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Also accessible through http://www.github.com/soymarwin/ee274/EE274_ProgEx03; for history tracking.

A.1-2. The Bilateral Z-Transform

Sequence (a) $x(n) = (\frac{4}{3})^n u(1-n)$

Manual Solution

$x(n) = (\frac{4}{3})^n u(-n+1)$

$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$X(z) = \sum_{n=-\infty}^{\infty} (\frac{4}{3})^n u(-n+1) z^{-n}$

$Let\ k = -n+1\ and\ n = 1-k$

$X(z) = \sum_{n=-\infty}^{\infty} (\frac{4}{3})^{1-k} u(k) z^{k-1}$

$$X(z) = \sum_{n=0}^{\infty}(\tfrac{4}{3}) \cdot ((\tfrac{4}{3})^{-1})^k \cdot ((1/z)^{-1})^k \cdot z^{-1}$$

$$X(z) = (\tfrac{4z^{-1}}{3})\ \sum_{n=0}^{\infty}(\tfrac{3}{4z^{-1}})^k$$

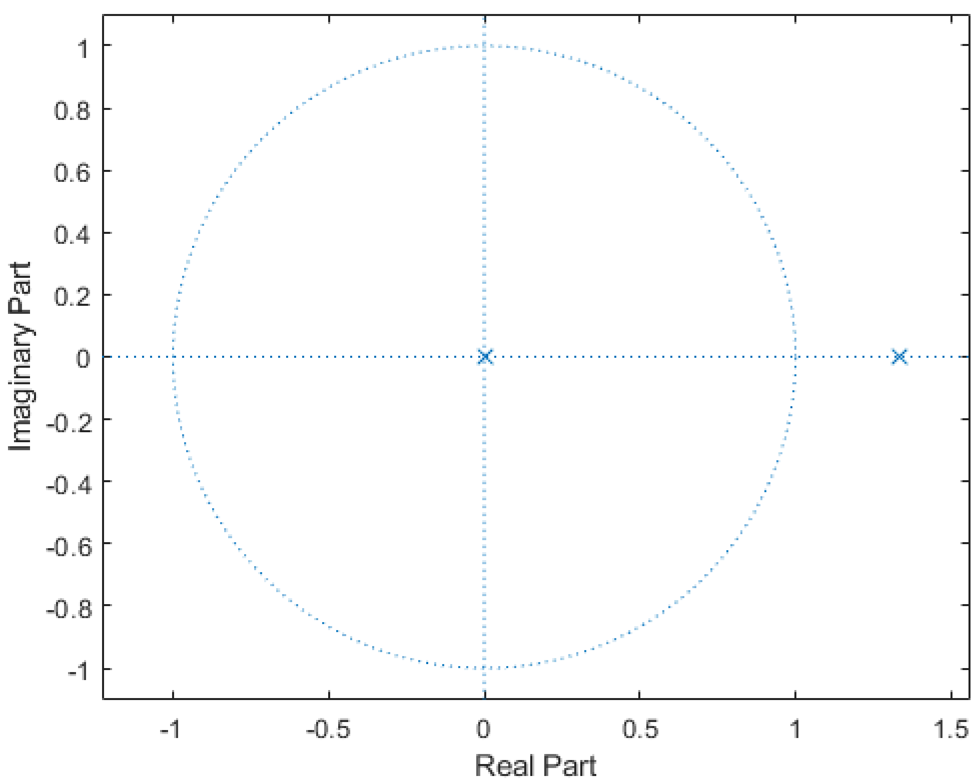
$$X(z) = (\tfrac{4z^{-1}}{3}) \cdot (\tfrac{1}{1-\frac{3}{4z^{-1}}}),\ 0\ <|\ z\ |<\ \tfrac{4}{3}$$

$$or\ X(z) = \tfrac{16z^{-2}}{-9+12z^{-1}},\ 0\ <|\ z\ |<\ \tfrac{4}{3}$$

$$or\ X(z) = \tfrac{-16z^{-2}}{9-12z^{-1}},\ 0\ <|\ z\ |<\ \tfrac{4}{3}$$

z-plane for 1.(a)

```
A1_a_a=[-9, 12, 0];
A1_a_b=[0, 0, -16];
zplane(A1_a_b,A1_a_a);
```



Verification of z-transform v. original sequence with first 8-coef.

```
[delta,n]= impseq(0,0,7);
A_a_Xz=filter(A1_a_b,A1_a_a,delta) %A_a_Xz is z-transform sequence
A_a_Xn=[(4/3).^n].*stepseq(1,0,7)
%A_a_Xn is the original sequence, see stepseq.m
```

```
A_a_Xz =

  Columns 1 through 7

         0         0    1.7778    2.3704    3.1605    4.2140    5.6187

  Column 8

    7.4915


A_a_Xn =

  Columns 1 through 7

         0         0    1.7778    2.3704    3.1605    4.2140    5.6187

  Column 8

    7.4915
```

Therefore, based on coef values generated from X(z) and x(n), the z-transform for sequence(a) is correct.

