

PRECISE summer school  
Warsaw

July 06, 2023

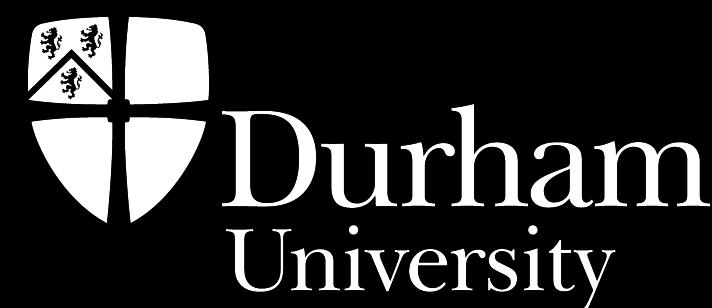
# hands-on cosmological simulations

session 3: identifying structures & basic predictions of simulations

Sownak Bose  
&  
Shaun Brown

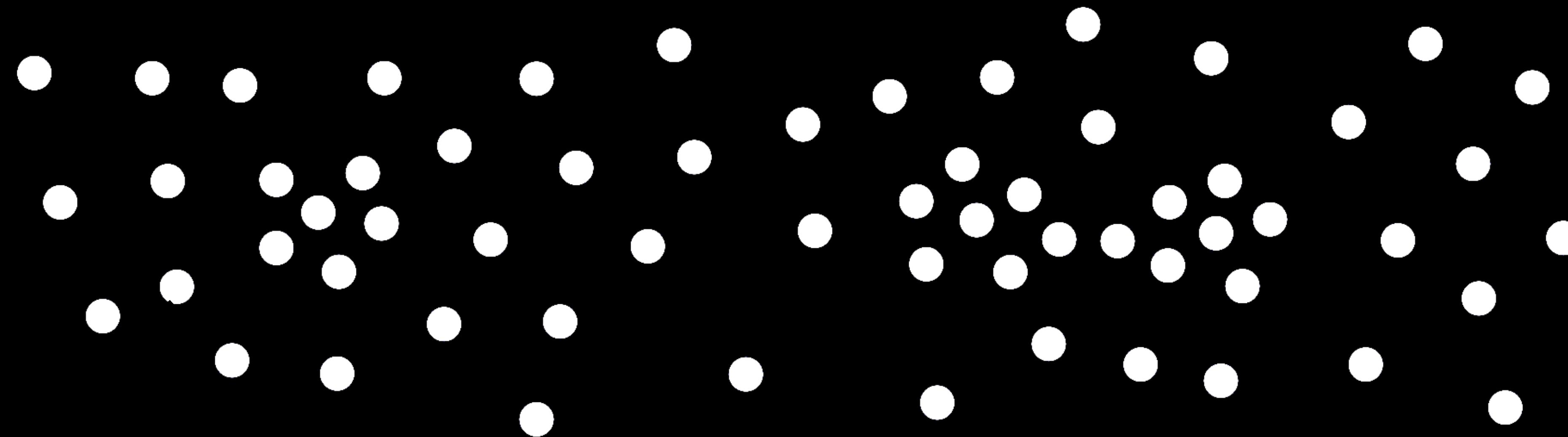
[sownak.bose@durham.ac.uk](mailto:sownak.bose@durham.ac.uk)

 @Swnk16



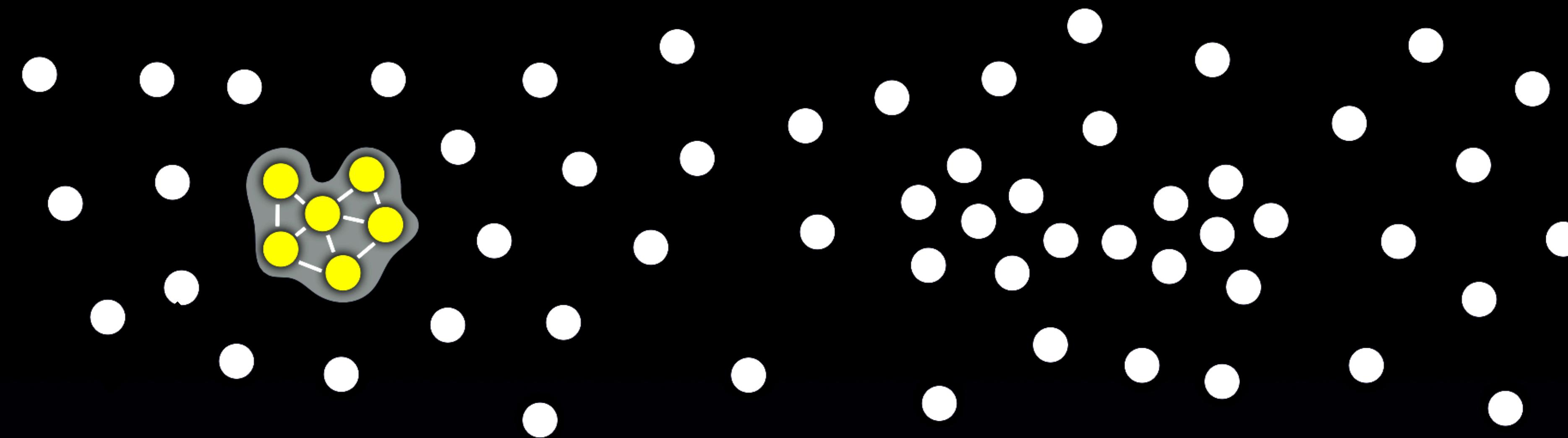
**identifying haloes & bound  
structures**

# the friends-of-friends algorithm



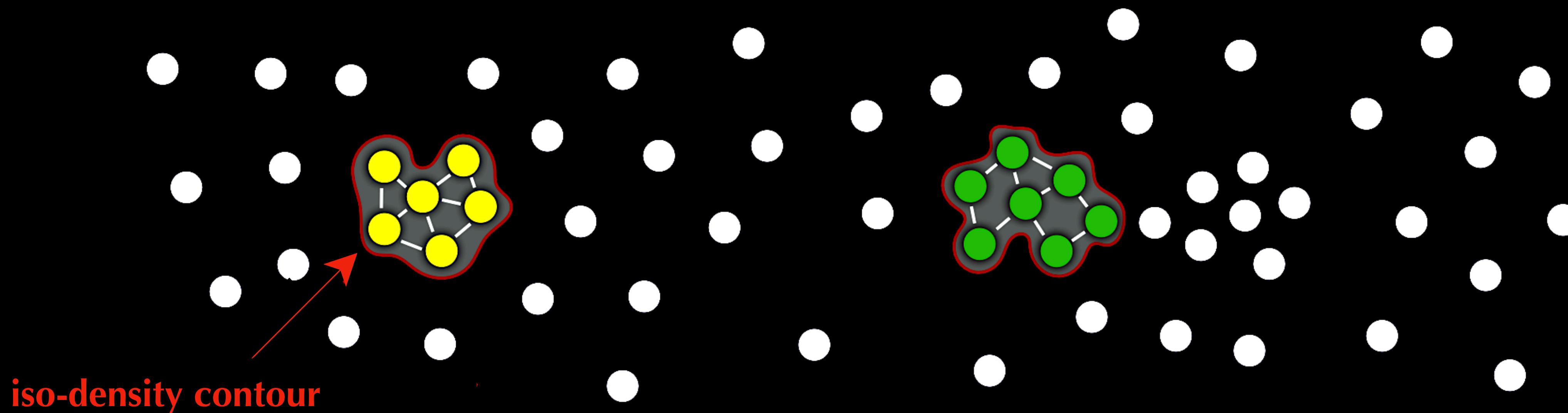
process: connect particles that are within some fraction,  $b$ , of the mean-interparticle separation,  $\Delta x$ , of one another. typical choice  $b = 0.2$

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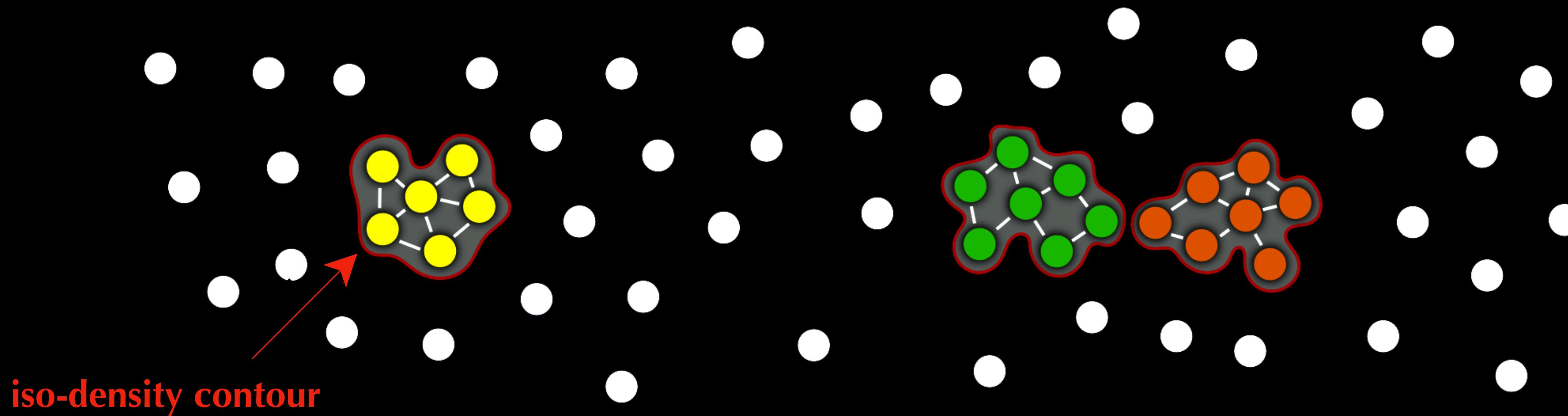
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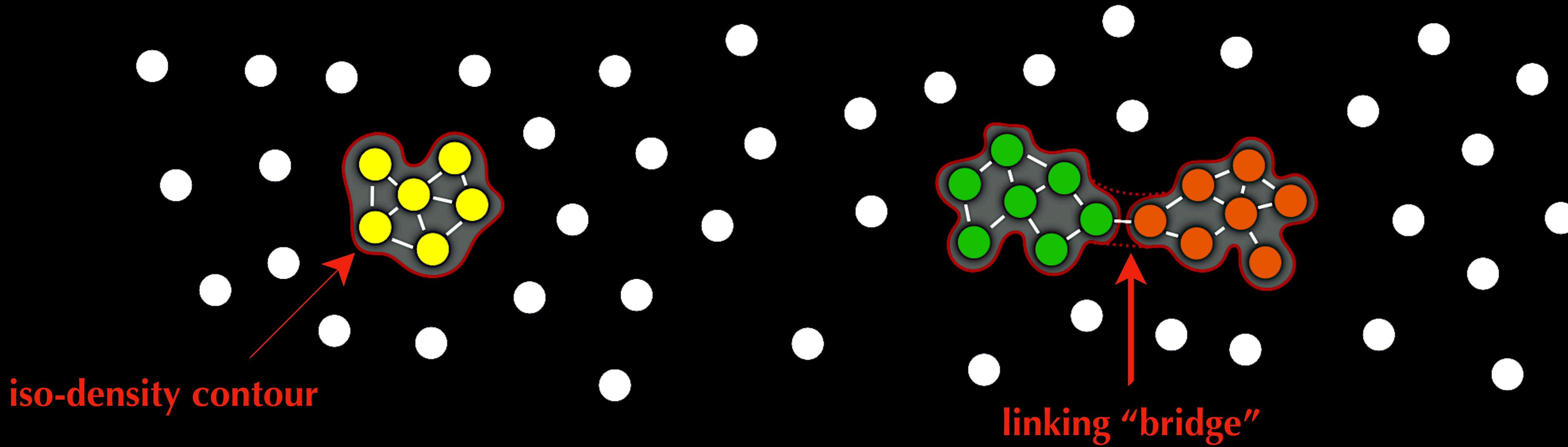


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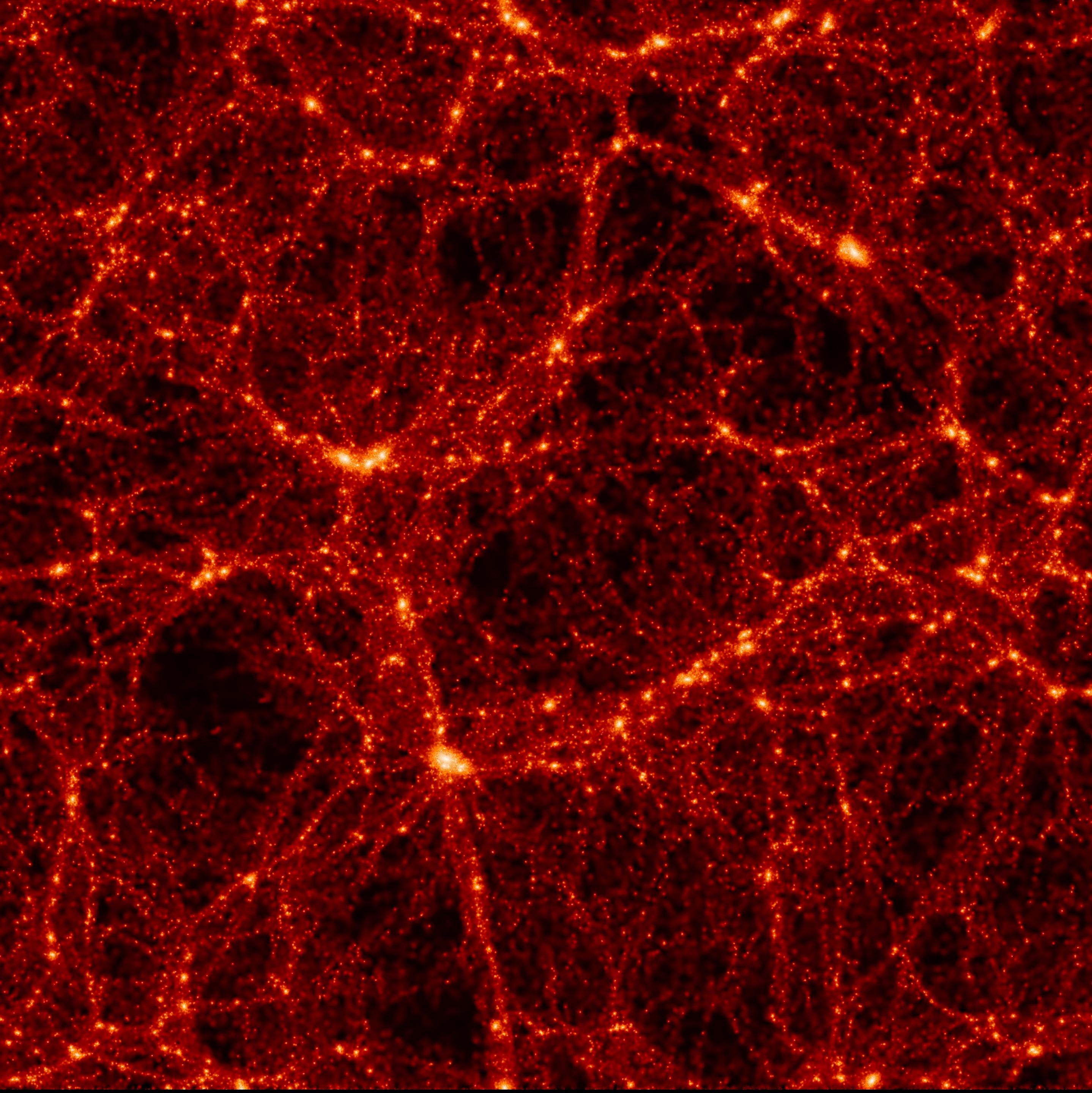
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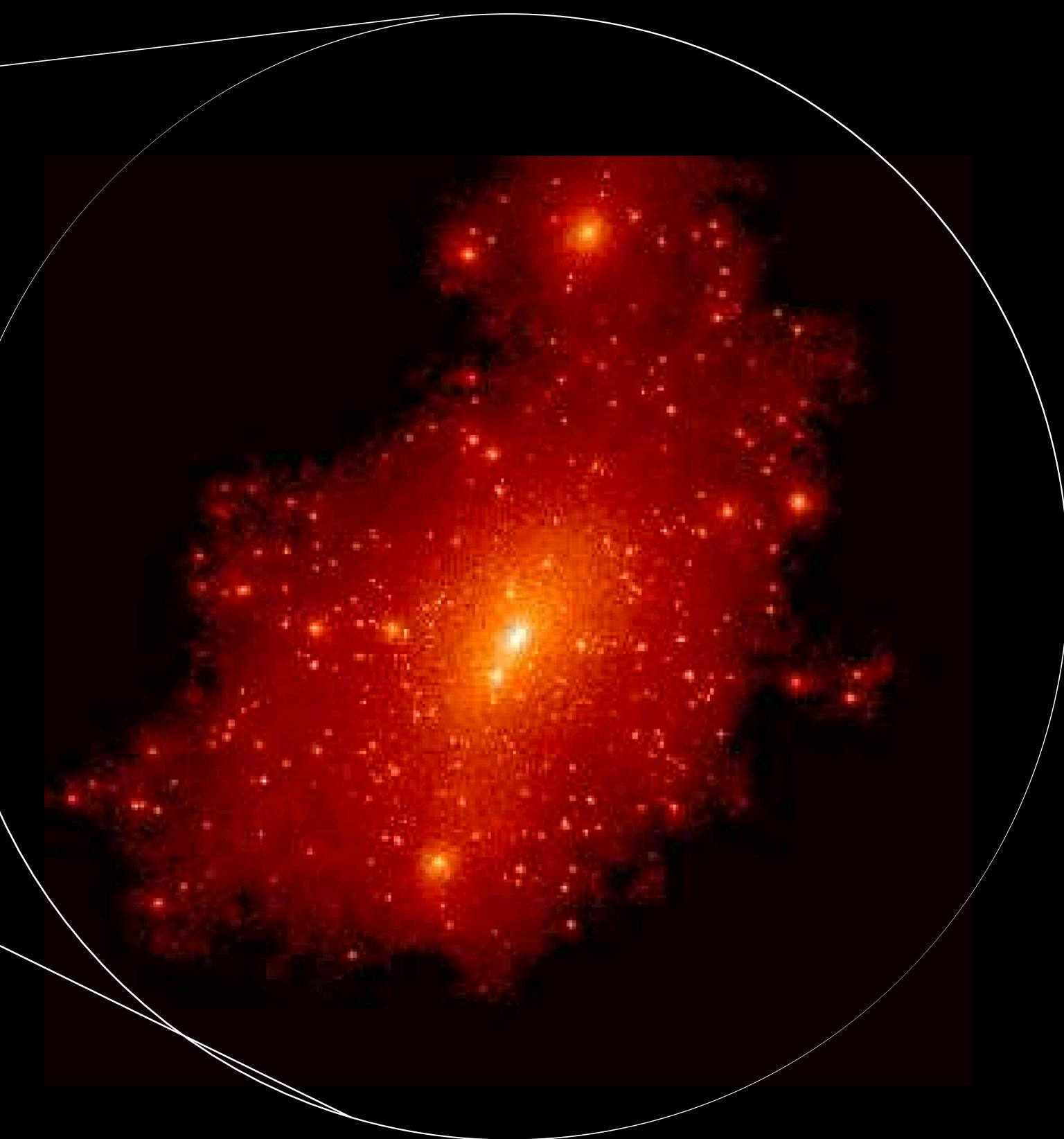
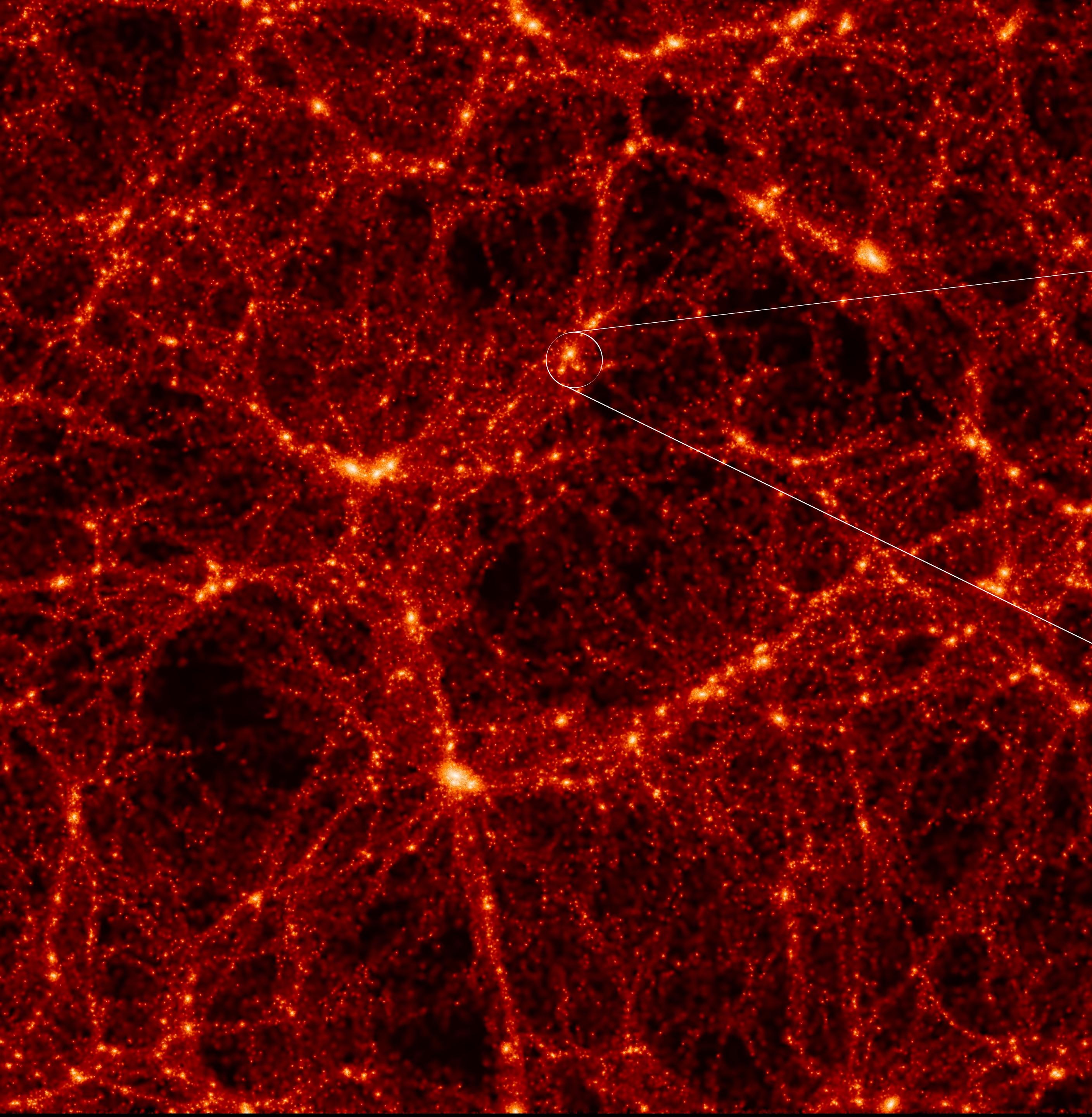
the FOF method is very simple & general, but exhibits unphysical pathologies

