4CS015
Lecture 3: Combinational Circuit

Prepared by: Uttam Acharya

1. Lecture 2 coverage

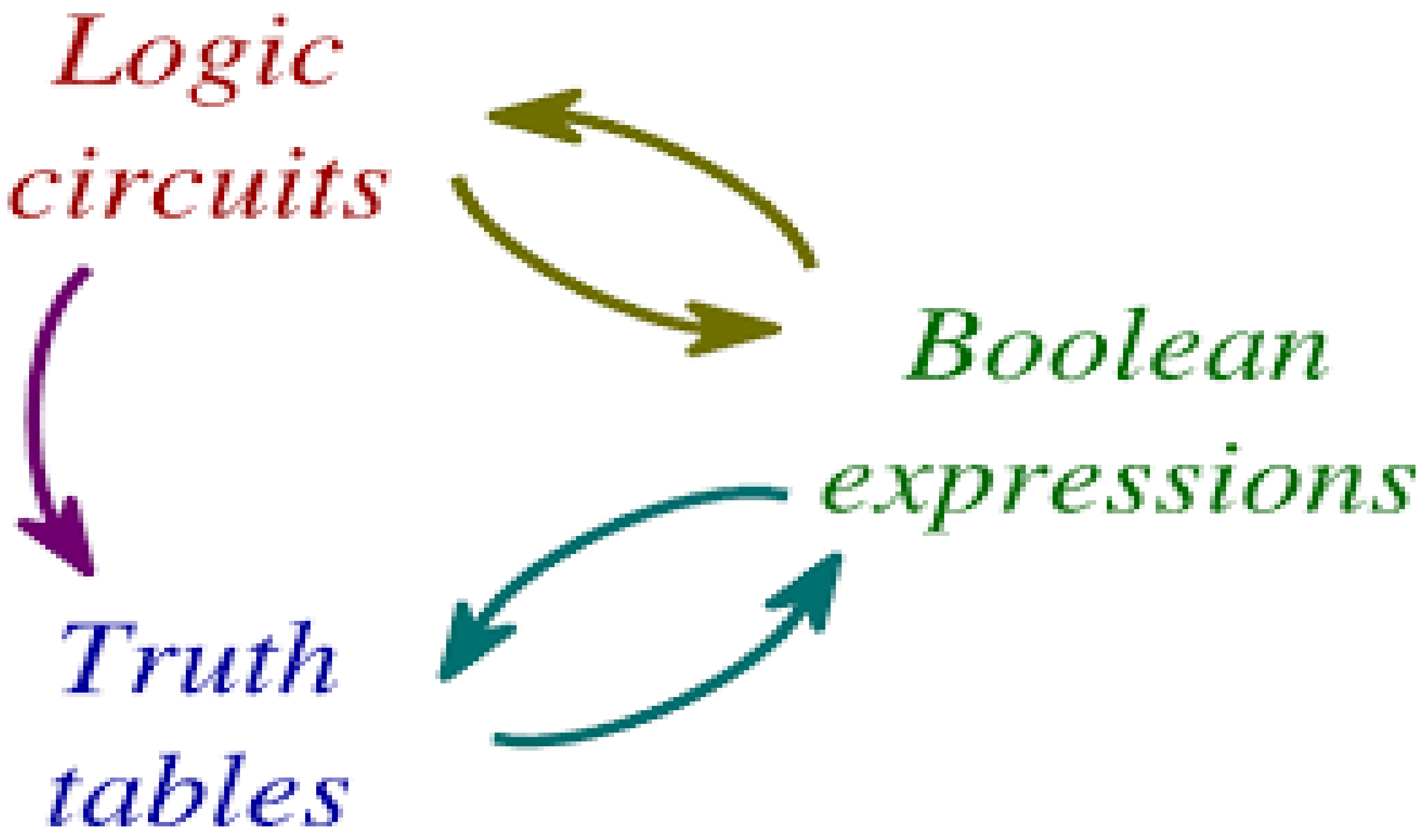
1.1 Review of Week 2

- Boolean gates and logic

1.2 Mathematical functions in logic!

- Understanding addition
- Making our own truth tables
- Building a circuit that can perform maths
- Reducing logic

2. Circuit Design:



2. Circuit Design (Contd.):

Three Steps starting from given circuit requirements in the form of a table.