Sequence (b) $x(n) = 2^{-|n|} + (\frac{1}{3})^{|n|}$

$X(z) = \sum_{n=0}^{\infty} 2^{-n} z^{-n} + \sum_{n=0}^{\infty} (\frac{1}{3})^{n} z^{-n}$

$X(z) = \sum_{n=0}^{\infty} (\frac{z^{-1}}{2})^{n} + \sum_{n=0}^{\infty} (\frac{z^{-1}}{3})^{n}$

$X(z) = \frac{1}{1-\frac{z^{-1}}{2}} + \frac{1}{1-\frac{z^{-1}}{3}}$

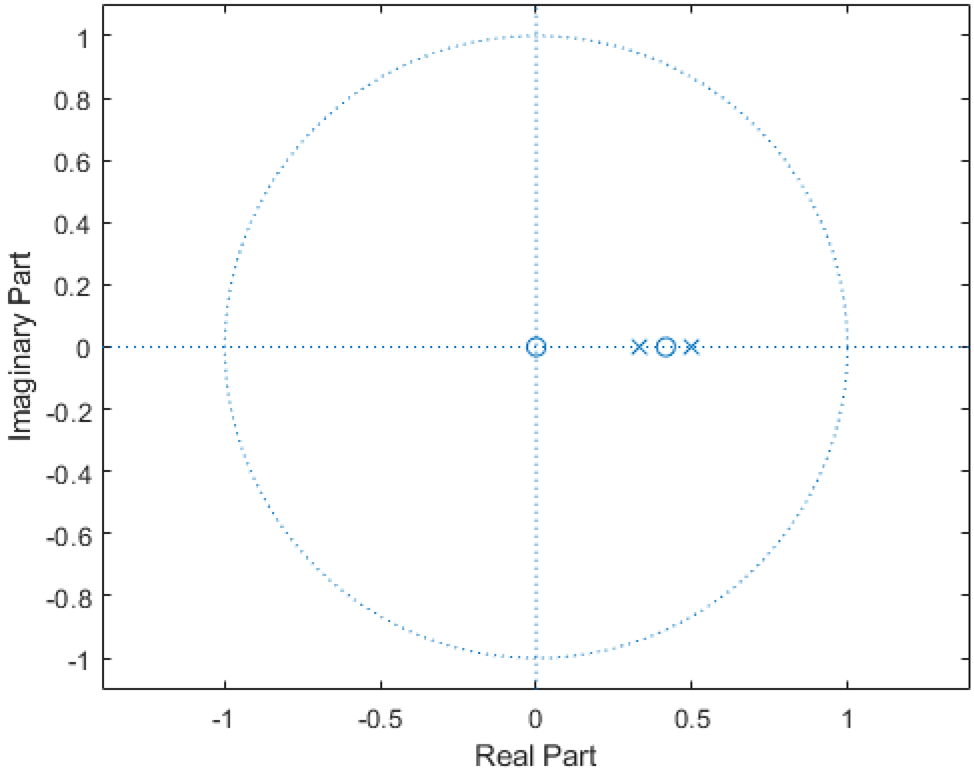
$X(z) = \frac{2}{2-z^{-1}} + \frac{3}{3-z^{-1}}$

$X(z) = \frac{12-5z^{-1}}{(2-z^{-1})(3-z^{-1})},\ |\ z\ | >\ \frac{1}{3} \cap\ |\ z\ | >\ \frac{1}{2}$

$or X(z) = \frac{12-5z^{-1}}{6-5z^{-1}+z^{-2}},\ |\ z\ | >\ \frac{1}{3} \cap\ |\ z\ | >\ \frac{1}{2}$

z-plane for 1.(b)

```
A1_b_a=[6 -5 1];
A1_b_b=[12 -5 0];
zplane(A1_b_b,A1_b_a);
```



Verification of z-transform v. original sequence with first 8-coef.

```
[delta,n]= impseq(0,0,7);
A_b_Xz=filter(A1_b_b,A1_b_a,delta) %A_b_Xz is z-transform sequence
A_b_Xn=((2).^(-abs(n)))+((1/3).^(abs(n))) %A_b_Xn is the original
 sequence
```

```
A_b_Xz =

  Columns 1 through 7

    2.0000    0.8333    0.3611    0.1620    0.0748    0.0354    0.0170

  Column 8

    0.0083


A_b_Xn =

  Columns 1 through 7

    2.0000    0.8333    0.3611    0.1620    0.0748    0.0354    0.0170

  Column 8
```

0.0083

Therefore, based on coef values generated from X(z) and x(n), the z-transform for sequence(b) is correct.

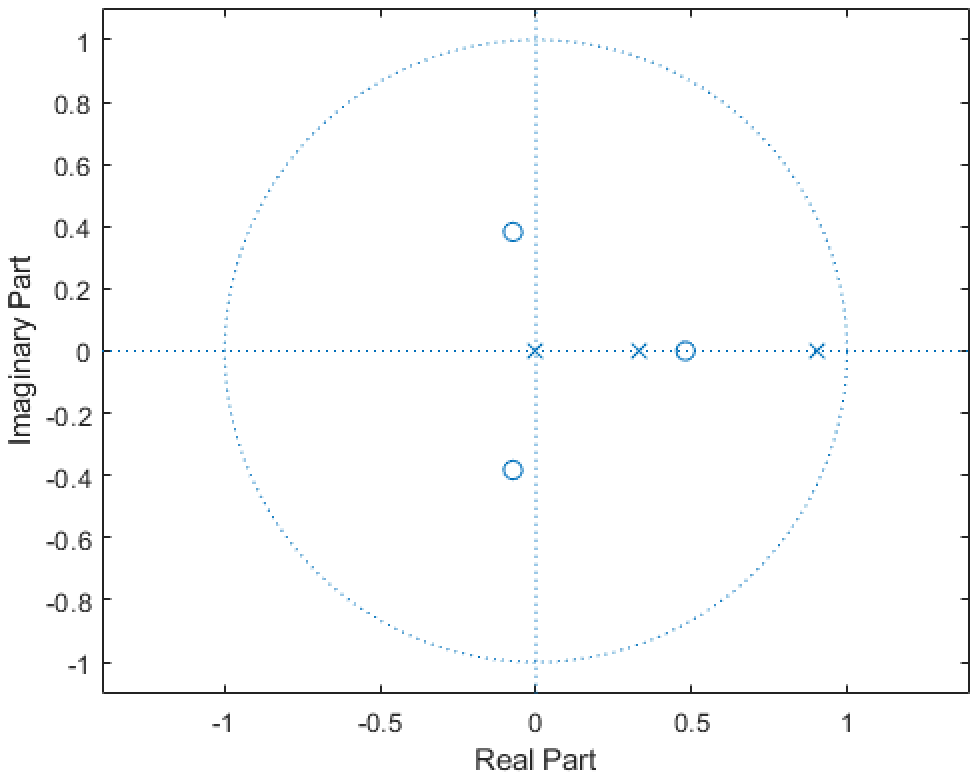
A.3. $x(n) = (\frac{1}{3})^n u(n-2) + (0.9)^{n-3} u(n)$

