PRECISE summer school
Warsaw

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hands-on cosmological simulations

session 2: initial conditions and cosmological integration

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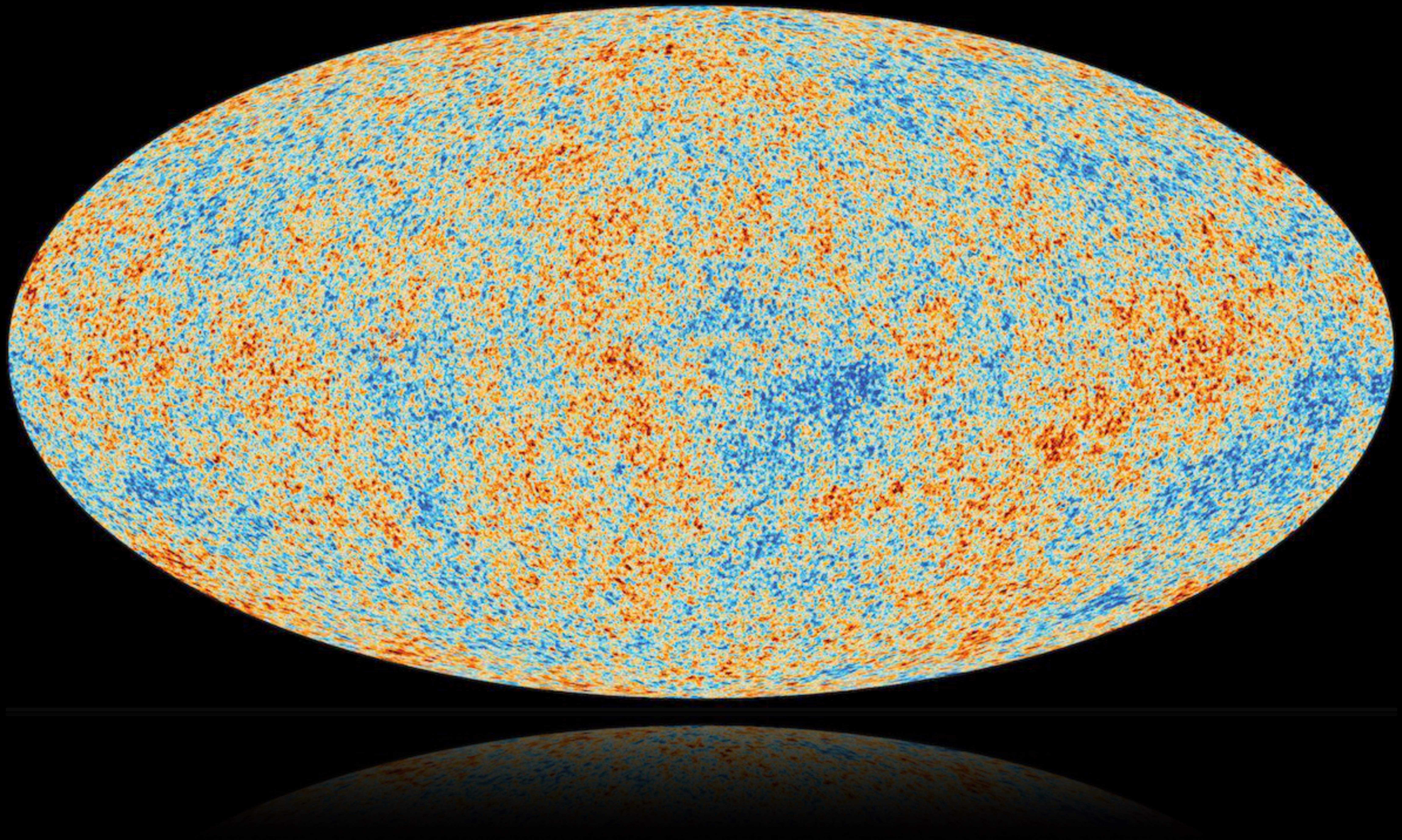
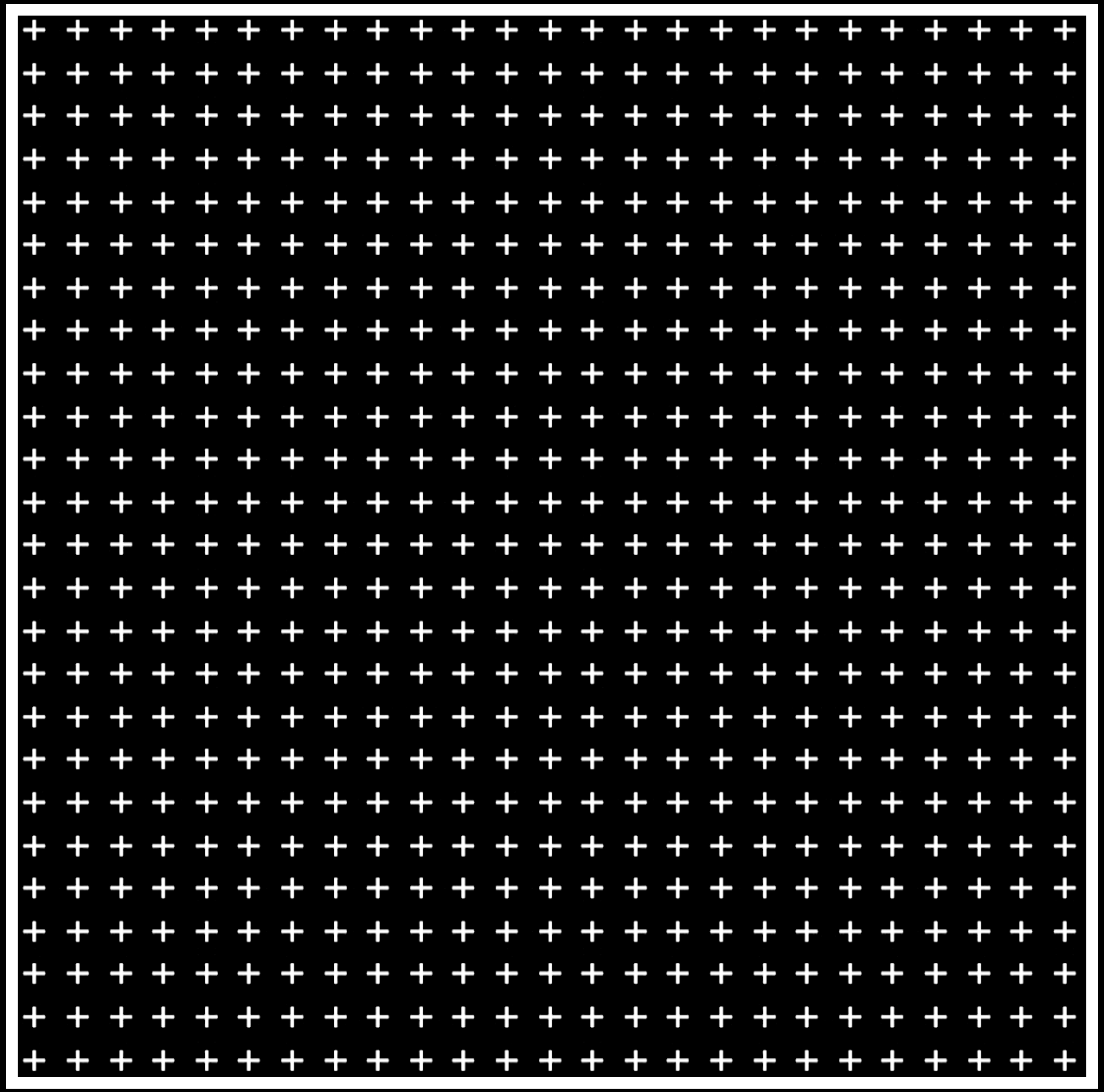
setting up initial conditions

