MatRANS: A Matlab-based RANS + k-ω model for turbulent boundary layer and sediment transport simulations

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1 Introduction

This note describes a Matlab-based implementation solving the horizontal component of the incompressible Navier-Stokes equations, combined with the two-equation k-ω turbulence closure model of Wilcox (2006, 2008). The hydrodynamic model is additionally coupled with a gradient diffusion model for suspended sediment calculations, subject to oscillatory and/or steady forcing. Bed load sediment transport can also be calcuated using various bed load formulae. The model is intended to be applicable for simple steady current and oscillatory wave boundary layer flows, including combined wave-current simulations. Its development began during the MSc project of Schløer & Sterner (2009), under the supervision of the present author, who has since made numerous additional modifications to the code.

By utilizing the familiar Matlab environment, it is hoped that the code makes state-of-the-art boundary layer simulations easily accessible for students studying turbulence and/or sediment transport processes, and that it might be useful for both educational and research purposes.

2 Model Description

2.1 Hydrodynamic and turbulence model

The model solves simplified versions of the horizontal component of the incompressible Reynolds-averaged Navier-Stokes (RANS) equations, combined with the two-equation k-ω turbulence closure model of Wilcox (2006). The considered RANS equation reads:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} + \frac{\partial}{\partial y}\left(\nu_T \frac{\partial u}{\partial y}\right) \underline{- \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) - \frac{2}{3}\frac{\partial k}{\partial x}}. \tag{1}$$

The turbulence model consists of two respective transport equations for the turbulent kinetic energy (per unit mass) $k = 1/2(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$, where the prime superscripted variables represent turbulent velocity fluctuations, as well as the specific dissipation rate ω:

$$\begin{aligned} \frac{\partial k}{\partial t} &= \nu_T \left(\frac{\partial u}{\partial y}\frac{\partial u}{\partial y}\right) - \beta^* k\omega + \frac{\partial}{\partial y}\left[\left(\nu + \sigma^* \frac{k}{\omega}\right)\frac{\partial k}{\partial y}\right] \\ &\quad \underline{-\left(u\frac{\partial k}{\partial x} + v\frac{\partial k}{\partial y}\right)} - \underline{\underline{\frac{\nu_T}{\sigma_p}N^2}}, \end{aligned} \tag{2}$$

$$\begin{aligned} \frac{\partial \omega}{\partial t} &= \alpha\frac{\omega}{k}\nu_T \left(\frac{\partial u}{\partial y}\frac{\partial u}{\partial y}\right) - \beta\omega^2 + \frac{\sigma_d}{\omega}\frac{\partial k}{\partial y}\frac{\partial \omega}{\partial y} + \frac{\partial}{\partial y}\left[\left(\nu + \sigma \frac{k}{\omega}\right)\frac{\partial \omega}{\partial y}\right] \\ &\quad \underline{-\left(u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y}\right)} - \underline{\underline{c_{e\epsilon}N^2}}. \end{aligned} \tag{3}$$

The eddy viscosity is defined by

$$\nu_T = \frac{k}{\tilde{\omega}}, \qquad \tilde{\omega} = \max\left\{\omega, C_{lim}\frac{|\partial u/\partial y|}{\sqrt{\beta^*}}\right\}, \tag{4}$$

where $C_{lim} = 7/8$. In (3)

$$\sigma_d = \mathcal{H}\left\{\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}\right\}\sigma_{do}, \tag{5}$$

where $\mathcal{H}\{\cdot\}$ is the Heaviside step function, taking a value of zero when the argument is negative, and a value of unity otherwise. In the right hand side of (2) the first term represents the **production** of turbulent kinetic energy (the rate at which kinetic energy is tranferred from the mean flow to the turbulence), the second term represents **dissipation** (the rate at which turbulent kinetic energy is converted into therman internal energy), the third (single underlined) term includes both molecular and turbulent **diffusion**, and the fourth term includes both horizontal and vertical **convection**. The final (double underlined) term additionally incorporates turbulence suppression effects. This term is not part of the standard Wilcox (2006) model, and is discussed in more detail in §2.3. The default model closure coefficients suggested by Wilcox (2006) are: $\alpha = 13/25$, $\beta = \beta_0 f_\beta$, $\beta_0 = 0.0708$, $\beta^* = 9/100$, $\sigma = 1/2$, $\sigma^* = 3/5$, $\sigma_{do} = 1/8$. Note that for two-dimensional flows $f_\beta = 1$.