1. Formulate a Boolean expression for the output function from the given table.
2. Simplify this expression as much as possible using Boolean algebra.
3. Draw the circuit corresponding to the simplified output function.

2. Circuit Design (Contd.):

2.1 Example:

We will design a circuit corresponding to the following truth table. The output function is labelled X.

A	B	X
1	1	1
1	0	1
0	1	0
0	0	1

2. Circuit Design (Contd.):

Step 1. First scan the output column for occurrences of 1. In this example there are three (lines 1, 2 and 4).

For **each** of these lines construct a sub-expression involving A and B and the operations AND (·) and NOT (¯) only that will return the value 1 for the corresponding input values.

In row 1, A = 1 and B = 1 so A · B will return the value 1 for these input values and for no others.

2. Circuit Design (Contd.):

In row 2, $A = 1$ and $B = 0$ so $A \cdot \overline{B}$ will return the required 1 for these values

Finally, with $A = 0$ and $B = 0$, row 4 will require $\overline{A} \cdot \overline{B}$

The three expression obtained are then combined together using OR (+) operations. The final expression

$$X = (A \cdot \overline{B}) + (\overline{A} \cdot \overline{B}) + (A \cdot B)$$

2. Circuit Design (Contd.):

Step 2: Simplify the Boolean Expression.

$$X = (A \cdot \overline{B}) + (\overline{A} \cdot \overline{B}) + (A \cdot B)$$

$$= A \cdot (B + \overline{B}) + (\overline{A} \cdot \overline{B}) \qquad \text{(law 3)}$$

$$= (A \cdot 1) + (\overline{A} \cdot \overline{B}) \qquad \text{(law 5)}$$

$$= A + (\overline{A} \cdot \overline{B}) \qquad \text{(law 4)}$$

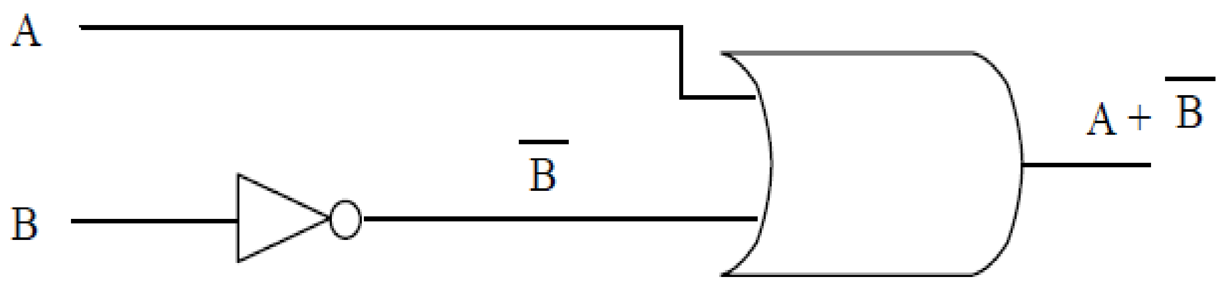
$$= (A + \overline{A}) \cdot (A + \overline{B}) \qquad \text{(law 3)}$$

$$= 1 \cdot (A + \overline{B}) \qquad \text{(law 5)}$$

$$\therefore X = A + \overline{B} \qquad \text{(laws 1 and 4)}$$

2. Circuit Design (Contd.):

Step 3: The circuit for the simplified output function X requires only two gates:



Check that the truth table for $X = A + \overline{B}$ agrees with the original.

The method extends easily to three or more input pulses.

2. Circuit Design (Contd.):

2.2 Another Example:

Design a Circuit corresponding to:

A	B	C	X
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

2. Circuit Design (Contd.):

Step 1 There is a 1 in lines 1, 3 and 5 of the output column.