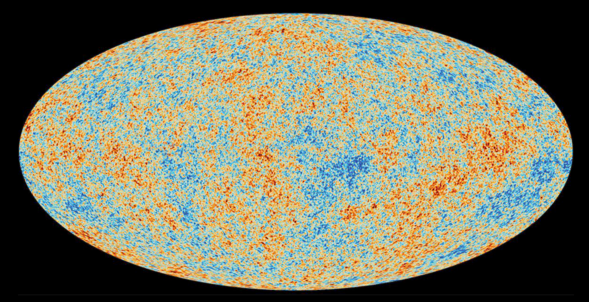
measurement of the cosmic microwave background radiation gives us *precise constraints* on cosmological initial conditions

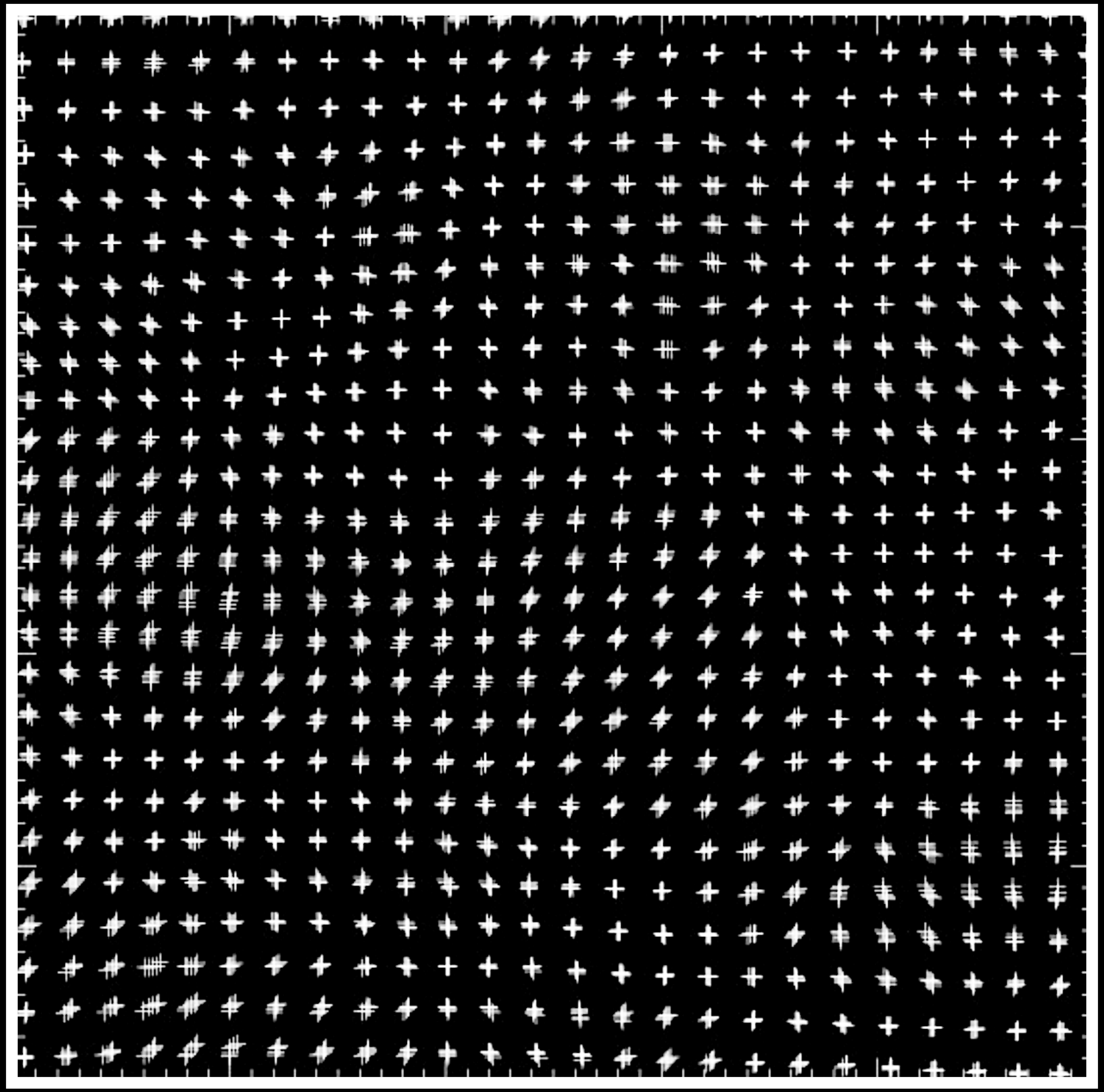


homogeneous & isotropic ICs



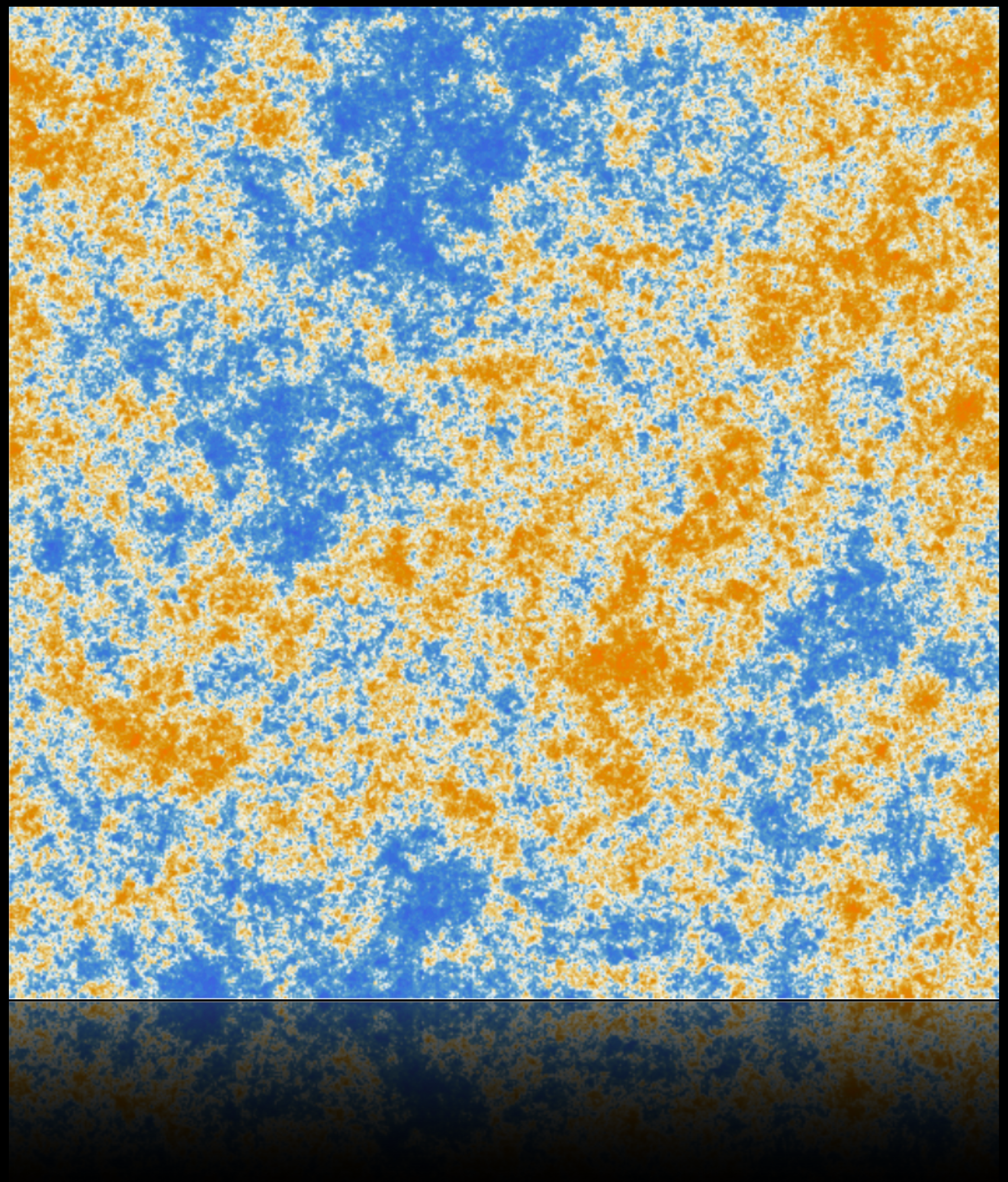
perturbed ICs





