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Also accessible through http://www.github.com/soymarwin/ee274/EE274_ProgEx03; for history tracking.

A.1-2. The Bilateral Z-Transform

Sequence (a) $x(n) = (\frac{4}{3})^n u(1-n)$

Manual Solution

$x(n) = (\frac{4}{3})^n u(-n+1)$

$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$X(z) = \sum_{n=-\infty}^{\infty} (\frac{4}{3})^n u(-n+1) z^{-n}$

$Let\ k = -n+1\ and\ n = 1-k$

$X(z) = \sum_{n=-\infty}^{\infty} (\frac{4}{3})^{1-k} u(k) z^{k-1}$

$X(z) = \sum_{n=0}^{\infty} (\frac{4}{3}) \cdot ((\frac{4}{3})^{-1})^k \cdot ((1/z)^{-1})^k \cdot z^{-1}$

$X(z) = (\frac{4z^{-1}}{3})\ \sum_{n=0}^{\infty} (\frac{3}{4z^{-1}})^k$

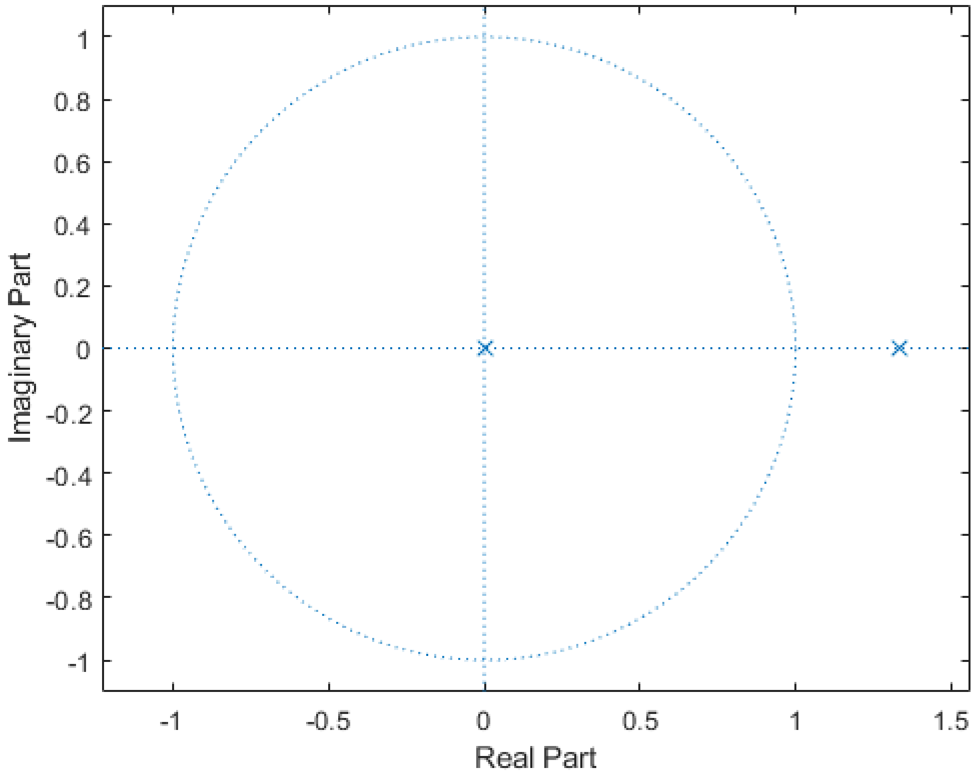
$X(z) = (\frac{4z^{-1}}{3}) \cdot (\frac{1}{1-\frac{3}{4z^{-1}}}),\ 0\ < |\ z\ | <\ \frac{4}{3}$