The sub-expressions which will return 1 in these lines are, respectively $A \cdot B \cdot C$, $A \cdot \overline{B} \cdot C$ and $\overline{A} \cdot B \cdot C$

The Boolean Expression is therefore given by

$$\mathbf{X = (A \cdot B \cdot C) + (A \cdot \overline{B} \cdot C) + (\overline{A} \cdot B \cdot C)}$$

2. Circuit Design (Contd.):

Step 2 : Simplify $X = (A \cdot B \cdot C) + (\bar{A} \cdot B \cdot C) + (A \cdot \bar{B} \cdot C)$

$$X = (A \cdot B \cdot C) + (A \cdot \bar{B} \cdot C) + (\bar{A} \cdot B \cdot C)$$

$$X = A.C\,(B + \bar{B}) + (\bar{A} \cdot B \cdot C)$$

$$X = A.C\,.1 + (\bar{A} \cdot B \cdot C) \qquad \text{(Complement law)}$$

$$X = A.C + (\bar{A} \cdot B \cdot C)$$

$$X = C\,.\,(A + \bar{A}\,.\,B)$$

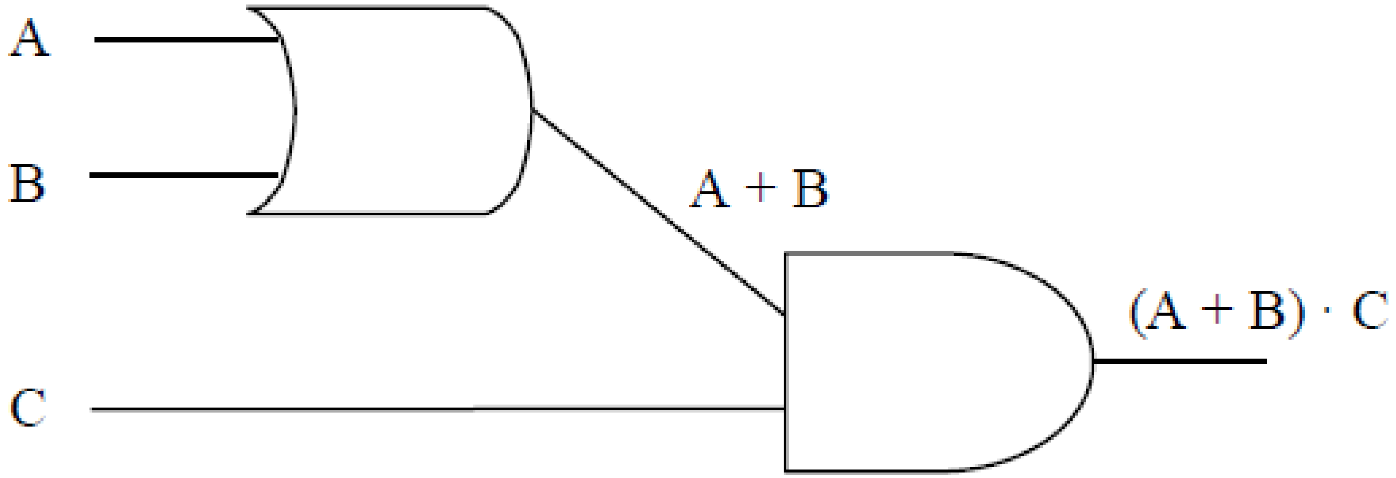
$$X = C\,.\,(A + B) \qquad \text{(Distributive law)}$$

The simplified Boolean Expression is therefore given by

$$\mathbf{X = C\,.\,(A + B)}$$

2. Circuit Design (Contd.):

Step 3 The circuit for (A + B) · C is



3. Exercises ……

1.Design a Circuit corresponding to following truth tables:

A	B	C	X
1	1	1	0
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	0

A	B	C	X
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	1

3. Exercises ……

Simplify and construct the logic circuit:

1. $A'.B' + (A.B)'$

2. $(A + B).(A + B) + A.(A + B')$

3. $(A'. B + A.B')'$

4. $((A + C).(AB)' + (BC + A')')'$

5. $(A.B'.C' + A'.B'.C+A.B.C+A'.B.C')$

4. Addition Rules as a Table

- Number 1 and Number 2 are the Inputs.
- Sum and Carry are the results after addition.

