4 (A) (1)

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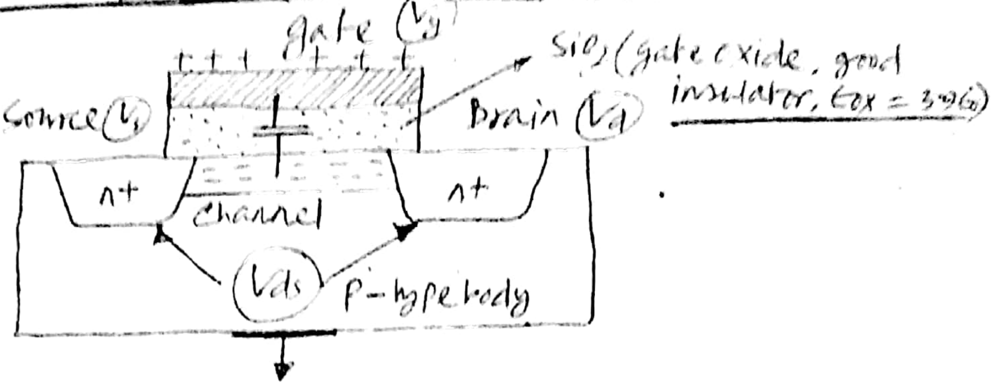
Developing I_{DS} equation for linear region:

Figure:



MOSFET structure looks like a parallel plate capacitor while operating in inversion. Since for n-type MOSFET a +ve voltage is implied to gate terminal (as V_g) and an electron sea channel (n-channel) is formed beneath the SiO_2 (gate oxide) layer. Hence the channel charge will be as follows —

$$Q_{channel} = C_g(V_{gc} - V_t) \quad \boxed{\because Q = CV} \quad \text{——— (1)}$$

where, C_g = Capacitance of the gate to the channel
$(V_{gc} - V_t)$ = the amount of voltage attracting charge channel which is not grounded

Again, $C_g = \epsilon_{ox} WL / t_{ox}$

$\Rightarrow C_g = C_{ox} WL$

where $C_{ox} = \dfrac{\epsilon_{ox}}{t_{ox}}$

where ϵ_{ox} = Dielectric permittivity of oxide layer
t_{ox} = thickness of oxide layer
W = width of the channel
L = length of the channel

Now to find Avg. $V_{gc} = \frac{V_{gs} + V_{gd}}{2}$

$$\Rightarrow V_{gc} = \frac{V_{gs}}{2} + \frac{V_{gd}}{2}$$

$$= V_{gs} - \frac{V_{gs}}{2} + \frac{V_{gd}}{2}$$

$$= V_{gs} - \left(\frac{V_{gs}}{2} - \frac{V_{gd}}{2}\right)$$

$$= V_{gs} - \frac{1}{2}(V_{gs} - V_{gd})$$

$$= V_{gs} - \frac{V_{ds}}{2} \qquad \boxed{\because V_{gs} - V_{gd} = V_{ds}}$$

$$\therefore \boxed{V_{gc} = V_{gs} - \frac{V_{ds}}{2}}$$

Therefore, from eqn. (I) we get $\Rightarrow$

$$Q_{channel} = C_g(V_{gc} - V_t)$$

$$= C_{ox}WL\left(V_{gs} - \frac{V_{ds}}{2} - V_t\right) \text{ —— (II)}$$

Now to find the time for the carrier to cross the channel is $\Rightarrow$

$$t = \frac{L \text{ (channel length)}}{v \text{ (velocity of the carrier)}} \qquad \boxed{\because v = \frac{ds}{dt} \text{ or } dt = \frac{ds}{v}}$$

$$\Rightarrow t = \frac{L}{\mu E}$$

($\because v = \mu E$, carrier velocity is proportional to the lateral E-field (electric field between source and the drain $\boxed{v \propto E}$)

$$\Rightarrow t = \frac{L}{\mu \cdot \frac{V_{ds}}{L}} \qquad \boxed{\because E = \frac{V_{ds}}{L}}$$

$$\Rightarrow t = L^2/\mu V_{ds} \text{ —— (III)}$$

Now we know, $I = \frac{Q}{t}$

$$\therefore I_{ds} = \frac{Q_{channel}}{t}$$

$$I_{ds} = \frac{Q_{channel}}{t}$$ [By putting the value from eqⁿ (II) and (III)]

$$= \frac{C_{ox} W \cancel{L} \left(V_{gs} - \frac{V_{ds}}{2} - V_t\right)}{L\, \cancel{L^2}/\mu V_{ds}}$$

$$I_{ds} = \mu C_{ox} \frac{W}{L}\left(V_{gs} - V_t - \frac{V_{ds}}{2}\right) V_{ds}$$