The most important differences between this version of the k-ω model and earlier versions created by Wilcox et al. are the addition of a "cross-diffusion" term (the term proportional to σ_d in (3)) and a built-in "stress-limiter" modification (the quantity proportional to C_{lim} in (4)). The cross-diffusion is added to reduce the model's sensitivity to the free stream value of ω, whereas the effect of the stress-limiter is most important for compressible flows (Wilcox, 2006). Fuhrman *et al.* (2010) have found that the stress-limiter can create undesirably large Reynolds number dependence for steady boundary layers under hydraulically rough conditions, when used in conjunction with a $k = 0$ wall boundary condition. Hence, in these conditions it can be switched off by simply setting $C_{lim} = 0$, whereby the eddy viscosity reduces to $\nu_T = k/\omega$.

2.2 Second-order terms

In (1)–(3) (and (11), below) the single-underlined terms are of so-called second-order importance. For wave boundary layers the importance of these terms (relative to leading-order terms) can be shown to scale with $a\kappa = U_{1m}/c$, where $a = U_{1m}/\omega_w$ is the amplitude of free stream orbital motion (for a sinusoidal velocity variation of magnitude U_{1m} with angular frequency ω_w) and $\kappa = 2\pi/L$ is the wave number (L being the wave length), with $c = \omega_w/\kappa$ the wave celerity. These terms are responsible *e.g.* for the phenomenon of boundary layer streaming within oscillatory flows under progressive waves. Alternatively, for flow in an oscillatory u-tube (having uniformity in the x-direction) $\kappa = 0$ (that is $L = \infty$) and the second-order terms drop out entirely. These terms may be omitted within the model simply by setting the `streaming` flag to zero. Additional third-order terms are neglected in the model, and are not described here.

Assuming constant form wave propagation, all x-derivatives appearing in the second-order terms are actually replaced by time derivatives in the model via the relation

$$\frac{\partial}{\partial x} = -\frac{1}{c}\frac{\partial}{\partial t}. \tag{6}$$

This is convenient, as it allows potential effects from x-variations (*e.g.* boundary layer streaming) to be incorporated, while only requiring discretization in the vertical y-direction.

The vertical velocity v is approximated by solving the local continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \approx -\frac{1}{c}\frac{\partial u^{(1)}}{\partial t} + \frac{\partial v}{\partial y} = 0, \tag{7}$$

where $\partial u^{(1)}/\partial t$ is the leading-order (*i.e.* non-underlined) part of $\partial u/\partial t$ from (1). This is justifiable because all terms containing v are already of second-order importance, hence the remaining errors are at third-order or higher.

2.3 Turbulence supression terms

The final double-underlined terms in (2) and (3) describe turbulence suppression, due to density gradients in the fluid-sediment mixture. These terms are implemented analogously to recent k-ϵ modelling undertaken by Ruessink *et al.* (2009), where

$$N = \sqrt{-\frac{g}{\rho_m}\frac{\partial \rho_m}{\partial y}} \tag{8}$$

is the so-called Brunt-Vaisala frequency, $\rho_m = s\rho c + \rho(1-c)$ is the density of the fluid-sediment mixture, and s is the relative density of the sediment. The default closure coefficients are $\sigma_p = 0.7$ with $c_{3\epsilon} = 1$ for $N^2 <= 0$ and $c_{3\epsilon} = 0$ for $N^2 > 0$. These terms can be important for flows giving rise to high suspended sediment concentrations near the bed. These may be omitted within the model simply by setting the `turbsup` flag equal to zero.

