Lecture 25: Graph Algorithms: Dijkstra

BT 3051 – Data Structures and Algorithms for Biology

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Dijkstra's algorithm

- How to find the shortest path from u to v in a graph?
- Different algorithms exist for
  - Negative weights (Bellman–Ford)
  - All pairs shortest paths (Floyd–Warshall – dynamic programming)
- BFS works when all weights are unity!

Dijkstra's algorithm

Algorithm Design

- Applying the greedy method to the single-source, shortest-path problem, results in an algorithm known as *Dijkstra's algorithm*
- Main idea: applying the greedy method to perform a weighted BFS starting at the source vertex s
- Greedy method: iteratively grows a *cloud* of vertices out of s, with the vertices entering the cloud in order of their distances from s
- In each iteration, the next vertex chosen is the vertex outside the cloud that is closest to s, and we perform an *edge relaxation*
- Algorithm terminates when no more vertices are outside the cloud*
- We then have a shortest path from s to every vertex of G that is reachable from s

Dijkstra's algorithm

Edge Relaxation

For all v, `dist[v]` is the length of some path (*best path so far*) from s to v

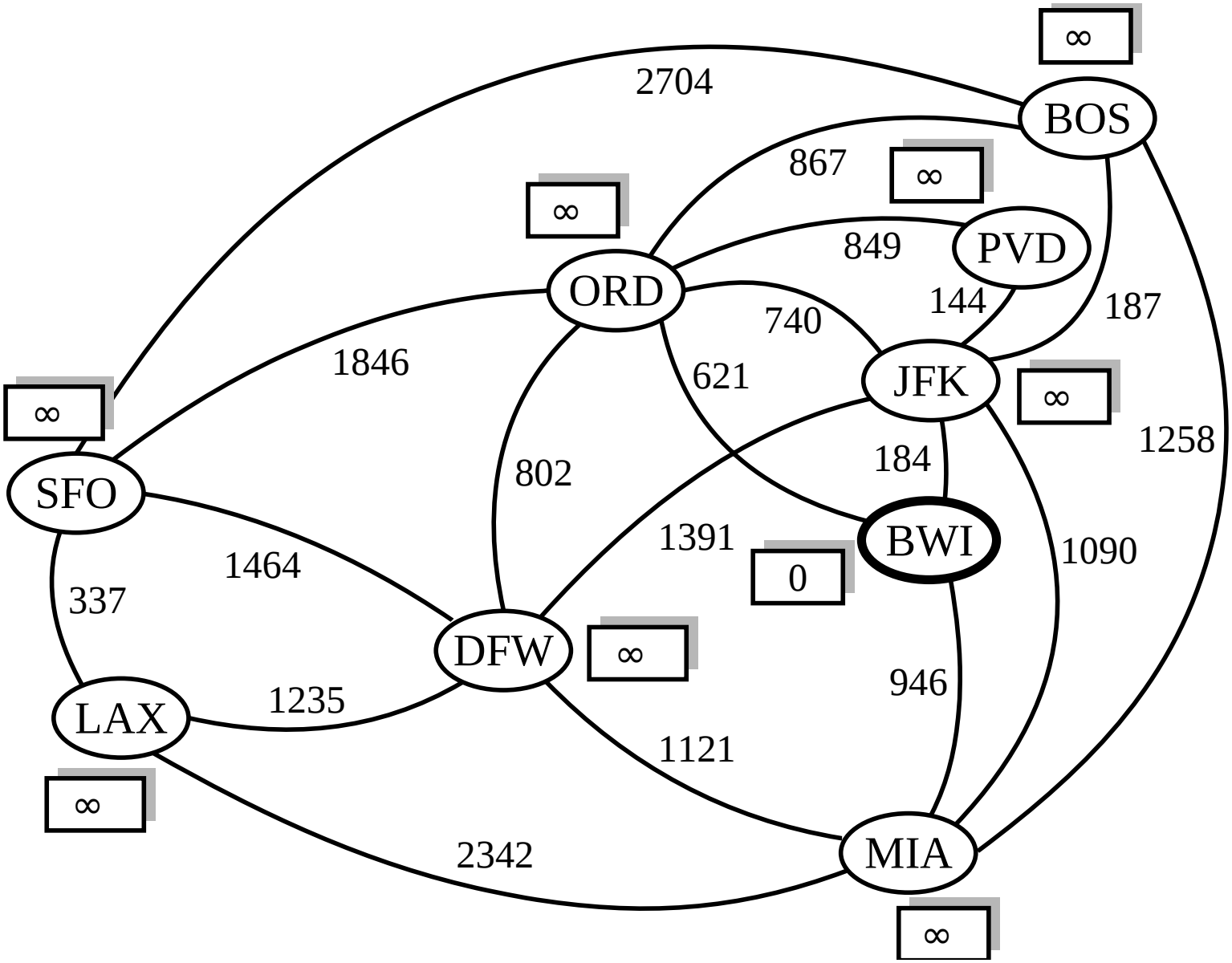
Relaxation along edge e from v to w

- `dist[v]` is length of some path from s to v
- `dist[w]` is length of some path from s to w
- if $v \rightarrow w$ gives a shorter path to w through v, update `dist[w]` and `pred[w]`

```
if dist[u] + edge_weight(u,v) < dist[v]:
    dist[v] = dist[u] + edge_weight(u,v)
```

Dijkstra's algorithm: Illustration

Goodrich *et al* (2013), Data Structures and Algorithms in Python



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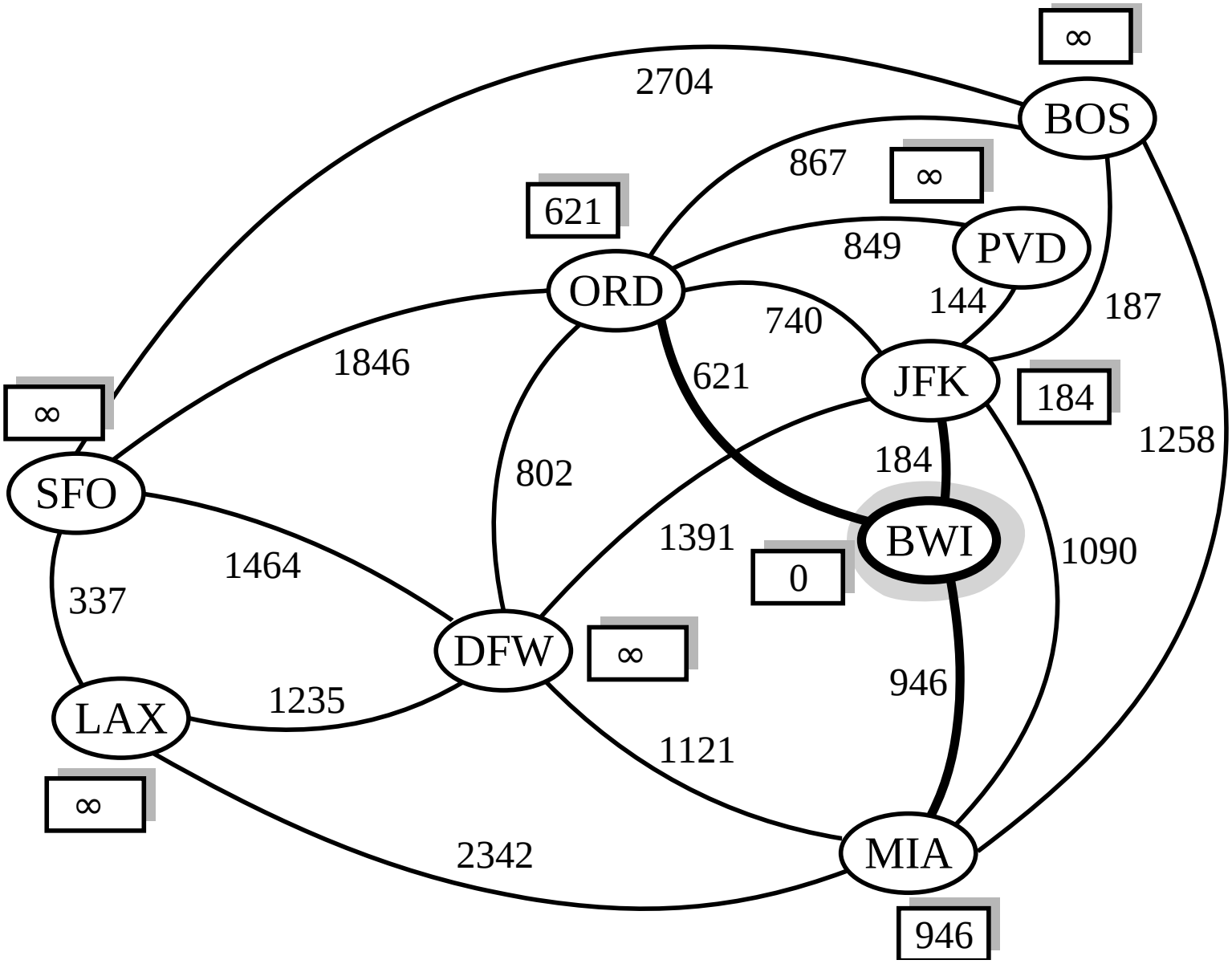

