $\boldsymbol{t_o}$:= Initial Time (s)
- $\boldsymbol{T}$:= Time Period (Δs)

$$I_C=\frac{1}{T}\int_{t_o}^{t_o+T} i_C(t)dt$$

NOTE: When a sinusoidal voltage is applied through a capacitor, the average current is zero ($I_C = 0$) for each period.

- $\boldsymbol{I_C}$:= Average Current Through a Capacitor(A)
- $\boldsymbol{i_C(t)}$:= Instantaneous Current Through a Capacitor (A)
- $\boldsymbol{t_o}$:= Initial Time (s)
- $\boldsymbol{T}$:= Time Period (Δs)

1.3.2 Power and Energy

$$w_C(t) = \frac{1}{2}Cv_C^2(t)$$

- $\boldsymbol{w_L(t)}$:= Instantaneous Work a Capacitor Does at Time t (J)
- $\boldsymbol{v_C(t)}$:= Instantaneous Voltage Across a Capacitor (V)
- $\boldsymbol{C}$:= Capacitance (F)

$$P_C = \frac{1}{T}\int_{t_o}^{t_o+T} p_L(t)dt$$

NOTE: The average power produced/consumed by a capacitor is zero ($P_C = 0$) for each period when a periodic sinusoidal current is applied through the inductor.

- $\boldsymbol{P_C}$:= Average Power a Capacitor Produces/Consumes (W)
- $\boldsymbol{p_C(t)}$:= Instantaneous Power a Capacitor Produces/Consumes (W)
- $\boldsymbol{t_o}$:= Initial Time (s)
- $\boldsymbol{T}$:= Time Period (Δs)

1.4 Root Mean Square (RMS)

The Root Mean Square (RMS) magnitude of voltages and currents identify the effective power applied to a circuit from a source. This is usually compared to the power consumed by each element in the circuit to calculate the efficiency of a circuit.

$$V_{RMS} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} v_S^2(t)dt}$$

$$V_{RMS} = \sqrt{V_{1,RMS}^2 + V_{2,RMS}^2 + V_{3,RMS}^2 + ...}$$

$$I_{RMS} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} i_S^2(t)dt}$$

$$I_{RMS} = \sqrt{I_{1,RMS}^2 + I_{2,RMS}^2 + I_{3,RMS}^2 + ...}$$

- $\boldsymbol{V_{RMS}}$:= RMS Voltage (V)
- $\boldsymbol{I_{RMS}}$:= RMS Current (A)
- $\boldsymbol{v_s(t)}$:= Voltage Emitted from Source (V)
- $\boldsymbol{i_s(t)}$:= Current Emitted from/to a Source (A)
- $\boldsymbol{t_o}$:= Initial Time (s)
- $\boldsymbol{T}$:= Time Period (Δs)

1.5 Apparent Power

Apparent power relates the true and reactive powers of a circuit. The true power is what is usually calculated with the fundamental formula ($P = IV$), and is measured in watts. The reactive power is represented by the variable Q, and describes how inductors and capacitors change the voltage and current of a circuit without actually drawing power. Reactive power is usually returned back to the source and is measured in units Volt-Amp-Reactive (VAR). Apparent power relates these two phenomenons in a trigonometric fashion with true power (P) and reactive power (Q) on the x and y axis, and apparent power (S) on the hypoteneuse. The angle used to relate each magnitude to eachother is the impedence phase angle, with apparent power equaling the tangent of the reactive power divided by true power ($S = tan(\frac{Q}{P})$).

$$S = V_{RMS} I_{RMS}$$

- $\boldsymbol{V_{RMS}}$:= RMS Voltage (V)
- $\boldsymbol{I_{RMS}}$:= RMS Current (A)

1.6 Power Factor

$$pf = \frac{P}{S} = \frac{P}{V_{RMS} I_{RMS}}$$

- $\boldsymbol{pf}$:= Power Factor
- $\boldsymbol{P}$:= True Power (W)
- $\boldsymbol{V_{RMS}}$:= RMS Voltage (V)
- $\boldsymbol{I_{RMS}}$:= RMS Current (A)

