Astro PG Course

cosmological simulations

session 2: cosmological integration and initial conditions

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choice of softening

$$\Phi(x) = -G\sum_{j=1}^{N}\frac{m_j}{\left[\left(\mathbf{x}-\mathbf{x}_j\right)^2+\epsilon^2\right]^{1/2}}$$

gravitational softening, ϵ, is introduced in N-body simulations to avoid large-angle scattering due to close encounters. higher-resolution simulations use smaller softening [and therefore also shorter timesteps]

$$\boxed{\text{simple choice: } \epsilon = \alpha \cdot d = \alpha\frac{L_{\text{box}}}{N_p^{1/3}}}$$

$$\alpha = 0.01 - 0.05$$

mean inter-particle separation

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stochastic acceleration from close encounters

stricter choice: $\frac{G m_{\mathrm{DM}}}{\epsilon^2} \lesssim \frac{G M_{200}}{r_{200}^2} \Rightarrow \epsilon \gtrsim \frac{r_{200}}{\sqrt{N_{200}}}$

minimum mean-field acceleration in DM halo

Power+ (2003)

higher-order time integration

once forces on particles have been calculated, we need to propagate these forward in time and update their (1) positions and (2) velocities

kick $$\mathbf{v}_j^{n+1/2} = \mathbf{v}_j^n + \frac{1}{2}\mathbf{a}_j^n \cdot \Delta t$$

drift $$\mathbf{x}_j^{n+1} = \mathbf{x}_j^n + \mathbf{v}_j^{n+1/2} \cdot \Delta t$$

kick $$\mathbf{v}_j^{n+1} = \mathbf{v}_j^{n+1/2} + \frac{1}{2}\mathbf{a}_j^{n+1} \cdot \Delta t$$

2nd order
"leapfrog" method

much more stable

"symplectic"
[~energy is conserved]

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Hubble parameter

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kick—drift—kick

$$E\left(\Delta\tau\right) = K\left(\frac{\Delta\tau}{2}\right) \circ D\left(\Delta\tau\right) \circ K\left(\frac{\Delta\tau}{2}\right)$$

cosmological scale factor

Hubble parameter

a note on the Hubble parameter

we define the Hubble parameter as $H(t) \equiv \dfrac{\dot{a}(t)}{a(t)}$. denoting the present-day value as

$$H_0 \equiv H(t = t_{\mathrm{now}}) = 100h\,\mathrm{km s^{-1} Mpc^{-1}}$$

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$$H(a) = H_0\,E(a)$$

$$E(a) = \sqrt{\Omega_m\,a^{-3} + \Omega_r\,a^{-4} + \Omega_k\,a^{-2} + \Omega_\Lambda \exp\left(3\tilde{w}(a)\right)}$$

$$\tilde{w}(a) = (a-1)\,w_a - \left(1 + w_0 + w_a\right)\log(a)$$

$$w(a) \equiv w_0 + w_a\,(1-a)$$

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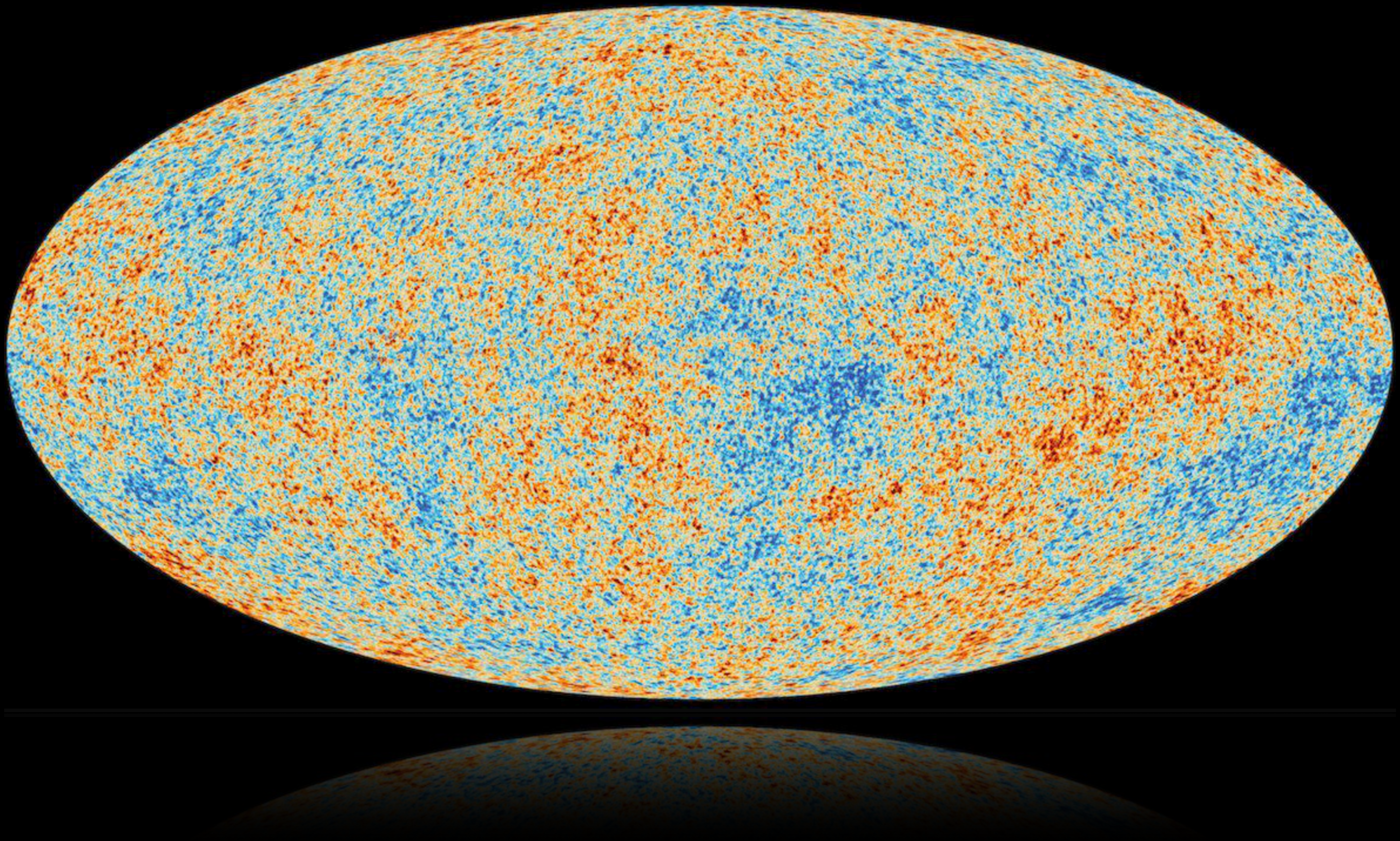
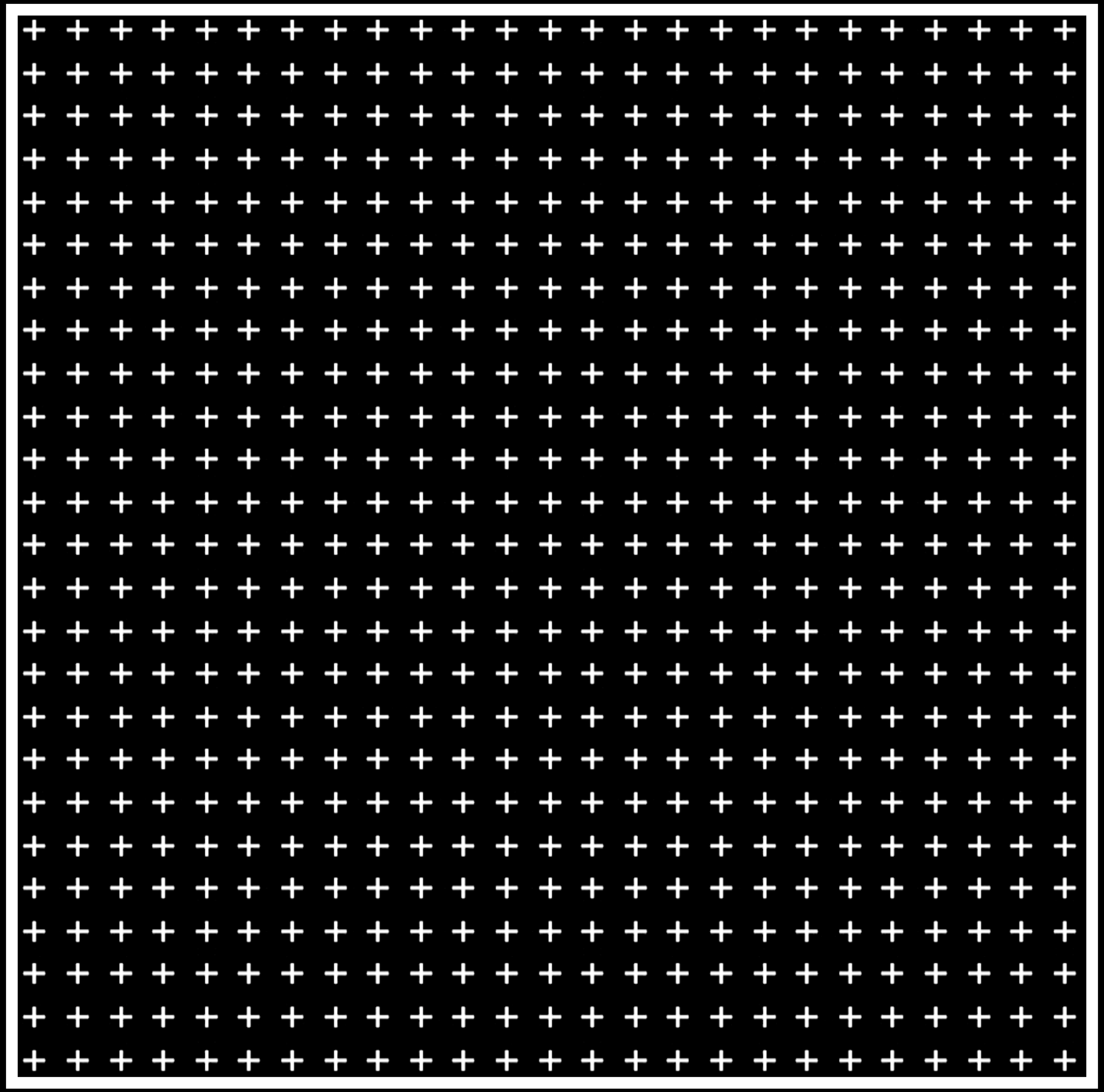
the (background) cosmological model is therefore *fully specified* through $\Omega_m, \Omega_r, \Omega_k, \Omega_\Lambda, h, w_0$ and w_a