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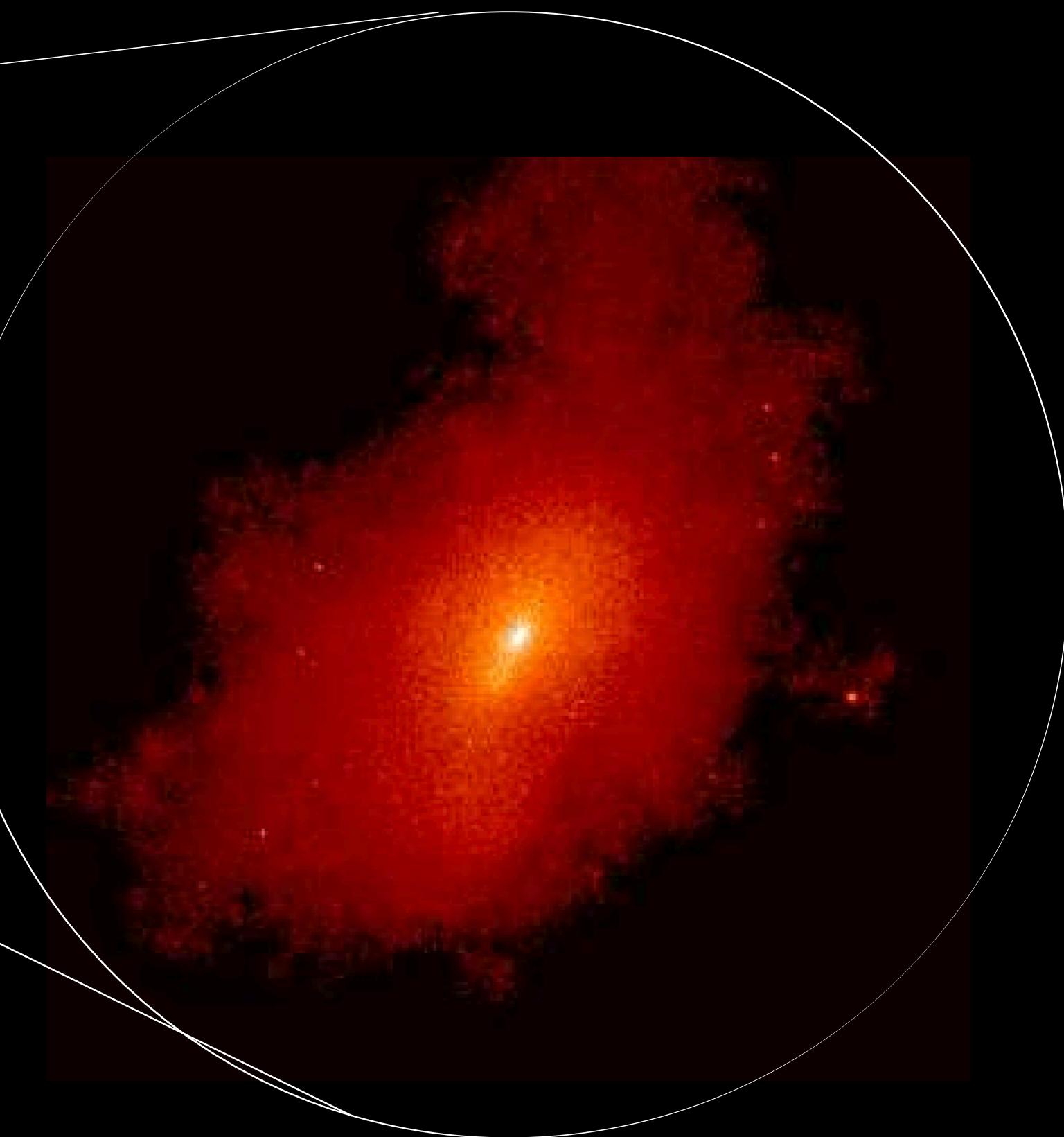
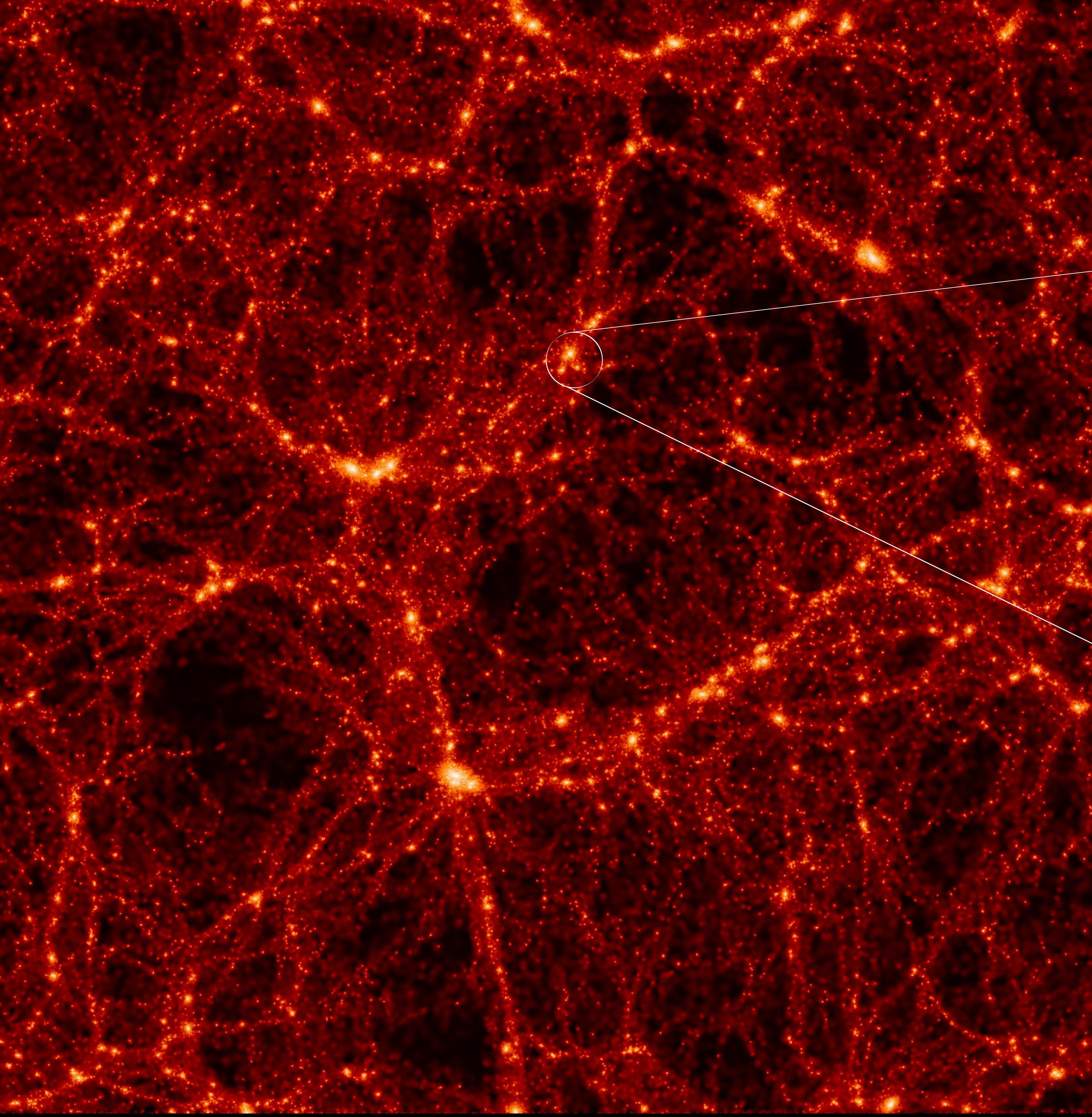


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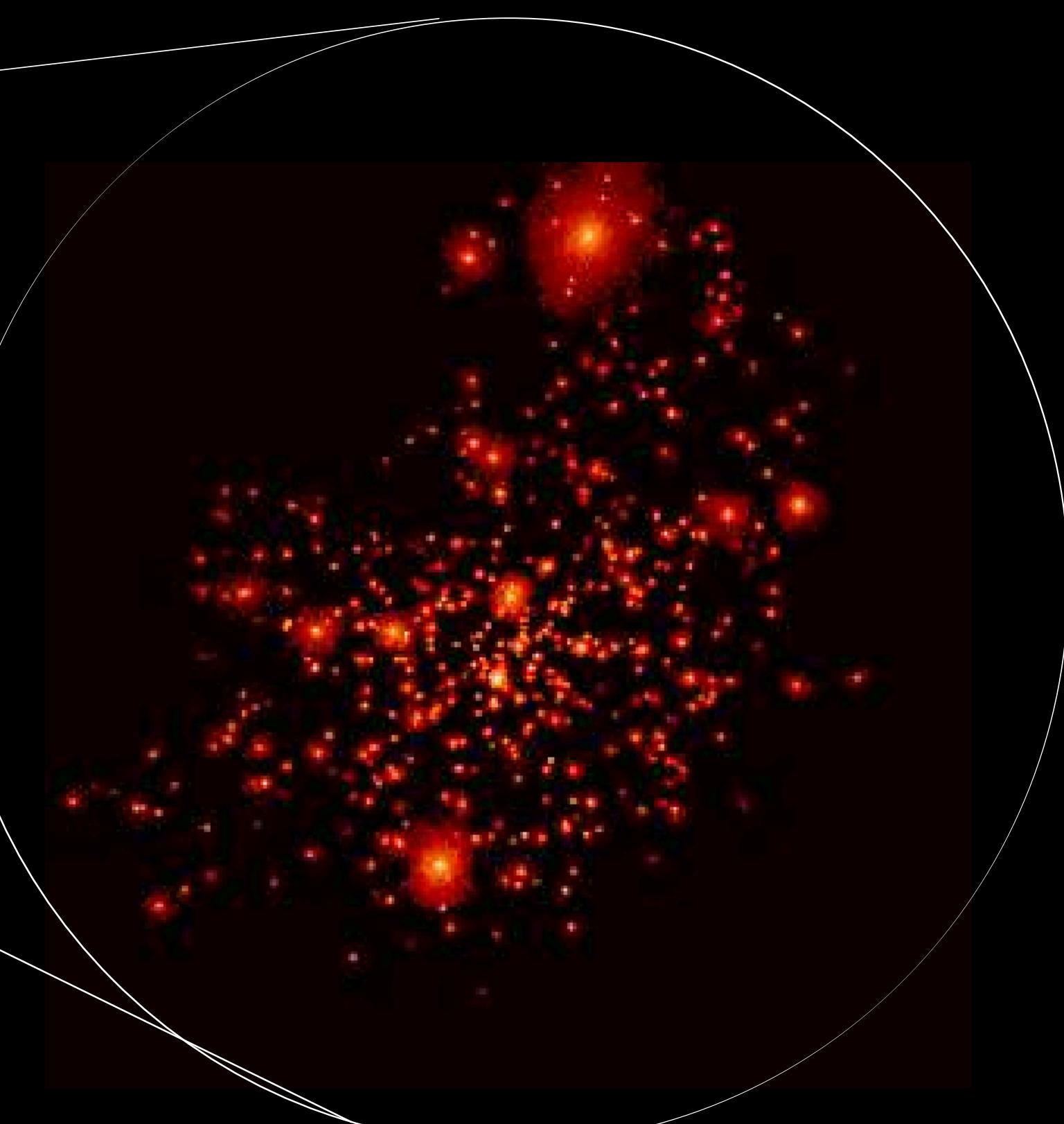
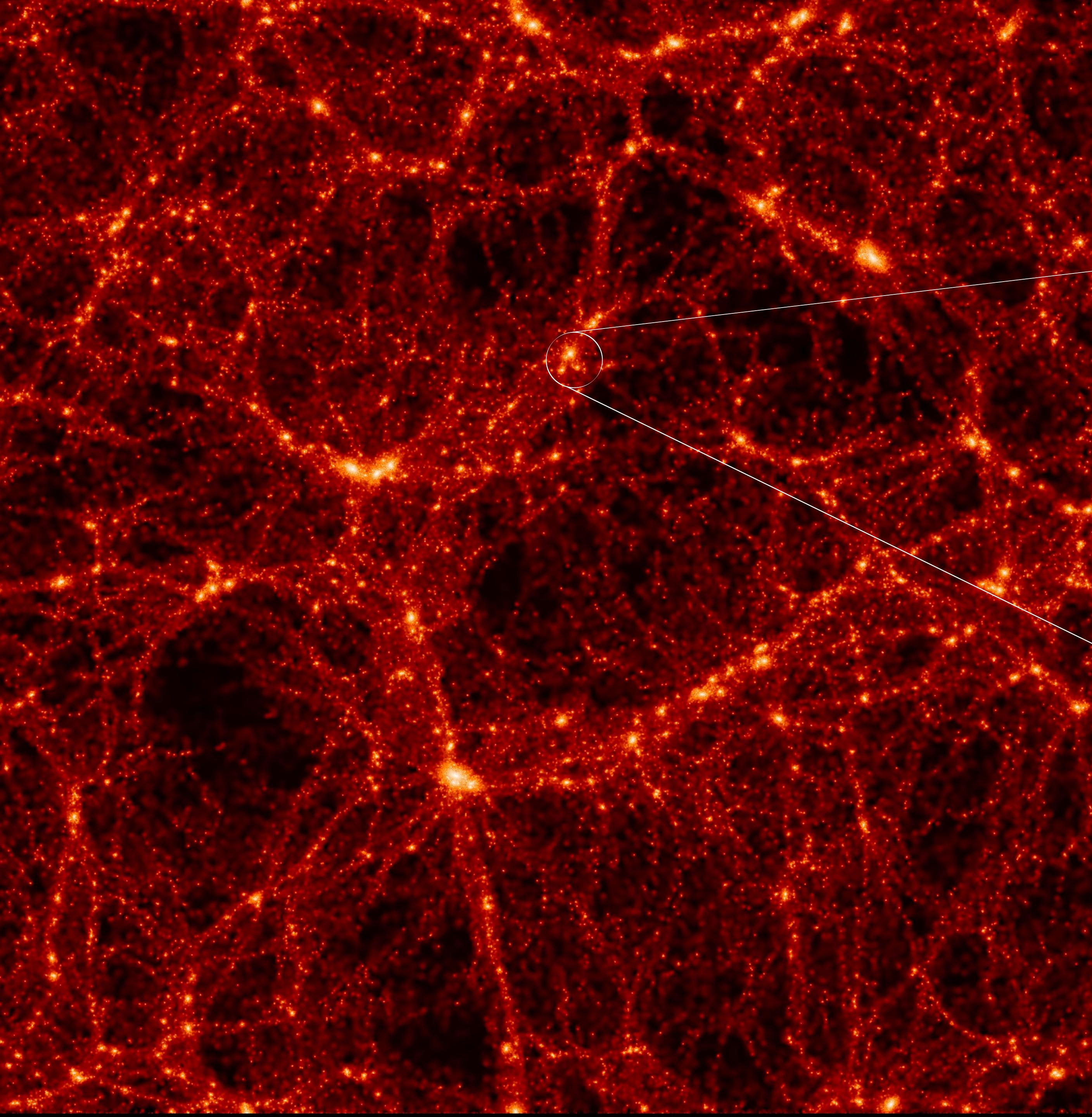




