The Macroeconomic Effects of Debt Relief Policies during Recessions

Soyoung Lee *

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Abstract

I study debt relief as a stimulus policy using a dynamic stochastic general equilibrium model that captures the rich heterogeneity in households' balance sheets. In this environment, a large-scale mortgage principal reduction can amplify a recovery, support house prices, and lower foreclosures. The nature of this intervention, in terms of its eligibility, liquidity, and financing, shapes its macroeconomic impact. This impact rests on how resources are redistributed across households that vary in their marginal propensities to consume. Moreover, through its effects on precautionary savings, the availability of bankruptcy on unsecured debt quantitatively changes the macroeconomic response to large-scale mortgage relief.

Keywords: Business cycles, Heterogeneous agent models, Large recessions, Household leverage, Debt relief, Bankruptcy, Foreclosure

JEL codes: E21, E32, E6

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*Bank of Canada. Email: soyoung.lee.n@gmail.com

1 Introduction

What are the effects of introducing debt relief programs during recessions? If a severe downturn is associated with high household leverage, does reducing household debt help stabilize the economy? Since the Great Recession, a widely held view is that alleviating underwater borrowers' financial distress could have prevented the sharp rise in foreclosure and dampened the fall in house prices.[1] Moreover, preventing large initial declines in house prices might have reduced subsequent foreclosures, thereby supporting house prices at later dates and household spending over time. To date, however, there is little quantitative analysis of debt relief as a stimulus policy.

My goal is to provide a comprehensive quantitative framework to evaluate debt relief programs during a recession characterized by elevated levels of household debt. I find that in a recession that involves an unusually large drop in house prices, a large-scale mortgage principal reduction can lower foreclosure but does not mitigate the recession. Instead, it amplifies a recovery. I show that general equilibrium responses in prices play an important role in propagating the effects of the policy intervention over time. Additionally, the design of the policy – who is eligible, the types of transfer, and how it is financed – shapes its macroeconomic consequences. The magnitude of these effects rests on the extent to which debt relief policies redistribute resources across households that vary in their marginal propensity to consume (MPC).

I develop a model that is designed to reproduce the heterogeneity in households' assets and liabilities. Households may borrow or save in liquid assets and can also hold illiquid assets and liabilities in the form of houses and mortgages. Housing provides service flows, which are valued alongside non-durable consumption. Households face idiosyncratic risks and have limited ability to insure themselves. Borrowers may declare bankruptcy on unsecured debt and foreclose on mortgages.

The model has three distinctive features. First, it includes aggregate business cycle risk in

[1] For example, see Eberly and Krishnamurthy (2014), Mian et al. (2015), Posner and Zingales (2009), and Agarwal et al. (2017).

the form of shocks to total factor productivity, credit conditions, and housing preferences. As we consider debt relief as a macroeconomic policy tool to mitigate the severity of downturns, the model must have aggregate uncertainty. While it is costly to solve a stochastic equilibrium, the decisions of households under aggregate uncertainty may significantly differ from those under perfect foresight.

Second, all prices (interest rates, wages, house prices, and loan rates) are determined endogenously in my model. This allows me to carefully consider the general equilibrium responses to large-scale debt relief programs, both for the macroeconomy and individual households. For all exercises, I show how the responses of households to a policy intervention impact house prices, wages, and interest rates, and how these changes in prices feed back to household decisions and shape the paths of aggregate variables.

Third, the model allows default on unsecured and secured debt separately. While Mitman (2016) and Li et al. (2011) show that there are significant interactions between these two types of debt, as well as their respective defaults, including both bankruptcy and foreclosure in a model is rare. However, not including bankruptcy may affect policy evaluation. First, as bankruptcy allows households to shed debt, its availability may reduce the benefits of additional debt relief policies. Second, the availability of bankruptcy influences household decisions, thereby shifting the distribution of households across assets and liabilities. As an example, in the absence of bankruptcy, households tend to hold more liquid assets to smooth consumption. Resulting changes in the distribution of households affect the relevance and effectiveness of policies. To assess its quantitative importance, I compare debt relief policies in environments with and without bankruptcy.

My model is calibrated to match a large set of relevant features of the US economy. The model successfully reproduces the household wealth distribution, not only for net worth but also for key components: financial assets, housing wealth and mortgages. It also generates business cycle moments similar to the US data.

In a severe recession involving a large fall in house prices, I study an unanticipated intervention where households with loan-to-value (LTV) ratios above 95% have them reduced to this level via a one-time mortgage reduction. This is a large intervention that affects about 11.2% of the mortgagors and costs 1.1% of GDP. Crucially, the policy is debt financed and involves increases in future taxes.

I find that the mortgage reduction lowers foreclosures but does not stimulate consumption and house prices immediately. However, it amplifies the recovery, raising output and consumption over the long run. These findings differ from those of Kaplan et al. (2020), who consider the same mortgage forgiveness program. In their model, the program reduces foreclosure rates significantly but has little effect on house prices or consumption. Among other things, the presence of capital, how the policy is funded, and equilibrium responses in wages and interest rates in my model help explain this difference. A detailed analysis of a series of counterfactual exercises allows me to draw the following lessons.

First, the effects of the policies depend on how households expect the costs to be covered. Since the policy is debt-financed, it does not result in a net transfer to the economy. Instead, the principal reduction redistributes resources from low MPC households to high MPC households, as highly indebted households tend to have high MPCs. This raises the consumption of those who receive a principal reduction. However, all households anticipate future tax hikes, leading them to reduce their consumption. The overall impact on aggregate consumption depends on the extent of these future tax increases. If households expect a more gradual adjustment of taxes, the reduction in consumption due to anticipated tax hikes is smaller, thereby providing better support for consumption during severe recessions.

Later, as the economy moves into a recovery, the government raises tax rates and begins to repay its debt. This, in turn, increases investment in physical capital. As capital initially falls, then rises, wages fluctuate as well. While the principal reduction directly benefits those who receive the transfer, changes in wages affect all households. These income effects build up over time, moving house prices and consumption beyond the direct effects of the mortgage principal reduction. I compare different rates of reducing public debt over the recovery. Slower repayment is associated with more gradual tax hikes. While the latter provides better support for consumption during the recession, it slows the government's debt repayment schedule.

This hampers capital and output growth over the recovery, weakening the general equilibrium effects of wage and interest rate changes.

Regardless of the pace of tax hikes, the principal reduction promotes homeownership and significantly reduces foreclosure rates.

Second, how the government transfers resources to households matters. I compare the effects of the mortgage debt relief with those of a tax rebate of the same size. Mortgage forgiveness targets highly indebted homeowners while tax rebates are not a targeted policy. The former provides illiquid assets by increasing home equity while the other provides liquid assets. These differences lead to different distributional and aggregate implications.

In the aggregate, the tax rebate is more effective than the principal reduction at supporting consumption because it directly increases households' liquid wealth and distributes benefits to a larger group of high MPC households including those who do not own houses. In terms of distributional effects, tax rebates benefit poorer households the most, whereas principal reductions primarily aid lower-middle-class homeowners—often at the expense of consumption for poorer households.

In addition to mortgage forgiveness, I consider a mortgage payment reduction. For this policy, I assume that per-period payments of principal are reduced to zero for three years, either for households with LTV higher than 80%, or with payment-to-income (PTI) higher than 50%. This effectively extends loan durations by slowing amortization. The resulting increase in disposable income boosts aggregate consumption and supports homeownership, similar to a principal reduction. However, the two policies differ in their impact on foreclosure. While payment reductions initially lower foreclosure rates, they lead to a sharp rise once the relief period ends due to increased household indebtedness. In contrast, principal reduction maintains low foreclosure rates even after the economy begins to recover.

While the policy effects are qualitatively similar in models with and without bankruptcy, there are noticeable, quantitative differences. In the absence of bankruptcy, households cannot discharge unsecured debt, leading them to cut consumption more quickly as their liquid assets decline or mortgages increase. As a result, mortgage reductions or increases in liquid assets lead to stronger consumption responses in a model without bankruptcy.

Literature This work is part of the literature on household credit and default, in particular, the branch that studies debt relief as a stimulus policy tool. Since the Great Recession, a large body of work has shown that the high leverage of households can exacerbate an economic downturn or slow down a recovery.[2] Following these findings, a set of papers have estimated the effectiveness of mortgage debt relief programs. For example, Agarwal et al. (2017) show that although participation rates were low, the Home Affordable Mortgage Program (HAMP) reduced foreclosure and increased spending. Further examining HAMP, Ganong and Noel (2020) found that principal reduction was less effective than payment reduction in reducing default and increasing consumption.

The principal reduction I consider is comparable to Kaplan et al. (2020). In contrast to HAMP, it affects a larger population and provides more substantial debt reduction that, critically, does not leave borrowers underwater. Thus, Ganong and Noel's (2020) findings are not inconsistent with mine. Empirical evidence supporting the effectiveness of larger principal reductions can be found in Cespedes et al. (2021). Examining cramdowns that discharged the underwater portion of mortgages during Chapter 13 bankruptcy proceedings between 1989 and 1993, they find that foreclosure rates fell. Lastly, Piskorski and Seru (2021) show that alleviating frictions affecting debt relief (e.g. refinancing, loan renegotiation) could have reduced foreclosure rates and resulted in up to twice as fast a recovery in house prices, consumption, and employment during the Great Recession.

Two papers, both assuming nominal rigidities, are most closely related to my work. Examining unsecured loans, Auclert et al. (2019) find that debt forgiveness provided by the US consumer bankruptcy system increases consumption and this increased consumption helps to stabilize employment. Gete and Zecchetto (2023) argue that non-recourse mortgages in the US contributed to a faster recovery in the US compared to Europe, where most countries only have recourse mortgages. I complement their work by showing that debt relief programs, over and beyond existing bankruptcy and foreclosure arrangements, can have a lasting impact on

[2]Examples include Mian et al. (2013), Mian and Sufi (2014), Guerrieri and Lorenzoni (2017), Jones et al. (2011), and Verner and Gyöngyösi (2020).

the economy.

My work is broadly related to research on fiscal policy with heterogeneous agents and incomplete markets. Oh and Reis (2012) show that increases in transfers over the Great Recession exceeded that in goods and services. In a model with incomplete markets, Heathcote (2005) studies the effects of temporary tax changes and finds large real effects. Consistent with these findings, the stimulus policies I study have large real effects, and the policy effects depend on the distribution of households over assets and liabilities.

The model developed here is related to existing quantitative analyses of housing, mortgages, and foreclosures.[3] My model contains the same elements while also allowing for default on unsecured debt.[4] To my knowledge, Mitman (2016) and Meghir et al. (2025) are among the few papers examining the interaction between bankruptcy and foreclosure, both incorporating key institutional features of bankruptcy. While my model simplifies certain aspects of the bankruptcy system, it introduces aggregate uncertainty to assess the impact of debt relief policies during recessions.

The design of mortgage principal reduction follows the seminal contribution of Kaplan et al. (2020), discussed above. Although my model shares many elements with theirs, the introduction of capital and bankruptcy, and equilibrium movements in wages and interest rates lead to different aggregate effects of a debt relief program in my framework.

The rest of the paper is organized as follows. In Section 2, I describe the model economy. Section 3 explains the calibration and examines untargeted moments. I study the effects of debt relief policies in Section 4, and Section 5 concludes.

[3] See, for example, Jeske et al. (2013), Corbae and Quintin (2015), Hatchondo et al. (2015), Chatterjee and Eyigungor (2015), Kaplan et al. (2020), and Garriga and Hedlund (2020)

[4] In this sense, I integrate models with secured debt and models with consumer credit and bankruptcy. Examples of the latter include Athreya (2002), Li and Sarte (2006), Livshits et al. (2007), Chatterjee et al. (2007), and Nakajima and Ríos-Rull (2019).

2 Model

2.1 Overview

The economy consists of a continuum of infinitely lived households, banks, non-durable goods producers and government. Households are indexed by their liquid assets, a, housing, h, mortgage, b, labor productivity, ε, and a combined homeownership status and credit history indicator, o. They are subject to uninsurable idiosyncratic shocks to their labor productivity, which they supply inelastically to competitive firms. Households can save or borrow in a financial asset whose return is determined in equilibrium. They consume non-durable goods and service flows from their housing.

Housing services are derived from renting or owning a home. If a household buys a house, it can take out a mortgage to fund the purchase. Mortgages are long-term debt and subject to loan-to-value (LTV) and payment-to-income (PTI) constraints at origination. Houses and mortgages are illiquid in the sense that costs are incurred when buying or selling a house, refinancing or prepaying a mortgage. Both types of debt (unsecured debt and mortgages) are defaultable.

Fixed costs and indivisibilities in house size lead households to make discrete choices. Households' portfolios involve a large change when they i) buy or sell houses, ii) refinance or prepay mortgages, iii) change the size of a rental unit, iv) default on unsecured debt, v) default on a mortgage, or vi) default on both debts. When households do not make a substantial change to their portfolio, they choose how much to consume.

There are a large number of identical firms that produce using capital, labor and a constant returns-to-scale technology. Their output is consumed or invested in physical capital. The supply of housing is fixed.

The financial sector is competitive. Banks price unsecured and secured debt, taking into account households' default risk. Perfect competition leads to zero expected profit on each loan. The government collects taxes from households, which funds transfers alongside government consumption.

The aggregate states of the economy are g, the distribution of households, and z, a vector containing aggregate shocks to total factor productivity (TFP), housing preferences, and credit conditions. For brevity, I summarize the household state as $\omega \equiv (a, b, \varepsilon, h, o)$ and set $\Omega \equiv (g, z)$ to represent the aggregate state.

2.2 Households

2.2.1 Environment

Labor productivity Each household's labor productivity follows a Poisson process. With frequency λ_ε, households receive a shock and draw new productivity from a time-invariant distribution.

Liquid assets Households can save or borrow using a liquid asset, a. When a is negative, it represents unsecured debt, and households may default on such debt. When a household chooses to default, the debt is erased. However, the borrower has a record of bankruptcy in her credit history and pays a utility cost, ξ_a. The possibility of default by households means that the price of unsecured borrowing will depend on individual and aggregate states as these determine the probability of default. Borrowing costs include a unit cost of lending, $\iota(z)$.