Number 1	Number 2	Result	Carry Over
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

5. Addition as Logical Functions

Input A ~~Number 1~~	Input B ~~Number 2~~	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

- Assume that SUM is now a truth table Entry then:

$$\mathrm{SUM} = \mathrm{A} \bullet \overline{\mathrm{B}} + \overline{\mathrm{A}} \bullet \mathrm{B} = \mathrm{A} \oplus \mathrm{B}$$

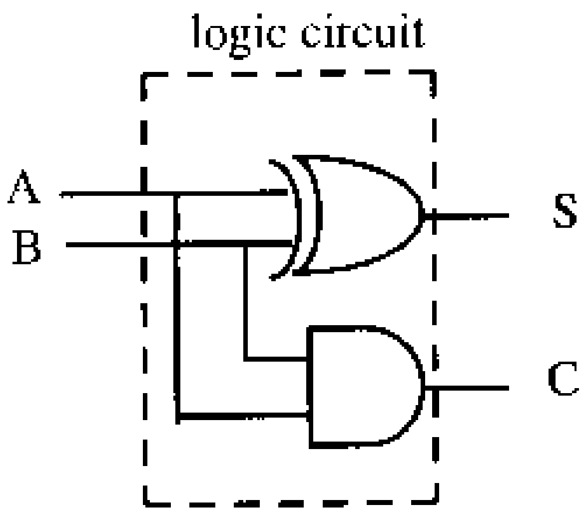
- Doing the same for CARRY we get:

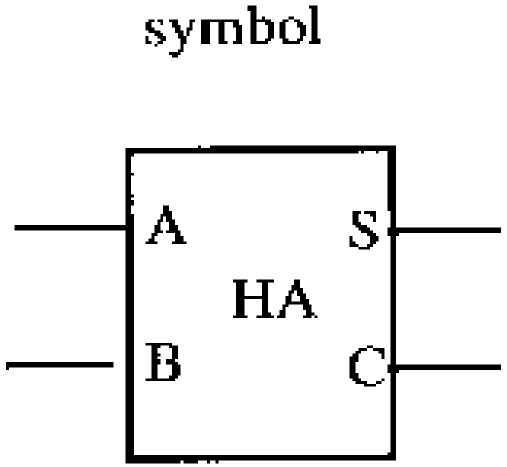
$$\mathbf{CARRY} = \mathbf{A} \bullet \mathbf{B}$$

6. Half Adder

- Combinational logic circuits give us many useful devices.
- One of the simplest is the *half adder*, which finds the sum of two bits.
- We can gain some insight as to the construction of a half adder by looking at its truth table, shown at the right.

6. Half Adder





Input A	Input B	S (Sum)	C (Carry)
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

7. Full Adder Function

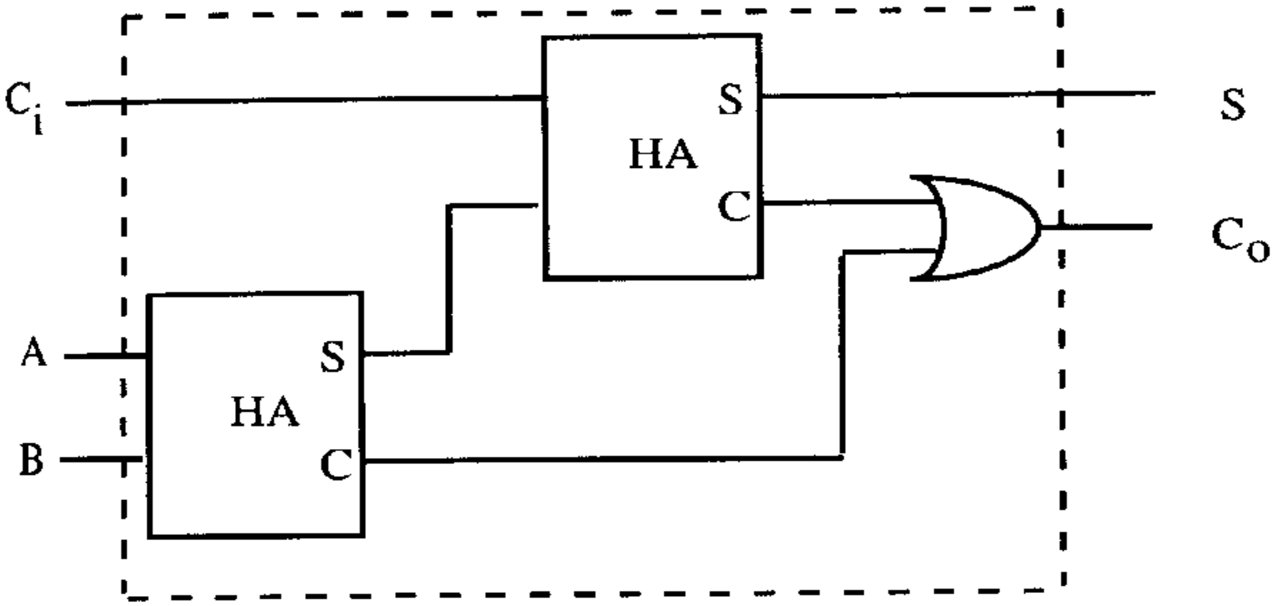
Input A	Input B	Carry IN	Sum	Carry OUT
0	0	0	0	0
1	0	0	1	0
0	1	0	1	0
1	1	0	0	1
0	0	1	1	0
1	0	1	0	1
0	1	1	0	1
1	1	1	1	1