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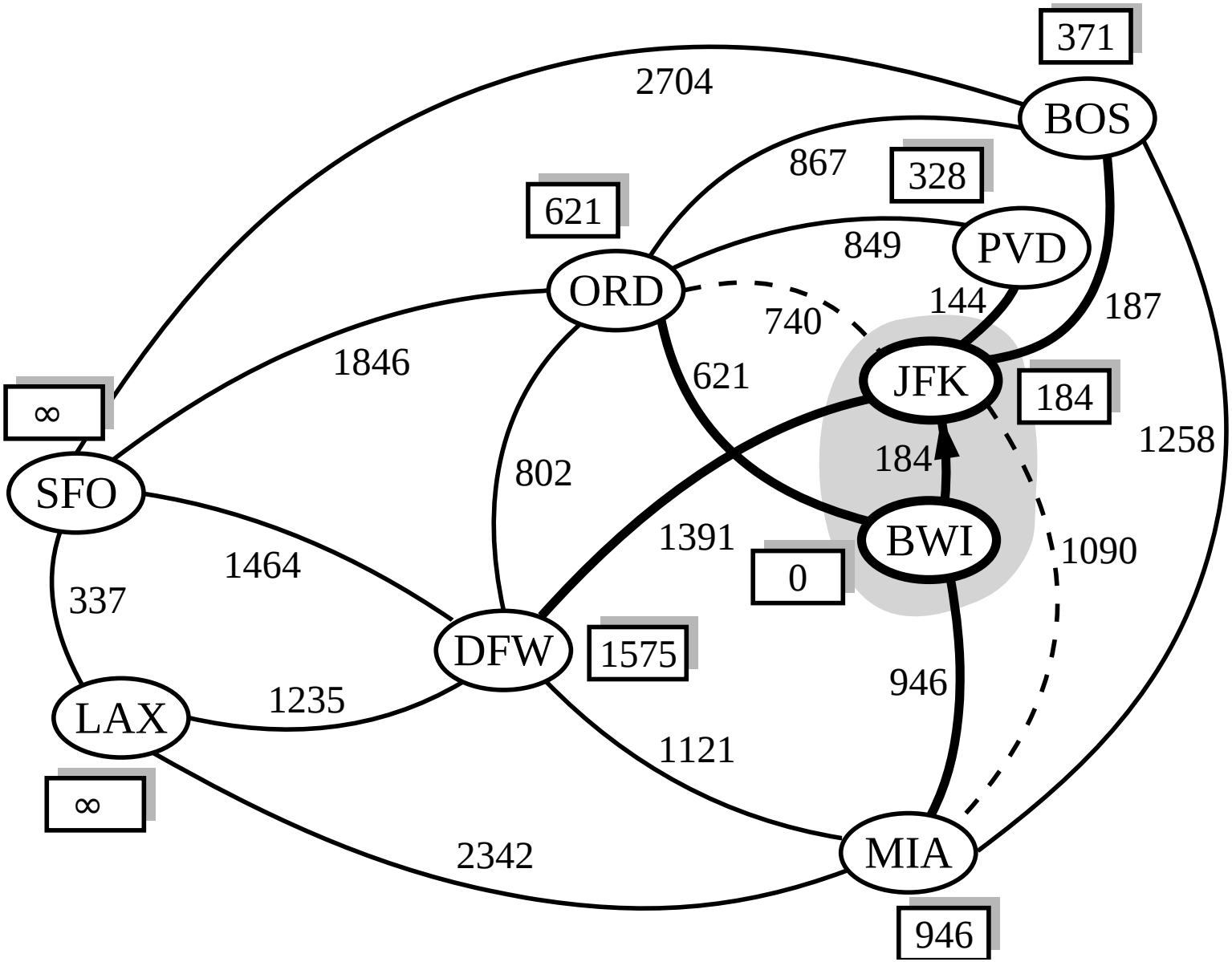