2.4 Boundary conditions

The above equations are (typically) solved starting from motionless initial conditions in a single vertical dimension, subject to the following boundary conditions. The bottom boundary is considered a friction wall, and a no-slip boundary condition is imposed, *i.e.* all velocity variables are set to zero. The bottom boundary condition for ω is adopted from Wilcox (2006), where

$$\omega = \frac{U_f^2}{\nu} S_R, \qquad y = 0. \tag{9}$$

The factor S_R is based on the roughness Reynolds number $k_N^+ = k_N U_f/\nu$, where k_N is Nikuradse's equivalent sand grain roughness, and $U_f = \sqrt{|\tau_b|/\rho}$ is the instantaneous friction velocity, according to

$$S_R = \begin{cases} \left(\frac{200}{k_N^+}\right)^2, & k_N^+ \leq 5, \\ \frac{K_r}{k_N^+} + \left[\left(\frac{200}{k_N^+}\right)^2 - \frac{K_r}{k_N^+}\right] e^{5-k_N^+}, & k_N^+ > 5, \end{cases} \tag{10}$$

where Wilcox (2006) suggests using $K_r = 100$. A frictionless rigid lid is imposed at the top boundary, whereby vertical derivatives of all variables (u, k, ω, and C) are set to zero. Note that better agreement with standard law of the wall solutions for steady boundary layers under hydraulically rough conditions (using a $k = 0$ wall boundary condition; see below) has been found by modifying the rough wall coefficient in (10) slightly to $K_r = 80$, as shown by Fuhrman *et al.* (2010).

Both $k = 0$ and $dk/dy = 0$ bottom wall boundary conditions are implemented within the model. The latter condition, while not conventionally used, is supported by experimental measurements on rough beds (Sumer *et al.*, 2003; Fuhrman *et al.*, 2010). Fuhrman *et al.* (2010) have also recently argued that the zero gradient condition is more generally consistent with near-wall physics than a $k = 0$ condition, as it allows a viscous sublayer to develop near smooth walls, while conveniently avoiding a forced viscous sublayer near rough walls. This can yield significant numerical advantages, especially for large roughness simulations. Hence, the zero-gradient condition is preferable is most cases. If this conditions is utilized, then the closure coefficient $K_r = 180$ is recommended (Fuhrman *et al.*, 2010).

2.5 Sediment transport model

2.5.1 Bed load models

Bed load sediment transport is calculated from the bed shear stress obtained from the hydrodynamics model. As options, the widely used Meyer-Peter &

Müller (1948) bedload formula is included, as are formulae from Engelund & Fredsøe (1976), and Nielsen (1992). Other bedload formulas of interest may be easily added, if desired.

The model also incorporates potential effects of a longitudinal bed slope S on the critical Shields parameter, as described in Fredsøe & Deigaard (1992), see their p. 205. The specified slope is that of the bottom, hence if it is positive it will increase the critical Shields parameter (*i.e.* make sediment more difficult to transport) when the flow is positive (uphill), while decreasing the critical parameter when the flow is negative (downhill). The adjustment is made dynamically, based on the sign of the instantaneous bed shear stress.

2.5.2 Suspended sediment model

The above hydrodynamic model is also coupled with a turbulent-diffusion equation for the possible simulation of the suspended sediment concentration C (see *e.g.* Fredsøe & Deigaard, 1992, p. 238):

$$\frac{\partial C}{\partial t} = \frac{\partial (w_s C)}{\partial y} + \frac{\partial}{\partial y}\left(\epsilon_s \frac{\partial C}{\partial y}\right) - \underline{\left(u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y}\right)}, \tag{11}$$

where w_s is the settling velocity, and $\epsilon_s = \beta_s \nu_T + \nu$ is the diffusion coefficient, where commonly $\beta_s = 1$ is used, though sometimes values corresponding up to $\beta_s = 2$ (*e.g.* Ruessink *et al.*, 2009) are utilized with success. The evaluation of the various terms in this equation is the same as described above.

Regarding the bottom boundary condition for the suspended sediment concentration C, a number of options are provided in the model, which will not be described in detail here. The implemented options include the reference concentration methods of Engelund & Fredsøe (1976), Zyserman & Fredsøe (1994), Einstein (2006), and O'Donoghue & Wright (2004), as well as the pickup function of van Rijn (1984). The option to prevent un-physical overloading conditions (*i.e.* where the reference concentration $C_b = C(y = b)$ is forced to be smaller than the concentration immediately above) is also included, and is controlled by the `iextrap` flag. If this flag is set to unity, then the reference concentration is taken as the maximum of that computed using the selected formula and that extrapolated from the two grid points nearest the bed. This option *e.g.* avoids directly forcing the reference concentration $C_b = 0$ during times of flow reversal in oscillatory flows.