$$X(z) = \frac{3z^{-2}}{27-9z^{-1}} + \frac{1.3717}{1-0.9z^{-1}}$$

$$X(z) = \frac{37.0359-12.3453z^{-1}+3z^{-2}-2.7z^{-3}}{27-33.3z^{-1}+8.1z^{-2}} \quad |z| > \frac{1}{3} \cap |z| > 0.9$$

z-plane for A.3

```
A3_b=[37.0359, -12.3453, 3, -2.7];
A3_a=[27, -33.3, 8.1];
zplane(A3_b,A3_a);
```



Verification of z-transform v. original sequence with first 20-coef.

```
[delta,n]= impseq(0,0,19);
A3_Xz=filter(A3_b,A3_a,delta) %A3_Xz is z-transform sequence
A3_Xn=(((1/3).^n).*(stepseq0(2,0,19))+(((0.9).^(n-3)).*(stepseq0(0,0,19))))
%A3_Xn is the original sequence, see stepseq0.m
```

```
A3_Xz =

  Columns 1 through 7

    1.3717    1.2345    1.2222    1.0370    0.9123    0.8141    0.7303

  Columns 8 through 14

    0.6565    0.5906    0.5315    0.4783    0.4305    0.3874    0.3487

  Columns 15 through 20

    0.3138    0.2824    0.2542    0.2288    0.2059    0.1853


A3_Xn =

  Columns 1 through 7

    1.3717    1.2346    1.2222    1.0370    0.9123    0.8141    0.7304

  Columns 8 through 14

    0.6566    0.5906    0.5315    0.4783    0.4305    0.3874    0.3487

  Columns 15 through 20

    0.3138    0.2824    0.2542    0.2288    0.2059    0.1853
```

Therefore, based on coef values generated from X(z) and x(n), the z-transform for sequence in (A.3.) is correct.

B.4. Inverse Z-Transform

Sequence(c) $X(z) = \frac{1-z^{-1}-4z^{-2}+4z^{-3}}{1-\frac{11}{4}z^{-1}+\frac{13}{8}z^{-2}-\frac{1}{4}z^{-3}}$

```
B4_b=[1, -1, -4, 4];
B4_a=[1, (-11/4), (13/8), (-1/4)];
[B4_R, B4_p, B4_C]=residuez(B4_b,B4_a);
```

$$X(z) = \frac{0z}{z-2} - \frac{10z}{z-0.5} + \frac{27z}{z-0.25} - 16$$

$$X(n) = u(-n) - (2^{-2n}(5 \times 2^{n+1} - 27)(1 - u(-n)))$$

Verification of z-transform v. ans sequence with first 8-coef.

Disclaimer: First element is a garbage value. Thus, array(2:9)

```
[delta,n]= impseq(0,0,8);
```

```
B4_Xz=filter(B4_b,B4_a,delta); %B4_Xz is z-transform sequence
%B4_Xn is inv. ztrans sequence
B4_Xn=-heaviside(-n)-((2.^(-2*n)).*(5.*(2.^(n+1))-27).*(1-heaviside(-
n)));
B4_Xz(2:8)% First 8 coef of B4_Xz - Z-transf
B4_Xn(2:8)% First 8 coef of B4_Xn - Inv. Z-transf
```

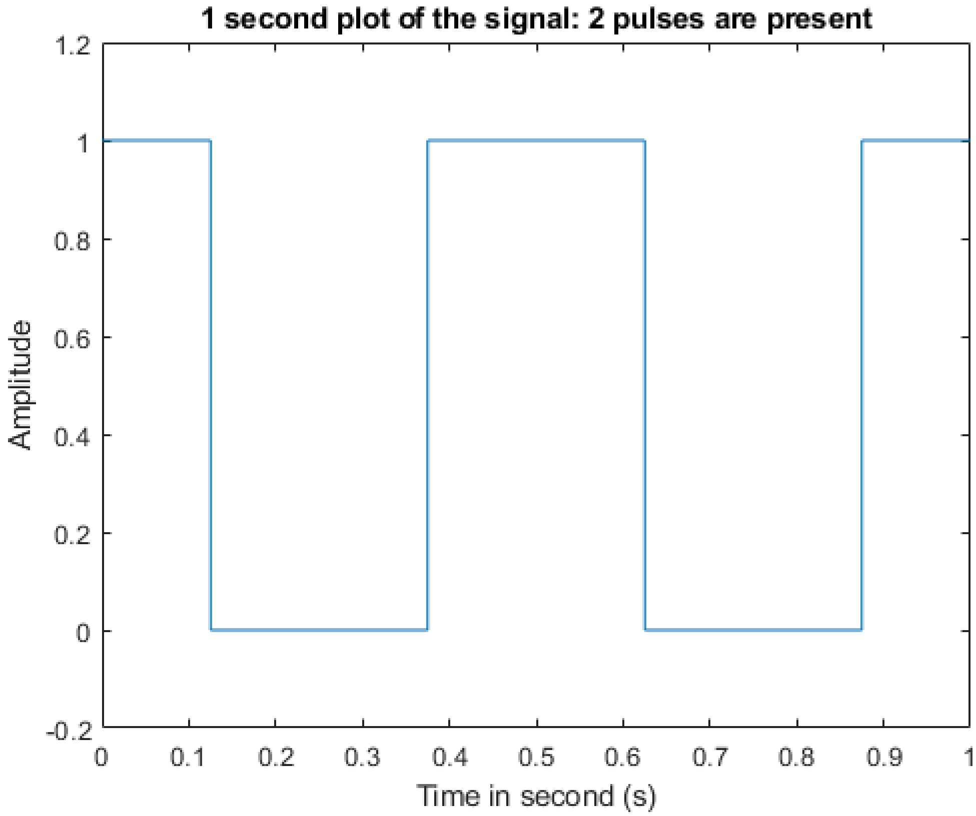
```
ans =

    1.7500   -0.8125   -0.8281   -0.5195   -0.2861   -0.1497   -0.0765


ans =

    1.7500   -0.8125   -0.8281   -0.5195   -0.2861   -0.1497   -0.0765
```

