galaxy formation theory

lecture 1

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outline of the course

- a brief review of the observational background
- assembly of dark matter haloes
- gas cooling
- angular momentum
- star formation
- feedback
- galaxy mergers & morphology
- evolution of supermassive black holes

lecture notes:
https://github.com/sownakbose/AstroPGCourse-Galform

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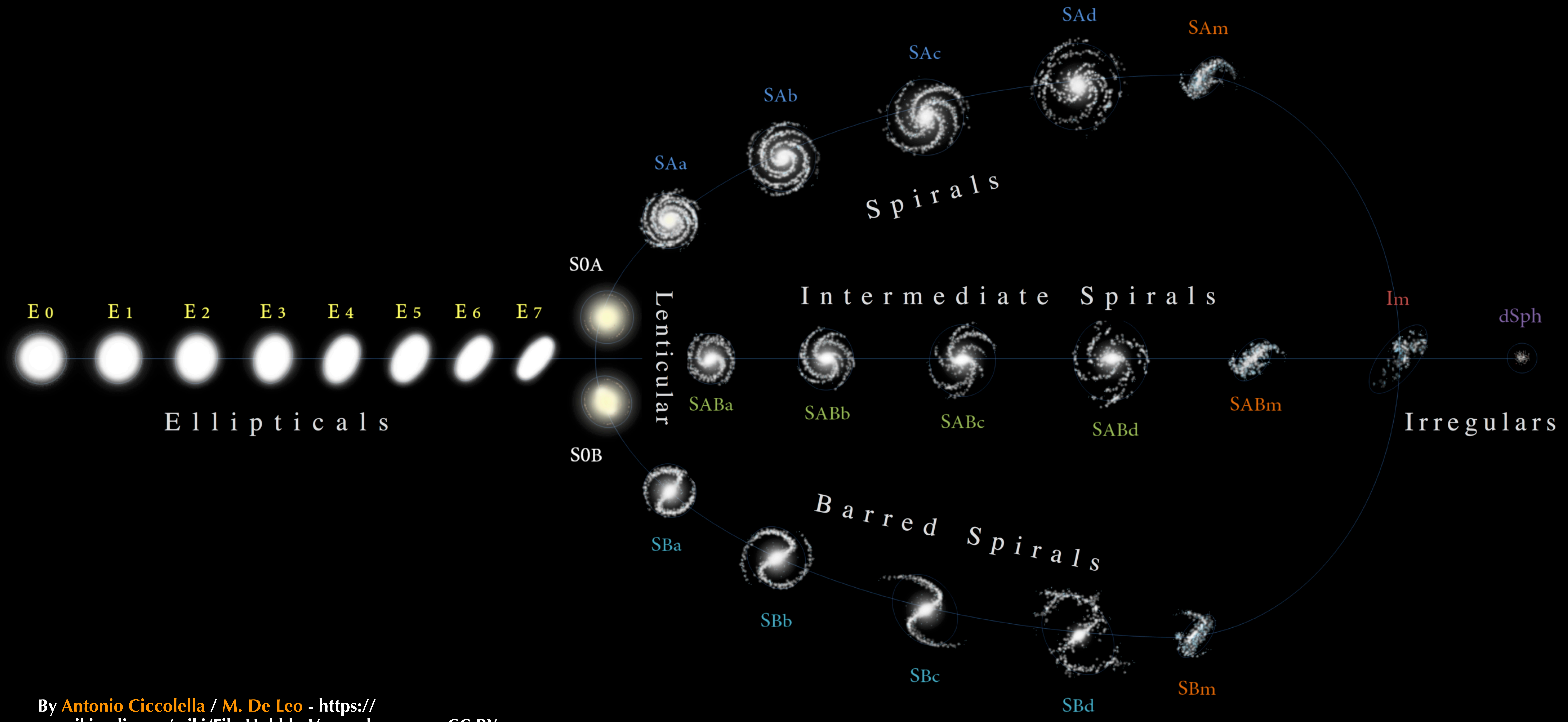
1. the observational background

a broad classification of galaxy morphologies

discs

spheroids

HUBBLE-DE VAUCOULEURS DIAGRAM



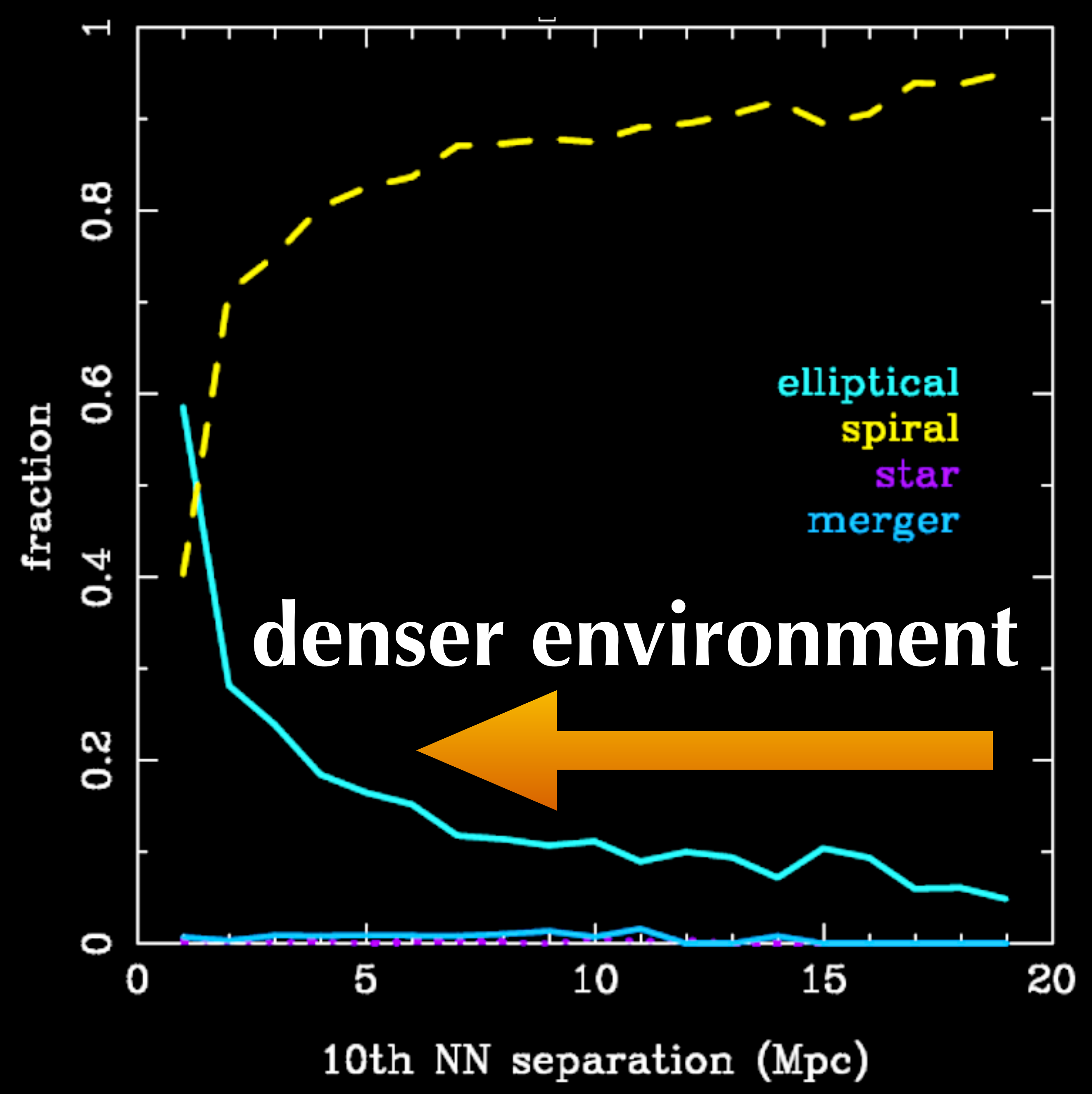
morphological correlations

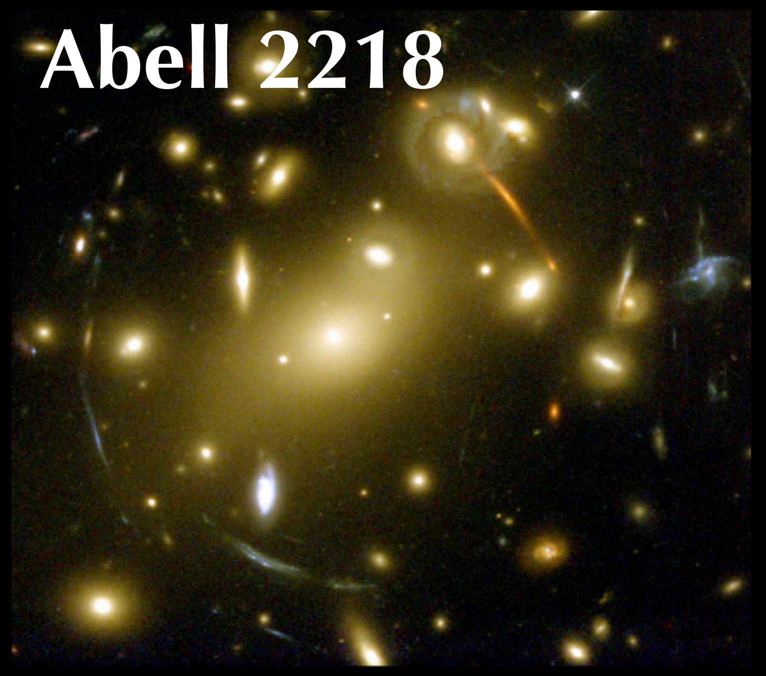
morphology varies most strongly with:

- luminosity/mass: most massive/ luminous galaxies are ellipticals
- environment: high-density regions are dominated by E/S0 while low-density "field" is where we find spirals and irregulars.

and correlates with:

- colour: most E/S0 galaxies are red, while most spirals/Irr are blue
- spectral type: most spiral/Irr have strong emission lines, while most E/S0 are absorption line systems