**friends-of-friends halo**

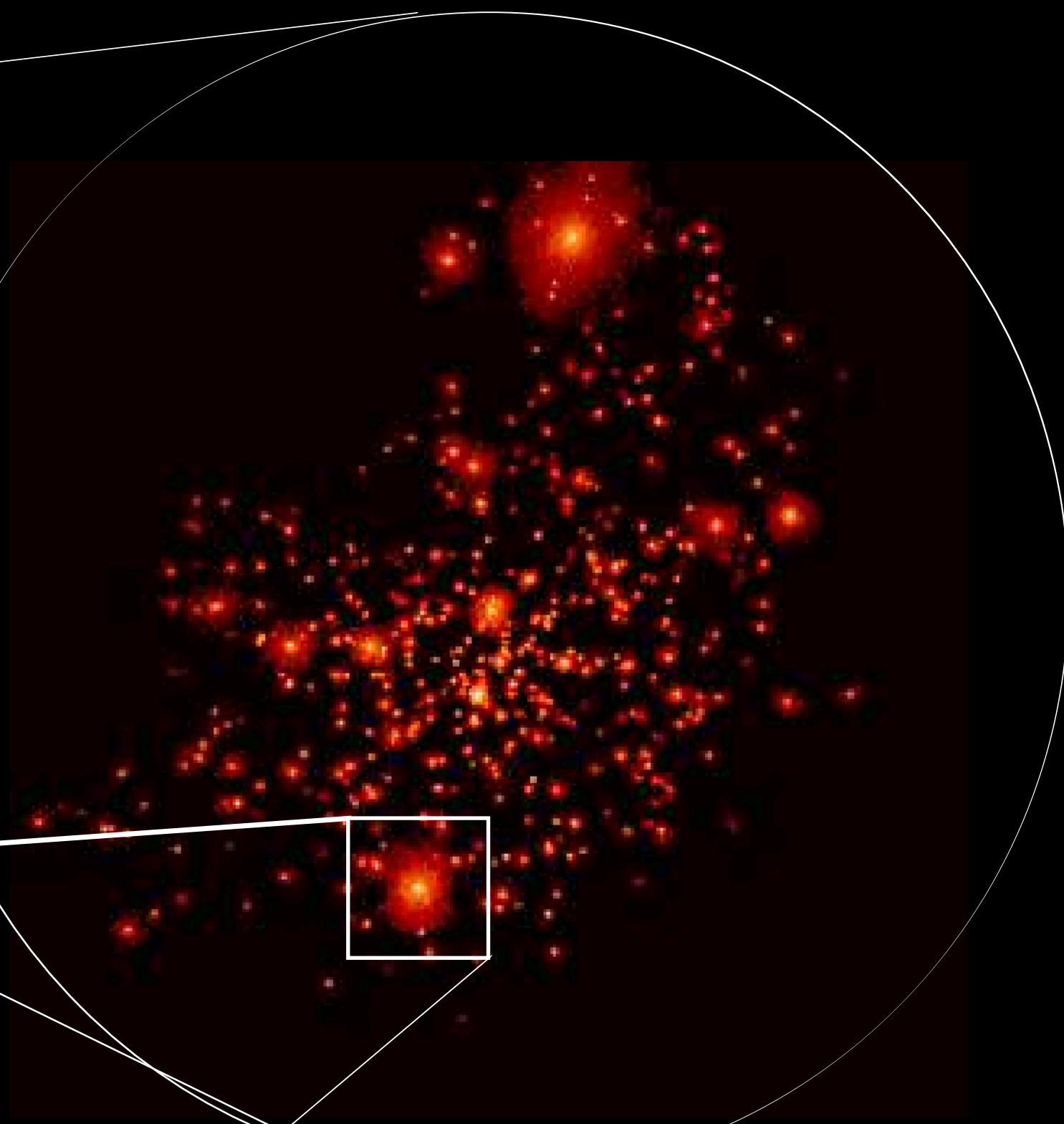
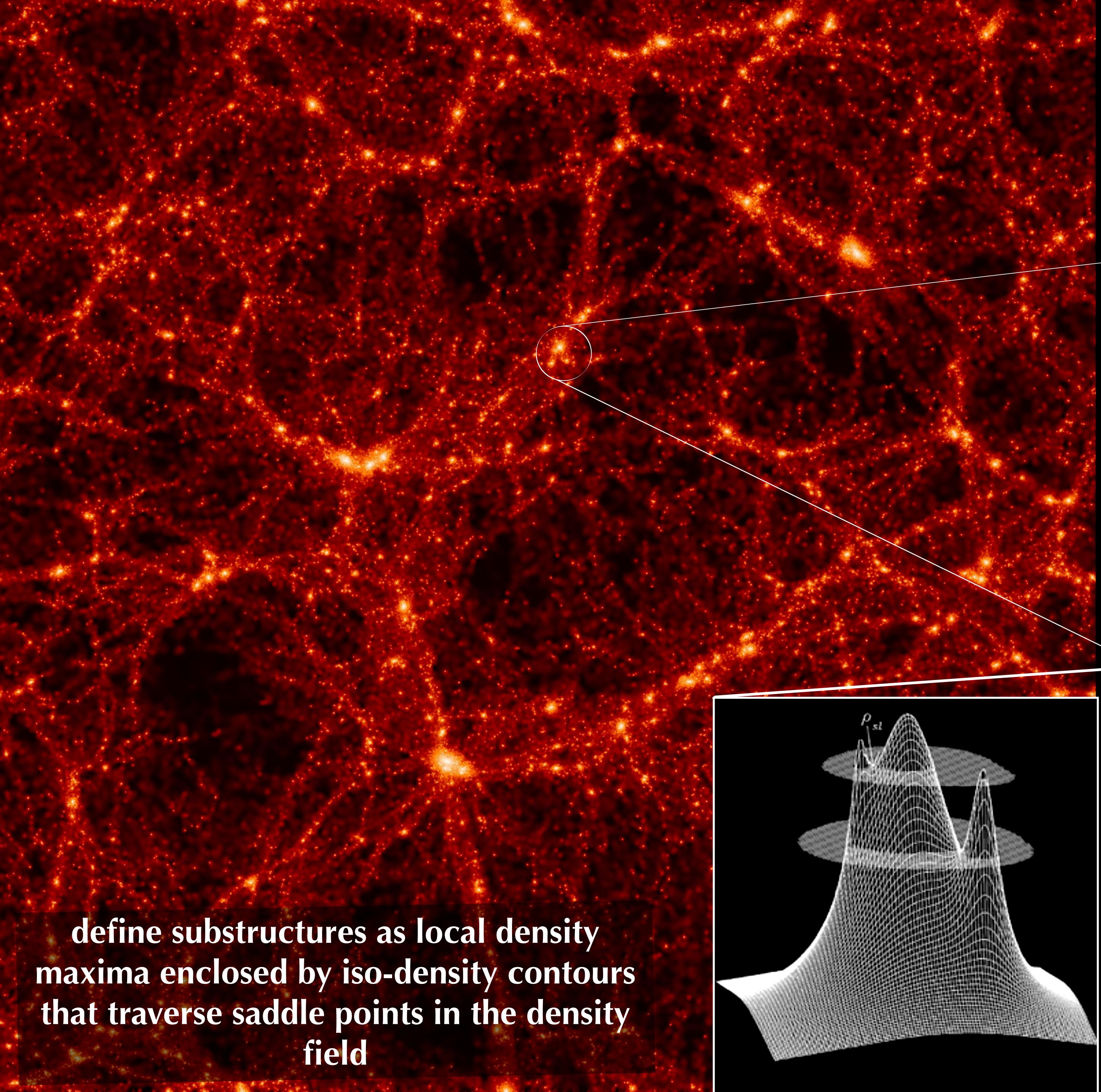
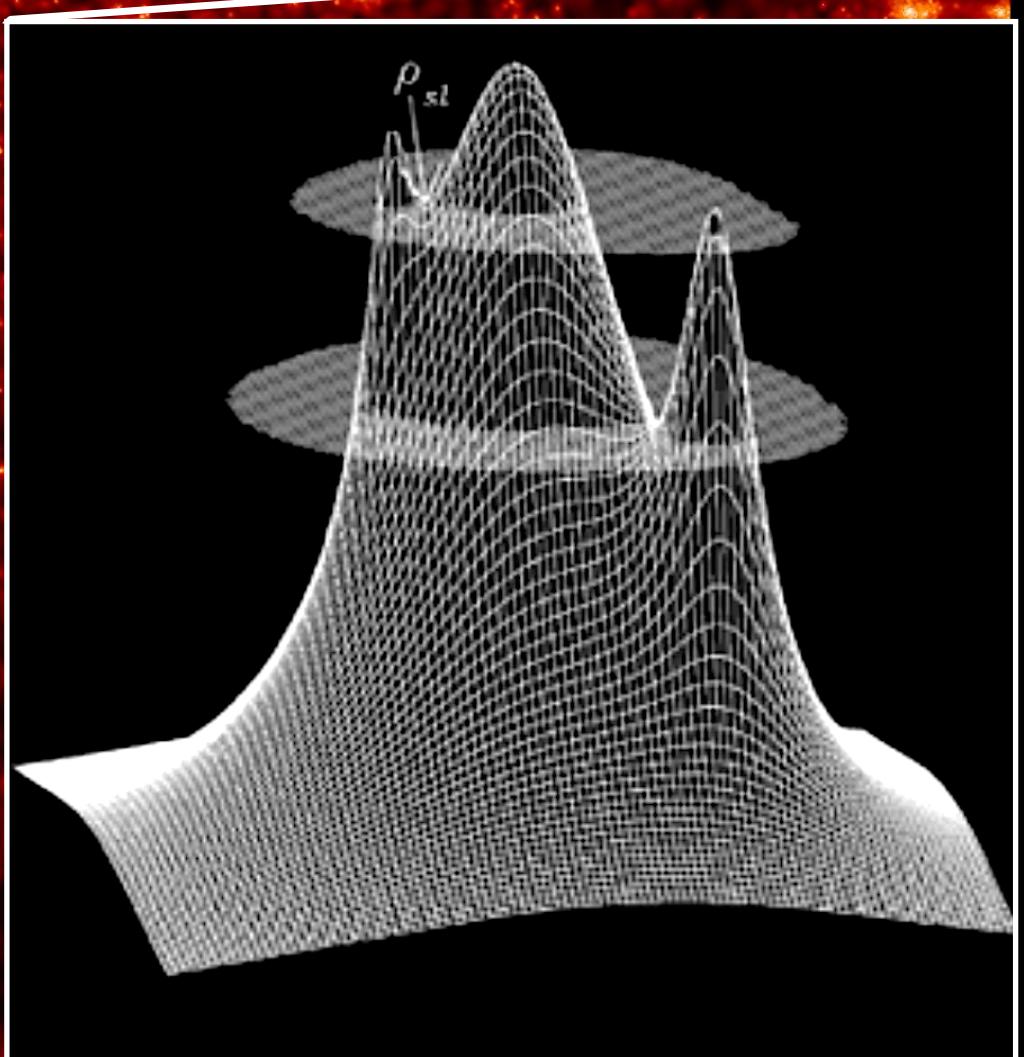


**“smooth” halo [central]**

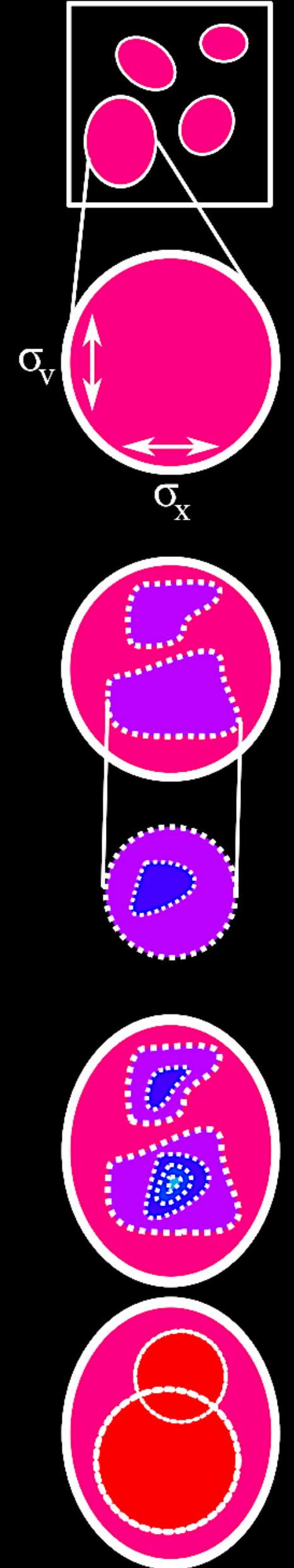


substructures [satellites]

define substructures as local density  
maxima enclosed by iso-density contours  
that traverse saddle points in the density  
field



substructures [satellites]

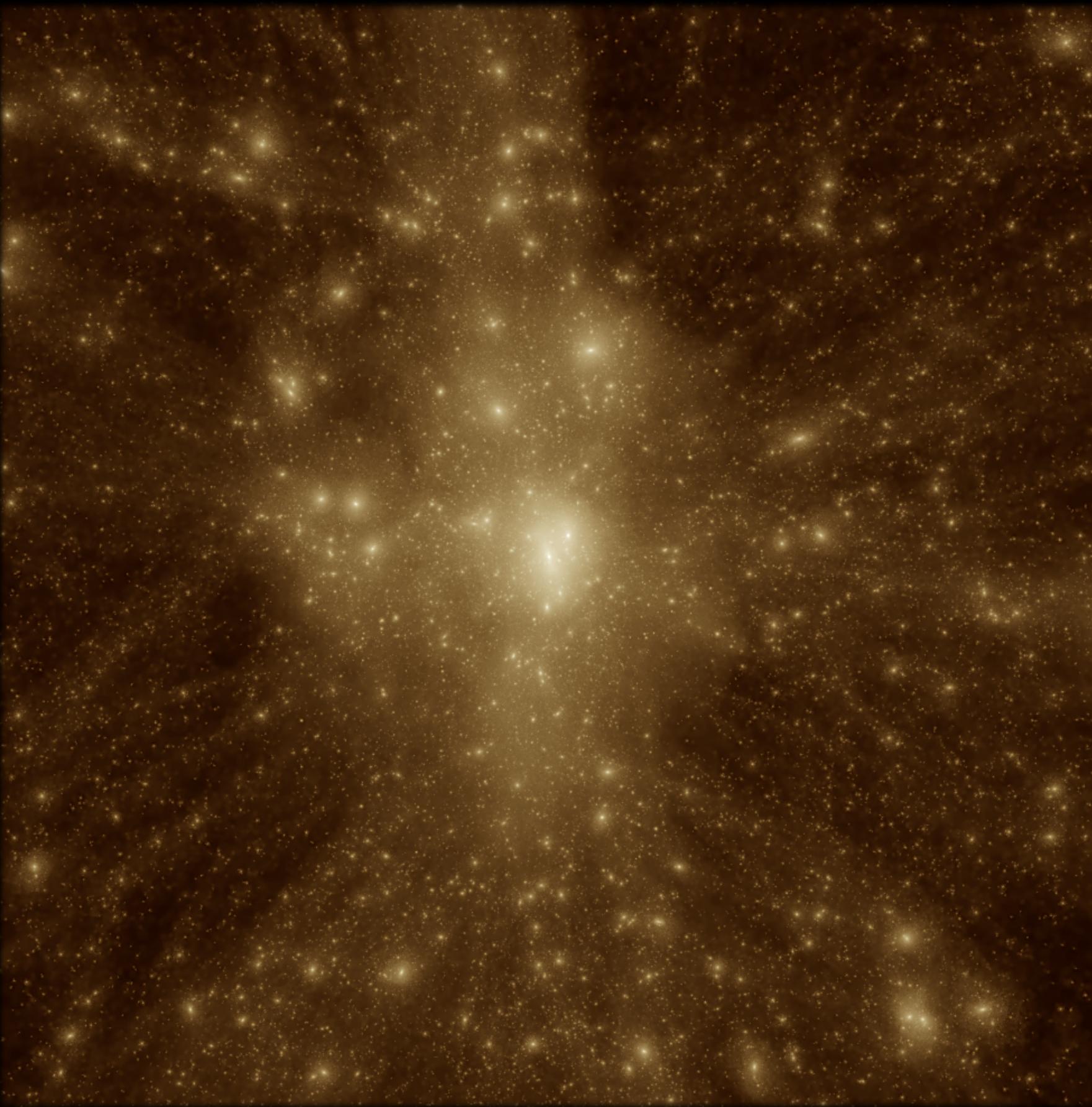


1. The simulation volume is divided into 3D Friends-of-Friends groups for easy parallelization.
2. For each group, particle positions and velocities are divided (normalized) by the group position and velocity dispersions, giving a natural phase-space metric.
3. A phase-space linking length is adaptively chosen such that 70% of the group's particles are linked together in subgroups.
4. The process repeats for each subgroup: renormalization, a new linking-length, and a new level of substructure calculated.
5. Once all levels of substructure are found, seed halos are placed at the lowest substructure levels and particles are assigned hierarchically to the closest seed halo in phase space.
6. Once particles have been assigned to halos, unbound particles are removed and halo properties (positions, velocities, etc.) are calculated.

**6D phase-space  
structure finding not  
always used**

**Behroozi+ (2013)**

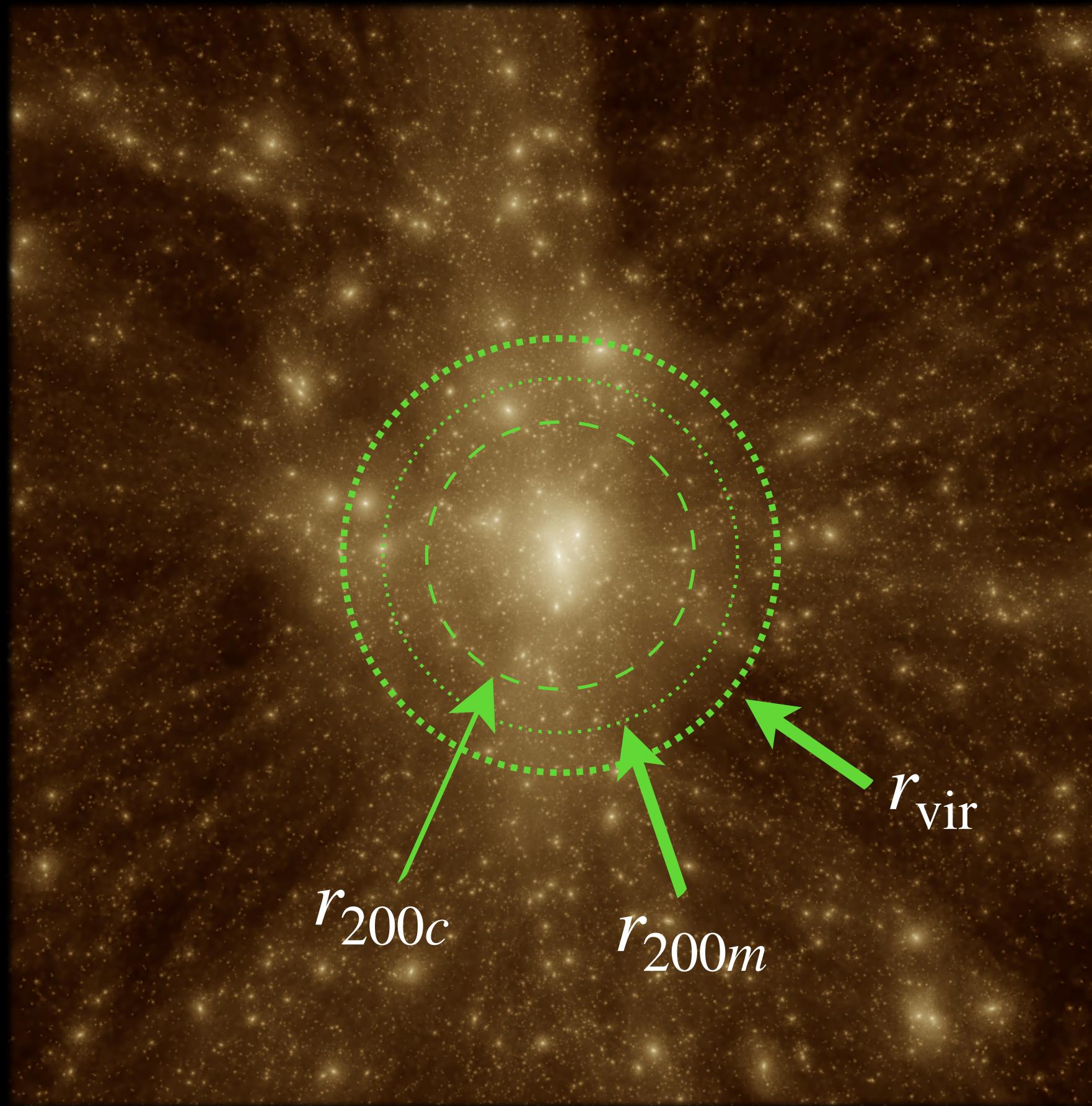
# definitions of halo mass



DM haloes identified in N-body simulations are highly irregular objects. how do we define their mass / extent?

