1. Negate the following formulae

a) $p \vee q \wedge \neg r$

$\cancel{\neg p \wedge \neg q}$

$(p \vee q) \wedge \neg r$

Negation of $p \vee q$ is $\neg p \wedge \neg q$

$(\neg p \wedge \neg q) \wedge \neg r$

Negation of A and B is $\neg A \vee \neg B$

$\Rightarrow \neg(\neg p \wedge \neg q) \wedge \neg(\neg r)$

$= p \vee q \wedge r$

(b) $(p \rightarrow \neg q) \wedge \neg p$

Negation of $p \rightarrow \neg q = p \wedge \neg(\neg q)$

$= p \wedge q$

$(p \wedge q) \wedge \neg p$

Negation of $A \wedge B = \neg A \vee \neg B$

$\Rightarrow \neg(p \wedge q) \vee \neg(\neg p)$

$= \neg p \vee \neg q \vee p$

$(p \vee \neg p) \vee \neg q$

2) For all $n > 0$, $5^n - 1$ is divisible by 4

1. For a basis, let $n = 1$. Then

$$5^1 - 1 = 4,$$

and clearly $4 \mid 4$.

2. Assume that $5^n - 1$ is is divisible by 4 for $n = k$, $k \in \mathbb{N}$, Then by this assumption,

$$4 \mid (5^k - 1) \Rightarrow 5^k - 1 = 4m, \; m \in \mathbb{Z}$$

(This notationally means than $5^k - 1$ is an integer multiple of 4.)

3. Let $n = k + 1$. Then

$$\begin{aligned} 5^{k+1} - 1 &= 5^k \cdot 5 - 1 \\ &= 5^k (4 + 1) - 1 \\ &= 4 \cdot 5^k + 5^k - 1 \\ &= 4 \cdot 5^k + 4m \\ &= 4(5^k + m). \end{aligned}$$

Since $4 \mid 4(5^k + m)$, we may conclude, by the axiom of induction, that the property holds for all $n \in \mathbb{N}$.

3.

Five character password

1^{st} ~~character is~~

For 1 digit

number of passwords are $26^4 \times 10$

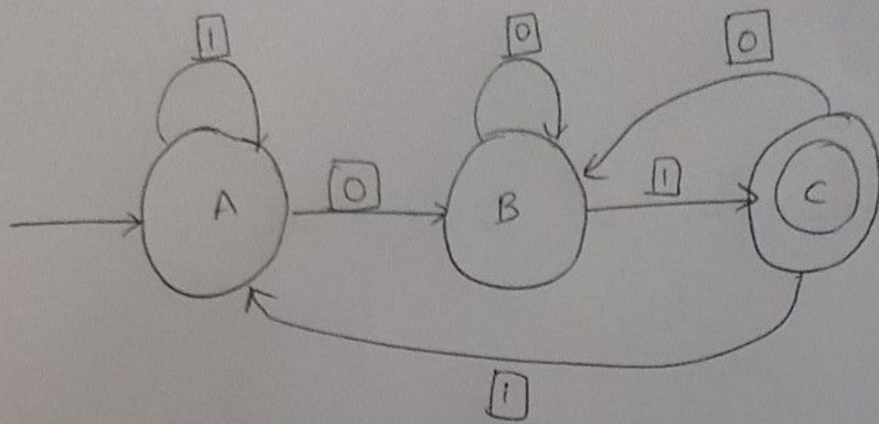
with 2 digits

number of passwords are $3 \times 26^3 \times 100$

Therefore the total number of passwords

with atmost 2 digits are $26^3 \times 560$

(4)



a) Example of a string accepted by this automaton

11001

(b) Example of a string rejected by this automaton

11100

(c) <u>Language</u>: the set of all strings accepted by an automaton is called the language of the automaton.

If M is an automaton on alphabet Σ, then $L(M)$ is the language of M:

$$L(M) = \{ x \in \Sigma^* \mid M \text{ accept } x \}$$

Accepting stage is c

Any string ending with 01 outputs the accepting stage.

Eg: 1001

11000101

10011101

5

(a) Language of $(0^* 1 0^+)^+$

= (010, 0010, 00010, 000010, ...)

= Example of a string in the language

is 000010

(b) Example of a string not in the language

→ ε

→ 001010

(c) Regular Expression that accepts the language of all binary strings with exactly one occurrence of aa.

$$(b^* a b^+)^* \, aa \, (b^+ a b^*)^*$$