these tiny fluctuations will grow into non-linear structures (e.g. filaments, walls, haloes, clusters) through gravitational instability

goal: create a Gaussian random distribution that is statistically consistent with the fluctuations measured in the CMB



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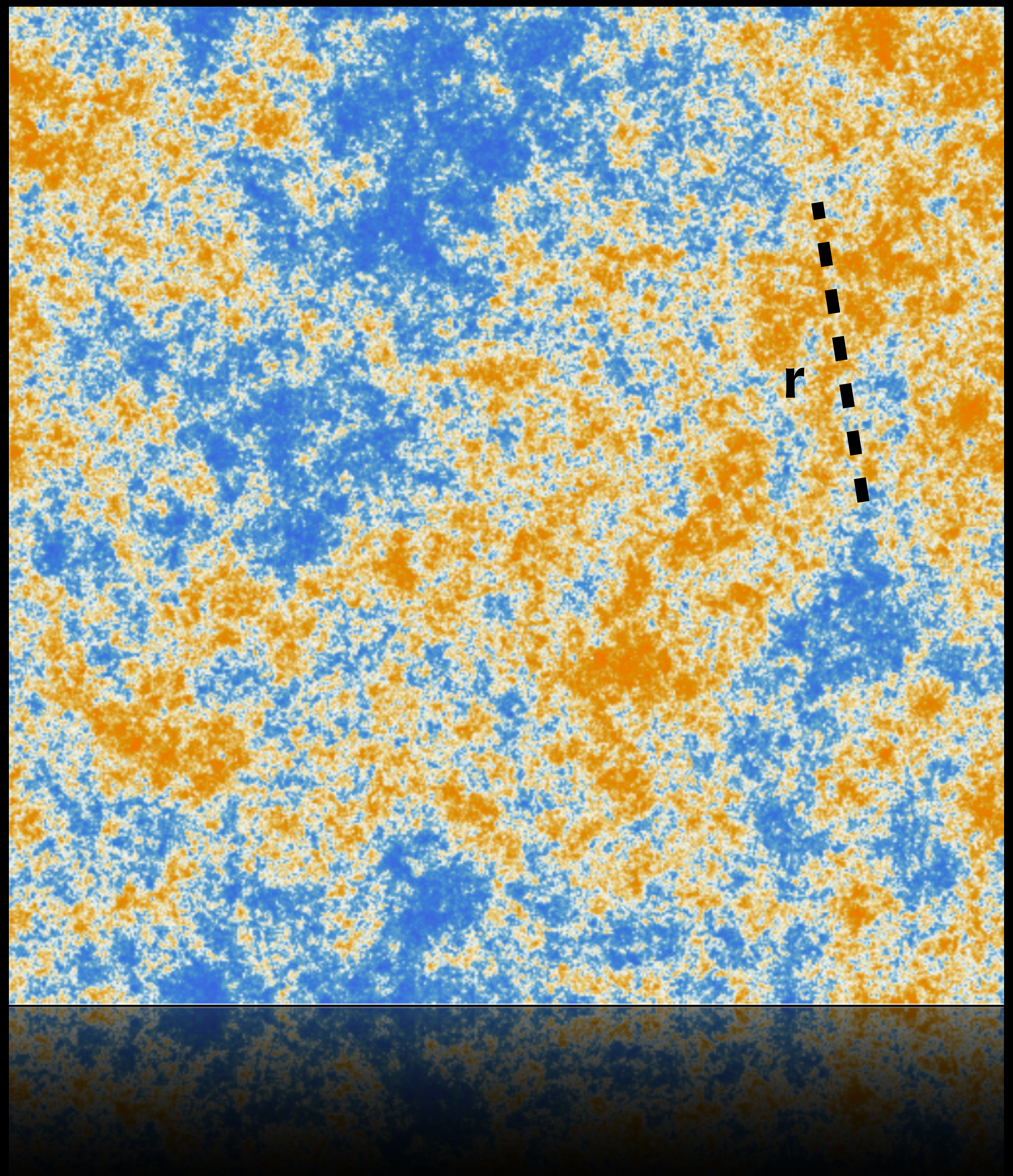