⇒ do what astronomers do best and assume everything is spherical

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$$M_\Delta = \frac{4}{3}\pi\bar{\rho}_\text{ref}r_\Delta^3 \text{ where } \bar{\rho}( < r_\Delta ) = \Delta \cdot \rho_\text{ref}$$

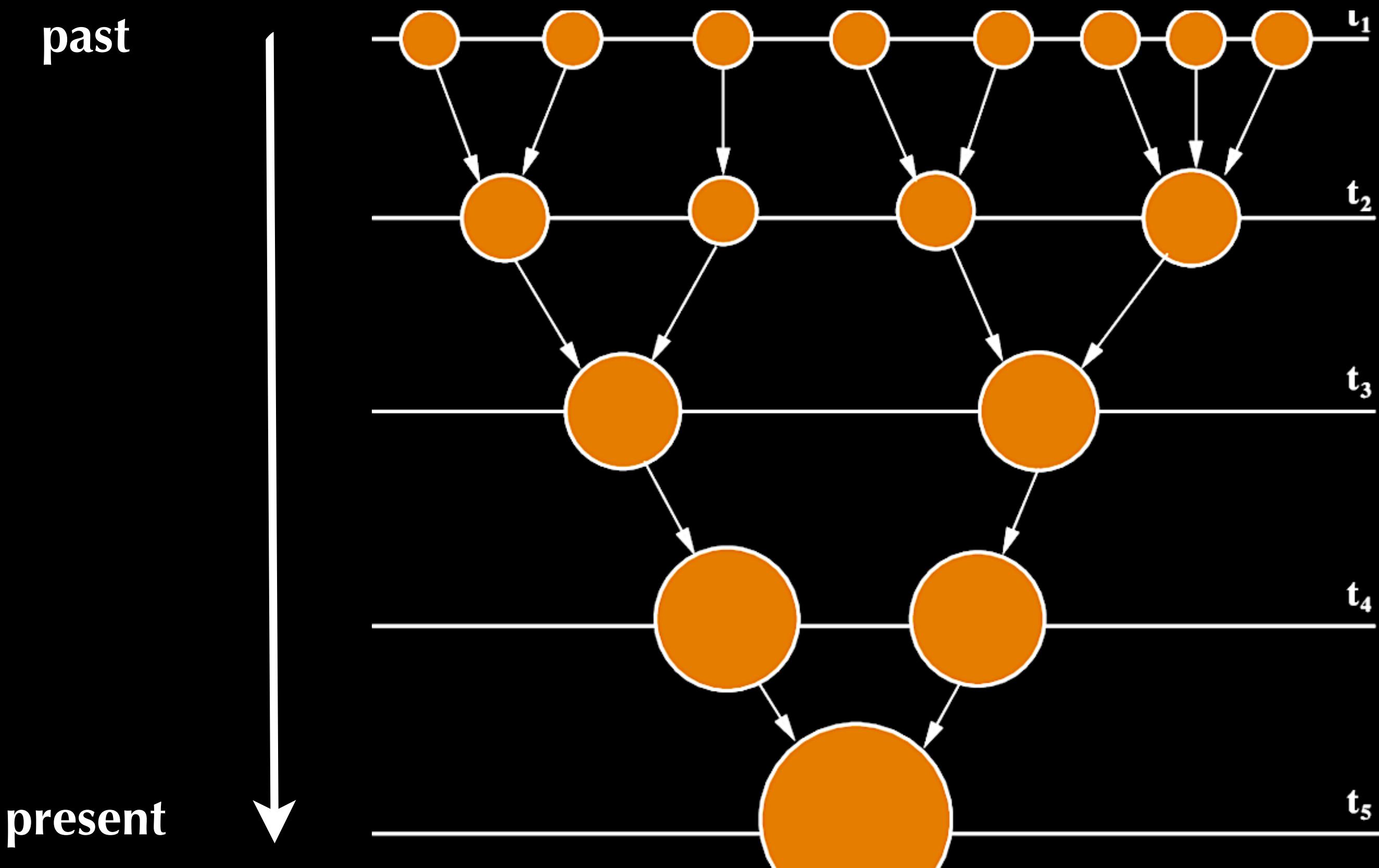
virial mass/radius



# commonly used definitions

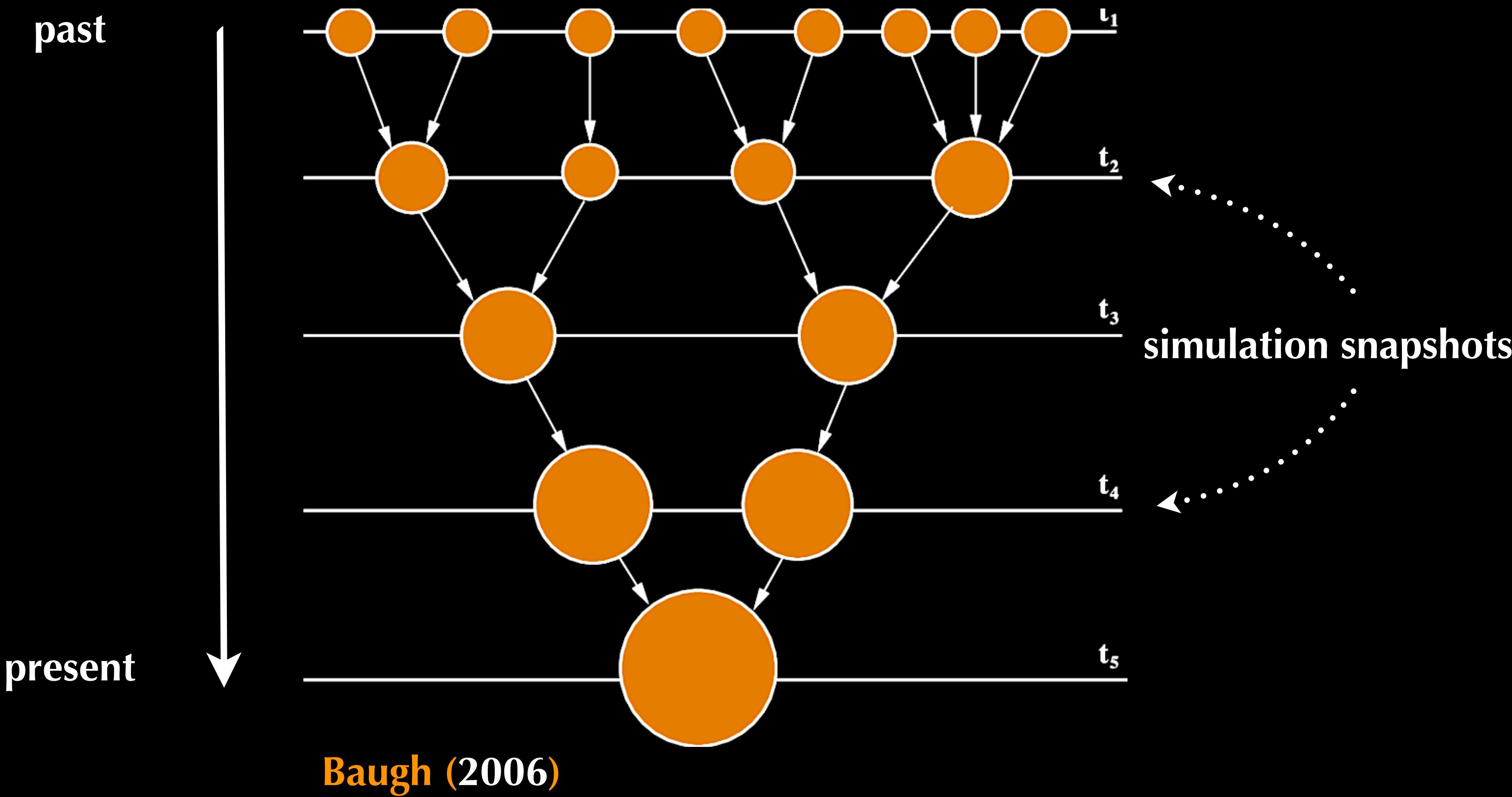
label	$\Delta$	$\rho_{\text{ref}}$	spherical	cosmology dependent?	binding information?
$M_{\text{FOF}}$	—	—	✗	✓	✗
$M_{\text{sub}}$	—	$\rho_{\text{host}}$	✗	✗	✓
$M_{200c}$	200	$\rho_{\text{crit}}$	✓	✓	✗
$M_{200m}$	200	$\rho_{\text{mean}}$	✓	✓	✗
$M_{\text{vir}}$	$18\pi^2 + 82x - 39x^2$ $x \equiv \Omega(z) - 1$	$\rho_{\text{mean}}$	✓	✓	✗

# growth histories of haloes

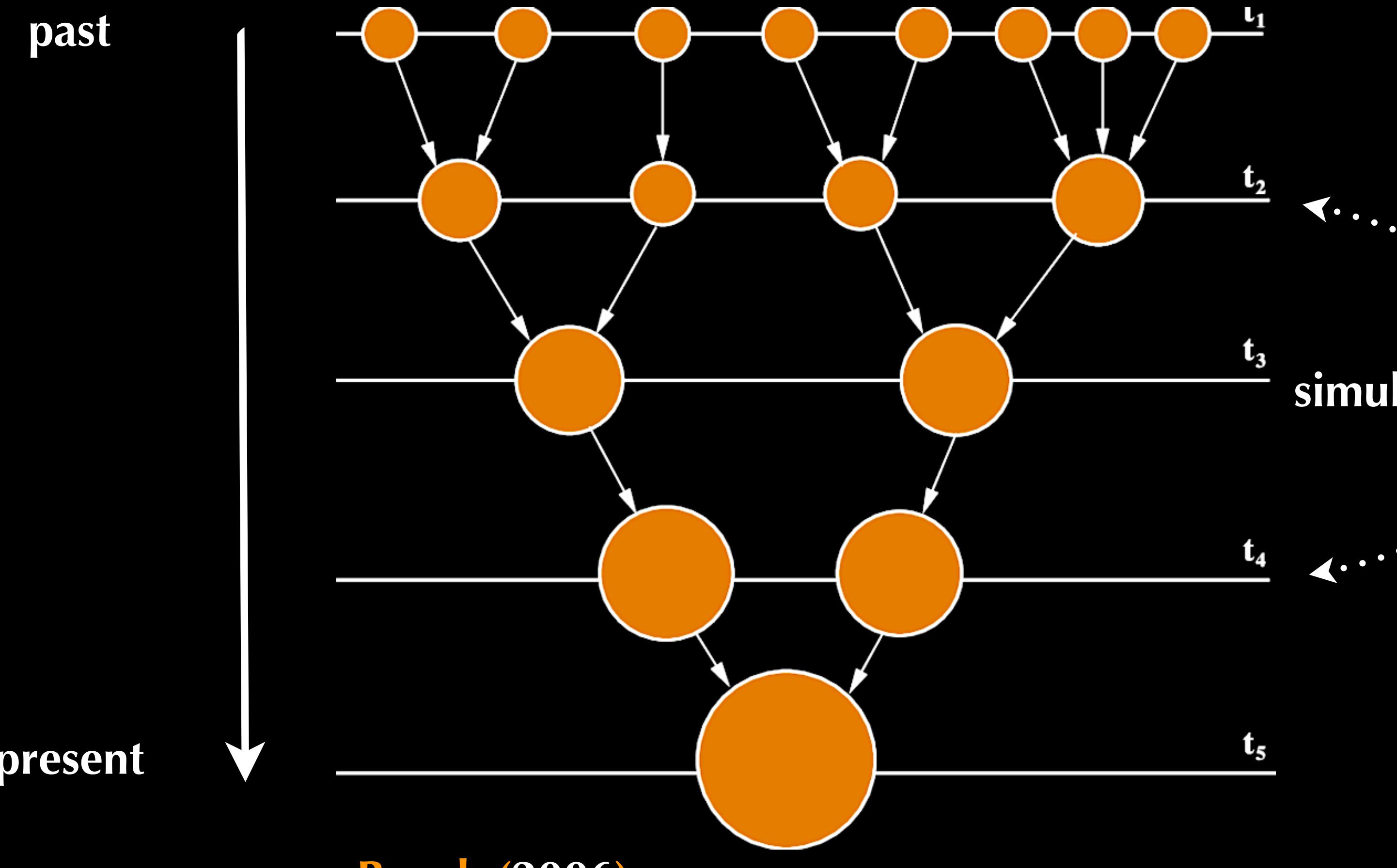


Baugh (2006)

# growth histories of haloes



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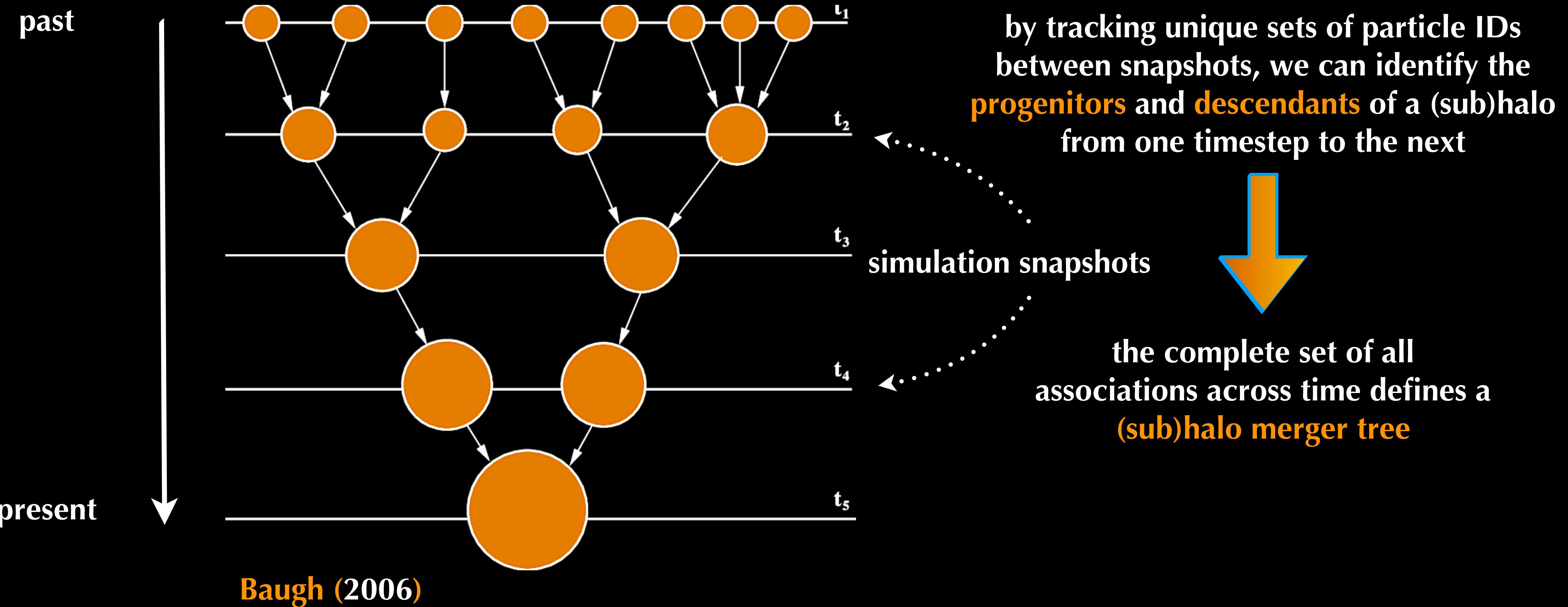


by tracking unique sets of particle IDs between snapshots, we can identify the **progenitors** and **descendants** of a (sub)halo from one timestep to the next

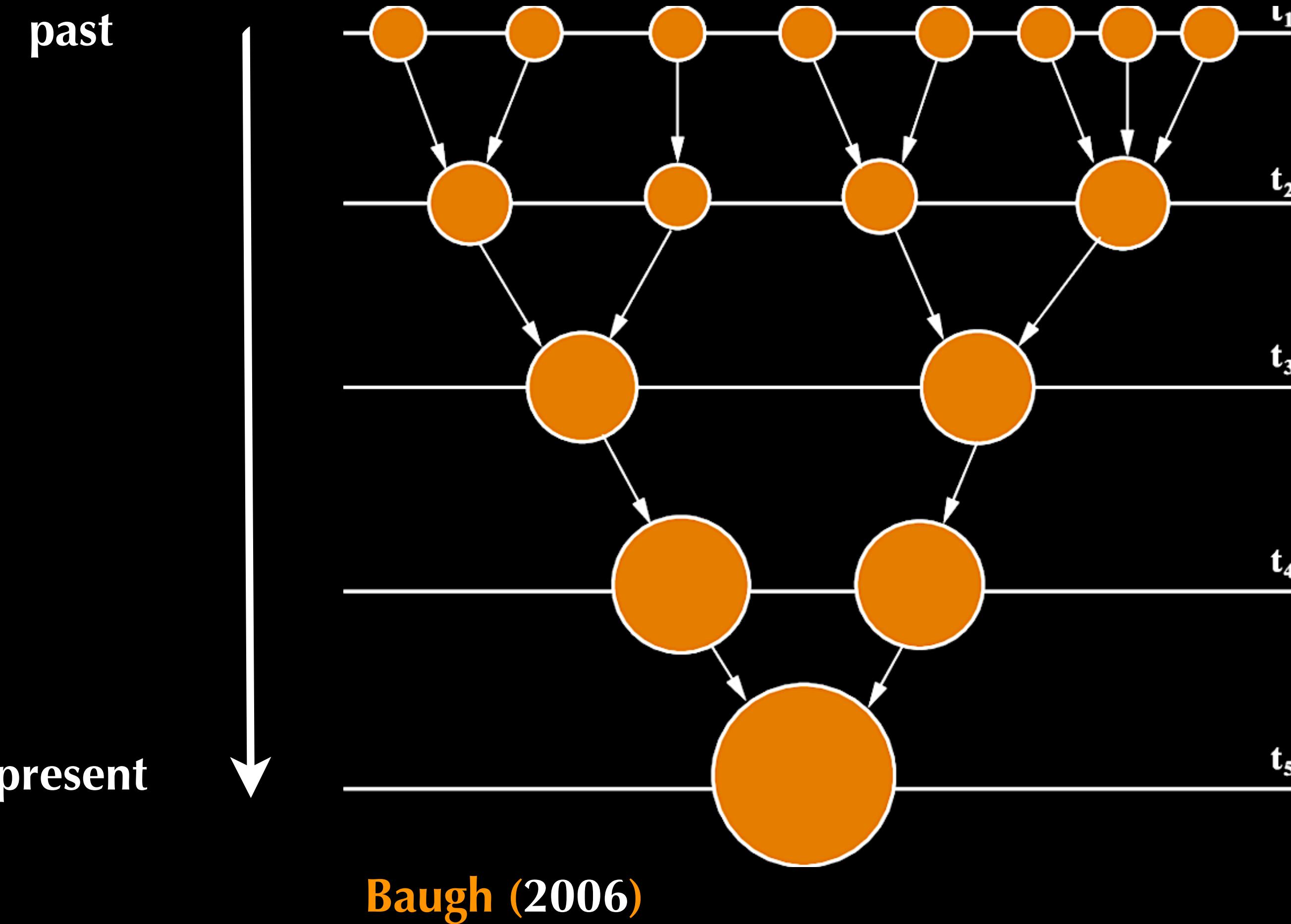
simulation snapshots

Baugh (2006)

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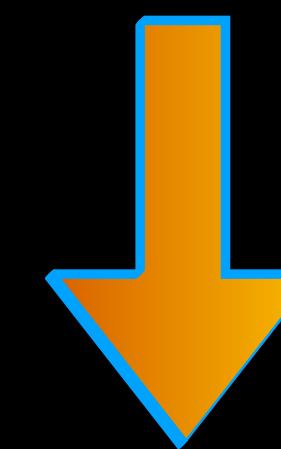


