Assignment1: NMM

1.a) Given Constant values substituted and equation simplified, s1 at A=0,s2 at A=0.5,s3 at A=1.k taken from 0 to 5

Program Commands:

```
>> k=(0:0.5:5)
s1=(0.7*k.^4)./(-6*k.^2-2.4)
s2=(0.7*k.^4-13.2*k.^2)./(141.6-6*k.^2)
s3=(0.7*k.^4-26.4*k.^2)./(-6*k.^2+285.6);
plot(s1,k,'--ro','LineWidth',0.25,'MarkerSize',5)
hold on
plot(s2,k,'-.b^','LineWidth',0.15,'MarkerSize',8)
plot(s3,k,':gs','LineWidth',0.15,'MarkerSize',9)
legend('A=0','A=0.5','A=1')
%labels
xlabel('s')
ylabel('k')
title('s vs k')
grid on
```

Command Window Execution Statement:

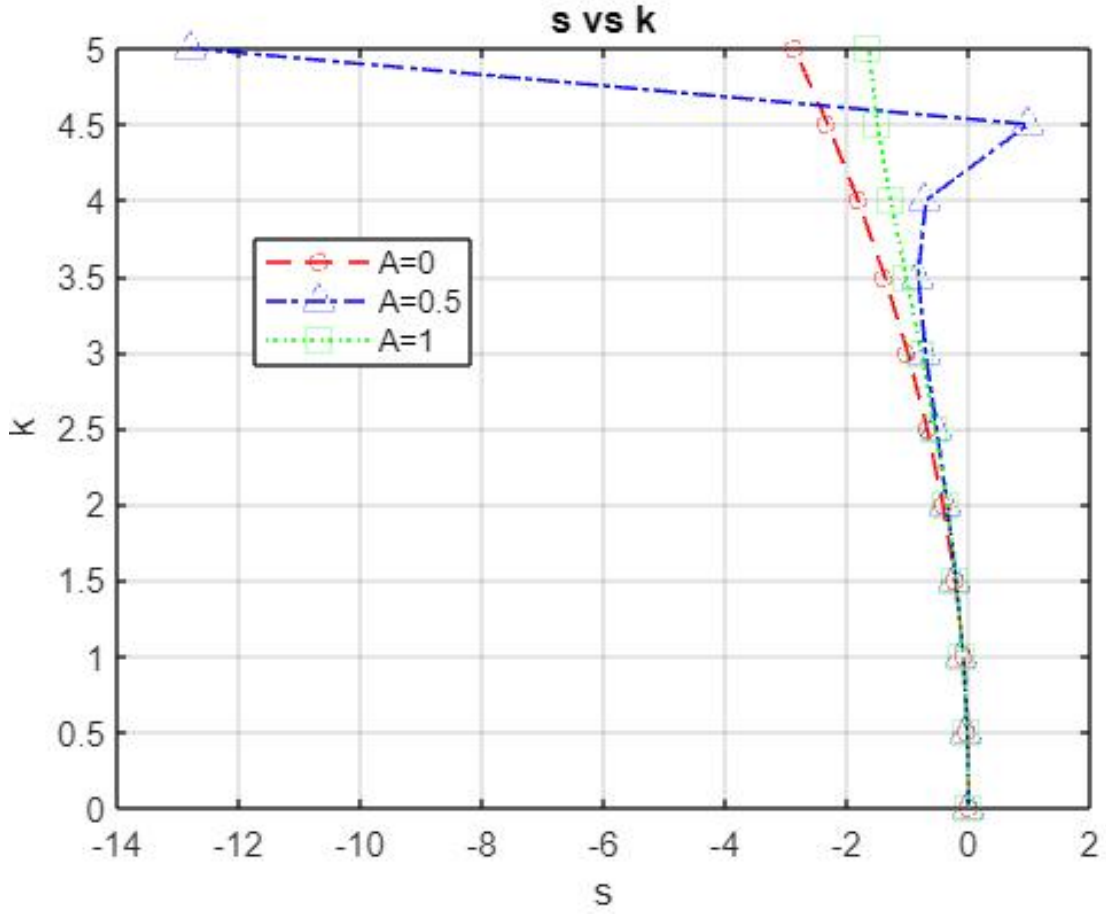
```
k =

0 0.5000 1.0000 1.5000 2.0000 2.5000 3.0000 3.5000 4.0000 4.5000 5.0000
s1 =

0 -0.0112 -0.0833 -0.2229 -0.4242 -0.6853 -1.0053 -1.3840 -1.8211 -2.3167 -2.8707
s2 =

0 -0.0232 -0.0922 -0.2042 -0.3537 -0.5298 -0.7089 -0.8320 -0.7018 0.9823 -12.7976
s3 =

0 -0.0231 -0.0919 -0.2053 -0.3609 -0.5548 -0.7811 -1.0295 -1.2827 -1.5086 -1.6409
```