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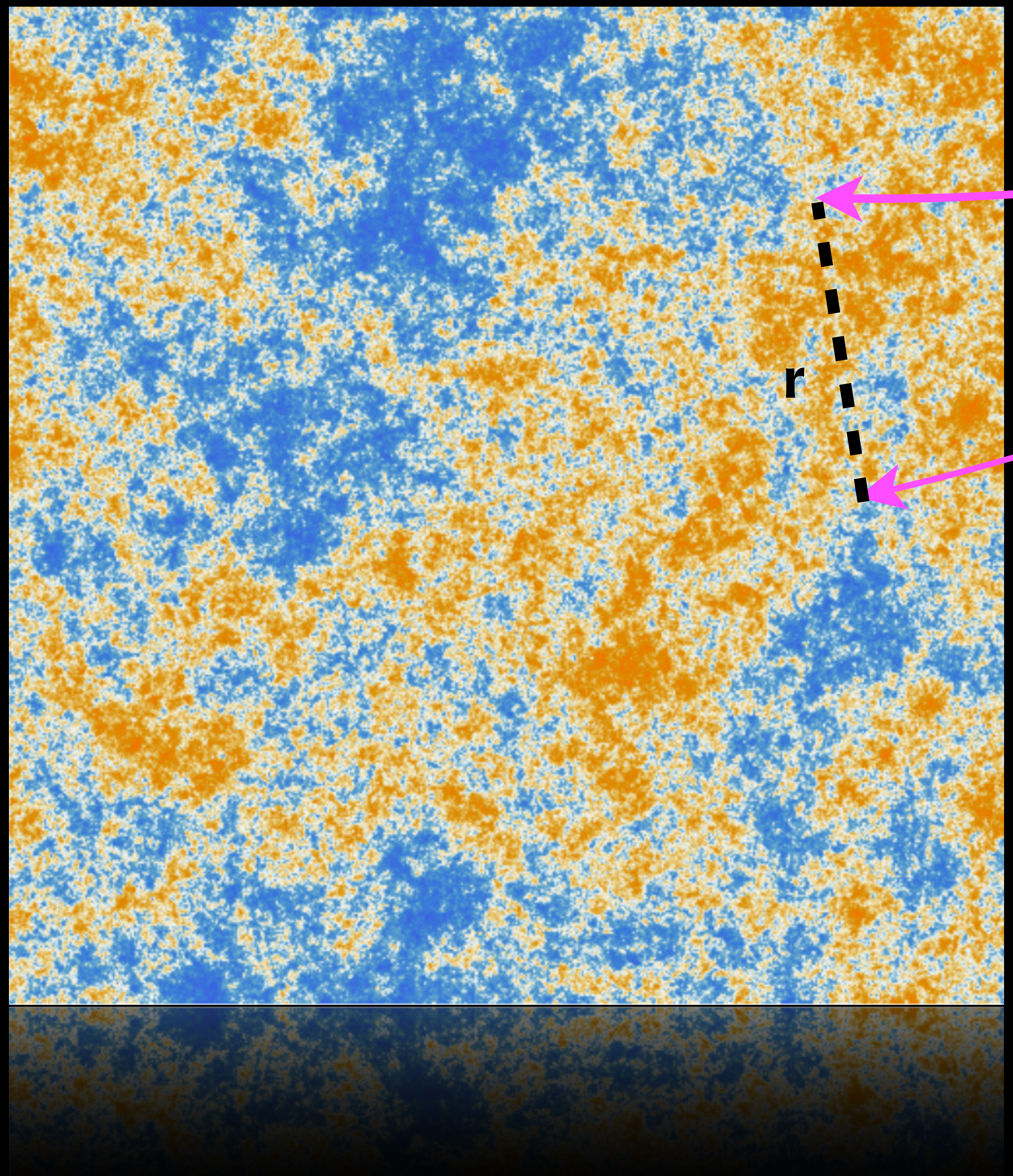

$$\langle \delta(\mathbf{x})\, \delta(\mathbf{x}') \rangle$$

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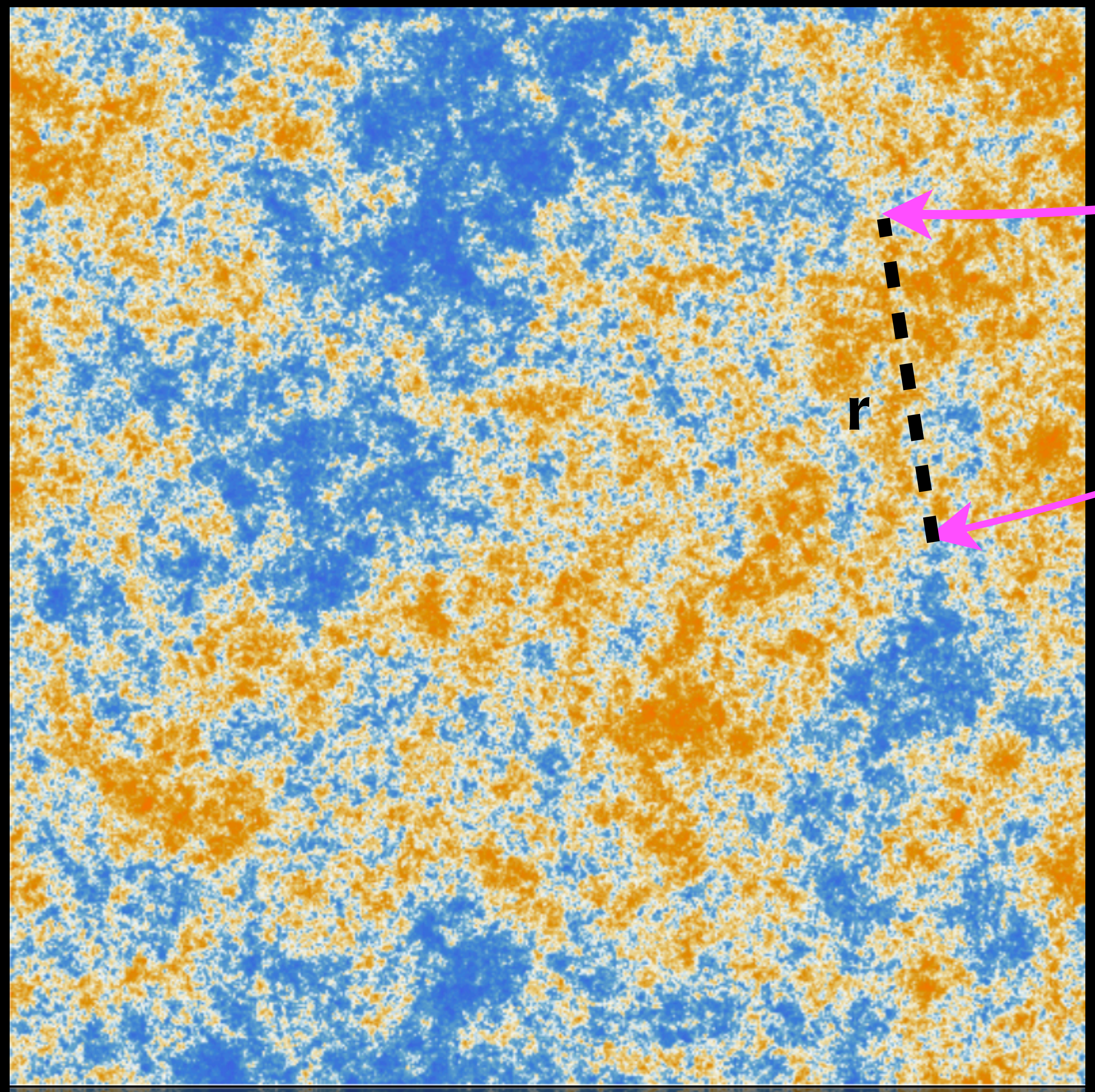