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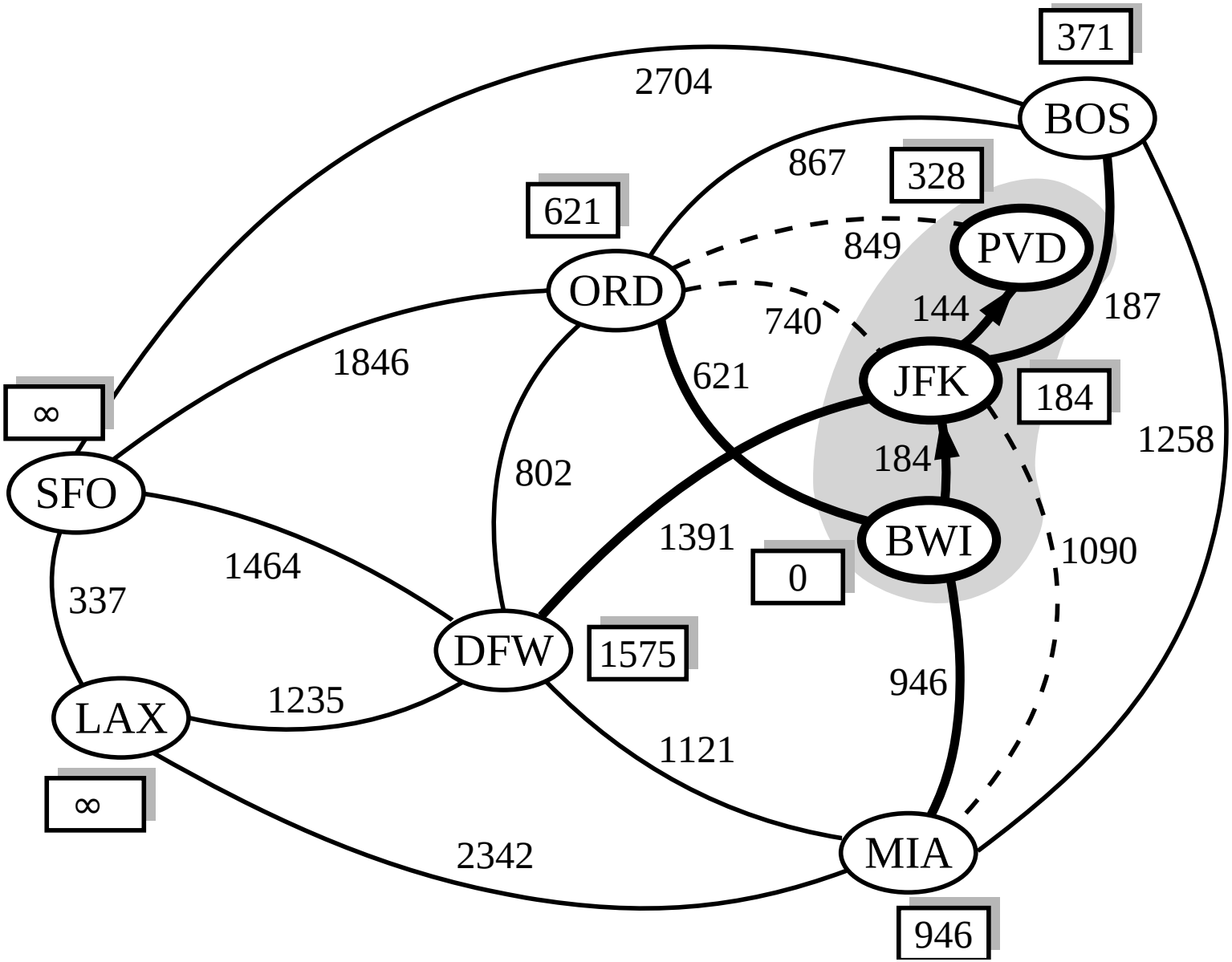