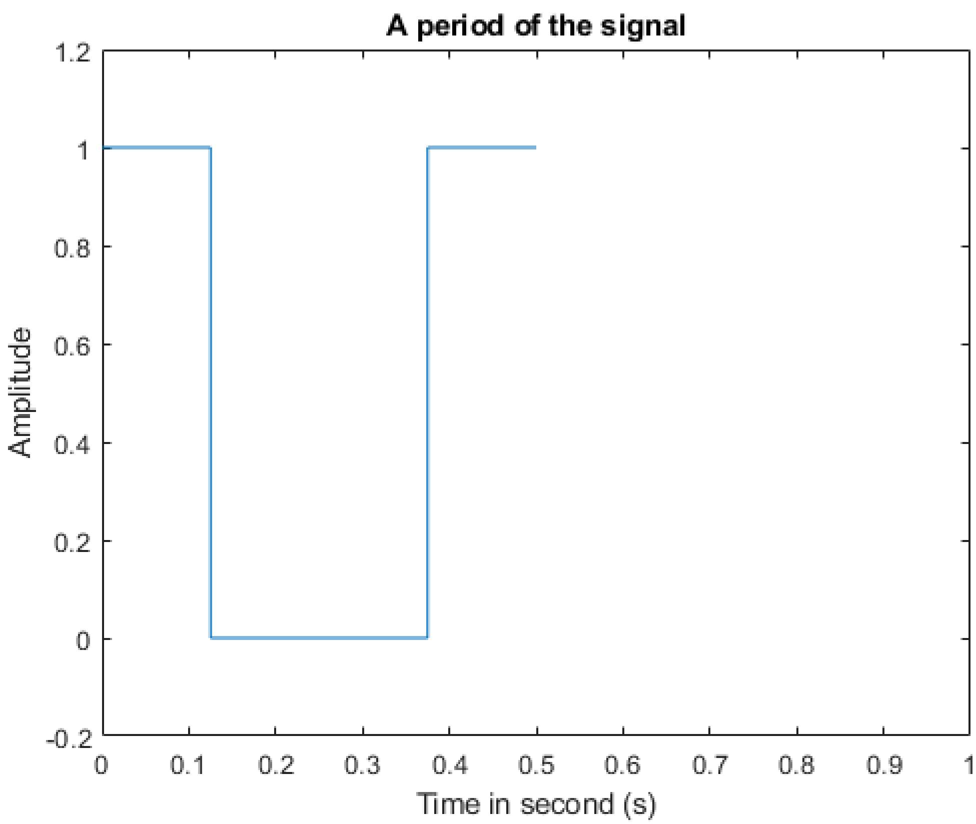
C.5. Signal Generation

Generate the periodic even symmetric square pulse x(n) from [0, 1]. Thepriod pulse is 1 second and a pulse with of 250ms with sampling freq. of 8KHz. Plot one period of x(n) and verify if you have the correct waveform.

```
C5_t=0:1/8e3:1; % time | x-axis
% C5_x is our x(n)
C5_x=square(cos(4*pi*C5_t)); % since 1pw = 250ms; 1period = 2pw; 4pw
 in 1s.
C5_x=(abs(C5_x)+C5_x)/2; % eliminating all -1 with 0.
figure();
plot(C5_t, C5_x);
axis([0 1 -0.2 1.2]); title("1 second plot of the signal: 2 pulses are
 present"); % 2 periods w/ 250ms pw each 1,0.
xlabel("Time in second (s)");ylabel("Amplitude");
C5_samp_prd = (length(C5_x))/2;
figure();
plot(C5_t(1:C5_samp_prd), C5_x(1:C5_samp_prd));
axis([0 1 -0.2 1.2]); title("A period of the signal");
xlabel("Time in second (s)");ylabel("Amplitude");
```

```
Warning: Integer operands are required for colon operator when used as
 index.
Warning: Integer operands are required for colon operator when used as
 index.
```


1 second plot of the signal: 2 pulses are present
Amplitude
Time in second (s)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


A period of the signal
Amplitude
Time in second (s)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

There are two periods in 1s of specified pulse conf.

5.a. How many samples in one period?

```
sample_periodic=(length(C5_x)-1)/2 % samples in one period
sample_1second=length(C5_x)-1 % samples in 1s
```

```
sample_periodic =

        4000


sample_1second =

        8000
```

There are 4000 samples in one period while 8000 samples for the entire 1s.

5.b. How many samples with a value of 1?