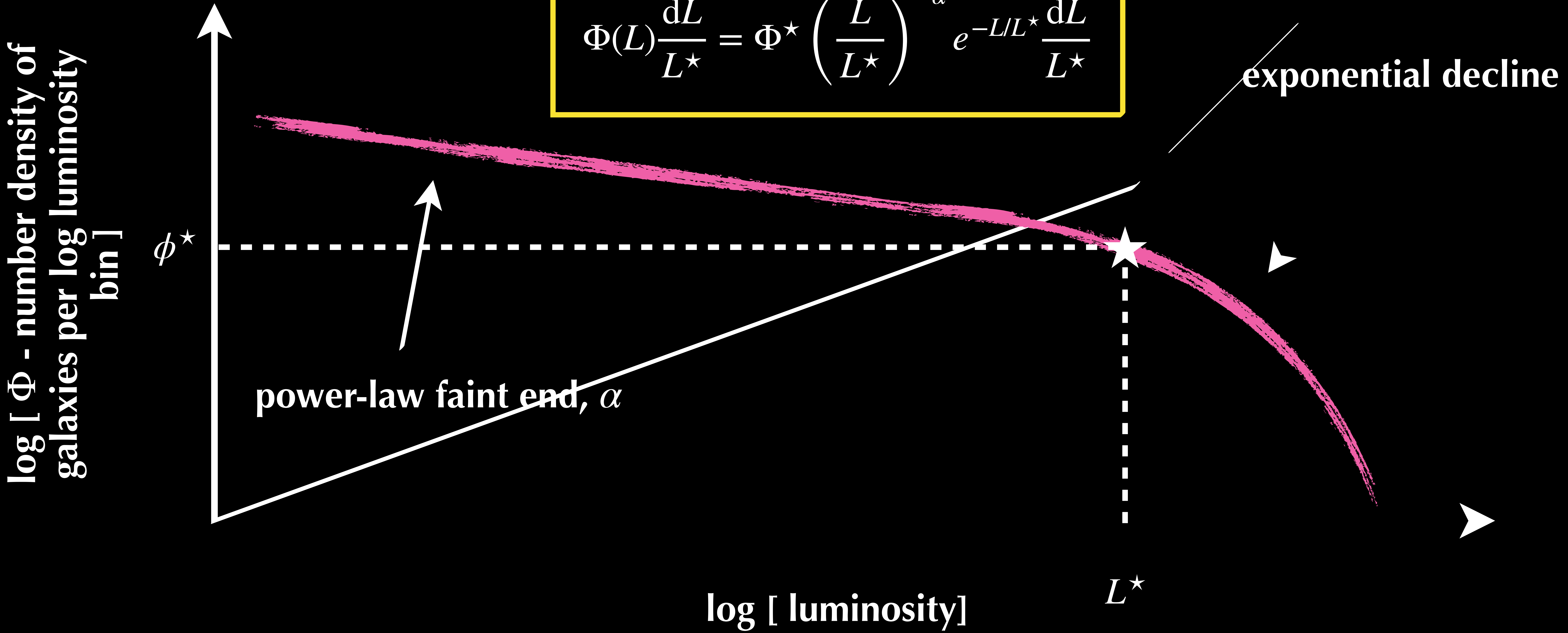


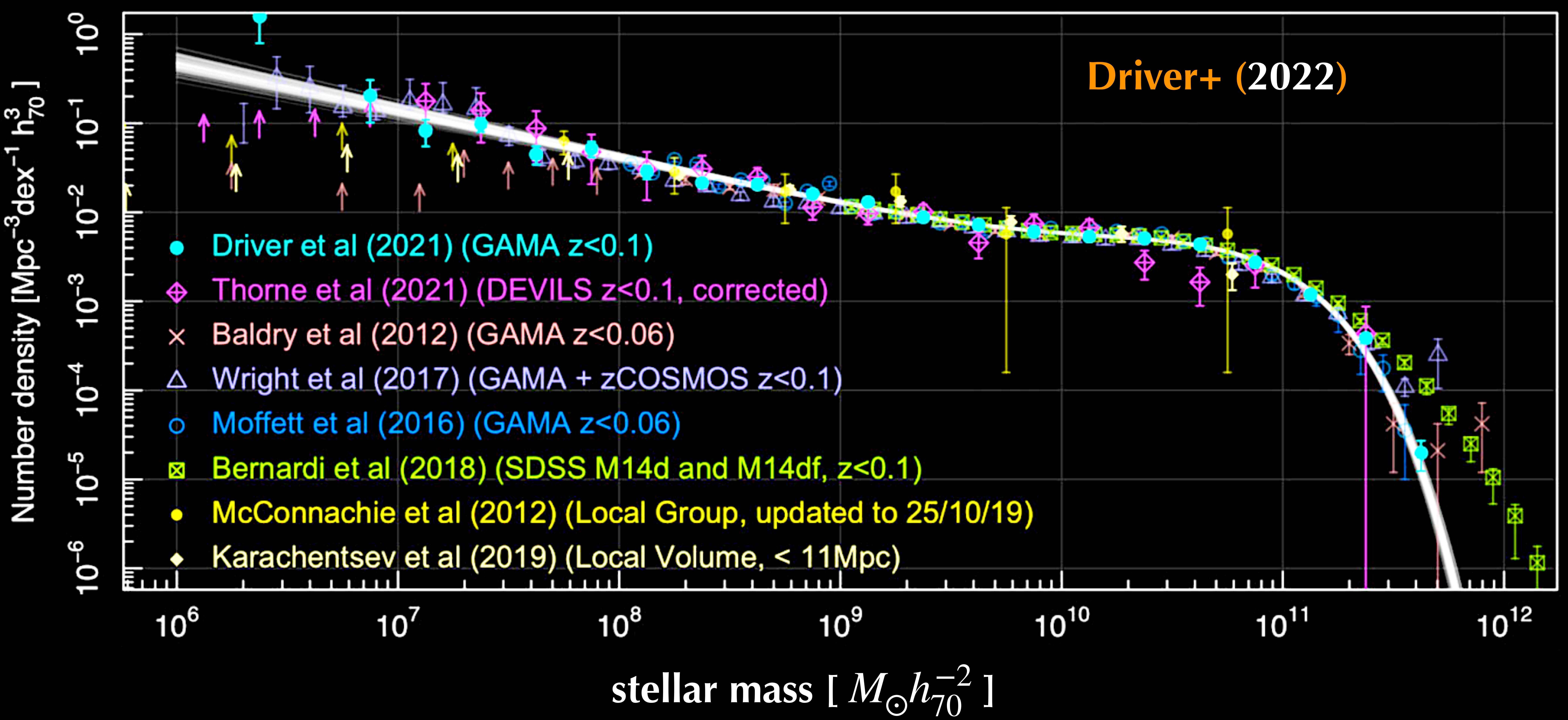
distribution of galaxy luminosity / masses

the galaxy luminosity function

Schechter (1976)

$$\Phi(L)\frac{\mathrm{d}L}{L^\star} = \Phi^\star \left(\frac{L}{L^\star}\right)^{-\alpha} e^{-L/L^\star}\frac{\mathrm{d}L}{L^\star}$$

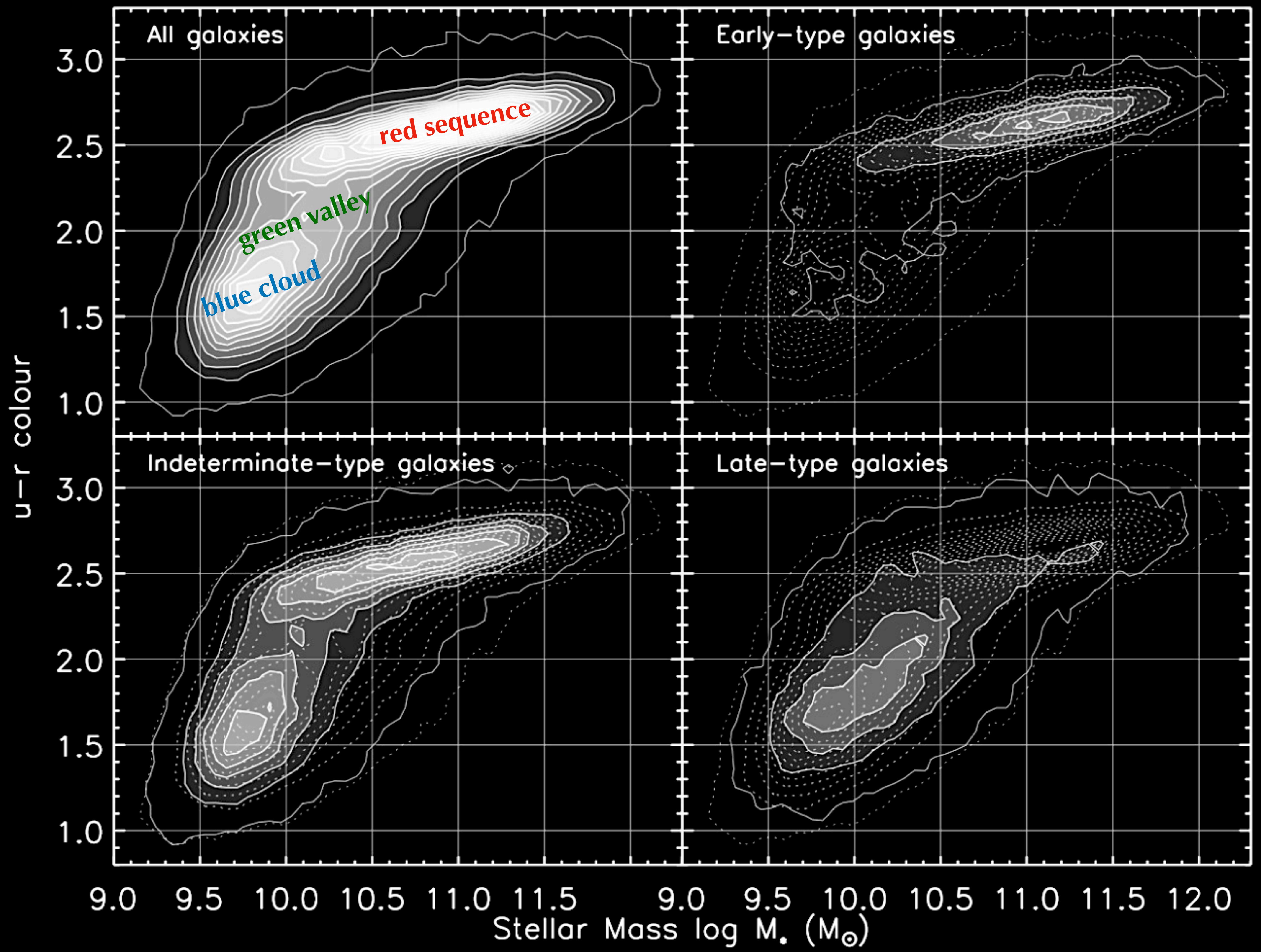




the present-day **stellar mass function** is obtained by fitting **stellar population models** to the broad-band spectral energy distributions (**SEDs**) of galaxies — also fit well with a Schechter function (though, more commonly, a double-Schechter function is used)

correlations/scaling relations of galaxy properties

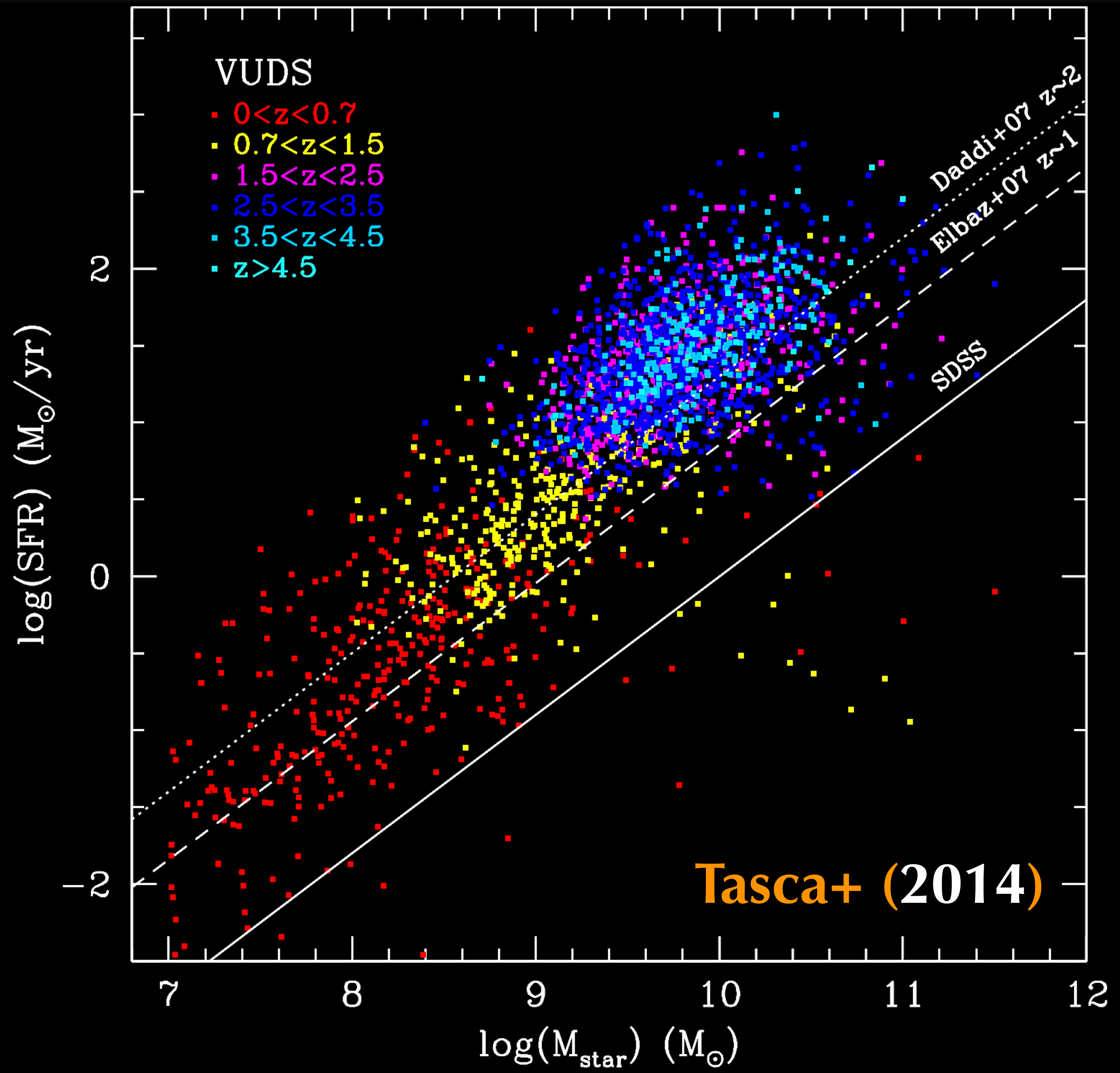
the distribution of galaxy colours at present day