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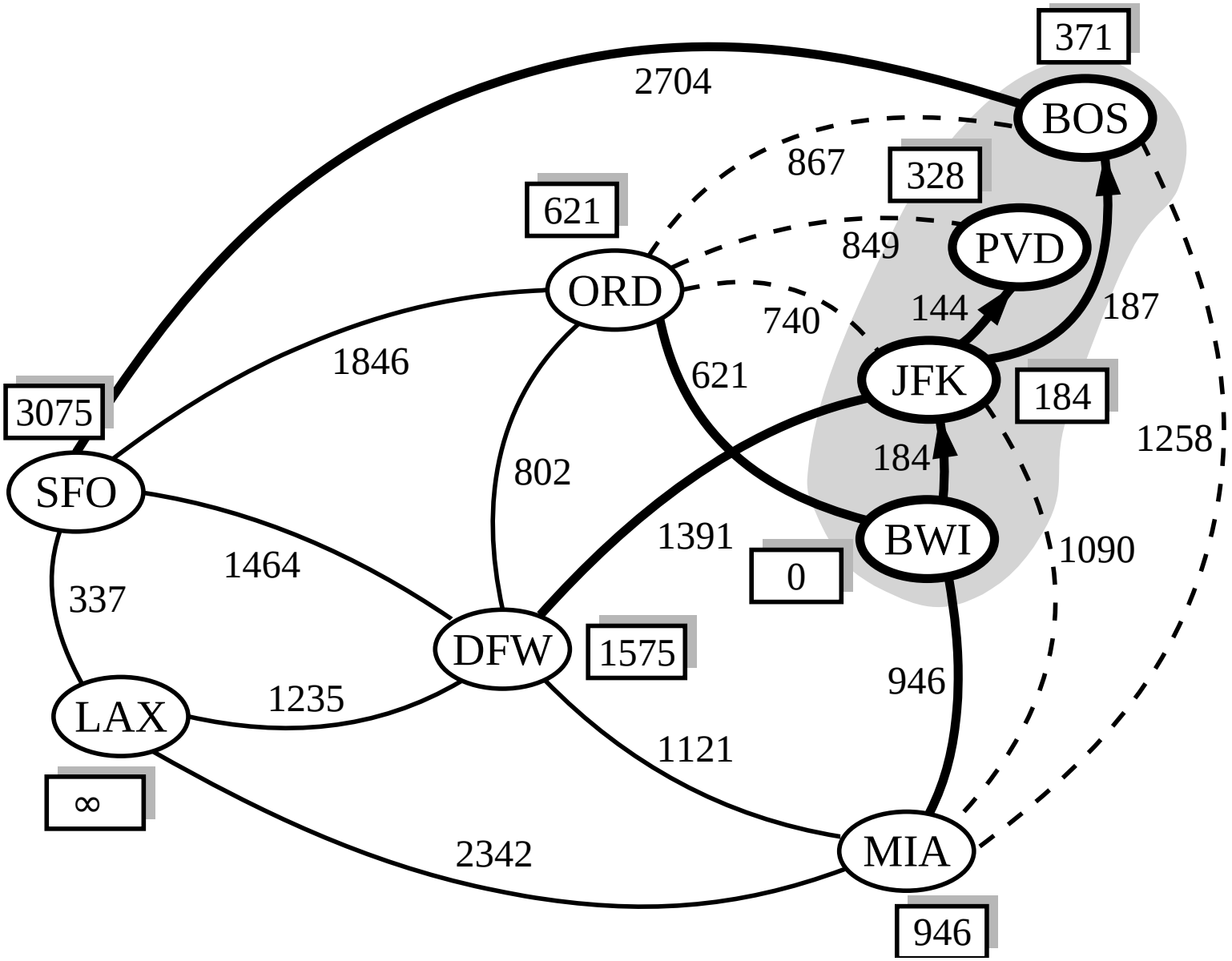

