Continuous Simulation of Galactic Collision

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1 Problem Description

We aim to utilize continuous simulation techniques to model the interactions between two galaxies. When galaxies come in close contact with one another, tidal forces (a secondary effect of the gravitational force) cause their shape and structure to change. Common deformities include a narrow bridge between the two galaxies or the formation of a tail. We can study these effects by numerical integration of the restricted three-body equations of motion for a collection of test particles. Time value will be an input, and a plot of point masses representing stars will be presented as output. We ultimately sought to recreate three cases seen in the Toomre and Toomre paper:
Case 1) Main galaxy mass ≈ Disruptor galaxy mass
Case 2) Main galaxy mass » Disruptor galaxy mass
Case 3) Main galaxy mass « Disruptor galaxy mass

2 Conceptual Model

State Variables: Two point masses indicating the two galaxies on a collision course. We set our frame of reference to be one of the two galaxies, and call it the "main" galaxy. As a consequence, the "main" galaxy is stationary in our reference frame. The other galaxy is considered to be "disruptor" galaxy, whose relative motion to the main galaxy will be modeled.

We also consider test particles in the main galaxy and model how the interaction between the galaxies will disrupt their velocity and orbital path in reference to the main galaxy.

The main galaxy will have 300 stars, represented by massless point particles. The stars will be distributed within 5 concentric rings on th base layer around the central mass of the galaxy. These rings will contain 12, 18, 24, 30, and 36 stars and will be placed at exactly 12, 18, 24, 30, and 36 percent of R (the distance between the two galaxies). Additionally, over each of the above concentric, we have also layered similar sized rings, to represent the bulkiness of the galaxies near their center. The extra rings at each level are added with a specified gap. We will only be modeling the stars of the main galaxy so we can clearly see how the disrupting galaxy changes its form.

Since, for the purpose of this project, we've tried to model and replicate the results of the Toomre paper, we have used similar initial conditions used in the said paper. There are 3 distinct initial values (w.r.t the masses of the galaxies) we have experimented with:

- Main galaxy mass = 300 solar masses, disruptor galaxy mass = 300 solar masses
- Main galaxy mass = 300 solar masses, disruptor galaxy mass = 82.5 solar masses
- Main galaxy mass = 82.5 solar masses, disruptor galaxy mass = 300 solar masses

Additionally for all the above cases, approaching velocity of the disruptor galaxy = 0.85 km/s in x direction, 0.65 km/s in y direction. Initial position (x, y, z)= (-8 parsecs, -9 parsecs, 0)

To compute the initial velocities of the stars in the main galaxy, we'll use the vis-viva equation:

$$v_0 = \sqrt{\frac{2GM}{r_0^2}}$$

Differential Equations: Differential equations will govern the changing values of state variables per each simulated entity. A deductive approach is the most feasible - the behavior of the system will arise from applied physical laws. Equations modeling the gravitational forces among objects will be utilized to maintain the velocities of these objects, along with other factors such as spin and shape of the galaxies.

Using the 3-body equations of motion, the positions of each star in the main galaxy will be represented by the differential equation (1):

$$\ddot{\mathbf{r}} = -G(\frac{M}{r^3}\mathbf{r} - \frac{S}{\rho^3}\boldsymbol{\rho} + \frac{S}{R^3}\mathbf{R})$$

