

Columbia University  
IEOR 4732: Computational Methods in Finance  
Case Study 1

**Problem 1:** The characteristic function of the log of stock price in Black-Scholes framework is given by:

$$\begin{aligned}\mathbb{E}(e^{iu \ln S_t}) &= \mathbb{E}(e^{ius_t}) \\ &= \exp\left(i(\ln S_0 + (r - q - \frac{\sigma^2}{2})t)u - \frac{1}{2}\sigma^2 u^2 t\right) \\ &= \exp\left(i(s_0 + (r - q - \frac{\sigma^2}{2})t)u - \frac{1}{2}\sigma^2 u^2 t\right)\end{aligned}$$

For the following parameters:

Spot price,  $S_0 = \$1900$ ; maturity,  $T = 0.25$  year; volatility,  $\sigma = 0.36$ ; risk-free interest rate,  $r = 2.00\%$ , continuous dividend rate,  $q = 1.87\%$  and strike range of  $K = 2000, 2100, 2200$  price European call options via the following transform techniques:

- (a) Fast Fourier transform (FFT): consider  $\eta = \Delta\nu = 0.25$ ,  $\alpha = 0.4, 1.0, 1.4, 3.0$ ,  $N = 2^n$  for  $n = 9, 11, 13, 15$ , and  $\beta = \ln K - \frac{\lambda N}{2}$
- (b) Fractional fast Fourier transform (FrFFT): consider  $\eta = \Delta\nu = 0.25$ ,  $\lambda = \Delta k = 0.1$ ,  $\alpha = 0.4, 1.0, 1.4, 3.0$ ,  $N = 2^n$  for  $n = 6, 7, 8, 9$ , and  $\beta = \ln K - \frac{\lambda N}{2}$
- (c) Fourier-cosine (COS) method: consider values  $[-1, 1]$ ,  $[-4, 4]$ ,  $[-8, 8]$ ,  $[-12, 12]$  for the interval  $[a, b]$  and find the sensitivity of your results to the choice of  $[a, b]$

Compare and conclude.