1.b) Given Constant values substituted and equation simplified, s1 at M=0 ,s2 at M=1, s3 at M=5, s4 at M=10. k taken from 0 to 5

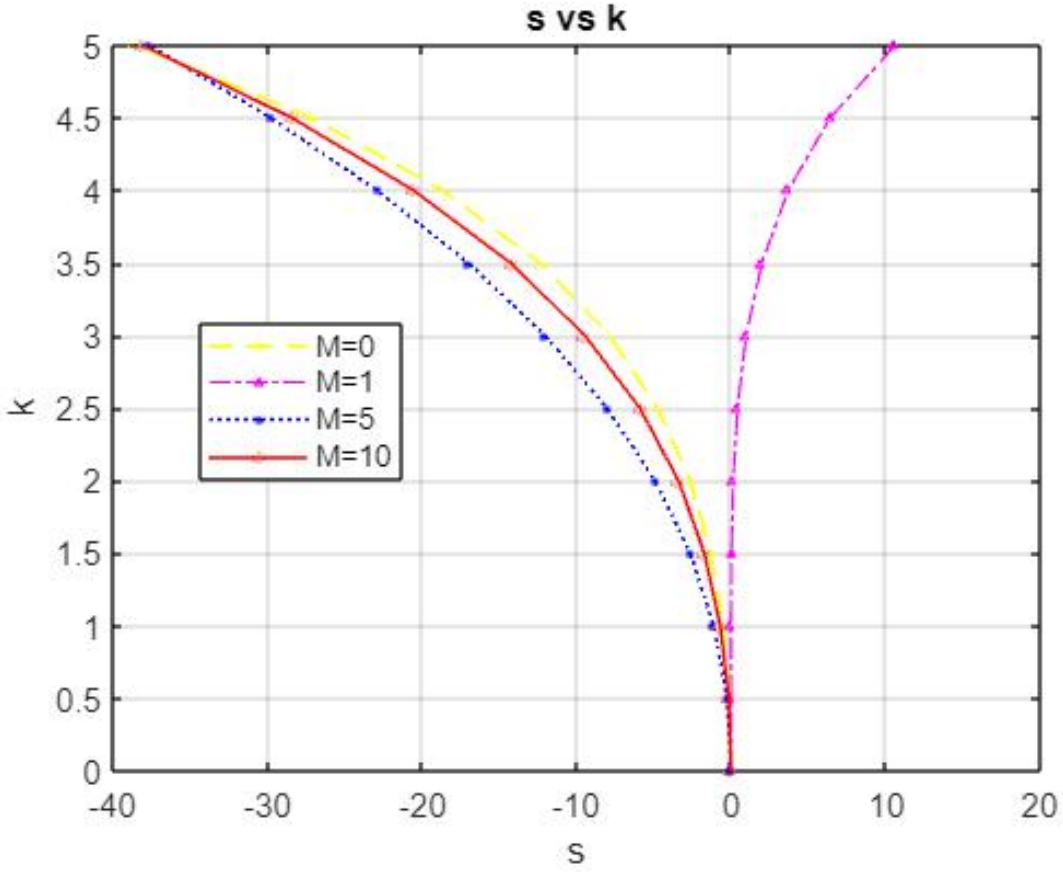
Program Commands:

```
>> k=(0:0.5:5)
s1=(k.^4+12*k.^2)./(-24)
s2=(1.3*k.^4-2.4*k.^2)./(-k.^2+96)
s3=(10.6*k.^4+105.6*k.^2)./(-6*k.^2-96)
s4=(20.6*k.^4+225.6*k.^2)./(-6*k.^2-336)
plot(s1,k,'--yo','LineWidth',0.25,'MarkerSize',3)
hold on
plot(s2,k,'-.m^','LineWidth',0.75,'MarkerSize',2)
plot(s3,k,':bs','LineWidth',0.95,'MarkerSize',1.5)
plot(s4,k,'-rhexagram','LineWidth',0.15,'MarkerSize',4)
legend('M=0','M=1','M=5','M=10');
xlabel('s')
ylabel('k')
title('s vs k')
grid on
```