Houses A house, h, is chosen from a discrete set, $\mathcal{H}$, and buying or selling a house involves adjustment costs. Houses provide utility flows to households and incur maintenance costs. One component of these maintenance costs is a property tax, which is tax deductible. I assume the total supply of houses for owners is fixed to $\overline{H}$. Importantly, house prices, $p(\Omega)$, are determined in equilibrium and vary as a function of the aggregate state.

Households are not required to own a home; those without h can rent housing services instead. Similar to Garriga and Hedlund (2020), there is a technology that converts goods to rental services at a rate of $\frac{1}{r_h}$, leading to a unit rental rate of r_h.[5] Renters choose unit sizes from a discrete set, $\nu\mathcal{H}$, and can move between different unit sizes by paying a utility cost, ξ_m. Rental units are smaller than owner-occupied units ($\nu < 1$).

Mortgages House purchases can be funded using mortgages, b. These are distinct from

[5] Rental rates have been stable over the past 30 years while house prices fluctuates, as shown in Sommer et al. (2013).

unsecured debt. Thus, similarly to Mitman (2016) and differently from Kaplan et al. (2020), I allow for both unsecured credit and mortgages. This debt is refinanceable, long-term, secured, and defaultable. When choosing their mortgage size, households are subject to both LTV and PTI constraints.

Mortgage loans are discounted by $q(a', b', \varepsilon, h, \Omega)$ at the time of origination. That is, a household that takes debt with a face value of b' receives a loan, $q(\cdot)b'$, where $q(\cdot)$ reflects the probability of payment. Notice the loan discount rate depends not only on the choice of b' but on the full individual state and the aggregate states as these determine the probability of default.

Households can refinance a mortgage loan by paying a cost, $\xi_{r_0} b'$, in units of the consumption good, and a utility cost ξ_{r_1}. When refinancing, they first pay the remaining balance on the current loan and then take out a new loan.[6] All mortgage interest rates are adjustable. Thus, refinancing is only used to extract equity or prepay the outstanding balance.

While a household is holding a mortgage, it pays the interest rate as well as a fraction (θ) of the principal at each instant. The mortgage interest rate is equal to the return on saving plus a premium reflecting the unit cost of lending, ι_m.[7]

As b is long-term debt, there is no requirement that the size of this loan remains less than γ times the current value of a house. Thus, the LTV limit only applies at origination. If the price of houses decrease, a household could find itself with negative equity. However, as long as the household pays off the required amount of the outstanding balance and interest on the remaining loan, it is not forced to default or refinance.

When a household chooses to default, the remaining balance of debt is forgiven and a financial intermediary takes the house. I assume the financial intermediary suffers a loss when foreclosing, and the sale value is $(1 - \delta_h)p(\Omega)h$. As will become evident in the financial intermediaries' problem, these expected foreclosure losses are factored into mortgage pricing, meaning mortgagors ultimately bear the cost of additional depreciation due to foreclosure.

[6]This implies that new loans are always subject to the same LTV and PTI limit and discounting.

[7]For unsecured credit, the unit cost of lending, $\iota(z)$, varies with the aggregate shock. This helps the model reproduce the procyclicality of credit. In contrast, for mortgages, a fixed lending cost, ι_m, helps match the observed share of mortgagors.

The household will have a foreclosure recorded on their credit history, which precludes them from buying houses. Additionally, households that enter foreclosure incur a utility cost, ξ_b, at that moment.

Bankruptcy and foreclosure histories Households can default on both unsecured debt and mortgages. However, mortgagors may foreclose without defaulting on unsecured debt, they cannot only default on unsecured debt without foreclosing.[8]

While bankruptcy remains on a household's credit history, unsecured borrowing, and purchasing a house are not allowed. I assume, for tractability, that the bankruptcy flag is removed stochastically with intensity λ_d.

A household that has defaulted on a mortgage is unable to purchase a new house while its credit history contains foreclosure. They are excluded from taking new mortgages. However, they can still take on unsecured debt or choose to default on any such unsecured debt they already have. If these households go bankrupt, their foreclosure flag will be replaced by a bankruptcy flag. For tractability, I assume that the foreclosure flag is removed stochastically with intensity λ_f.

2.2.2 Household Problem

Households receive flow utility from consuming non-durable goods and from the service flow from their housing. Their utility function, $u(c, h)$, is strictly increasing and strictly concave in c and h.

A household's problem is given by (1):

$$
\begin{aligned}
v(\omega_t) &= \max_{\{c_t\},\tau} \mathbf{E_0} \int_0^\tau e^{-\rho t} u(c_t, h_t) dt + \mathbf{E_0} e^{-\rho\tau} v^*(\omega_\tau) \\
\dot{a}_t &= w_t \varepsilon_t + r_{at}(\omega) a_t - (r_{bt} + \theta) b_t - c_t - T(b_t, \varepsilon_t, p_t h_t) - \xi_h p_t h_t|_{o=0} - r_h h_t|_{o>0} \\
\dot{b}_t &= -\theta b_t, \quad \omega_0 = \omega.
\end{aligned} \tag{1}
$$

Households choose non-durable consumption $\{c_t\}$ and their optimal stopping time τ.

[8] Allowing default only on unsecured debt for mortgagors significantly increases computational costs, but no households choose this option when it is available.

Table 1: Available discrete choices

	Home owners	Renters		
		w/o default	w/ foreclosure	w/ bankruptcy
	$(o=0)$	$(o=1)$	$(o=2)$	$(o=3)$
Buy a house	o	o	x	x
Sell a house	o	x	x	x
Refinance a mortgage	o	x	x	x
Change a rental unit size	x	o	o	o
Bankruptcy	o	o	o	x
Foreclosure	o	x	x	x

Before τ, households that have mortgages repay them at the rate θb. A household's income tax is given by the function $T(b, \varepsilon, p(\Omega)h, \Omega)$. Taxable income is a function of earnings, mortgage interest payments and property tax. Homeowners ($o = 0$) also need to pay the maintenance cost of their houses, $\xi_h ph$, and renters ($o = 1, 2, \text{or } 3$) pay rent, $r_h h$. Across renters, $o = 1$ indicates those without any default flag. Renters who have recently foreclosed without bankruptcy have $o = 2$, while those with bankruptcy have $o = 3$.

Stopping involves a household making a discrete choice that causes a large shift in its asset position or credit history. When a household chooses to stop, they do one of the following: i) buy or sell a house, ii) refinance their mortgages, iii) change the size of a rental unit, iv) bankruptcy, or v) foreclosure. The set of available options depends on a household's status, o, and is summarized in Table 1.

The Hamilton-Jacobi-Bellman (HJB) equation prior to stopping is

$$\begin{aligned}
\rho v(\omega, \Omega) = & \max_c u(c, h) + \partial_a v(\omega, \Omega)\dot{a} + \partial_b v(\omega, \Omega)\dot{b} + \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v(\omega^{\varepsilon_j}, \Omega) \\
& + \sum_{k=1}^{n_z} \lambda_{zz_k} v(\omega, \Omega^{z_k}) + \lambda_d (v(\omega, \Omega | o = 1) - v(\omega, \Omega | o = 3))|_{o=3} \\
& + \lambda_f (v(\omega, \Omega | o = 1) - v(\omega, \Omega | o = 2))|_{o=2} + \int \frac{\delta v(\omega, \Omega)}{\delta g(\omega)} \mathcal{K} g(\omega) d\omega \qquad (2) \\
\dot{a} = & \; w(\Omega)\varepsilon + (r_a(\omega, \Omega) + \iota(z)|_{a<0})a - (r(\Omega) + \iota_m + \theta)b - c \\
& - T(b, \varepsilon, p(\Omega)h, \Omega) - \xi_h p(\Omega)h|_{o=0} - r_h h|_{o>0}, \\
\dot{b} = & -\theta b, \quad a \geq 0|_{o=3}, \quad b = 0|_{o>0}, \quad v(\omega, \Omega) \geq v^*(\omega, \Omega).
\end{aligned}$$

Above, $\lambda_{\varepsilon\varepsilon_j}$ describes the labor productivity process and $|_{o=j}$ means that a term in front of it only applies to the households with $o = j$.[9]

Aggregate productivity follows a stochastic process described by λ_{zz_k}. I define ω^{ε_j} as $(a, b, \varepsilon_j, h, o)$ and Ω^{z_k} as (g, z_k). $\mathcal{K}$ is a Kolmogorov forward operator that operates on the distributions of households, g_t, which evolves according to shocks and households' decisions, $\frac{dg_t(\omega)}{dt} = \mathcal{K}g_t(\omega)$.[10]

The stopping value $v^*(\omega_\tau, \Omega_\tau)$ is the maximum of the values listed below in (1)–(5).

1. Buying or selling a house ($o_\tau = 0$ or 1)

$$
\begin{aligned}
&v^m(a_\tau, b_\tau, \varepsilon, h_\tau, o_\tau, \Omega) = \max_{h' \in \mathcal{H}, a', b', o'} v(a', b', \varepsilon, h', o', \Omega) - \xi_m \\
&a' = a_\tau - b_\tau + (1 - \xi_0)p(\Omega)h_\tau - p(\Omega)h' + q(a', b', \varepsilon, h', 0, \Omega)b' - \xi_{r_0}b' \\
&b' \leq \gamma p(\Omega)h', \quad \frac{(r + \iota_m + \theta)b'}{w(\Omega)\varepsilon + (r_a(\omega, \Omega) + \iota(z)|_{a<0})a} \leq \gamma_p
\end{aligned}
$$

When buying or selling a house , a household chooses the optimal house size (h'), the amount of the mortgage (b'), and liquid assets (a'). In the first constraint, h_τ, a_τ, and b_τ, are the corresponding current values at the stopping time. The remaining mortgage balance on the current house has to be repaid. Housing transactions costs are given by $p(\Omega))\xi_0 h$, and $\xi_{r_0}b'$ is a mortgage origination fee. Additionally, moving—whether due to buying/selling a house or changing rental units—incurs a utility cost, ξ_m.[11] In the second and third constraints, γ is the LTV limit and γ_p is the PTI limit.

[9] When it is the intensity of jumping to ε_j from ε_i, and $\varepsilon_i \neq \varepsilon_j$, $\lambda_{\varepsilon_i\varepsilon_j} > 0$. In contrast, the intensity of losing the current level of labor productivity is $\lambda_{\varepsilon_i\varepsilon_i} < 0$. We also have $\sum_j \lambda_{\varepsilon_i\varepsilon_j} = 0 \quad \forall i = 1, ..., n_\varepsilon$.

[10] Ahn et al. (2018) describe the recursive formulation of a model with aggregate uncertainty using the Kolmogorov forward operator.

[11] All discrete choices in the model incur utility costs. These costs are included not only for calibration purposes but also for computation. See Section A.2 in the Technical Appendix for further discussion.

2. Refinancing a mortgage ($o = 0$)

$$v^r(a_\tau, b_\tau, \varepsilon, h, o, \Omega) = \max_{b'} v(a', b', \varepsilon, h, o, \Omega) - \xi_r$$

$$a' = a_\tau - b_\tau + q(a', b', \varepsilon, h, o, \Omega)b' - \xi_{r_0} b'$$

$$b' \leq \gamma p(\Omega) h, \quad \frac{(r + \iota_m \theta)b'}{w(\Omega)\varepsilon + (r_a(\omega, \Omega) + \iota(z)|_{a<0})a} \leq \gamma_p$$

Households that hold a mortgage have the option to refinance by paying the remaining balance, costs $\xi_{r_0} b'$, and enduring an utility cost ξ_r. They do not change their house size but simply originate a new loan, b', which is subject to the LTV and PTI limit.

3. Changing a rental unit ($o = 1, 2,$ or 3)

$$v^c(a, b, \varepsilon, h_\tau, o, \Omega) = \max_{h' \in \nu\mathcal{H}} v(a, b, \varepsilon, h', o, \Omega) - \xi_m$$

As is the case when buying or selling a house, renters who change the size of their housing, incur the utility cost of moving, ξ_m.

4. Bankruptcy ($o_\tau = 0, 1,$ or 2)

$$v^a(a_\tau, b_\tau, \varepsilon, h_\tau, o_\tau, \Omega) = \begin{cases} \max_{h' \in \upsilon\mathcal{H}} v(0, 0, \varepsilon, h', 3, \Omega) - \xi_a - \xi_m, & \text{if } o_\tau = 0 \\ v(0, b_\tau, \varepsilon, h_\tau, 3, \Omega) - \xi_a, & \text{if } o_\tau \in [1, 2] \end{cases}$$

As mentioned above, when a homeowner with a mortgage declares bankruptcy, they must move to a rental unit. In contrast, a renter's bankruptcy does not involve moving. The utility costs associated with bankruptcy is ξ_a.

5. Foreclosure ($o_\tau = 0$)

$$v^b(a, b_\tau, \varepsilon, h_\tau, o_\tau, \Omega) = \max_{h' \in \upsilon\mathcal{H}} v(a, 0, \varepsilon, h', 2, \Omega) - \xi_b - \xi_m$$

Foreclosure incurs a utility cost of ξ_b.

The overall stopping value for a household is v^*, where

$$\begin{aligned}
v^*(a, b, \varepsilon, h, 0, \Omega) &= \max\{v^m, v^r, v^a, v^b\}, \\
v^*(a, b, \varepsilon, h, 1, \Omega) &= \max\{v^m, v^c, v^a\}, \\
v^*(a, b, \varepsilon, h, 2, \Omega) &= \max\{v^c, v^a\}, \\
v^*(a, b, \varepsilon, h, 3, \Omega) &= v^c.
\end{aligned} \tag{3}$$

As mentioned above, the available stopping options depend on households' credit history and homeownership status. The household's problem can be compactly written as an HJB variational inequality (HJBVI). See Appendix B for details.

