# Marwin B. Alejo 2020-20221 EE274\_ProgEx03

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Also accessible through <a href="http://www.github.com/soymarwin/ee274/EE274">http://www.github.com/soymarwin/ee274/EE274</a> <a href="ProgEx03">ProgEx03</a>.

# A. The Bilateral Z-Transform

(a) 
$$x(n) = (\frac{4}{3})^n u(1-n)$$

#### **Manual Solution**

$$x(n) = (\frac{4}{3})^n u(-n+1)$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

$$X(z) = \sum_{n=-\infty}^{\infty} (\frac{4}{3})^n u(-n+1)z^{-n}$$

Let 
$$k = -n + 1$$
 and  $n = 1 - k$ 

$$X(z) = \sum_{n=-\infty}^{\infty} (\frac{4}{3})^{1-k} u(k) z^{k-1}$$

$$X(z) = \sum_{n=0}^{\infty} (\frac{4}{3}) \cdot ((\frac{4}{3})^{-1})^k \cdot ((1/z)^{-1})^k \cdot z^{-1}$$

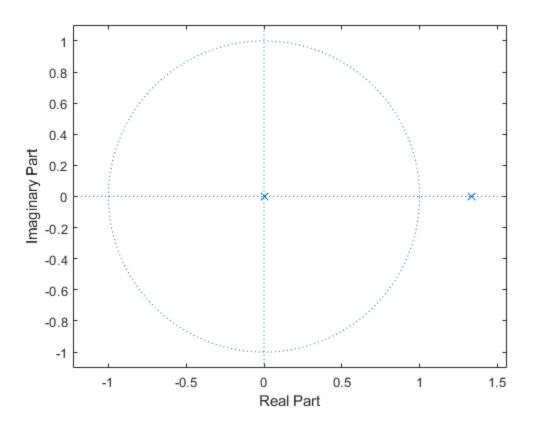
$$X(z) = (\frac{4z^{-1}}{3}) \sum_{n=0}^{\infty} (\frac{3}{4z^{-1}})^k$$

$$X(z) = (\frac{4z^{-1}}{3}) \cdot (\frac{1}{1 - \frac{3}{4z^{-1}}}), \ 0 \ < \mid z \mid \ < \ \frac{4}{3}$$

or 
$$X(z) = \frac{16z^{-2}}{-9+12z^{-1}}, \ 0 \ < \mid z \mid < \ \frac{4}{3}$$

or 
$$X(z) = \frac{-16z^{-2}}{9-12z^{-1}}, \ 0 \ < \mid z \mid < \ \frac{4}{3}$$

#### z-plane for 1.(a)



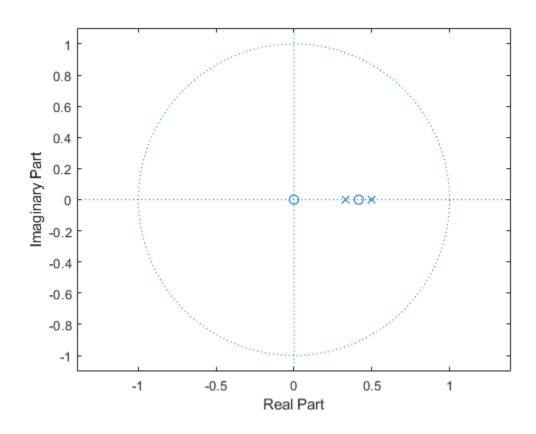
#### Verification of z-transform v. original sequence with first 8-coef.

[delta,n] = impseq(0,0,7);A\_a\_Xz=filter(A1\_a\_b,A1\_a\_a,delta) %A\_a\_Xz is z-transform sequence  $A_a_{n=[(4/3).^n].*stepseq(1,0,7) %A_A_Xn is the original sequence$  $A_a_Xz =$ Columns 1 through 7 0 1.7778 2.3704 3.1605 4.2140 5.6187 Column 8 7.4915  $A_a_Xn =$ Columns 1 through 7 0 1.7778 2.3704 3.1605 4.2140 5.6187 Column 8

7.4915

Therefore, based on coef values generated from X(z) and x(n), the z-transform for sequence(a) is correct.

$$\begin{split} (b) \ \ x(n) &= 2^{-|n|} + (\frac{1}{3})^{|n|} \\ X(z) &= \sum_{n=0}^{\infty} 2^{-n} z^{-n} + \sum_{n=0}^{\infty} (\frac{1}{3})^n z^{-n} \\ X(z) &= \sum_{n=0}^{\infty} (\frac{z^{-1}}{2})^n + \sum_{n=0}^{\infty} (\frac{z^{-1}}{3})^n \\ X(z) &= \frac{1}{1 - \frac{z^{-1}}{2}} + \frac{1}{1 - \frac{z^{-1}}{3}} \\ X(z) &= \frac{2}{2 - z^{-1}} + \frac{3}{3 - z^{-1}} \\ X(z) &= \frac{12 - 5z^{-1}}{(2 - z^{-1})(3 - z^{-1})}, \ \frac{1}{3} < |z| < \frac{1}{2} \\ or X(z) &= \frac{12 - 5z^{-1}}{6 - 5z^{-1} + z^{-2}}, \ \frac{1}{3} < |z| < \frac{1}{2} \\ \% \ \ z - \text{plane for 1. (b)} \\ \text{Al\_b\_a = [6 -5 1];} \\ \text{Al\_b\_b = [12 -5 0];} \\ \text{zplane (Al\_b\_b, Al\_b\_a);} \end{split}$$



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