BT6270 Computational Neuroscience

# Computational Neuroscience Assignment 2

N Sowmya Manojna | BE17B007
Department of Biotechnology,
Indian Institute of Technology, Madras

The FitzHugh Nagumo model is a relaxation oscillator [1] that is obtained from simplification of the detailed Hodgkin-Huxley model. This simplification is obtained by condensing the ion channel activations into a $w$ variable.

The two variable FitzHugh-Nagumo model can be simulated using the following equations:

$$\frac{dv}{dt} = v(a-v)(v-1) - w + I_{ext}$$

$$\frac{dw}{dt} = bv - rw$$

The parameters used in the first three Cases are are:

$$a = 0.5, b = 0.1, r = 0.1$$

# 1 Case 1: $I_{ext} = 0$

## 1.1 Phase Plot

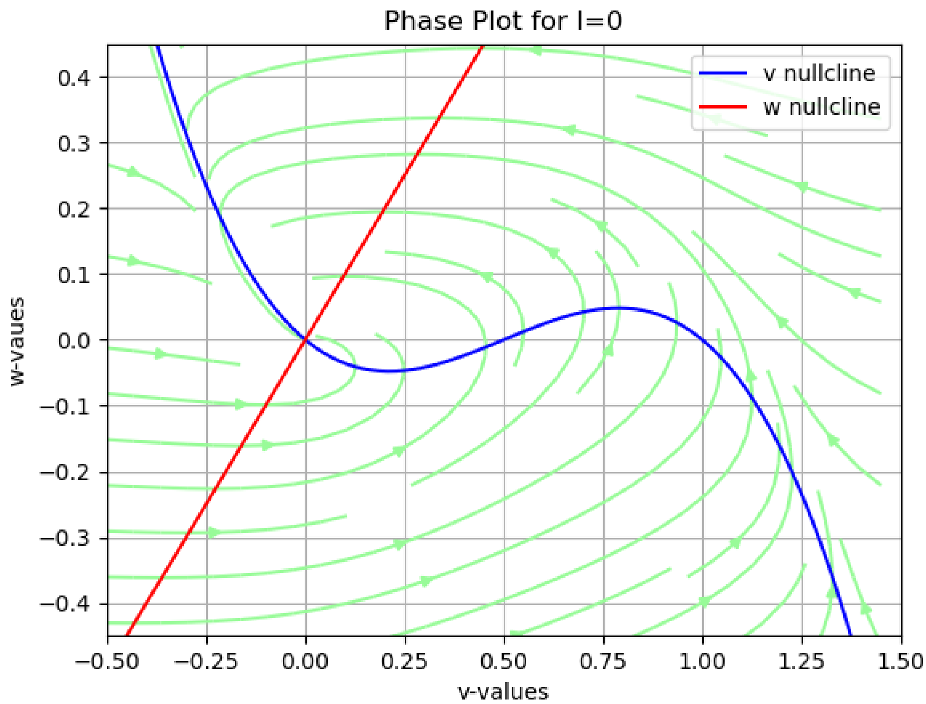


**Figure 1:** Phase Plot of the system when $I_{ext} = 0$. The stationary point obtained is a stable point.

Analyzing the trajectories by using initial points - $[0, 0.4, 0.6, 1], w = 0$, we can see that even for perturbations in the initial start point, we approach the equilibrium point at $[0, 0]$. Hence, the point $[0, 0]$ is a stable fixed point.