2.3 Financial intermediaries

Financial intermediaries are perfectly competitive and risk neutral. These banks issue short-term deposits and loans, as well as mortgages, to households. They also lend capital to firms. The possibility of default leads banks to offer loan rates based on a household's portfolio and income. Bank loans have expected discounted profits equal to zero. While individual loans may generate a profit or a loss, ex post, banks will have zero profit on their total portfolio in the absence of aggregate shocks. However, there will be systematic profits and losses as a result of aggregate risk. I assume that the government absorbs any realized profits or losses using taxes or subsidies paid to intermediaries.[12]

Unsecured debt As shown in Bornstein (2018), the expected interest on lending in the region of no default, $(D_a(\omega, \Omega) = 0)$, is a return minus a default probability. In our context, the expected return of an unsecured loan is the interest payment net of the expected loss from the borrower moving into the default region. Over an interval of duration dt, we then have

[12]See Ozkan et al. (2017) for a similar assumption.

$$E[dr_a(\omega,\Omega)] = r_a(\omega,\Omega)dt - \lambda_z \sum_{z'} p_{zz'} \lambda_\varepsilon \sum_{\varepsilon'} p_{\varepsilon\varepsilon'} D_a(\omega^{\varepsilon'}, \Omega^{z'})dt,$$

where $p_{\varepsilon\varepsilon'}$ is the probability of moving from ε to ε' conditional on receiving a labor productivity shock and, as defined above, ω^{ε_j} is $(a, b, \varepsilon_j, h, o)$ and Ω^{z_k} is (g, z_k). In the default region $(D_a(\omega,\Omega) = 1)$, we set

$$r_a(\omega,\Omega) = \infty. \tag{4}$$

The zero profit condition in the region of no default implies that the return, $r_a(\omega,\Omega)$, should be equal to the risk free rate, $r(\Omega)$:

$$r_a(\omega,\Omega) = r(\Omega) + \lambda_z \sum_{z'} p_{zz'} \lambda_\varepsilon \sum_{\varepsilon'} p_{\varepsilon\varepsilon'} D_a(\omega^{\varepsilon'}, \Omega^{z'}). \tag{5}$$

When considering savings, because no household defaults on a in the region where a is positive, $r_a(\omega,\Omega)) = r(\Omega)$.

Mortgages As explained above, households holding a mortgage pay an interest rate, $r(\Omega) + \iota_m$, and a fraction, θ, of the remaining balance b at each instant, where ι_m is a cost. Therefore, before a stopping decision, $t \in [0,\tau)$, the flow income from a loan is $(r_t + \theta)b_t$. Banks discount the loan with a rate, $r_t + \theta$, as the loan matures at the rate θ. Recall that if a household defaults on its secured debt, the bank recovers the depreciated value of the house, $(1 - \delta_d)ph$.

Since banks expect zero profit on each loan, the discounted value of the loan at origination has to be equal to its expected cash flow. The price of the loan in the non-default region is given by

$$q_0(\omega,\Omega))b_0 = \mathbb{E}\Big[\mathbb{E}_\tau \int_0^\tau e^{-\int_0^s (r_s+\theta)ds}(r_t + \theta)b_0 dt + e^{-\int_0^\tau r_s ds} b(\omega_\tau, \Omega_\tau)\Big].$$

The scrap value $b(\omega_\tau, \Omega_\tau)$ at the stopping point depends on a household's discrete choice. In the case of foreclosure, $b(\omega_\tau, \Omega_\tau) = (1 - \delta_d)p(\Omega)h$.

When a household prepays the loan due to refinancing or a new house transaction, the scrap value is $b(\omega_\tau, \Omega_\tau) = e^{-\int_0^\tau \theta_s ds} b_0$.

Applying the Feynman-Kac formula, the above equations can be written as the following partial differential equation.[13] Before the stopping time,

$$
\begin{aligned}
(\theta + r(\Omega))q(\omega, \Omega) &= \theta + r(\Omega) + q_a(\omega, \Omega)\dot{a} + q_b(\omega, \Omega)\dot{b} \\
&+ \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} q(\omega^{\varepsilon_j}, \Omega) + \sum_{k=1}^{n_z} \lambda_{zz_k} q(\omega, \Omega^{z_k}) + \int \frac{\delta q(\omega, \Omega)}{\delta g(\omega)} \mathcal{K} g(\omega) d\omega.
\end{aligned} \tag{6}
$$

At the stopping time,

$$
q(\omega, \Omega) = \begin{cases} \frac{(1-\delta_d)p(\Omega)h}{b} & \text{if foreclose,} \\ 1 & \text{otherwise.} \end{cases} \tag{7}
$$

Firms' problems and the government's budget constraint, as well as the definition of equilibrium, are described in Appendix A.

2.4 Computation of equilibrium

The aggregate state of the model contains the distribution of households and is high dimensional. The solution algorithm I use is based on the finite difference method in Achdou et al. (2022) with several notable differences. First, there are multiple stopping choices including two types of default, the buying and selling of houses, changing the size of a rental unit, and refinancing and prepayment. Second, the model solution is nonlinear in both the individual and aggregate state vectors. Importantly, I do not assume certainty equivalence but allow for aggregate uncertainty. I solve stochastic equilibrium following the approach in Krusell and Smith (1998). Technical Appendix A provides a detailed description of the computation and the accuracy of the forecasting functions is reported in Table 13 in Technical Appendix A.

[13] The Feynman-Kac formula establishes a connection between partial differential equations and stochastic processes. See Hurtado et al. (2023) and Kaplan et al. (2018) for similar usages of the Feynman-Kac formula.

3 Mapping Model to Data

I choose model parameters to match key cross-sectional features of the US economy in the early 2010s. A quantitative study of debt relief programs requires reproducing the distribution of assets and liabilities across households. Moreover, as changes in income drive changes in households' balance sheets and labor income shocks are the source of uninsurable risk, the calibration of the stochastic process for labor earnings must be consistent with the data.

A subset of parameters is assigned in advance of solving the model's stationary state. In addition, the earnings process is estimated outside of the model. Finally, 11 parameters are jointly calibrated in the steady state by targeting 12 moments. These are parameters specifying household preference ($\sigma, \sigma_h, \kappa, \rho$), utility costs of moving, refinancing, bankruptcy, and foreclosure ($\xi_m, \xi_r, \xi_a, \xi_b$), relative size of rental units (ν), unit cost of mortgages (ι_m), and τ_0 in the tax function.

Table 5 lists parameters and Table 4 reports targeted data moments and model moments. I associate targets with specific parameters, but this correspondence is only suggestive as the parameters are jointly determined. The parameters describing aggregate shocks are chosen to match the cyclical properties of output and credit and to replicate large house price declines during severe recessions.

Earnings process I model the labor earnings process as a combination of two independent processes, $\varepsilon_{ij} = \varepsilon_i^p(1 + \varepsilon_j^t)$. In calibrating this earnings process, I target both cross-sectional inequality and individual risk. The former involves the earnings distribution described in Table 2, and the latter uses moments from the distribution of earnings growth listed in Table 3. Detailed information about the calibration of the earnings process and the labor productivity parameters in Table 5 can be found in Appendix C.1.

Housing The survey by Davis and Van Nieuwerburgh (2015) finds maintenance costs to be between 1% and 3% of the value of a house and I set maintenance costs, ξ_h, to 2%. The parameter for the housing transactions cost function, ξ_0, is 0.07 to match the average transaction cost of 7% in Delcoure et al. (2002).[14] The fixed stock of houses, $\overline{H}$, is set to

[14]Other estimates are similar to 7%. Smith et al. (1988) estimate the transaction costs of changing houses,

Table 2: Earnings distribution

	Variance	Quintiles (%)					Top (%)		
		1q	2q	3q	4q	5q	90-95	95-99	99-100
Data	0.92	-0.1	4.2	11.7	20.8	63.5	11.7	16.6	18.7
Model	0.73	5.4	6.9	8.6	17.7	61.3	13.7	12.7	17.2

Note: The numbers in Quintiles and Top represent the shares of earnings for the corresponding quintiles and percentiles.
Data: Song et al. (2018), SCF (2007)

Table 3: Earnings growth rates statistics

	Std.		Skewness		Kurtosis		$\mathbf{P}(\lvert\Delta y\rvert) < \mathbf{x}$			$\mathbf{P}(\lvert\Delta y\rvert) \in [\,\underline{x}, \overline{x}\,]$	
	1y	5y	1y	5y	1y	5y	x = 0.2	0.5	1.0	[0,0.25)	[0.25,1)
Data	0.51	0.78	-1.07	-1.25	14.93	9.51	0.67	0.83	0.93	0.31	0.16
Model	0.29	0.57	-0.58	-0.16	21.28	11.68	0.61	0.97	0.98	0.41	0.19

Note: $|\Delta y|$: Absolute change in log earnings. 1y' and '5y' indicate the earnings growth rate over one year and five years, respectively. To compute the corresponding moments, I simulate the discretized earnings process with a panel size of 4,000 and a simulation length of 5,000 periods. The first 800 periods of each simulated series are discarded before computing the statistics.
Data: Guvenen et al. (2015)

the value of housing wealth in SCF (2007). Normalizing this by average labor income, the corresponding ratio in the model implies $\overline{H}$ is 2.89.[15] The rental rate, r_h, is set to 0.15 to match a price-to-rent ratio of 15.7, consistent with data reported in Garner and Verbrugge (2009).[16] Turning to jointly calibrated parameters, the utility cost of moving, ξ_m, is set to 4.5, and the parameter determining the relative size of rental units, ν, is 0.15 to match moments in Table 4.

Mortgages The LTV limit, γ, is set to 1.0 based on the observations that mortgages are available with zero down payment and home equity lines of credit are available to households.[17] The PTI limit, γ_p, is set to 0.65, corresponding to the maximum observed PTI of newly

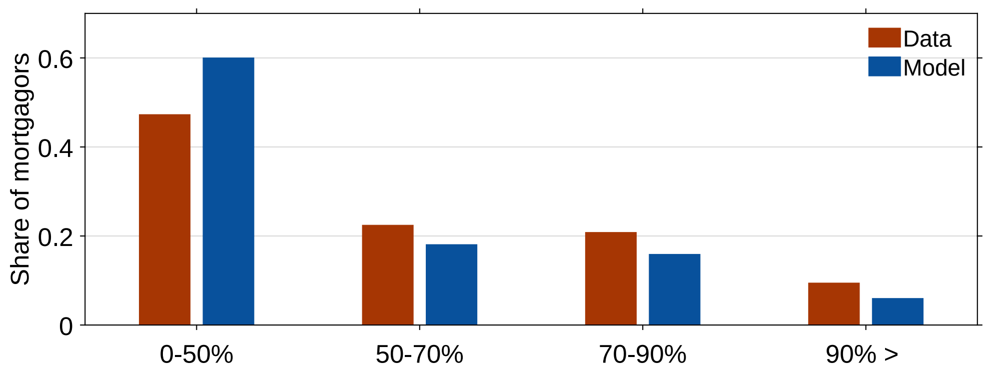
and their estimate is approximately 8–10%. Martin (2003) shows that cost of buying a new home is 7–11% of the purchase price of a house.

[15]To convert the 2007 dollars to model consistent values, I use average labor income in SCF (2007) and the corresponding value in the model.

[16]Garner and Verbrugge (2009), using Consumer Expenditure Survey (CE) data from five cities (Chicago, Houston, Los Angeles, New York, and Philadelphia) during 1982–2002, report house price-to-rent ratios ranging from 8 to 15.5. The equilibrium house price in the model is 2.36, yielding a price-to-rent ratio of $\frac{2.36}{0.15}$.

[17]An LTV limit above 1 is consistent with the following observations. First, 100% LTV loans are available. For example, the United States Department of Veterans Affairs and the United States Department of Agriculture guarantee purchase loans to 100%, and the Federal Housing Administration (FHA) insures purchase loans to 96.5%. Further, in the data, there are households with negative home equity. In the 1989–2013 waves of the SCF, the size of secured debt exceeded the value of non-financial assets among the poorest 20% of households.

Figure 1: LTV distribution



Note: Each bar at x-y% shows the share of mortgagors with LTV greater than x% and less than equal to y%. Data: SCF (2007)

originated mortgages in 2016 Q1, as reported in Greenwald (2018). The amortization rate of mortgages, θ, is set to 0.025, which implies that the duration of a loan is approximately 40 years if a household fully finances the purchase of the house. The refinancing cost, ξ_{r_0}, is 1% of the loan principal.[18] Turning to the parameters involving mortgages that are jointly calibrated, the unit cost of mortgage, ι_m, is set to -0.3%, representing a subsidy, which helps match the observed share of mortgagors in Table 4. I assume this subsidy is provided by the government. This assumption does not lead to excessive household leverage, as the share of households with LTV ratios exceeding 50% is lower than that observed in the data (Figure 1). The utility cost of refinancing, ξ_r is set to 1.5, to match moments in Table 4.

Bankruptcy and foreclosure The utility costs ξ_a and ξ_b mainly affect the bankruptcy and foreclosure rates in Table 4. The bankruptcy rate target is 1.06%, which is the number of Chapter 7 and Chapter 13 bankruptcy filings as a fraction of the total population (averaged over 2000–2017). The foreclosure rate target is 0.55% (average during the late 1990s). The intensities at which the bankruptcy and the foreclosure flags are removed are set to match the following. After filing for Chapter 7 bankruptcy, households cannot file again for 6 years. Households that file for Chapter 13 bankruptcy enter into repayment plans that last 3–5 years. Accordingly, I choose λ_d to 0.167 to match an average bankruptcy duration of 6 years. For

[18] See `https://www.federalreserve.gov/pubs/refinancings/default.htm` for more information about refinancing costs. A fraction of a loan principal ranges from 0.0 to 1.5%, and I chose a value within that range.