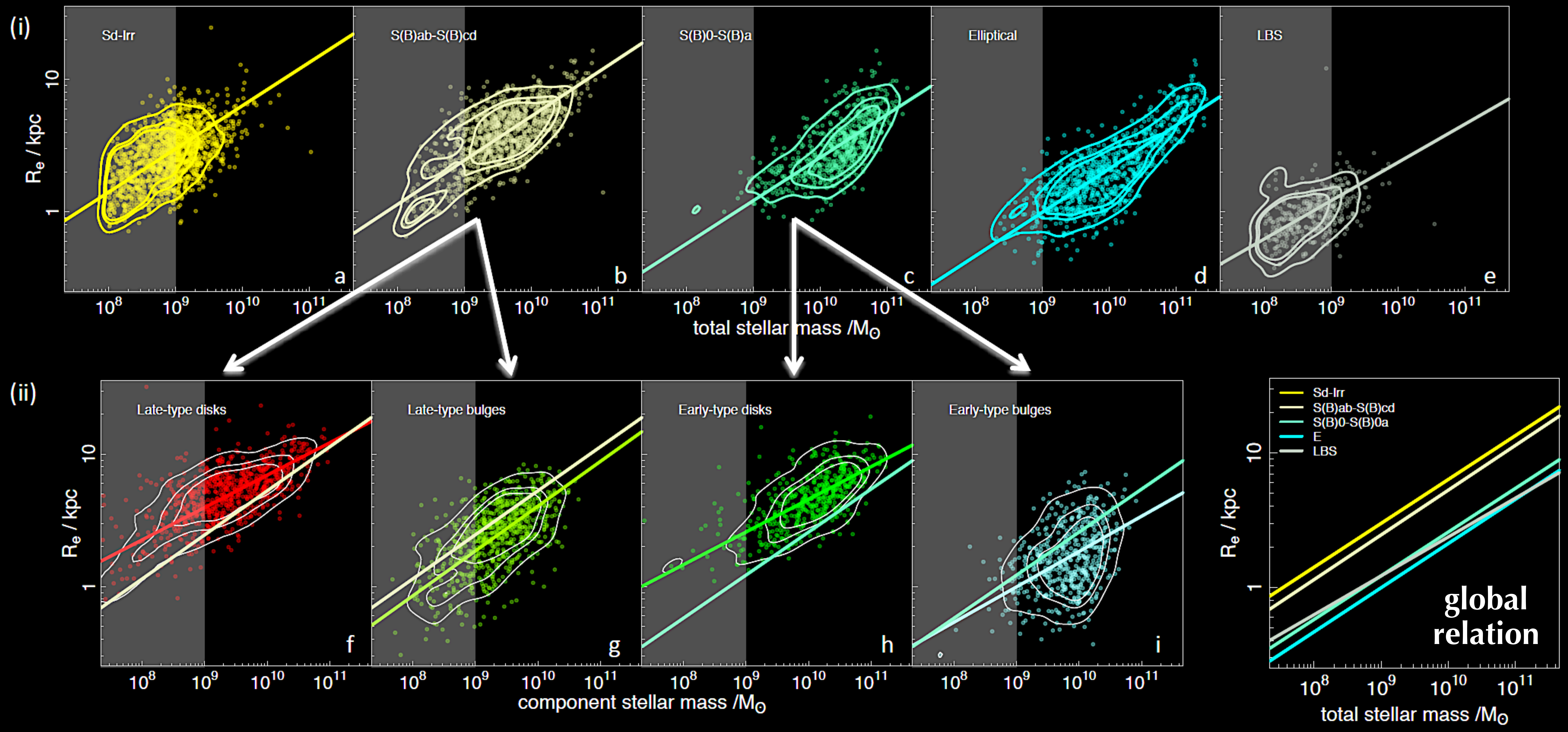
galaxy colours roughly delineate star-forming vs passive/quenched galaxies

SDSS data [credit: https://pages.astronomy.ua.edu/keel/galaxies/systematics.html]

star-forming galaxies lie along a narrow relation



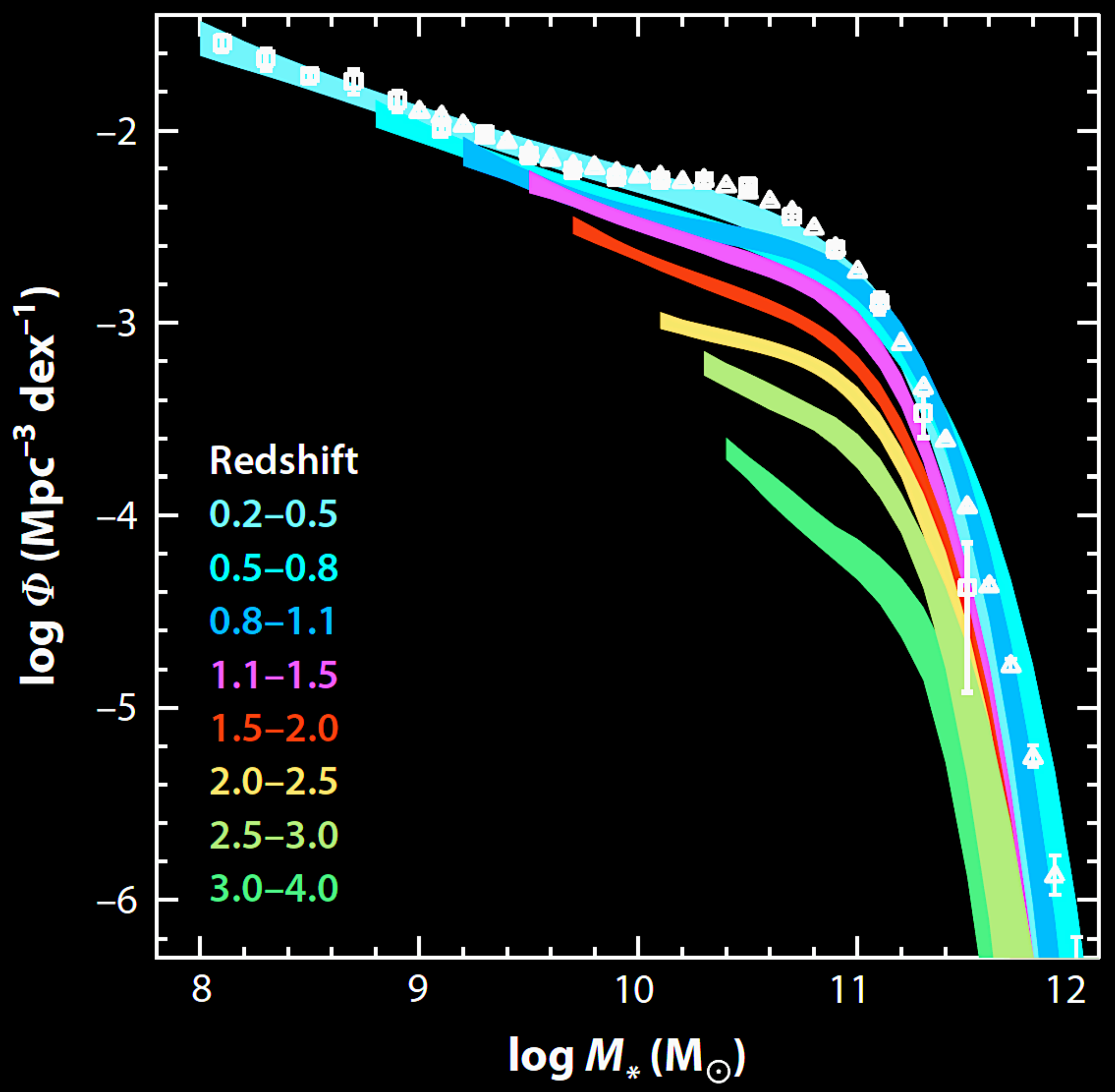
[sometimes known as the "star-forming main sequence"]

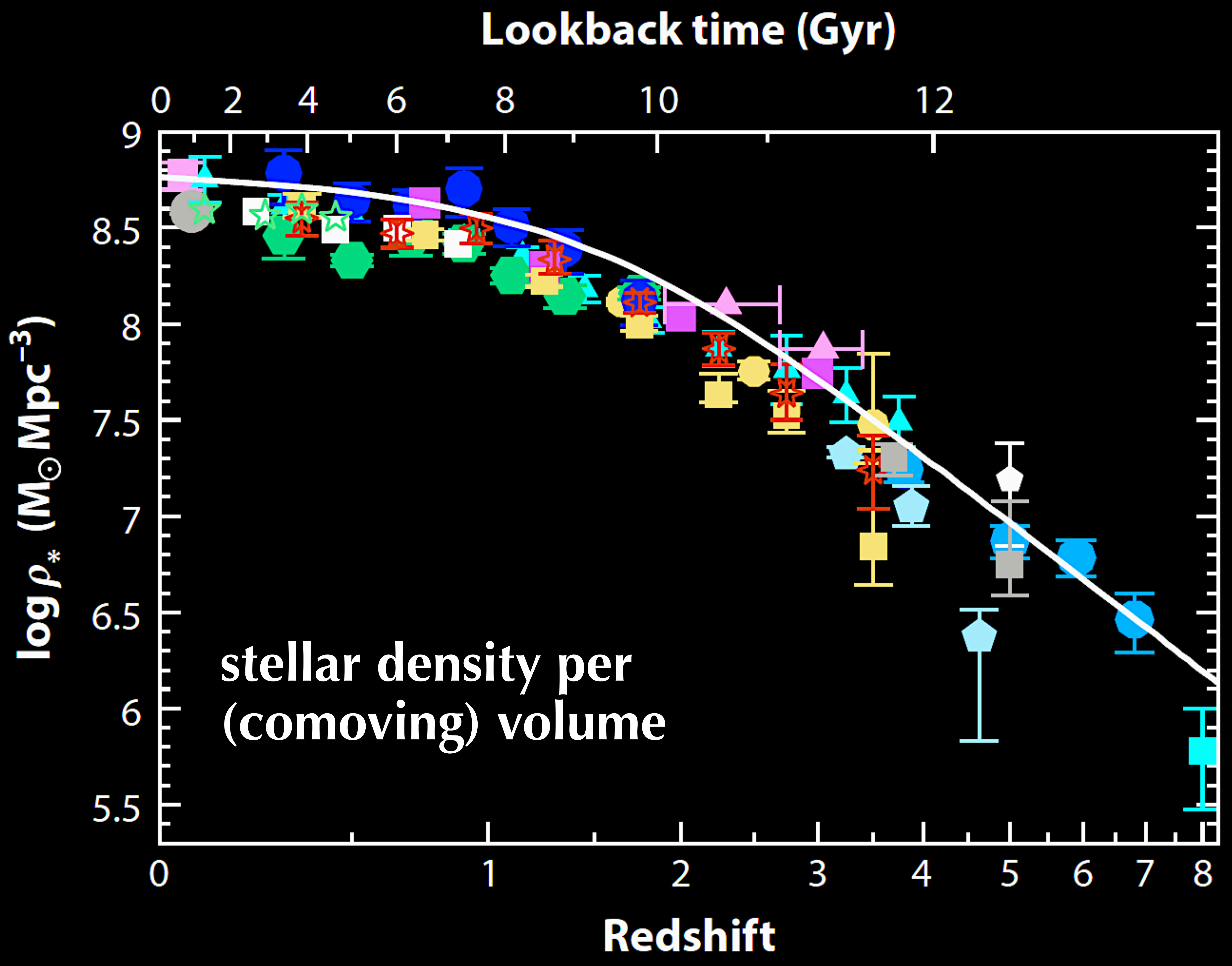

(i)
Sd-Irr
S(B)ab-S(B)cd
S(B)0-S(B)a
Elliptical
LBS
R_e / kpc
total stellar mass /M_⊙
(ii)
Late-type disks
Late-type bulges
Early-type disks
Early-type bulges
component stellar mass /M_⊙
Sd-Irr
S(B)ab-S(B)cd
S(B)0-S(B)0a
E
LBS
global relation
total stellar mass /M_⊙

Lange+ (2016)

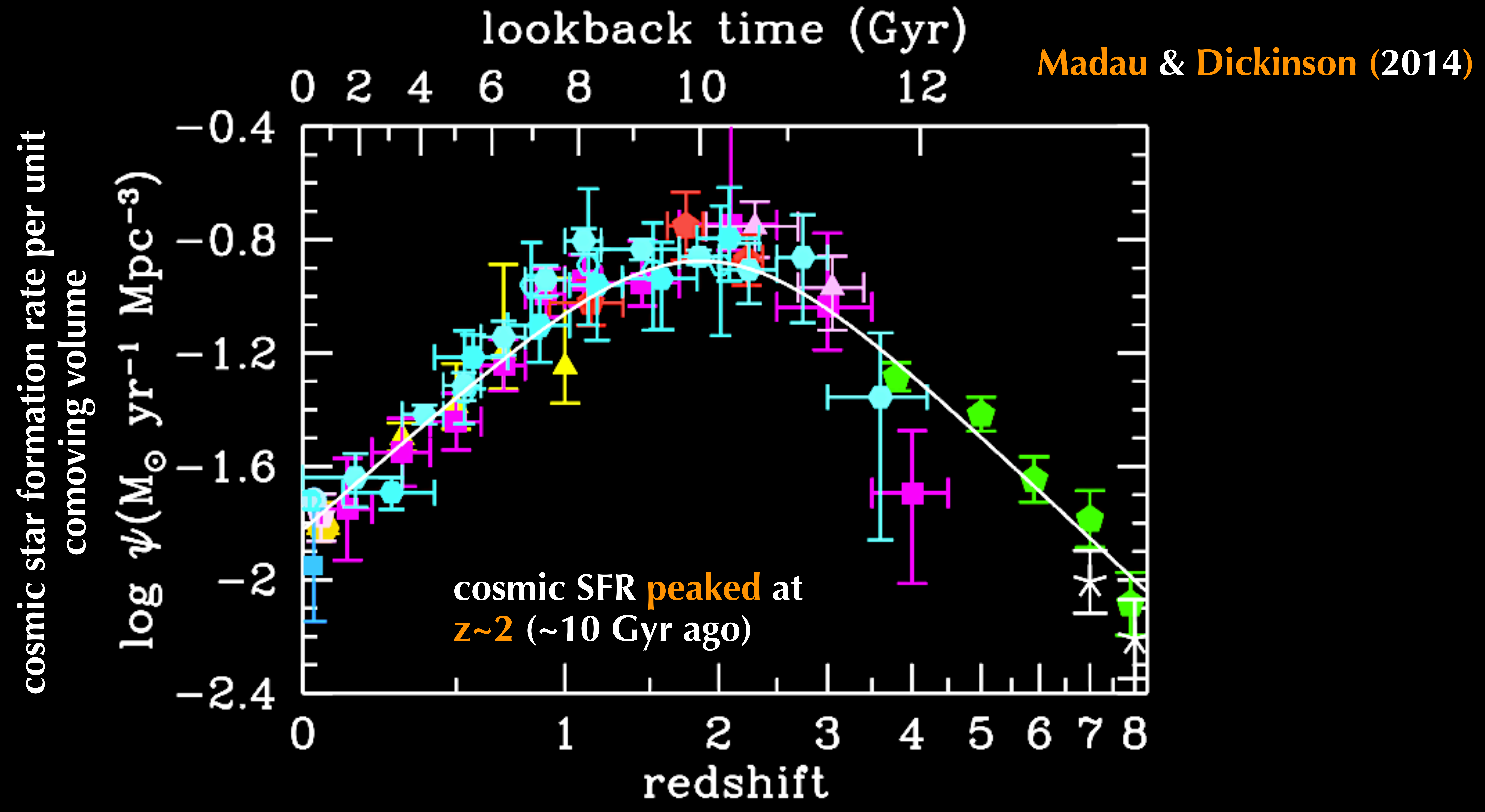
the evolution of galaxy properties

Ilbert+ (2013)