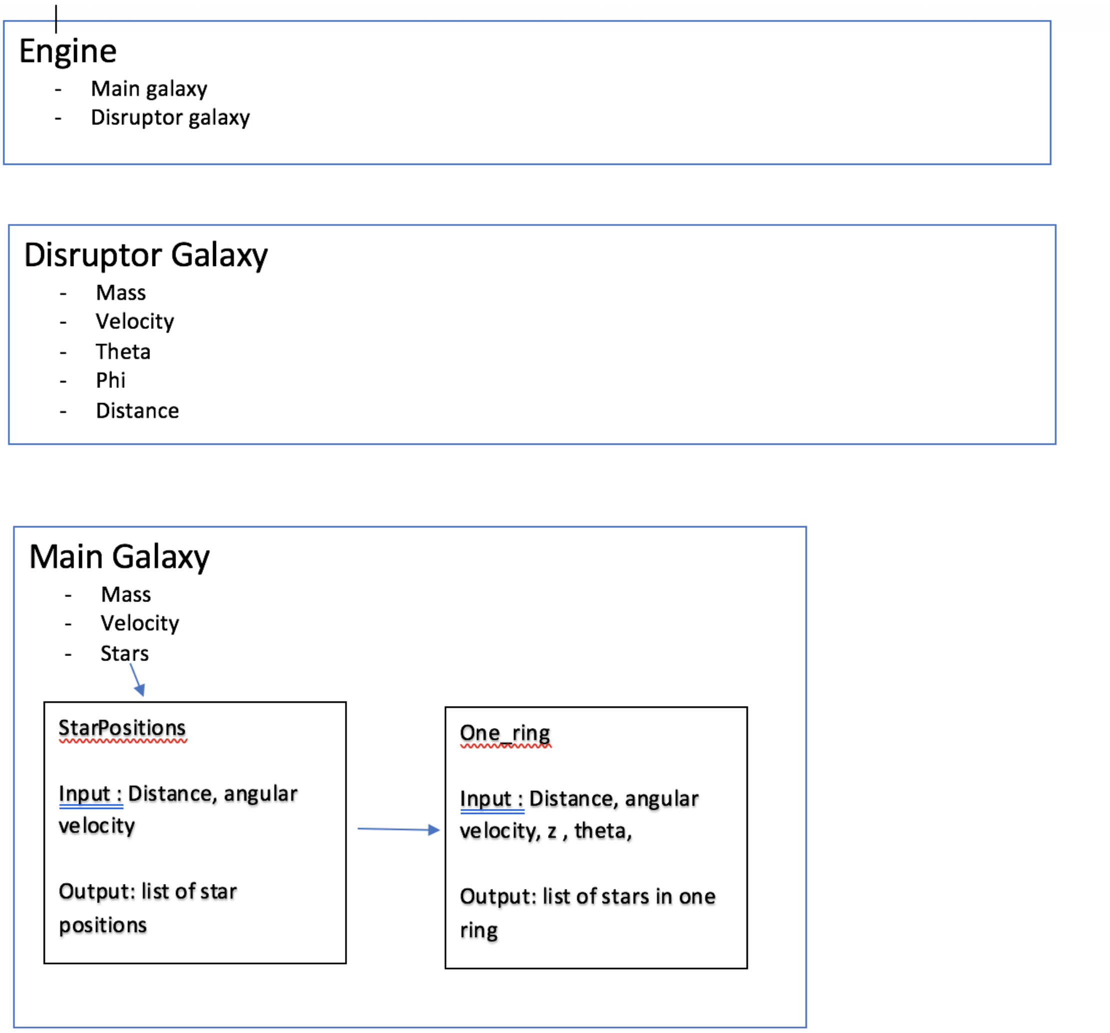
and the position of the disruptor galaxy will be represented by the differential equation (2):