Command Window Execution Statement:

```
k =
0 0.5000 1.0000 1.5000 2.0000 2.5000 3.0000 3.5000 4.0000 4.5000 5.0000

s1 =
0 -0.1276 -0.5417 -1.3359 -2.6667 -4.7526 -7.8750 -12.3776 -18.6667 -27.2109 -38.5417
s2 =
0 -0.0054 -0.0116 0.0126 0.1217 0.3987 0.9621 1.9783 3.6800 6.3958 10.5986
s3 =
0 -0.2776 -1.1392 -2.6599 -4.9333 -8.0454 -12.0600 -17.0163 -22.9333 -29.8164 -37.6626
s4 =
0 -0.1709 -0.7199 -1.7508 -3.4222 -5.9296 -9.4846 -14.2976 -20.5630 -28.4496 -38.0967
```



Note: .m file for above 1a, 1b added in folder the name of question1apolt.m which contains **both figures (1a,1b)**script

2.

2a)

$\sqrt{x+1} - \sqrt{x}$

For large values of x. (ex $x = 10^9$) $\sqrt{x+1} - \sqrt{x}$ leads to severe cancellations because

$\sqrt{x+1} \cong \sqrt{x}$ so to avoid this formula is re written by using calculus, that is multiply and divide by the conjugate

$$\Rightarrow \left(\sqrt{x+1} - \sqrt{x}\right) \frac{\sqrt{x+1} + \sqrt{x}}{\sqrt{x+1} + \sqrt{x}} = \frac{\left(\sqrt{(x+1)}\right)^2 - \left(\sqrt{x}\right)^2}{\sqrt{x+1} + \sqrt{x}}$$

$$\Rightarrow \frac{1}{\sqrt{x+1} + \sqrt{x}} - \text{(Eq) new}$$

∴ In the above Eq new has no cancellation.

Note:- Matlab verification for $x = 10^9$ given added in the folder, file name - 'roundoff2a.m'