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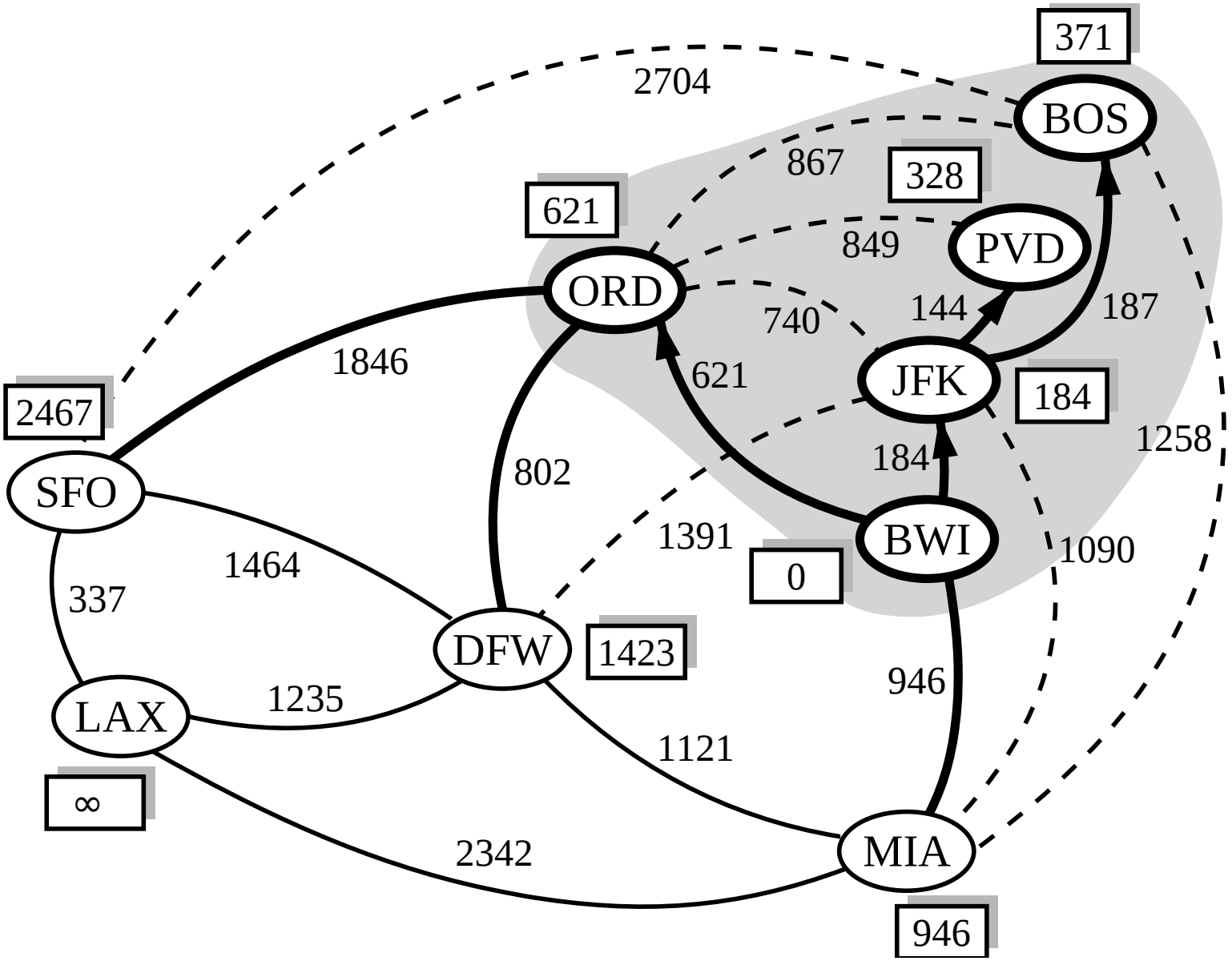

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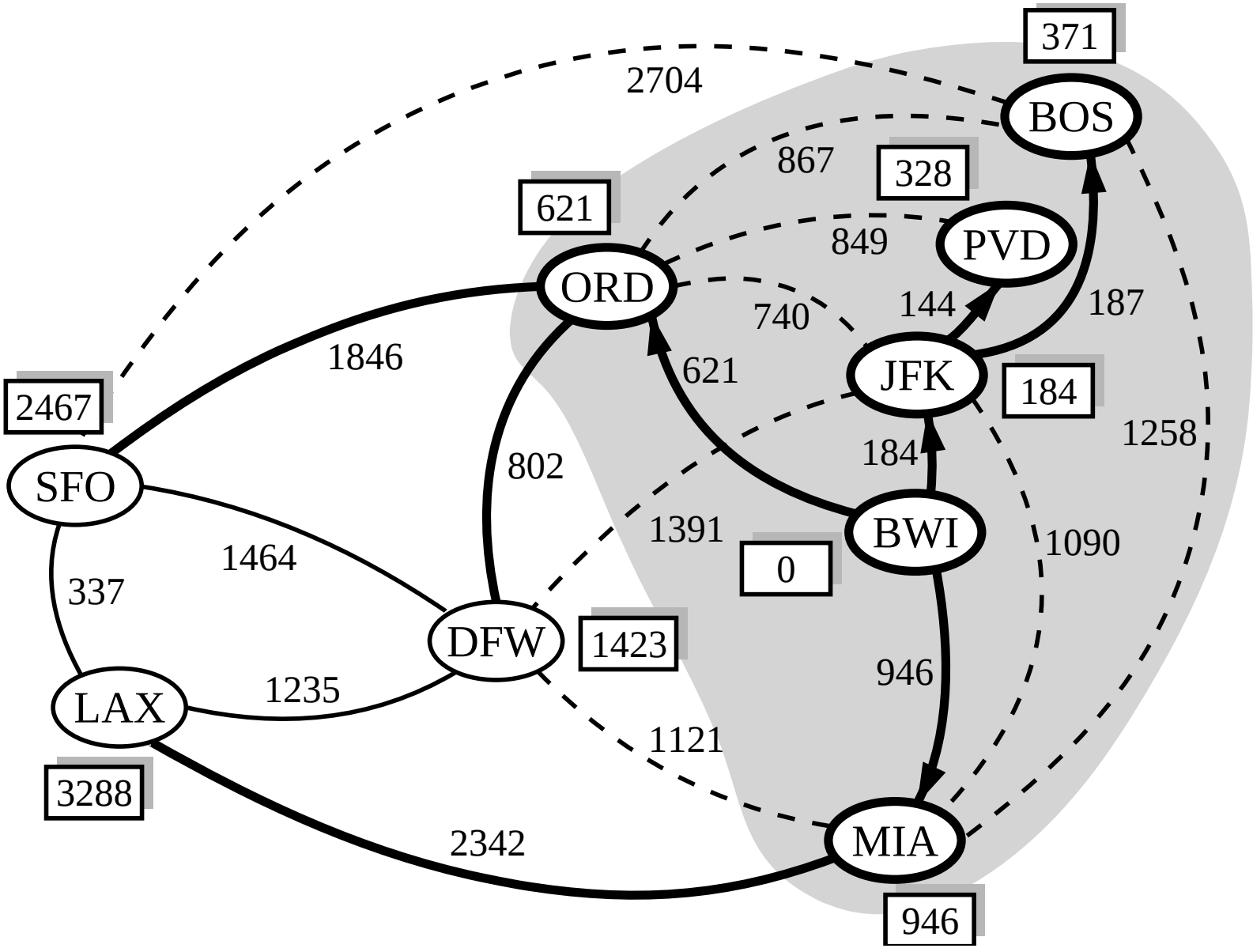