Table 4: Targeted moments and model values

Moment	**Data**	**Model**	**Data Source**
Foreclosure rate	0.005	0.001	Mortgage Banker's Association
Bankruptcy rate	0.010	0.010	US Courts & Census
Tax revenue to output	0.16	0.16	CBO
30th pctl housing wealth to NW	0.75	0.65	SCF (2007)
50th pctl housing wealth to NW	1.08	0.95	SCF (2007)
70th pctl housing wealth to NW	2.52	2.29	SCF (2007)
30th pctl debt payment to income	0.00	0.00	SCF (2007)
50th pctl debt payment to income	0.06	0.05	SCF (2007)
70th pctl debt payment to income	0.17	0.21	SCF (2007)
Share of homeowners	0.68	0.74	SCF (2007)
Households with mortgage	0.49	0.47	SCF (2007)
Share of mortgagors with LTV < 0.5	0.47	0.60	SCF (2007)

Note: NW indicates net worth. Business assets (debt) and vehicles (loans) are excluded from non-financial assets (debt). More details on the categorization of assets and debt can be found in Appendix C.2.

foreclosure, Fair Issac reports that households' FICO scores can recover in as little as 2 years (see Mitman (2016)). Hence, λ_f is set to 0.5. The depreciation rate when foreclosing, δ_d, is 22% and is taken from Pennington-Cross (2006).

Preferences Households receive a utility flow from consuming non-durable goods and a service flow from their houses. Their utility function is

$$u(c,h) = \frac{c^{1-\sigma}}{1-\sigma} + \frac{(\kappa(o)h)^{1-\sigma_h}}{1-\sigma_h}.$$

The parameter κ is is set to 1.0 for renters ($o = 1, 2, 3$) and 4.0 for homeowners ($o = 0$), chosen to match the share of homeowners. The curvature parameters, σ and σ_h, along with the discount rate ρ, primarily affect households' asset and debt composition. They are calibrated to match the total debt-to-asset ratio and the debt payment-to-income ratio across households.

Production The production technology is Cobb-Douglas and constant returns to scale. The labor share of output is taken from Giandrea and Sprague (2017).[19] I use the average value between 1989 and 2013, 60.5%; thus, α, capital's share is 0.395. The depreciation rate δ is 0.069 (see Khan and Thomas (2013)).

[19]The labor share of output is the sum of employees' and proprietors' labor compensation. They calculate labor's share in the non-farm business sector from 1947 through 2016.

Table 5: Parameter values

Parameter	Value	Internal	Description
Preferences and production			
ρ	0.054	Y	Discount rate
σ	2.0	Y	Curvature of the utility function
σ_h	0.45	Y	Curvature of the utility function
κ	4.0	Y	Weight on houses
α	0.395	N	Capital share
δ	0.069	N	Depreciation rate
Tax			
τ_0	0.8	Y	Tax rate
τ_1	0.181	N	Tax progressivity
τ_h	0.01	N	Property tax rate
$\overline{b}$	$1,000,000	N	Max. debt to deduct interest payments
$\overline{\tau_h}$	$10,000	N	Max. deduction on property tax
Labor productivity			
$\overline{\varepsilon}^p$	8.5	N	Upper bound of Pareto distribution
$\underline{\varepsilon}^p$	0.08	N	Lower bound of Pareto distribution
η_ε^p	1.526	N	Shape of Pareto distribution
$\eta_{\varepsilon_i^p}$	[1.9,1.5,1.3,0.6]	N	Shape of Pareto distribution
λ^p	0.048	N	Shock intensity
λ^t	1.260	N	Shock intensity
χ	0.239	N	Size of the ε^t shock
p^t	0.600	N	Probability of drawing negative ε^t
Assets and debts			
ξ_h	0.02	N	Depreciation rate of h
ξ_0	0.07	N	h transaction cost
$\overline{H}_s$	2.89	N	Supply of housing
h	[1.2,2.5,6.8,14.9]	N	House sizes
ν	0.15	Y	Relative size of rental units
γ	1.0	N	Loan-to-value ratio
γ_p	0.65	N	Payment-to-income ratio
θ	0.025	N	Amortization rate of b
ι_m	-0.003	Y	Mortgage rate premium
r_h	0.15	N	Price-rent ratio
ξ_{r_0}	0.01	N	Refinancing cost
ξ_r	1.5	Y	Utility cost of refinancing
ξ_m	4.5	Y	Utility cost of moving
ξ_a	9.0	Y	Utility cost of bankruptcy
ξ_b	-1.0	Y	Utility cost of foreclosure
λ_d	0.16	N	Removal of bankruptcy flag
λ_f	0.5	N	Removal of foreclosure flag
δ_d	0.22	N	Depreciation due to foreclosure

Note: The dollar values in the table are scaled by the ratio of the average wage in the model to the average wage in the data.

Table 6: Aggregate shocks

	TFP	LTV	PTI	$\iota(z)$	κ	Mean duration
Expansions (z_1)	1.02	1.0	0.65	0.0	4.0	16.4 quarters
Recessions (z_2)	0.98	1.0	0.65	0.03	4.0	11.2 quarters
Severe recessions (z_3)	0.92	0.8	0.35	0.03	1.0	13.0 quarters

Note: A recession is when the deviation of the output from its trend is below -1%, and a severe recession is a recession that accompanies large house price falls: the deviation of the house price from its trend is lower than -10%.

Government The income tax function $T(y) = y - \tau_0 y^{1-\tau_1}$ is taken from Heathcote et al. (2017) where y is taxable income. Taxable income is labor income minus the tax deductible interest payments on mortgages and property taxes. Taxable income is $y = w(g,z)\varepsilon - r(g,z)\min(b,\bar{b}) - \min(\tau_h p(g,z)h, \overline{\tau_h})$.

The property tax rate (τ_h) is set to 1%, which is the median tax rate across US states.[20] The IRS's rules imply that the maximum size of a mortgage ($\bar{b}$) for interest rate payment deduction is \$1,000,000 and the maximum value of tax deductible property tax ($\overline{\tau_h}$) is \$10,000.[21,22] The parameter τ_1, determining the degree of progressivity of the tax system, is 0.181 as in Heathcote et al. (2017).[23] Next, τ_0 is set to 0.8 to match the tax revenue-output ratio of 16.7%.[24]

Aggregate shocks The shock process is in Table 6. The aggregate shock z follows a three-state Poisson process, $z \in [z_1, z_2, z_3]$. These states represent an expansion (z_1), a recession (z_2), and a severe recession with a housing bust (z_3), similar to Glover et al. (2020). The shock process are designed to replicate the cyclical properties of output, consumption, and credit while generating large house price fluctuations to provide a setting for analyzing debt relief policies. Towards that, each state is mapped to different levels of TFP, credit conditions,

[20]See Kaplan et al. (2020), who reference data from the Tax Policy Center.

[21]When considering $\bar{b}$, in 2018 the deduction for home mortgage interest was limited to the first \$750,000 (\$375,000 if married filing separately) of debt. If a household was deducting mortgage interest incurred on or before December 15, 2017, the corresponding limits were \$1,000,000 (\$500,000 if married filing separately). Since I target 2007, I apply the limit before 2017.

[22]The parameter $\overline{\tau_h}$ matches the limit on deductions of state and local taxes. These include general sales taxes, real estate taxes and personal property taxes.

[23]They estimate this parameter using the Panel Study of Income Dynamics (PSID) for survey years 2000, 2002, 2004, and 2006, in combination with the NBER's TAXSIM program.

[24]I use the value of tax revenue-output ratio (Congressional Budget Office) between 2000 and 2014.

Table 7: Cyclical properties

	Data		Model	
	std (%)	corr. with output	std (%)	corr. with output
Output	1.8	1.0	1.7	1.0
Consumption	1.3	0.9	0.6	0.8
Investment	6.9	0.8	3.6	0.9
Unsecured debt	5.8	0.4	5.0	0.3
Mortgage	4.4	0.2	0.3	0.0
Bankruptcy	11.1	0.1	24.5	-0.4
Foreclosure	28.4	-0.3	1.2	-0.5
House price	4.3	0.2	2.8	0.8

Note: Logs of the data are filtered using the HP filter with a smoothing parameter of 100. Output, consumption and investment data combine information from NIPA tables 1.1.5 and 2.3.5. from 1963 to 2007. Output: real GDP minus net export; consumption: real private consumption expenditures minus housing service; investment: real gross domestic investment; unsecured debt: consumer credit (flow of funds) deflated by GDP deflator; secured debt: home mortgages (flow of funds) deflated by GDP deflator; house price: Freddie Mac US house price index divided by durable good GDP deflator (NIPA table 1.1.4, line 4). Unlike other data, house price statistics are computed using 1975–2007 data. The standard deviation and the correlation with output of bankruptcy are taken from Nakajima and Ríos-Rull (2019). Number of bankruptcies (from US courts) is normalized by the number of households (from Census) using 1995–2004 data. For foreclosure, I use the number of consumers with new foreclosures (from New York Fed Consumer Credit Panel/Equifax, 1999–2019), normalized by the number of households (Census). The model series are simulated quarterly and aggregated to the annual frequency to compute the statistics.

Table 8: Share of assets and debt

		Asset		Debt	
		Non-financial	Financial	Secured	Unsecured
Data	Q1	0.59	0.16	2.22	14.71
	Q2	4.42	1.13	11.40	18.92
	Q3	11.14	3.47	20.52	21.83
	Q4	19.29	9.72	23.94	23.41
	Q5	64.58	85.53	41.97	21.13
Model	Q1	0.17	-0.29	0.79	86.20
	Q2	4.38	3.34	14.63	7.43
	Q3	11.82	12.47	26.19	1.99
	Q4	19.99	20.49	16.95	1.48
	Q5	63.61	63.98	41.41	2.88

Note: Share of assets and debt by net worth quintiles. Data: SCF (2007) and housing preferences.[25]

To pin down parameters in the transition matrix, I classify the U.S. economy as being in a recession when the deviation of GDP per capita (U.S. Bureau of Economic Analysis) from a Hodrick-Prescott trend falls below -1%. A severe recession is defined as a recession accompanied by a significant decline in house prices, where the deviation of the average sales price of houses sold (Census, U.S. Department of Housing and Urban Development) from its trend is below -10% while the output is also below the trend. Using quarterly data from 1963 to 2019, this classification identifies six recessions and one severe recession. To reduce the number of free parameters, I impose the following restrictions: i) both a recession and a severe recession happen only after an expansion ($\lambda_{23} = 0$), ii) recovery from a severe recession is gradual ($\lambda_{31} = 0$). The average duration of each state determines λ_{ii}, which represents the intensity of shock arrival. Combined with the assumptions, the average durations pin down the values of $\lambda_{11}, \lambda_{22}, \lambda_{21}, \lambda_{33}$, and λ_{32}. The transition intensity from an expansion to either a recession or a severe recession is determined by the relative frequency of the two types of recessions: $\lambda_{12} = \lambda_{11} \times \frac{6}{7}$, $\lambda_{13} = \lambda_{11} \times \frac{1}{7}$.

I selected values for total factor productivity (TFP) to match the standard deviation of U.S. output, given the calibrated shock intensities, while ensuring that its average value over time remains one. The unsecured loan rate premium in recessions is set to 3.0% to capture the procyclicality of credit. To generate a large decline in house prices in a severe recession, I impose tighter borrowing constraints by reducing the LTV limit to 0.8 and the PTI limit to 0.35. Additionally, I set κ to one during a severe recession (z_3). Since κ represents the weight of housing services in the utility function and is equal to one for renters, this adjustment makes homeownership less attractive. However, households may still prefer owning due to the ability to occupy larger housing units.

Calibration results and discussion Table 4 shows that the model does a reasonable job in reproducing households' portfolio allocation across houses and financial assets, their use

[25] These variables are commonly used in studies examining large house price movements, such as Kaplan et al. (2020), Garriga and Hedlund (2018) and Jones et al. (2022).

of debt, and default behavior in the steady state.[26] In addition to the targeted moments, Table 8 shows the distribution of assets and debt. The share held by each net worth quintile of households of non-financial assets, financial assets and secured debt from the model is reasonably close to the data.[27] However, unsecured debt is primarily concentrated among low-net-worth households in the model, whereas it is more evenly distributed across net worth quintiles in the data. Despite the model's rich asset structure, households in the model cannot have unsecured debt and liquid savings at the same time. As a result, given that 9.6% of households in the data have net negative financial assets, it is hard to simultaneously match the distribution of financial assets and unsecured debt.

The model captures the cyclical properties of the economy. Table 7 compares key aggregate statistics from U.S. data with model outcomes, demonstrating that the model successfully replicates important empirical features. In particular, both consumption and investment show positive correlations with output, and investment is more volatile than consumption. Furthermore, both secured and unsecured credit are procyclical, while foreclosure filings are countercyclical in both the model and the data. However, overall credit usage and default rates are less volatile in the model compared to the data.[28]

House prices in the model are procyclical, as in the data, and their volatility aligns with the data as well. Due to the combined effects of productivity, credit, and preference shocks, house prices decline rapidly during severe recessions. I simulate 2,400 quarters of data and I observe 8 severe recessions. On average, house prices fall by 12.9% during severe recessions, with each fall ranging from 11.6 to 16.9%. The largest fall, 16.9%, is comparable to the 20% decline in the real house prices in the data (Freddie Mac US house price index) during 2007 – 2011.[29] Overall, the model reproduces two-thirds of the house price volatility seen in the data.

[26] Table 4 reports the 30th, 50th, and 70th percentiles of the debt payment-to-income ratio, where total debt payments include both unsecured debt and mortgage payments. Additional details on separate unsecured and mortgage payment-to-income ratios are provided in Table 9 in Appendix E. Since the 30th percentile values are zero, I instead report the 50th, 70th, and 90th percentiles for these measures. Overall, households in the model tend to have higher mortgage payments and lower unsecured debt payments compared to the data.

[27] Appendix C.2 provides detailed information about categorization of assets and debts.

[28] In the data, foreclosure volatility is significantly higher than bankruptcy volatility, which may partly be due to the relatively short span of foreclosure data (1999–2019).

[29] I used the average sales price of houses sold (Census) to define the aggregate shock process due to its long data span. During the 2007–2011 period, the real average sales price of houses sold declined by 10.6%.

4 Debt relief programs in recessions

Above, I have established that my model is consistent with salient empirical regularities characterizing the distribution of households (Section 3). Furthermore, my model generates business cycles that resemble the data along important margins. These make the model useful for the analysis of debt relief programs during recessions. In this section, I analyze such programs using a series of policy experiments.