Hindered settling effects may optionally be included within the model (using the `Hind_Set` flag). If this flag is set to unity, then the fall velocity w_s depends on the local suspended sediment concentration according to

$$w_s = w_{s0}(1 - C)^n. \tag{12}$$

The exponent n may either be input directly, or computed according to the expressions of Richardson & Zaki (1954); see Fredsøe & Deigaard (1992), p. 200. Likewise the base settling velocity w_{s0} may be either input directly, or calculated automatically for a given grain diameter d according to (7.9), (7.10), and (7.12) of Fredsøe & Deigaard (1992).

Suspended sediment transport is calculated for each time as

$$q_S = \int_b^{h_m} ucdy, \tag{13}$$

where $y = b$ is the reference level, and h_m is the total height of the modelled domain. Typically $b = 2d$ is utilized.

2.6 Pressure gradient

A prescribed pressure gradient in (1) is used to drive the flow within the model domain. To obtain a desired free stream velocity signal U_0, this can be implemented generally as

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = \left(\underline{\frac{U_0}{c}} - 1\right)\frac{\partial U_0}{\partial t} + P_x - \underline{\underline{\underline{SU_0^2/h}}}, \tag{14}$$

where the underlined term is again of secondary importance *i.e.* it is $O(a\kappa)$. Note that a constant pressure gradient P_x is also added, which can be used *e.g.* to drive steady currents or within combined wave-current simulations. For pure wave simulations simply set $P_x = 0$. Within the model a free stream velocity having the form of a second-order Stokes signal is utilized, defined according to

$$U_0 = U_{1m}\sin(\omega_w t') - U_{2m}\cos(2\omega_w t') \tag{15}$$

$$\frac{\partial U_0}{\partial t} = U_{1m}\omega_w\cos(\omega_w t') + 2U_{2m}\omega_w\sin(2\omega_w t') \tag{16}$$

where $t' = t + t_0$, where t_0 represents a time shift in the signal, which is automatically determined to ensure that $U_0(t = 0) = 0$. Note that a sinusoidal free stream velocity signal is achieved by setting $U_{2m} = 0$, whereby $t_0 = 0$. The more flexible wave form shape proposed recently by Abreu *et al.* (2010) is also implemented in the model, though this will not be described in detail here. This option allows forcing velocity signals ranging from highly skewed to highly front-back asymmetric (*e.g.* as commonly found in the surf zone).

For steady current simulations the oscillatory part of the pressure gradient can be switched off by setting $U_{1m} = U_{2m} = 0$. The constant pressure gradient term will then be $-P_x = U_f^2/h_m$ once steady-state conditions are achieved, where h_m is the height of the model domain, hence it can be prescribed based on a desired friction velocity.

The final triple-underlined term in (14) represents a modification of the pressure gradient due to potential converging-diverging effects induced *e.g.* by a sloping bed, where S is the bed slope, and h is the flow depth (or in the case of a converging-diverging tunnel, the tunnel half depth). For convenience, this is left as a free input parameter, since it may be undesirable to resolve a given flow depth in its entirety (*i.e.* h is not necessarily taken as equal to h_m). Such converging-diverging effects can be important within cross-shore boundary layer and sediment transport dynamics, as discussed by Sumer *et al.* (1993) and Fuhrman *et al.* (2009*a*,*b*). This term may be switched off by setting the `iconv` flag to zero.

2.7 Filtering

Generally, the model is quite robust and stabil, and does not usually require any filtering for stability purposes. However, experience has shown that the inclusion of the turbulence supression terms can de-stabilize the k-equation, sometimes leading to the development of significant high-wavenumber noise in the vertical profiles. For this reason, optional filtering has been added to the evaluation of time derivative for each of the governing equations (1), (2), (3), and (11). These, respectively, utilize three-point filter stencils of the form:

$$[\alpha_f \qquad 1 - 2\alpha_f \qquad \alpha_f]. \tag{17}$$

Setting $\alpha_f = 0$ for a given variable leads to the stencil [0 1 0], and hence shuts the filter off. Alternatively, setting $\alpha_f = 0.25$ eliminates Nyquist frequency modes (see *e.g.* Abbott & Minns, 1998, p. 228). Hence, when turbulence suppression is not included it is suggested to turn off filtering completely, *i.e.* set all filter coefficients to zero. Alternatively, when turbulence suppression is included, it is suggested to set the filter coefficient for dk/dt to 0.25 and that on du/dt to 0.05, which has been found to significantly reduce noise levels in the model.