2b) $\cos^2 x - \sin^2 x$

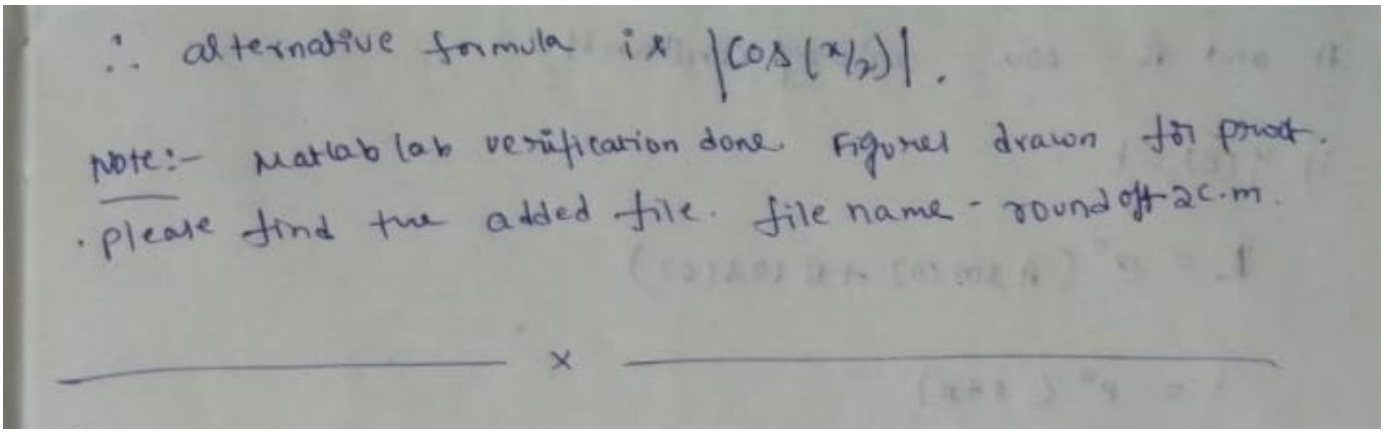
if "$x \geq \pi/4$" / above formula leads to large round-off error

∴ alternative Formula is $\cos 2x$

Note:- Matlab verification done, figures drawn for proof. please find the addfile, file name - roundoff2b.m

2c) $\sqrt{\frac{1+\cos(x)}{2}}$

If "$x \geq \frac{2\pi}{3}$" above formula leads to large round-off error.

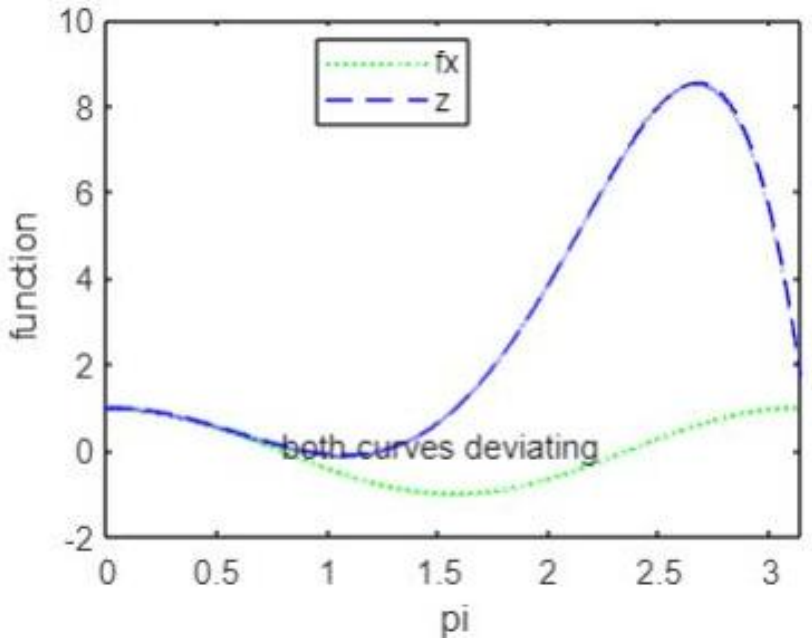


2a)

```
%% Question2.a
% The formula rearranged,Matlab verification for
x=10.^9;
sqrt(x+1)-sqrt(x);
% ans= 1.5811e-05;
1./(sqrt(x+1)+sqrt(x));
% ans=1.5811e-05;
```