-1 was added to negate matlab's indexing rule that starts with 1.

```
value1_periodic=(sum(C5_x(:)==1)-1)/2 % in one period
value1_1second=(sum(C5_x(:)==1)-1) % in 1s
```

```
value1_periodic =

        2000


value1_1second =

        4000
```

There are 2000 samples with a value of 1 in one period and 4000 samples in 1s.

5.c. How many zeros?

```
value0_periodic=sum(C5_x(:)==0)/2 % in one period
value0_1second=sum(C5_x(:)==0) % in 1s
```

```
value0_periodic =

        2000
```

```
value0_1second =

        4000
```

There are 2000 samples with a value of 0 in one period and 4000 samples in 1s.

C.6.Fourier Series Analysis Equation

Using the analysis equation of the Fourier series, write a program that will compute the Fourier series coefficients of the periodic square pulse signal. Plot the magnitude and phase of the first 10 Fourier coef.

```
% plot of 100000 harmonic
C6_t = (0:1/8000:1);
C6_NHarmonics=100000; C6_Ncycles=2; C6_Nsamples=8000;
C6_y(1:C6_Nsamples)=0.5;C6_j=1:C6_Nsamples;
for C6_k=1:C6_NHarmonics
 C6_x(C6_j)=(2*sin(0.5*pi*C6_k)/
(pi*C6_k))*cos(C6_k*2*pi*C6_Ncycles*C6_j/C6_Nsamples);
 C6_y=C6_y+C6_x;
end
figure(), plot(C6_t(1:8000),C6_y);axis([0 1 -0.2 1.2]);
title("Illustration of the Fourier synthesis with 100000 harmonics");
xlabel("Time in second (s)"); ylabel("Amplitude");
% plot of 10 harmonic
C6_t = (0:1/8000:1);
C6_NHarmonics=10; C6_Ncycles=2; C6_Nsamples=8000;
C6_y(1:C6_Nsamples)=0.5;C6_j=1:C6_Nsamples;
for C6_k=1:C6_NHarmonics
 C6_x(C6_j)=(2*sin(0.5*pi*C6_k)/
(pi*C6_k))*cos(C6_k*2*pi*C6_Ncycles*C6_j/C6_Nsamples);
 C6_y=C6_y+C6_x;
end
figure(), plot(C6_t(1:8000),C6_y);axis([0 1 -0.2 1.2]);
title("Illustration of the Fourier synthesis with 10 harmonics");
xlabel("Time in second (s)"); ylabel("Amplitude");

% compute the magnitude and phase of 10 harmonics signal
C6y_fft = fft(C6_y);
C6_mag = abs(C6y_fft);
C6_ang = angle(C6y_fft);

figure(), stem(C6_mag(1:10)); title("Magnitude Spectrum of first 10
 Coef"); xlabel("coef"); ylabel("Magnitude Spectrum");
figure(), stem(C6_ang(1:10)); title("Phase Spectrum of first 10
 Coef"); xlabel("coef"); ylabel("Phase Spectrum");
```

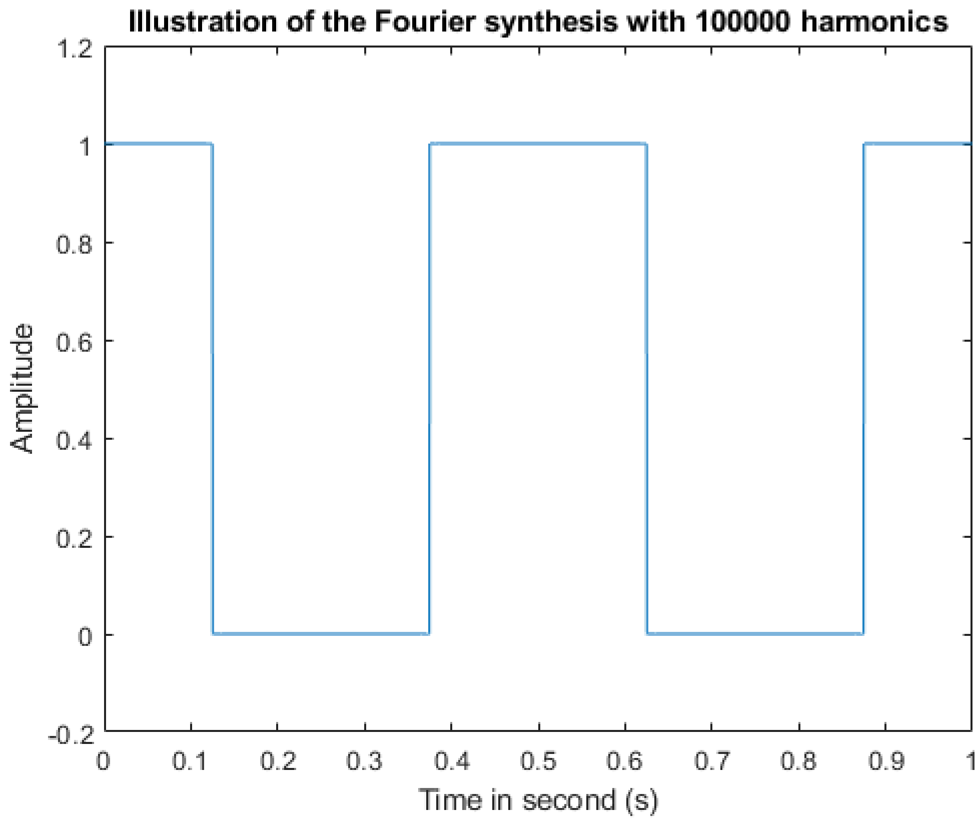


Illustration of the Fourier synthesis with 100000 harmonics
Amplitude
Time in second (s)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