setting up initial conditions

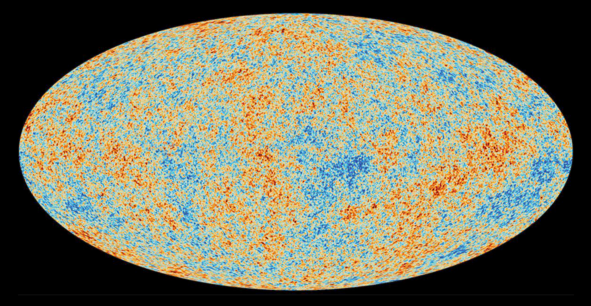
measurement of the cosmic microwave background radiation gives us *precise constraints* on cosmological initial conditions

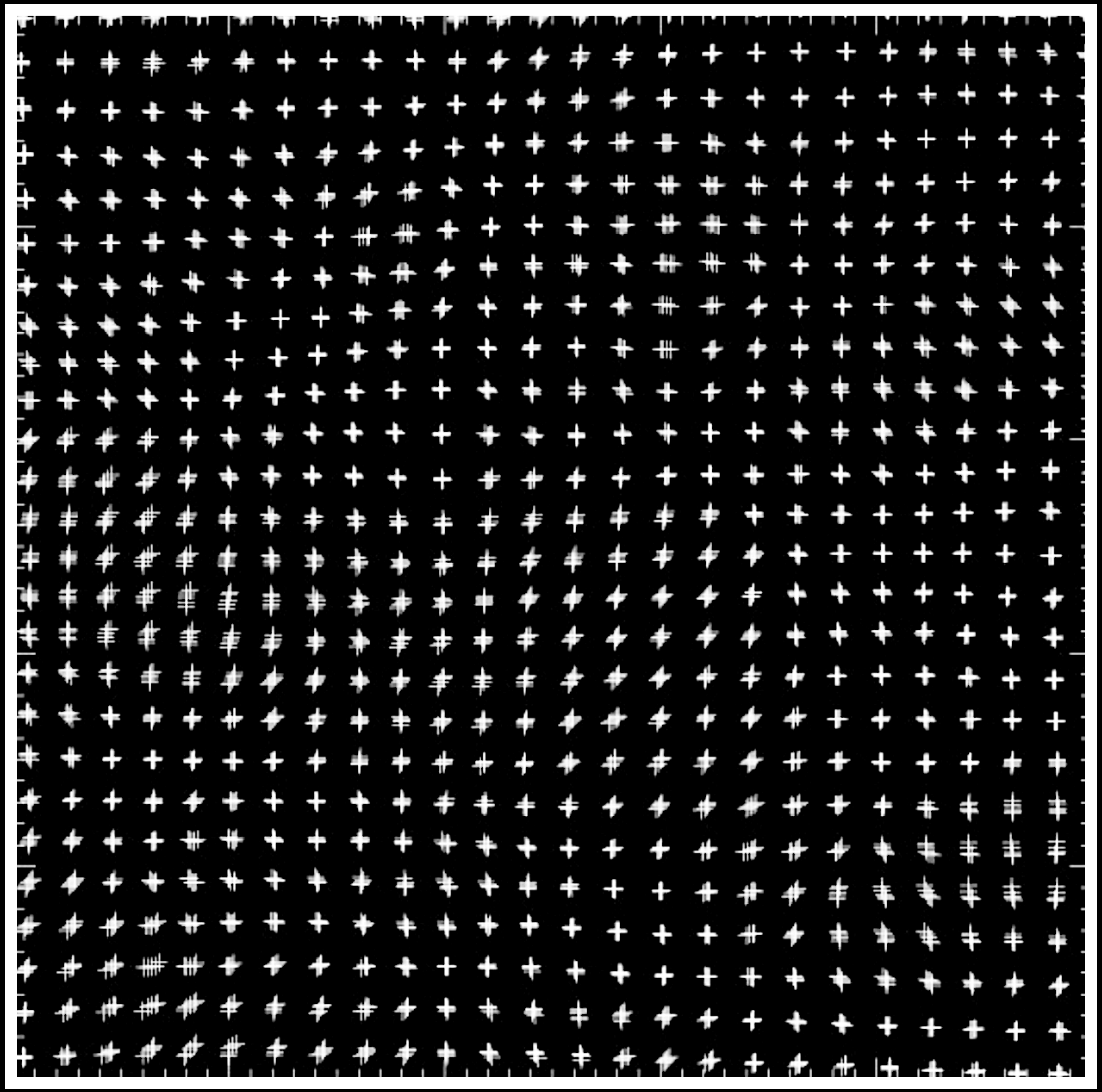


homogeneous & isotropic ICs



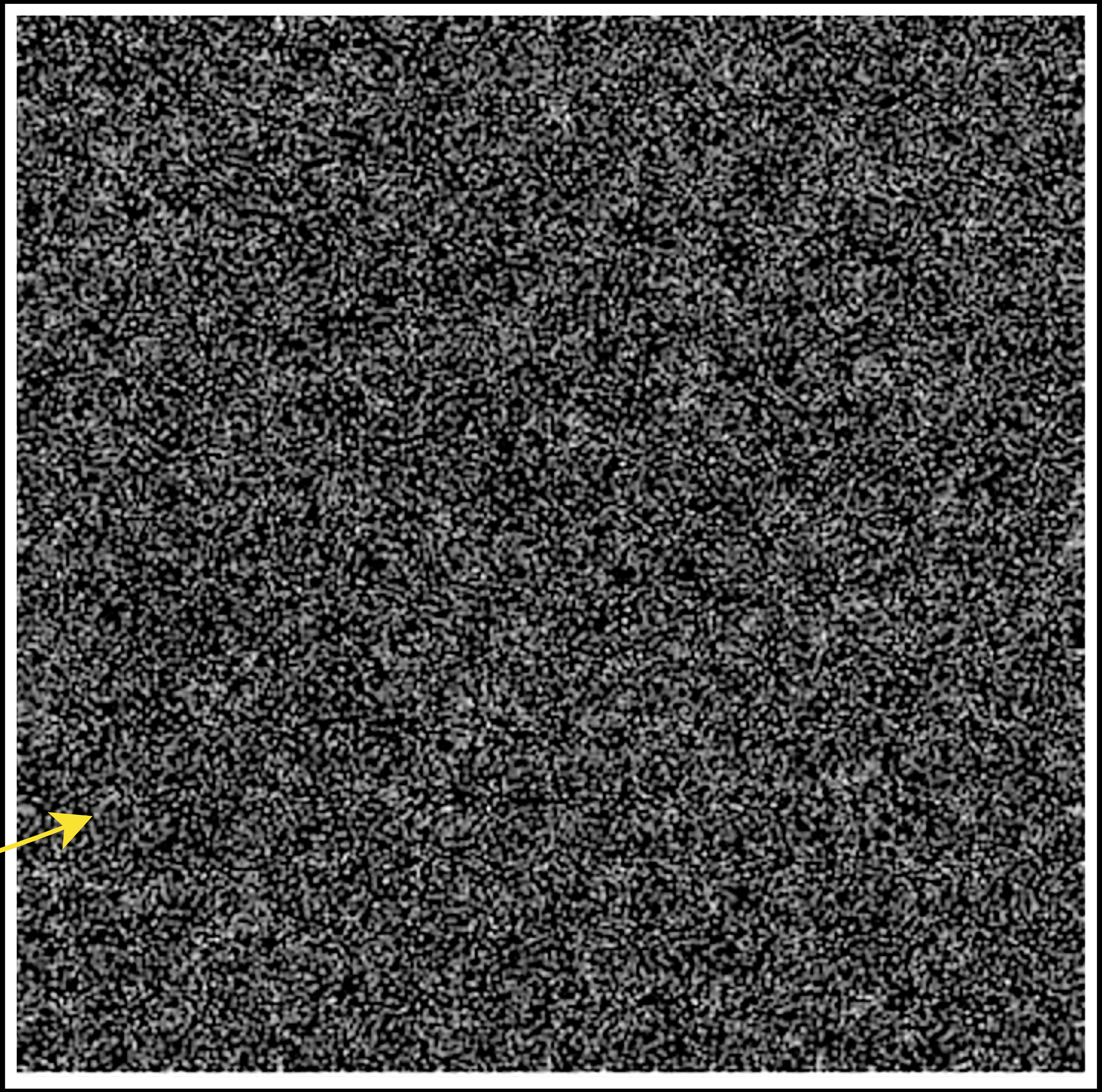
perturbed ICs





