

Lecture 17,18: Introduction to String Matching Algorithms

BT 3051 – Data Structures and Algorithms for Biology

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INTRODUCTION

String Matching

Problem Definition

- ▶ Given the text

*We should continually be striving to transform every art into a science:
in the process, we advance the art.*

Find the string “science”

- ▶ More challenging: How many times does CATCC appear in
GATAAGGACCAGCGCAGTGGTAATTTAGGAACAAAGTATAGAAAT
GCTCATTTGAATTCGTAAGTTCAACTATAGTGTTCAGGATCGGTATCT
GAAAGGATCGATGCCTACTGAGTAATACGCGTGTGGACTGGTCGACCCTC
ATACGCTGCCACATCTCACACTTGTTAAGATGTTGGGCCTATGGG
TCAGCAACCATTGGATCGGAACTTGAACAAGTCTGCGCCTCAGGTACGG
AACCCCCACCTTCTGGCGTGGCACTCGGTGACTGTGTTGAGAGCCGGAA
GTCAGGGTCGGTACTCCGCCGCGGTGTCGATACGTAACACACAGACATTC
AGTCTCTTAGTCTCCAAACCATGAGGAAATGTTGCCGCCGAGGCTTTTTT

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ATACGCTGCCACATCTCACACTTGTTAACATCCGATGTTGGGCCTATGGG
TCAGCAACCATTGGATCGGAACTTGAACAAGTCTGCGCCTCAGGTACGG
AACCCCCACCTTCTGGCGTGGCACTCGGTGACTGTGTTTCAGAGCCGGAA
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GAAAGGATCGATGCCTACTGAGTAATACGCGTGTGGACTGGTCGACCCTC
ATACGCTGCCACATCTCACACTTGTTAA**CATCC**GATGTTGGGCCTATGGG
TCAGCAACCATTGGATCGGAACTTGAACAAGTCTGCGCCTCAGGTACGG
AACCCCCACCTTCTGGCGTGGCACTCGGTGACTGTGTTTCAGAGCCGGAA
GTCAGGGTCGGTACTCCGCCGCGGTGTCGATACGTAAACACACAGACATTC
AGTCTCTTAGTCTCCAAACCATGAGGAAATGTTGCCGCCGAGGCTTTTTTT

String Matching

Formal Definition

- ▶ Given a **text** T
 - ▶ $T \in \Sigma^*$: finite alphabet Σ
 - ▶ $|T| = n$
- ▶ and a **pattern** P
 - ▶ $P \in \Sigma^*$: same finite alphabet Σ
 - ▶ $|P| = m$
- ▶ Assuming both T and P can be represented using arrays
 - ▶ $T[1 \dots n]$ and $P[1 \dots m]$
- ▶ Pattern P occurs with shift s in T iff
 - ▶ $0 \leq s \leq n - m$
 - ▶ $T[s + i] = P[i]$ for all positions $1 \leq i \leq m$
 - ▶ Problem: Find all s

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Formal Definition

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String T

A	T	C	G	A	T	C	A	G	A	T	C	G	A	A
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

$n = 15$

Pattern P

String Matching

Formal Definition

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---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

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Pattern P

A	T	C
---	---	---

$m = 3$

$s = 0$

- ▶ Hits: $[0]$

String Matching

Formal Definition

- ▶ Problem: find all s such that
 - ▶ $0 \leq s \leq n - m$
 - ▶ $T[s + i] = P[i]$ for $1 \leq i \leq m$

String T

A	T	C	G	A	T	C	A	G	A	T	C	G	A	A
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

$n = 15$

Pattern P

A	T	C
---	---	---

$s = 4$

$m = 3$

- ▶ Hits: $[0, 4]$

String Matching

Formal Definition

- ▶ Problem: find all s such that
 - ▶ $0 \leq s \leq n - m$
 - ▶ $T[s + i] = P[i]$ for $1 \leq i \leq m$

String T

A	T	C	G	A	T	C	A	G	A	T	C	G	A	A
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

$n = 15$

Pattern P

A	T	C
---	---	---

 $s = 9$

$m = 3$

- ▶ Hits: $[0, 4, 9]$

String Matching

Applications

- ▶ String matching is universally used in several applications
- ▶ Searching words in a document
- ▶ Searching for genes in an organism
- ▶ Spell-check
- ▶ ...