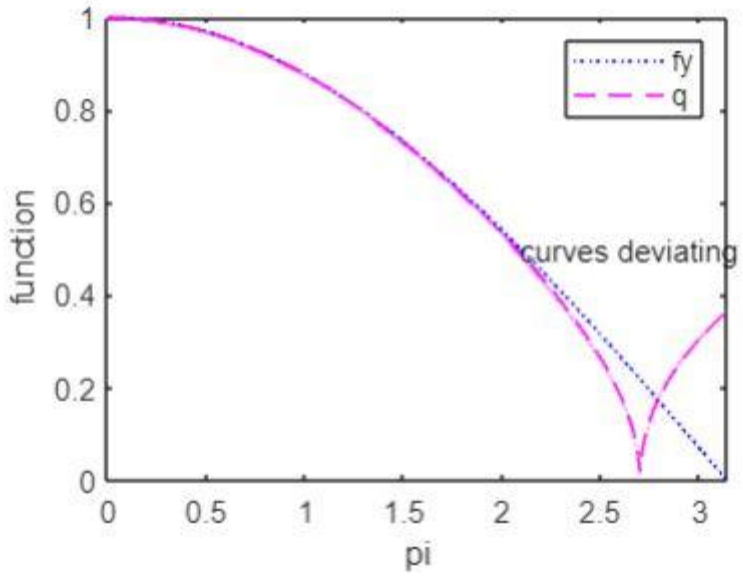
2b)

```
% f(x)=f(a)+(x-a)/1!f^1(a)+(x-a)^2/2!f^2(a)+…+(x-a)^n/n!f^n(a)+....
%Elementary Functions such as cos(x), sin(x) is calculated using polynomial
%approximation
x=(0:0.01:pi);
fx=(cos(x).^2-sin(x).^2);
z=(-0.08*x.^6+0.999*x.^4-2*x.^2+1);
plot(x,fx,':go','MarkerSize',0.1)
hold on
plot(x,z,'--b*','MarkerSize',0.2);
legend('fx','z');
xlabel('pi');
ylabel('function');
text(0.7854,0.1276,'both curves deviating');
%% therefore x≥pi/4, formula leads large round of errors
% The formula rearranged,Matlab verification for (x=) k=1.5785
k=1.5785
cos(k).^2-sin(k).^2;
%ans = -0.9999
cos(2*k)
%ans = -0.9999
```