these tiny fluctuations will grow into non-linear structures (e.g. filaments, walls, haloes, clusters) through gravitational instability glass initial conditions

start with a Poisson [random] particle distribution and evolve this with the sign of gravity “inverted” [repulsive force] until particles “freeze” in comoving coordinates



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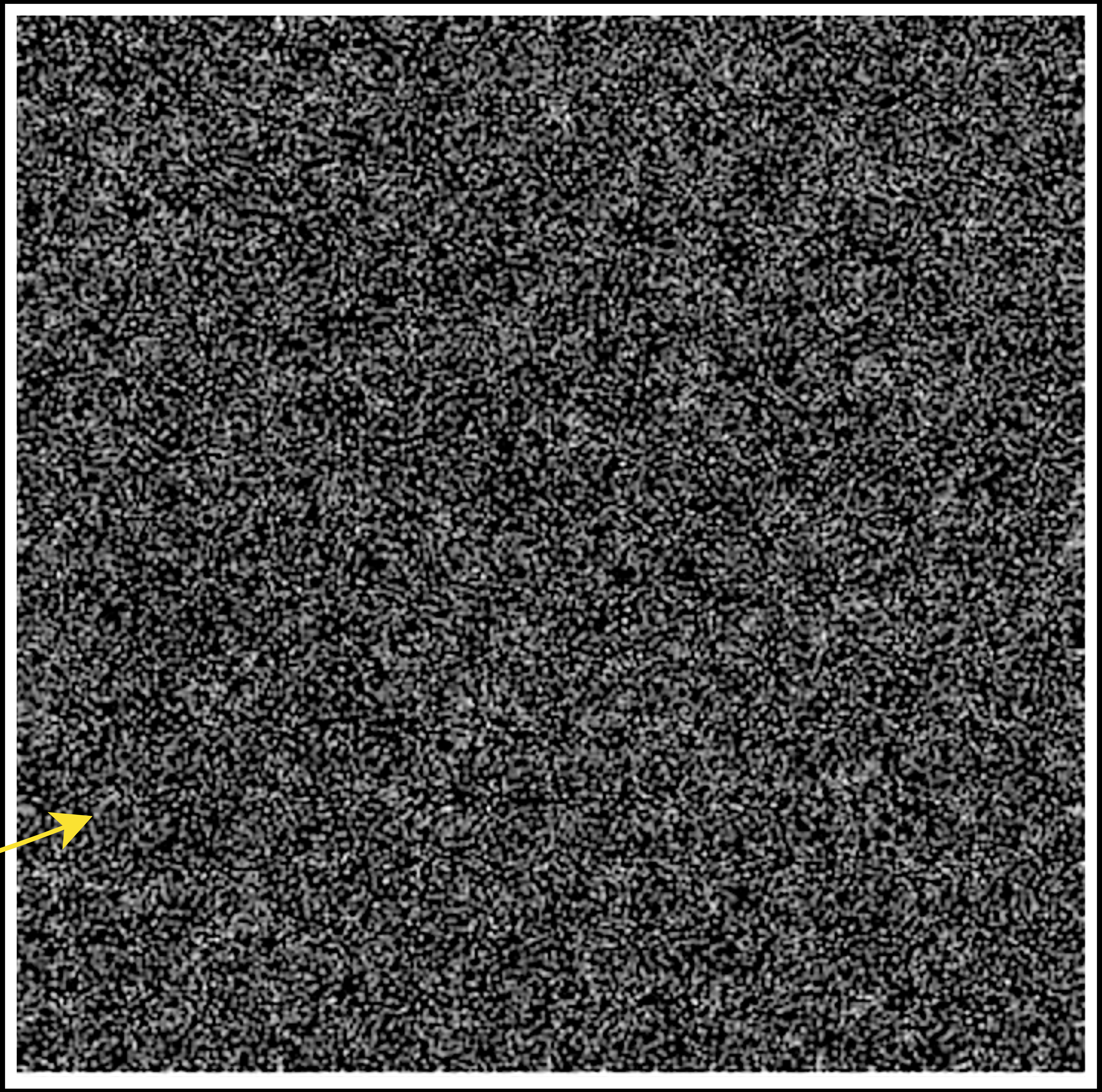