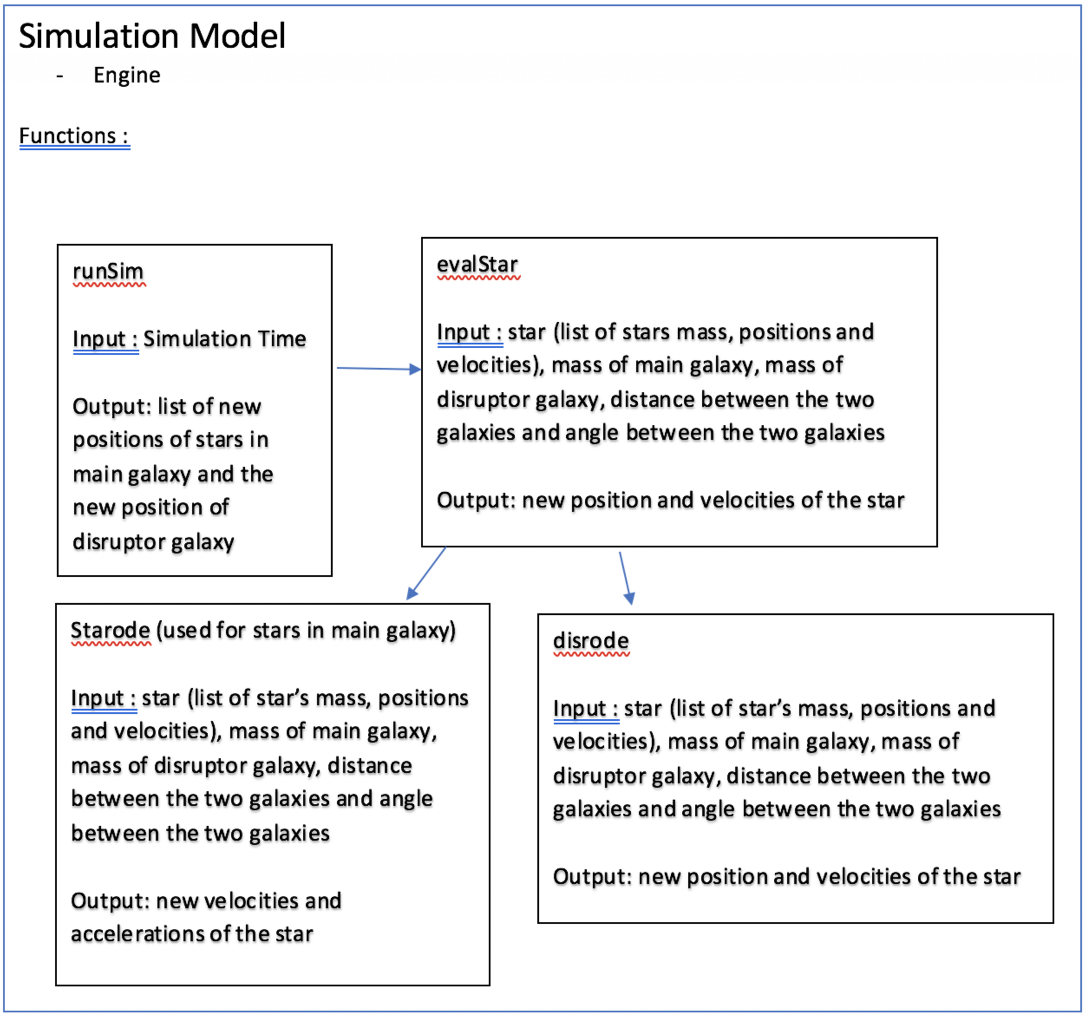
$$\ddot{\mathbf{R}} = -G(\frac{M+S}{R^3}\mathbf{R})$$

where

- G = Gravitational constant
- M = Central mass of main galaxy
- S = Central mass of disruptor galaxy
- $\mathbf{r}$ = Radius vector from main galaxy to a massless point particle (star)
- $\mathbf{R}$ = Radius vector from main galaxy to disruptor galaxy
- $\boldsymbol{\rho}$ = $\mathbf{R}$ - $\mathbf{r}$

3 Block Diagram





The software is essentially composed of a few initialization methods that are utilized to create an environment for which the main simulation methods may calculate changes in state variables using differential equations in an optimal manner.

4 Project Plan

The project checkpoint requires that an initial implementation of the simulator be completed, where code is modular and well-documented. Given these requirements, we will build our code in a modular fashion where work can be divided among group members and merged into a single simulation. We anticipate utilizing several Python-based numerical tools including NumPy, Scipy, and MatPlotLib. In particular, SciPy offers a differential equation integrator which will be useful for our simulation.

Task division:

- Coding the initialization steps for the simulation model: Matthew
- Coding the interactions between the galaxies using differential equations : Mollene and Sowmya
- Coding the graphical visualization methods: Matthew, Mollene, Sowmya

5 Software Description

For the first project checkpoint, we were focused on demonstrating functionality of a modular implementation of the galactic collision simulation. For the final report, we hope to have more advanced functionality added as well as actual galactic data in order to simulate the real-world collision of the Milky Way and Andromeda, but for now we are using simple hard-coded initial star positions and velocities.

Our code is structured such that the two separate galaxies are initialized separately where the main galaxy is centered at the origin with five concentric rings of stars of sizes 12 through 36, increasing by a count of six stars at each ring.

User Input: Mass of the two galaxies (M, S); Initial distance between two galaxies; Time at which to display plot visualization; Number of time steps required. The inputs are fed to the program with input.txt file. As mentioned before, for this project we experimented with the following (M, S) = (300, 300), (300, 82.5), (82.5, 300)
Program Output: A plot representing the positions of all stars at a specific timestep.