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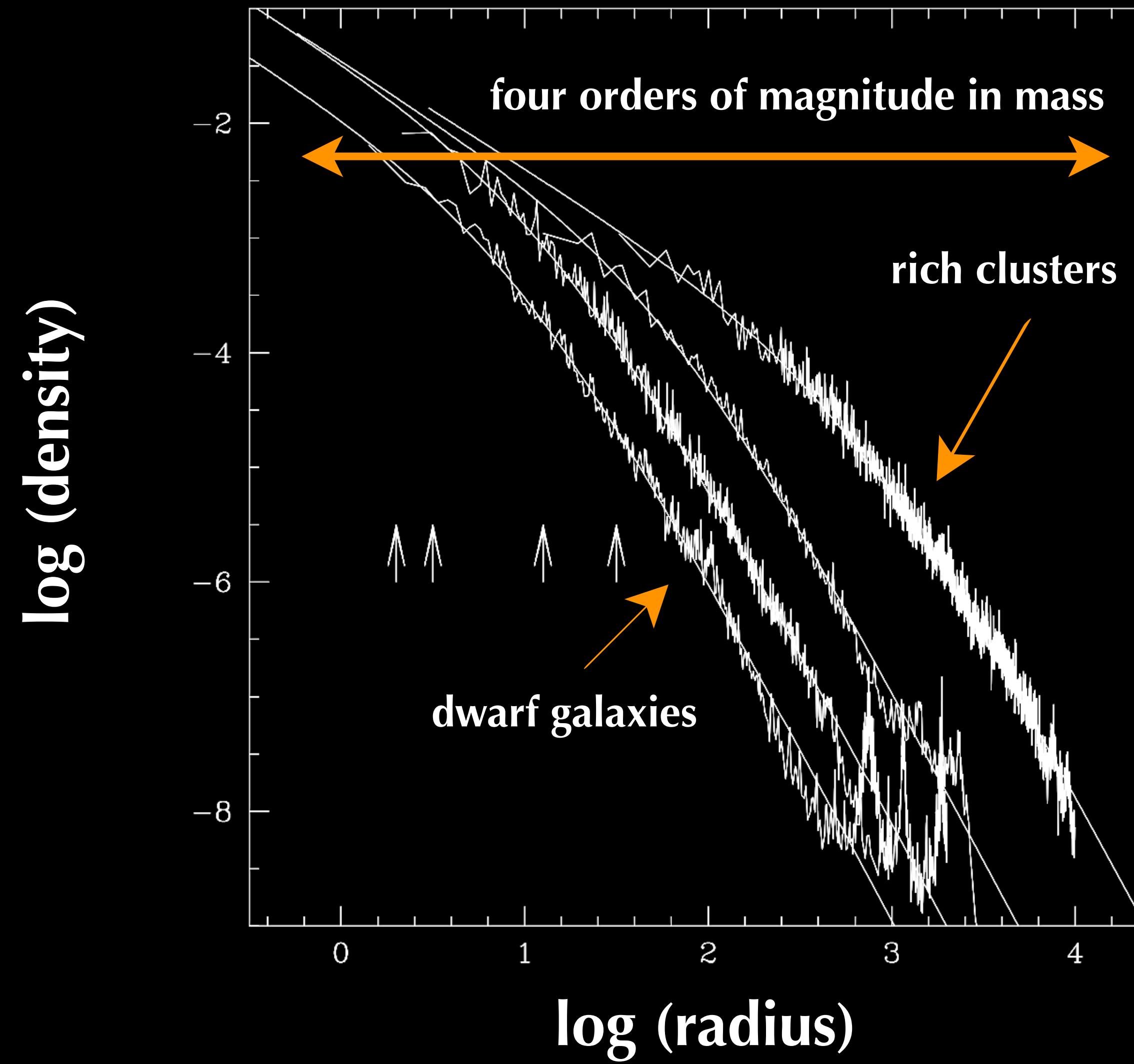


the complete set of all associations across time defines a **(sub)halo merger tree**

⇒ merger trees act as the skeleton on which many galaxy formation models (semi-analytic, empirical models etc.) are built

**some universal predictions**

# self-similar structure



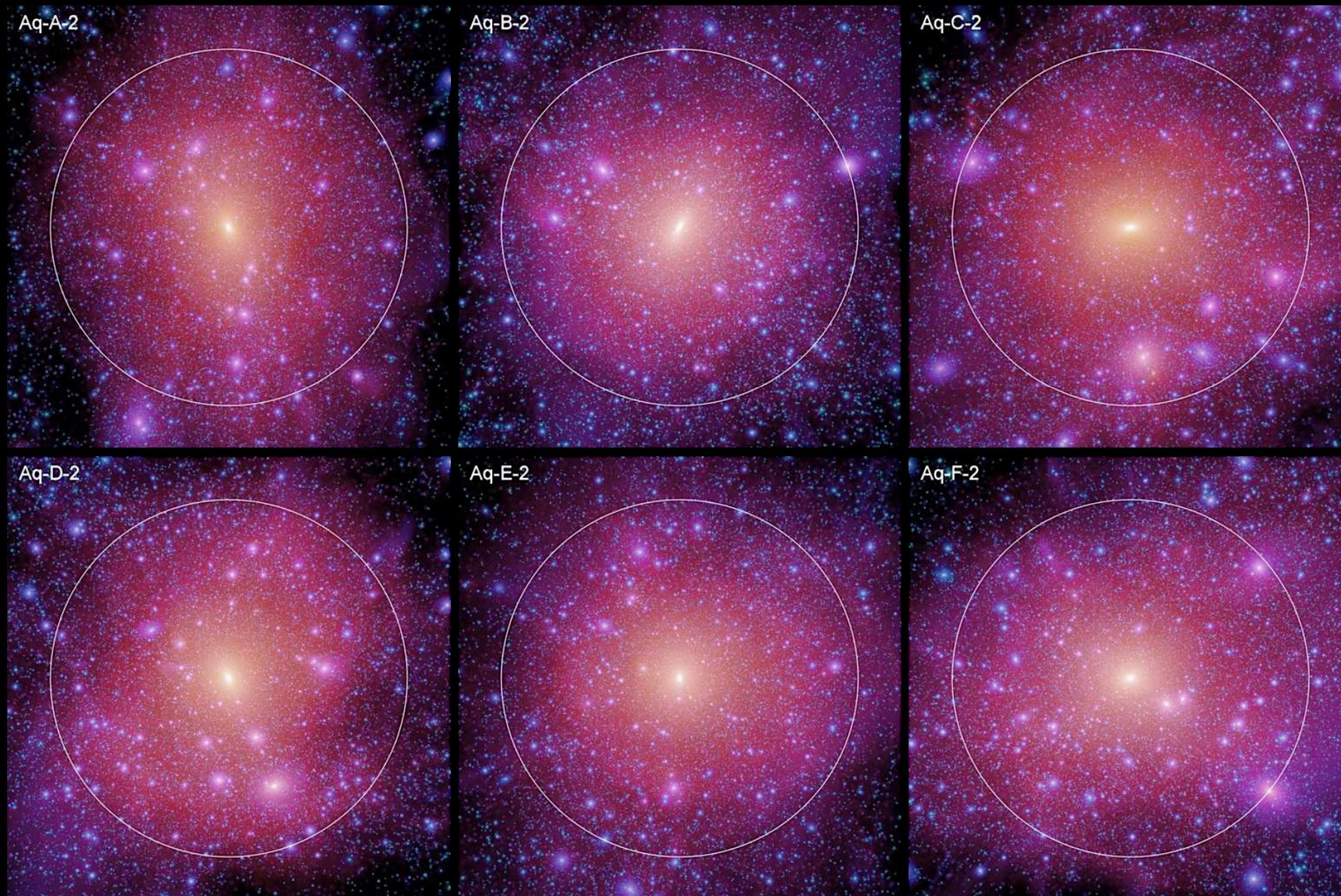
centre:  $\rho \propto r^{-1}$

middle:  $\rho \propto r^{-2}$

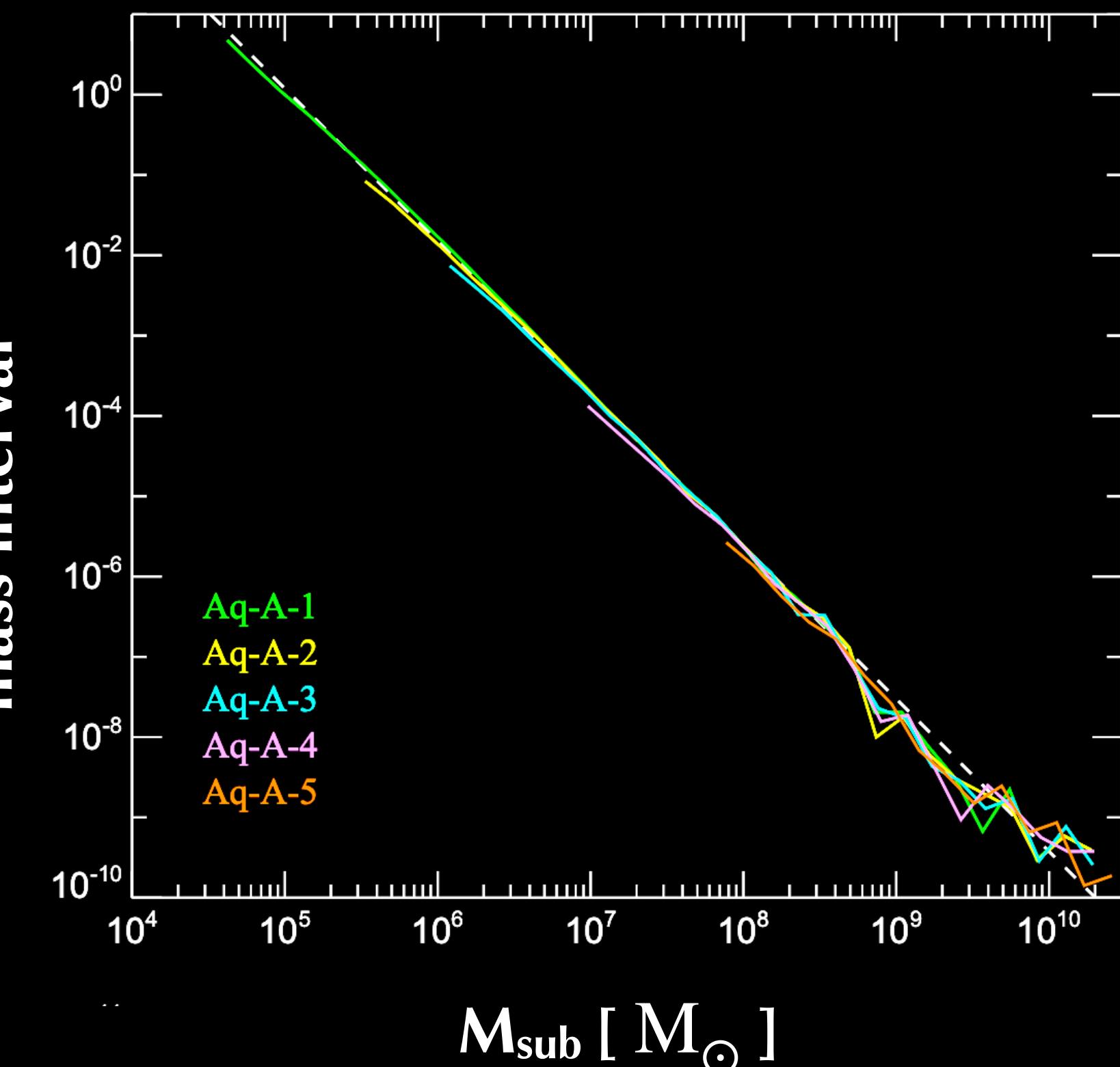
outskirts:  $\rho \propto r^{-3}$

Navarro, Frenk & White (1996)

# self-similar abundance



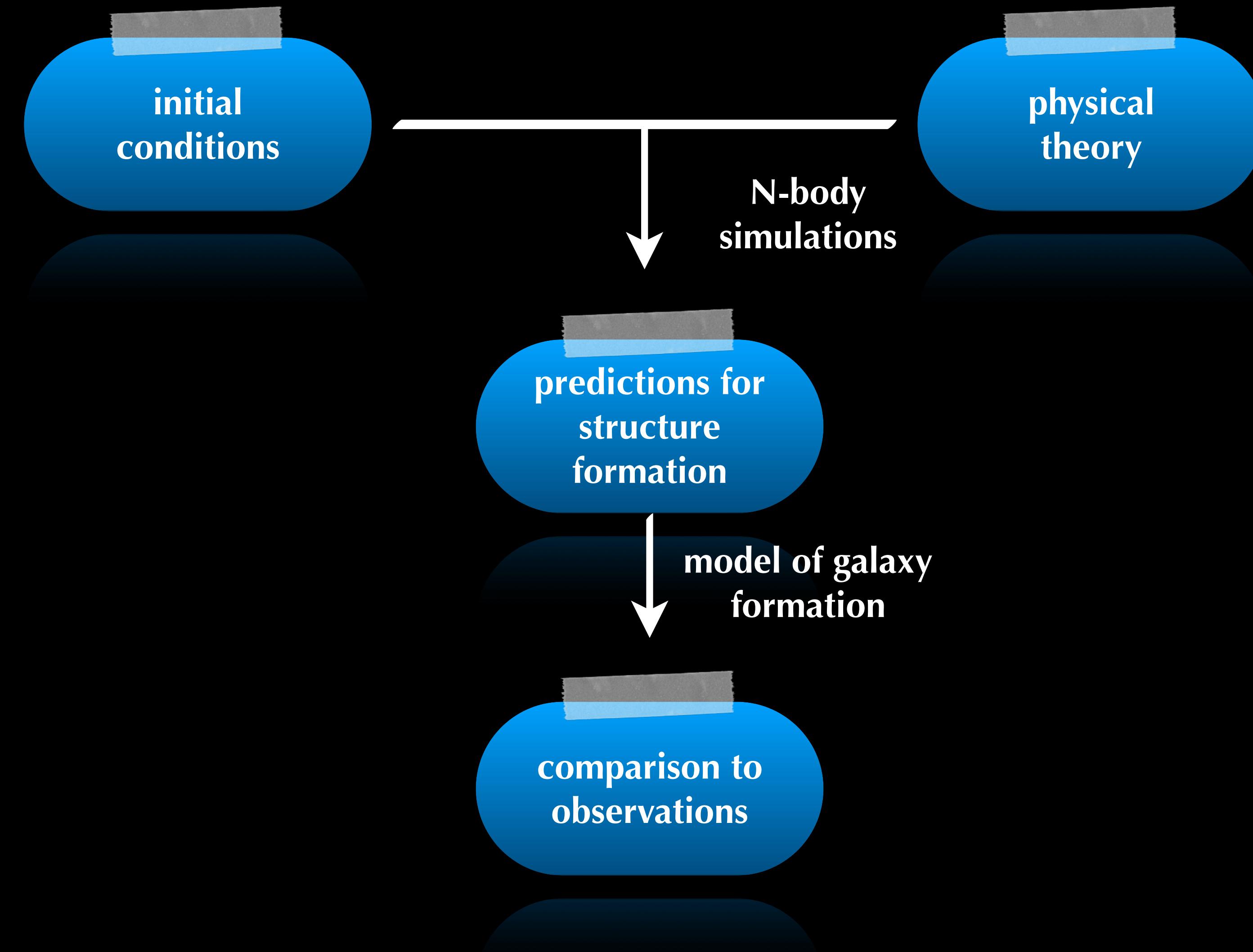
number of subhaloes per  
mass interval



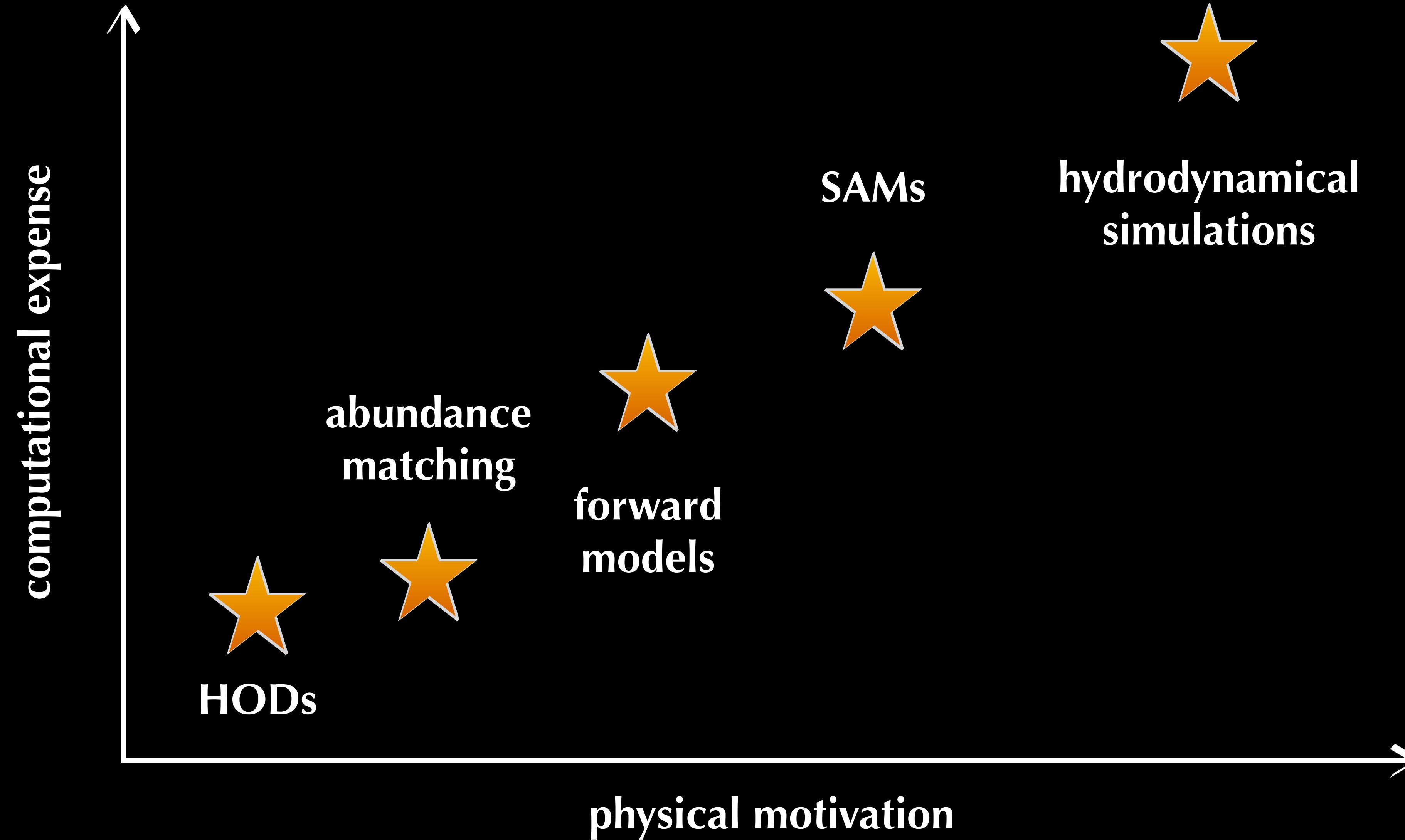
Springel+ (2008)