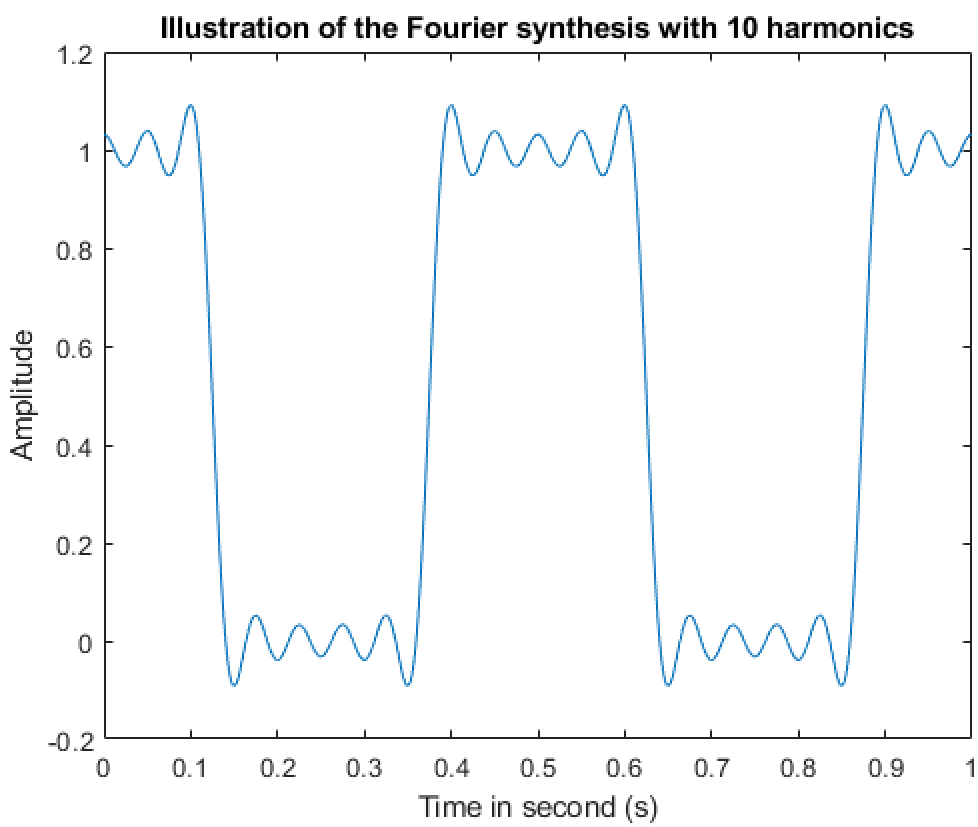
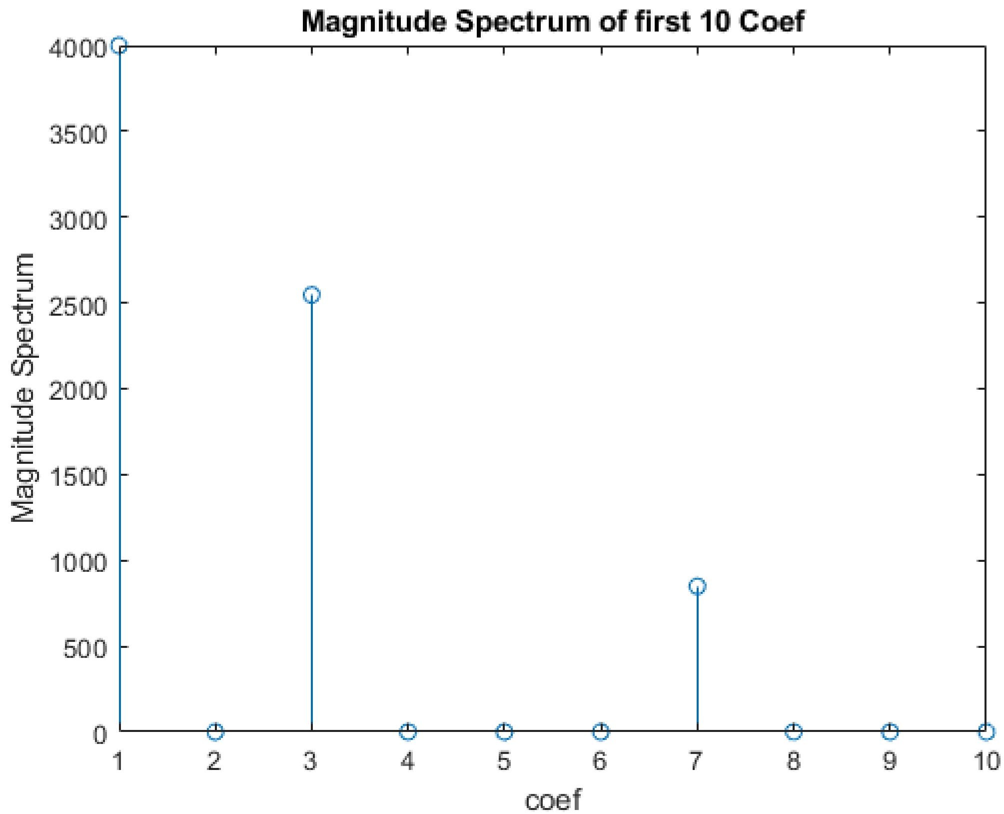
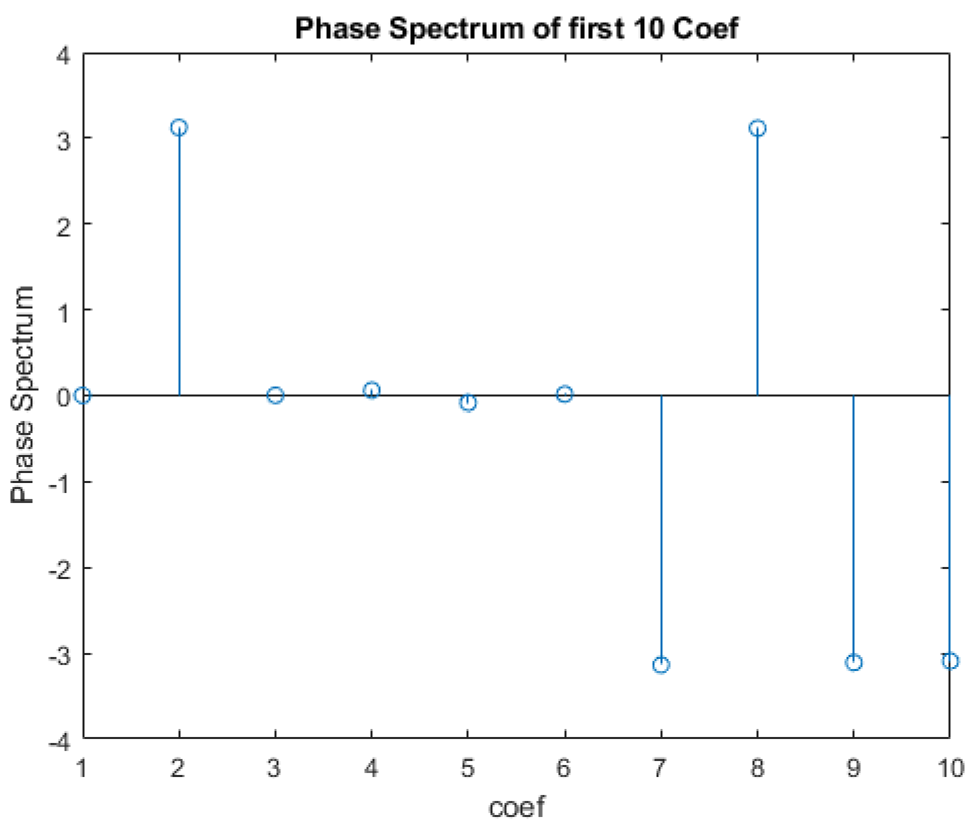