For the policy experiments below, I simulate a recession in which, absent any intervention, house prices fall by 17.0% from their peak.[30] This no-policy scenario serves as the benchmark. I then compare it to an alternative scenario in which the government intervenes once house prices have declined by 13.2%. In this intervention, all households with LTV ratios above 95% become eligible for a mortgage principal reduction. These households receive partial debt forgiveness, reducing their LTVs to 95%. I assume that this intervention is unanticipated.[31] While the policy intervention is unanticipated, it is important to note that household decisions are affected by aggregate uncertainty.

The mortgage debt relief studied here is chosen to allow comparability with the principal reduction examined by Kaplan et al. (2020).[32] This program affects 11.2% of the mortgagors. The average mortgage principal reduction for eligible households is \$8,190.7 (in 2007 dollars), and its total cost is 1.1% of GDP.

The government covers the cost of the policy by issuing debt, B_0, which is repaid by increasing taxes over the subsequent recovery. Therefore, when the government intervenes, households expect future taxes to rise. Once the debt is repaid, the government returns to the pre-intervention tax regime. From the time the government intervenes to the beginning of

[30]While the model closely replicates the nearly 20% decline in real house prices observed during 2007–2011, it does not generate households with extremely negative equity (e.g. LTV > 200%). However, Ganong and Noel (2023) who studied drivers of mortgage default using data from 2008 - 2015, mention that 'only 0.5% of defaulters have LTVs above 200' in their sample. They conclude that households with substantial negative equity were rare.

[31]While forecasting rules post-intervention are consistent with temporary changes in tax policy, simulations before and after the intervention are based on forecasting rules estimated in environments without policy intervention. Thus, these policies are unanticipated. This is intended to capture the unusual nature of the Great Recession, whose severity was unexpected by most policy makers and market participants.

[32]The size of intervention is larger than policies that were implemented during the Great Recession and Appendix D.6 discusses mortgage relief policies in the Great Recession.

the recovery, government debt evolves as

$$\dot{B}_t = r_t B_t.$$

Once the economy transitions to the recovery, the government starts to repay its debt and it evolves as

$$\dot{B}_t = r_t B_t - \int (T_z(b, \varepsilon, p(\Omega)h, \Omega) - T(b, \varepsilon, p(\Omega)h, \Omega)g(\omega)d\omega,$$

where $T_z(b, \varepsilon, p(\Omega)h, \Omega)$ is the procyclical tax function. Specifically, τ_0 is constant at 0.8 for $T(\cdot)$, whereas for $T_z(\cdot)$, $\tau_0(z)$ is less by 0.004 during states z_1 and z_2, which implies approximately 2% higher tax in aggregates.[33] The government uses the increase in its tax revenue to repay debt, leaving the government consumption as it would have been if there had been no change in tax rates for a given distribution of households. For tractability, I assume households are uncertain about the exact timing of full debt repayment but expect the procyclical tax regime to end with a $\frac{1}{6}$ probability at any given time, reflecting an expected repayment period of approximately six years. A full characterization of the procyclical tax rate economy can be found in Appendix D.1.

The way in which the policy is funded – whether it is financed through debt or unfunded – plays a crucial role in determining its effects. I demonstrate this by comparing the outcomes of a funded intervention with those of an unfunded program.

I also compare the effects of the mortgage debt relief program with those of a tax rebate, a cash transfer of equal amount to all households. While the tax rebate is not a direct debt relief policy, it is a common stimulus policy. In addition, there are important differences between these two policies. First, mortgage forgiveness specifically targets highly indebted homeowners, whereas tax rebates are distributed universally without targeting. Second, mortgage forgiveness provides relief in the form of an illiquid asset (reduced debt), while a

[33] Since the magnitude of tax hikes influences household decisions, I also analyze cases where $\tau_0(z)$ is reduced by 0.002 and 0.008 during the recovery to conduct policy experiments with varying degrees of tax changes.

tax rebate increases households' liquid assets.[34] By comparing principal reduction with a tax rebate, I highlight the relative benefits and trade-offs of debt reduction versus a more general stimulus policy. For comparability, I set the size of the tax rebate to match the cost of the mortgage reduction. This implies each household receives a lump sum transfer equivalent to \$432.4 (2007 dollars). I assume the tax rebated is financed the same way as the principal reduction.

Instead of reducing mortgage principal, an alternative approach to alleviate household financial distress is to reduce mortgage payments. I study a policy in which eligible households have their per-period principal payments reduced to zero for 12 quarters. The timing of this intervention is same as that of the principal reduction program. However, the payment reduction policy does not require government financing as it simply delays payments. By comparing the effects of this payment reduction alongside the principal reduction, I assess the relative effectiveness of easing liquidity constraints versus lowering overall indebtedness.

In Section 4.1, I examine the effects of the mortgage principal reduction policy. Section 4.2 turns to the analysis of the payment reduction program. Finally, to highlight the importance of allowing for bankruptcy, I study the principal reduction and other policies in an economy where foreclosure is the only form of default, in Section 4.3.

4.1 Mortgage forgiveness

To better understand the effects of the policies, I first examine the characteristics of households eligible for the principal reduction. As noted earlier, eligibility is determined by the LTV ratio. The majority of eligible households own relatively small homes—83.4% have the smallest housing size, while 15.6% have the second smallest. However, these households are not exclusively low-income; aside from the very top of the income distribution, eligibility is fairly evenly spread across income levels.

The MPC of eligible households varies widely, ranging from 0.001 to 0.88, with approximately 3.9% exhibiting an MPC above 0.1. At the time of intervention, most households

[34] Mortgage forgiveness does have an indirect liquidity effect, as reducing the outstanding loan balance lowers required mortgage payments.

at risk of foreclosure do so because they are unable to meet mortgage principal and interest payments (cash-flow defaults) rather than because their mortgage exceeds their home value (strategic defaults). Specifically, 91.4% of foreclosures are cash-flow defaults, while 40.3% result from both cash-flow constraints and negative equity (double-trigger defaults).[35] This is consistent with Ganong and Noel (2023), who report that 94% of mortgage defaults are cash-flow driven, with 24% classified as double-trigger defaults.

Aggregate responses Figure 2 shows an economy initially in a severe recession transitioning to a recovery after 6 years. Throughout all the exercises, the figures show variables as *percent deviations from the corresponding value in the economy without policy intervention.*[36][37] I begin by examining the response in the model economy to an unfunded mortgage principal reduction, as studied by Kaplan et al. (2020).

The dotted lines in Figure 2 illustrate that a one-time mortgage principal reduction delivers an immediate and persistent stimulus to the economy. Within the first year following the intervention, house prices rise by approximately 0.4% and aggregate consumption increases by 0.1%. As the partial mortgage reduction lowers interest and principal payments, the consumption of recipients rises. However, the effects are not limited to directly affected households—general equilibrium (GE) effects transmit the policy's impact more broadly, influencing the decisions of the entire population.

In the model with an unfunded policy intervention, the reduction in mortgages shifts savings towards capital investment. The resulting rise in capital leads to a protracted increase in real wages and a decline in interest rates.[38] As a result, households with fewer financial assets tend to increase their consumption, while those with greater financial assets reduce

[35]Households are classified as cash-flow defaulters if, following a negative income shock, their PTI exceeds one, leading them to foreclosure. If a household's LTV remains below one, the default is considered strategic.

[36]Levels of aggregate variables in the baseline (no-policy) economy are shown in Appendix E, Figure 9.

[37]In all figures showing the effects of policies, some series show non-smoothness. These non-monotonic patterns are driven by the presence of multiple discrete choices in the model. In particular, since these choices involve utility costs, the distribution of households tends to concentrate near stopping thresholds. When such households simultaneously make discrete choices, there may follow abrupt changes in the distribution of households types. Nonetheless, the model is solved to a high level of accuracy, see Table 13 in the Technical Appendix. In addition, Section D.5 in the Appendix discusses errors in the forecasting functions following policy interventions.

[38]While the model does not have a labor supply choice, the increase in wages and the positive wealth effect experienced by recipients of the mortgage reduction lead to higher consumption.

theirs. Overall, the wage effects are more pronounced, supporting both house prices and consumption. In Appendix E, Figure 10, I compare the effects of the unfunded policy with and without changes in wages and interest rates. We see that, without the changes in prices, consumption and house prices rise by less.

I now examine the same mortgage debt relief program under the assumption that it must be fully funded. As before, the policy intervention occurs at time $t = 0$, when the economy is in a severe recession. In this case, however, the government finances the intervention by issuing debt, with the proceeds immediately used to fund the mortgage principal reductions. Households understand that taxes will increase once the recovery begins to repay the government debt.

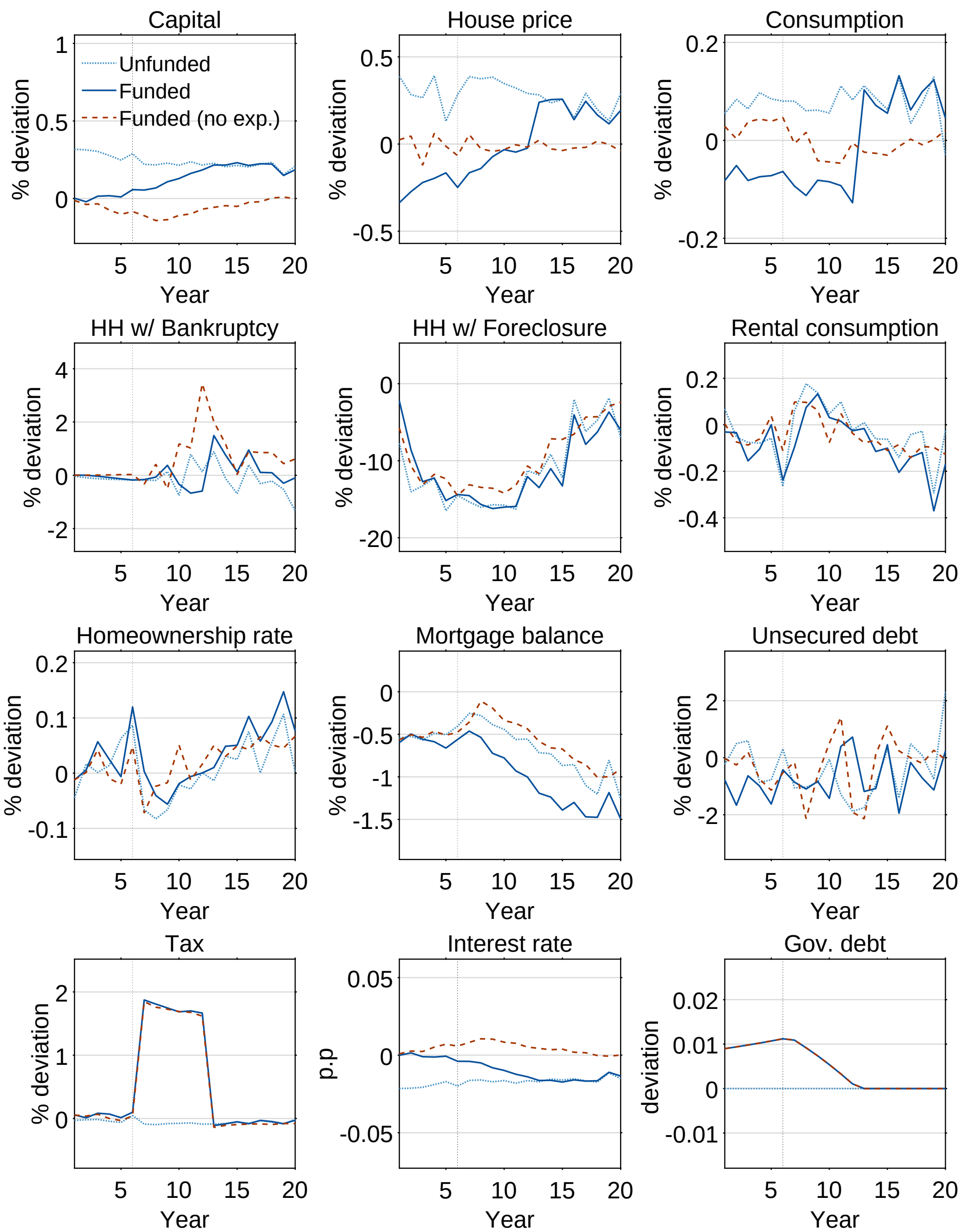
The private sector balance sheet sees no net change following the intervention. As the fall in mortgage debt is offset by a rise in government debt, total resources available to the economy do not change. In effect, the mortgage principal reduction tends to be a redistribution from low MPC households to heavily indebted households with higher MPCs. This raises consumption of those who receive the principal reduction. However, this increase is partly offset by expectations of higher future taxes, which reduce future disposable income and leads these households to increase current savings to smooth consumption. Moreover, households not receiving relief also cut back on consumption. Together, these effects result in lower aggregate consumption during the recession compared to the benchmark without the policy intervention.

Households' expectations about future taxes play a critical role. The dashed lines in Figure 2 show the effects when households do not anticipate future tax increases and instead experience them as unexpected shocks. In this case, consumption rises during the severe recession, while capital declines as a result.

At the onset of the recovery, in period 6, the government raises taxes and uses the additional revenue to begin repaying its debt. As shown in the lower left panel of Figure 2, taxes increase by approximately 2% for six years. As government debt declines, the associated crowding-out effect weakens, resulting capital to accumulate more rapidly than in the no-policy scenario.[39]

[39] Appendix D.5 discusses the accuracy of the forecast rules related to the debt policy.

Figure 2: Effects of the principal reduction



Note: Aggregate series are shown as percentage deviations, percentage point changes, or differences relative to the corresponding series in the no-policy intervention economy. Dotted lines indicate responses to the unfunded principal reduction, solid lines indicate the funded principal reduction, and dashed lines indicate the funded principal reduction under the assumption that households do not anticipate future tax increases.

This accelerates the recovery of GDP, while consumption and house prices rise as the tax rate returns to its pre-intervention level.

Summarizing the effects on aggregate quantities and house prices, the anticipated rise in future taxes that accompanies a funded debt relief shifts the timing of responses to the stimulus. There is less effect on consumption and output during the severe recession, but growth in these series is weakly amplified over the recovery. This is a result of assuming that the government will begin to repay its debt, which will increase investment in physical capital, only once a recovery begins.