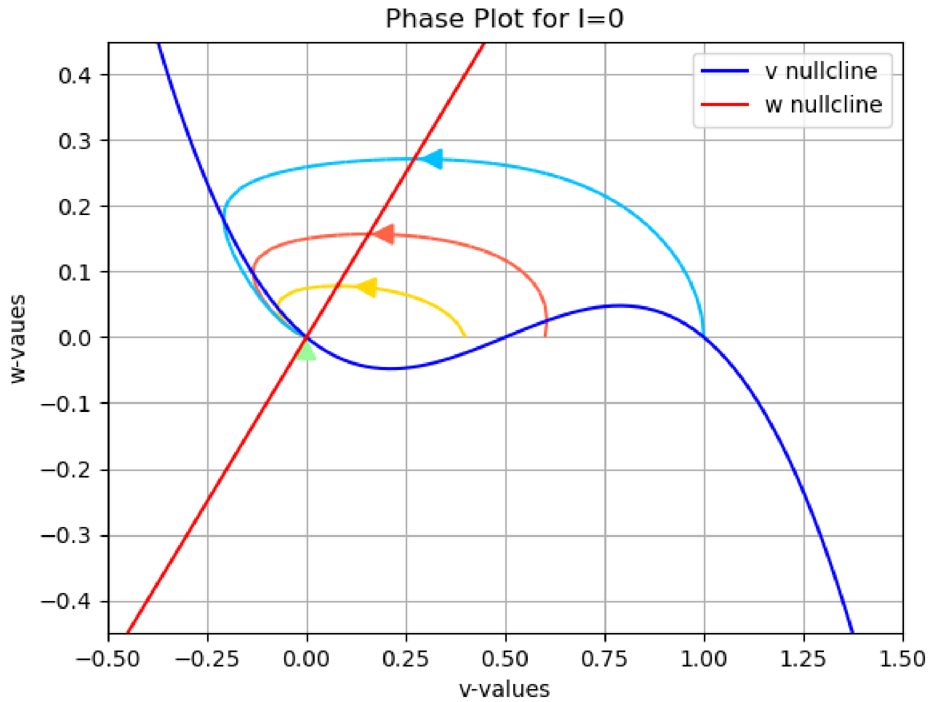


**Figure 2:** Stability analysis of the equilibrium point. The model approaches the equilibrium point irrespective of the initial conditions. Hence, the equilibrium point is a stable fixed point.

## 1.2 $V(t), W(t)$ across $t$, Trajectories

For an $I_{ext}$ value of 0, no action potentials are observed.

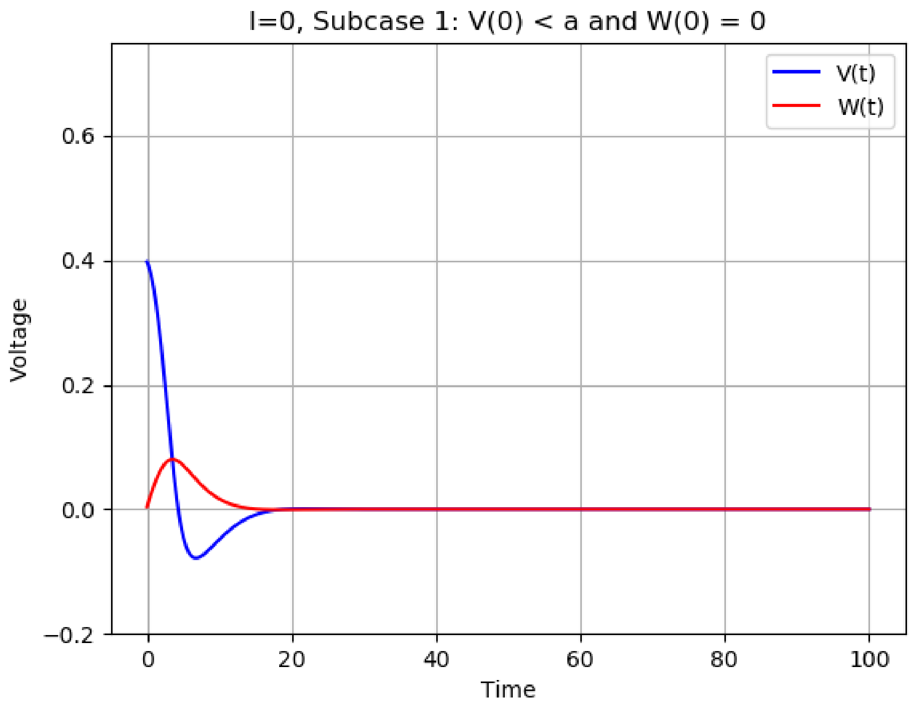


**Figure 3:** $V(t), W(t)$ across $t$, when $V(0) < a$. With sub-threshold pulse injections, no action potentials are observed.

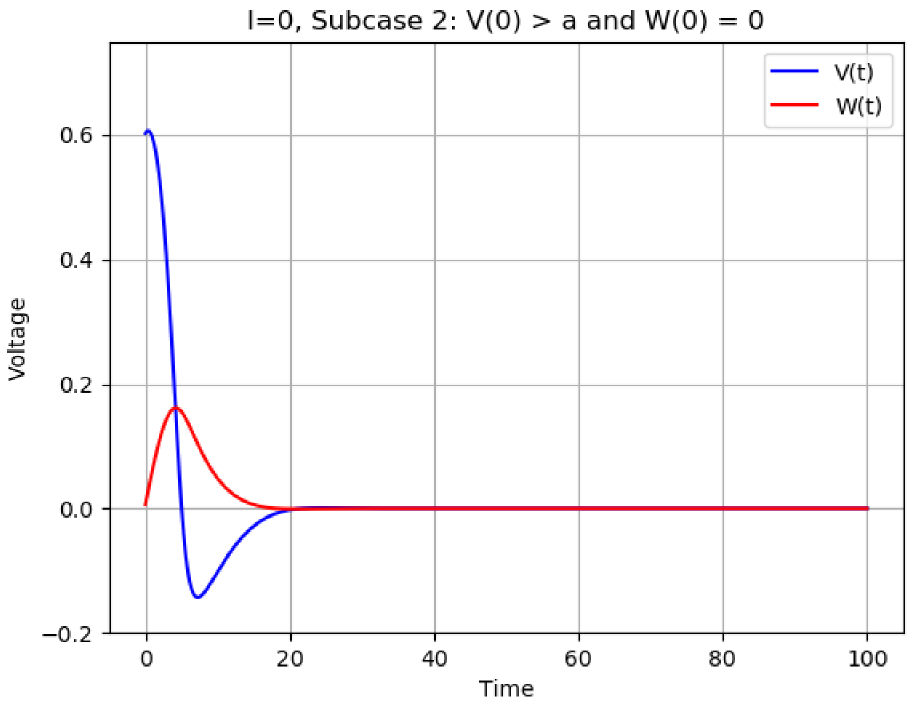


**Figure 4:** $V(t), W(t)$ across $t$, when $V(0) > a$. With sub-threshold pulse injections, no action potentials are observed.