$or\ X(z) = \frac{16z^{-2}}{-9+12z^{-1}},\ 0\ < |\ z\ | <\ \frac{4}{3}$

$or\ X(z) = \frac{-16z^{-2}}{9-12z^{-1}},\ 0\ < |\ z\ | <\ \frac{4}{3}$

z-plane for 1.(a)

```
A1_a_a=[-9, 12, 0];
A1_a_b=[0, 0, -16];
zplane(A1_a_b,A1_a_a);
```



Verification of z-transform v. original sequence with first 8-coef.

```
[delta,n]= impseq(0,0,7);
A_a_Xz=filter(A1_a_b,A1_a_a,delta) %A_a_Xz is z-transform sequence
A_a_Xn=[(4/3).^n].*stepseq(1,0,7)
%A_a_Xn is the original sequence, see stepseq.m
```

```
A_a_Xz =

  Columns 1 through 7

         0         0    1.7778    2.3704    3.1605    4.2140    5.6187

  Column 8

    7.4915


A_a_Xn =
```

```
Columns 1 through 7

     0         0    1.7778    2.3704    3.1605    4.2140    5.6187

Column 8

  7.4915
```

Therefore, based on coef values generated from X(z) and x(n), the z-transform for sequence(a) is correct.

Sequence (b) $x(n) = 2^{-|n|} + (\frac{1}{3})^{|n|}$

$X(z) = \sum_{n=0}^{\infty} 2^{-n} z^{-n} + \sum_{n=0}^{\infty} (\frac{1}{3})^n z^{-n}$

$X(z) = \sum_{n=0}^{\infty} (\frac{z^{-1}}{2})^n + \sum_{n=0}^{\infty} (\frac{z^{-1}}{3})^n$

$X(z) = \frac{1}{1-\frac{z^{-1}}{2}} + \frac{1}{1-\frac{z^{-1}}{3}}$

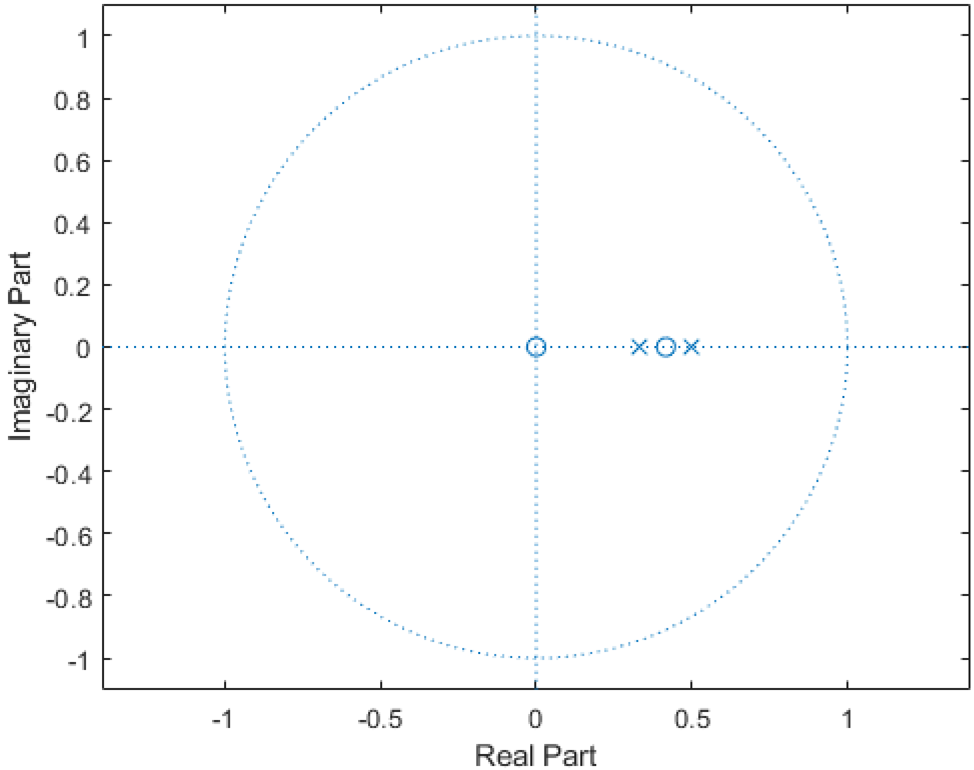
$X(z) = \frac{2}{2-z^{-1}} + \frac{3}{3-z^{-1}}$

