BT 2020 — Numerical Methods for Biology
Jan–May 2019
Quiz 1

February 19, 2019

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Instructions: This examination is 'open notes.' You can only use your own hand-written notes. Answer all questions. **Keep your answers brief and to the point.**

There are a total of 3 pages in this question paper. Allotted time is 50 minutes.

Maximum marks: 40

1. (14 marks) **Justify** (as briefly as possible) whether each of the following statements is true or false. If the justification is incorrect, no credit will be awarded. **Answer sub-questions in the correct order.**

Write in pen!

(a) The matrix $A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ is not in echelon form

2

True

In a matrix that is in echlon form, the numbers in a colum under the first 1 should be zero

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \end{pmatrix} \checkmark \quad \rightarrow \neq 0$$

(b) There are some special floating-point values that do not have a unique representation False

0

All floating point numbers have unique representation

(c) The float `0xfffabcdefabcdefa` is normalised False

2

~~fff~~ abcdefabcdefa

1 | 111 1111 1111 |

s e

e = 111 1111 1111

⇒ The float is NaN. It is neither normalised nor denormalised

(d) For a linear system given by $Ax = b$, if our measurements of the values of b_i are 99.0% accurate, then our estimates of x will have a relative error of at most 1% False

2

$\frac{\|\Delta b\|}{\|b\|} = 0.01$ and $\frac{\|\Delta x\|}{\|x\|} \leq cond(A) \cdot \frac{\|\Delta b\|}{\|b\|}$

$\frac{\|\Delta x\|}{\|x\|} \leq cond(A) \cdot (0.01)$ but $cond(A) \geq 1$

$\Rightarrow \frac{\|\Delta x\|}{\|x\|}$ will atleast be 0.01

(e) The condition number of a matrix $\|A\|$ depends only on its norm, $\|A\|$ False

$cond(A) = \|A\| \cdot \|A^{-1}\|$

2

It depends on $\|A^{-1}\|$

(f) The L_0-norm of $[1 \quad 0 \quad 0 \quad -3 \quad 9 \quad -2 \quad -11]$ is 26.0 False

L_0 norm = number of non zero elements

2

$= 5$

and $5 \neq 26$

(g) The single-precision floats 0x00000000 and 0x80000000 represent the same number False

00000000 ⟶ 0

(True zero)

↓

When a value is numerically equal to 0

80000000 ⟶ −0

(signed zero/rounded off zero)

↓

When a value is rounded off to zero

2

2. (4 marks) What is the floating-point double (hex) representation of the number −0.7?

$-0.7 = -(0.5 + 0.2)$

4

In binary

$s = 1$

$e = 1022$

$f = 613$

$(0.5 + 0.2)$

↓

$0.10110011001 1 \ldots$

$1.0\overline{1100}$ $\quad (2)^{-1} = 1.\overline{0110}$

$-1 = x - 1023$

$\Rightarrow x = 1022$

double

$1022 = 1023 - 1$

$= 011 \; 1111 \; 1110$

$f = (0110)_{13}$

$= \underline{6_{13}}$

$\frac{51}{4}$

52

49

$-0.7 = 1|011 \; 1111 \; 1110| \; (0110)_{13}$

$= \underline{bfe \; 6_{13}}$

3. (4 marks) IEEE also has a 16-bit notation where $|e| = 5$. What is the value of the smallest positive normalised floating point number that can be represented in 16 bits?

5 bits : e

smallest normalized

Total = $2^5 - 2$

$1 \cdot (0)_{10} \ (2)^{e}$

$u = (2^5 - 2)/2$

smallest normalized = $1.0 \, (2)^{-14}$

$\Rightarrow u = 2^4 - 1 = 16 - 1 = 15$

1 − 15 and

(−14) − 0

— **Answer the remaining problems on a separate sheet** —

4. (6 marks) You are given vectors of observations v and S, corresponding to the initial velocity of the reaction and substrate concentration for an enzyme-catalysed reaction that follows Michaelis–Menten kinetics. Write a small (MATLAB) function to return the values of $v_{\max}$ and K_M, given v and S.