Python was selected as our language of choice and numerical tools were used from the packages NumPy, MatPlotLib, and SciPy. The `odeint` function from SciPy was utilized to solve the ordinary differential equations described in our conceptual model in order to update star positions. MatPlotLib's 3D scatter plot is utilized for program output of star plots.

The current implementation is handled by two Python files, `SimulationModel.py` and `Engine.py`.

`SimulationModel.py`

- class SimulationApplication
  - **function runSim**
    In this function, we loop through time at each time step instance, for the disruptor galaxy and all the stars of the main galaxy and compute the changed position and velocity values at the end of each time-step, by calling evalStar functions. It also repeats this for the disruptor galaxy. Note that disruptor galaxy is considered to be a point mass, as discussed before.
  - **function starode**
    This function solves differential equation (1) from the conceptual model for a single star within the main galaxy. It takes an initial position of a star along with the physical constants associated with the galaxies to compute a new vector containing the velocity and acceleration in the x, y, and z positions.
  - **function disrode**
    This function solves the differential equation (2) from the conceptual model for the position of the disrupter galaxy. It takes the distance vector between the two galaxies along with other physical constants and from that computes the velocity and acceleration of the disruptor galaxy. We chose to treat the disruptor galaxy as a point mass, as we are more concerned with the effects to the shape of the main galaxy. A point mass will have a similar impact as breaking the galaxy up into its many stars.
  - **function evalstar**
    This function uses the previous ode functions to compute the new position of a star (or of the disruptor galaxy) for a specified number of time steps.

`Engine.py`

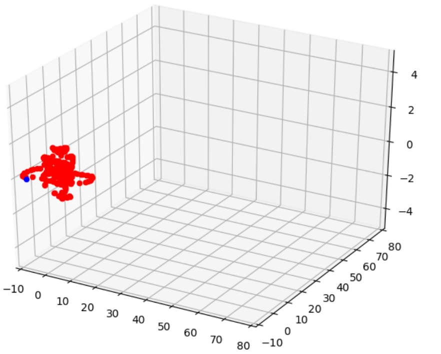
- class MainGalaxy
  - **function starPositions**
    Initializes an array of star positions determined by currently hardcoded number of stars in each ring and radii sizes.

- **function onering**
  Called by starPositions() and places stars in a single ring determined by the radius of the ring and the number of stars in the ring.

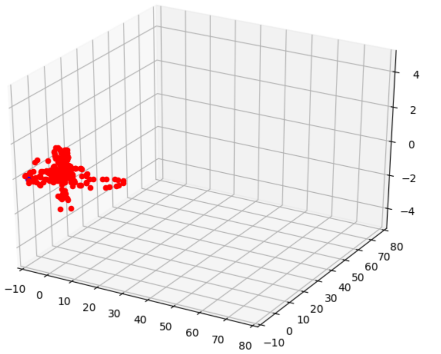
- class DisruptorGalaxy
  Initializes the point mass representing the disruptor galaxy.

6 Experimental Results

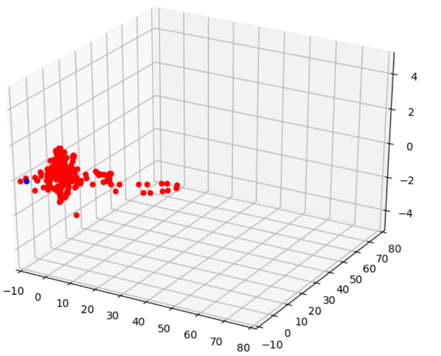
Below are the experimental results for the case where the disruptor mass is a 4x smaller than the mass of the main galaxy. The disruptor galaxy is represented as the blue point and the main galaxy is composed of all of the red points in the scatterplot. The axes are fixed to preserve relative scale.