# 2 Case 2: $I_{ext} = 0.6$

## 2.1 Phase Plot

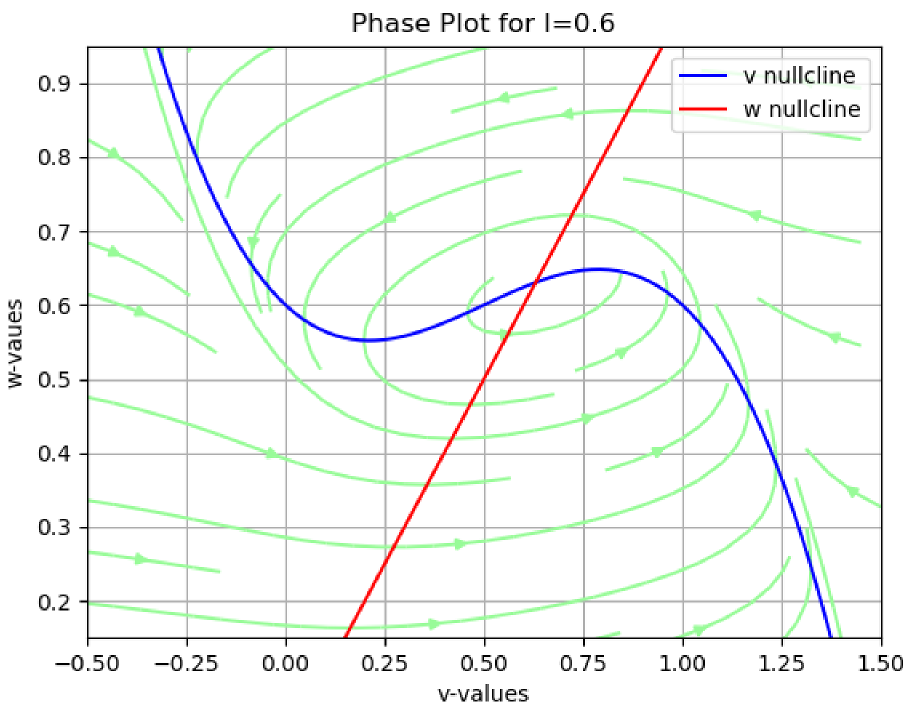


**Figure 5:** Phase Plot of the system when $I_{ext} = 0.6$. This stationary point is found to be a unstable point.

The trajectories were analyzed by using initial points - $[0, 0.4, 0.6, 1], w = 0$. We can see that at the point of intersection of the nullclines, there are circulating fields around the unstable stationary point. Additionally we also see limit cycle enclosing the stationary point.

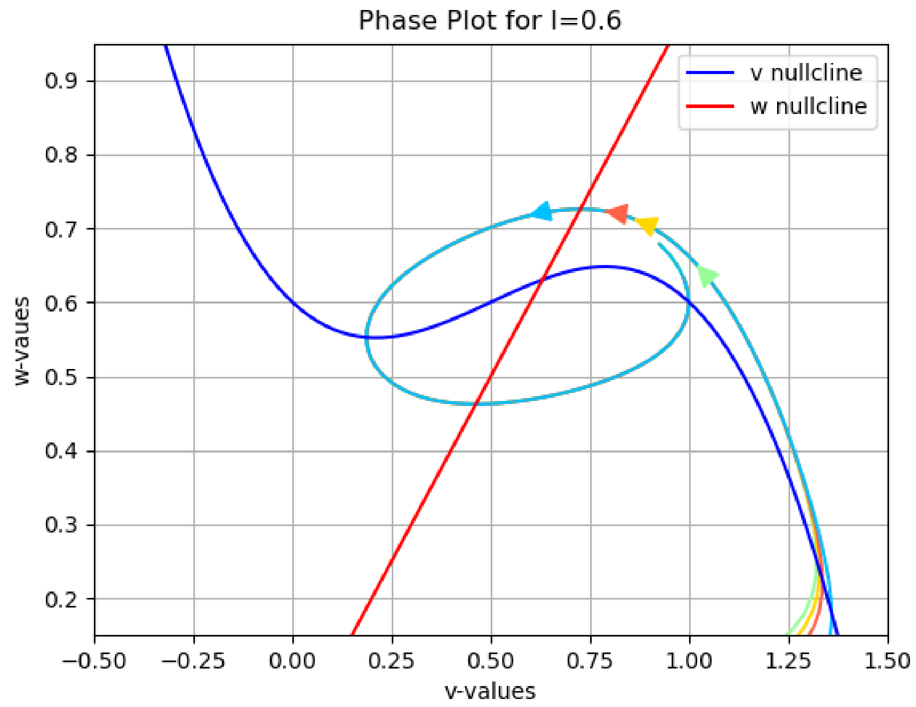


**Figure 6:** Stability analysis of the stationary point. The stationary point is found to be unstable and limit cycle behavior is also observed.