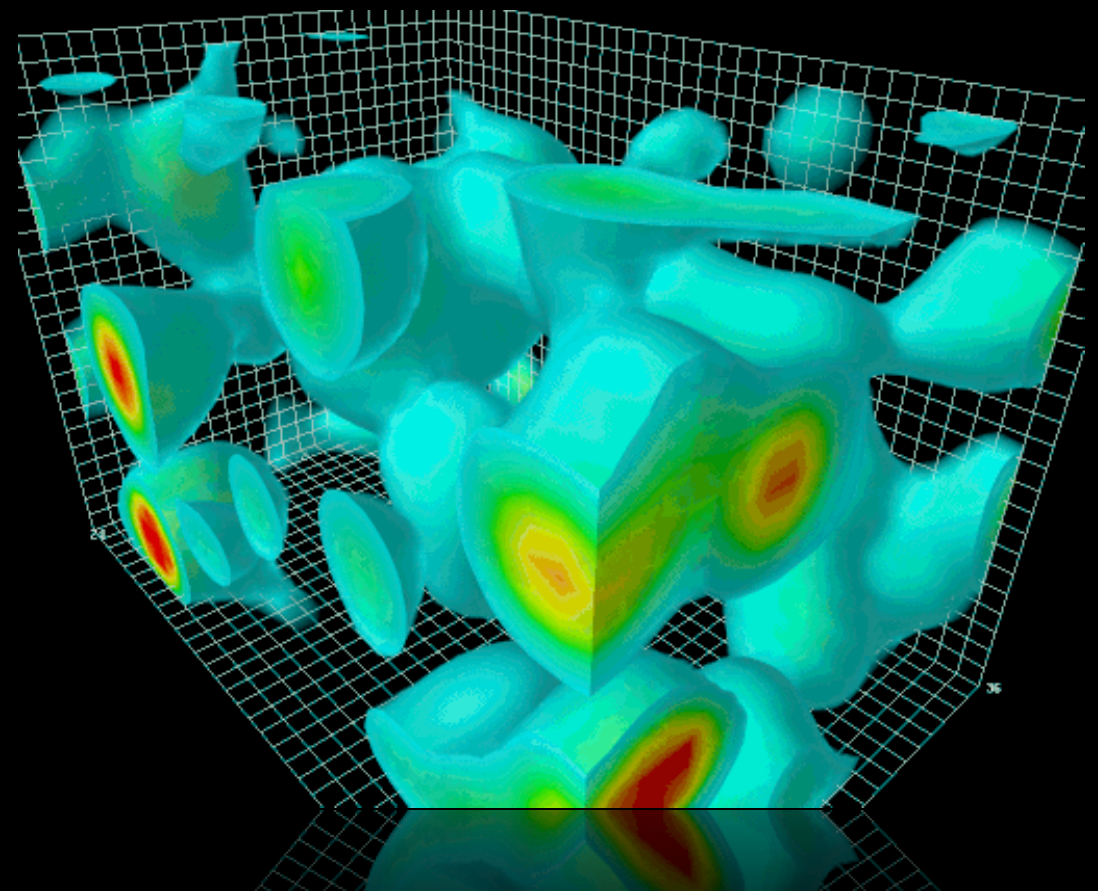
50% of the stars formed over the history of the Universe have formed since z~1 (~ 8 Gyr ago) — 90% since z~3



Madau & Dickinson (2014)
lookback time (Gyr)
0 2 4 6 8 10 12
−0.4
−0.8
−1.2
−1.6
−2
−2.4
log ψ(M⊙ yr−1 Mpc−3)
cosmic star formation rate per unit comoving volume
0 1 2 3 4 5 6 7 8
redshift
cosmic SFR peaked at z~2 (~10 Gyr ago)

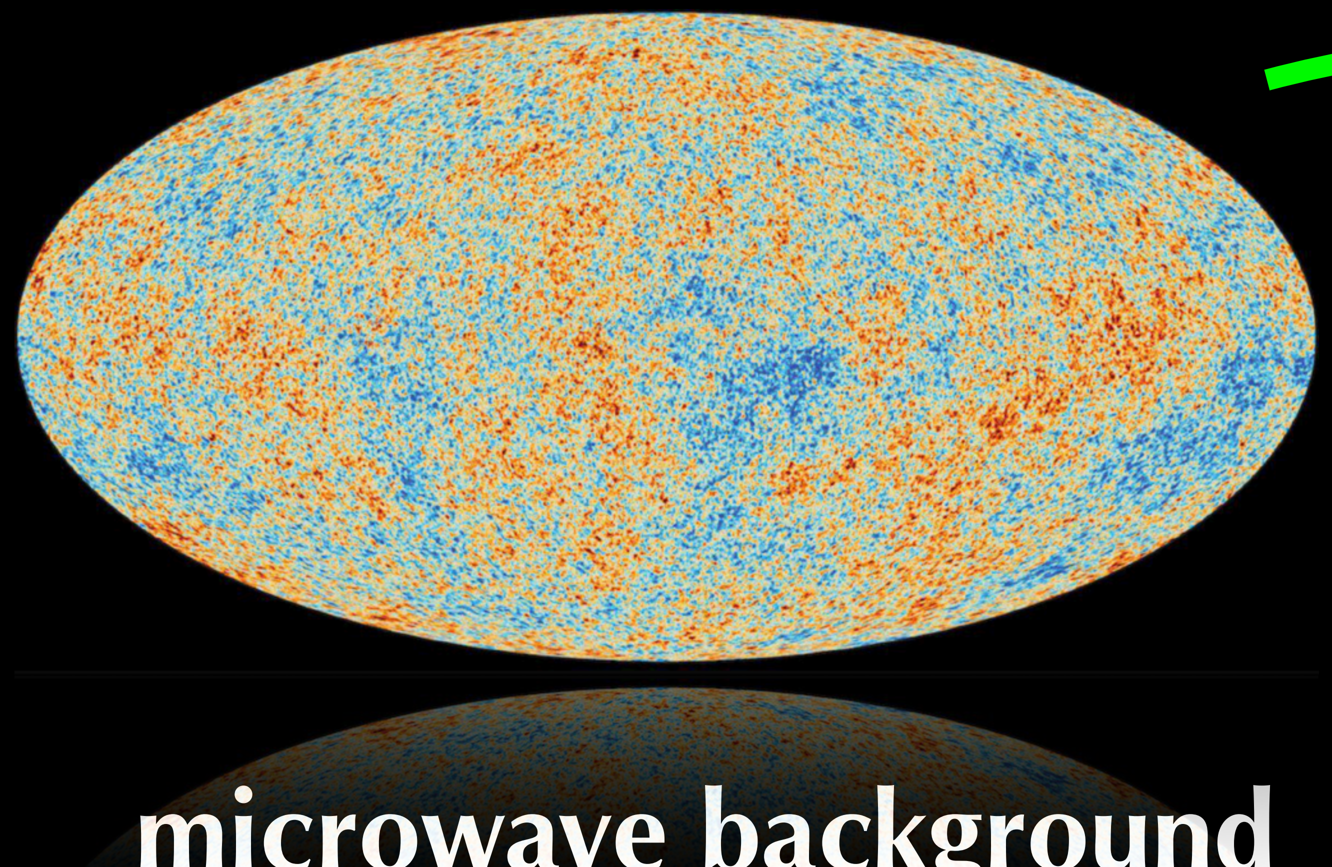
the physics of galaxy formation

the growth of structure from start to finish



quantum fluctuations

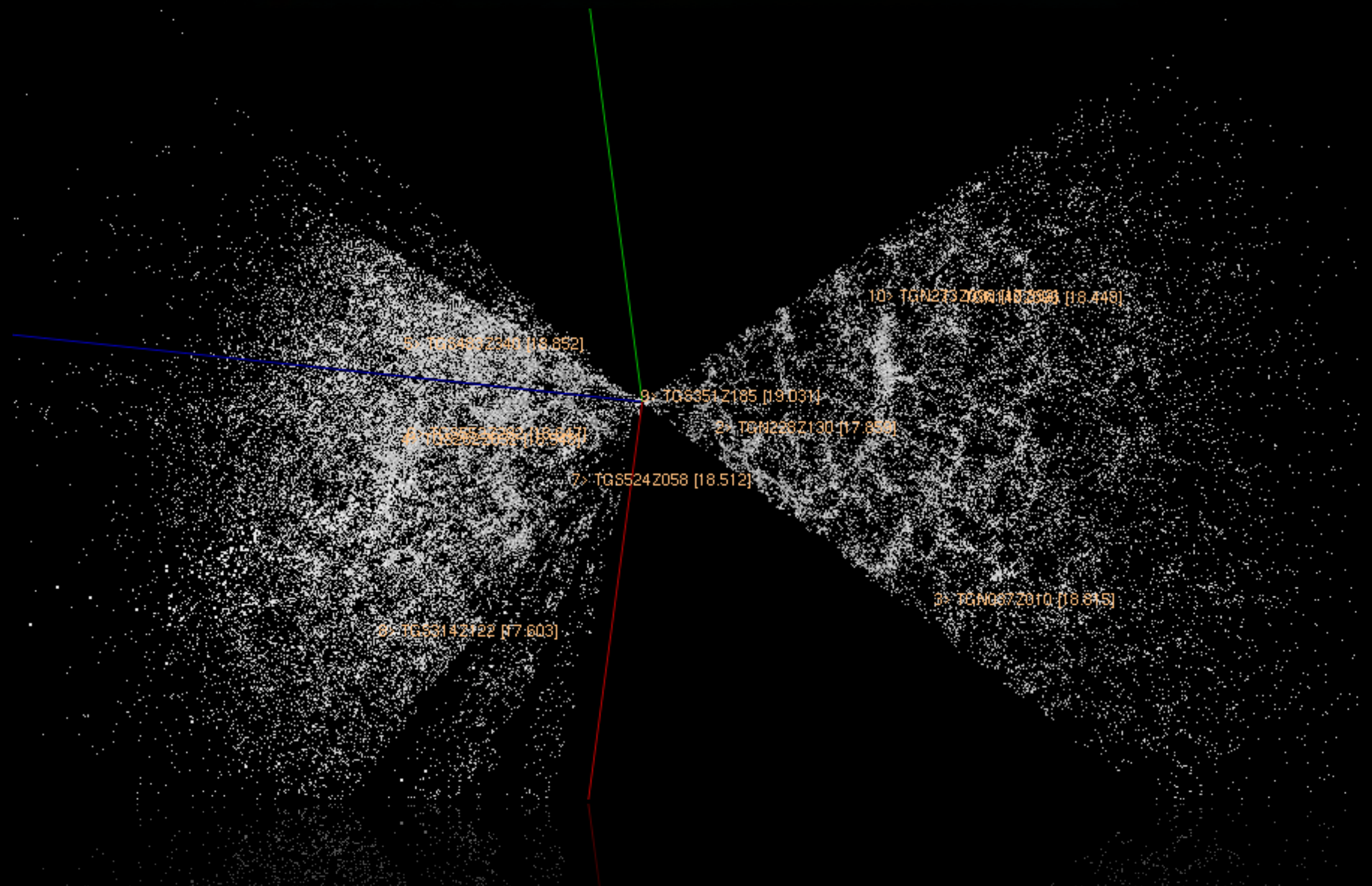
inflation?