By forgiving a portion of eligible households' mortgages, the program reduces outstanding mortgage balances relative to the benchmark during the severe recession. As the economy recovers, mortgage levels adjust in response to changes in homeownership and interest rates. By easing financial distress, the intervention leads to a significant decline in foreclosure rates.[40] While the effects on most macroeconomic variables vary depending on how the policy is funded and whether households anticipate future tax increases, foreclosure dynamics remain largely consistent across all three scenarios. As discussed earlier, most foreclosures are driven by the lack of cash-flow and households on the verge of foreclosure are insensitive to future tax changes or price fluctuations.

Lastly, I examine how the speed of tax adjustment and the size of the policy intervention influence aggregate outcomes. The eligibility criterion – an LTV higher than 95% – results in an intervention size of 1.1% of GDP. To explore larger interventions, I also consider cases where the LTV target is lowered to 90% and 85%, corresponding to intervention sizes of 2.5% and 3.9% of GDP, respectively (Figure 11 in Appendix E). When the intervention reaches 3.9% of GDP, consumption rises even during the severe recession, despite the dampening effect of anticipated future tax increases. Mortgage balances remain lower, homeownership is sustained at higher levels for longer, and foreclosure rates decline more substantially.

In Appendix E, Figure 12, I present scenarios in which households anticipate both smaller and larger tax hikes relative to the benchmark, corresponding to aggregate tax increases of 1%

[40]This result is consistent with the negative relationship between the amount of negative equity and mortgage default rates as in Haughwout et al. (2009) and Gerardi et al. (2017).

and 4%, respectively. When households expect a more gradual tax adjustment, the incentive to save in anticipation of future taxes diminishes, resulting in relatively stable consumption during the severe recession. Moreover, as the recovery begins and taxes rise only modestly, consumption shows little change due to the small size of the tax adjustment. In contrast, when households expect larger tax increases, both house prices and consumption are more depressed during the severe recession. However, they recover more quickly once the economy begins to rebound. Notably, foreclosure dynamics remain largely unaffected by the expected path of future taxes.

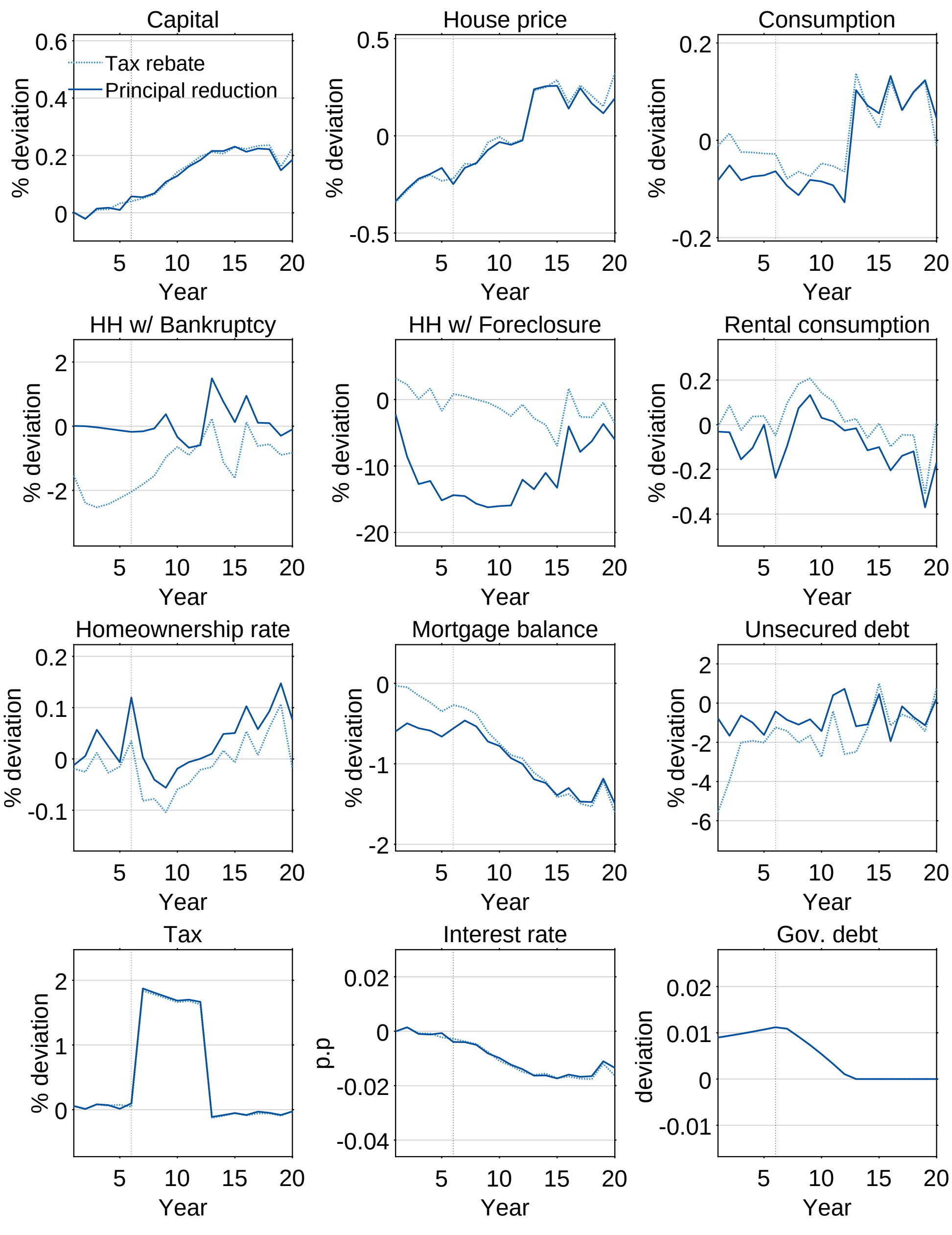
To assess the effectiveness of each funded policy using a common measure, I compute fiscal multipliers. Following standard practice in the literature, the cumulative fiscal multiplier over time T is calculated as the ratio of the discounted cumulative change in output or consumption to the government spending:

$$M_T = \frac{\int_{t=0}^{T} e^{-r_s t}(y_t^p - y_t^b)dt}{\int_{t=0}^{T} e^{-r_s t} G_t dt},$$

where y_t^p and y_t^b denote output or consumption in the policy and benchmark economies, respectively, G_t is the cost of the intervention, and r_s is the average interest rate over time, following Mountford and Uhlig (2009). Consumption is sum of non-durable consumption and housing consumption ($\xi_h p(\Omega)h|_{o=0} + r_h h|_{o>0}$) and T is 50 years. In the funded principal reduction case in Figure 2, the output multiplier is 0.07, while the consumption multiplier is -0.01. I find that larger interventions and smaller tax increases tend to produce higher consumption multipliers but lower output multipliers. Table 12 in Appendix E reports the full set of output and consumption multipliers across all scenarios.

Comparison to Kaplan et al. (2020) As discussed in Section 1, Kaplan et al. (2020) perform a similar experiment. They also consider a policy that forgives a fraction of mortgages, leaving no households with an LTV ratio higher than 95%. They implement their policy two years into the bust and it affects over a quarter of homeowners with mortgages. In contrast, the same policy in my model affects 11.2% of homeowners with mortgages, resulting in a smaller intervention size.

Figure 3: Effects of the tax rebate



Note: Aggregate series are shown as percentage deviations, percentage point changes, or differences relative to the corresponding series in the no-policy intervention economy. The dashed lines indicate responses to funded tax rebate and the solid lines indicate funded principal reduction.

Their results are different from mine. They find a mortgage forgiveness program would not have prevented a sharp drop in house prices and aggregate expenditures but would have significantly dampened the rise in foreclosures. While my household environment shares several features with theirs, there are several important differences that lead to different policy outcomes. These include i) the presence of capital, ii) equilibrium wages and interest rates, iii) differences in the preferences for housing, iv) the availability of bankruptcy, and vi) the structure of the rental market. As Kaplan et al. (2020) examine an unfunded program, I focus on comparing their results to the effects of the unfunded principal reduction shown in Figure 2 (dotted lines). Both models predict that the principal reduction leads to a large decrease in foreclosures. In my model, there is also a rise in house prices. Turning to real quantities, in contrast to their results, we see a persistent increase in aggregate consumption.

In the Kaplan et al. (2020) model, households' beliefs on taste for housing varies over time, and changes in beliefs account for most of their house price dynamics. Leverage has a limited role in affecting house prices. In addition, only house prices are determined in equilibrium. In contrast, in my model, interest rates and wages are also determined in equilibrium. Figure 2 shows that with the intervention, capital is persistently higher than the benchmark economy. The resulting higher wage benefits all households, and the associated fall in interest rates helps net borrowers at the expense of net savers. Overall, the income and substitution effect arising from changes in aggregate capital help to support consumption and house prices. Higher house prices reduce subsequent foreclosures and loosens financial constraints, preventing further decline in house prices. In the absence of interest rate and wage responses, Kaplan et al.'s (2020) model delivers small responses in aggregate variables. Similarly, when I shut down wage and interest rate responses in my model, the results become close to theirs. Appendix E, Figure 10, shows that without general equilibrium adjustments (dotted lines), the house price response is near zero and the consumption response during the severe recession is considerably smaller.

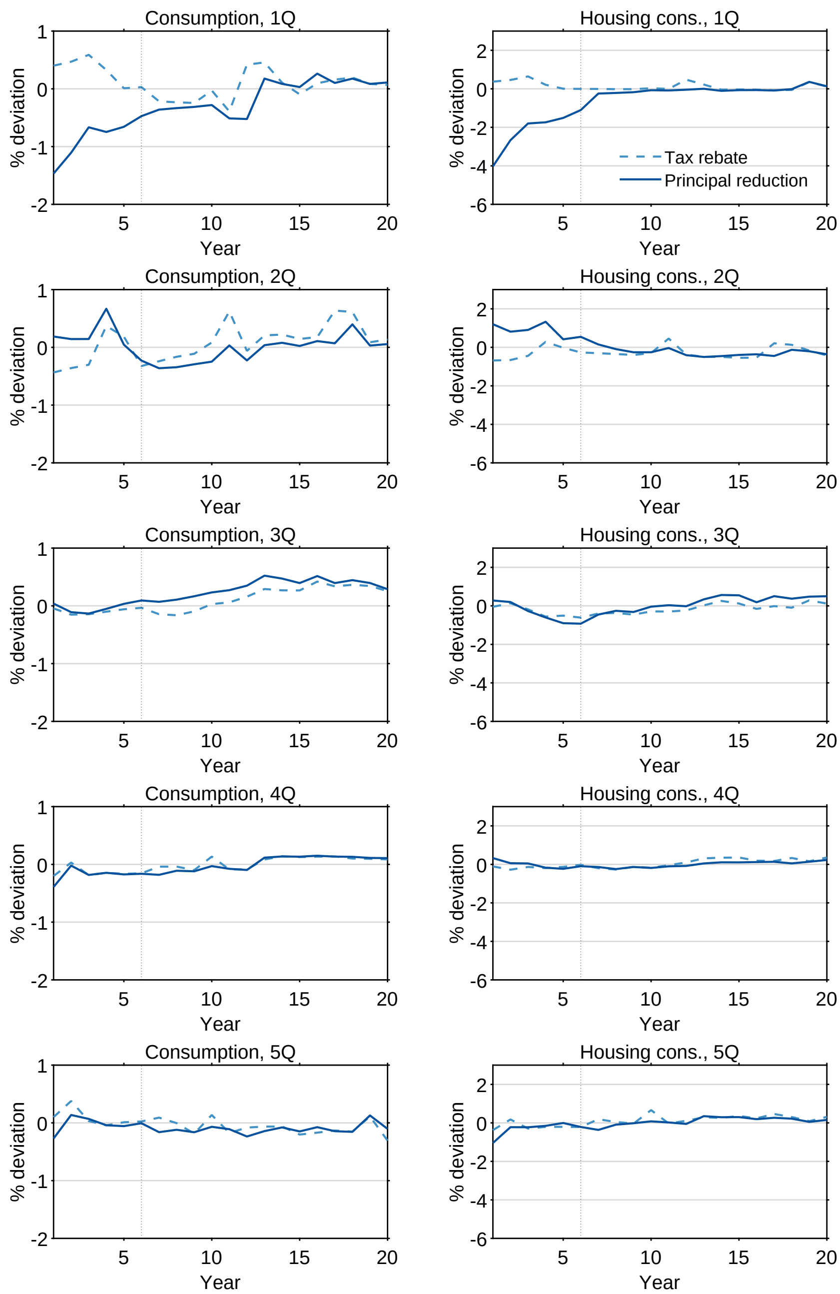
Comparison to a tax rebate I now compare the mortgage principal reduction program to a tax rebate. This transfers funds equally across households, directly adding to their liquid wealth. While the total cost of the tax rebate is the same as the principal reduction, the *equal* distribution of *liquid* wealth leads to different aggregate dynamics. Figure 3 illustrates the differences between the two policies. Following the tax rebate, consumption during the severe recession remains closer to its benchmark level, whereas the principal reduction has a stronger effect in supporting homeownership. The most striking contrasts arise in default behavior: the tax rebate significantly reduces bankruptcy filings but has little impact on foreclosures, while the principal reduction substantially reduces foreclosures but does not significantly affect bankruptcy.

Another key distinction between the two policies is in their distributional impacts. Figure 4 shows the responses of non-durable consumption (left panels) and housing consumption (right panels) by net worth quintile. Housing consumption is defined as spending on owner-occupied or rental housing, $\xi_h p(\Omega)h|_{o=0} + r_h h|_{o>0}$. While the consumption responses of the top 60% of households are similar across policies, the consumption of the bottom 40% show significant differences. The principal reduction boosts both non-durable and housing consumption for households in the second quintile, at the expense of those in the first quintile. This is because many recipients fall into the second quintile and receive substantial mortgage relief: 43.5% of eligible households are in the second quintile, and they receive 36.7% of the total reduction amount. In contrast, many households in the first quintile are renters and thus ineligible for the mortgage reduction, yet they reduce consumption in anticipation of future tax increases.[41]

I also examine whether the speed of tax adjustment and the size of the policy intervention influence the aggregate effects of the tax rebate. Appendix E, Figure 13 presents results for a larger intervention size, while Figure 14 shows outcomes under more moderate future tax increases. Compared to the baseline results in Figure 3, these scenarios show larger differences in consumption, homeownership, and rental housing consumption. In both cases, the tax rebate proves more effective than the principal reduction at supporting non-durable consumption. This is because liquid transfers can be used for consumption without paying

[41]Although households in the first quintile receive 49.3% of the total reduction, this amount is concentrated among a small number of recipients. Meanwhile, 44.8% of eligible households fall into the third quintile, but they receive only 11.1% of the total benefit.