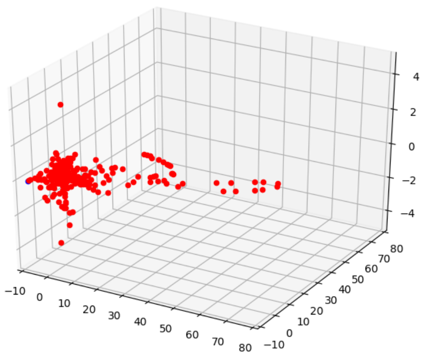
(a) first timestep



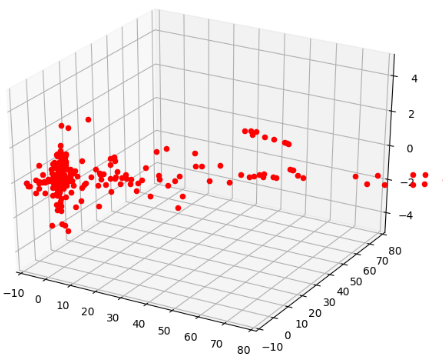
(b) second timestep



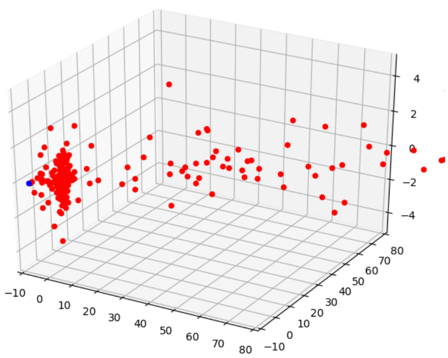
(c) third timestep



(d) fourth timestep

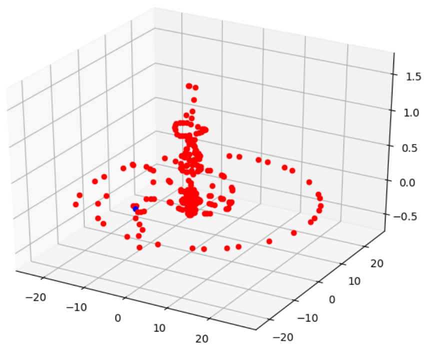


(c) fifth timestep

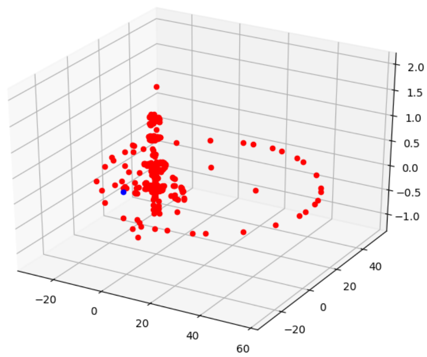


(d) sixth timestep

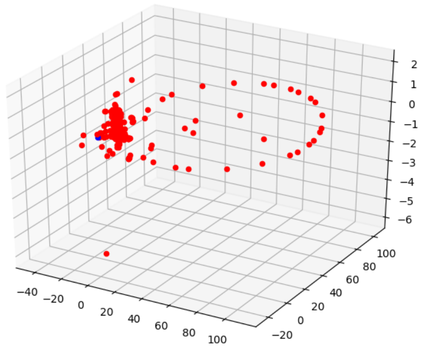
Below is the case for mass of the disruptor 4x greater than the mass of the main galaxy.