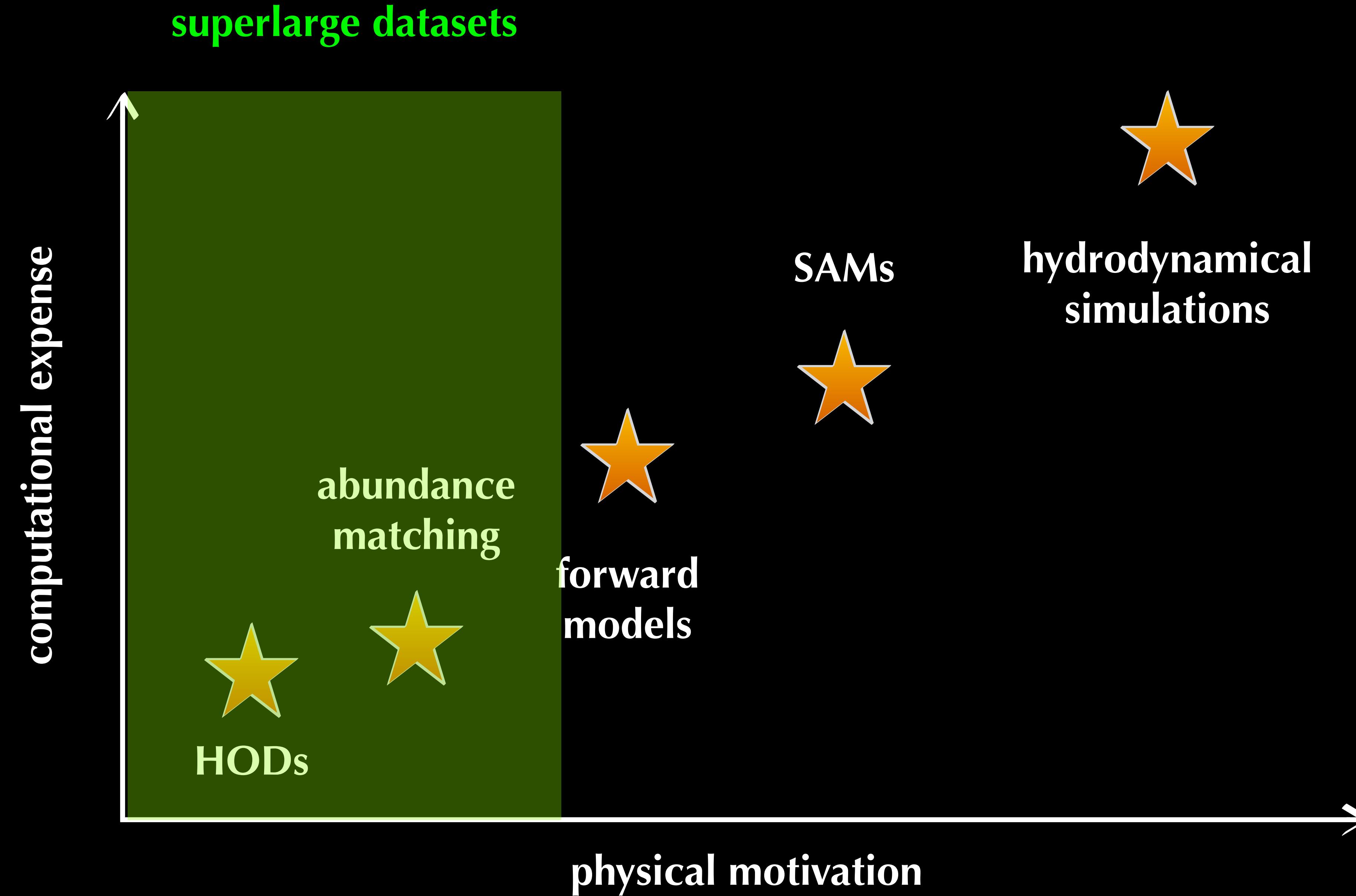
adding galaxies

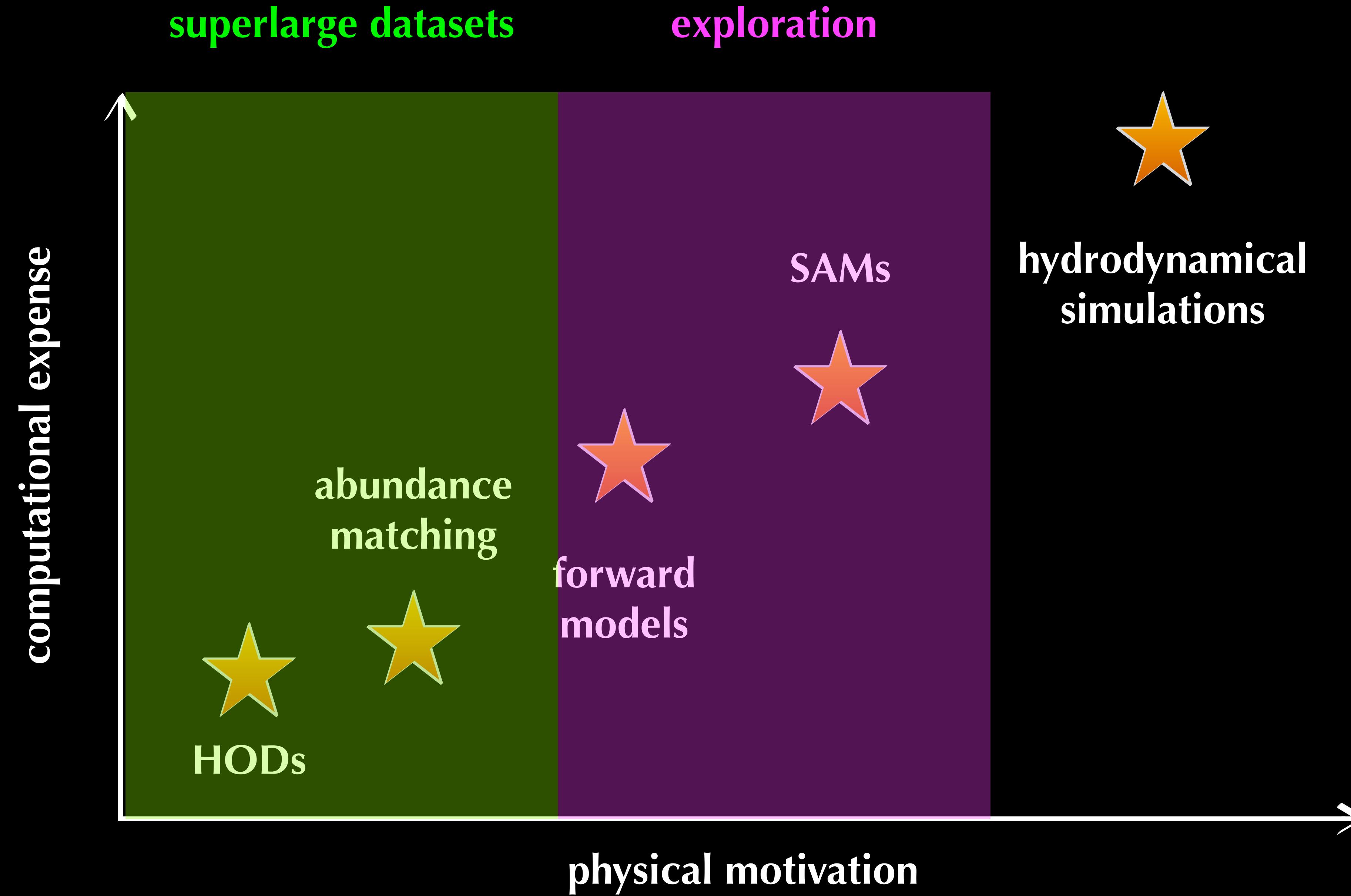
# the pathway to simulating the cosmos

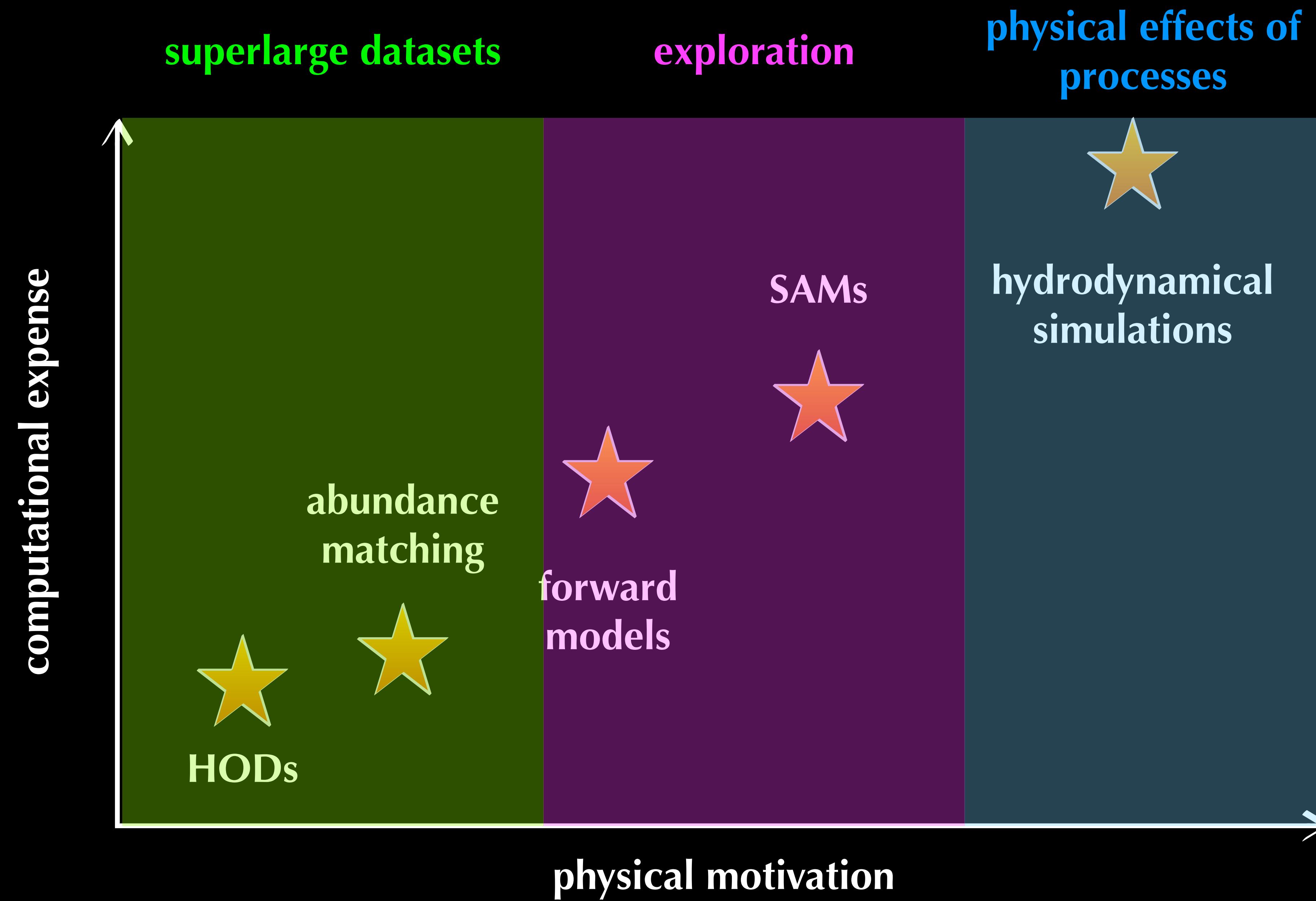






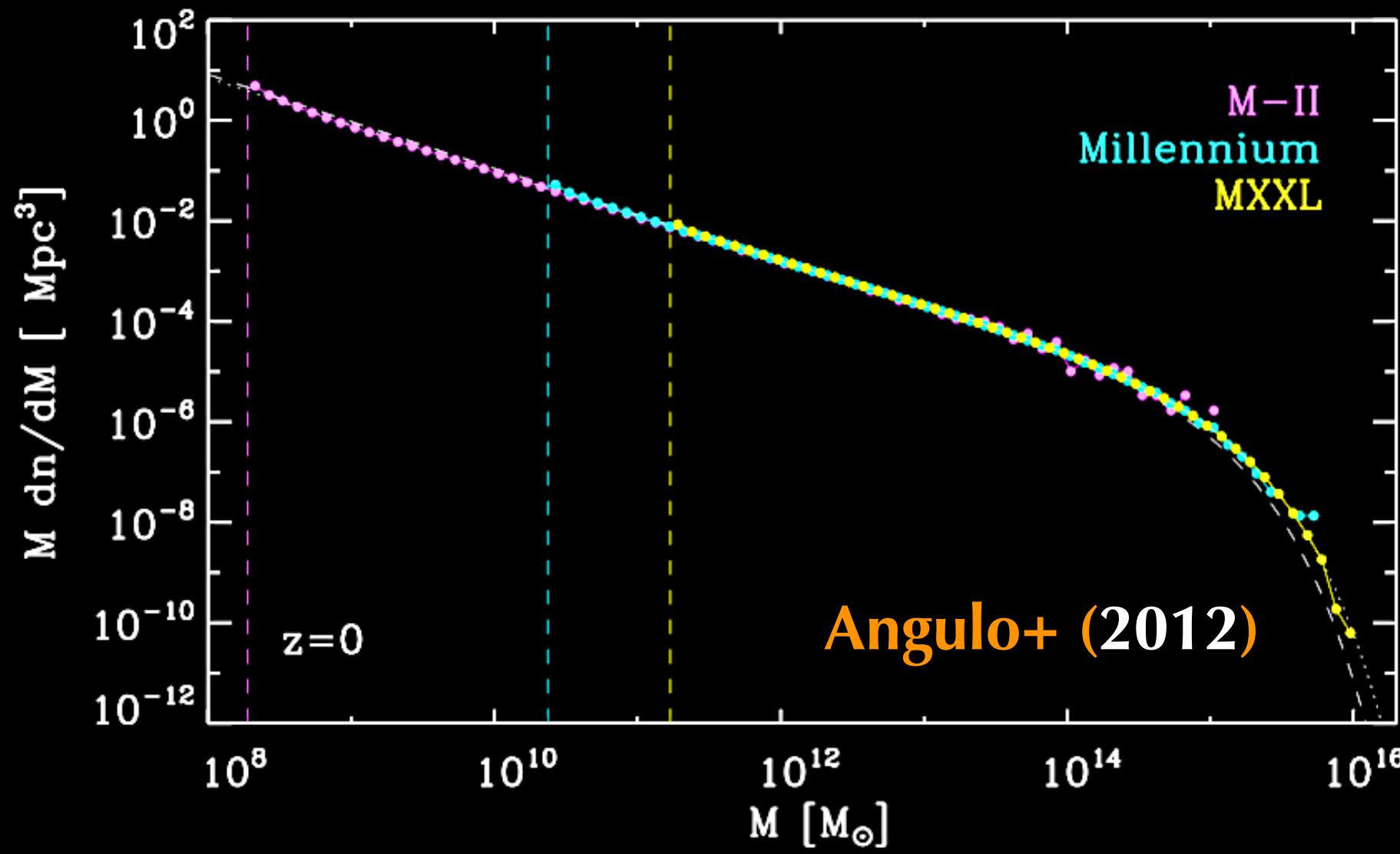






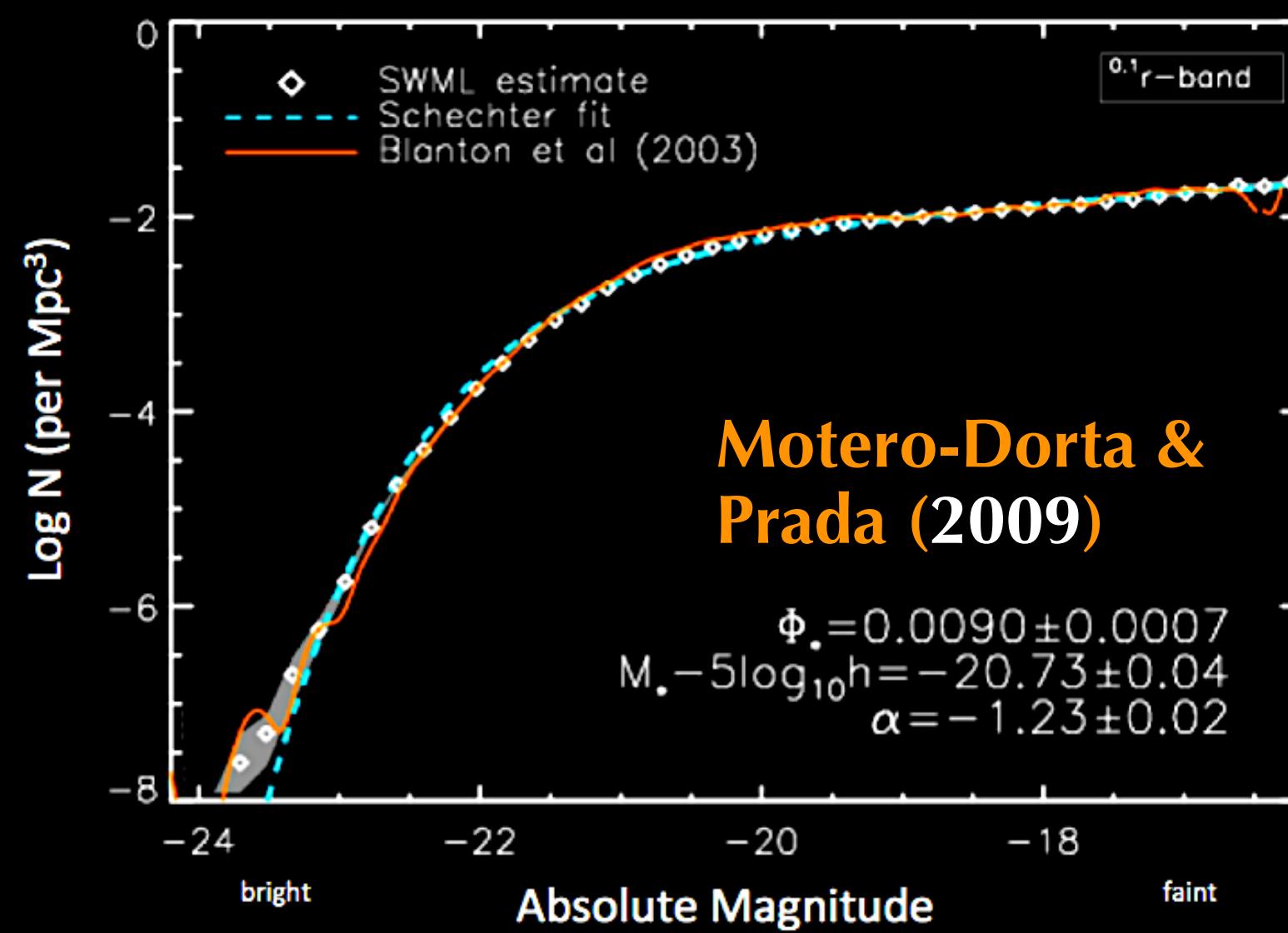
... but their  
objectives are  
similar

## DM halo mass function

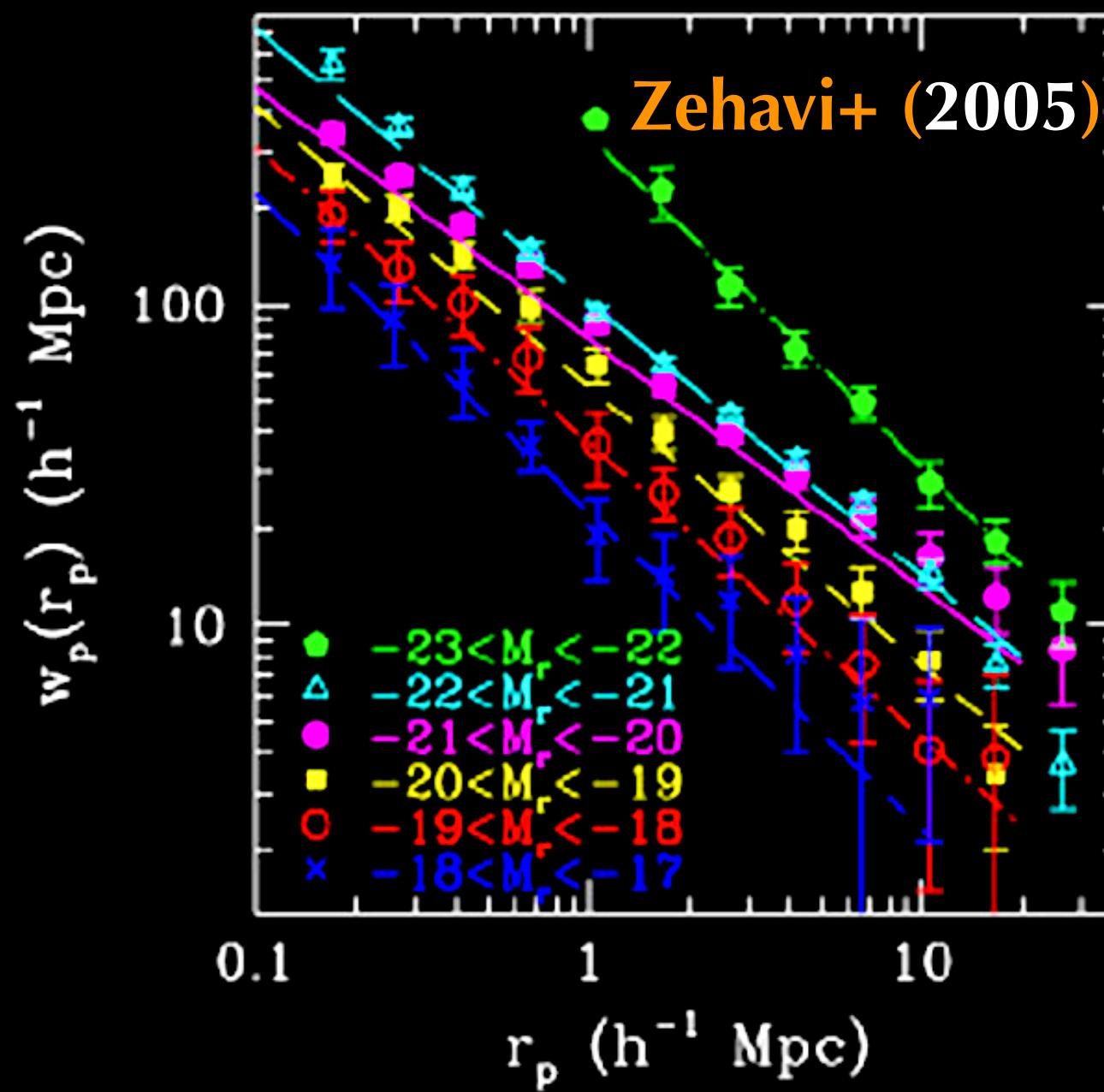


## galaxy clustering

### galaxy luminosity function



Motero-Dorta &  
Prada (2009)