String Matching

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String Matching

Algorithm Design Overview (Skiena)

- ▶ Is the search pattern/text short? *Naïve matching*
- ▶ Is the search pattern/text very long? *Knuth–Morris–Pratt*
- ▶ Do we expect to find the pattern or not? *Boyer–Moore*
- ▶ Will we perform multiple queries on the same text? *Suffix trees*
- ▶ Will we search many texts using the same pattern? *Complex algorithms ...*
- ▶ What if the input contains a spelling error? *Approximate string matching*

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ALGORITHMS

Naïve algorithm

```
def NaiveStringMatcher(Text, Pattern):  
    m = len(Pattern)  
    n = len(Text)  
  
    matches = []  
    for s in range(n-m):  
        if Text[s:s+m]==Pattern:  
            matches.append(s)  
  
    return matches  
  
if __name__ == '__main__':  
    print(NaiveStringMatcher('ATCGATCAGATCGAA', 'ATC'  
                               ))
```

What is the complexity?
Can we do better?

Naïve algorithm

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Can we do better?

Improvement Strategy

► Observation

T

A	T	C	A	A	T	A	A	T	A	T	A	C	A
---	---	---	---	---	---	---	---	---	---	---	---	---	---

P

► Motivation

► Example

Improvement Strategy

► Observation

T

A	T	C	A	A	T	A	A	T	A	T	A	C	A
---	---	---	---	---	---	---	---	---	---	---	---	---	---

P

A	T	A
---	---	---

► What now?

Improvement Strategy

► Observation

T

A	T	C	A	A	T	A	A	T	A	T	A	C	A
---	---	---	---	---	---	---	---	---	---	---	---	---	---

=

P

A	T	A
---	---	---

► What now?

Improvement Strategy

► Observation

T

A	T	C	A	A	T	A	A	T	A	T	A	C	A
---	---	---	---	---	---	---	---	---	---	---	---	---	---

= =

P

A	T	A
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► What now?

Improvement Strategy

► Observation

T

A	T	C	A	A	T	A	A	T	A	T	A	C	A
---	---	---	---	---	---	---	---	---	---	---	---	---	---

$= = \neq$

P

A	T	A
---	---	---

► What now?

► the naive algorithm goes back to the second position in T and starts from the beginning of P

► *What if we could skip some positions in T ?*

Improvement Strategy

► Observation

T

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---	---	---	---	---	---	---	---	---	---	---	---	---	---

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► What now?

- the naïve algorithm *goes back to the second position in T and starts from the beginning of P*
- can't we simply move along through T ?

Improvement Strategy

► Observation

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- the naïve algorithm *goes back to the second position in T and starts from the beginning of P*
- can't we simply move along through T ?

Careless String Matcher

```
def CarelessStringMatcher(Text, Pattern):  
    n = len(Text)  
    m = len(Pattern)  
    q = s = 0  
    while s < n:  
        if Text[s] == Pattern[q]:  
            q += 1 #increase match length  
            if q == m: #matched `m' characters  
                print (s - m + 1)  
            q = 0 #reset match length  
        else:  
            q = 0 #found a mismatch  
            s += 1 #move further in the text
```

Is there a bug?

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Is there a bug?

Boyer-Moore Algorithm

Smart Heuristics

- ▶ Main idea: improve running time of brute-force algorithm by adding two potentially time-saving heuristics
- ▶ Roughly stated, these heuristics are:
 - ▶ *Looking-Glass Heuristic:* When testing a possible placement of P against T , begin comparisons from the end of P and move backward to the front of P
 - ▶ *Bad Character Heuristic:* During the testing of a possible placement of P within T , if mismatch occurs at position i with the corresponding pattern character $P[i]$, it is handled as follows:

Boyer-Moore Algorithm

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 - ▶ *Looking-Glass Heuristic*: When testing a possible placement of P against T , begin comparisons from the end of P and move backward to the front of P
 - ▶ *Character-Jump Heuristic*: During the testing of a possible placement of P within T , a mismatch of text character $T[i] = c$ with the corresponding pattern character $P[k]$ is handled as follows:

Find the largest proper prefix of $P[k:]$ that is also a suffix of $P[k:]$. If the length of this prefix is l , then we can skip ahead by $l + 1$ positions in T .

Boyer-Moore Algorithm

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 - ▶ Otherwise, shift P until an occurrence of character c in P gets aligned with $T[i]$

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Boyer–Moore Algorithm

h	e	r	e		i	s		a		s	i	m	p	l	e		e	x	a	m	p	l	e
---	---	---	---	--	---	---	--	---	--	---	---	---	---	---	---	--	---	---	---	---	---	---	---

- ▶ We match the pattern right-to-left
- ▶ If we find a bad character α in the text, we can shift
 - ▶ so that the pattern skips α , if α is not in the pattern
 - ▶ or that the pattern starts with a character α that is in the pattern, if the pattern contains α
 - ▶ so that a pattern prefix lines up with a suffix of the current period (the longest such)

Boyer–Moore Algorithm

h	e	r	e		i	s		a		s	i	m	p	l	e		e	x	a	m	p	l	e
---	---	---	---	--	---	---	--	---	--	---	---	---	---	---	---	--	---	---	---	---	---	---	---

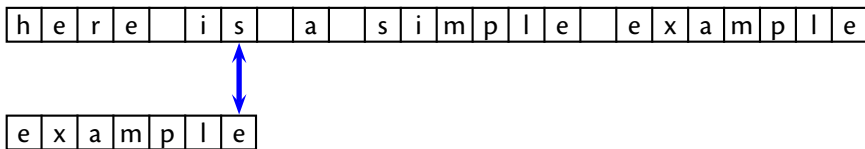
e	x	a	m	p	l	e
---	---	---	---	---	---	---

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Example: $\alpha = \text{p}$. The pattern contains α at index 4 (0-indexed), so we can shift the pattern by 4 positions.