2 Sinusoidal AC Circuits

2.1 Definitions

2.1.1 Voltage and Current Definitions for any and all Elements

$$v(t) = V_m cos(\omega t + \Theta)$$
$$i(t) = I_m cos(\omega t + \phi)$$

2.1.2 Real Power

$$p(t) = (\frac{V_m I_m}{2})[cos(2\omega t + \Theta + \phi) + cos(\Theta - \phi)]$$

$$P = (\frac{V_m I_m}{2})cos(\Theta - \phi)$$

2.1.3 Reactive Power

$$Q = V_{RMS} I_{RMS} sin(\Theta - \phi)$$

2.1.4 Apparent Power

$$S = P + jQ = (V_{RMS})(I_{RMS})^*$$

Variables

$\boldsymbol{C}$ Capacitance (F).
$\boldsymbol{I_C}$ Average Current Through a Capacitor(A).
$\boldsymbol{I_L}$ Average Current Through an Inductor (A).
$\boldsymbol{I_{RMS}}$ RMS Current (A).
$\boldsymbol{I}$ Average Current (A).
$\boldsymbol{L}$ Inductance (H).
$\boldsymbol{P_C}$ Average Power a Capacitor Produces/Consumes (W).
$\boldsymbol{P_L}$ Average Power an Inductor Produces/Consumes (W).
$\boldsymbol{P}$ Average Power (W).
$\boldsymbol{T}$ Time Period (Δs).
$\boldsymbol{V_C}$ Average Voltage Across a Capacitor (V).
$\boldsymbol{V_L}$ Average Voltage Across and Inductor (V).
$\boldsymbol{V_{RMS}}$ RMS Voltage (V).
$\boldsymbol{V}$ Average Voltage (V).
$\boldsymbol{W_C}$ Work a Capacitor Does over a Period of Time (J).
$\boldsymbol{W_L}$ Work an Inductor Does over a Period of Time (J).
$\boldsymbol{W}$ Work (J).
$\boldsymbol{i(t)}$ Instantaneous Current (A).
$\boldsymbol{i(t_o)}$ Initial Current at Time t_o (A).
$\boldsymbol{i_C(t)}$ Instantaneous Current Through a Capacitor (A).
$\boldsymbol{i_L(t)}$ Instantaneous Current Through an Inductor (A).
$\boldsymbol{p(t)}$ Instantaneous Power (W).
$\boldsymbol{p_C(t)}$ Instantaneous Power a Capacitor Produces/Consumes (W).
$\boldsymbol{p_L(t)}$ Instantaneous Power an Inductor Produces/Consumes (W).
$\boldsymbol{t_o}$ Initial Time (s).
$\boldsymbol{t}$ Instantaneous Time (s).
$\boldsymbol{v(t)}$ Instantaneous Voltage (V).
$\boldsymbol{v(t_o)}$ Initial Voltage at Time t_o (V).
$\boldsymbol{v_C(t)}$ Instantaneous Voltage Across a Capacitor (V).
$\boldsymbol{v_L(t)}$ Instantaneous Voltage Across an Inductor (V).
$\boldsymbol{w_L(t)}$ Instantaneous Work a Capacitor Does at Time t (J).
$\boldsymbol{w_L(t)}$ Instantaneous Work an Inductor Does at Time t (J).

Glossary

Average Current The average current going through a reference point throughout a period of time T.
Average Power The average power an element is consuming or producing throughout a period of time T.
Average Voltage The average voltage across two reference points throughout a period of time T.

Capacitance A measurement of the phenomenon that change in voltage is resisted in a capacitor dut to the establishment of an electric field.

Inductance A measurement of the phenomenon that change in current is resisted in an inductor due to the establishment of a magnetic field.
Initial Time Describes the initial time that a period in time is taken.
Instantaneous Current The current going through a reference point at time t.
Instantaneous Power The power an element is consuming or producing at time t.
Instantaneous Time An instant in time.
Instantaneous Voltage The voltage across two reference points at time t.

Time Period A period between two points in time. Usually describes a full period in a sinusoidal wave.

Work The power a system is able to output over a period in time.

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