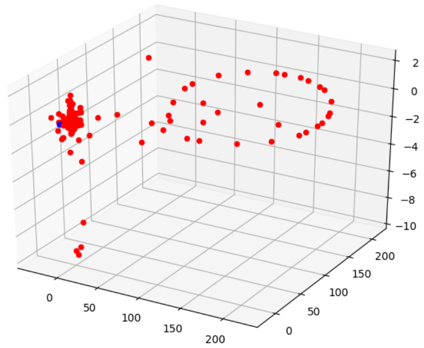
(a) first timestep



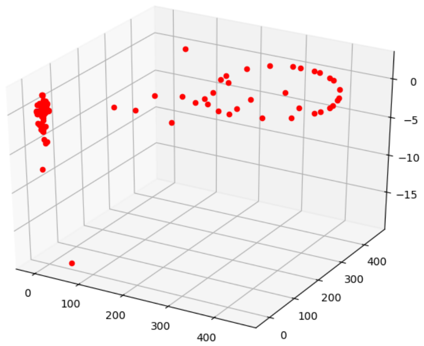
(b) second timestep



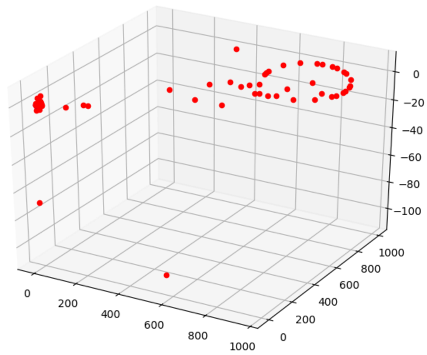
(c) third timestep



(d) fourth timestep



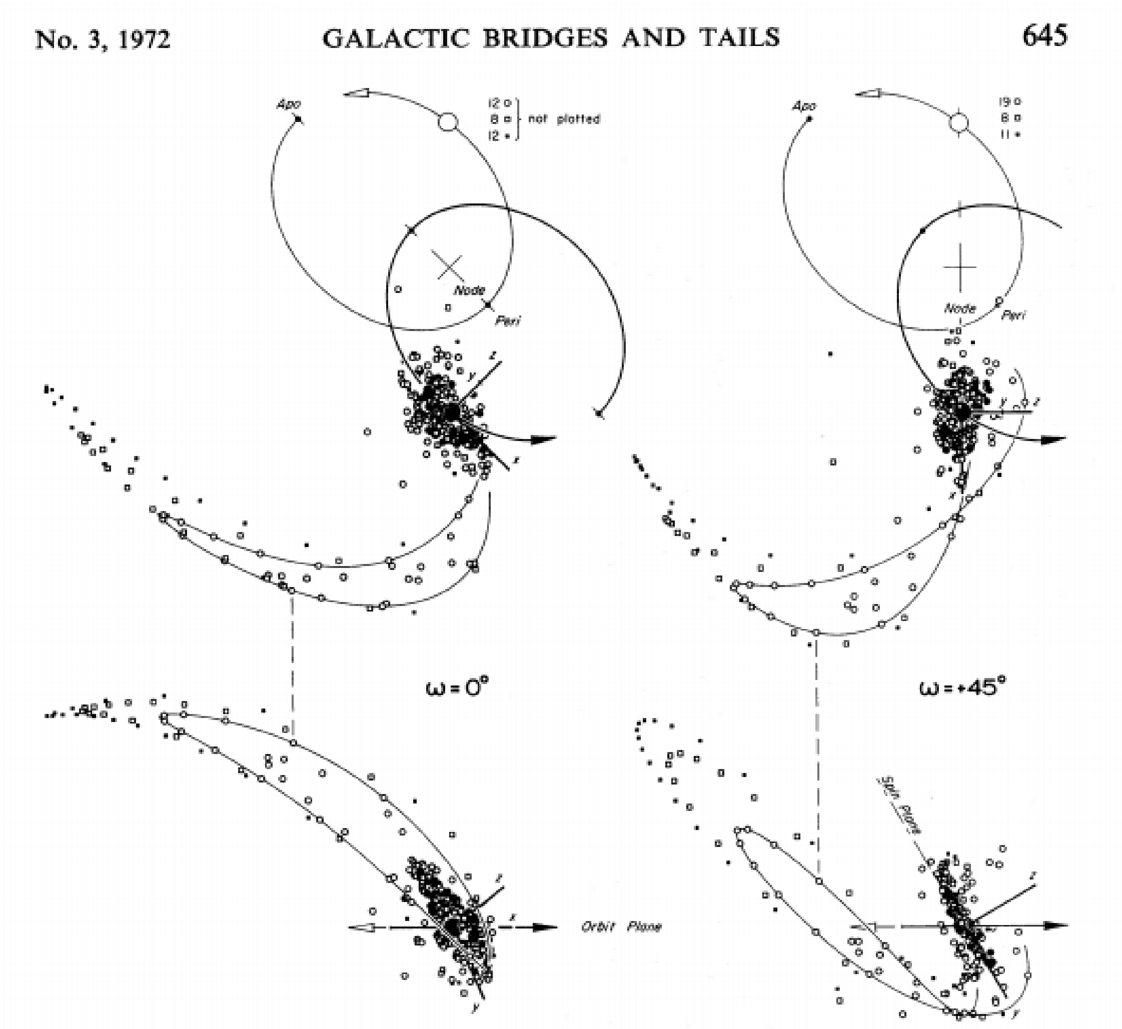
(c) fifth timestep



(d) sixth timestep

7 Validation and Verification

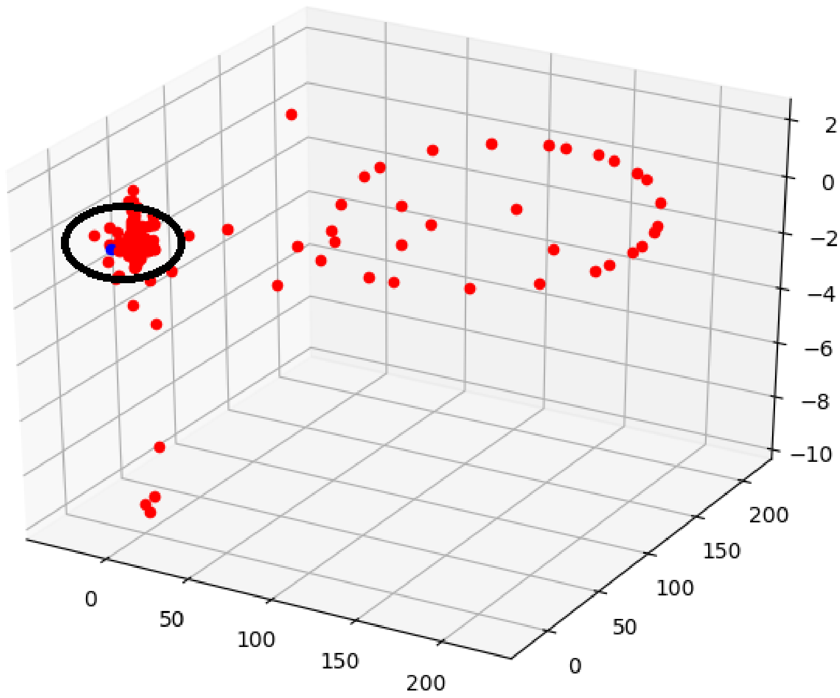
To validate our code, we altered the values of our input variables and compared the results of the simulation. We know from the Toomre paper that the passage of an equally or more massive disruptor will result in a galactic "tail", whereas the passage of a smaller