Illustration of the Fourier synthesis with 10 harmonics
Amplitude
Time in second (s)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Magnitude Spectrum of first 10 Coef
Magnitude Spectrum
coef
4000
3500
3000
2500
2000
1500
1000
500
0
1
2
3
4
5
6
7
8
9
10


Phase Spectrum of first 10 Coef
Phase Spectrum
coef
4
3
2
1
0
-1
-2
-3
-4
1
2
3
4
5
6
7
8
9
10

6.a. What is the fundamental frequency of the square pulse?

The fundamental frequency is defined by f=1/0.5s as such one complete period is 0.5s or 2Hz and having 250ms of on and off.

6.b. Enumerate the Magnitude and Phase of first 10 coef.

```
disp("Magnitude");
C6_mag(1:10)

disp("Phase");
C6_ang(1:10)
```

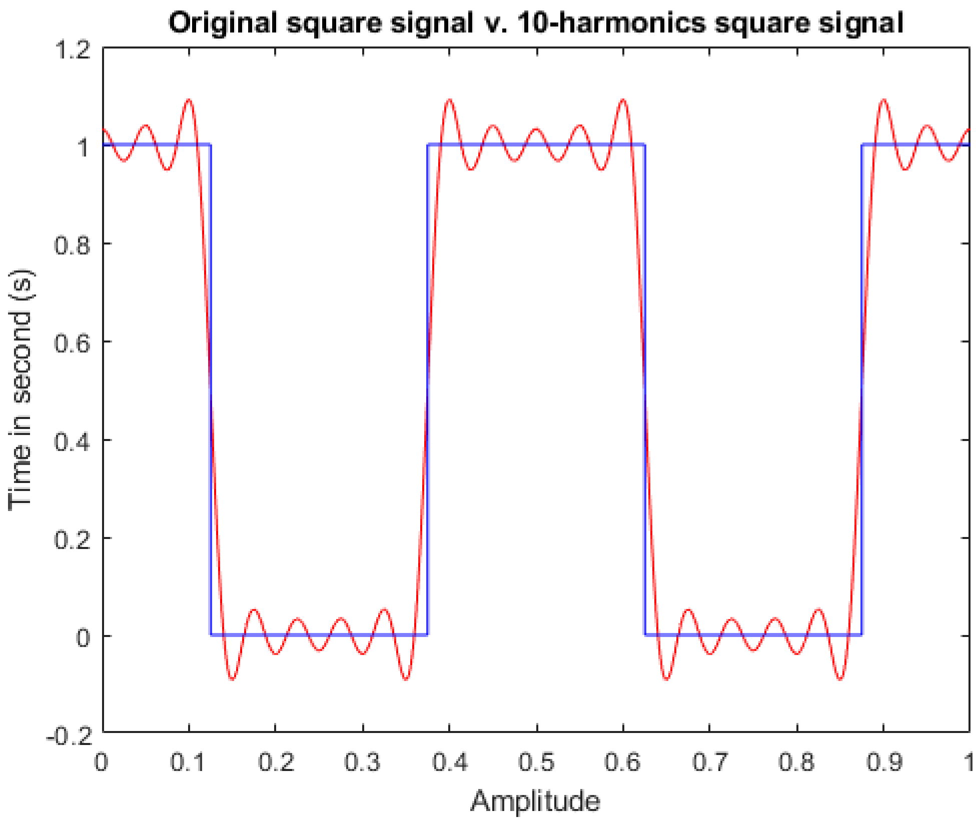
```
Magnitude

ans =

   1.0e+03 *

  Columns 1 through 7

    4.0000    0.0000    2.5465    0.0000    0.0000    0.0000    0.8488

  Columns 8 through 10

    0.0000    0.0000    0.0000

Phase

ans =

  Columns 1 through 7

         0    3.1250    0.0016    0.0603   -0.0834    0.0156   -3.1369

  Columns 8 through 10

    3.1156   -3.1093   -3.0906
```

