

dFBA

In the following derivation, I have used notation that is consistent with the book Sir.

$$\frac{dX}{dt} = \mu X \Rightarrow X = X_0 e^{\mu \Delta t} \quad (1)$$

$$\frac{dS_i}{dt} = -v_{S_i} X \quad (2)$$

$$\Rightarrow \int dS_i = \int -v_{S_i} X dt \quad (3)$$

Integrating from 0 to Δt ,

$$S_i - S_0 = -\frac{v_{S_i}}{\mu} (X_0 e^{\mu \Delta t}) \Big|_0^{\Delta t} \quad (4)$$

$$S_i = S_0 - \frac{v_{S_i} X_0}{\mu} (e^{\mu \Delta t} - 1) \quad (5)$$

$$\Rightarrow \boxed{S_i = S_0 + \frac{v_{S_i} X_0}{\mu} (1 - e^{\mu \Delta t})} \quad (6)$$