microwave background

gravitational instability

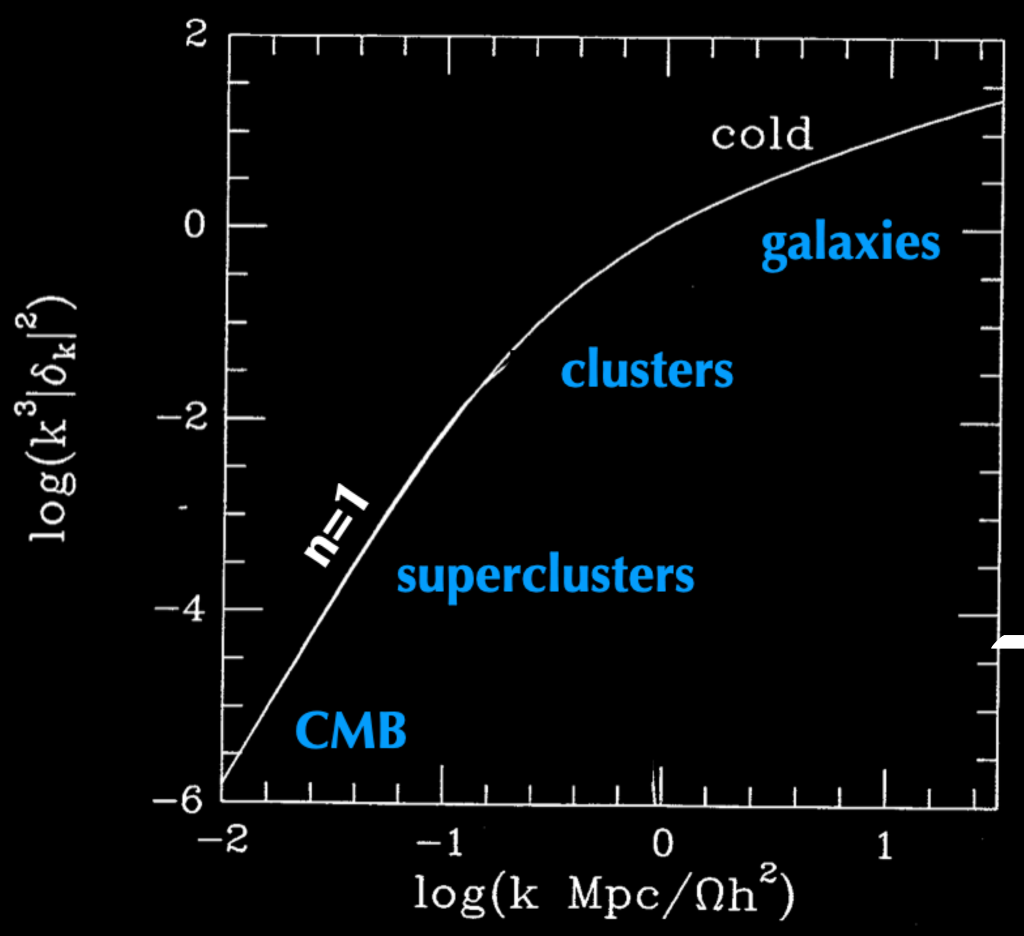
cosmic web



galaxy distribution

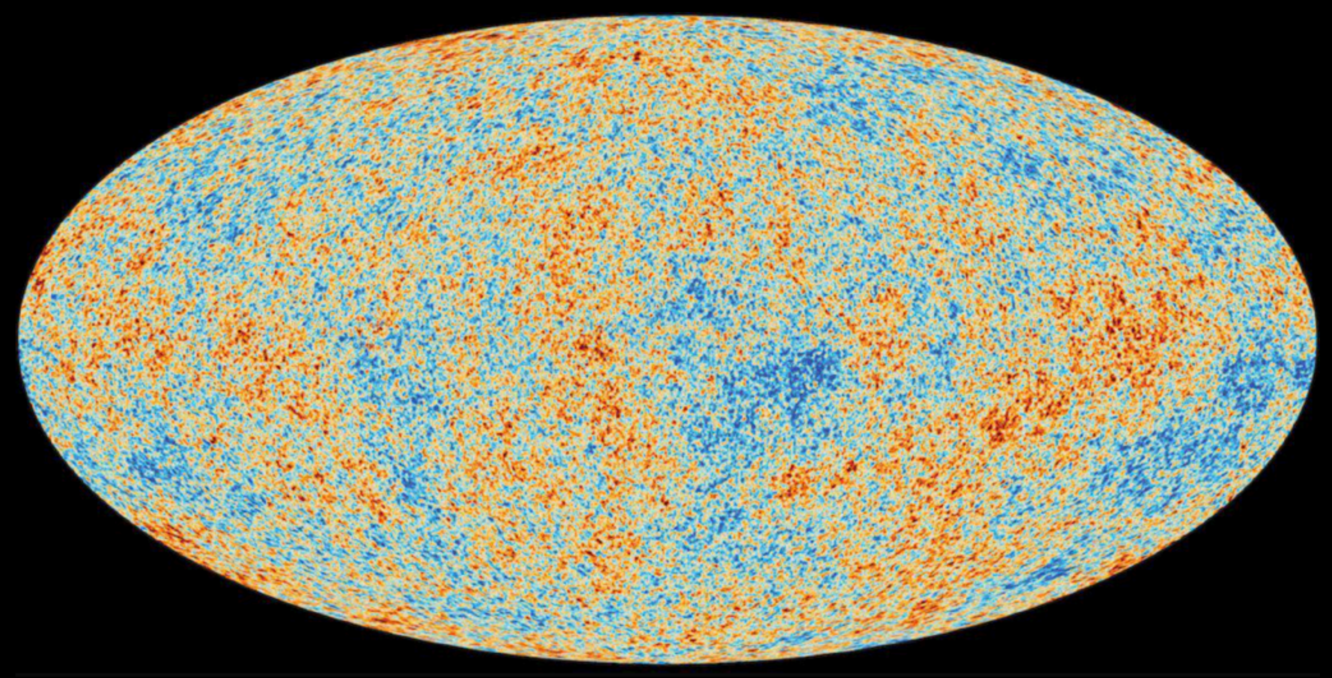
?

modelling galaxy formation



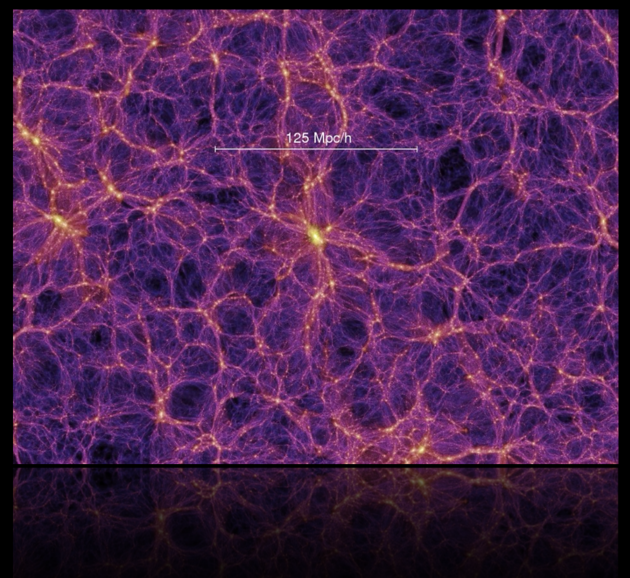
cosmological model
(Ω, Λ, h, DM)

primordial fluctuations
$\delta\rho/\rho\,(M,t)$



evolution of DM

N-body simulations



hydrodynamical simulations & semi-analytic models

dynamics of cooling gas

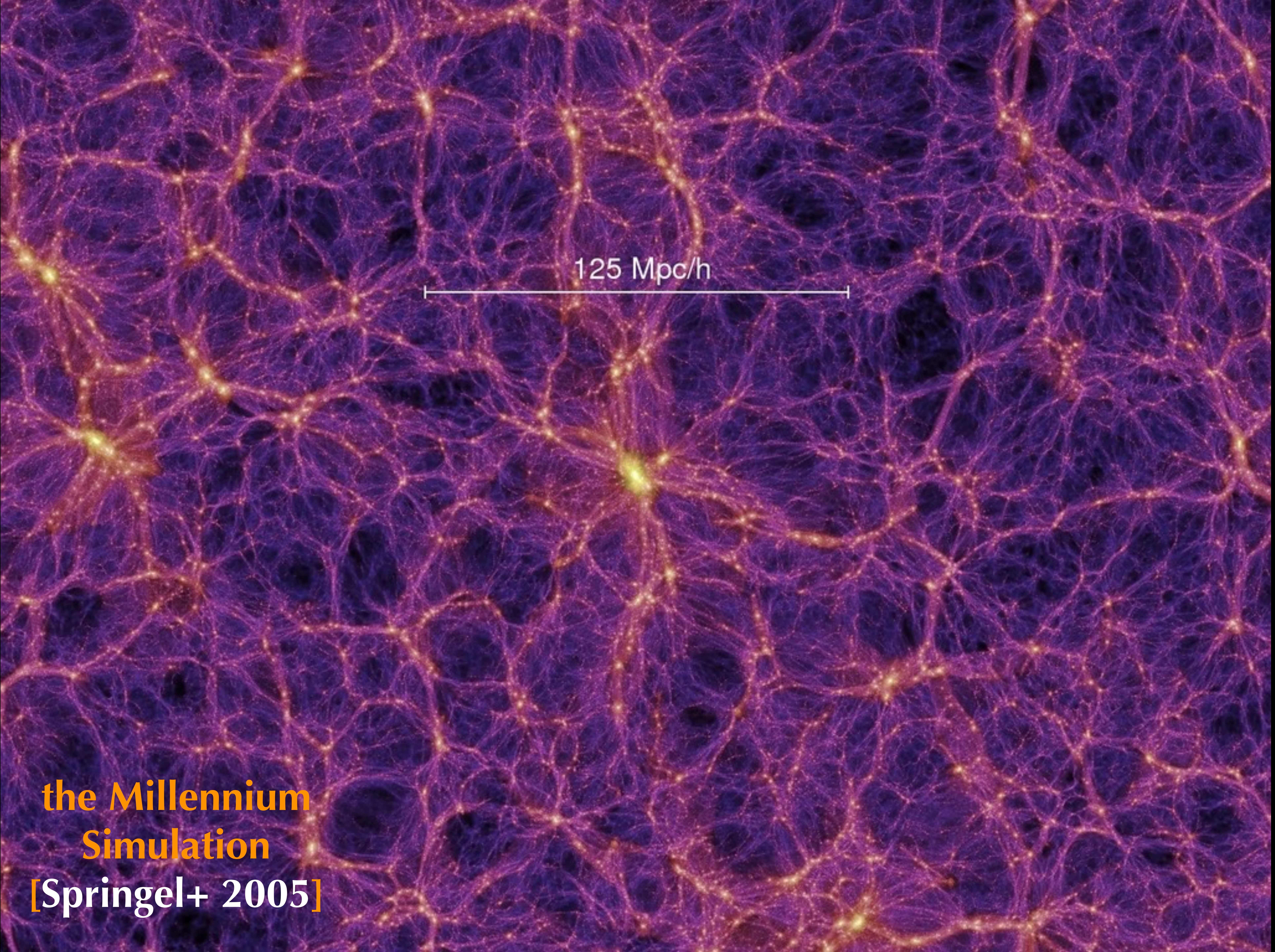
star formation, feedback, evolution of stellar populations

galaxy mergers

formation & evolution of galaxies

2. the assembly of dark matter haloes


z = 48.4
T = 0.05 Gyr
500 kpc



the Millennium Simulation [Springel+ 2005]

(cold) dark matter is the dominant mass component

dark matter haloes build-up hierarchically

these act as the gravitational potential wells within which baryons then condense to form galaxies

the **primordial** power spectrum

inflation makes a specific prediction for the primordial density field that is a power-law:

$$P(k) \propto k^{n_s}$$

where $n_s = 1$ corresponds to a **scale-invariant** power spectrum

depending on what the Universe is made of and how fluctuations grow in time and space, the inflationary power spectrum is modified by a **transfer function**

$$P(k,t) = A k^{n_s} \left| T(k,t) \right|^2$$

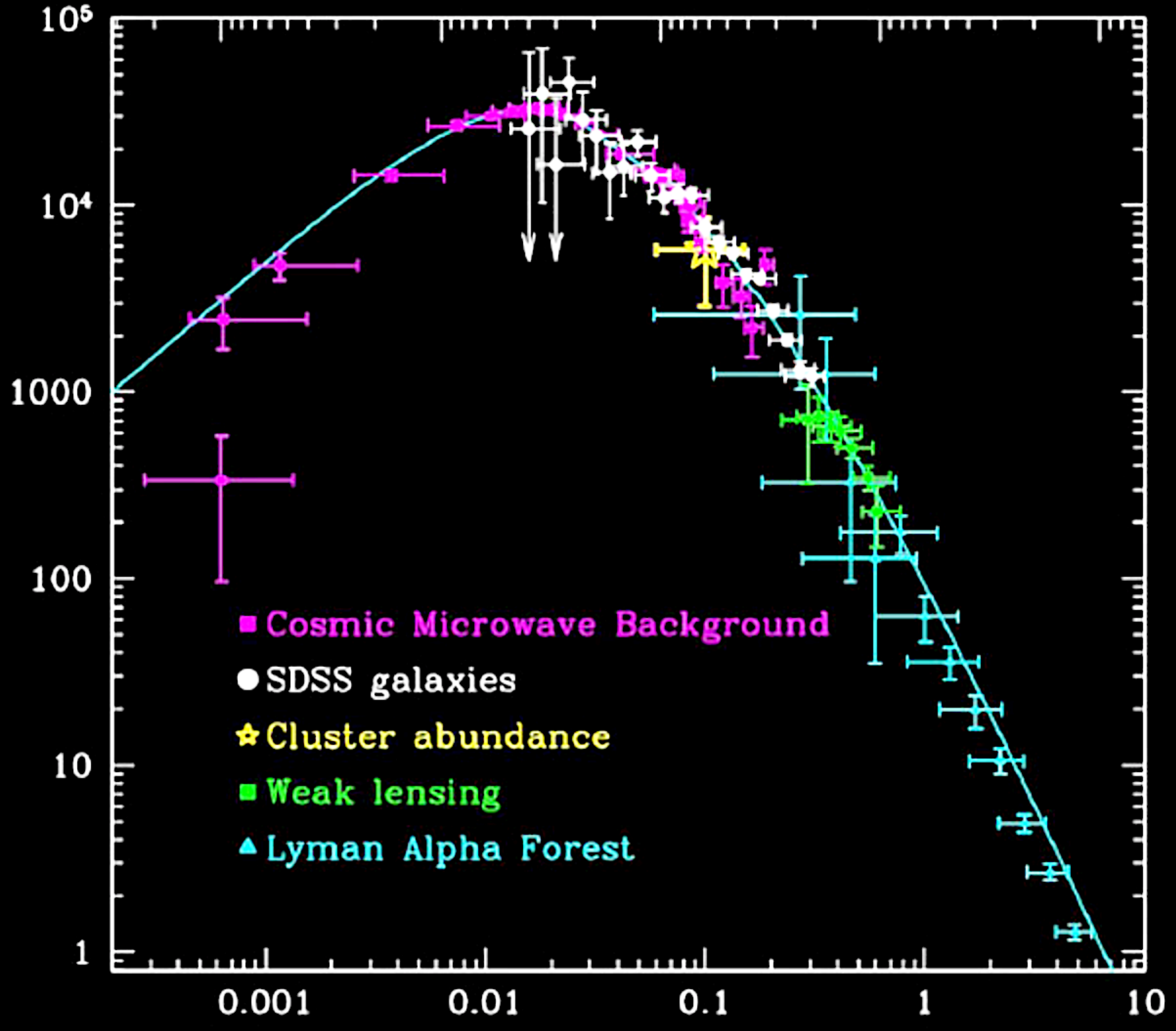
computed by public codes like CAMB, CLASS etc.