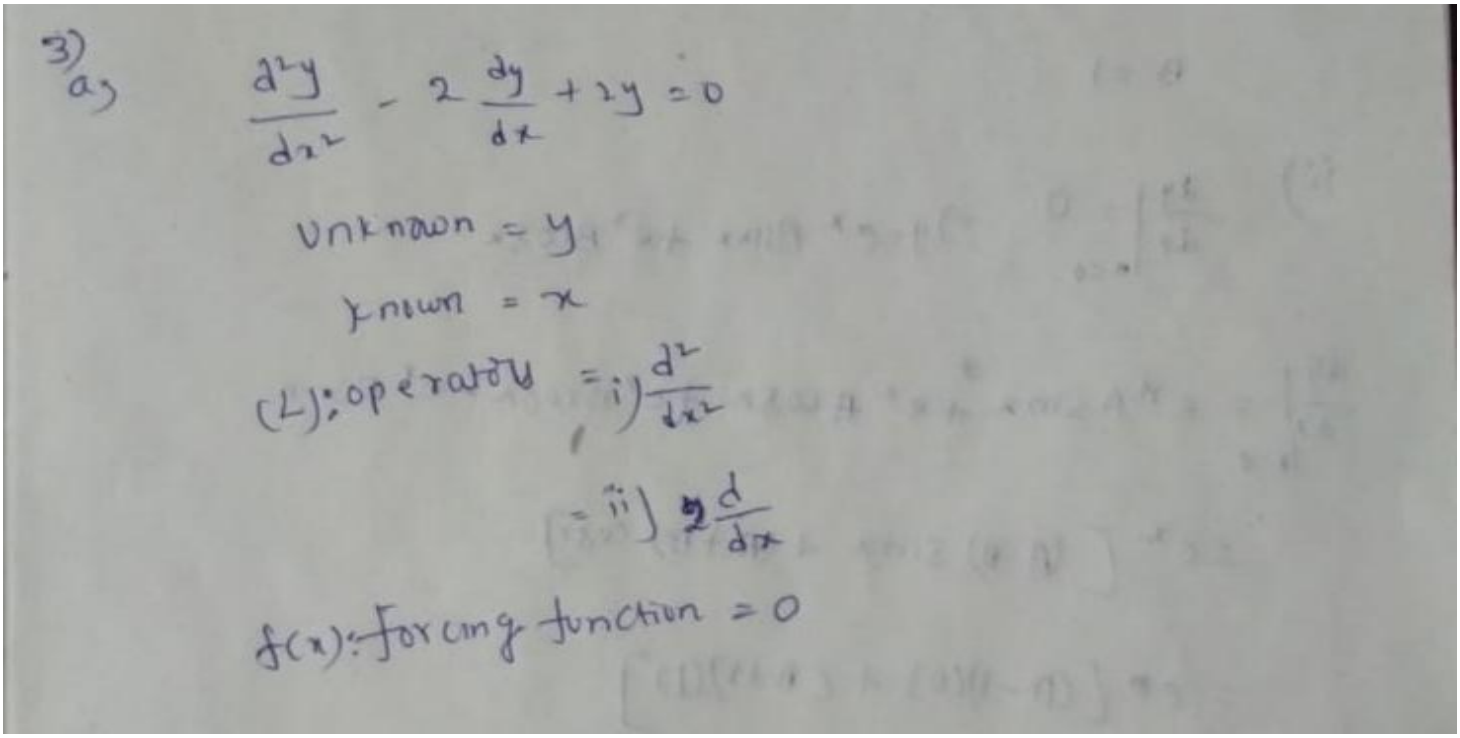


2c)

