Columbia University
IEOR 4732: Computational Methods in Derivatives Pricing
Case Study 2

Pricing an up-and-out call (UOC) – Let $w(x,\tau)$ be the value of a derivative security that satisfies the following PIDE

$$\frac{\partial w}{\partial \tau}(x,\tau) - (r-q)\frac{\partial w}{\partial x}(x,\tau) + rw(x,\tau)$$
$$-\int_{-\infty}^{\infty}\left[w(x+y,\tau) - w(x,\tau) - \frac{\partial w}{\partial x}(x,\tau)(e^y - 1)\right]k(y)dy = 0$$

where

$$k(y) = \frac{e^{-\lambda_p y}}{\nu y^{1+Y}}\mathbf{1}_{y>0} + \frac{e^{-\lambda_n |y|}}{\nu |y|^{1+Y}}\mathbf{1}_{y<0},$$

with

$$\begin{aligned}
\lambda_p &= \left(\frac{\theta^2}{\sigma^4} + \frac{2}{\sigma^2\nu}\right)^{\frac{1}{2}} - \frac{\theta}{\sigma^2},\\
\lambda_n &= \left(\frac{\theta^2}{\sigma^4} + \frac{2}{\sigma^2\nu}\right)^{\frac{1}{2}} + \frac{\theta}{\sigma^2}.
\end{aligned}$$

and $x = \ln(S)$ and $\tau = T - t$. For an up-and-out call (UOC) option premium, this PIDE must be solved subject to the initial condition

$$w(x,0) = (e^x - K)^+$$

and boundary conditions

$$\begin{aligned}
w(x_0,\tau) &= 0 \quad \forall\tau,\\
w(x_N,\tau) = w(B,\tau) &= 0 \quad \forall\tau.
\end{aligned}$$

Use explicit-implicit finite difference scheme covered during the lecture to solve the PIDE.

Calculate UOC option premium in this framework for the following parameters: spot price, $S_0 = \$1900$; strike price $K = 2000$; upper barrier $B = 2200$, risk-free interest rate, $r = 0.25\%$; dividend rate, $q = 1.5\%$; maturity, $T = 0.5$ year; $\sigma = 25\%$, $\nu = 0.31$, $\theta = -0.25$, and $Y = 0.4$.