$X(z) = \frac{12-5z^{-1}}{(2-z^{-1})(3-z^{-1})},\ |\,z\,| > \ \frac{1}{3} \cap \ |\,z\,| > \ \frac{1}{2}$

$or X(z) = \frac{12-5z^{-1}}{6-5z^{-1}+z^{-2}},\ |\,z\,| > \ \frac{1}{3} \cap \ |\,z\,| > \ \frac{1}{2}$

z-plane for 1.(b)

```
A1_b_a=[6 -5 1];
A1_b_b=[12 -5 0];
zplane(A1_b_b,A1_b_a);
```



Verification of z-transform v. original sequence with first 8-coef.

```
[delta,n]= impseq(0,0,7);
A_b_Xz=filter(A1_b_b,A1_b_a,delta) %A_b_Xz is z-transform sequence
A_b_Xn=((2).^(-abs(n)))+((1/3).^(abs(n))) %A_b_Xn is the original
 sequence
```

```
A_b_Xz =

  Columns 1 through 7

    2.0000    0.8333    0.3611    0.1620    0.0748    0.0354    0.0170

  Column 8

    0.0083


A_b_Xn =

  Columns 1 through 7

    2.0000    0.8333    0.3611    0.1620    0.0748    0.0354    0.0170

  Column 8
```

0.0083

Therefore, based on coef values generated from X(z) and x(n), the z-transform for sequence(b) is correct.

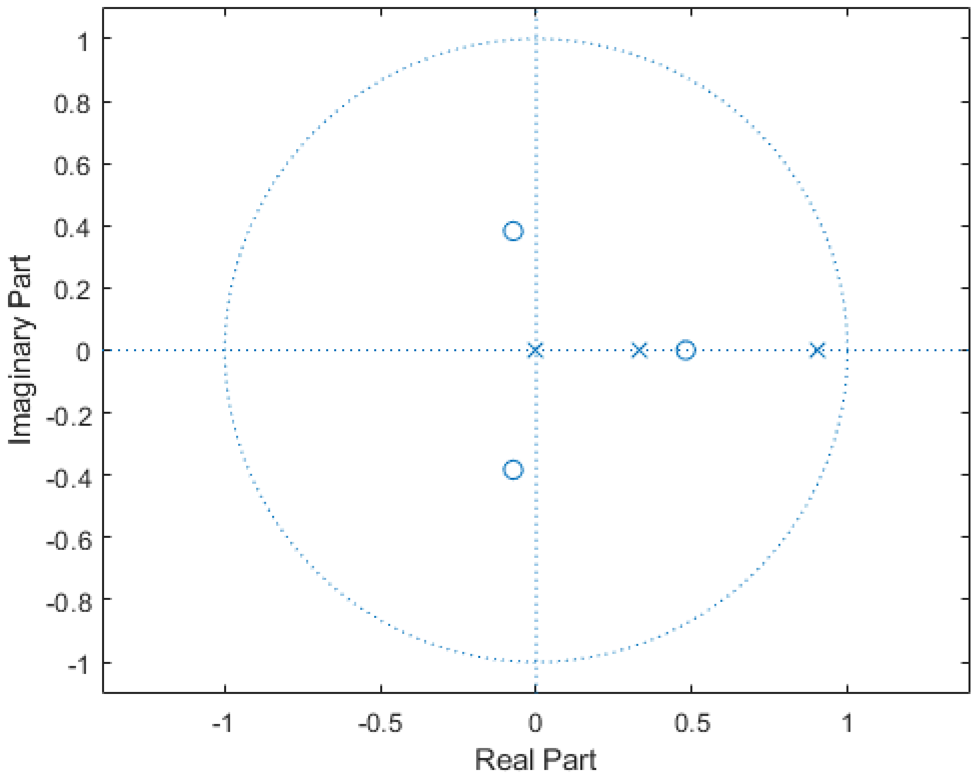
A.3. $x(n) = (\frac{1}{3})^n u(n-2) + (0.9)^{n-3} u(n)$