## 2.2 $V(t), W(t)$ across $t$, Trajectories

For $I_{ext} = 0.6$, oscillatory membrane potential is seen in the limit cycle region.

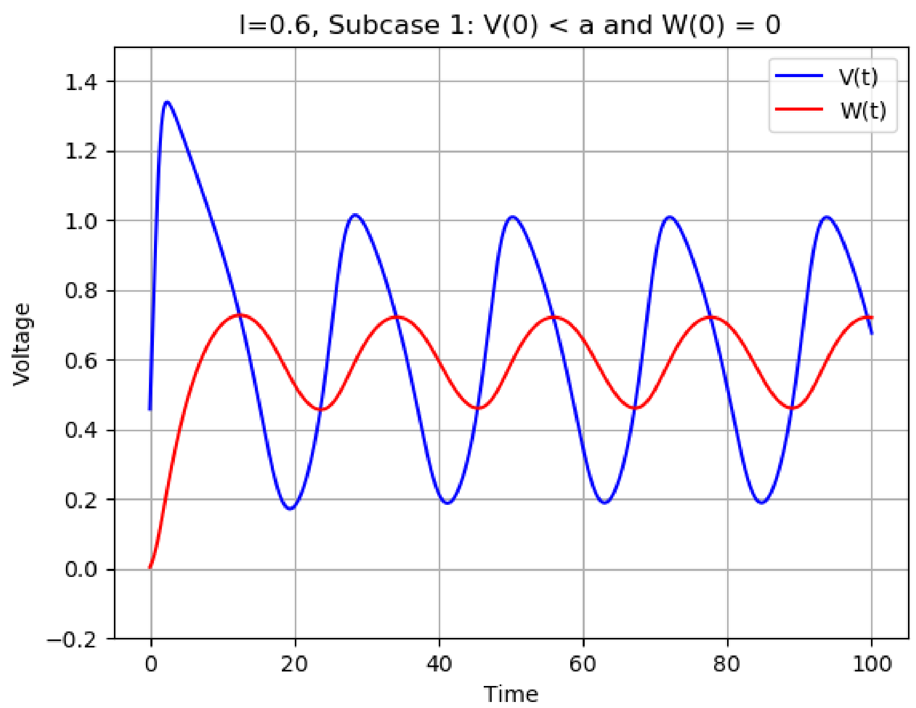


**Figure 7:** $V(t), W(t)$ across $t$, when $V(0) < a$. Sustained oscillations are observed.

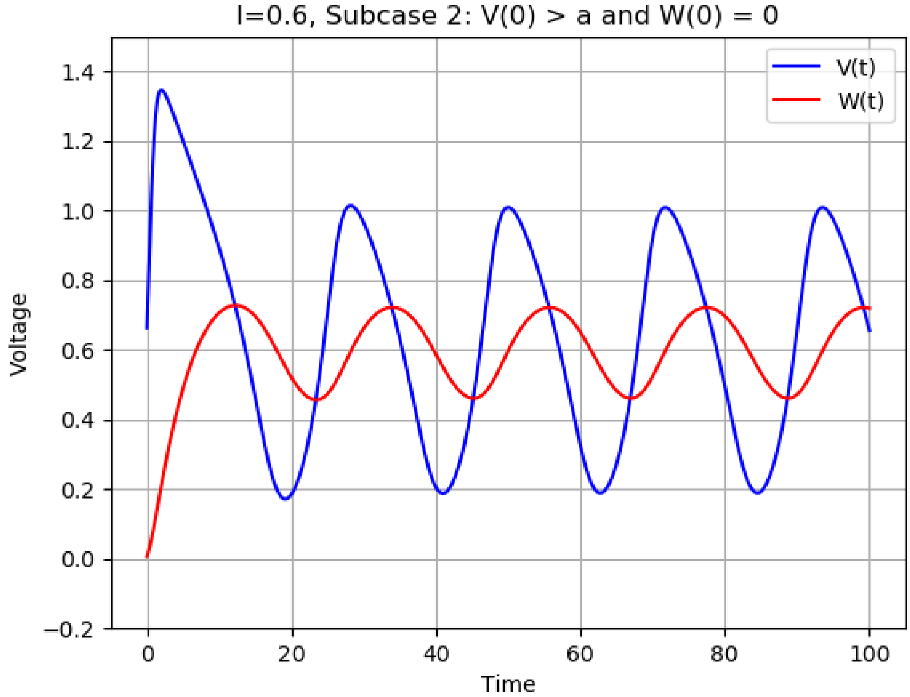


**Figure 8:** $V(t), W(t)$ across $t$, when $V(0) > a$. Sustained oscillations are observed.

# 3 Case 3: $I_{ext} = 1$

## 3.1 Phase Plot

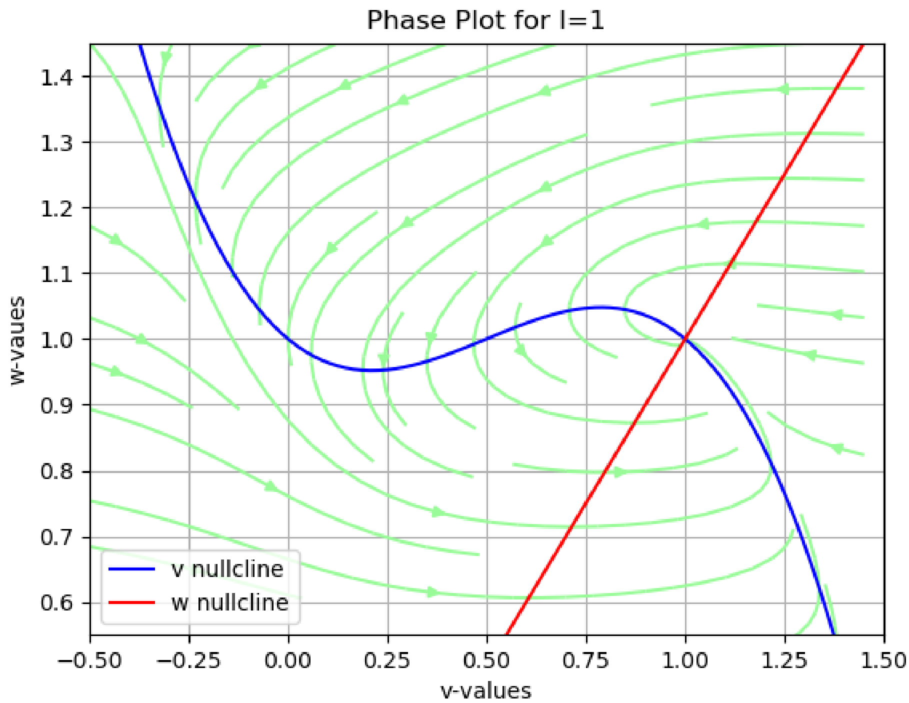


**Figure 9:** Phase Plot of the system when $I_{ext} = 1$. This stationary point is found to be a stable point.