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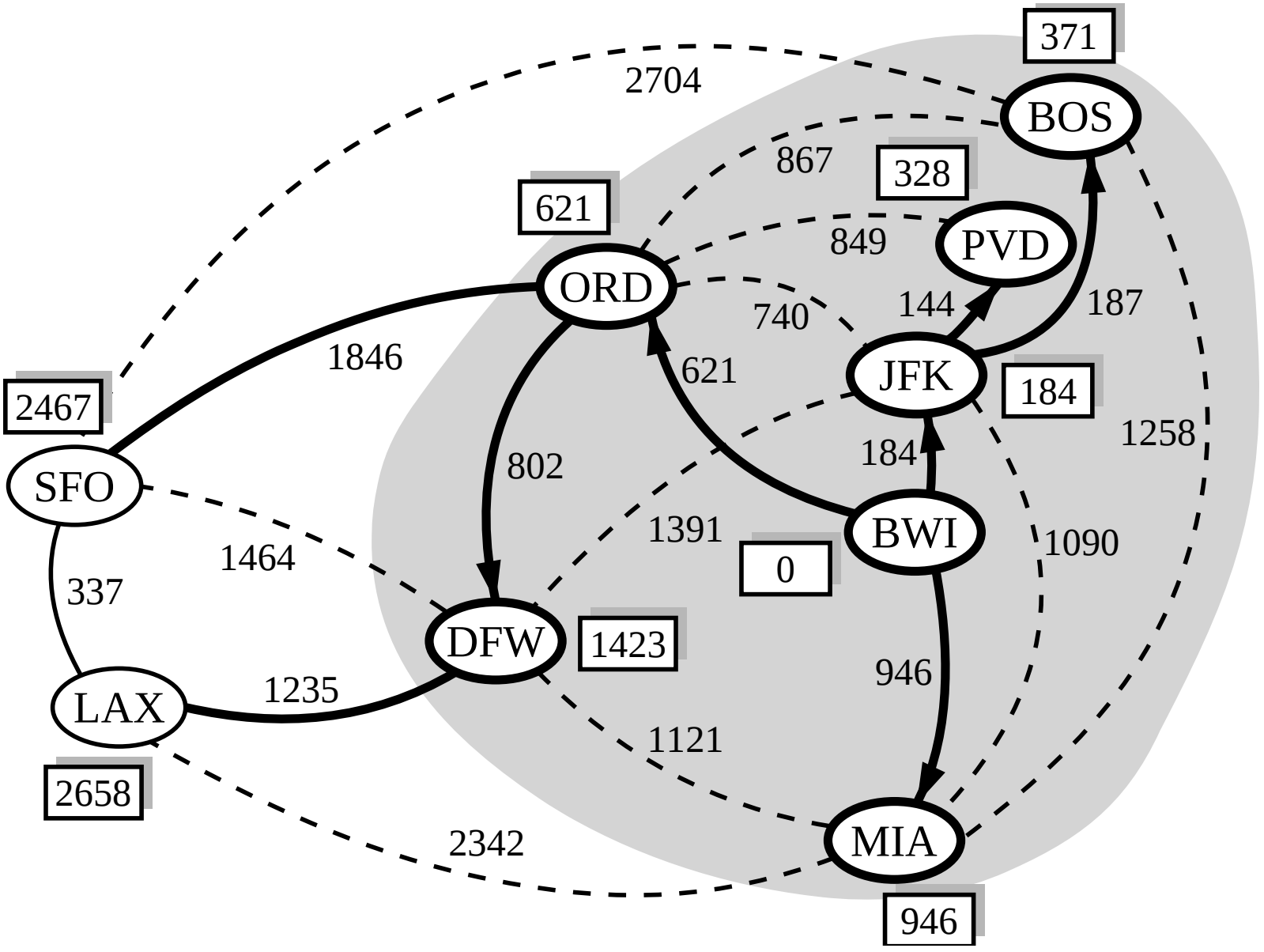