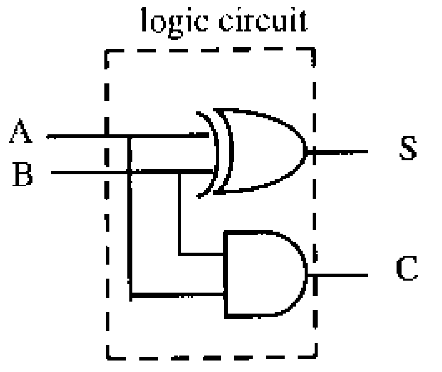
7.1 Full Adder

- We could derive the full Boolean expression for the Sum and Carry OUT.
- However, there is a great deal of symmetry associated with the half and full adder and we can simply build a FULL from two Halves.

7.2 Full Adder from Two Half Adders

- **The Full Adder:**

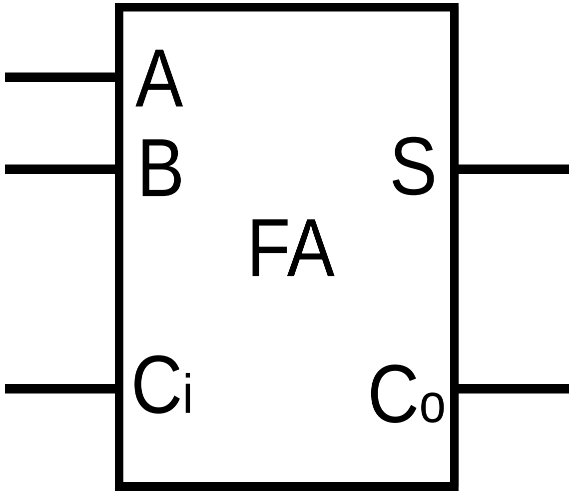




The Half Adder

		1	1	0
+		1	1	1
	1	1	0	
carry:				
	1	1	0	1

7.3 Symbol for a Full Adder



A	B	C_{in}	S	C_{out}
0	0	0	0	0
1	0	0	1	0
0	1	0	1	0
1	1	0	0	1
0	0	1	1	0
1	0	1	0	1
0	1	1	0	1
1	1	1	1	1

8. Reduction.

If you develop the sum of products for the full adder.

Sum = $ABC+\bar{A}\bar{B}C+\bar{A}\bar{B}C+\bar{A}B\bar{C}$

Carry OUT = $ABC+\bar{A}BC+AB\bar{C}+A\bar{B}C$

These show very little resemblance to the circuits we are using.

A	B	C_{in}	S	C_{out}
0	0	0	0	0
1	0	0	1	0
0	1	0	1	0
1	1	0	0	1
0	0	1	1	0
1	0	1	0	1
0	1	1	0	1
1	1	1	1	1

By applying the laws and theorems of Boolean algebra we should be able to get from the above to our circuits.