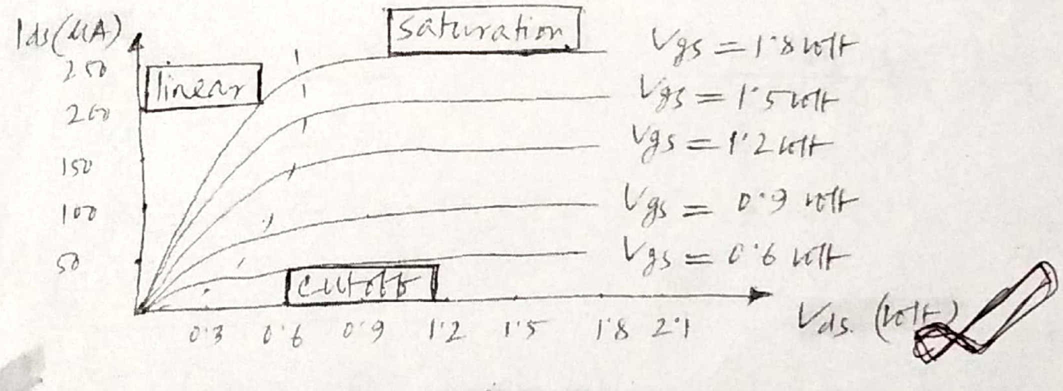
where, $\mu C_{ox} \frac{W}{L}$ = process gain factor, it can be represented by β.

μ = process dependent

W/L = geometry dependent

$$\therefore \boxed{I_{ds} = \beta\left(V_{gs} - V_t - \frac{V_{ds}}{2}\right) V_{ds}}$$

I_{DS} vs V_{DS} Curve for n-channel MOSFET (enhancement type):



I_{DS} equation for saturation region:

We know that saturation occurs when $V_{ds} \geq V_{gs} - V_t$ where $(V_{gs} - V_t)$ is the effective gate voltage for the MOSFET. Now at the saturation region if

$V_{ds.sat} = (V_{gs} - V_t)$ then from the equation of I_{ds} at linear region we can write as follows →

$$I_{ds} = \beta\left(V_{gs} - V_t - \frac{V_{ds.sat}}{2}\right) V_{ds.sat} \quad \text{——} (1)$$

Now drain voltage no longer increases the current, I_{ds}. Hence I_{ds} is independent of V_{ds} and can be a perfect current source

Now from equation ① by putting $V_{ds,sat} = (V_{gs} - V_t)$ we can achieve $\Rightarrow$

$$I_{ds} = \beta\left(V_{gs} - V_t - \frac{V_{ds,sat}}{2}\right)V_{ds,sat}$$

$$= \beta\left(V_{gs} - V_t - \frac{(V_{gs} - V_t)}{2}\right)(V_{gs} - V_t)$$

$$= \beta\left(\frac{2V_{gs} - 2V_t - V_{gs} + V_t}{2}\right)(V_{gs} - V_t)$$

$$= \beta\frac{(V_{gs} - V_t)}{2}.(V_{gs} - V_t) = \beta\frac{(V_{gs} - V_t)^2}{2}$$

$$\boxed{\therefore\ I_{ds} = \beta\frac{(V_{gs} - V_t)^2}{2}}$$

(I_{ds} becomes independent of V_{ds})

