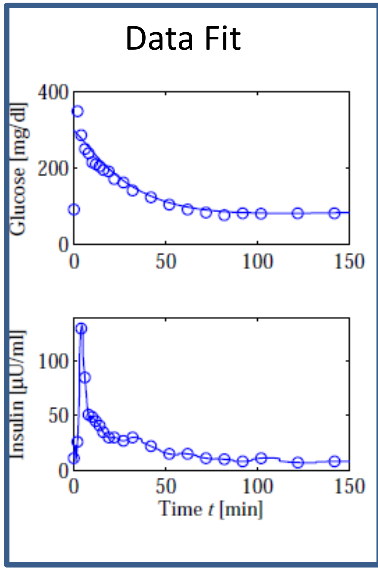
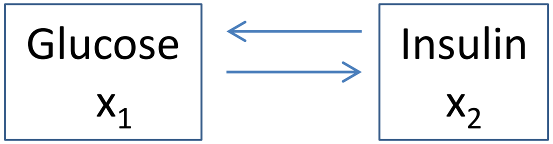
Insulin-Glucose Dynamics *(The Minimal Model)*

- **What does it model?**
  - The concentrations of interstitial insulin and glucose over time, and the interactions between the two.
- **What does it neglect?**
  - The reaction of the pancreas to glucose, among many other finer, sub-first-order interactions.
- **Characteristics of the Model**
  - **Mathematical**
  - **Deterministic –** The result and equation are deterministic for a given person
  - **Closed –** All necessary interactions are encompassed within the model
  - **Correlative –** The curve is fit and equations are derived based on experimental values
  - **Empirical** – The models are typically tested with data from experiments where glucose is injected intravenously and insulin and glucose concentrations are measured at regular time intervals.
  - **Continuous** – The result is a differential equation, which can provide concentrations of insulin and glucose at any given time, given the initial concentrations.
- **Salient Features of the Model**
  - **Two compartment model** – Lowest complexity known to model this system
  - Parameters change depending on the person
  - Use in diagnosis and in patient treatment
  - Should ideally lead to an artificial pancreas in the future
- **Mathematics of the model**





- $$\frac{dx_1}{dt} = -(p_1 + x_2)x_1 + p_1 g_e \quad ; \quad \frac{dx_2}{dt} = -p_2 x_2 + p_3(u - ie)$$

where g_e and i_e represent the equilibrium values of glucose and insulin, x_1 is the concentration of blood glucose and x_2 is proportional to the concentration of interstitial insulin. p_1, p_2 and p_3 are parameters defined by the characteristics of the person of interest.

Kalman Filter

- Kalman filter is an empirical model used in finding a statistically optimal state estimates of the system from noisy input data.
- It is a recursive estimator with two phases.

Assumption

- The underlying system dynamics is linear.
- All noises have a Gaussian distribution.

Applications

- Navigation systems
- Time series analysis
- Radar tracker

Advantages

- It can run real time with present input measurements and previous state estimate.
- There is no need for storage of past estimates.

Disadvantages

- Most noises are not gaussian.
- Covariance of noise cannot be easily estimated.