disruptor will result in a "bridge" between the two galaxies. By changing the mass ratio of the two galaxies, we were able to observe similar effects in our simulation as Toomre and Toomre did in their paper, pictured on the next page.



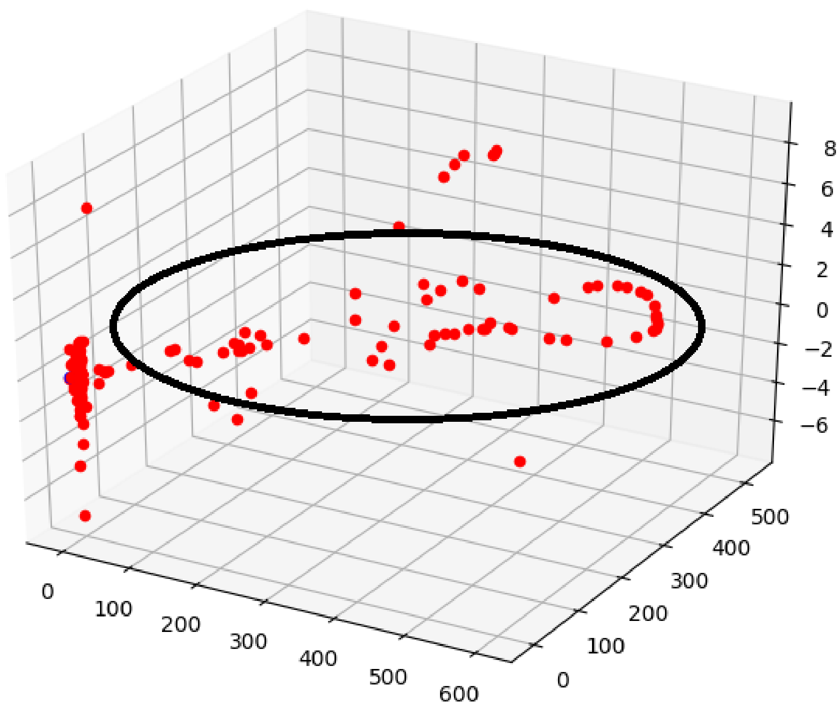
For verification, interactive debugger was utilized to monitor variable values. All inputs were varied and program operation was monitored. Small time steps were used in order to verify that the differential equations were properly affecting positions and velocities. The model output was also examined for reasonableness under a variety of input parameters.