$$X(z) = \frac{3z^{-2}}{27-9z^{-1}} + \frac{1.3717}{1-0.9z^{-1}}$$

$$X(z) = \frac{37.0359-12.3453z^{-1}+3z^{-2}-2.7z^{-3}}{27-33.3z^{-1}+8.1z^{-2}} \quad |z| > \frac{1}{3} \cap |z| > 0.9$$

z-plane for A.3

```
A3_b=[37.0359, -12.3453, 3, -2.7];
A3_a=[27, -33.3, 8.1];
zplane(A3_b,A3_a);
```



Verification of z-transform v. original sequence with first 20-coef.

```
[delta,n]= impseq(0,0,19);
A3_Xz=filter(A3_b,A3_a,delta) %A3_Xz is z-transform sequence
A3_Xn=(((1/3).^n).*(stepseq0(2,0,19))+(((0.9).^(n-3)).*(stepseq0(0,0,19))))
%A3_Xn is the original sequence, see stepseq0.m
```

```
A3_Xz =

  Columns 1 through 7

    1.3717    1.2345    1.2222    1.0370    0.9123    0.8141    0.7303

  Columns 8 through 14

    0.6565    0.5906    0.5315    0.4783    0.4305    0.3874    0.3487

  Columns 15 through 20

    0.3138    0.2824    0.2542    0.2288    0.2059    0.1853


A3_Xn =

  Columns 1 through 7

    1.3717    1.2346    1.2222    1.0370    0.9123    0.8141    0.7304

  Columns 8 through 14

    0.6566    0.5906    0.5315    0.4783    0.4305    0.3874    0.3487

  Columns 15 through 20

    0.3138    0.2824    0.2542    0.2288    0.2059    0.1853
```

Therefore, based on coef values generated from X(z) and x(n), the z-transform for sequence in (A.3.) is correct.

B.4. $X(z) = \frac{1-z^{-1}-4z^{-2}+4z^{-3}}{1-\frac{11}{4}z^{-1}+\frac{13}{8}z^{-2}-\frac{1}{4}z^{-3}}$

```
B4_b=[1, -1, -4, 4];
B4_a=[1, (-11/4), (13/8), (-1/4)];
[B4_R, B4_p, B4_C]=residuez(B4_b,B4_a);
```

$$X(z) = \frac{0z}{z-2} - \frac{10z}{z-0.5} + \frac{27z}{z-0.25} - 16$$

$$X(n) = u(-n) - (2^{-2n}(5 \times 2^{n+1} - 27)(1 - u(-n)))$$

Verification of z-transform v. ans sequence with first 8-coef.

Disclaimer: First element is a garbage value. Thus, array(2:9)

```
[delta,n]= impseq(0,0,8);
B4_Xz=filter(B4_b,B4_a,delta); %B4_Xz is z-transform sequence
%B4_Xn is inv. ztrans sequence
B4_Xn=-heaviside(-n)-((2.^(-2*n)).*(5.*(2.^(n+1))-27).*(1-heaviside(-
n)));
B4_Xz(2:8)% First 8 coef of B4_Xz - Z-transf
```

```
B4_Xn(2:8)% First 8 coef of B4_Xn - Inv. Z-transf
```

```
ans =

    1.7500   -0.8125   -0.8281   -0.5195   -0.2861   -0.1497   -0.0765


ans =

    1.7500   -0.8125   -0.8281   -0.5195   -0.2861   -0.1497   -0.0765
```

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