Figure 4: Consumption and housing consumption by net worth quintile



Note: Aggregate series are shown as percentage deviations, percentage point changes, or differences relative to the corresponding series in the no-policy intervention economy. The dashed lines indicate responses to funded tax rebate and the solid lines indicate funded principal reduction. Housing consumption is $\xi_h p(\Omega) h|_{o=0} + r_h h|_{o>0}$.

adjustment costs. Moreover, renters—who are ineligible for the principal reduction but do receive the tax rebate—tend to have high MPCs (see Figure 8 in Appendix E). These factors amplify the aggregate consumption response, particularly when the intervention is larger or when future tax hikes are more gradual. Table 12 in Appendix E reports the output and consumption multipliers for all tax rebate scenarios.

4.2 Payment reduction

In addition to mortgage forgiveness, I consider a mortgage payment reduction. This policy does not requires government funding, and the tax regime does not change upon implementation.

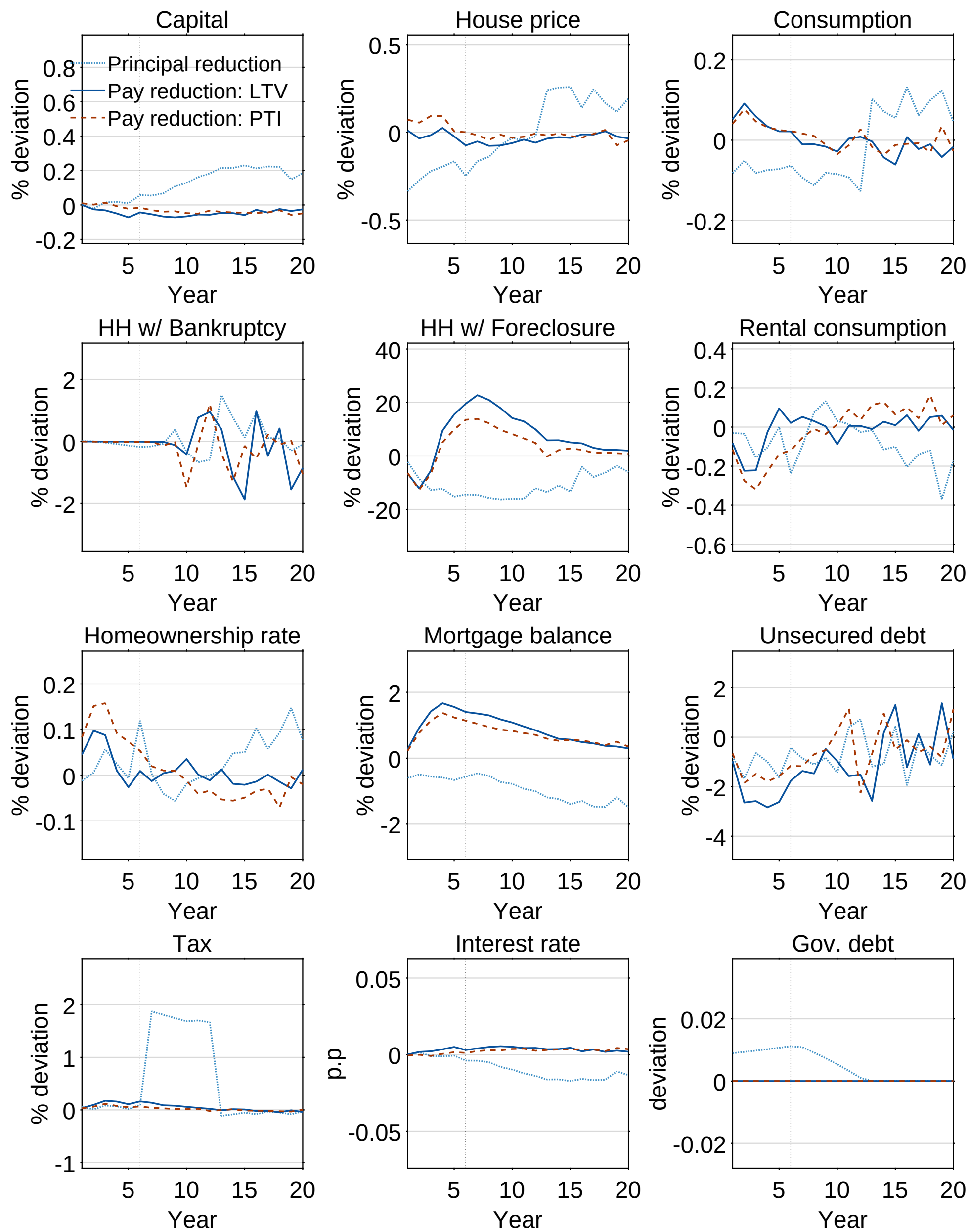
Several papers compare policies that provide wealth (principal reduction) against those providing liquidity (payment reduction). Ganong and Noel (2020) find that debt relief is less effective at reducing default and increasing consumption than payment reductions.[42] In contrast, examining cramdowns that discharged the underwater portion of mortgages during Chapter 13 bankruptcy, Cespedes et al. (2021) find that foreclosure rates fell. Due to the differences in implementation, these findings are not comparable to the results in my work.[43] While Ganong and Noel (2020) and Cespedes et al. (2021) focus on the responses of debt relief recipients, this paper emphasizes aggregate outcomes, including the indirect effects on non-eligible households through general equilibrium (GE) channels. Nonetheless, their findings highlight the potentially important role of payment reduction policies and I explore that below.

For this policy experiment, I assume that per period repayments of principal are reduced to zero for 12 quarters. I consider two eligibility criteria: (i) households with LTV ratios

[42] Indarte (2019), who studies bankruptcy, also finds payment reduction is more effective at reducing default.

[43] Ganong and Noel (2020) use HAMP as a natural experiment. In contrast to the cramdowns in Cespedes et al. (2021) and the mortgage reduction policy I study, HAMP reduced mortgages to 115% of house value, still leaving households underwater. Moreover, the share of households that modified their mortgages via HAMP was far lower than the 15% who receive mortgage reductions in my experiment. Conducting a HAMP exercise in my model might have helped validate it, however it is infeasible. HAMP reduces payment to 31% of income by lowering mortgage rates, lowering mortgage balance, and extending mortgage duration. Implementing this policy in my recursive model requires a large increase in households' state vectors that is computationally prohibitively costly.

Figure 5: Effects of the payment reduction



Note: Aggregate series are shown as percentage deviations, percentage point changes, or differences relative to the corresponding series in the no-policy intervention economy. The dotted lines indicate responses to the principal reduction, the solid lines indicate responses to the payment reduction of households with LTV higher than 80%, and the dashed lines indicate responses to the payment reduction of households with PTI higher than 50%.

above 80%, and (ii) those with PTI ratios above 50%. During this period, eligible households are required to pay only the interest on their mortgages.[44] This policy effectively extends the duration of a loan by allowing slower amortization of debt.

Figure 5 compares the effects of payment reductions with those of principal reductions. The solid lines represent the response under payment reduction for households with LTV ratios above 80%, while the dashed lines correspond to households with PTI ratios over 50%. By increasing disposable income during the period of relief, the payment reduction policy raises aggregate consumption relative the no-policy benchmark during the severe recession. Unlike the principal reduction, households do not expect future tax hikes. As a result, non eligible households are only indirectly affected through changes in interest rates and wages, both of which are small, given the small change in aggregate capital.

The payment reduction supports homeownership, similar to the principal reduction, but the impact of the two policies on foreclosure are different. Payment reductions lead to an initial decline in foreclosures; however, as the policy ends and households face higher debt burdens due to delayed amortization, foreclosure rates increase sharply. In contrast, the principal reduction lead to a persistent decline in foreclosures.

Between the two eligibility criteria, targeting households with high PTI ratios is more effective than targeting those with high LTV ratios. Targeting high-PTI households results in a smaller increase in mortgage balances compared to the high-LTV group, but it provides stronger support for both house prices and homeownership.

Overall, mortgage payment reductions can be an effective debt relief policy. While they may entail modest long-run costs –such as slight output losses and elevated foreclosure rates once the relief period ends – they offer meaningful short-run benefits by supporting consumption and homeownership during a severe recession.

[44] During the payment reduction period, households who want to buy a house or refinance must still satisfy the PTI constraint. Reduced payments are not considered when evaluating PTIs for new loans.

4.3 The role of bankruptcy

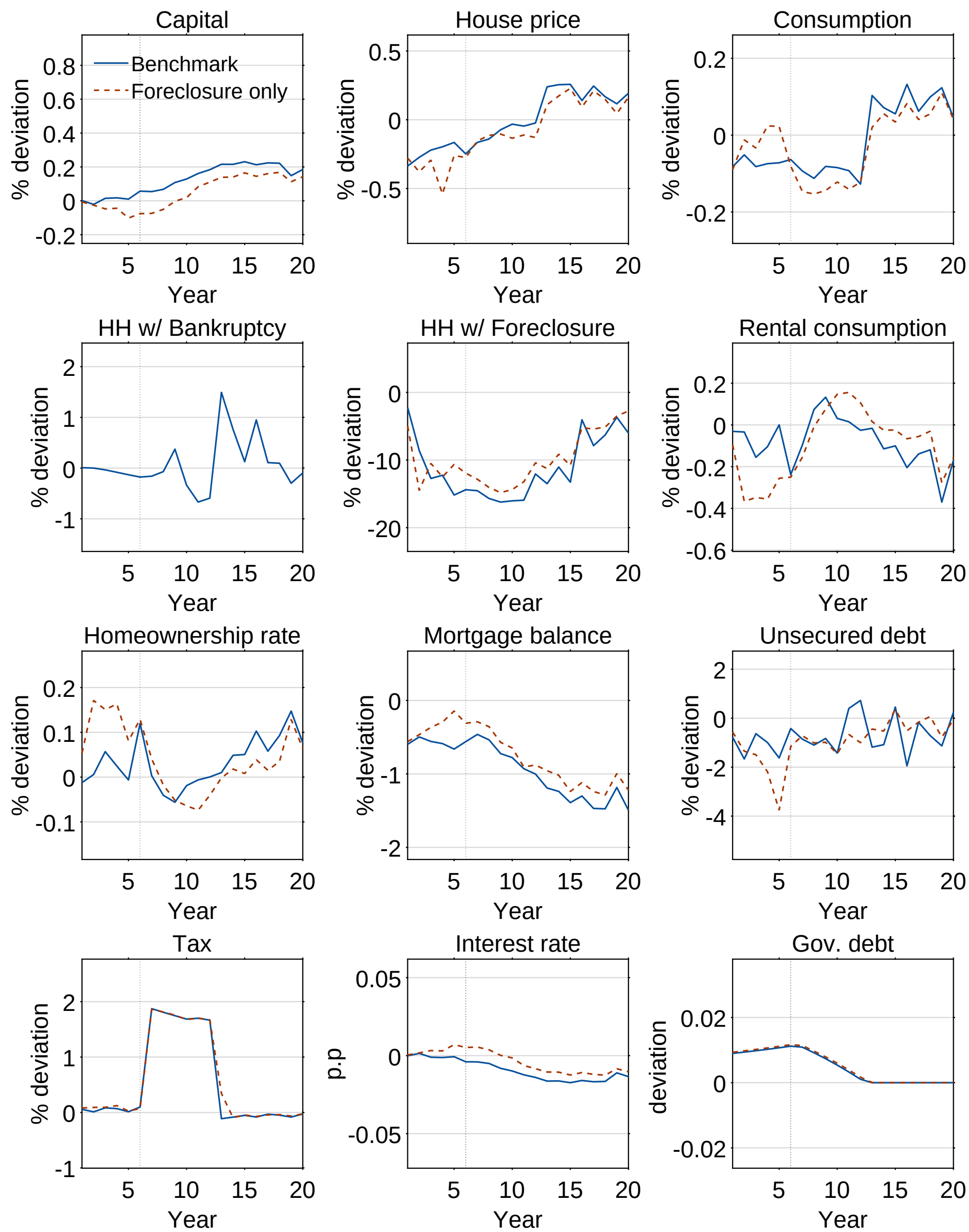
Including an option to declare bankruptcy – default on unsecured debt – to a model with housing and mortgages is rare. However, to study the effects of extraordinary debt relief programs during crises, it is arguably important to include bankruptcy. After all, bankruptcy, along with foreclosure, exists as partial consumption insurance for households that are unable to pay their debt.[45] It follows that the effects of mortgage relief policy may be overstated in an environment without the option to declare bankruptcy. Moreover, a lack of bankruptcy may make households save more to better insure themselves, altering the distribution of households and, as a result, the effects of policy.

To assess the role of bankruptcy in shaping the effects of debt relief and tax rebate policies, I examine an economy with foreclosure but without bankruptcy. Given the absence of bankruptcy, a borrowing limit on unsecured loans is introduced to ensure that a positive level of consumption is feasible across households. The mortgage reduction policy design remains the same; households with LTV ratios above 95% are eligible and receive a partial reduction in their mortgage balances. The size and scope of the intervention is similar to the baseline model. In the model without bankruptcy, the policy reaches 0.4 percentage points more households than in the baseline case and the average mortgage reduction per recipient is \$515 higher (2007 dollars). Overall, the policy directly affects 11.6% of mortgagors at an overall cost of 1.3% of GDP. Re-examining the tax rebate policy, again in an economy without bankruptcy, each household receives \$477.6 – \$45.2 more than in the model with the bankruptcy option.