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correlation function

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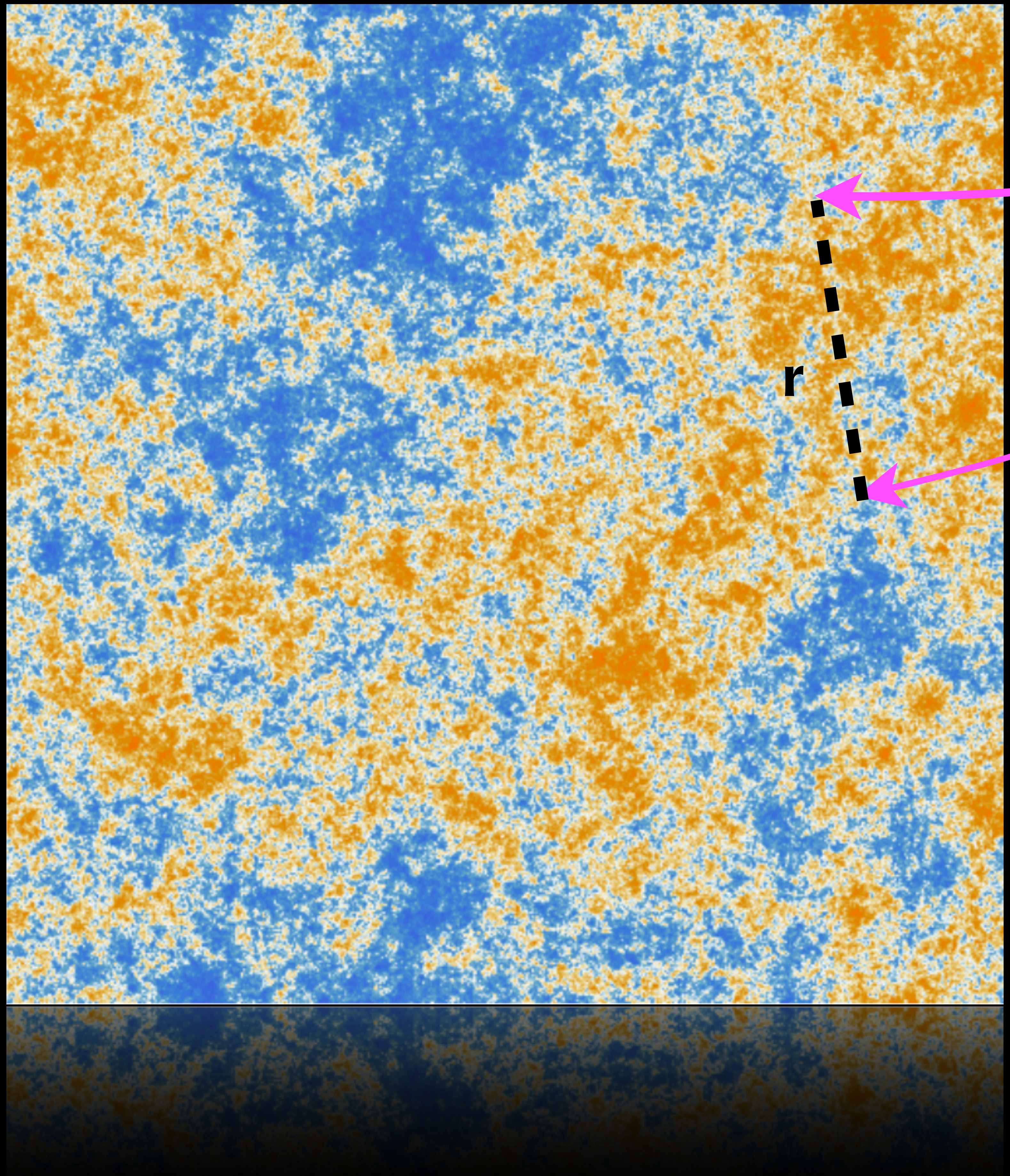

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$$\langle \delta(\mathbf{k})\, \delta^*(\mathbf{k}') \rangle = P(k)$$