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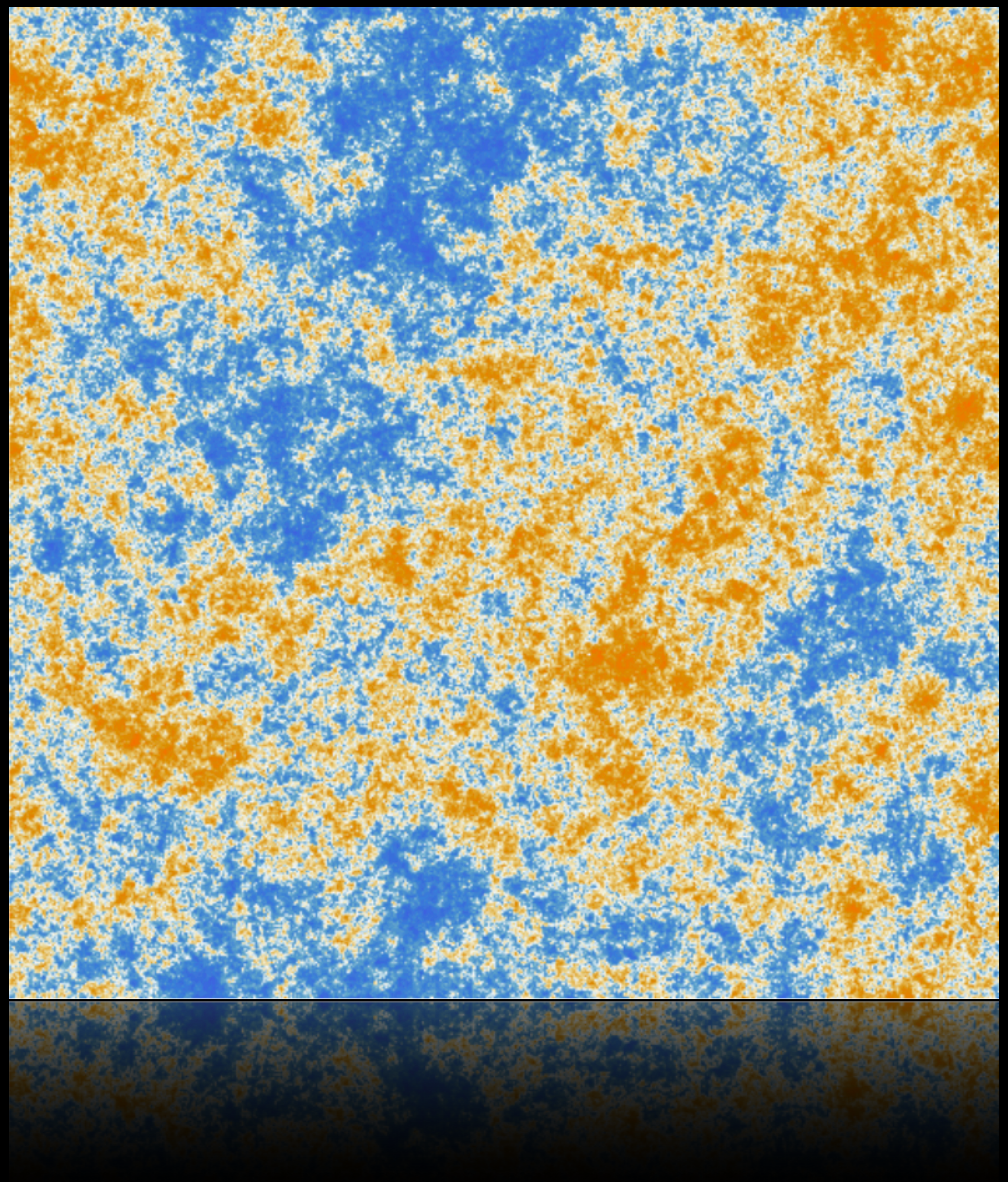
goal: create a **Gaussian random distribution** that is statistically consistent with the fluctuations measured in the CMB



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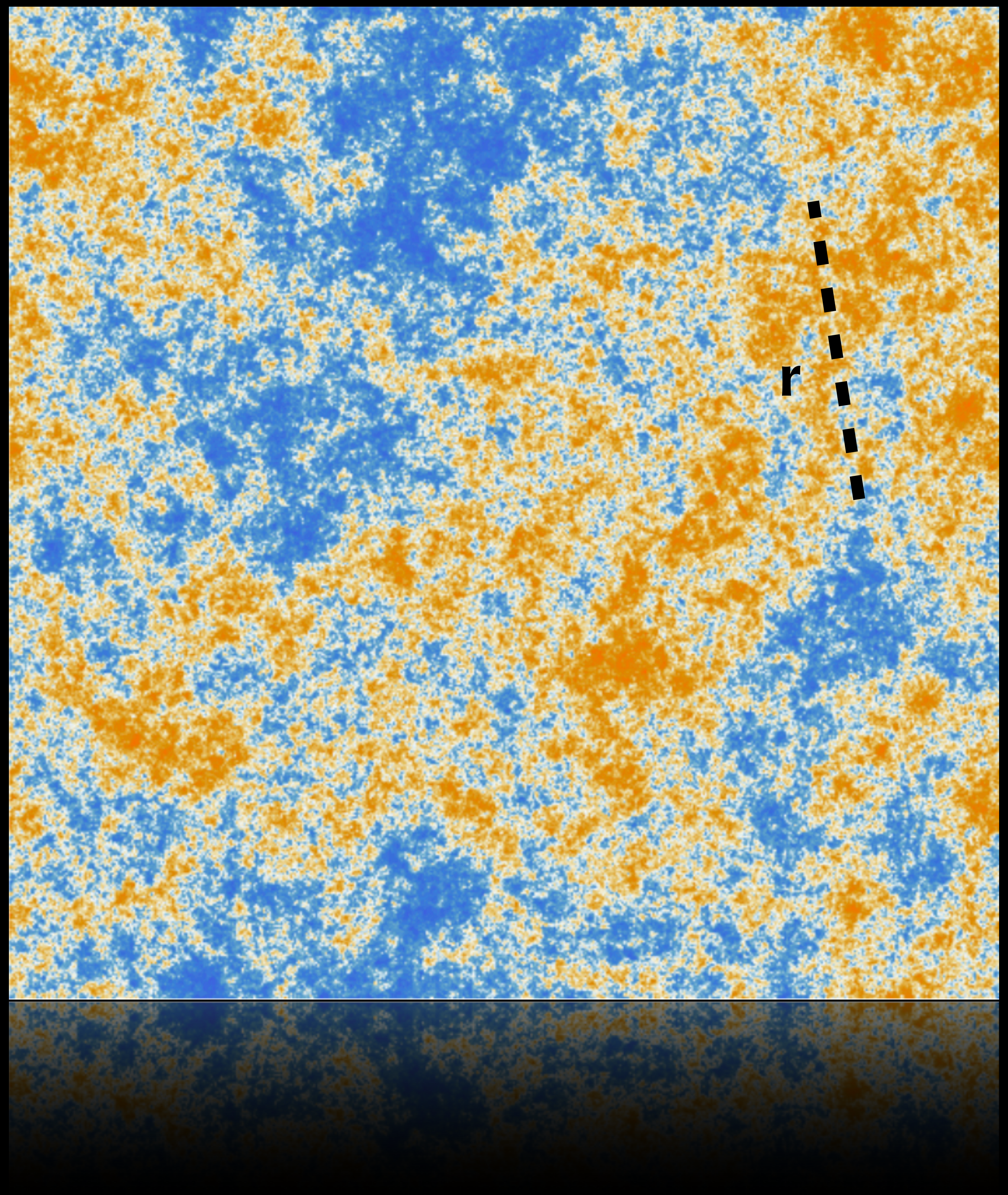