Example: $\alpha = \text{e}$. The pattern contains α at index 0 and 6 (0-indexed), so we can shift the pattern by 1 or 7 positions.

Boyer–Moore Algorithm



- ▶ We match the pattern right-to-left
- ▶ If we find a bad character α in the text, we can shift
 - ▶ so that the pattern skips α , if α is not in the pattern

For example, if we find a bad character α at index i in the text, we can shift the pattern to the right by $i - \text{last_occurrence}(\alpha)$ positions, where $\text{last_occurrence}(\alpha)$ is the index of the last occurrence of α in the pattern. If the pattern contains α , then the pattern will skip α and match the next character in the text. If the pattern does not contain α , then the pattern will skip α and match the next character in the text.

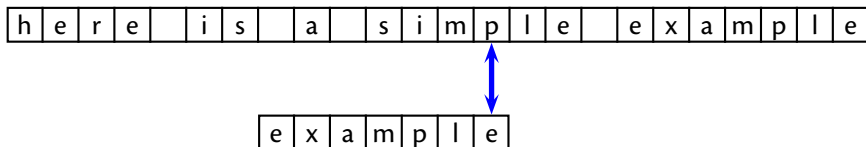
Boyer–Moore Algorithm

h	e	r	e		i	s		a		s	i	m	p	l	e		e	x	a	m	p	l	e
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e	x	a	m	p	l	e
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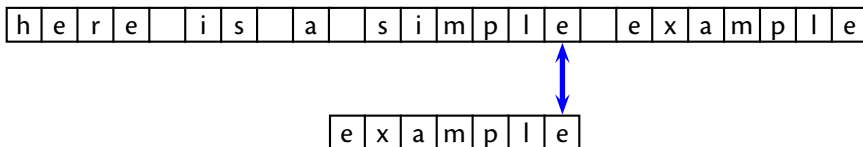
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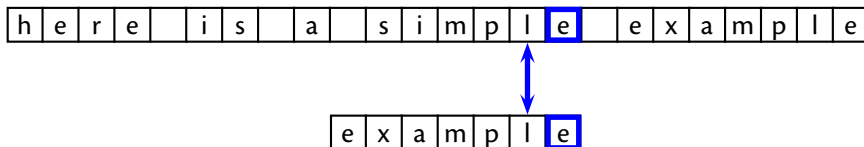
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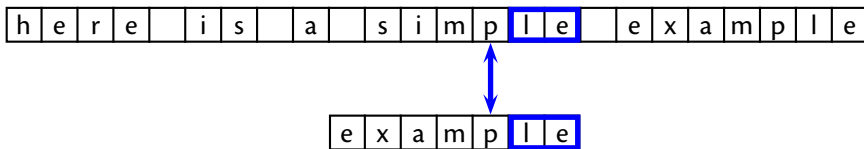
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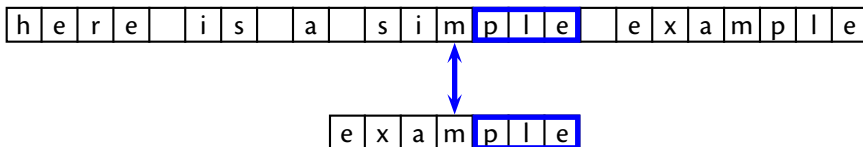
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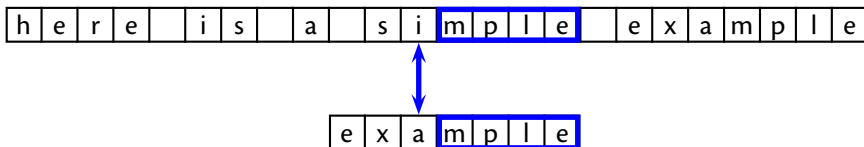