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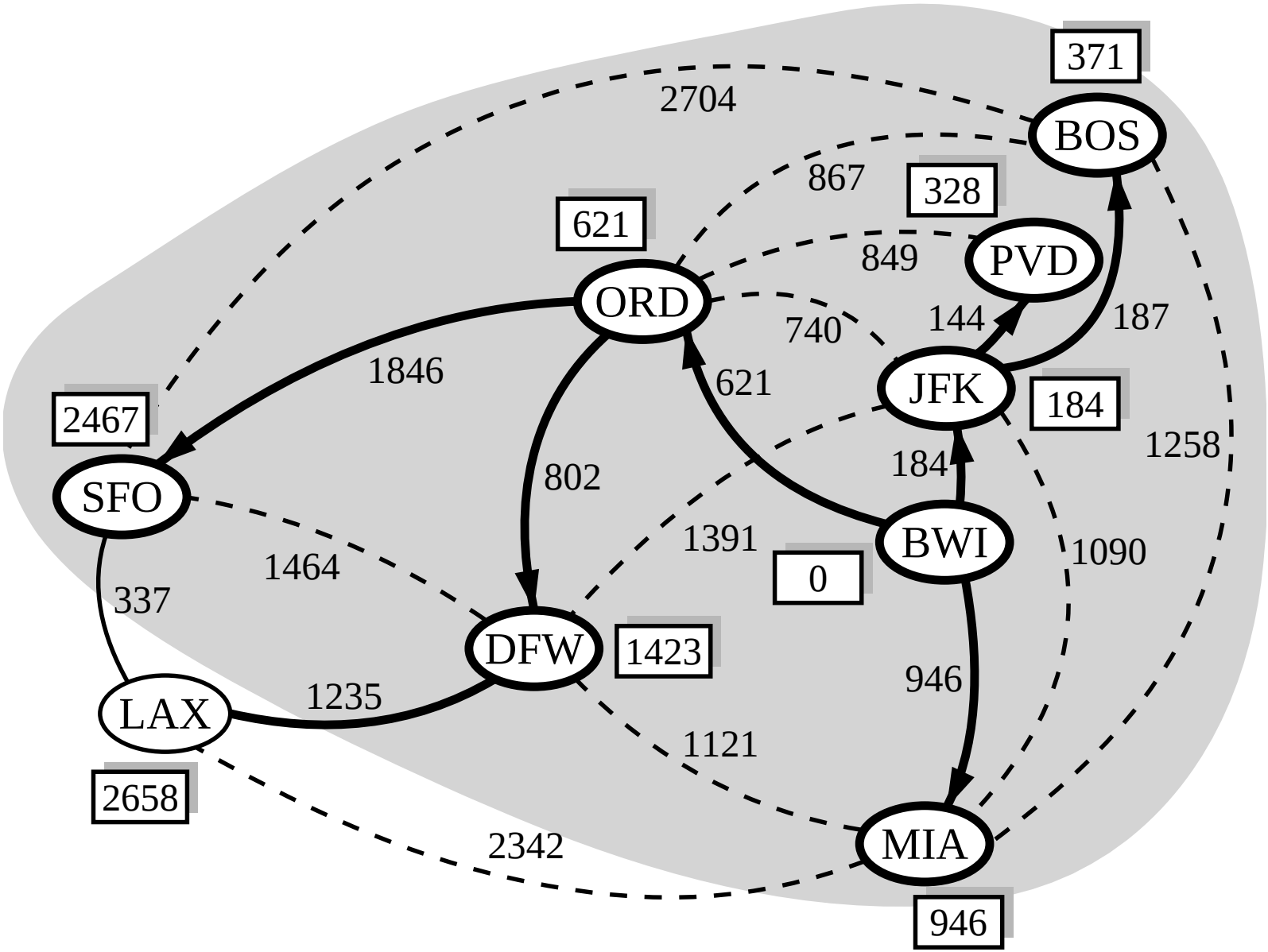

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