$$P(N_{\text{gal}} \mid M_{\text{halo}})$$

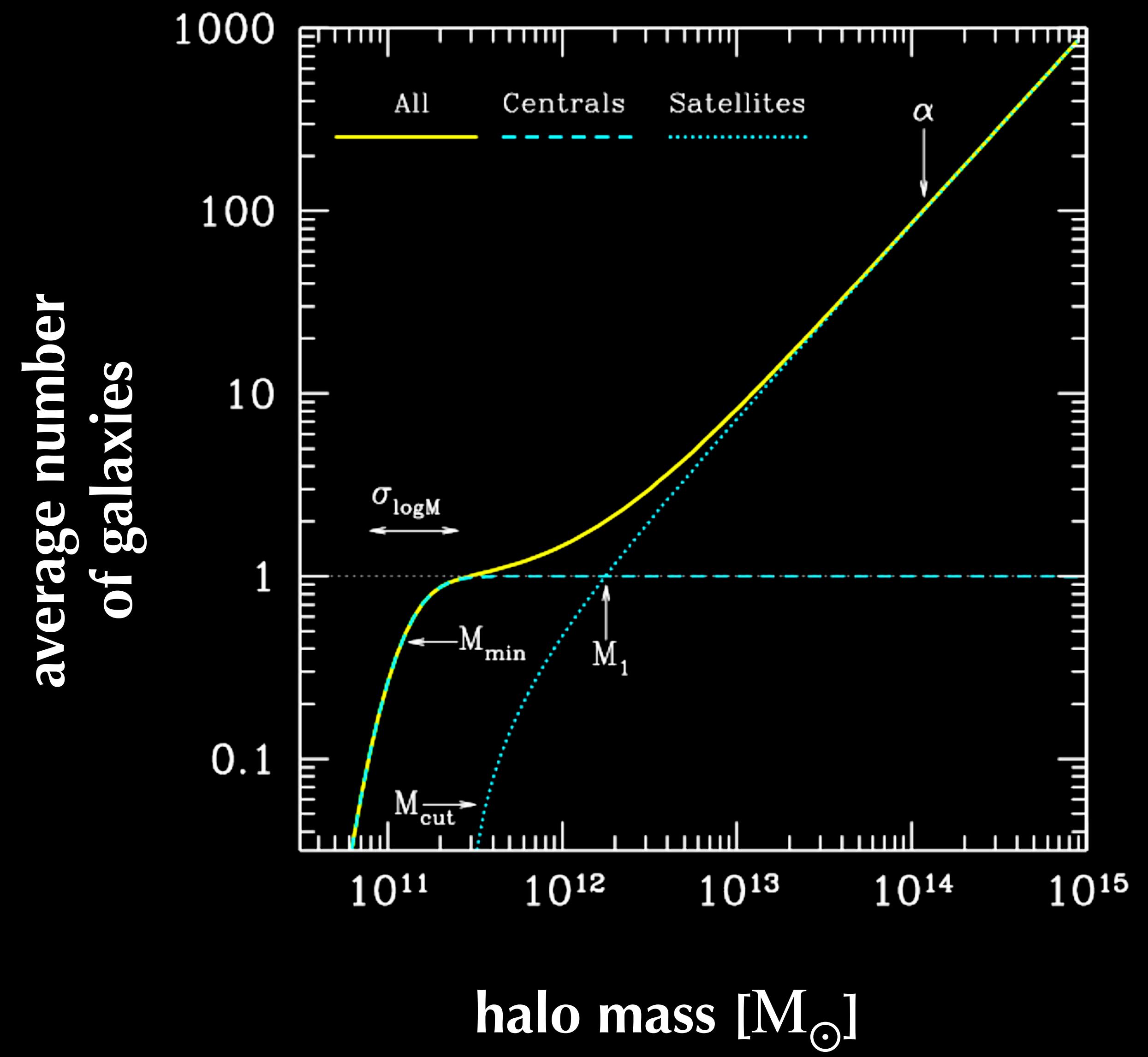
“halo occupation distribution”

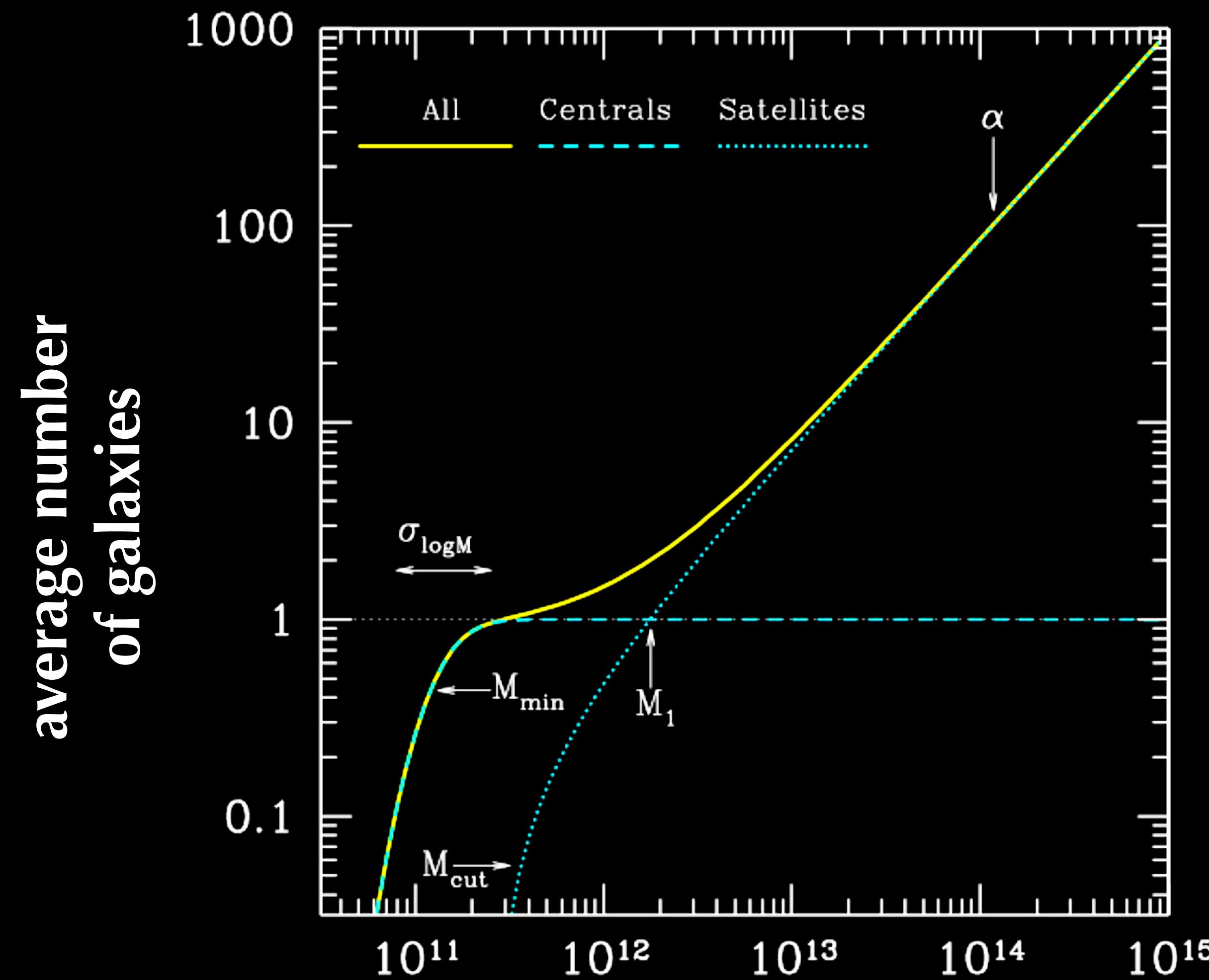
$$P(N_{\text{gal}} \mid M_{\text{halo}}, z_{\text{form}}, \dots)$$

“halo occupation distribution”

+

“assembly bias”

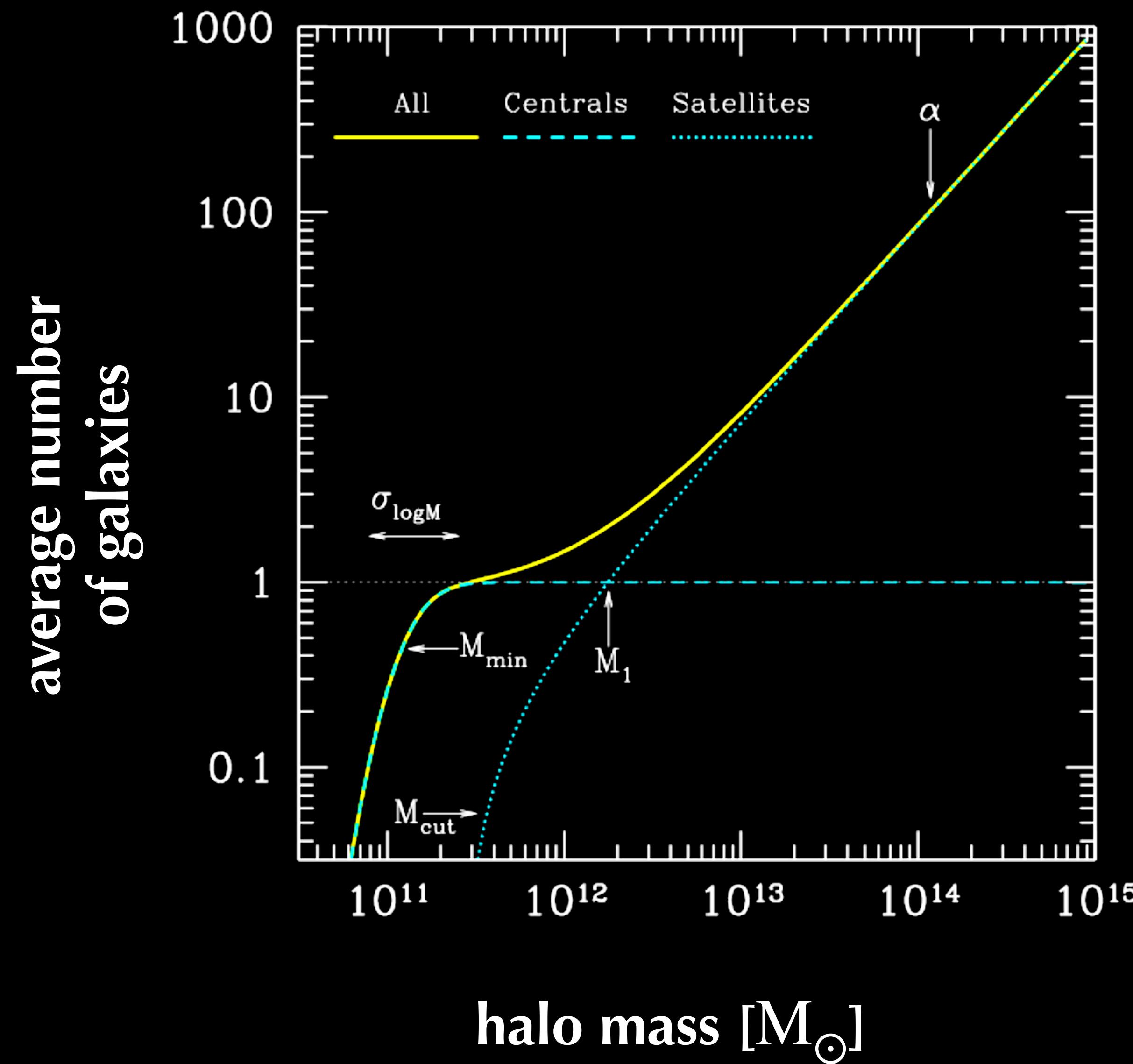




Zheng+ (2005)

$$N_{\text{cen}}(M_{\text{halo}}) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{\log M_{\text{halo}} - \log M_{\min}}{\sigma_{\log M}} \right) \right]$$

$$N_{\text{sat}}(M_{\text{halo}}) = \left( \frac{M_{\text{halo}} - M_{\text{cut}}}{M_1} \right)^\alpha$$



Zheng+ (2005)

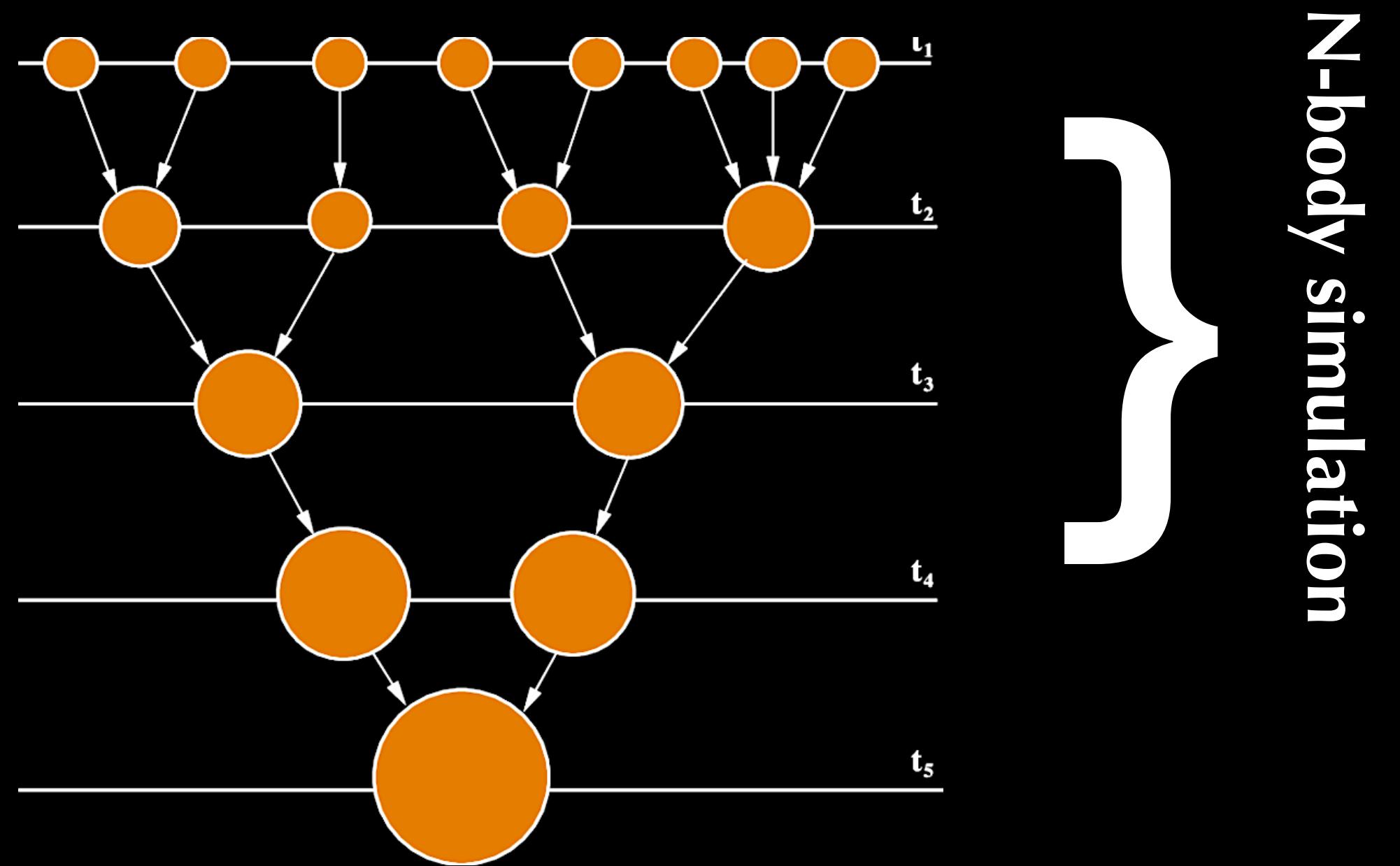
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the 5 free parameters of this model  
are tuned so as to reproduced  
observed data