2.8 Matlab files

The model described above is essentially contained within two Matlab M-files: a main execution script `main_MatRANS.m`, which calls the function `ddt_MatRANS.m`. A third script `GlobalVars.m` simply declares global variables. Input for individual simulations is intended to be set up within a separate `MatRANS.m` file, which should be fairly self explanatory. This script then calls the `main_MatRANS.m` script at the end, hence its location must be within the Matlab path. If it is not already, then this can be achieved from the command line using the `addpath` command. The implementation allows various terms to be easily incorporated or shut off, as described in some instances above. For example, the inclusion of the turbulence model itself is controlled by the `turb` flag in `MatRANS.m` *i.e.* if this flag is set to 0 the simulation will be laminar, whereas if it is 1 then the turbulence model is used. Similarly, the inclusion of the second-order $O(a\kappa)$ terms underlined above is controlled by the `streaming` flag (*i.e.* for flow in a u-tube, these should be switched off by setting `streaming`$= 0$). Finally, the inclusion of the suspended sediment calculation is controlled by the `susp` flag. All input variables are in SI units.

The evaluation of the time derivatives (1)–(3) and (11) for a given state is performed within the the function `ddt_MatRANS.m`, making use of finite difference approximations for the required vertical spatial derivatives. This function is utilized by Matlab's stiff ODE solver `ode15s`, which provides dynamic time step control, and is used for the time integration. The call to this internal function is made in the `main_MatRANS.m` script.

For a given set up, the model can be run by executing the `MatRANS.m` script from the Matlab command line. After a simulation is completed the important variables are exported to a file having the name specified by the `OutFileName` variable, which can then be easily loaded into Matlab. An example of how to load the data, plot time series, and make animations from the model results is provided in a separate script `PostProcess.m`.

3 Provided examples

A number of examples are provided as part of the model. These have been selected to include basic (laminar, smooth turbulent, and rough turbulent) steady current and oscillatory flows. These include the following cases, listed together with relevant references:

- Laminar current
- Laminar wave boundary layer
- Laminar wave boundary layer streaming (Longuet-Higgins, 1953)
- Smooth turbulent current flow (Fuhrman *et al.*, 2010)
- Rough turbulent current flow (Fuhrman *et al.*, 2010)
- Vanoni suspended sediment distribution in a current
- Smooth turbulent wave boundary layer (Jensen *et al.*, 1989)
- Rough turblent wave boundary layer flow (Jensen *et al.*, 1989)
- Rough turblent wave boundary layer streaming (Holmedal & Myrhaug, 2009)

The model is a work in progress, and additional test cases are presently under consideration!

The individual test cases can be simulated simply by executing the associated `MatRANS.m` file, again assuming that the basic model described above are in the Matlab path. Comparison against relevant theory or experiments can be obtained afterwards simply by executing the associated `Compare.m` script.

4 Conclusions

This note describes a Matlab implementation of a Reynolds-averaged Navier-Stokes model, combined with the k-ω turbulence closure of Wilcox (2006), which can be used for performing simple (wave plus current) boundary layer simulations. The hydrodynamic model is also coupled with a suspended sediment description, and includes several features believed to play an important role within cross-shore wave boundary layer and sediment transport dynamics. These include *e.g.* non-sinusoidal wave shape, boundary layer streaming, bed slope modifications to critical Shields parameter, converging-diverging effects, hindered settling of sediments, as well as turbulence suppression due to density gradients. These various features can easily be switched on or off, as desired. It is again hoped that, by utilizing the Matlab environment, the model will be easily accessible to students for demonstration of turbulence and sediment tranport processes, including possible research purposes.

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