The trajectories were analyzed by using initial points - $[0, 0.4, 0.6, 1], w = 0.6$ and $[0, 0.4, 0.6, 1], w = 1.4$. We can see that even for large perturbations in the initial start point, we approach the equilibrium point at $[1, 1]$. Hence, the point $[1, 1]$ is a stable fixed point.

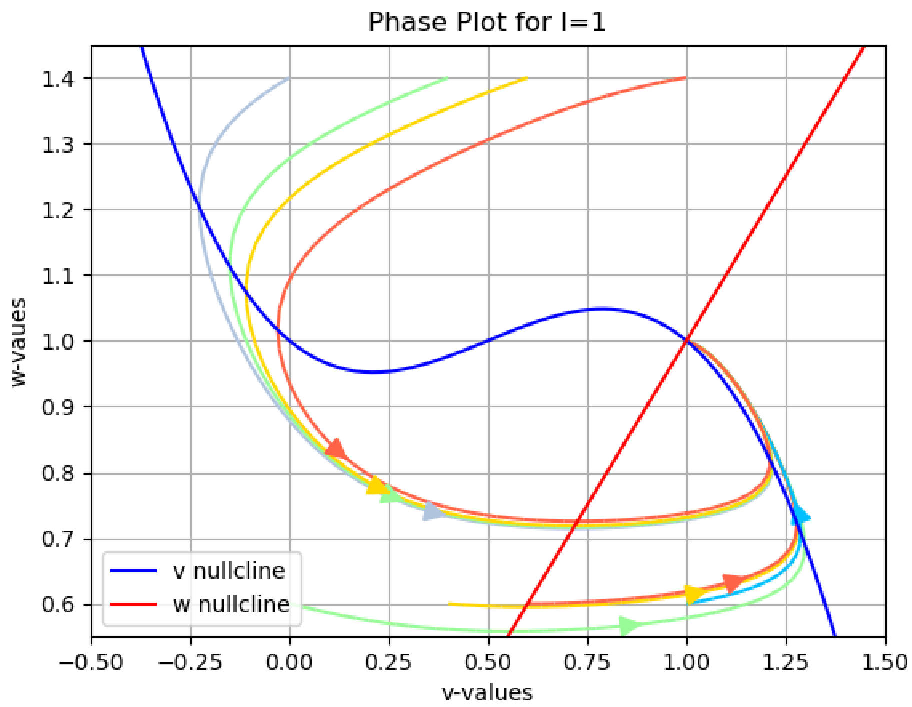


**Figure 10:** Stability analysis of the stationary point. This stationary point is found to be a stable point.

## 3.2 $V(t), W(t)$ across $t$, Trajectories

For $I_{ext} = 1$, depolarization is observed in the membrane potential. The voltage initially rises and then stays at a high value.