large scales

small scales

size of density fluctuations in the universe

log [power spectrum]



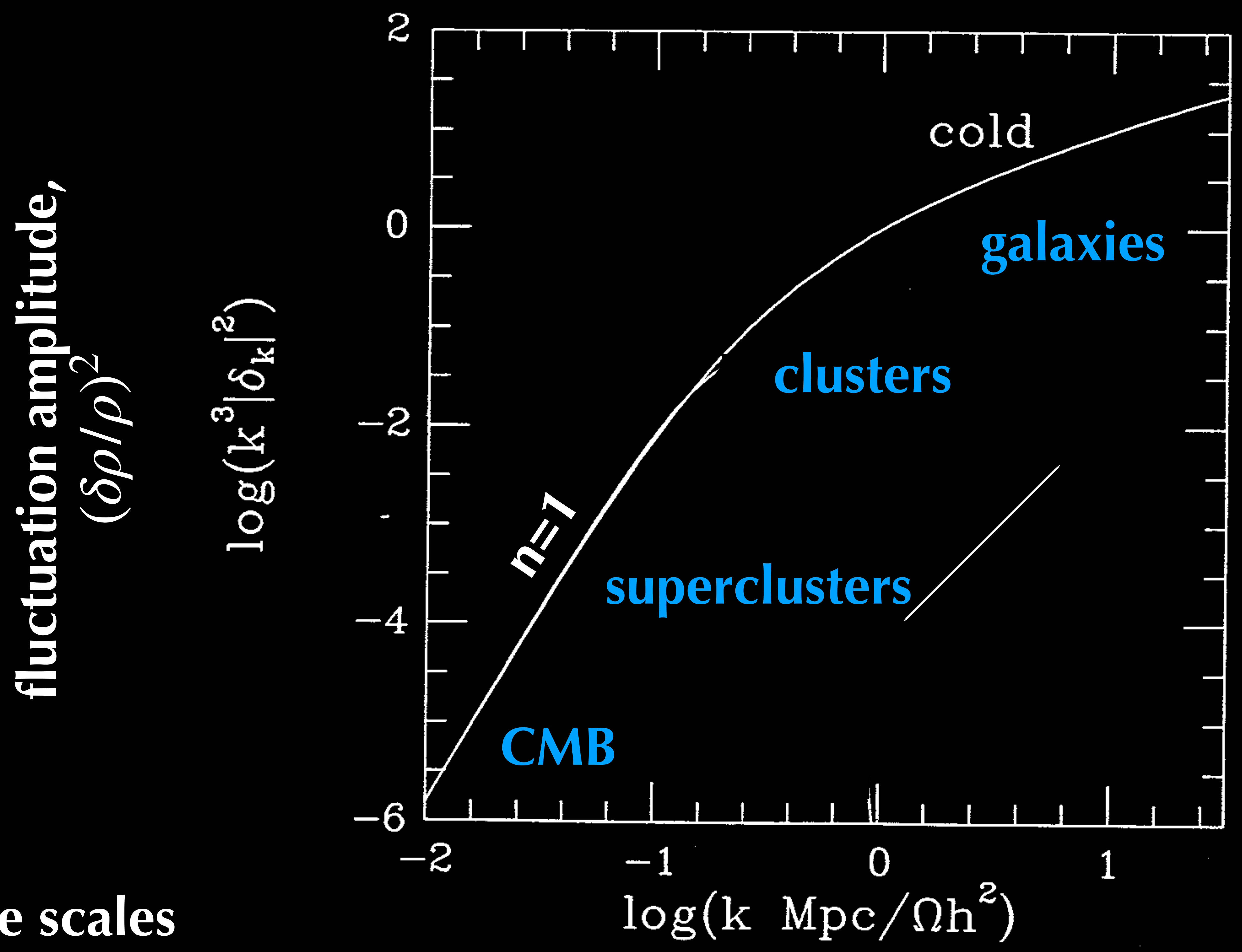
spatial scale [h Mpc^{-1}]

solid curve: ΛCDM prediction

symbols: data from multi-scale probes

Tegmark+ (2004)

the amplitude of linear density fluctuations in CDM

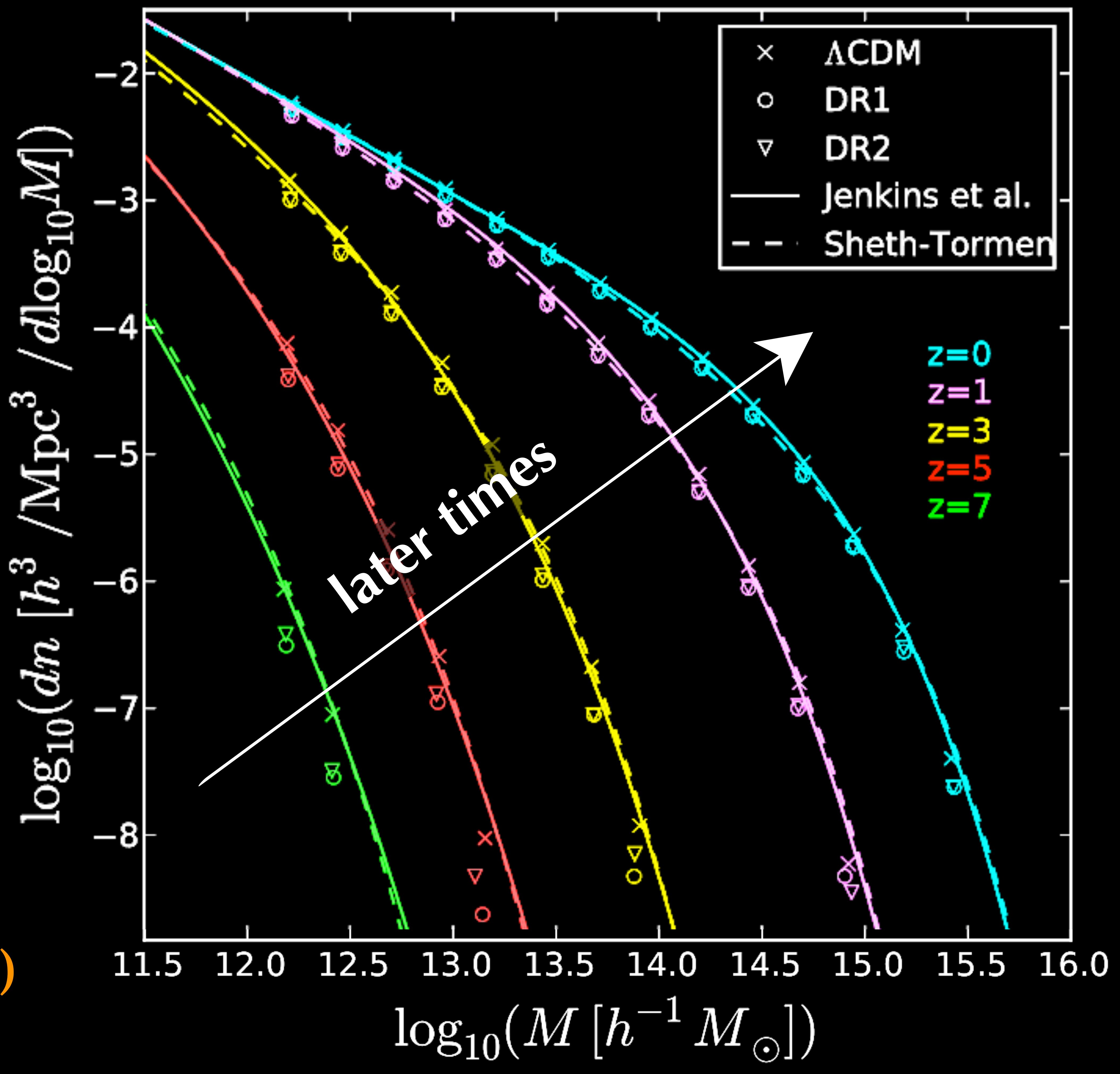


$k^3 \left|\delta_k\right|^2 \sim (\delta\rho/\rho)^2$ on some length scale $\lambda \sim 1/k$ according to linear perturbation theory at z=0

large scales

small scales

the halo mass function: theory vs simulations

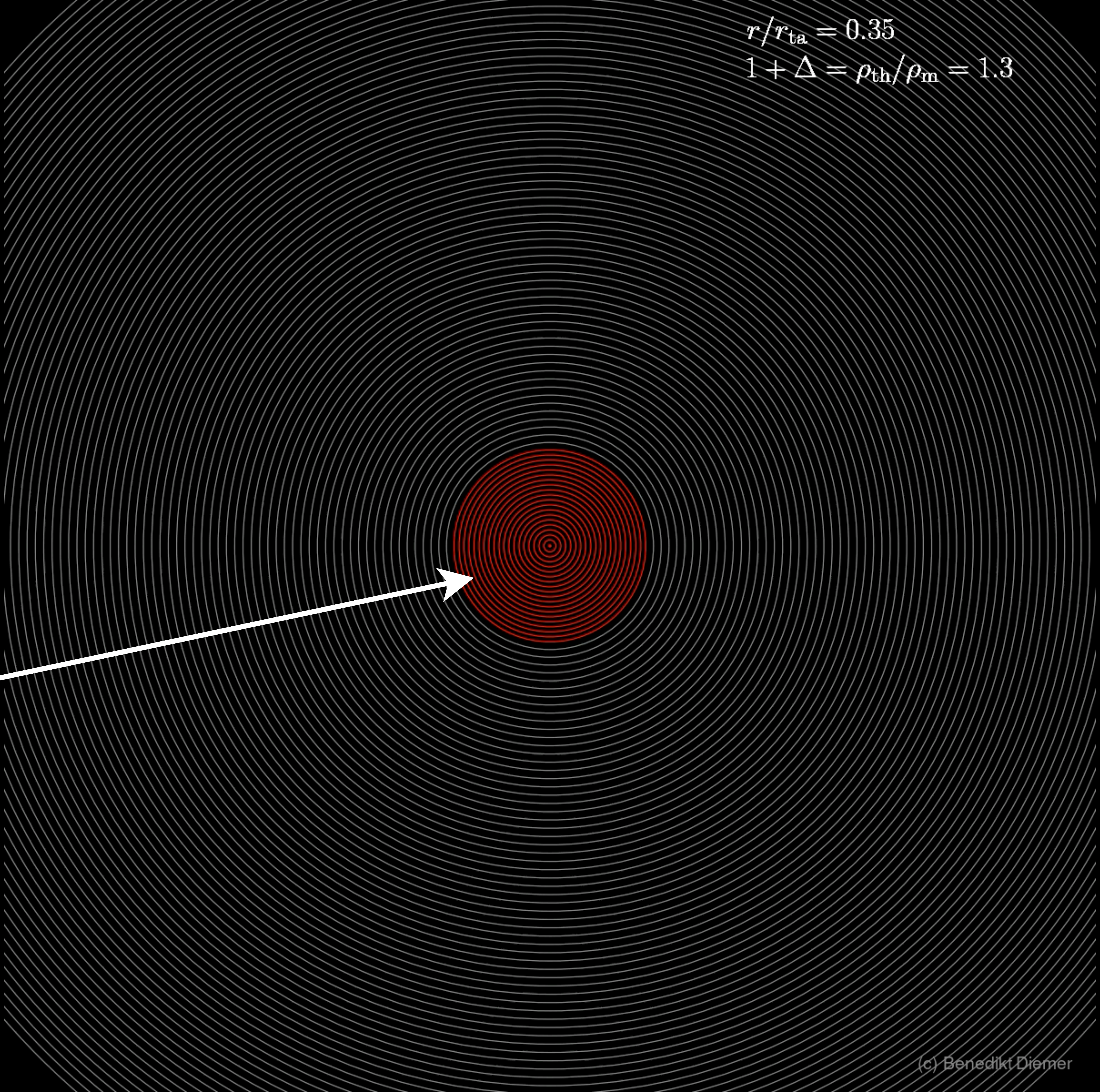


Shi & Baugh (2016)