C.7. Fourier Series Synthesis Equation

Using the synthesis equatin for the Foyrier series, synthesiez the original square pulse using the first 10 Fourier coefficients. Generate a plot of the original square pulse and the synthesized square pulse.

```
figure(), plot(C6_t(1:8000),C6_y, 'color', 'r');
hold on; plot(C6_t(1:8000),C5_x(1:8000), 'color', 'b');
```

```
hold off; title("Original square signal v. 10-harmonics square
 signal");
xlabel("Amplitude"); ylabel("Time in second (s)");
```



7.a. What is the average MSE of original square pulse vs synthesized pulse?

MSE is 1.0100%

```
C7_mse_10harm = immse(C5_x(1:8000),C6_y)

C7_mse_10harm =

    0.0101
```

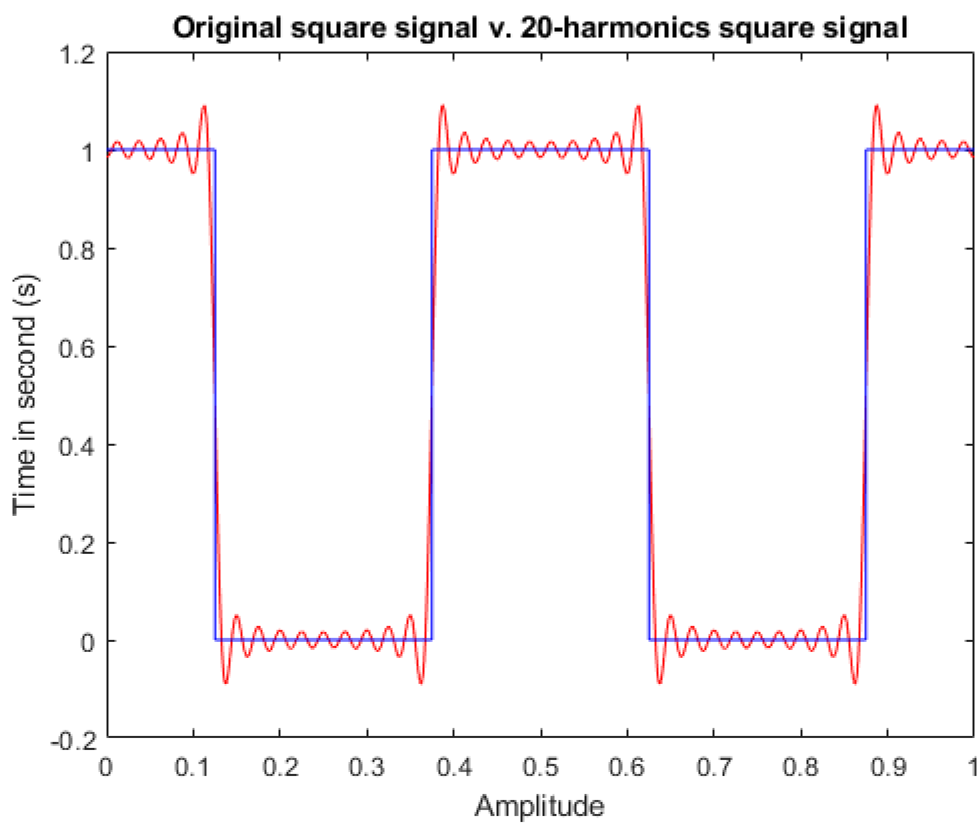
7.b. If you use 20 Fourier coef, what will be the MSE?

MSE will be 0.51%

```
C6_t = (0:1/8000:1);
```

```
C6_NHarmonics=20; C6_Ncycles=2; C6_Nsamples=8000;
C6_y(1:C6_Nsamples)=0.5;C6_j=1:C6_Nsamples;
for C6_k=1:C6_NHarmonics
 C6_x(C6_j)=(2*sin(0.5*pi*C6_k)/
(pi*C6_k))*cos(C6_k*2*pi*C6_Ncycles*C6_j/C6_Nsamples);
 C6_y=C6_y+C6_x;
end

C7_mse_20harm = immse(C5_x(1:8000),C6_y)

figure(), plot(C6_t(1:8000),C6_y, 'color', 'r');
hold on; plot(C6_t(1:8000),C5_x(1:8000), 'color', 'b');
hold off; title("Original square signal v. 20-harmonics square
 signal");
xlabel("Amplitude"); ylabel("Time in second (s)");
```

```
C7_mse_20harm =

    0.0051
```



7.c. What is the effect on the fundamental freq if I increase the pulse width to 300ms? Explain.

Fundamental frequency will be shortened to 1.6667Hz from 2.000Hz. One period is composed of two pulse width in this activity hence, changing the pulse width from 250ms to 300ms yields to an increased period/time. Moreover, the original 4 pulses seen in 1 second tends to expand making a portion of the original cycle invisble under the 1sec timeframe.

7.d. What is the effect on the Fourier coef if I change the pulse width?

Depending on the rate of change of values of pulse width but when it is altered, it inversely alter the value of Fourier coef.

7.e. What is the effect on the Fourier coef if I change the period?

Depending on the rate of change of values of period, being it as twice as the pulse width in a periodic signal, but when it is altered, it inversely alter the value of Fourier coef.

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