5. (4 marks) Find matrices P and Q such that $P \times \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \times Q = \begin{bmatrix} 6 & 4 & 5 \\ 9 & 7 & 8 \\ 3 & 1 & 2 \end{bmatrix}$

6. (8 marks) Perform a Cholesky decomposition of the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 5 & 0 & 0 \\ 0 & 0 & 16 & 4 \\ 0 & 0 & 4 & 65 \end{bmatrix}$$

*** END OF QUIZ 1 ***

$$\frac{1}{v} = \frac{(S) + k_m}{k_2 (E_0)(S)}$$

$$\frac{1}{v} = \frac{1}{c} + \frac{k_m}{k_2 (E_0)(S)}$$

v_{max}

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B

Roll No. ____________

Name : ____________ Total No. of Pages ____

Quiz I ☐ Quiz II/ Mid-Sem ☐ End-Semester ☐ Make-up ☐ Date : ____________

Semester & Degree : ____________ Course No. ____________ Part : ____________

Question No.	1	2	3	4	5	6	7	8	9	10
Marks	12	4	4	0	4	8				

11	12	13	14	15	16	17	18	19	20	Total
										82

Answer on both sides of the paper including the space below

6)
$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 5 & 0 & 0 \\ 0 & 0 & 16 & 4 \\ 0 & 0 & 4 & 65 \end{pmatrix} = \begin{pmatrix} a & 0 & 0 & 0 \\ b & c & 0 & 0 \\ d & e & f & 0 \\ g & h & i & j \end{pmatrix} \begin{pmatrix} a & b & d & g \\ 0 & c & e & h \\ 0 & 0 & f & i \\ 0 & 0 & 0 & j \end{pmatrix} = LL^T$$

$a^2 = 1 \Rightarrow \underline{\underline{a = 1}}$

$ab = 1 \Rightarrow \underline{\underline{b = 1}}$

$ad = 0 \Rightarrow \underline{\underline{d = 0}}$

$ag = 0 \Rightarrow \underline{\underline{g = 0}}$

$b^2 + c^2 = 5$, $c^2 = 4 \Rightarrow \underline{\underline{c = 2}}$

$db + ec = 0$, $0 + ec = 0$, $e(2) = 0 \Rightarrow \underline{\underline{e = 0}}$

$d^2 + e^2 + f^2 = 16$, $f^2 = 16 \Rightarrow \underline{\underline{f = 4}}$

$gb + hc = 0$, $0 + h(2) = 0 \Rightarrow \underline{\underline{h = 0}}$

$dg + he + if = 4$, $0 + 0 + i(4) = 4 \Rightarrow \underline{\underline{j =}}$ [illegible]

$g^2 + h^2 + i^2 + j^2 = 65$, $1 + j^2 = 65 \Rightarrow \underline{\underline{j = 8}}$

$$L = \underline{\underline{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & 8 \end{pmatrix}}}$$

Hence.

$$\underline{\underline{A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 5 & 0 & 0 \\ 0 & 0 & 16 & 4 \\ 0 & 0 & 4 & 65 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 1 & 8 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 8 \end{pmatrix}}}$$

8

5) $PAQ = A'$

(4) $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ $A' = \begin{pmatrix} 6 & 4 & 5 \\ 9 & 7 & 8 \\ 3 & 1 & 2 \end{pmatrix}$

↳ permutation of rows, columns of A

$A \longrightarrow A_1$ (by pre mul. with P_1)

where

$A_1 = \begin{pmatrix} 4 & 5 & 6 \\ 7 & 8 & 9 \\ 1 & 2 & 3 \end{pmatrix}$ $\Rightarrow$ $P_1 A = A_1$

permute rows

$P_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$A_1 \longrightarrow A_2$ (by post mul with Q_1)

where

$A_2 = \begin{pmatrix} 6 & 4 & 5 \\ 9 & 7 & 8 \\ 3 & 1 & 2 \end{pmatrix}$ $\Rightarrow Q_1 A_1 = A_2$

permute columns

$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

He $A_2 = A'$

Hence,

$P = P_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ and $Q = Q_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$$\Rightarrow \underline{\underline{\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}}}$$