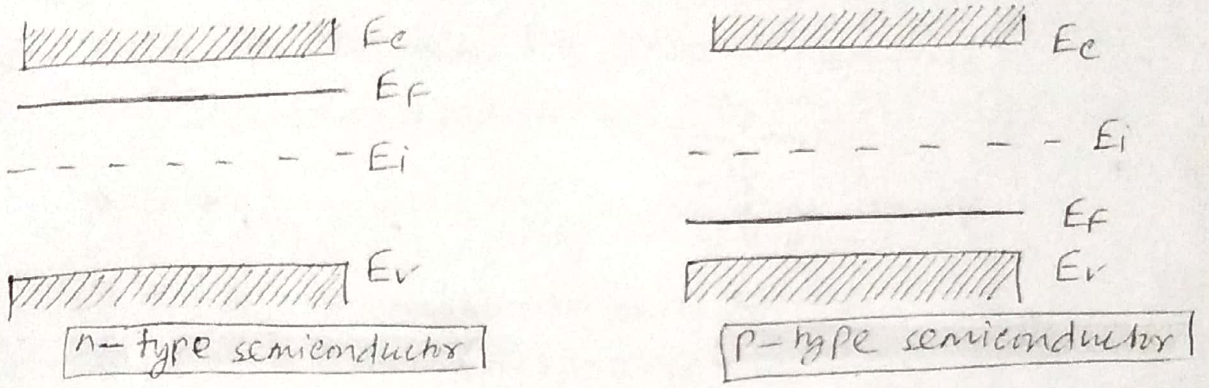
- **Summerization:**

  - Cut off region:
    - ① $I_{ds} = 0$
    - ⓪ $V_{gs} < V_t$ (or if $V_{gs} > V_t$ but $V_{ds} = 0$) and $V_{ds} = 0$

  - Linear region:
    - ① $I_{ds} = \beta\left(V_{gs} - V_t - \frac{V_{ds}}{2}\right)V_{ds}$
    - ⓪ $V_{gs} > V_t$ and $0 < V_{ds} < V_{gs} - V_t$

  - Saturation region:
    - ① $I_{ds} = \beta\frac{(V_{gs} - V_t)^2}{2}$
    - ⓪ $V_{gs} > V_t$ and $V_{ds} > V_{gs} - V_t$

The symbol ϕ_b is called bulk potential. It represents the difference between the Fermi level of the dopped semiconductor and Fermi energy level for intrinsic semi-conductor. The intrinsic level is midway between the valance-band edge and the conduction-band edge of the semiconductor. In n-type semiconductor the Fermi level is closer to the conduction band and in p-type semiconductor the Fermi level is closer to the valance band.



Again, $Q_b = \sqrt{2 \epsilon_{si} . q . N_A . 2\phi_b}$ which is called bulk charge term,

where, $\epsilon_{si} = 11.7 \epsilon_0$ or 1.06×10^{-12} F/cm which is the permittivity for silicon.

Now the flatband voltage, V_{fb} is given by $\Longrightarrow$

$$V_{fb} = \phi_{ms} - \frac{Q_{fc}}{C_{ox}}$$

Where, Q_{fc} = represents the fixed charge due to surface states that aries due to imperfection in the silicon oxide interface and doping.

ϕ_{ms} = The work function difference between the gate material and silicon substrate ($\phi_{gate} - \phi_{si}$)

- ϕ_{ms} is -ve voltage for both N-MOS & P-MOS

Threshold voltage equation, V_t (without body effect, $V_{sb}=0$)

The threshold voltage, V_t may be expressed as

$$V_t = V_{tmos} + V_{fb}$$

where, V_{tmos} = ideal threshold voltage of an ideal MOS capacitor (no workfunction difference)

V_{fb} = termed as flat-band voltage.

V_{tmos} is the threshold voltage where there is no work functi-on difference between the gate and the substrate materi-als. This ideal threshold voltage can be expressed as →

$$V_{t-mos} = 2\phi_b + \frac{Q_b}{C_{ox}}$$

where, ϕ_b = Channel inversion voltage or voltage for inverted surface (from p-type to n-type or vice versa)

Q_b = Accomodate charge in depletion region (+ve voltage for N-mos)

C_{ox} = oxide capacitance

Again, $$\phi_b = \frac{KT}{q} \ln\left(\frac{N_A}{N_i}\right)$$

where, K = Boltzman's constant 1.380×10^{-23} J/K

$\frac{KT}{q} = 0.02586$ volt at 300° K

q = charge for electron 1.602×10^{-19} C

N_A = Carrier concentration for acceptor atom (The hole concentration of p-type material after adding impurity in it)

N_i = Carrier concentration in intrinsic (undopped) silicon. 1.45×10^{10} cm^{-3} at 300 K,

* For n-mos transistor:

$$\phi_{ms} = -\left(\frac{E_g}{2} + \phi_b\right) \simeq -0.9V \; (N_A = 1\times10^{16}\,cm^{-3})$$

* For p-mos transistor:

$$\phi_{ms} = -\left(\frac{E_g}{2} - \phi_b\right) \simeq -0.2V \; (N_A = 1\times10^{16}\,cm^{-3})$$

