

2) For all  $n > 0$ ,  $5^n - 1$  is divisible by 4

1. For a basis, let  $n = 1$ . Then

$$5^1 - 1 = 4,$$

and clearly  $4 \mid 4$ .

2. Assume that  $5^n - 1$  is divisible by 4

for  $n = k$ ,  $k \in \mathbb{N}$ , Then by this assumption,

$$4 \mid (5^k - 1) \Rightarrow 5^k - 1 = 4m, m \in \mathbb{Z}$$

(This notationally means that  $5^k - 1$  is an integer multiple of 4.)

3. Let  $n = k + 1$ . Then

$$5^{k+1} - 1 = 5^k \cdot 5 - 1$$

$$= 5^k (4 + 1) - 1$$

$$= 4 \cdot 5^k + 5^k - 1$$

$$= 4 \cdot 5^k + 4m$$

$$= 4(5^k + m).$$

Since  $4 \mid 4(5^k + m)$ , we may conclude, by the axiom of induction, that the property holds for all  $n \in \mathbb{N}$ .