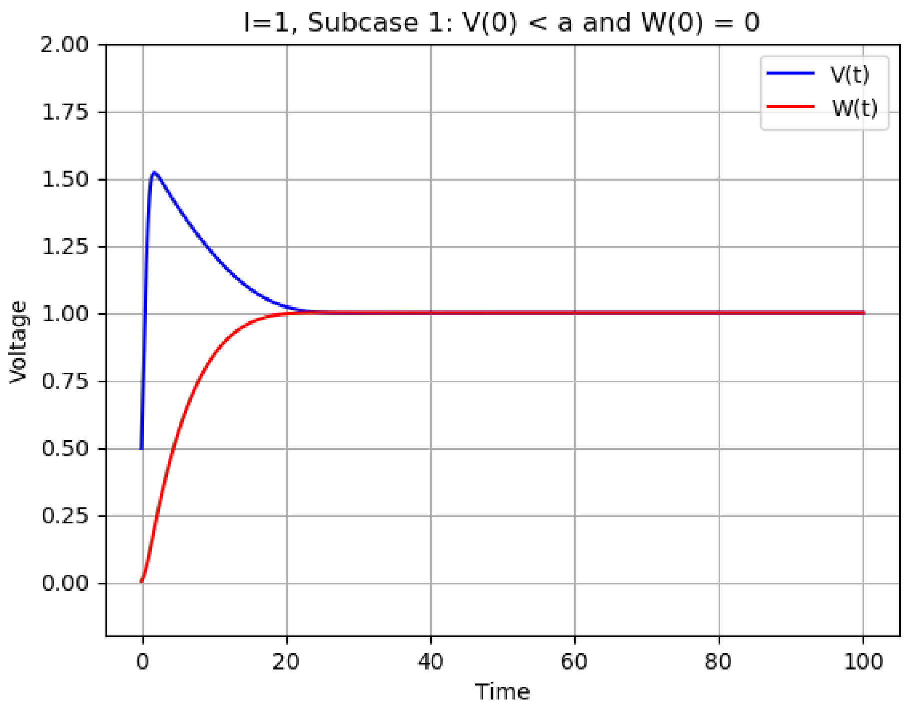


**Figure 11:** $V(t), W(t)$ across $t$, when $V(0) < a$. With sub-threshold pulse injections, depolarization in the action potential can be observed.

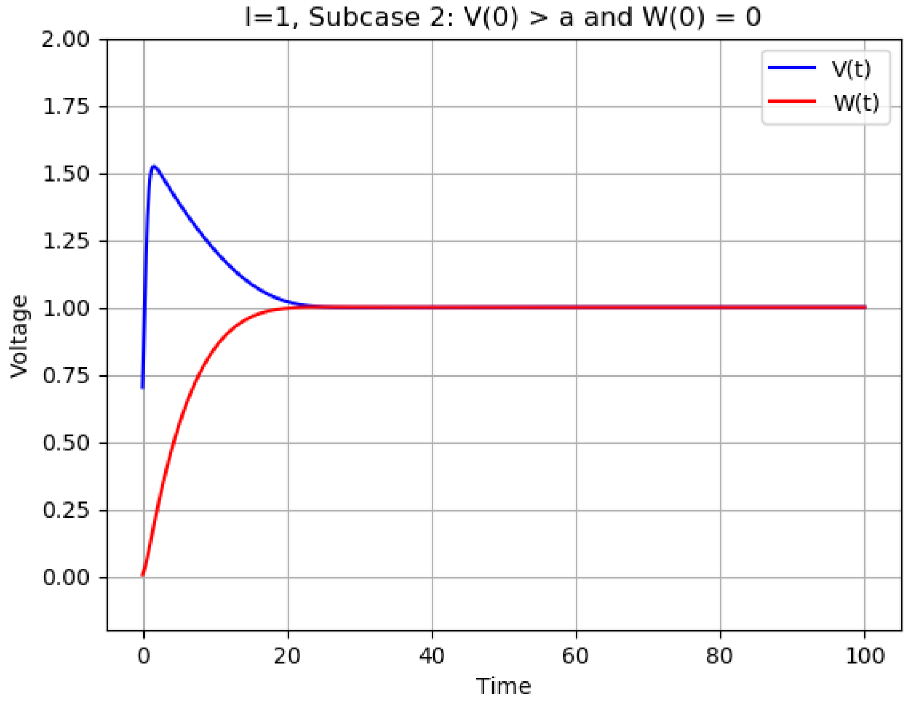


**Figure 12:** $V(t), W(t)$ across $t$, when $V(0) > a$. With sub-threshold pulse injections, depolarization in the action potential can be observed.

# 4 Case 4: $I_{ext} = 0.02$

The parameter values used to simulate this case are: $b = 0.01, r = 0.8$. Hence, $b/r = 0.0125$.