Therefor, the equation for V_t we get as follows →

$$V_t = V_{tmos} + V_{fb}$$
$$= 2\phi_b + \frac{Q_b}{C_{ox}} + \phi_{ms} - \frac{Q_{fc}}{C_{ox}}$$

$\boxed{\because \text{generally } Q_{fc} = 0}$

$$\therefore V_t = 2\phi_b + \frac{Q_b}{C_{ox}} + \phi_{ms}$$

* Remember: ϕ_s (surface potential) $= 2\phi_b$ (Bulk potential) $= 2\phi_f$ (Fermi level potential)

$$\phi_b = \phi_f = \frac{KT}{q}\ln\left(\frac{N_A}{N_i}\right)$$

$$\boxed{\therefore V_t = \phi_s + \frac{Q_b}{C_{ox}} + \phi_{ms}}$$

* Note: For intrinsic normal semiconductor the bult potential will be, ϕ_b
For Extrinsic semiconductor the bult potential will be written as, $2\phi_b$

* Explanation: The Fermi level ϕ_f (or ϕ_b) is usually about 0.3V. After the potential in the silicon reaches $2\phi_f$ (or $2\phi_b$), it further increases in gate voltage and produce no further changes in the depletion layer but instead induce a thin layer of electrons in the depletion layer at the surface of the silicon directly under the oxide and forms conducting channel between drain & source.

Threshold voltage equation (Derivation for V_t):

- Here the derivation is done with the body effect. Threshold voltage is not constant with respect to voltage difference between the substrate and the source. This is known as substrate bias effect or body effect.

- (For body effect $V_{sb} \neq 0$)

We know that without — body effect the equation for

$$V_t = 2\phi_b + \frac{Q_b}{C_{ox}} + V_{fb} \quad \text{———} \quad (I)$$

Now the expression for the threshold voltage may be modified to incorporate, V_{sb} (The difference between the source and the substrate)

$$\therefore V_t \text{ (with body effect)} = V_{fb} + 2\phi_b + \frac{Q_b + \sqrt{2\epsilon_{si} q N_A |V_{sb}|}}{C_{ox}}$$

$$\Rightarrow V_t = V_{fb} + 2\phi_b + \frac{\sqrt{2\epsilon_{si} q N_A (2\phi_b + |V_{sb}|)}}{C_{ox}} \quad \text{———} \quad (II)$$

$$\boxed{\because Q_b = \sqrt{2\epsilon_{si} q N_A 2\phi_b}}$$

- By putting $V_{sb} = 0$ in eqn (II) we get →

$$V_{t0} = V_{fb} + 2\phi_b + \frac{\sqrt{2\epsilon_{si} q N_A 2\phi_b}}{C_{ox}}$$

$$\Rightarrow V_{t0} = V_{fb} + 2\phi_b + \frac{Q_b}{C_{ox}}$$

[Same like equation (I) without body effect, $V_{sb} = 0$]

Now from eqn. (II) we get,

$$V_t = V_{fb} + 2\phi_b + \frac{\sqrt{2\epsilon_{si} q N_A (2\phi_b + |V_{sb}|)}}{C_{ox}}$$

$$\Rightarrow V_t = \underbrace{V_{fb} + 2\phi_b + \frac{Q_b}{C_{ox}}}_{V_{t0}} - \frac{Q_b}{C_{ox}} + \frac{\sqrt{2\epsilon_{si} q N_A (2\phi_b + |V_{sb}|)}}{C_{ox}}$$

$$\Rightarrow V_t = V_{t0} - \frac{\sqrt{2\epsilon_{si} q N_A 2\phi_b}}{C_{ox}} + \frac{\sqrt{2\epsilon_{si} q N_A (2\phi_b + |V_{sb}|)}}{C_{ox}}$$

$$= V_{t0} + \frac{\sqrt{2\epsilon_{si} q N_A}}{C_{ox}} \left[\sqrt{(2\phi_b + |V_{sb}|)} - \sqrt{2\phi_b}\right]$$

$$\Rightarrow V_t = V_{t0} + \gamma \left[\sqrt{(2\phi_b + |V_{sb}|)} - \sqrt{2\phi_b}\right]$$