power spectrum

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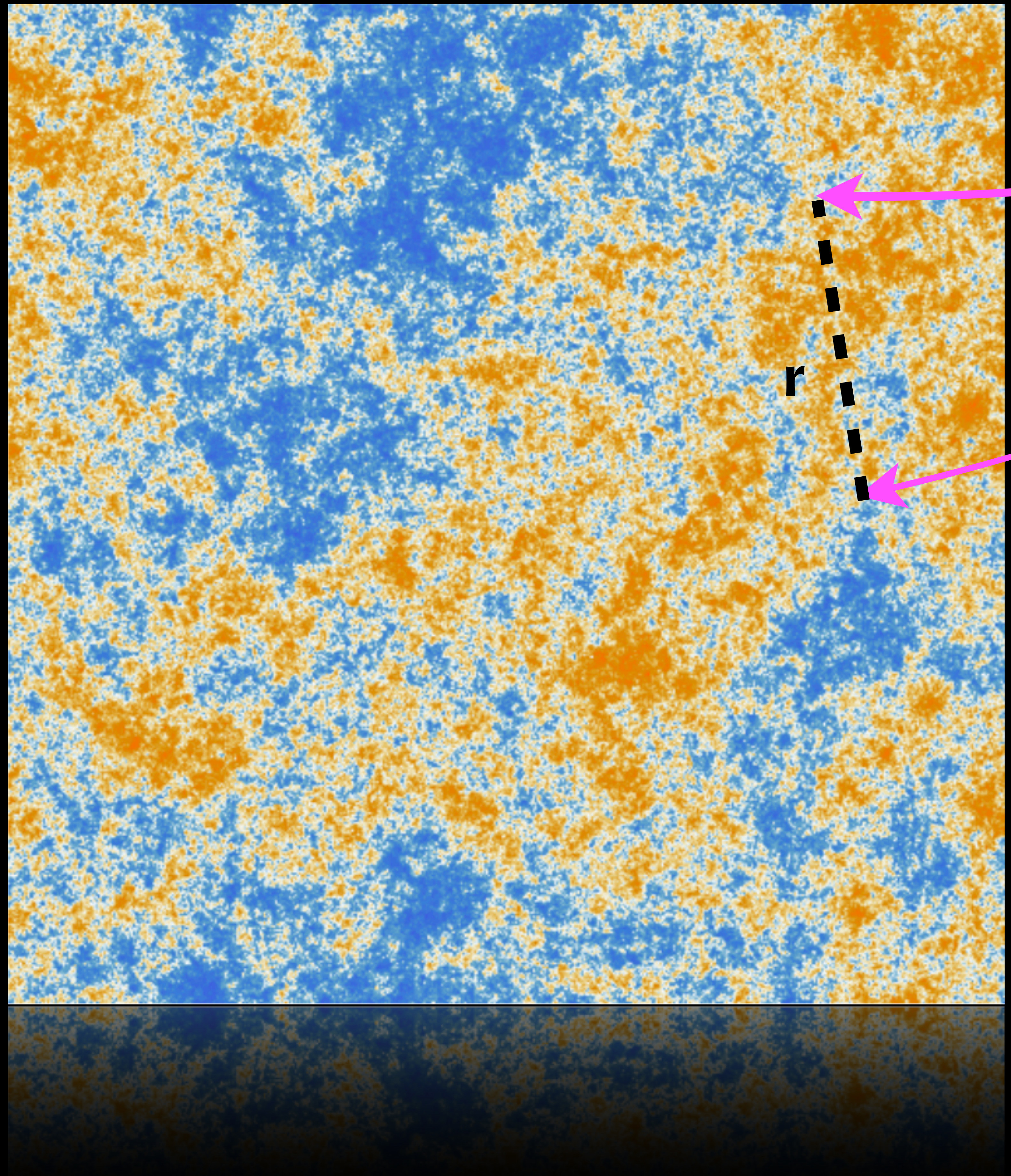

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