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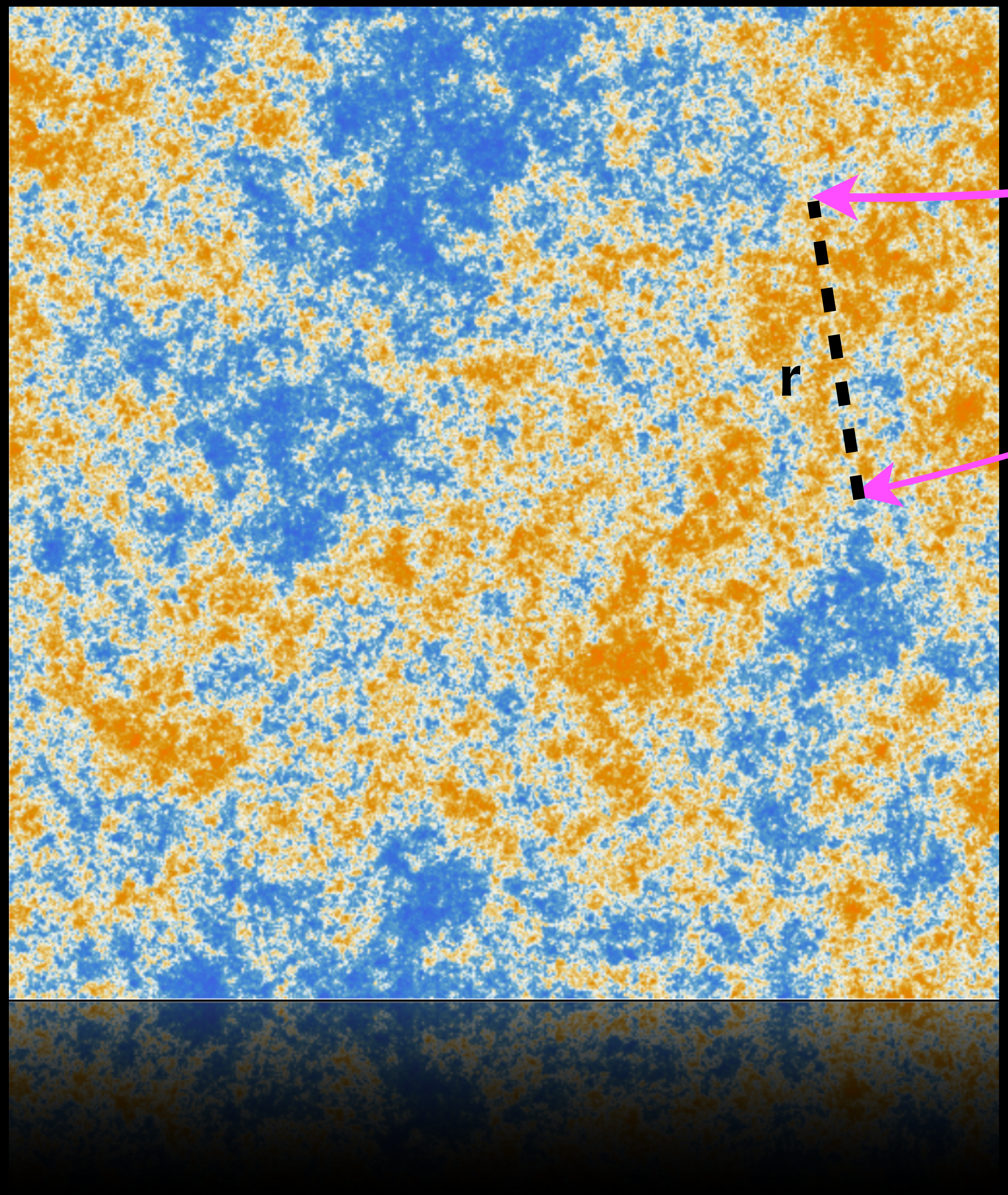

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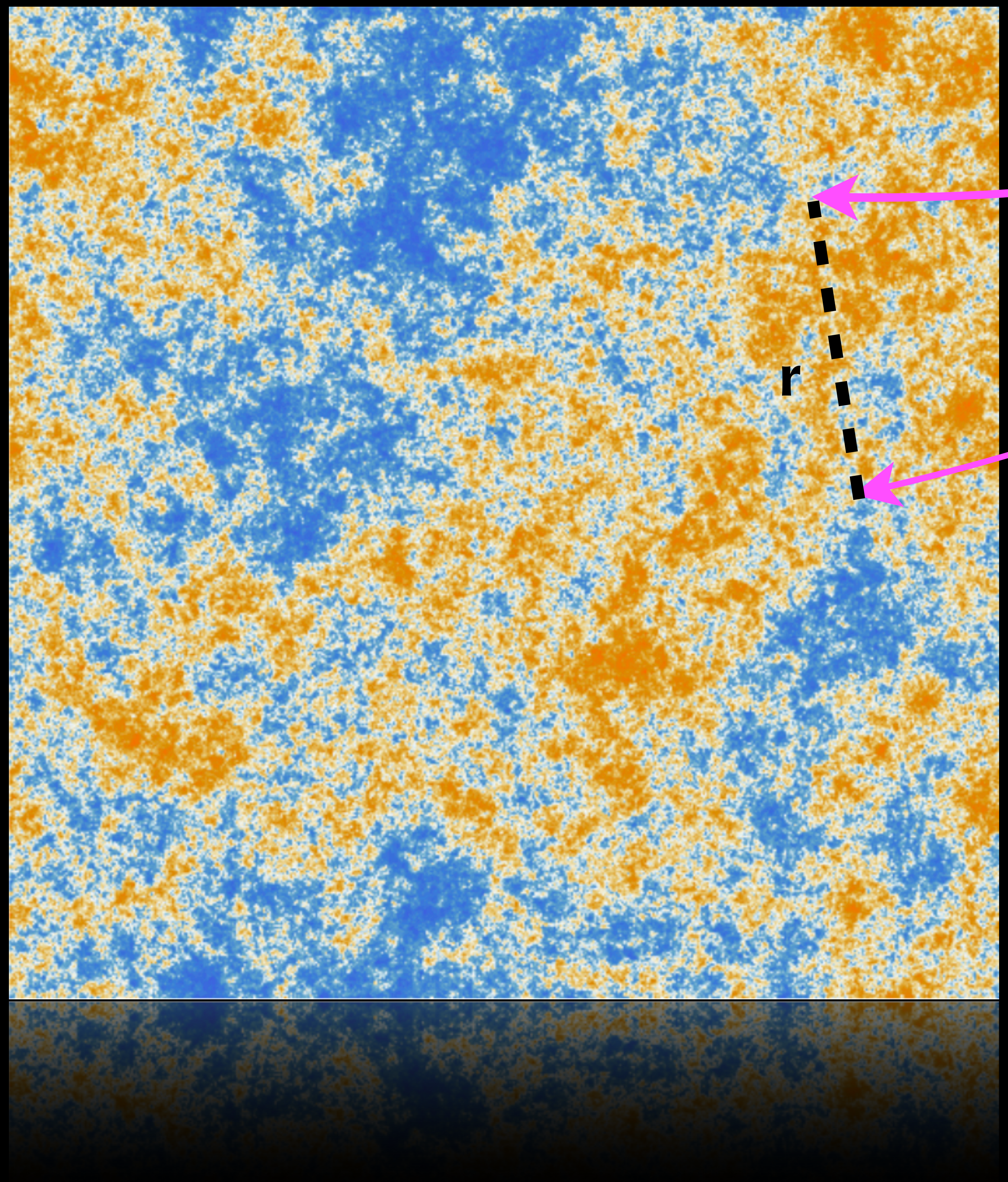