The underlying system model is of the form

$$x_k = A\, x_{k-1} + \mathrm{B}\, u_k + w_k$$
$$y_k = C x_k + v_k$$

where

A is the state transition model

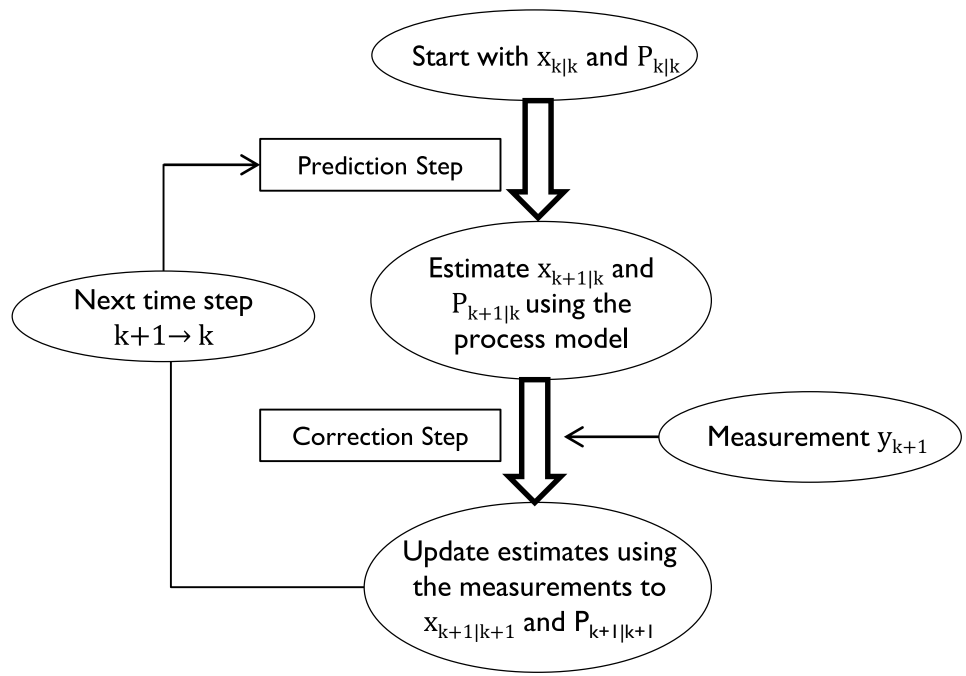
B is the input control model

y_k is the measurement of the true state x_k

$w_k \sim N(0, Q_k)$ is the process noise

$v_k \sim N(0, R_k)$ is the observation noise

Q_k and R_k are the covariance



$P_{k+1|k} = E(x_{k+1} - x_{k+1|k})^2$

$x_{k+1|k+1} = x_{k+1|k} + K_{k+1}(y_{k+1} - C\, x_{k+1|k})$

Where K_{k+1} is the Kalman Gain

Gambler's Ruin

- Stochastic model that employs random walks to predict the outcome of a game of gambling
- Parameters in the model are the probabilities of winning and losing a particular game, say p and q and the amount placed as bet by the gambler
- For a given initial amount of money the model predicts whether the gambler reaches his/her objective or goes broke
- This model can be reduced to a Markov chain:

  for an initial state *i*, the probability of reaching a state *a*, before state *b* based on p and q

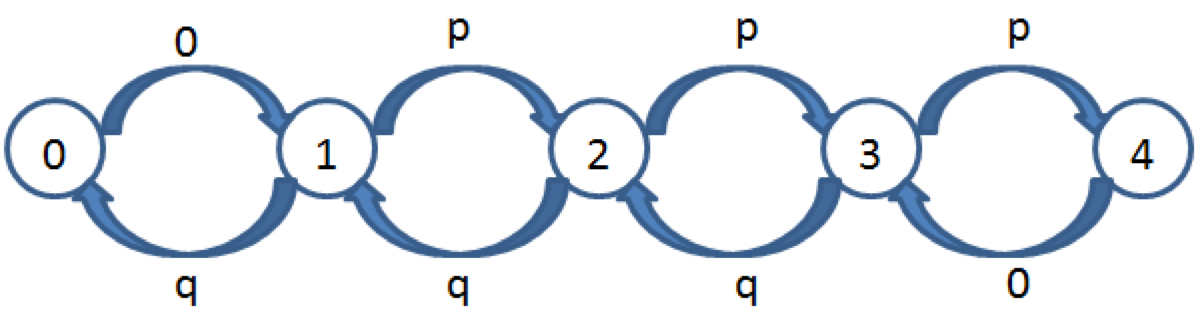


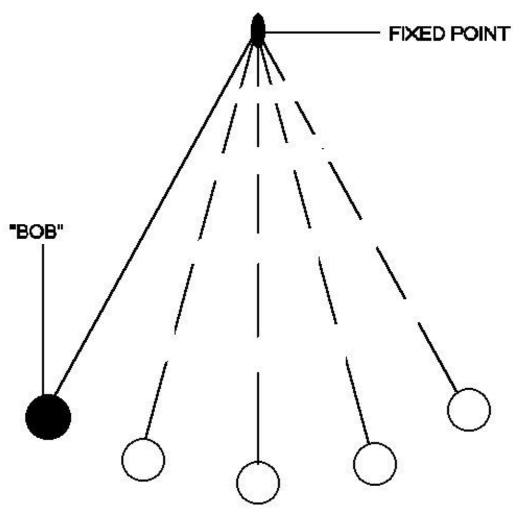
Figure showing the chain when the objective is 4 and stakes are of worth 1

- Model used to predict probability of win/ruin in a fair game, i.e p=q=0.5 when
  - Opponent is infinitely rich
  - Stakes are increased or reduced

Newton’s model of Motion

A continuous and analytical model developed by Isaac Newton to describe the motion of bodies due to forces.

Governing Equations



- This model is governed by the three laws of motion developed by him:
  1. Every body persists in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by force impressed
  2. The alteration of motion is ever proportional to the motive force impress'd; and is made in the direction of the right line in which that force is impress'd.
  3. To every action there is always an equal and opposite reaction: or the forces of two bodies on each other are always equal and are directed in opposite directions.

  - Isaac Newton, *The Principia*, A new translation by I.B. Cohen and A. Whitman, University of California press, Berkeley 1999.