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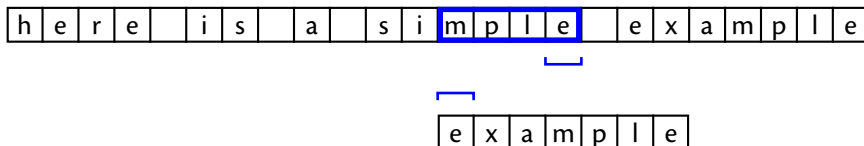
h	e	r	e		i	s		a		s	i	m	p	l	e		e	x	a	m	p	l	e
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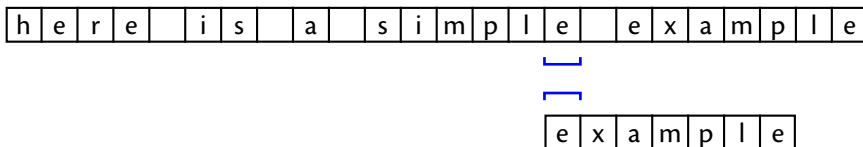
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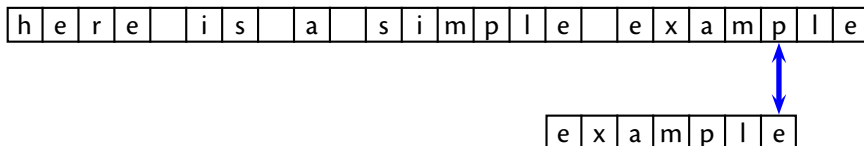
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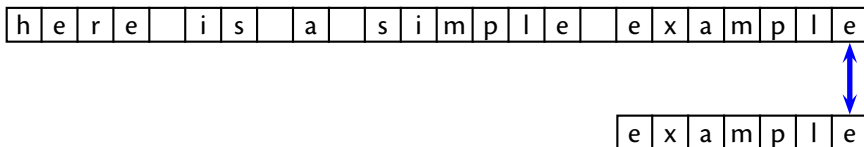
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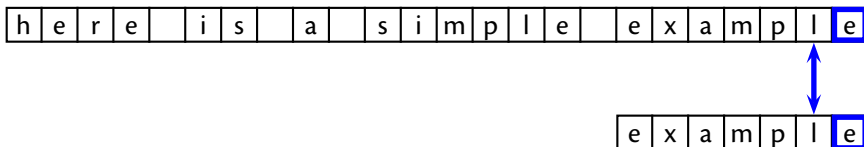
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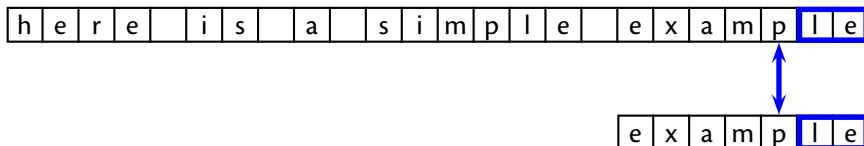
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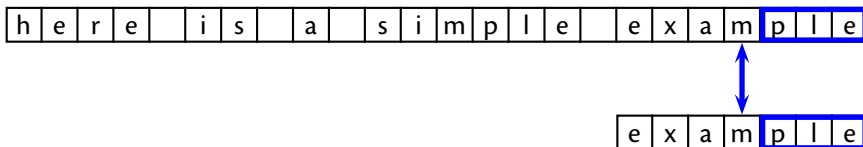
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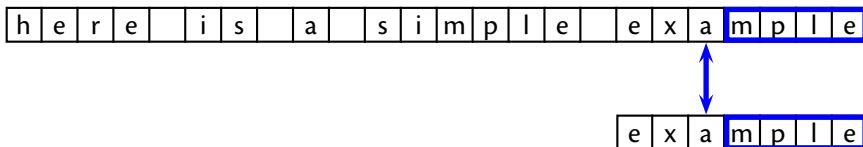
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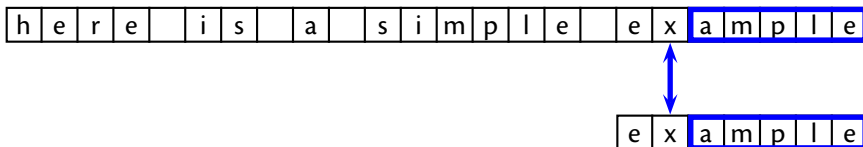
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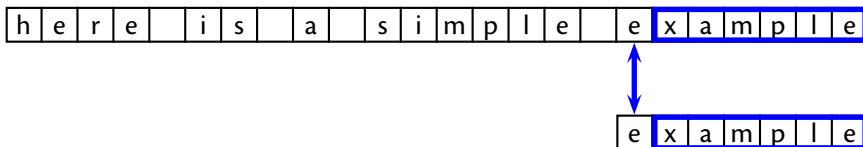
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KNUTH–MORRIS–PRATT ALGORITHM

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- ▶ For a certain alignment of the pattern, if we find several matching characters but then detect a mismatch, we ignore all the information gained by the successful comparisons after restarting with the next incremental placement of the pattern!
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