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correlation function

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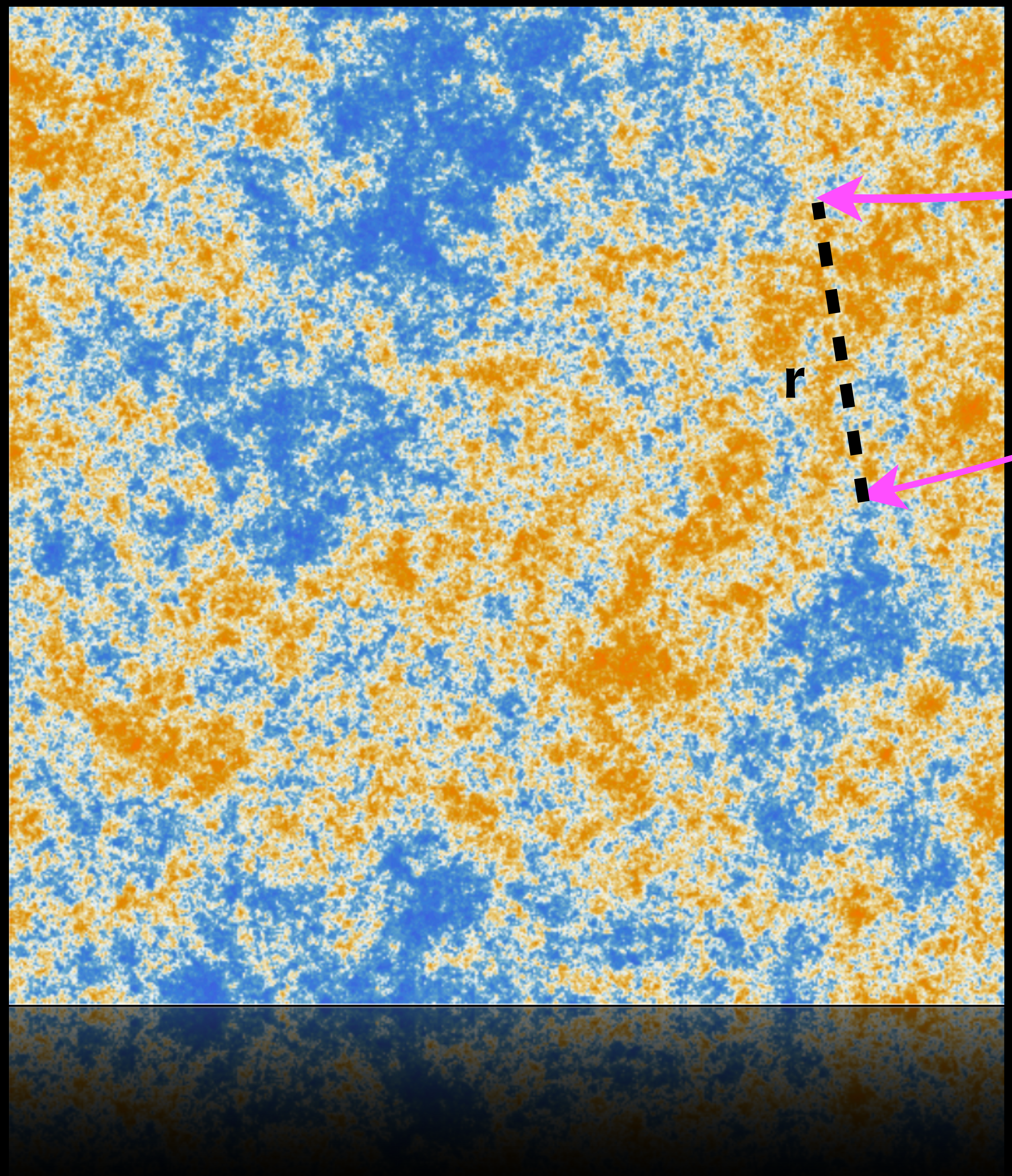

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$$\langle \delta(\mathbf{k})\, \delta^{*}(\mathbf{k}') \rangle = P(k)$$

power spectrum

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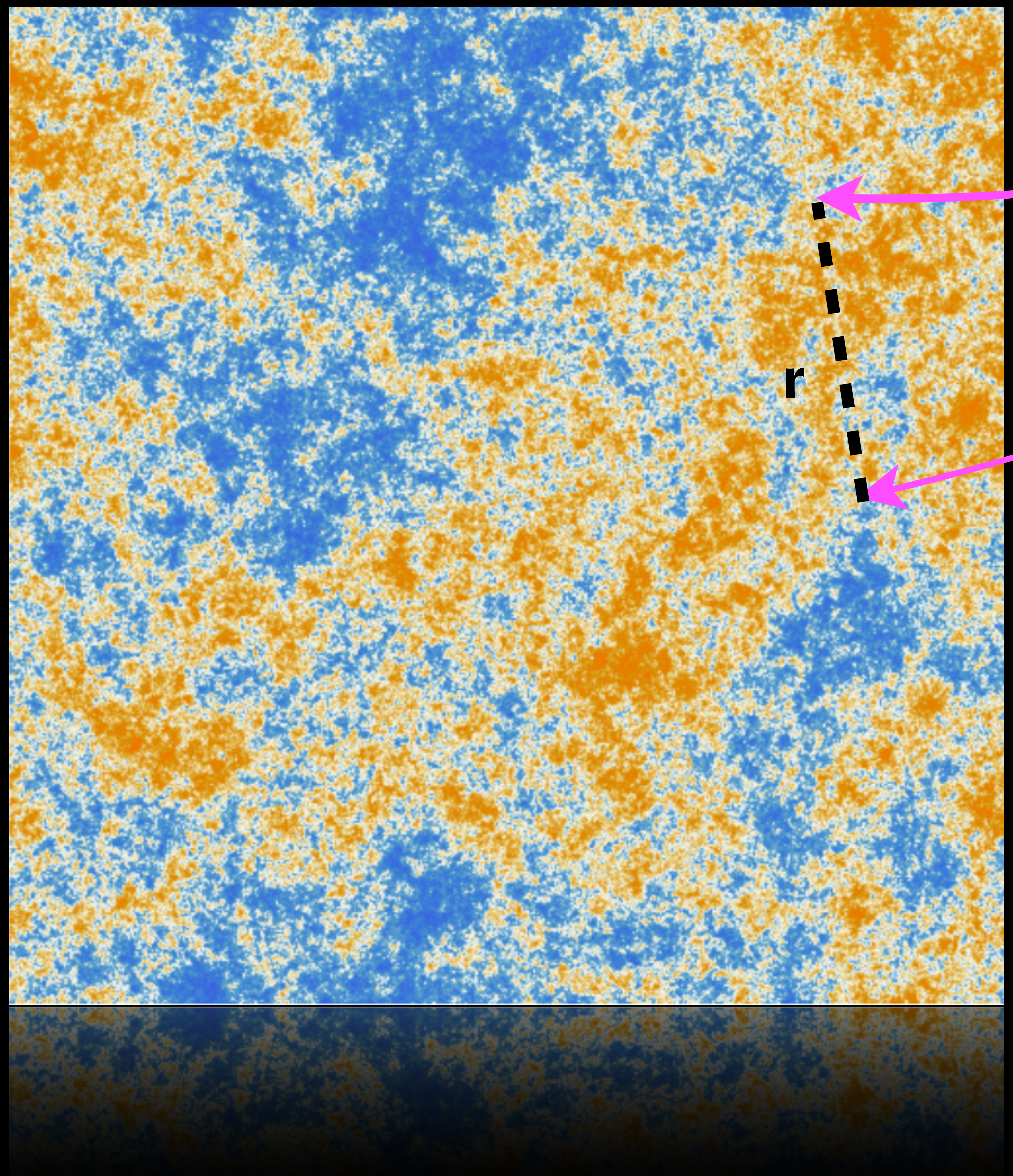

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power spectrum

the statistical properties of a Gaussian random field are characterised fully by $P(k)$

fluctuation amplitudes $\propto \sqrt{P(k)}$

phases $\in [0, 2\pi]$

the **primordial** power spectrum

inflation makes a specific prediction for the primordial density field that is a power-law:

$$P(k) \propto k^{n_s}$$

where $n_s = 1$ corresponds to a **scale-invariant** power spectrum

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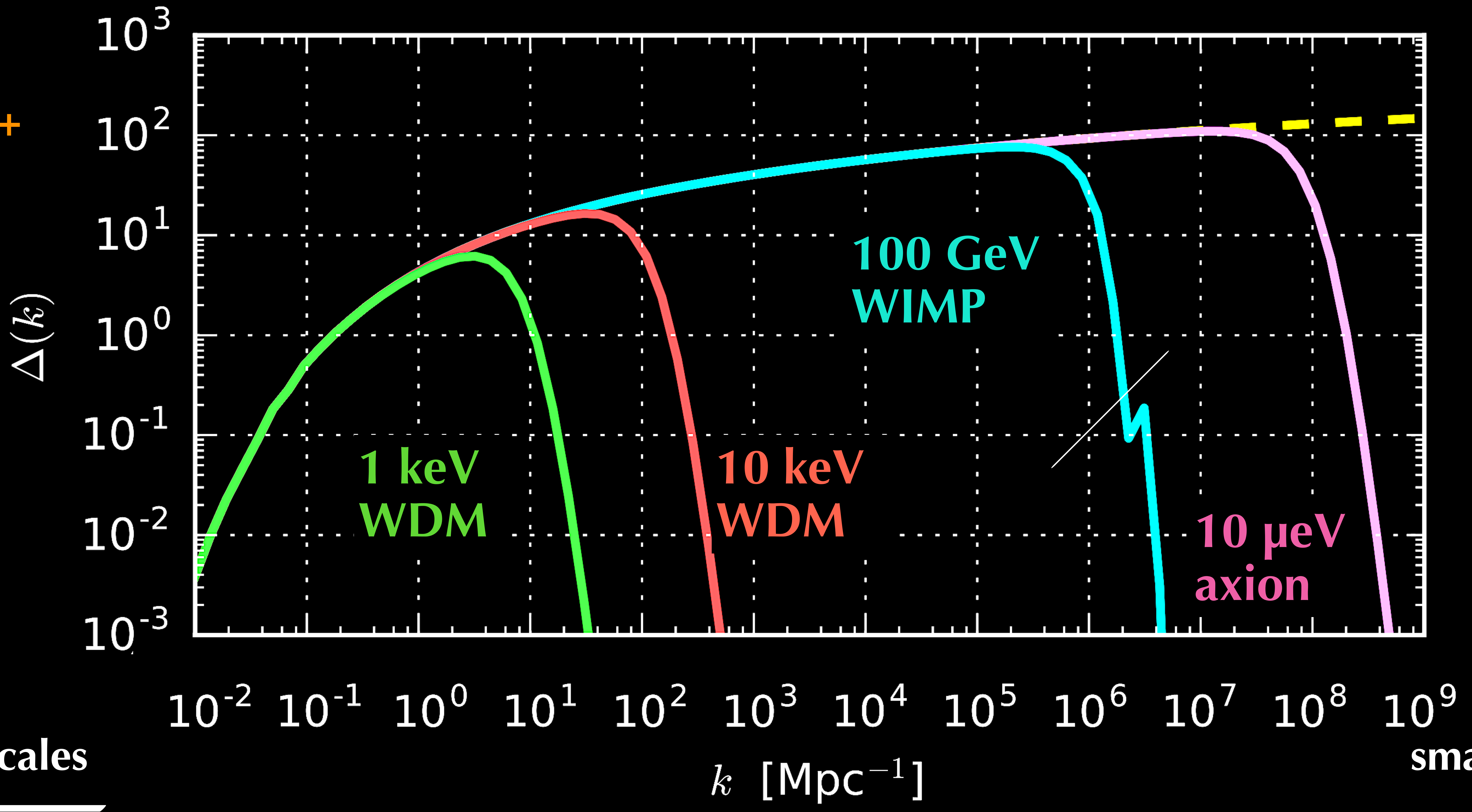
$$P(k,t) = Ak^{n_s} \left| T(k,t) \right|^2$$

computed by public codes like CAMB, CLASS etc.

Stücker+ (2018)



large scales ← → small scales

one way to model different DM candidates is to change the $P(k)$ used to generate ICs

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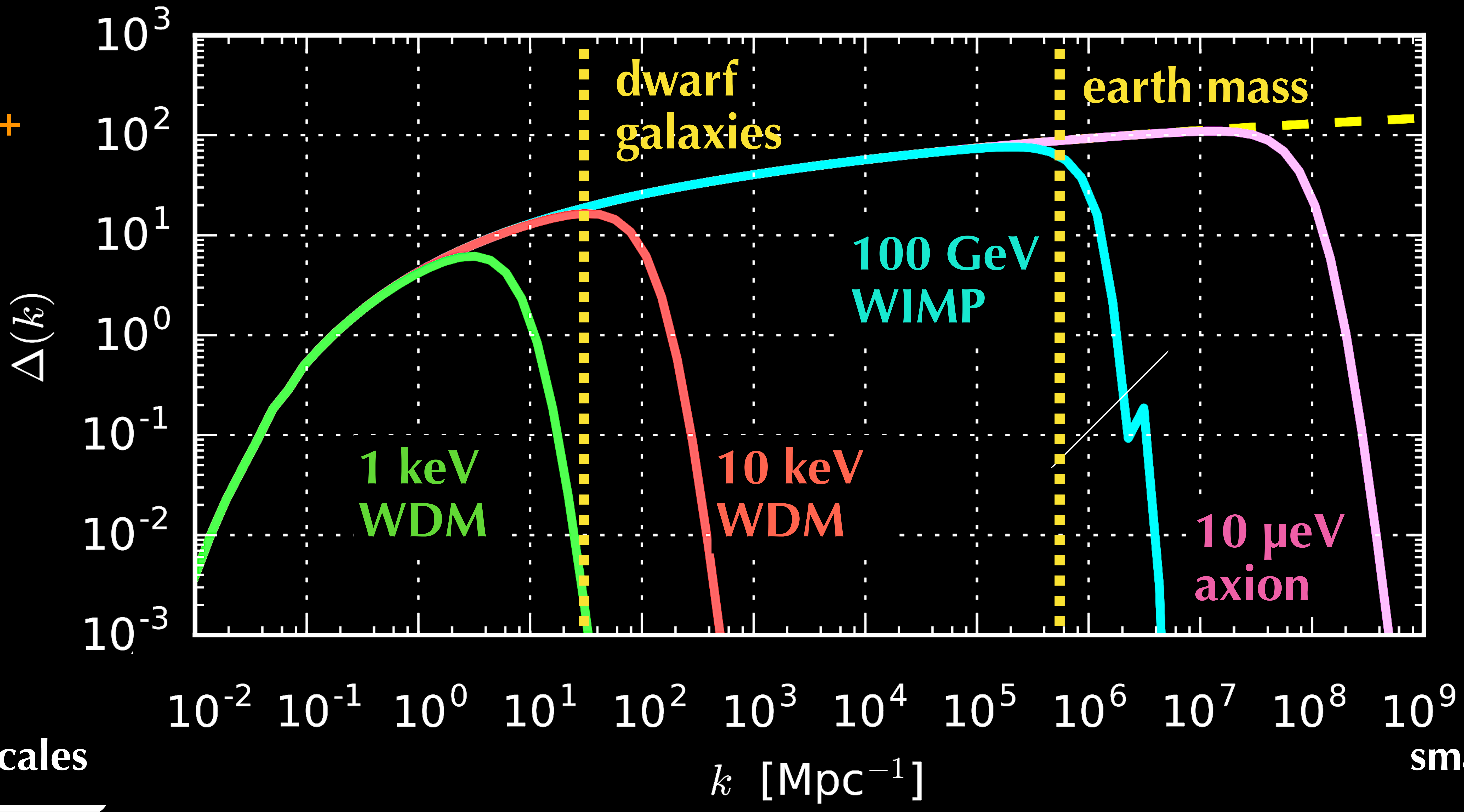

large scales

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one way to model different DM candidates is to change the $P(k)$ used to generate ICs

now, we more or less have everything we need: we just need to perturb the initial particle configuration according to these fluctuations. the position of a particle initially at coordinate q after some time t is given by:

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Zel′dovich approximation

Eulerian [final] position

Lagrangian [unpertubed] position

growth function

displacement field

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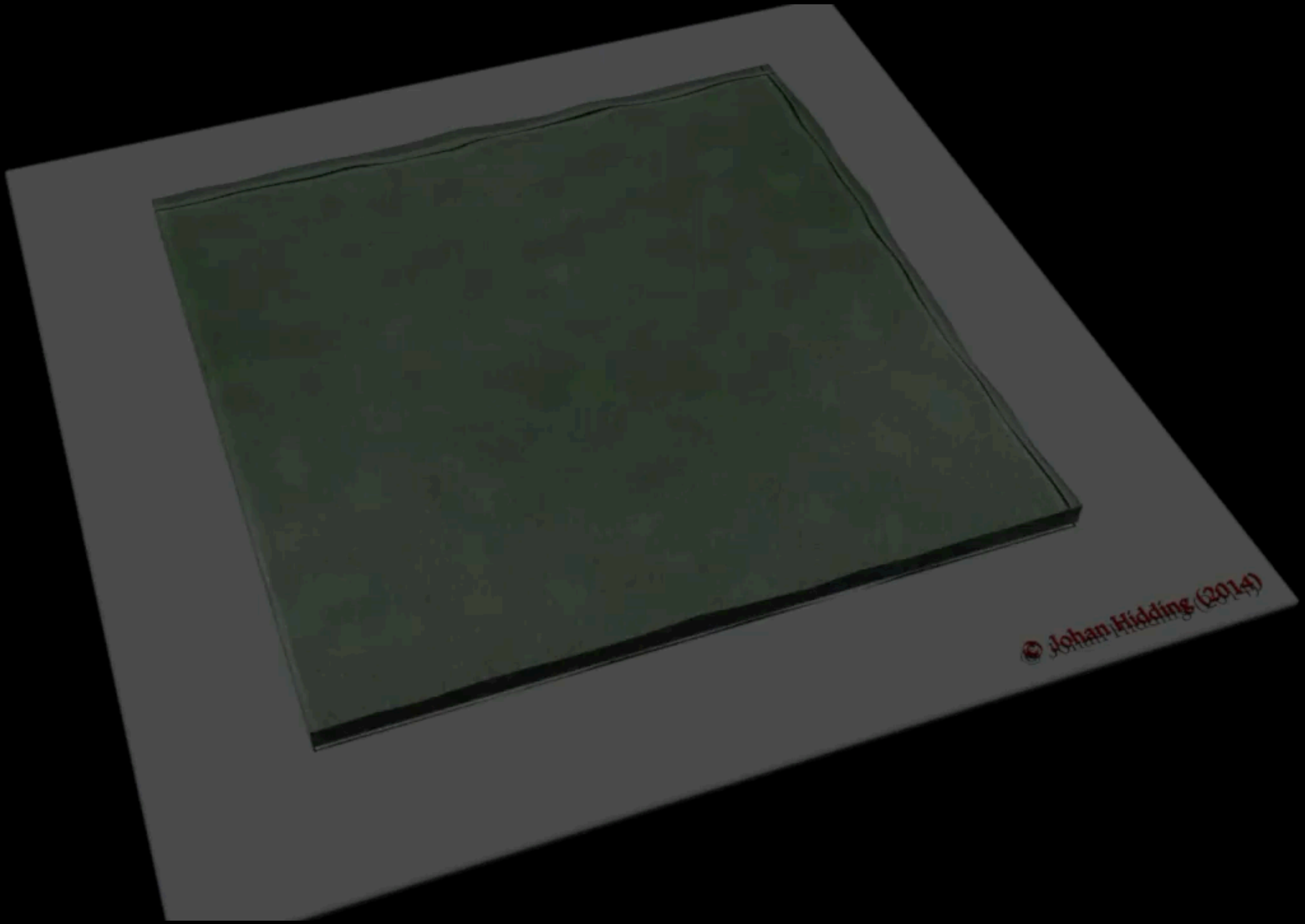
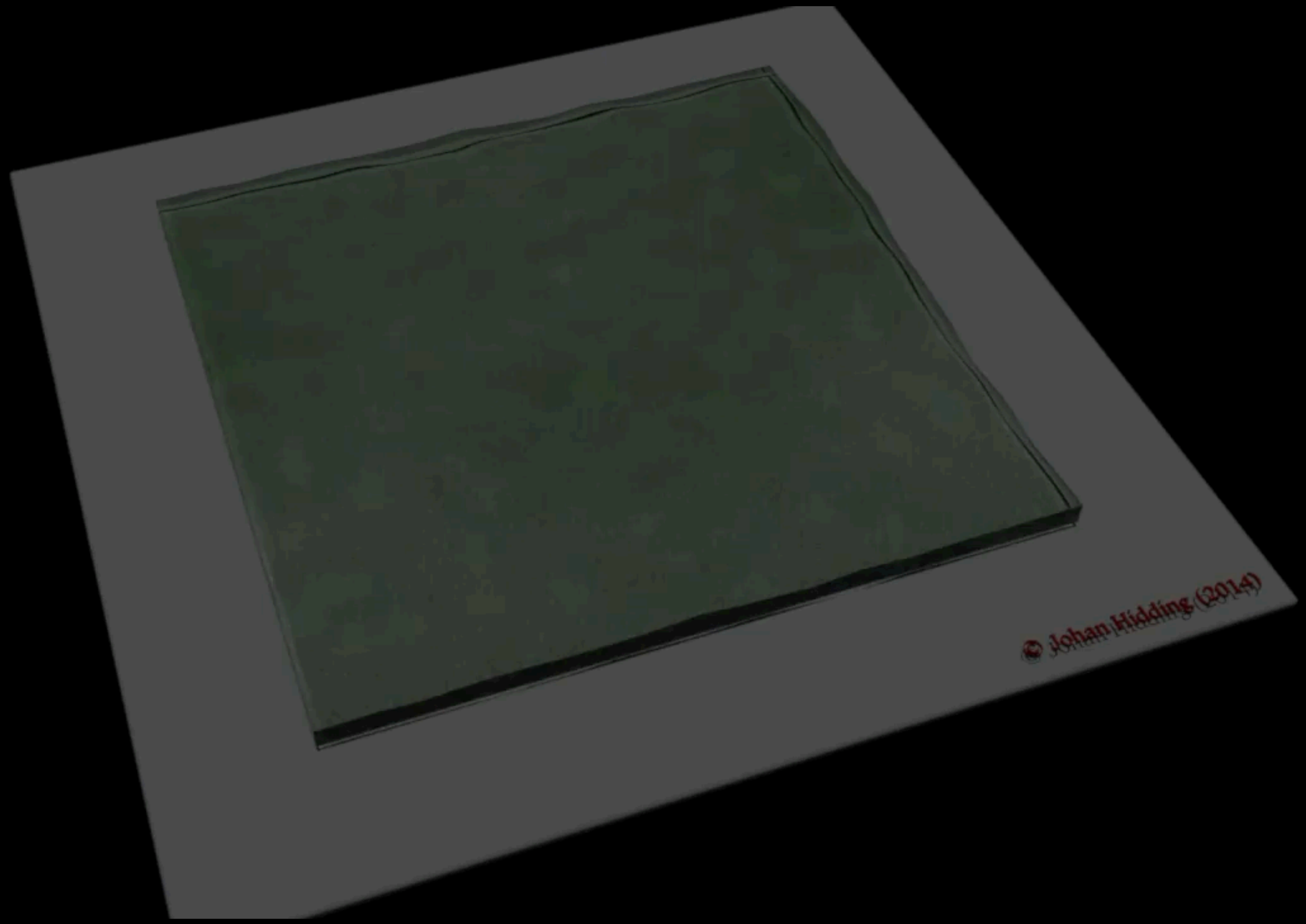
$\mathbf{S}(\mathbf{q}) \propto -\nabla\Phi(\mathbf{q})$, and we know that Φ is related to $\delta(\mathbf{k})$ through Poisson's equation



credit: Johan Hidding



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the non-linear density field

while the initial conditions of our cosmos may be approximated as a Gaussian random field, the late-time, non-linear density is anything but Gaussian! gravitational instability naturally transitions the matter field away from Gaussianity

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given a density field, $\delta(\mathbf{x})$, you can also quantify the degree of non-Gaussianity by computing moments:

size of the grid used to smooth the density field

$$\langle \delta^n \rangle = \frac{1}{N_g} \sum_i^{N_g} \left(\delta^i - \langle \delta \rangle \right)^n$$

$n = 0$ (mean)
$n = 2$ (variance)
$n = 3$ (skewness)
$n = 4$ (kurtosis)

lecture notes:

https://github.com/sownakbose/AstroPGCourse-Nbody

jupyter notebooks:

https://github.com/Shaun-T-Brown/Summer_school

[link also on lecture notes Github page]