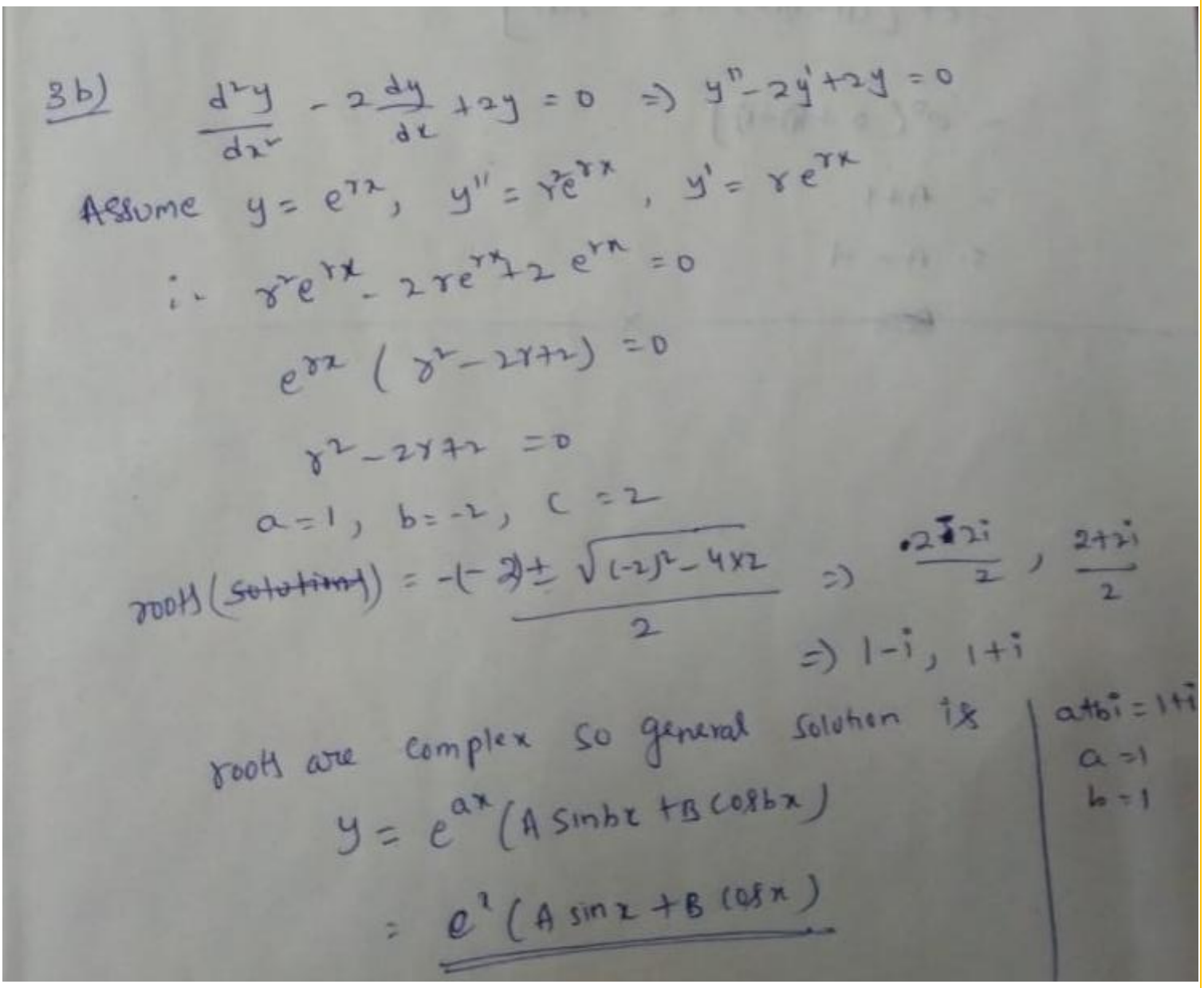
```
%% question 2.c
x=(0:0.01:pi);
fy= sqrt((cos(x)+1)./2);
q=sqrt((-1.38*10.^-3*x.^6+0.041*x.^4-0.5*x.^2+2)./2);
plot(x,fy,':bo','MarkerSize',0.1);
hold on
plot(x,abs(q),'--m*','MarkerSize',0.2);
legend('fy','q');
xlabel('pi');
ylabel('function');
text(2.0944,0.496,'curves deviating')
%%therefore x≥2pi/3, formula leads large round of errors
% The formula rearranged,Matlab verification for (x=)or p=2.1256
p=2.1256;
sqrt((cos(p)+1)./2);
%ans = 0.4864
abs(cos(p/2))
%ans = 0.4864
```



3a)



3b)



3b) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0 \Rightarrow y'' - 2y' + 2y = 0$

Assume $y = e^{rx}$, $y'' = r^2e^{rx}$, $y' = re^{rx}$

$\therefore r^2e^{rx} - 2re^{rx} + 2e^{rx} = 0$

$e^{rx}(r^2 - 2r + 2) = 0$

$r^2 - 2r + 2 = 0$

$a = 1, \quad b = -2, \quad c = 2$

roots (~~solutions~~) $= \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 2}}{2} \Rightarrow \frac{2 \pm 2i}{2}, \frac{2+2i}{2}$

$\Rightarrow 1 - i, \quad 1 + i$

roots are complex so general solution is

$y = e^{ax}(A \sin bx + B \cos bx)$

$= e^{x}(A \sin x + B \cos x)$

$a \pm bi = 1 + i$

$a = 1$

$b = 1$

A and B are arbitrary constants

i) $y(0) = 1$

$y = e^0 (A \sin(0) + B \cos(0))$

$1 = e^0 (0 + B)$

$B = 1$

ii) $\left.\frac{dy}{dx}\right|_{x=0} = 0 \Rightarrow y = e^x A \sin x + e^x B \cos x$

$\left.\frac{dy}{dx}\right|_{x=0} = e^x A \sin x + e^x A \cos x + e^x B \cos x - e^x B \sin x$

$= e^x [(A-B) \sin x + (A+B) \cos x]$

$= e^0 [(A-1)(0) + (A+1)(1)]$

$= e^0 (0 + (A+1))$

$= A+1$

$\therefore A = -1$

END