Where γ is the constant that describes the substrate bias effect. It ranges from 0.4 ~ 1.2. Where,

$$\gamma = \frac{t_{ox}}{\epsilon_{ox}} \sqrt{2\epsilon_{si} q N_A} \quad \text{or} \quad \frac{\sqrt{2\epsilon_{si} q N_A}}{C_{ox}}$$

- V_t without body effect, $V_{sb} = 0$:

$$V_t = \phi_s + \frac{Q_b}{C_{ox}} + \phi_{ms}$$

- V_t with body effect, $V_{sb} \neq 0$:

$$V_t = V_{t0} + \gamma \left[\sqrt{2\phi_b + |V_{sb}|} - \sqrt{2\phi_b}\right]$$

- Math problem: Calculate V_T for a NMOS transistor given Si substrate, $N_A = 1.8 \times 10^{16}\ cm^{-3}$ at 300°K, $N_i = 1.45 \times 10^{10}\ cm^{-3}$. SiO_2 gate thickness 200Å. Assume $\phi_{ms} = -0.9$ volt, $Q_{fc} = 0$ V. Find V_t.

- Soln: We find, $Q_b = \frac{KT}{q} \ln\left(\frac{N_A}{N_i}\right) = 0.025 \ln\left(\frac{1.80 \times 10^{16}}{1.45 \times 10^{10}}\right)$

$$= 0.36 \text{ volt}.$$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.9 \times 8.854 \times 10^{-14}}{200 \times 10^{-8}} = 1.726 \times 10^{-7}\ F/cm^2$$

$$Q_b = \sqrt{2\epsilon_{si} q N_A 2\phi_b} = \sqrt{2 \times 1.06 \times 10^{-12} \times 1.602 \times 10^{-19} \times 1.80 \times 10^{16} \times 2 \times 0.36}$$

$$= 6.6344 \times 10^{-8}\ c/cm^2$$

$$\therefore V_T = (\phi_s \text{ or } 2\phi_b) + \frac{Q_b}{C_{ox}} + \phi_{ms}$$

$$= (2 \times 0.36) + \frac{6.6344 \times 10^{-8}}{1.726 \times 10^{-7}} + (-0.9) = 0.204 \text{ volt}$$

$\boxed{\text{Ans}}$

Remember: $K = 1.38 \times 10^{-23}$ J/K
$1 Å = 10^{-8}$ cm or 10^{-10} m
$\epsilon_{si} = 11.7\epsilon_0$ | $\epsilon_{ox} = 3.9\epsilon_0$

Math problem.

In a Cmos process NMos has nominal threshold voltage (while $V_{SB}=0$), $V_{t0} = 0.36$ volt and doping level, $N_A = 1.8 \times 10^{16}$ cm^{-3}, gate oxide thickness, $t_{ox} = 200$ Å, channel length modulation, $\lambda = 0.03$, $\epsilon_{si} = 1.04 \times 10^{-12}$ F/cm, $\epsilon_{ox} = 3.9 \times 8.854 \times 10^{-14}$ F/cm, $N_i = 1.45 \times 10^{10}$ cm^{-3} and $q = 1.6 \times 10^{-19}$ c.

(1) Calculate V_t for NMos when, $V_g = 4$ volt, $V_d = 3$ volt, $V_s = 2$ volt, $V_b = 0$ volt.

Soln: From the question, $V_{gs} = 2$ volt $(V_g - V_s)$

$$V_{ds} = (V_{gs} - V_{gd}) = (2-1) = 1V$$

$$V_{sb} = (V_s - V_b) = (2-0) = 2 \text{ volt}$$

To find out $V_t = V_{t0} + \gamma\left[\sqrt{2\phi_b + |V_{sb}|} - \sqrt{2\phi_b}\right]$

Since $\phi_s = 2\phi_b = 2\phi_f$

$$\therefore V_t = V_{t0} + \gamma\left[\sqrt{\phi_s + |V_{sb}|} - \sqrt{\phi_s}\right]$$

To find ϕ_s:

$$\phi_f = \phi_b = \frac{KT}{q}\ln\left(\frac{N_A}{N_i}\right) = 0.026 \times \ln\left(\frac{1.8\times10^{16}}{1.45\times10^{10}}\right)$$

(at room temperature $\frac{KT}{q} = 0.026$ volt)

$$\therefore \phi_f = \phi_b = 0.026 \times 14.031 = 0.364 \text{ volt.}$$

$$\therefore \phi_s = 2\phi_b = (0.364 \times 2) = 0.729 \text{ volt.}$$

① To find out : γ (Body effect coefficient)