Limitations

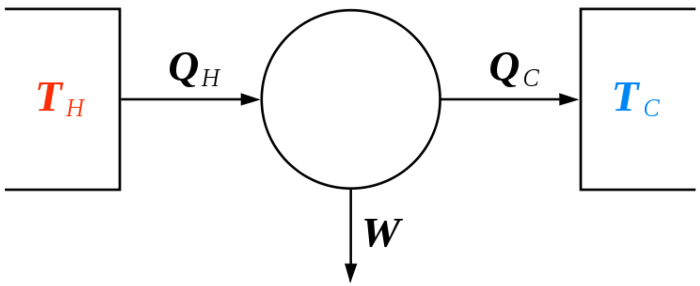
- Small particles and velocities close to the speed of light cannot be handled by this model.

Applications

- The conservation of momentum was derived using the third law from the first section. An analytical explanation could be given for Kepler’s laws.

Model: The Carnot Engine

- Its an continuous and theoretical system. First proposed by Sadi Carnot(1796-1832).



- Follows laws of thermodynamics;
- Carnot efficiency = $1-(T_H/T_C) = 1-(Q_H/Q_C)$
  (Where $_H$=heat in; $_C$ = heat out);
- dS= dQ/dT and classius inequality are notable contributions came out of this model;

- Carnot engine, though hypothetical, gave a great deal of theoretical understanding and mathematical equivalents for each concept, in thermodynamics, and helped us to understand the concept of entropy in particular.
- It gave engineers an upper limit, a functional end point, a limit that can be achieved by any heat engine.
- **But** Carnot engine considers uniform resistance and absence of friction thus equations derived has to be modified to include non – uniform combustion and friction to use in real life engines.

Zipf's law

- Empirical law in statistics. Originally found to describe word frequencies (1932) and city sizes (1949)
- Discrete power law distribution. $\log \mathrm{cf}_i = \log c + k \log i$

  i = rank, cf_i = collection frequency

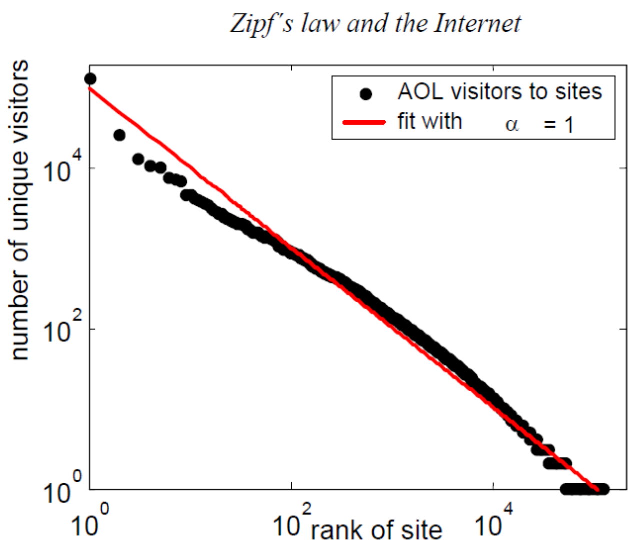


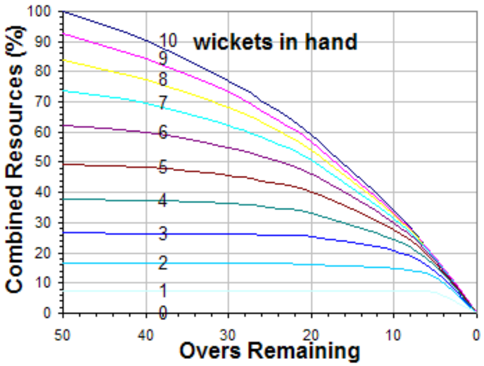
Figure 2. Sites ranked by the number of unique AOL visitors they received Dec. 1, 1997. AOL (America Online) is the largest Internet service provider in the United States. The fit is a Zipf distribution $n_r \sim r^{-1}$

Superseded Pareto distributions and provided a more general framework to Benford's law (financial fraud etc.)

Describes features of social networks such as the Internet; even used in web-caching strategies.

Unlike Gaussian distributions, these are scale-free

Duckworth-Lewis Model



- The basic principle is that each team in a limited-overs match has two available resources: wickets remaining and overs to play
- Attempts to set a statistically fair target for the second team's innings, based on the score achieved by the first team, taking their wickets lost and overs played into account
- In 2004, the D/L method was split into a Professional Edition and a Standard Edition
- The Standard Edition preserves the use of a single table and simple calculation
- The Professional Edition uses substantially more sophisticated statistical modelling, and requires the use of a computer (used in ODIs)