# empirical forward models

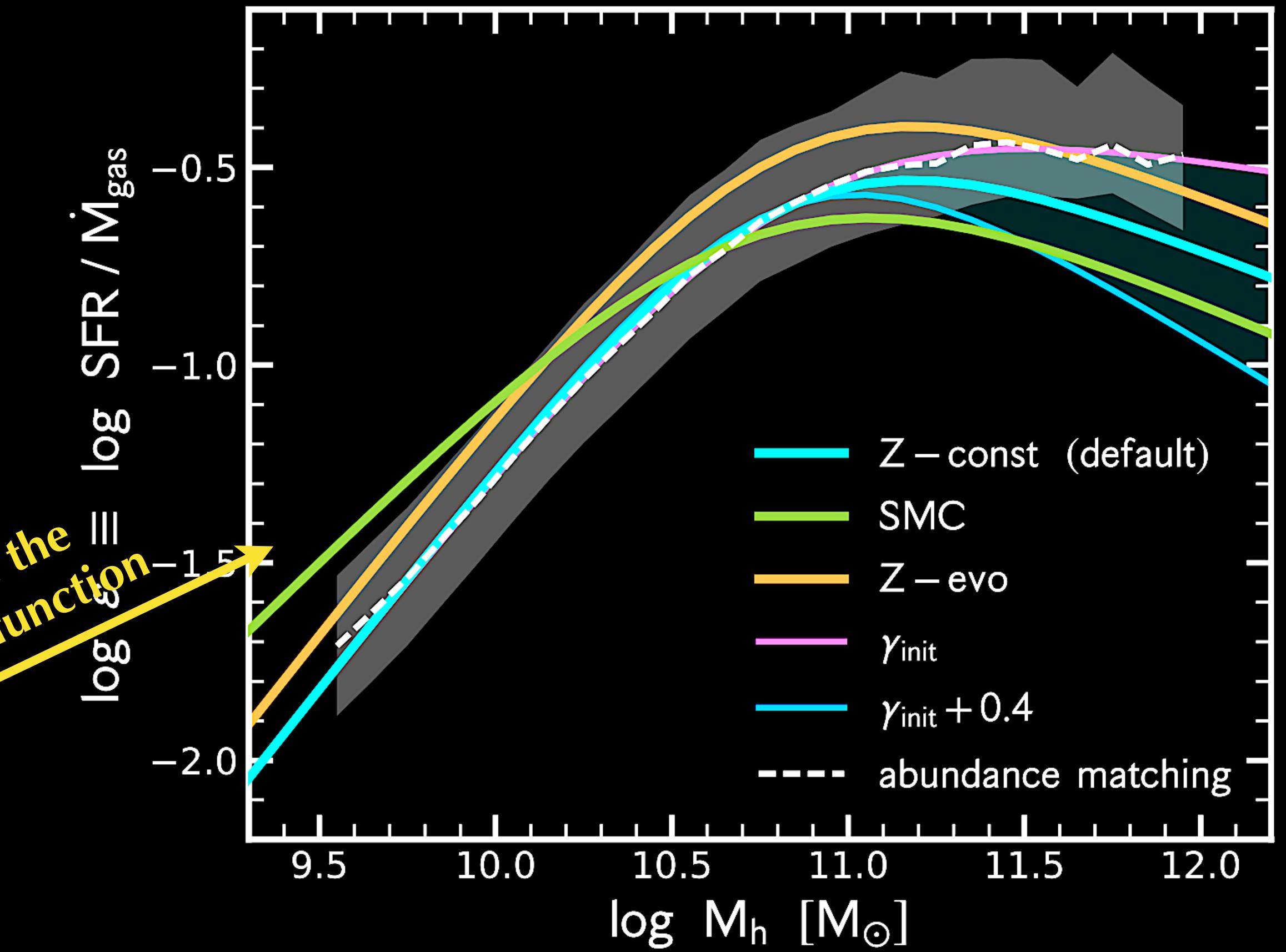
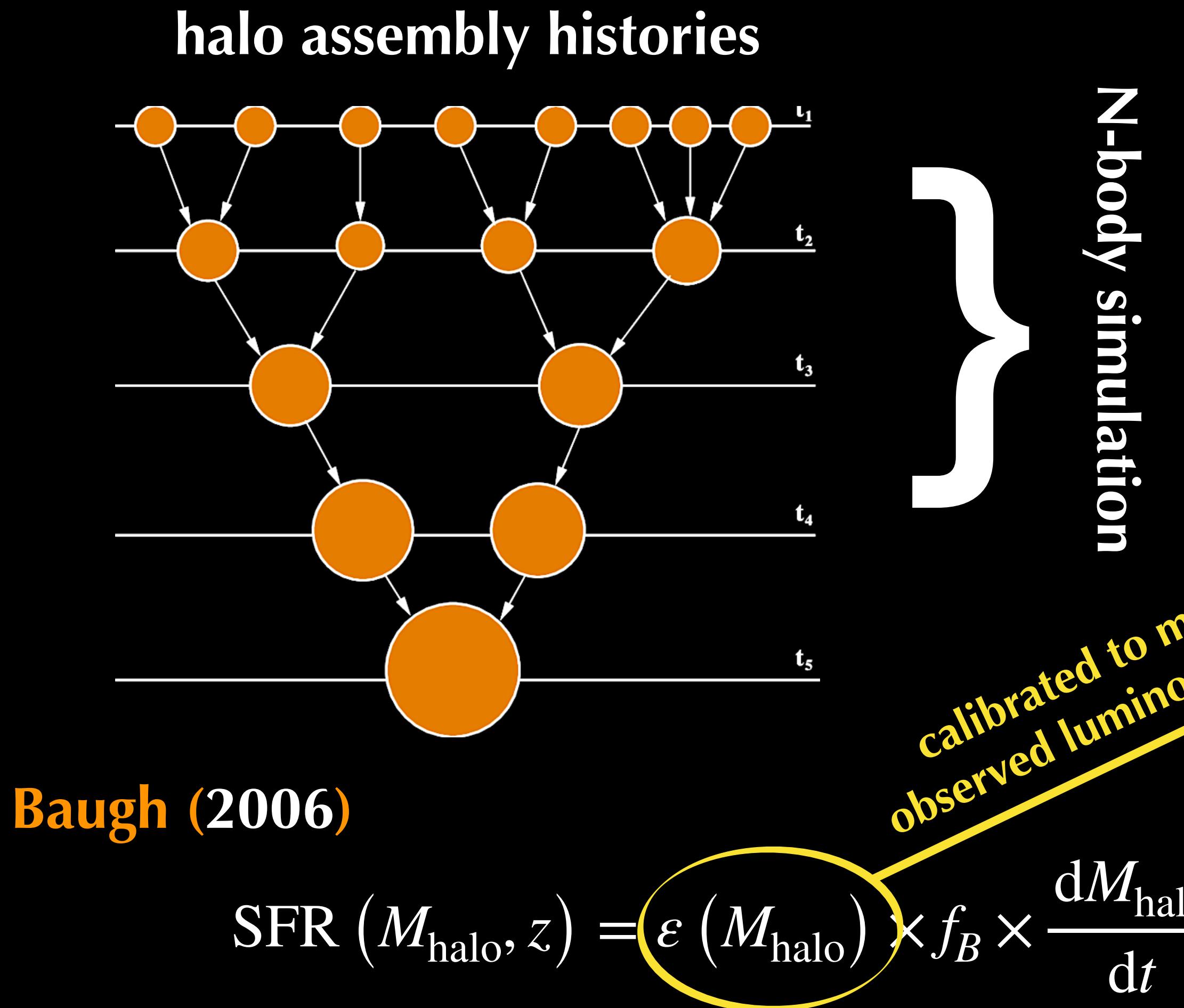
halo assembly histories



Baugh (2006)

$$\text{SFR} (M_{\text{halo}}, z) = \epsilon (M_{\text{halo}}) \times f_B \times \frac{dM_{\text{halo}}}{dt}$$

# empirical forward models



# modelling cosmic baryons

$$\frac{d\mathbf{v}}{dt} = - \frac{\nabla P}{\rho} - \nabla \Phi$$

$$\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{v} = 0$$

$$\frac{du}{dt} = - \frac{P}{\rho} \nabla \cdot \mathbf{v} - \frac{\Lambda(u, \rho)}{\rho}$$

$$P = (\gamma - 1) \rho u, \quad \gamma = 5/3$$

**Euler equation**  
[conservation of momentum]

**continuity equation**  
[conservation of mass]

**1st law of thermodynamics**  
[conservation of energy]

**equation of state of an ideal monoatomic gas**

**equations of  
hydrodynamics**

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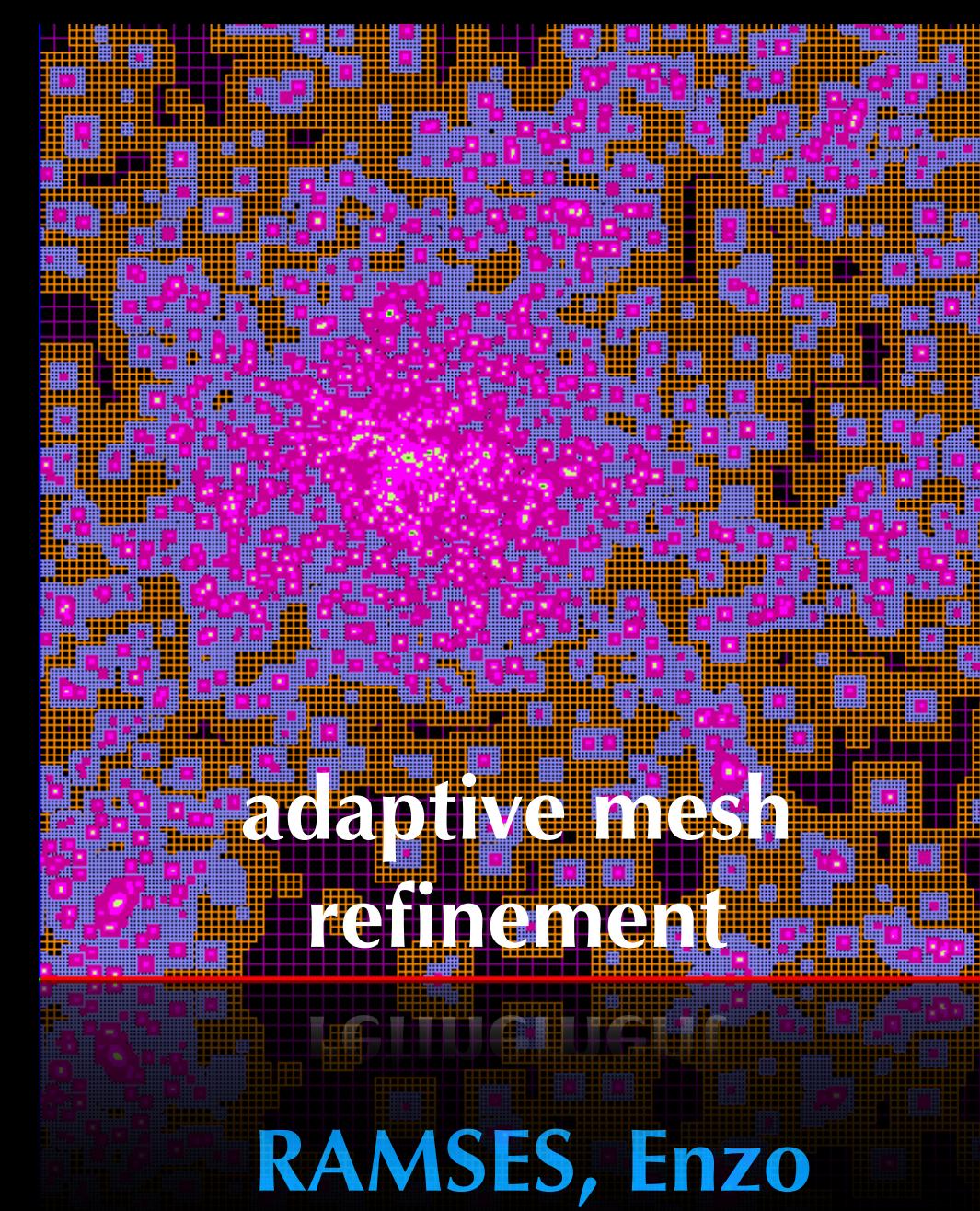
Eulerian methods

equations of  
hydrodynamics

уравнения  
гидродинамики

Lagrangian methods

hybrid methods



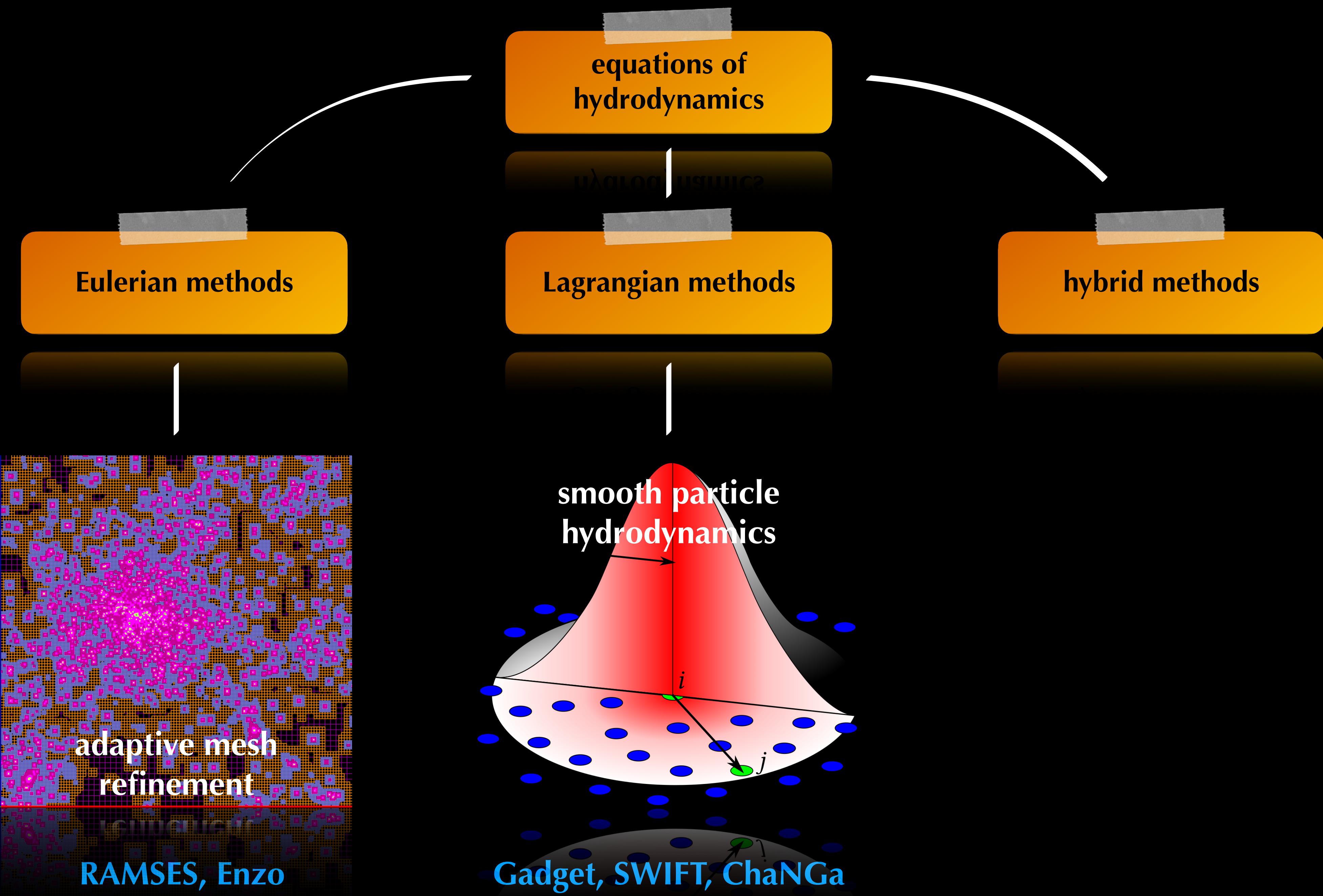
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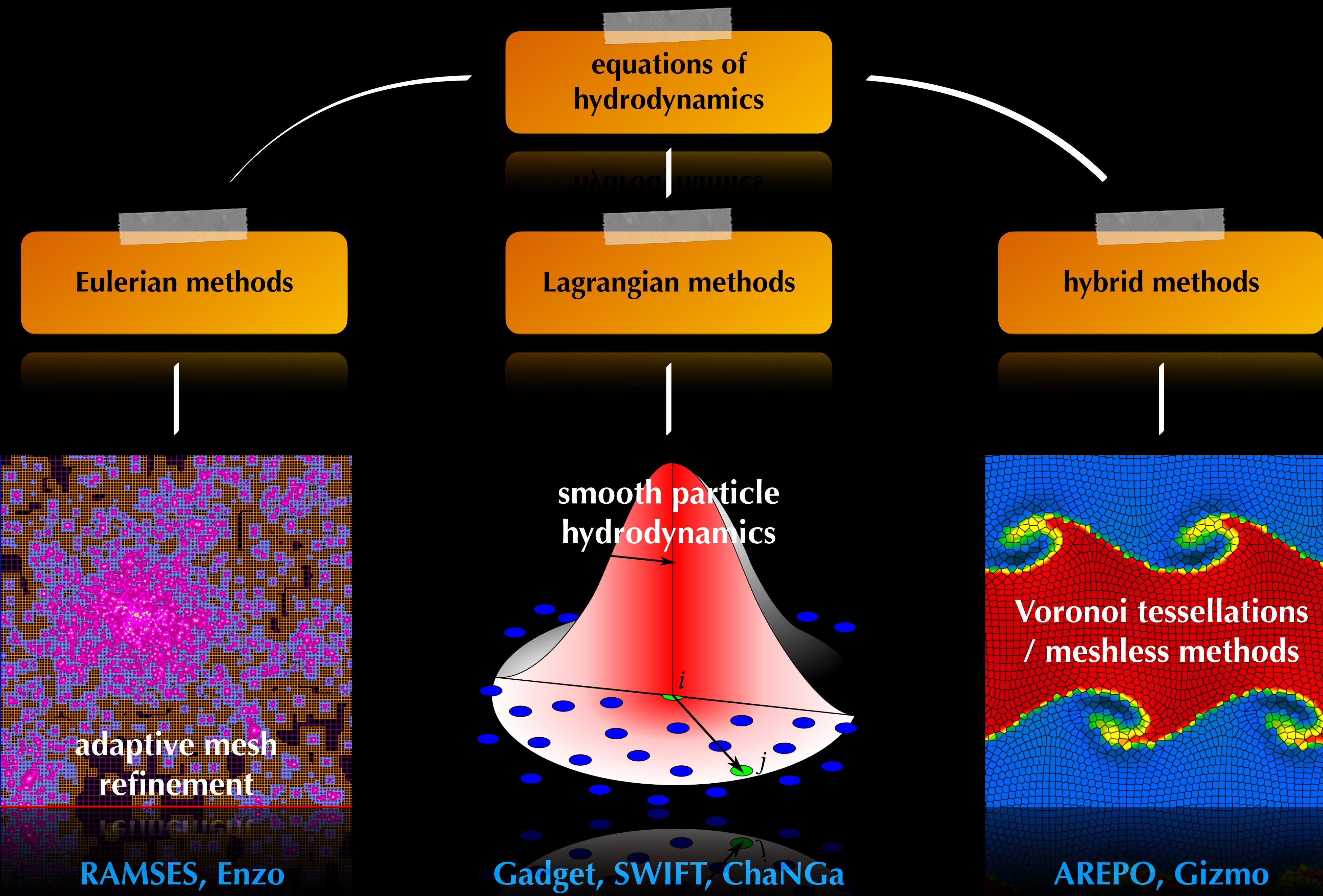
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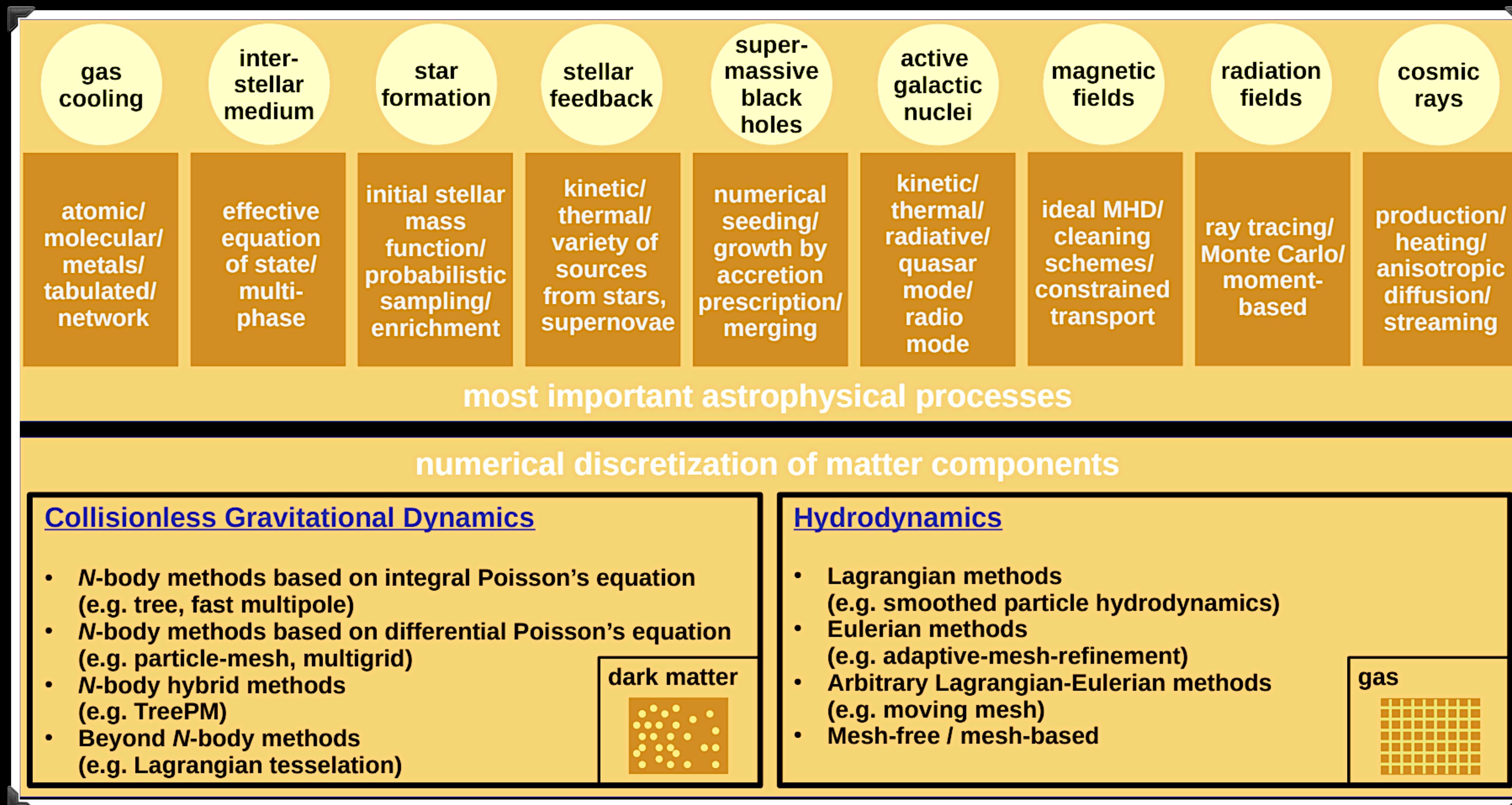
уравнения  
гидродинамики

Lagrangian methods

hybrid methods







**ultimately, what method / approach to modelling galaxies is used depends on the i) objectives and ii) the size of the problem being addressed!**

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subgrid model	mass + spatial resolution	“realism” of galaxy formation physics