We know that, $\gamma = \frac{t_{ox}}{\epsilon_{ox}} \sqrt{2q \epsilon_{si} N_A}$

$\therefore \gamma = \frac{200 \times 10^{-8}}{3.9 \times 8.854 \times 10^{-14}} \times \sqrt{(2 \times 1.602 \times 10^{-19} \times 1.04 \times 10^{-12} \times 1.8 \times 10^{16})}$

$= 0.4485 \simeq 0.5$

Now, $V_t = V_{t0} + \gamma\left(\sqrt{\phi_s + |V_{sb}|} - \sqrt{\phi_s}\right)$

$= 0.36 + 0.5\left(\sqrt{0.73 + 2} - \sqrt{0.73}\right)$

$= 0.758 = 0.76$ volt $\boxed{\text{Am}}$

(II) If the drain to source current, I_{ds} through NMOS for the above condition is $30\ \mu A$, then what will be the current in the NMOS if the body is connected to the source ($V_{sb} = 0$) ?

① Soln: For the above condition in math ① —

We find $V_{gs} - V_t = (2 - 0.76) = 1.24$ volt

$V_{ds} = 1$ volt

$\therefore V_{ds} < V_{gs} - V_t$ means the MOS is in linear region

From eq., $I_{ds} = \beta\left(V_{gs} - V_t - \frac{V_{ds}}{2}\right) \times V_{ds}$

$\Rightarrow 30 \times 10^{-6} = \beta\left(2 - 0.76 - \frac{1}{2}\right) \times 1$

$= 0.74\ \beta$

$\therefore \beta = 30 \times 10^{-6} / (0.74) = 4.05 \times 10^{-5}\ \frac{A}{V^2}$

Now, for math (II). $V_g = 4$ volt, $V_d = 3$ volt, $V_s = 2$ volt, $V_b = 2$ volt

$\therefore$ $V_{gs} = 2$ volt

$V_{ds} = (V_{gs} - V_{gd}) = (2-1) = 1$ volt

$V_{sb} = (2-2) = 0$ volt

if $V_{sb} = 0$ the we get

$$V_t = V_{to} + \gamma\left(\sqrt{\phi_s + |V_{sb}|} - \sqrt{\phi_s}\right)$$

$$= V_{to} + \gamma\left(\cancel{\sqrt{\phi_s + 0}} - \cancel{\sqrt{\phi_s}}\right)$$

$$V_t = V_{to}$$

$$\therefore V_t = 0.36$$

Now, $(V_{gs} - V_t) = (2 - 0.36) = 1.64$ volt

$V_{ds} = 1$ volt.

$\therefore$ $V_{ds} < (V_{gs} - V_t)$ therefore the mos is still in linear region.

$\therefore$ The new $I_{ds} = \beta\left(V_{gs} - V_t - \frac{V_{ds}}{2}\right) \times V_{ds}$

$= 4.05 \times 10^{-5}\left(2 - 0.36 - \frac{1}{2}\right) \times 1$

$= 46.17\ \mu A$ $\boxed{\text{Ans}}$

Note: The values given as $V_s = 0$, $V_g = 2$, $V_d = 3$, $V_b = 1.1$ for the same math as above we would find.

$V_{gs} = V_g - V_s = (2-0) = 2$ volt

$V_t = 0.76$ volt from the first calculation in no (I)

since $|V_{sb}|$ exist (not zero) $= |V_s - V_b| = |0 - 1.1| = 1.1$ volt

$V_{ds} = V_d - V_s = (3-0) = 3$ volt.

Now we see that V_{ds} (3 volt) $> (V_{gs} - V_t)$

Since $(V_{gs} - V_t) = (2 - 0{\cdot}76) = 1{\cdot}24$ volt

$\therefore$ $V_{ds} > (V_{gs} - V_t)$ concludes that I_{ds} is in saturation region. Since in this math → $\lambda = 0{\cdot}03$ is given, so in no. (11) math to find out the value for β we will consider channel length modulation equation for the saturation as

$$I_{ds} = \frac{\beta}{2}(V_{gs} - V_{ds})^2(1 + \lambda V_{ds})$$

but if λ is not given the we would simply consider the equation for saturation region as →

$$I_{ds} = \frac{\beta}{2}(V_{gs} - V_t)^2$$

Please do no. (11) math by considering channel length modulation by yourself