8.1 Reduction Basics

AND relationship

$0.X = 0$

$1.X = X$

$X.X = X$

$X.\overline{X} = 0$

OR relationship

$0+X = X$

$1+X = 1$

$X+X = X$

$X+\overline{X} = 1$

Not

$\overline{\overline{X}} = X$

Theorem

$(X+Y)(X+\overline{Y}) = X$

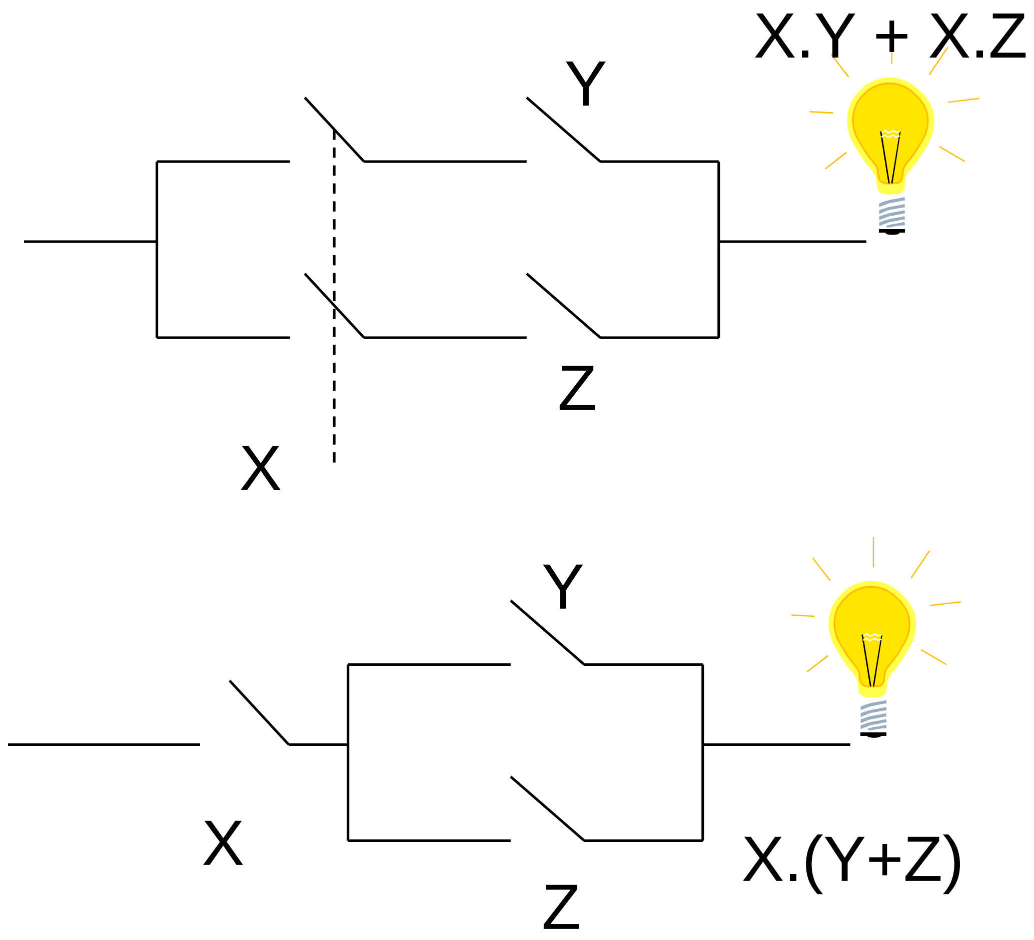
because

$(X+Y)(X+\overline{Y}) = X.(Y+\overline{Y})$

and $Y+\overline{Y} = 1$

and $X.1 = X$

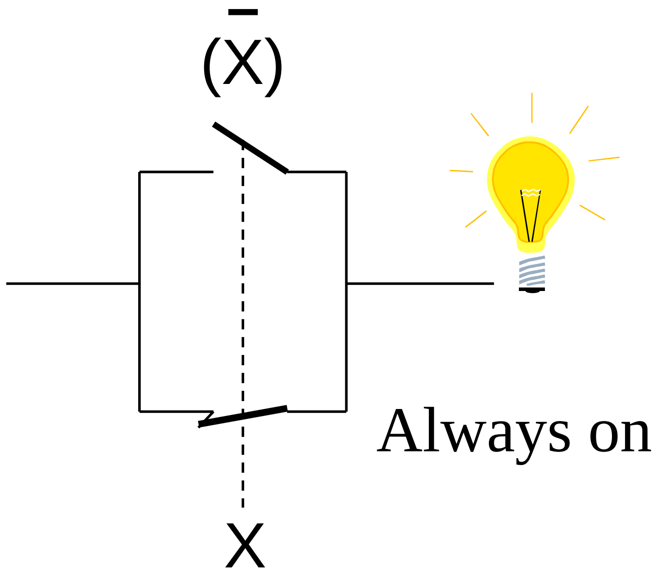
8.2 Reduction



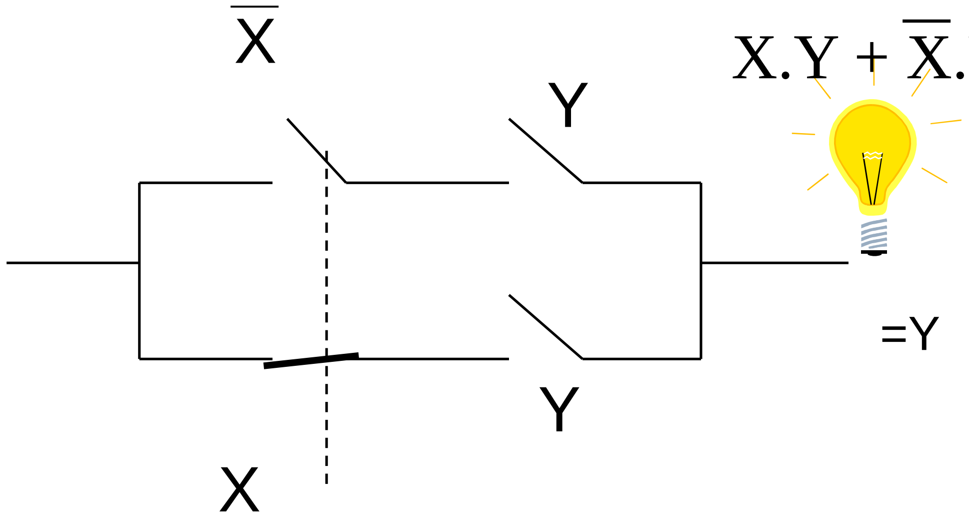
As the outputs are same for all inputs we can use this to reduce XY+XZ
To X(Y+Z)

Absorption

8.2 Reduction



Always on



$X.Y + \overline{X}.Y$

$=Y$

So $X+\overline{X}= 1$

1 = always on

9. Carry: Sum of Products

$$\text{Carry} = A\bar{B}C+\bar{A}BC+AB\bar{C}+ABC$$

$$\text{Carry} = A\bar{B}C+\bar{A}BC+AB\bar{C}+ABC+ABC+ABC$$

$$AB(\bar{C}+C)$$

$$BC(\bar{A}+A)$$

$$AC(\bar{B}+B)$$

We have simply applied Theorem 6 3 times.

$$\text{Carry} = A.B+B.C+A.C$$

10. Sum: sum of products

Sum = $\bar{A}\bar{B}C+\bar{A}B\bar{C}+A\bar{B}\bar{C}+ABC$

1. Sum =$\bar{A}(\bar{B}C+B\bar{C})+A\,(\bar{B}\bar{C}+BC)$
2. $\bar{B}C+B\bar{C} = B\oplus C$
3. $\bar{B}\bar{C}+BC = \overline{B\oplus C}$
4. Substitute X for $B\oplus C$
5. We get $\bar{A}X + A\bar{X}$
6. $A\oplus X$

7 substitute $B\oplus C = X$

8. Sum = $A\oplus(B\oplus C)$

11. Summary

- We have looked at the basic logic gates:
  - Identifying OR, AND, NOT, NAND, NOR and XOR.
- We have seen that gates can be joined together to form Combinatorial Logic.


Thank you!!