the formation of dark matter haloes

the spherical collapse model

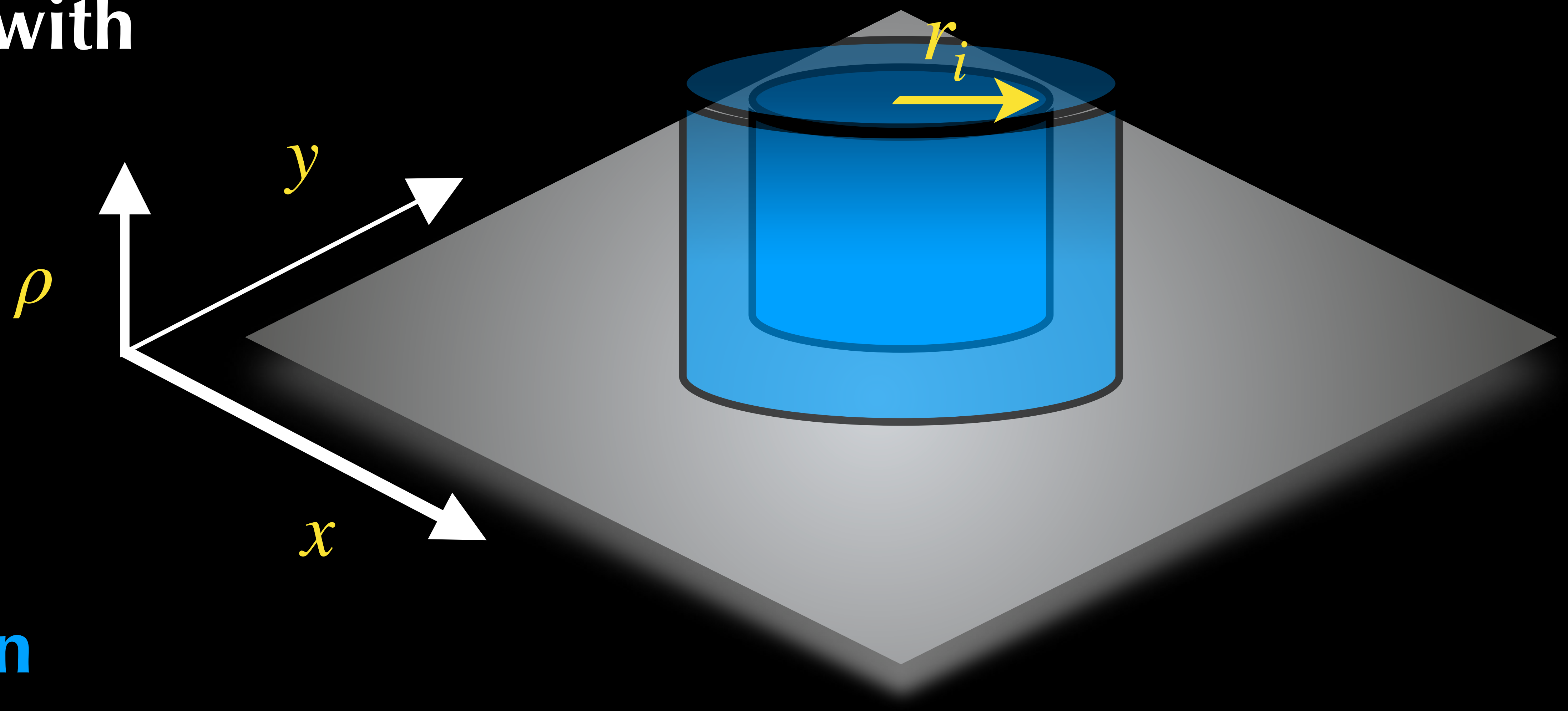
there will be a few equations. you don't need to remember all the steps, but it's useful to know where the basic results come from



$$\rho_{\text{background}} = \rho_m$$

$$\rho = \rho_m (1 + \Delta)$$

at some initial time, t_i , let r_i denote the (physical) radius of a mass shell contained within a spherical perturbation with overdensity δ_i



the mass enclosed within the shell is:

$$M(<r) = \frac{4}{3}\pi r_i^3 \bar{\rho}_i \left[1 + \delta_i\right]$$
$$= \frac{4}{3}\pi r^3(t)\, \bar{\rho}(t) \left[1 + \delta(t)\right]$$

mass conservation

the equation of motion of this mass shell under gravity can de derived using **Newton's shell theorem**:

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = -\frac{GM}{r^2}$$

"a spherically symmetric mass distribution outside a sphere exerts no force on it"

[also known as **Birkhoff's theorem**]

integrating this equation yields:

$$\frac{1}{2}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 - \frac{GM}{r} = E$$

energy per unit mass of the shell

$$\boxed{\frac{1}{2}\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 - \frac{GM}{r} = E}$$

the case where $E < 0$ corresponds to the gravitationally bound case, where the mass shell "collapses"

assuming that $r = 0$ at $t = 0$, we can write the solution parametrically as:

$$r = A\,(1 - \cos\theta) \qquad \theta \in [0, 2\pi]$$
$$t = B\,(\theta - \sin\theta)$$

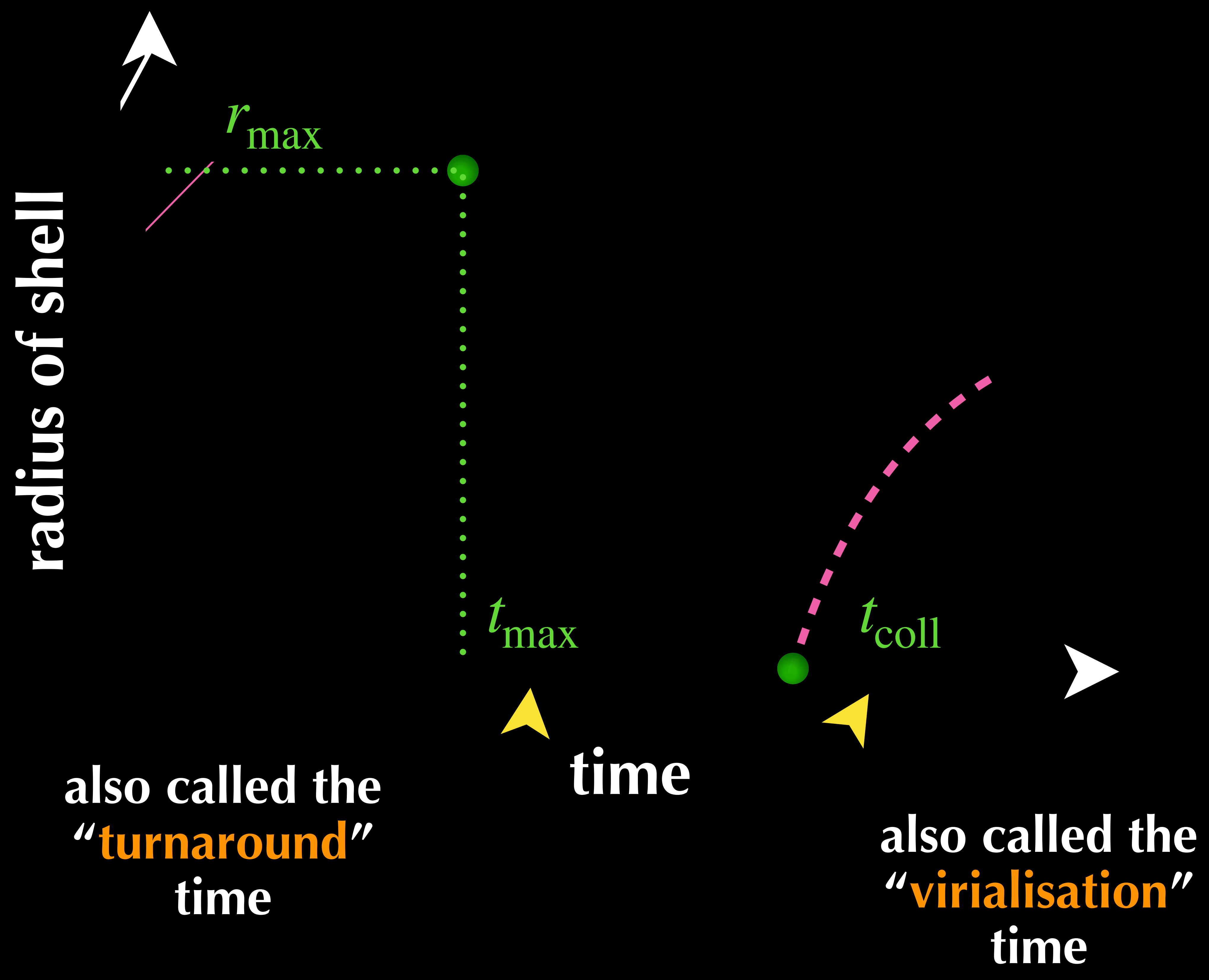
where $A = GM/2|E|$ and $B = GM/(2|E|)^{3/2}$

the solution implies the following evolution:

the shell expands from $r = 0$ at $\theta = 0$ ($t = 0$)

then reaches a maximum radius r_{max} at $\theta = \pi$ ($t = t_{\text{max}}$)

collapses back to $r = 0$ at $\theta = 2\pi$ ($t = t_{\text{coll}} = 2\,t_{\text{max}}$)



with our parametric solutions for r and t, we can now write the evolution of the overdensity itself:

$$\rho = \frac{3M}{4\pi r^3} = \frac{3M}{4\pi A^3}(1-\cos\theta)^{-3} \qquad \text{and} \qquad \bar{\rho} = \frac{1}{6\pi G t^2} = \frac{1}{6\pi G B^2}\left(\theta - \sin\theta\right)^{-2}$$

putting these together, we get: $$1+\delta = \frac{\rho}{\bar{\rho}} = \frac{9}{2}\frac{\left(\theta-\sin\theta\right)^2}{(1-\cos\theta)^3}$$

let's now consider what this means for the overdensity criterion for collapse. we know that, in linear theory ($\theta \ll 1$ and $\delta \ll 1$), density perturbations in the matter-dominated era grow as:

$$\boxed{\delta_{\rm lin} = \delta_i \left(\frac{t}{t_i}\right)^{2/3}}$$

. we can also Taylor expand the equation above:

$$\boxed{\delta_i = \frac{3}{20}(6\pi)^{2/3}\left(\frac{t_i}{t_{\rm max}}\right)^{2/3}}$$

$$\delta_{\rm lin} = \frac{3}{20}(6\pi)^{2/3}\left(\frac{t}{t_{\rm max}}\right)^{2/3}$$

$$\boxed{\delta_{\rm lin} = \frac{3}{20}(6\pi)^{2/3}\left(\frac{t}{t_{\rm max}}\right)^{2/3}}$$

so, according to linear theory, when collapse occurs ($t_{\rm coll} = 2\,t_{\rm max}$), the overdensity is:

$$\delta_{\rm lin} = \frac{3}{20}(12\pi)^{2/3} \simeq 1.686$$

great! now, we can estimate the size of the halo when virialisation occurs. for this we need to make the following assumptions:

virial equilibrium: $2K_f + W_f = 0$ and energy conservation: $E_f = K_f + W_f = E_i = E_{\rm max}$

$$E_{\rm max} = W_{\rm max} = -\frac{GM}{r_{\rm max}} \qquad E_f = W_f\,/\,2 = -\frac{GM}{2\,r_{\rm vir}}$$

equating these $\Rightarrow r_{\rm vir} = r_{\rm max}\,/\,2 \Rightarrow$ a mass shell virialises at half its maximum (turnaround) radius

$r_{\rm vir} = r_{\rm max}/2 \Rightarrow$ **a mass shell virialises at half its maximum (turnaround) radius. the average density is therefore 8 times larger than at turnaround.**

finally (!), we can compute the average overdensity of a virialised dark matter halo:

$$1 + \Delta_{\rm vir} \equiv 1 + \delta\left(t_{\rm coll}\right) = \frac{\rho\left(t_{\rm coll}\right)}{\bar{\rho}\left(t_{\rm coll}\right)}$$

using $\bar{\rho} \propto t^{-2}$ **and that** $t_{\rm coll} = 2\,t_{\rm max}$, **we have that:**

$$1 + \Delta_{\rm vir} = \frac{8\,\rho_{\rm max}}{\bar{\rho}_{\rm max}/4} = 32\left(1 + \delta_{\rm max}\right) = 18\pi^2 \simeq 178$$