## 4.1 Phase Plot

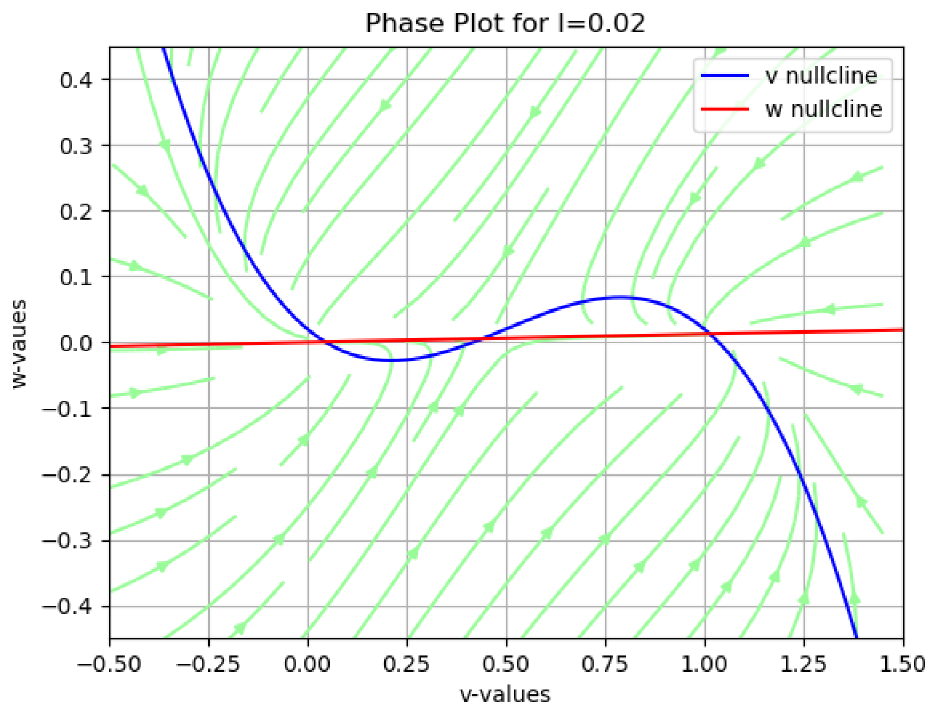


**Figure 13:** Phase Plot of the system when $I_{ext} = 0.02$. The stationary points P1, P2 and P3 in that order are stable, saddle and stable points respectively.

The trajectories were analyzed by using initial points - $[0, 0.4, 0.6, 1], w = 0.6$ and $[0, 0.4, 0.6, 1], w = 1.4$. The stationary points are P1, P2 and P3, in that order. In case of P1 and P3 - small and intermediate perturbations lead back to P1 and P3 respectively. Hence P1 is a stable point. In case of P2, small perturbations along one axis leads to large change in final point. Hence, P2 is a saddle node.

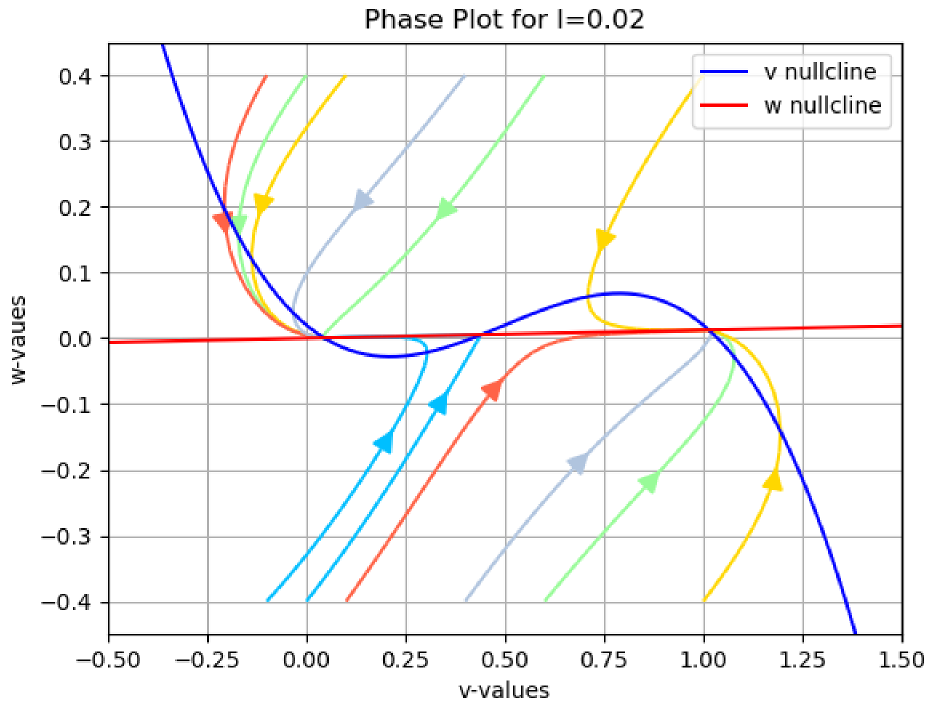


**Figure 14:** Stability analysis of the stationary point. The stationary point is found to be unstable and limit cycle behavior is also observed.