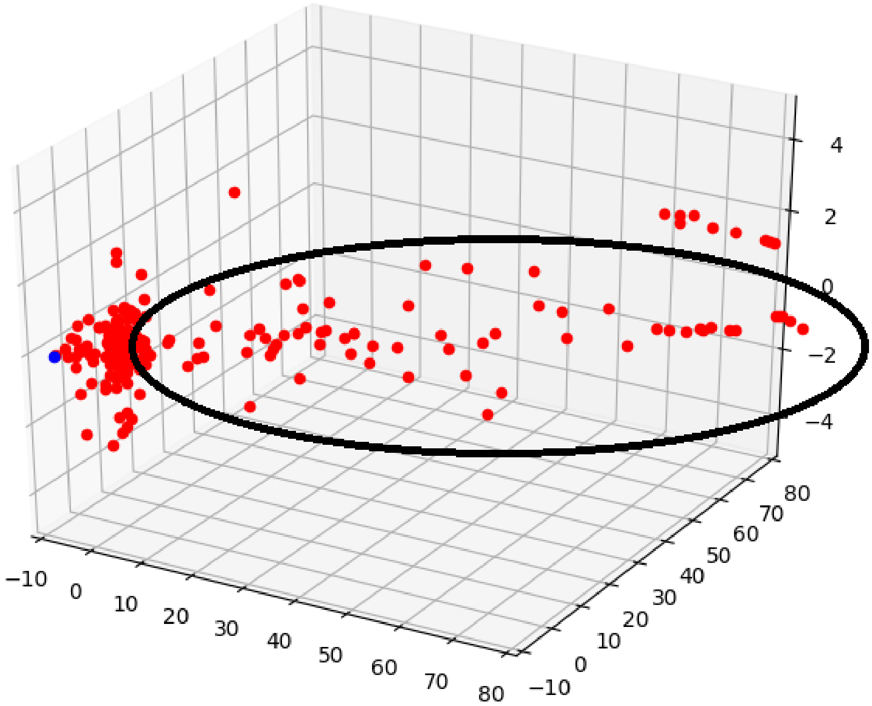
8 Conclusions

In these visualizations, we were able to see how galaxy interactions form bridges and tails. As in the Toomre paper, after a direct passage of a small companion (Case 2), the outer portions of the primary disk often deform both into a near-side spiral arm or "bridge". In the below image from our Case 2 simulation, we can see that the main galaxy is joined to the disruptor, showing a bridge between them. There is no evidence of a tail here, as the loose stars are no longer connected to the main galaxy, and are not in a thin, slender shape.



Alternatively, a similar encounter with an equal or more massive partner (Case 1 and 3) resulted in a long and curving "tail". In the first image below, we see a tail form with the Case 1 inputs. We see a similar tail form in Case 3, but this time with fixed axes. In both cases, instead of being pulled towards the disruptor galaxy, a long slender stream of stars is pulled away from the other side of the main galaxy.





Ultimately, we can conclude that the observation of a "bridge" between two nearby galaxies shows signs of a small mass disruptor. On the other hand, if a galaxy has a long "tail" coming out of it, it shows signs of a heavy or equal mass disruptor. Our results are again consistent with the Toomre paper.

9 Future Work

For this project to be a viable scientific use application, a few improvements should be made. There should be an easier way to customize inputs, which a graphical user interface can achieve. There should also be a higher quantity of stars on the order of millions so that more precise observations can be made, but this will require much more computing power.

10 References

A. Toomre, J. Toomre, “Galactic Bridges and Tails”, The Astrophysical Journal 178, 623-666 (1972)

Baiesi Pillastrini, Giovanni. (2009). On the Adromeda to Milky Way mass-ratio. Monthly Notices of the Royal Astronomical Society. 397. 1990 - 1994. 10.1111/j.1365-2966.2009.15109.x.