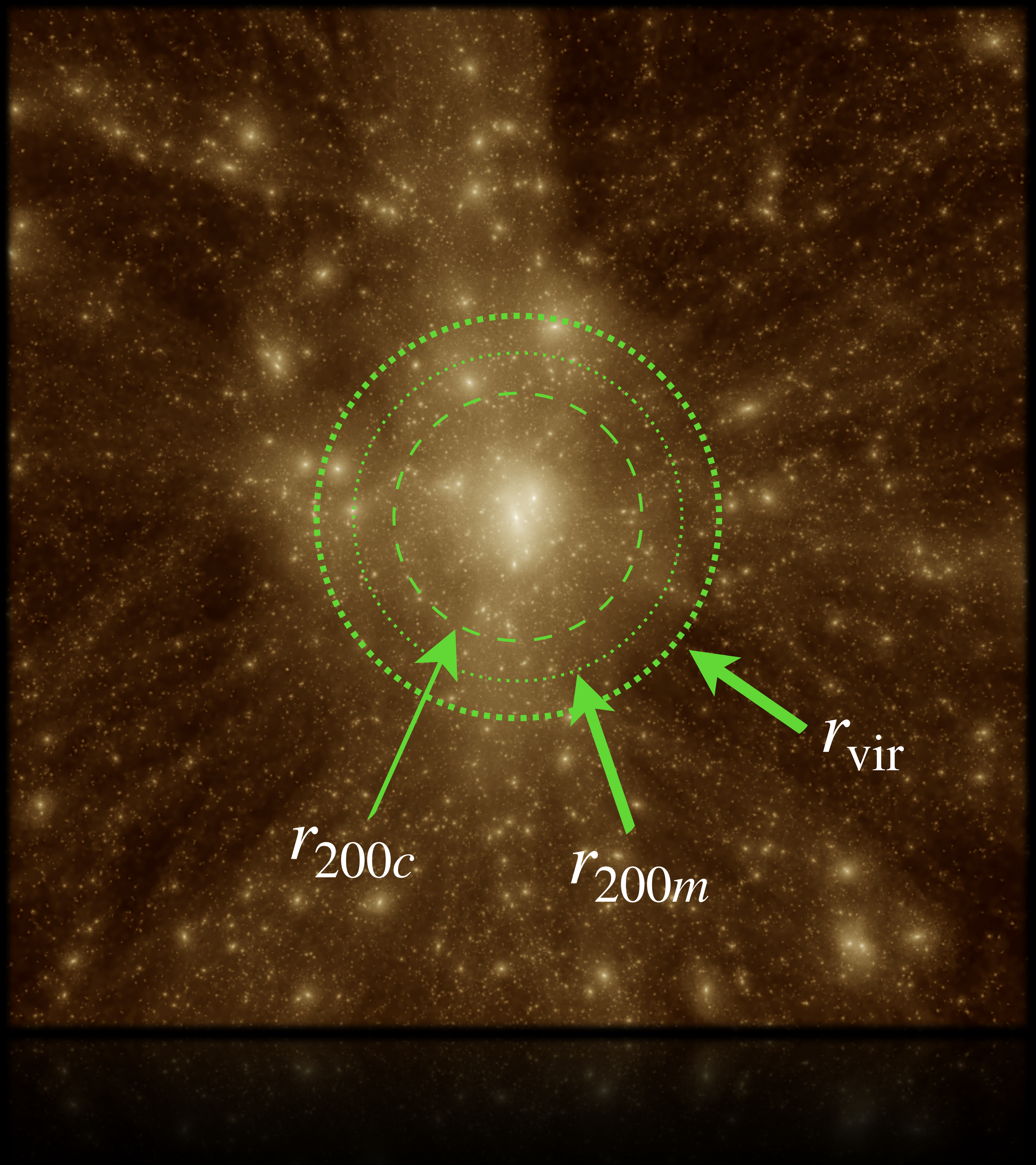
valid for a matter-dominated or Einstein-de Sitter cosmology. but similar for Λ**CDM cosmology, too**

so, haloes at redshift z **have a mean density**

$\rho_{\rm vir}(z) = \Delta_{\rm vir}\left(\Omega_m, \Omega_\Lambda\right)\rho_{\rm ref}(z) \sim (200 - 300)\,\rho_{\rm ref}(z) \propto (1+z)^3$

some reference background density (mean, critical)

definitions of halo mass



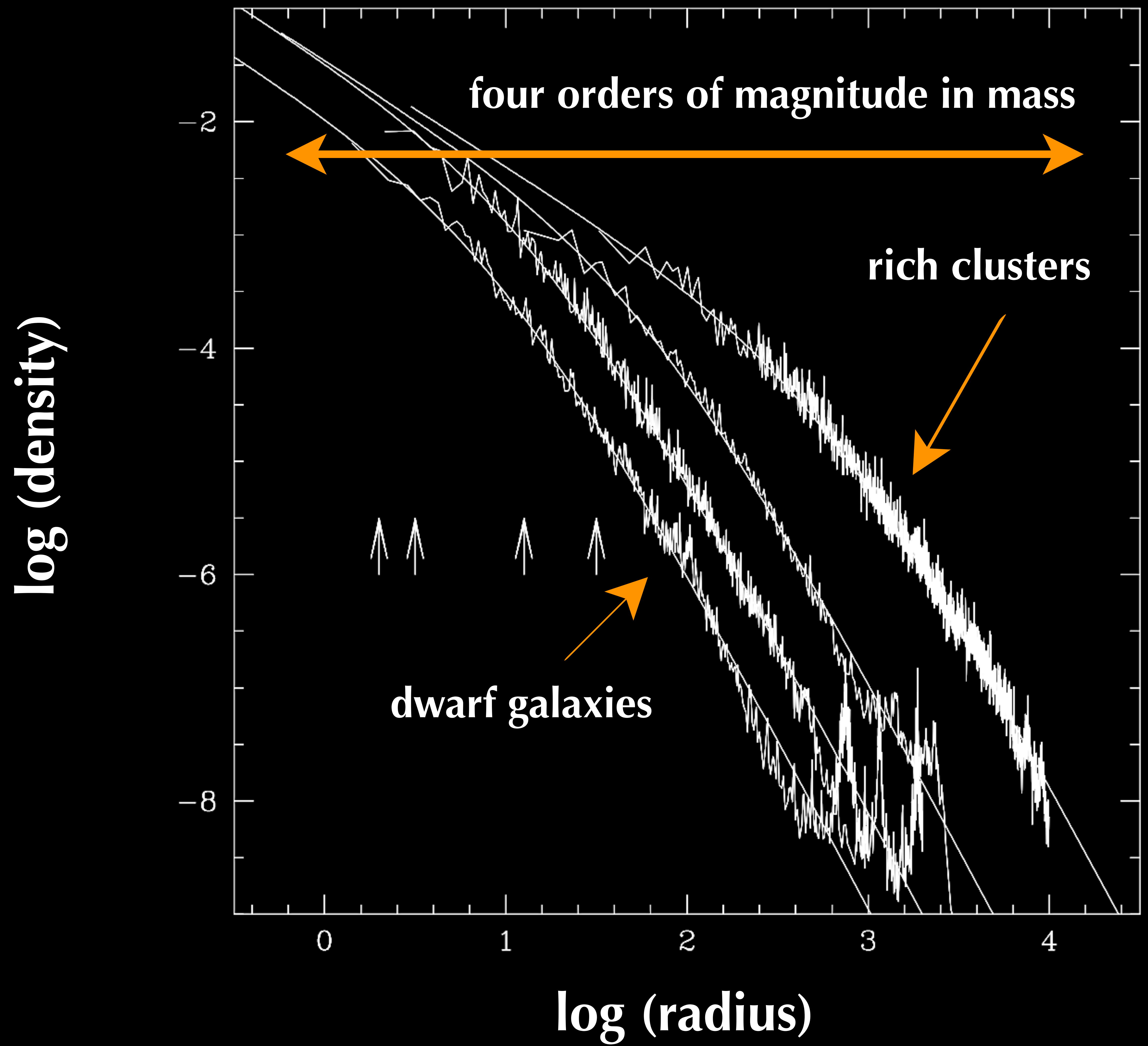
DM haloes identified in N-body simulations are highly irregular objects. how do we define their mass / extent?

$\Rightarrow$ do what astronomers do best and assume everything is spherical

$$M_\Delta = \frac{4}{3}\pi\,\Delta\,\rho_{\rm ref}\,r_\Delta^3 \quad \text{where } \bar{\rho}(<r_\Delta) = \Delta \cdot \rho_{\rm ref}$$

virial mass/radius

the structure of DM haloes in CDM



centre: $\rho \propto r^{-1}$

middle: $\rho \propto r^{-2}$

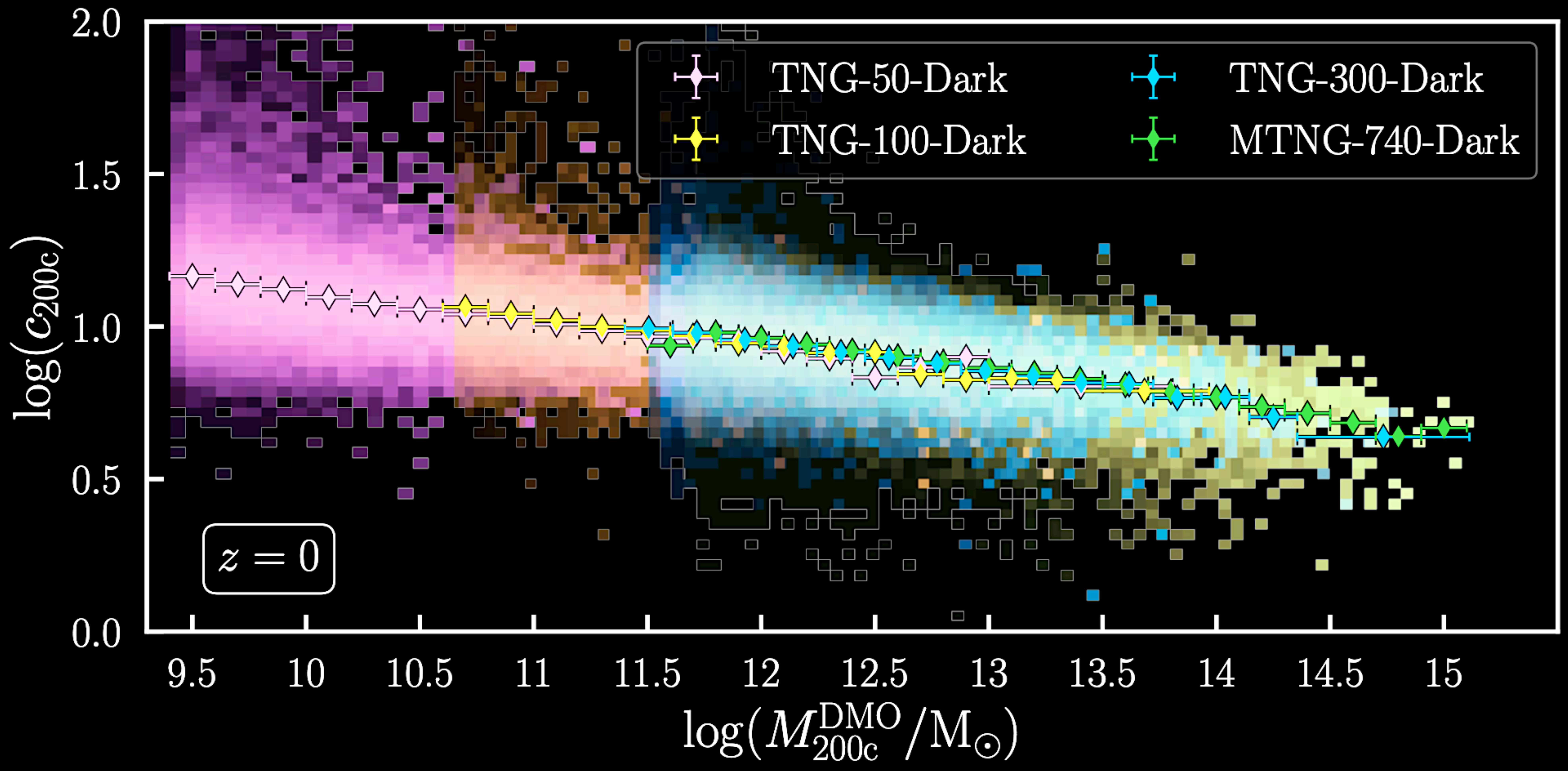
outskirts: $\rho \propto r^{-3}$

Navarro, Frenk & White (1996)

$$\rho(r) = \frac{\rho_s}{\frac{r}{r_s}\left(1 + \frac{r}{r_s}\right)^2}$$

$$c = \frac{r_{200c}}{r_s}$$

Sorini+ (2024)



halo concentrations vary weakly as a function of mass and redshift; lower mass haloes have higher concentrations, reflecting the mean density of the universe when they collapsed initially

characteristic quantities in DM haloes

- haloes in simulations are roughly in dynamical equilibrium at mean interior densities:
  $M\left(<r_{\rm vir}\right)/\left(4\pi/3r_{\rm vir}^3\right) = \rho_{\rm vir}(z)$

- $\rho_{\rm vir}$ is given by spherical collapse model:
  $\rho_{\rm vir}(z) = \Delta_{\rm vir}\,\rho_m(z) \sim 200\,\rho_m(z) \propto (1+z)^3$

- which also defines a characteristic radius for each halo: $r_{\rm vir} \propto M^{1/3}/(1+z)$

- characteristic circular velocity for halo:
  $V_{\rm vir} = \left(GM\left(<r_{\rm vir}\right)/r_{\rm vir}\right)^{1/2} \propto M^{1/3}\,(1+z)^{1/2}$