## 4.2 $V(t), W(t)$ across $t$, Trajectories

For $I_{ext} = 0.02, r = 0.8, b = 0.01$, bi-stability is observed.

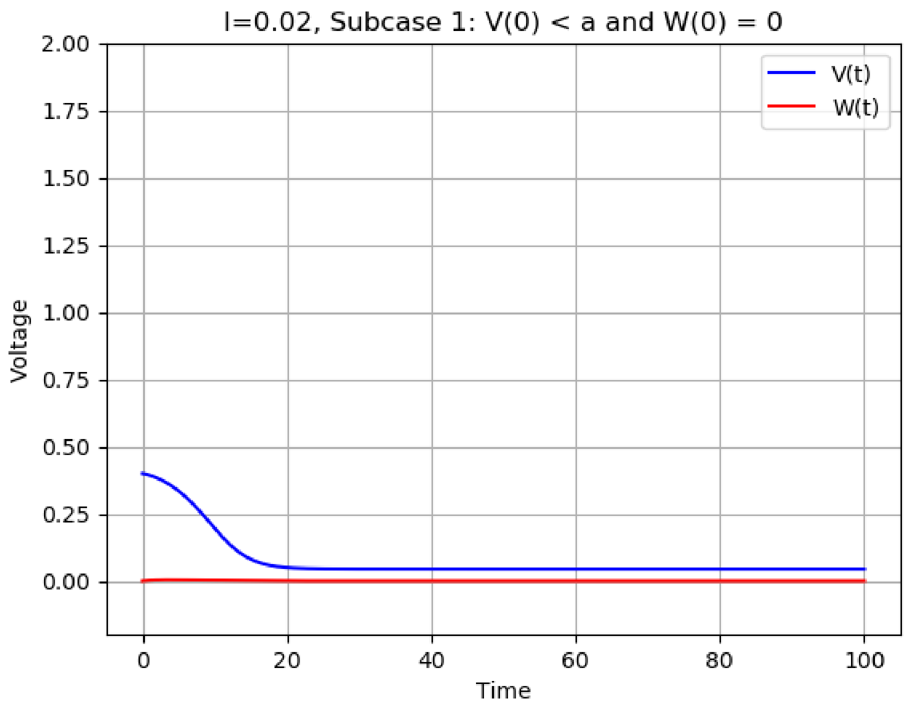


**Figure 15:** $V(t), W(t)$ across $t$, when $V(0) < a$. The neuron exists in a tonically down state.

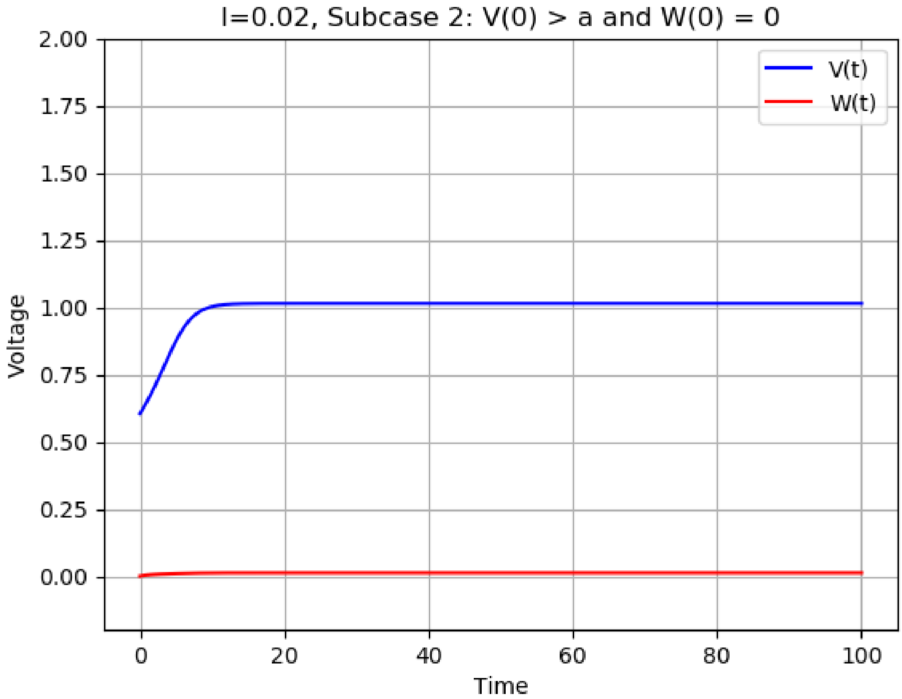


**Figure 16:** $V(t), W(t)$ across $t$, when $V(0) > a$. The neuron exists in a tonically up state.

# References

[1] Wikipedia, “FitzHugh-Nagumo model.” https://en.wikipedia.org/wiki/FitzHugh%E2%80%93Nagumo_model.