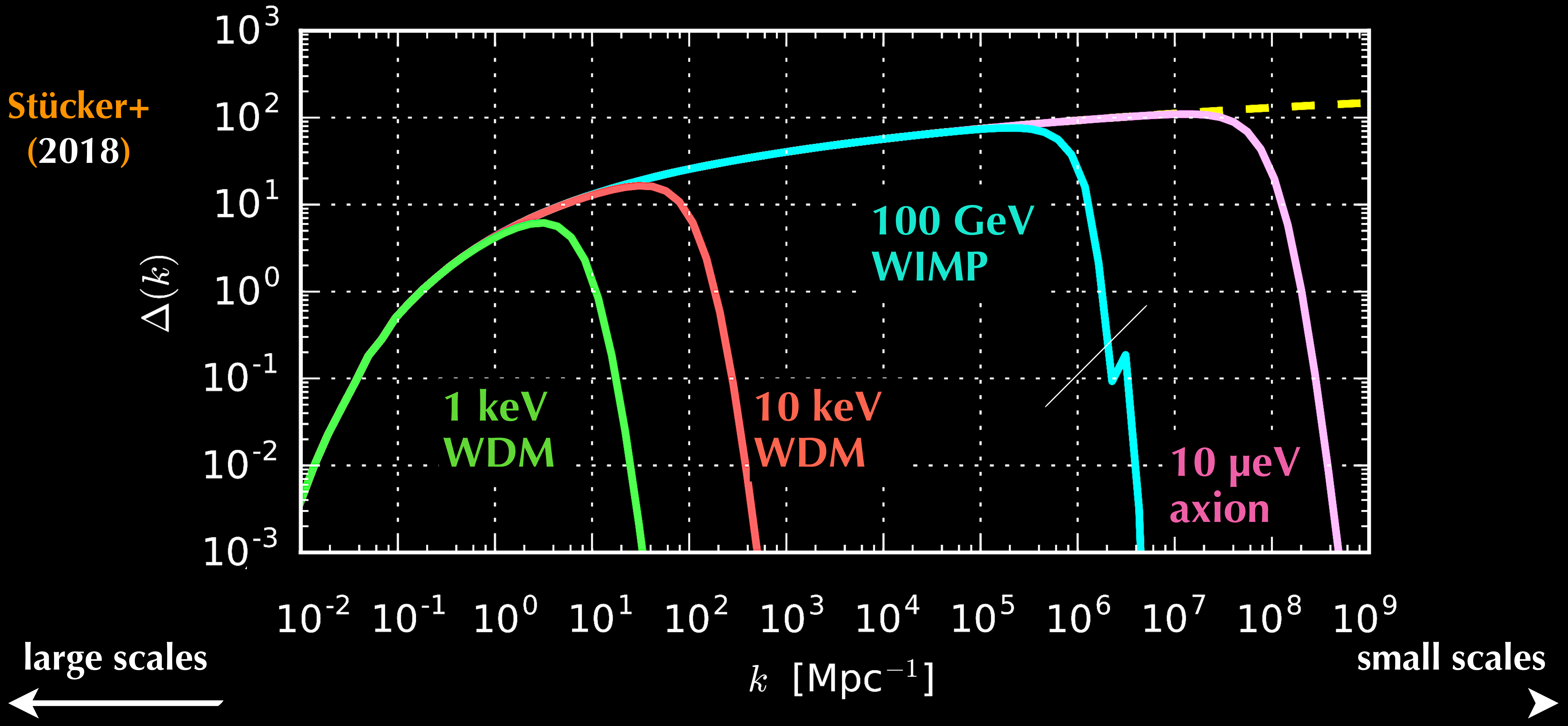
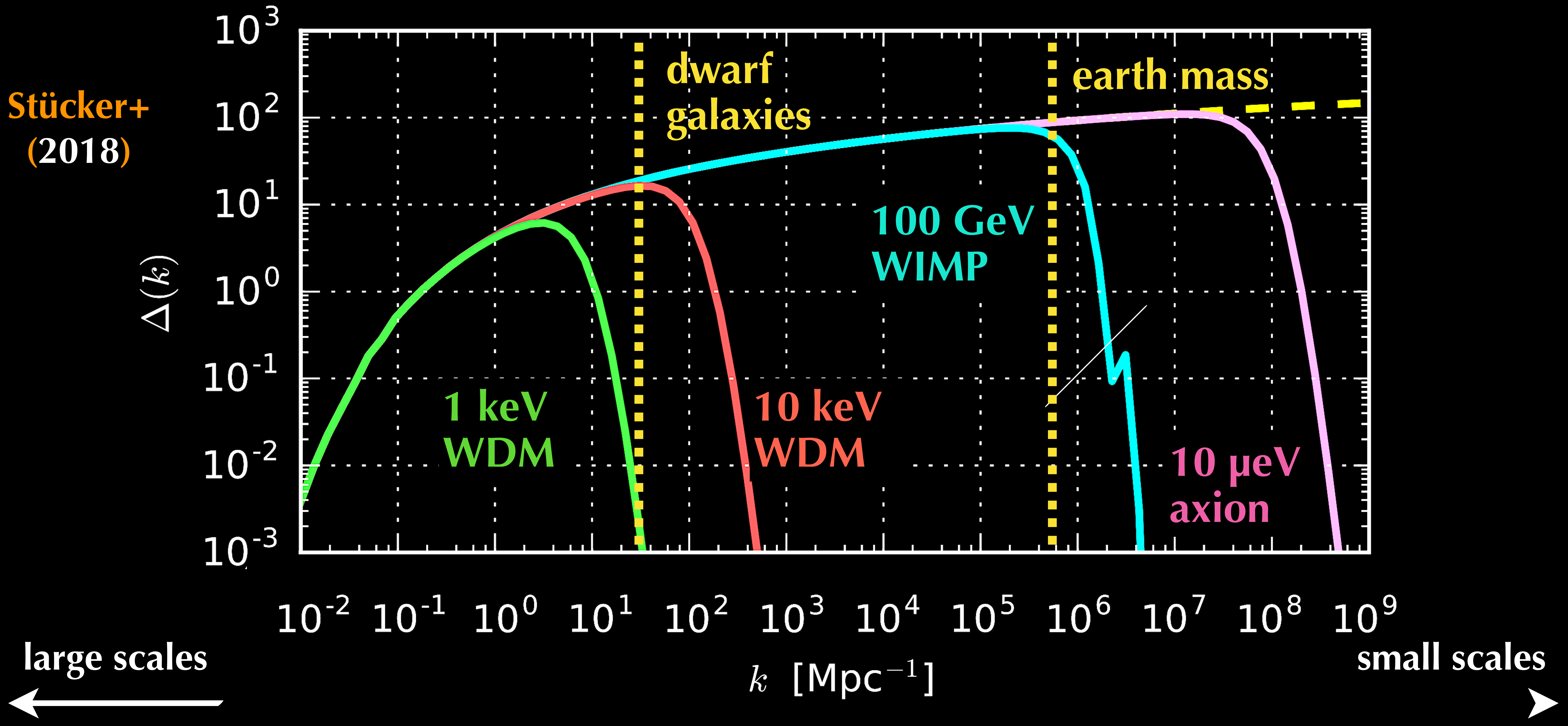
the statistical properties of a Gaussian random field are characterised fully by $P(k)$

fluctuation amplitudes $\propto \sqrt{P(k)}$ phases $\in [0, 2\pi]$

Stücker+ (2018)



one way to model different DM candidates is to change the $P(k)$ used to generate ICs



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now, we more or less have everything we need: we just need to perturb the initial particle configuration according to these fluctuations. the position of a particle initially at coordinate q after some time t is given by:

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Zel'dovich approximation

Eulerian [final] position

Lagrangian [unpertubed] position

growth function

displacement field

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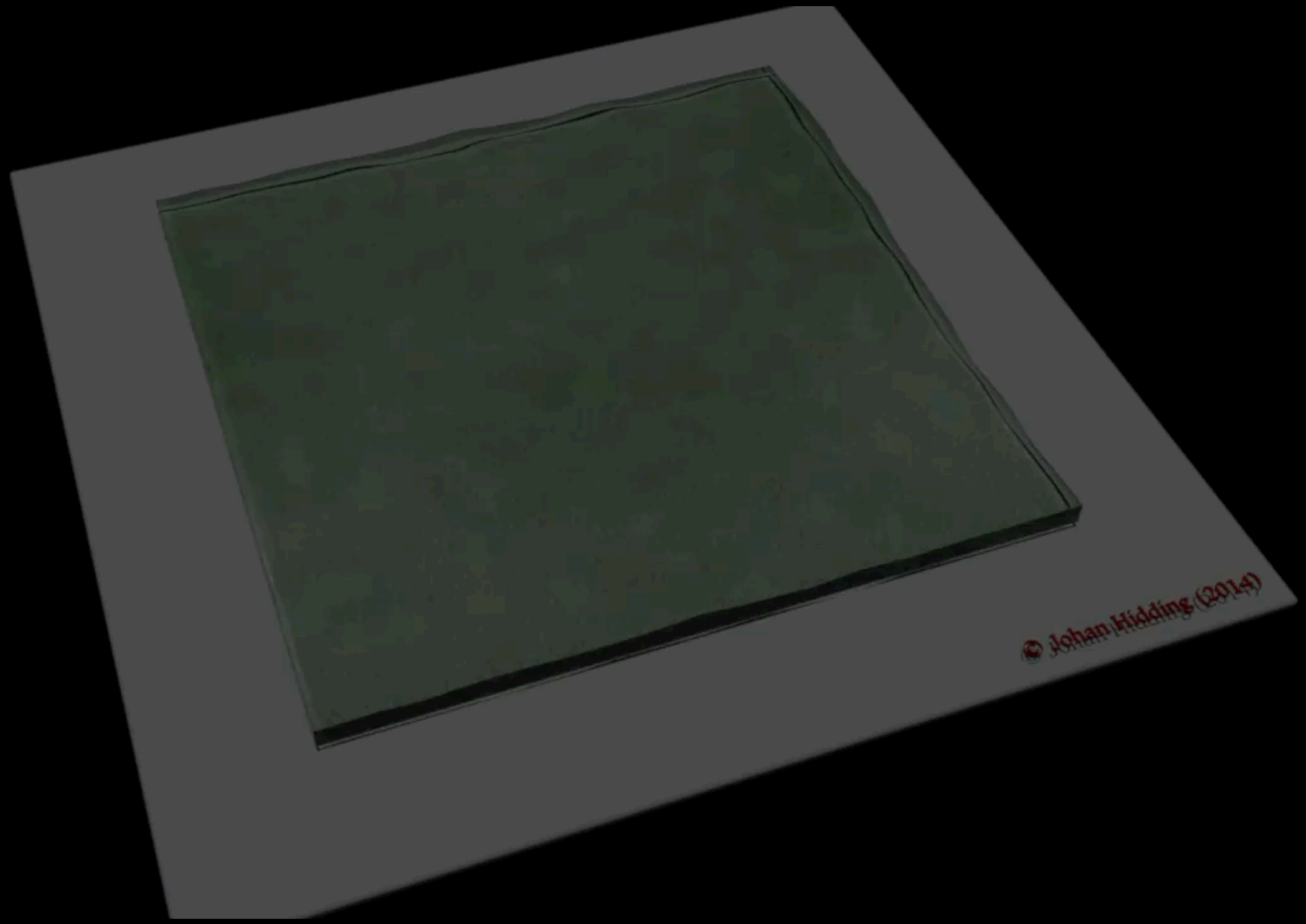
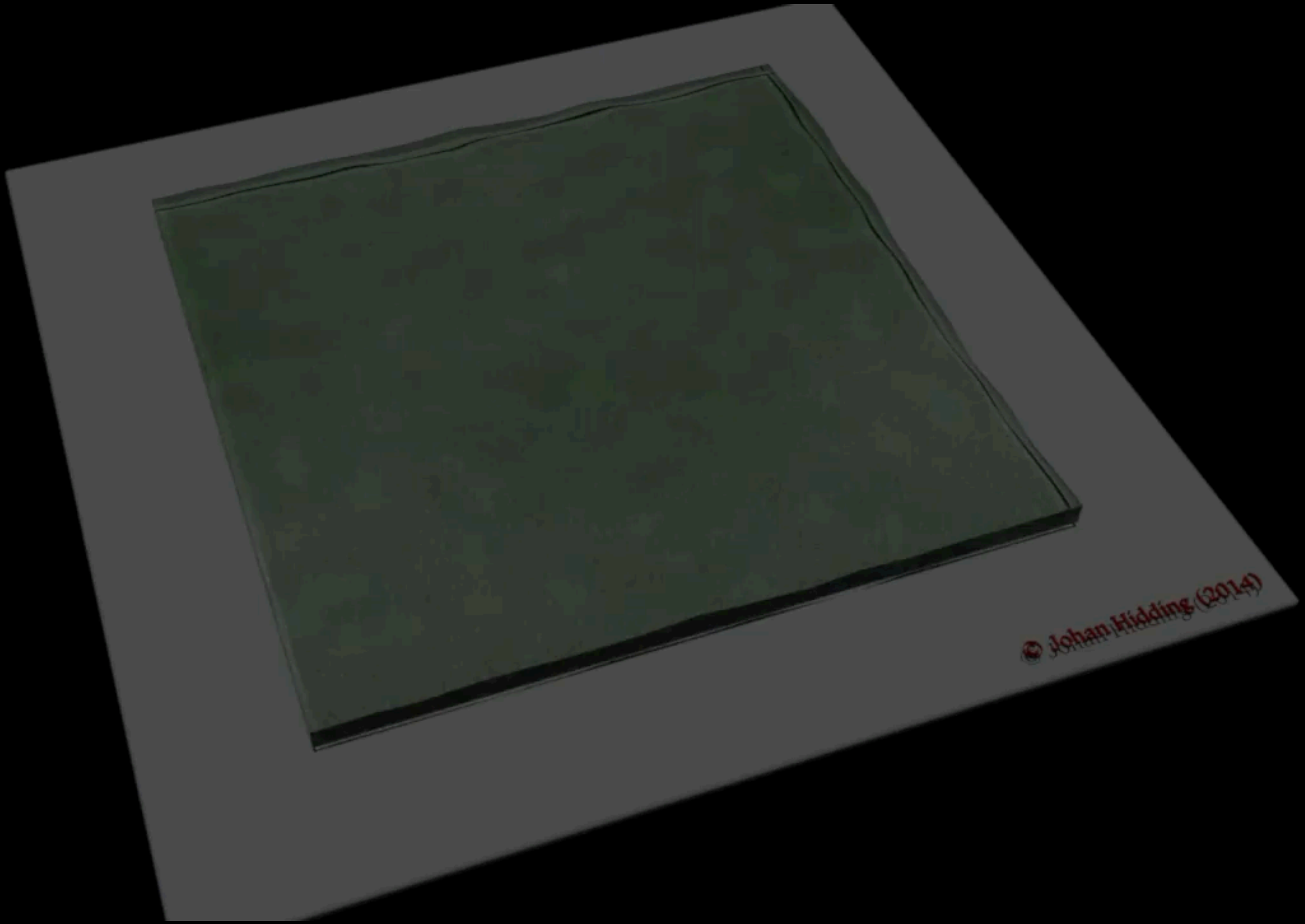
$\mathbf{S}(\mathbf{q}) \propto -\nabla\Phi(\mathbf{q})$, and we know that Φ is related to $\delta(\mathbf{k})$ through Poisson's equation



credit: Johan Hidding



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choice of softening

$$\Phi(x) = -G \sum_{j=1}^{N} \frac{m_j}{\left[\left(\mathbf{x} - \mathbf{x}_j\right)^2 + \epsilon^2\right]^{1/2}}$$

gravitational softening, ϵ, is introduced in N-body simulations to avoid large-angle scattering due to close encounters. higher-resolution simulations use smaller softening [and therefore also shorter timesteps]

simple choice: $\epsilon = \alpha \cdot d = \alpha \frac{L_{\rm box}}{N_p^{1/3}}$

$\alpha = 0.01 - 0.05$

mean inter-particle separation

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stochastic acceleration from close encounters

stricter choice: $\frac{G m_{\rm DM}}{\epsilon^2} \lesssim \frac{G M_{200}}{r_{200}^2} \Rightarrow \epsilon \gtrsim \frac{r_{200}}{\sqrt{N_{200}}}$

minimum mean-field acceleration in DM halo

Power+ (2003)

higher-order time integration

once forces on particles have even calculated, we need to propagate these forward in time and update their (1) positions and (2) velocities

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2nd order
"leapfrog" method

much more stable

"symplectic"
[~energy is conserved]

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kick—drift—kick

$$E\left(\Delta\tau\right) = K\left(\frac{\Delta\tau}{2}\right) \circ D\left(\Delta\tau\right) \circ K\left(\frac{\Delta\tau}{2}\right)$$

cosmological scale factor

Hubble parameter

a note on the **Hubble** parameter

we define the Hubble parameter as $H(t) \equiv \dfrac{\dot{a}(t)}{a(t)}$. denoting the present-day value as

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the cosmological model is therefore *fully specified* through $\Omega_m, \Omega_r, \Omega_k, \Omega_\Lambda, h, w_0$ and w_a