The principal reduction policy has qualitatively similar responses with and without bankruptcy, as shown in Figure 6. However, the consumption response is quantitatively larger, during the recession, in the model without bankruptcy. In this environment, households cannot walk away from unsecured debt therefore they tend to reduce their consumption more sharply when their liquid assets fall or their mortgages rise compared to households in the

[45] Bankruptcy is one of the largest social insurances in the US affecting households' motives for saving. Auclert et al. (2019) show that the amount of unsecured credit discharged in Chapter 7 bankruptcies is as large as the total payments of the unemployment insurance system. Mitman (2016) shows that the presence of bankruptcy and foreclosure affect each other.

Figure 6: Effects of the principal reduction



Note: Aggregate series are shown as percentage deviations, percentage point changes, or differences relative to the corresponding series in no-policy intervention economies, with and without bankruptcy. The solid lines indicate responses to the principal reduction in the model with both bankruptcy and foreclosure option, and the dashed lines indicate responses to the principal reduction in the model with foreclosure option only.

economy with bankruptcy.[46] This implies that households in the model with only foreclosure have a higher MPC out of mortgage reductions or tax rebates. As a result, consumption is better supported during the severe recession but capital falls more than in the no policy economy. This leads to a rise in interest rates. Eventually, the reduction in capital leads to lower house prices and weaker consumption.

Also, homeownership and foreclosure responses are larger during the severe recession in the model without bankruptcy. These differences – stronger responses in consumption, homeownership, and foreclosure during a severe recession – are consistent across different policy experiments. Figures 15 and 16 in Appendix E compare the effects of the tax rebate and the mortgage payment reduction policies in models with and without the bankruptcy option.

These results confirm that the availability of the bankruptcy option affects households' consumption–saving and homeownership decisions, and plays a quantitatively important role in evaluating the effects of large-scale debt relief policies.

5 Concluding remarks

I have quantitatively assessed the effects of debt relief programs during recessions. While a growing empirical literature studies effects of such policies on foreclosure and household consumption, there is little equilibrium analysis of the dynamic response in the aggregate economy to large-scale debt relief policies.

To understand the effects of debt relief programs better, I build a model with financial assets, unsecured debt, housing, and mortgages as well as the option to default on both forms of borrowing. The model successfully replicates the household distribution for overall net worth, as well as liquid assets, mortgages, and housing. My model captures key business cycle moments while reproducing households' default behavior.

I show that a large mortgage principal reduction has persistent effects on aggregate

[46]These differences are more pronounced when households' incomes are low, when their liquid assets are low, and the size of their houses are small.

consumption, output, foreclosure rates, and house prices. The overall impact of the policy depends critically on how it is financed—specifically, whether households expect future tax increases to cover the cost, the size of those tax increases, and the aggregate size of the intervention. Nevertheless, across all scenarios, the policy consistently delivers substantial declines in foreclosure. When comparing debt relief to a tax rebate – an untargeted, liquid income transfer – I find that the two policies have different distributional effects. The principal reduction increases both non-durable and housing consumption primarily among households in the second net worth quintile, often at the expense of those in the bottom quintile. In contrast, the tax rebate benefits households in the first quintile.

Instead of forgiving mortgage balances, temporarily reducing mortgage payments can be an effective debt relief policy. It boosts both consumption and homeownership during a severe recession by easing short-term financial distress. However, since this policy extends the duration of mortgage and keeps mortgage balances higher than in the absence of the payment reduction, it leads to increased foreclosure rates over the longer term.

Including the bankruptcy option is important when assessing large-scale debt relief policies, as it provides partial consumption insurance to households, potentially reducing the need for additional relief. The availability of bankruptcy also influences households' borrowing and saving behavior, which in turn shapes the aggregate response to policy interventions. I find that in the absence of a bankruptcy option, debt relief policies are more effective at supporting consumption and homeownership during a severe recession.

References

Achdou, Y., Han, J., Lasry, J.-M., Lions, P.-L., and Moll, B. (2022). Income and wealth distribution in macroeconomics: A continuous-time approach. *The Review of Economic Studies*, 89(1):45–86.

Agarwal, S., Amromin, G., Ben-David, I., Chomsisengphet, S., Piskorski, T., and Seru, A.

(2017). Policy intervention in debt renegotiation: Evidence from the home affordable modification program. *Journal of Political Economy*, 125(3):654–712.

Ahn, S., Kaplan, G., Moll, B., Winberry, T., and Wolf, C. (2018). When inequality matters for macro and macro matters for inequality. *NBER Macroeconomics Annual*, 32(1):1–75.

Athreya, K. B. (2002). Welfare implications of the bankruptcy reform act of 1999. *Journal of Monetary Economics*, 49(8):1567–1595.

Auclert, A., Dobbie, W. S., and Goldsmith-Pinkham, P. (2019). Macroeconomic effects of debt relief: Consumer bankruptcy protections in the great recession. Technical report, National Bureau of Economic Research.

Bornstein, G. (2018). A continuous time model of sovereign debt. *Available at SSRN 3058364*.

Cespedes, J. C., Parra, C. R., and Sialm, C. (2021). The effect of principal reduction on household distress: Evidence from mortgage cramdown. Technical report, National Bureau of Economic Research.

Chatterjee, S., Corbae, D., Nakajima, M., and Ríos-Rull, J.-V. (2007). A quantitative theory of unsecured consumer credit with risk of default. *Econometrica*, 75(6):1525–1589.

Chatterjee, S. and Eyigungor, B. (2015). A quantitative analysis of the US housing and mortgage markets and the foreclosure crisis. *Review of Economic Dynamics*, 18(2):165–184.

Corbae, D. and Quintin, E. (2015). Leverage and the foreclosure crisis. *Journal of Political Economy*, 123(1):1–65.

Crandall, M. G. and Lions, P.-L. (1983). Viscosity solutions of Hamilton-Jacobi equations. *Transactions of the American Mathematical Society*, 277(1):1–42.

Davis, M. A. and Van Nieuwerburgh, S. (2015). Housing, finance, and the macroeconomy. In *Handbook of regional and urban economics*, volume 5, pages 753–811. Elsevier.

Delcoure, N., Miller, N. G., et al. (2002). International residential real estate brokerage fees and implications for the US brokerage industry. *International Real Estate Review*, 5(1):12–39.

Eberly, J. and Krishnamurthy, A. (2014). Efficient credit policies in a housing debt crisis. *Brookings Papers on Economic Activity*, 2014(2):73–136.

Fernández-Villaverde, J., Hurtado, S., and Nuno, G. (2023). Financial frictions and the wealth distribution. *Econometrica*, 91(3):869–901.

Ganong, P. and Noel, P. (2020). Liquidity versus wealth in household debt obligations: Evidence from housing policy in the great recession. *American Economic Review*, 110(10):3100–3138.

Ganong, P. and Noel, P. (2023). Why do borrowers default on mortgages? *The Quarterly Journal of Economics*, 138(2):1001–1065.

Garner, T. I. and Verbrugge, R. (2009). Reconciling user costs and rental equivalence: Evidence from the us consumer expenditure survey. *Journal of Housing Economics*, 18(3):172–192.

Garriga, C. and Hedlund, A. (2018). Housing finance, boom-bust episodes, and macroeconomic fragility. In *2018 Meeting Papers*, volume 354. Society for Economic Dynamics.

Garriga, C. and Hedlund, A. (2020). Mortgage debt, consumption, and illiquid housing markets in the great recession. *American Economic Review*, 110(6):1603–34.

Gerardi, K., Herkenhoff, K. F., Ohanian, L. E., and Willen, P. S. (2017). Can't pay or won't pay? Unemployment, negative equity, and strategic default. *The Review of Financial Studies*, 31(3):1098–1131.

Gete, P. and Zecchetto, F. (2023). Mortgage design and slow recoveries: The role of recourse and default. *The Review of Economic Studies*, forthcoming.

Giandrea, M. D. and Sprague, S. A. (2017). Estimating the US labor share. *Monthly Labor Review, US Bureau of Labor Statistics*.

Glover, A., Heathcote, J., Krueger, D., and Ríos-Rull, J.-V. (2020). Intergenerational redistribution in the great recession. *Journal of Political Economy*, 128(10):3730–3778.

Greenwald, D. (2018). The mortgage credit channel of macroeconomic transmission.

Guerrieri, V. and Lorenzoni, G. (2017). Credit crises, precautionary savings, and the liquidity trap. *The Quarterly Journal of Economics*, 132(3):1427–1467.

Guvenen, F., Karahan, F., Ozkan, S., and Song, J. (2015). What do data on millions of US workers reveal about life-cycle earnings risk? Technical report, National Bureau of Economic Research.

Hatchondo, J. C., Martinez, L., and Sánchez, J. M. (2015). Mortgage defaults. *Journal of Monetary Economics*, 76:173–190.

Haughwout, A., Okah, E., and Tracy, J. S. (2009). Second chances: Subprime mortgage modification and re-default. *FRB of New York Staff Report*, (417).

Heathcote, J. (2005). Fiscal policy with heterogeneous agents and incomplete markets. *The Review of Economic Studies*, 72(1):161–188.

Heathcote, J., Storesletten, K., and Violante, G. L. (2017). Optimal tax progressivity: An analytical framework. *The Quarterly Journal of Economics*, 132(4):1693–1754.

Hurtado, S., Nuño, G., and Thomas, C. (2023). Monetary policy and sovereign debt sustainability. *Journal of the European Economic Association*, 21(1):293–325.

Indarte, S. (2019). The impact of debt relief generosity and liquid wealth on household bankruptcy. *Available at SSRN 3378669*.

Jeske, K., Krueger, D., and Mitman, K. (2013). Housing, mortgage bailout guarantees and the macro economy. *Journal of Monetary Economics*, 60(8):917–935.

Jones, C., Midrigan, V., and Philippon, T. (2011). Household leverage and the recession. Technical report, National Bureau of Economic Research.

Jones, C., Midrigan, V., and Philippon, T. (2022). Household leverage and the recession. *Econometrica*, 90(5):2471–2505.

Kaplan, G., Mitman, K., and Violante, G. L. (2020). The housing boom and bust: Model meets evidence. *Journal of Political Economy*, 128(9):3285–3345.

Kaplan, G., Moll, B., and Violante, G. L. (2018). Monetary policy according to HANK. *American Economic Review*, 108(3):697–743.

Khan, A. and Thomas, J. K. (2013). Credit shocks and aggregate fluctuations in an economy with production heterogeneity. *Journal of Political Economy*, 121(6):1055–1107.

Krusell, P. and Smith, Jr, A. A. (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106(5):867–896.

Li, W. and Sarte, P.-D. (2006). US consumer bankruptcy choice: The importance of general equilibrium effects. *Journal of Monetary Economics*, 53(3):613–631.

Li, W., White, M. J., and Zhu, N. (2011). Did bankruptcy reform cause mortgage defaults to rise? *American Economic Journal: Economic Policy*, 3(4):123–47.

Livshits, I., MacGee, J., and Tertilt, M. (2007). Consumer bankruptcy: A fresh start. *American Economic Review*, 97(1):402–418.

Martin, R. F. (2003). Consumption, durable goods, and transaction costs. *FRB International Finance Discussion Paper*, (756).

Meghir, C., Wenli, L., Oswald, F., et al. (2025). Consumer bankruptcy, mortgage default and labor supply.

Mian, A., Rao, K., and Sufi, A. (2013). Household balance sheets, consumption, and the economic slump. *The Quarterly Journal of Economics*, 128(4):1687–1726.

Mian, A. and Sufi, A. (2014). What explains the 2007–2009 drop in employment? *Econometrica*, 82(6):2197–2223.

Mian, A., Sufi, A., and Trebbi, F. (2015). Foreclosures, house prices, and the real economy. *The Journal of Finance*, 70(6):2587–2634.

Mitman, K. (2016). Macroeconomic effects of bankruptcy and foreclosure policies. *American Economic Review*, 106(8):2219–55.

Mountford, A. and Uhlig, H. (2009). What are the effects of fiscal policy shocks? *Journal of Applied Econometrics*, 24(6):960–992.

Nakajima, M. and Ríos-Rull, J.-V. (2019). Credit, bankruptcy, and aggregate fluctuations. Technical report, National Bureau of Economic Research.

Oh, H. and Reis, R. (2012). Targeted transfers and the fiscal response to the great recession. *Journal of Monetary Economics*, 59:S50–S64.

Øksendal, B. K. and Sulem, A. (2005). *Applied stochastic control of jump diffusions*. Springer.

Ozkan, S., Mitman, K., Karahan, F., and Hedlund, A. (2017). Monetary policy, heterogeneity, and the housing channel. Technical report, Society for Economic Dynamics.

Pennington-Cross, A. (2006). The value of foreclosed property. *Journal of Real Estate Research*, 28(2):193–214.

Piskorski, T. and Seru, A. (2021). Debt relief and slow recovery: A decade after Lehman. *Journal of Financial Economics*, 141(3):1036–1059.

Posner, E. A. and Zingales, L. (2009). A loan modification approach to the housing crisis. *American Law and Economics Review*, 11(2):575–607.

Smith, L. B., Rosen, K. T., and Fallis, G. (1988). Recent developments in economic models of housing markets. *Journal of Economic Literature*, 26(1):29–64.

Sommer, K., Sullivan, P., and Verbrugge, R. (2013). The equilibrium effect of fundamentals on house prices and rents. *Journal of Monetary Economics*, 60(7):854–870.

Song, J., Price, D. J., Guvenen, F., Bloom, N., and Von Wachter, T. (2018). Firming up inequality. *The Quarterly Journal of Economics*, 134(1):1–50.

Verner, E. and Gyöngyösi, G. (2020). Household debt revaluation and the real economy: Evidence from a foreign currency debt crisis. *American Economic Review*, 110(9):2667–2702.

Appendix For Online Publication

This Appendix is organized as follows. Section A provides additional details for Section 2 in the paper. Section B reformulates the household problem towards computation. Section C describes the estimation of the earnings process and categorization of assets and debt used in computing calibration targets. Section D describes the procyclical tax-rate economy that is used in experiments with funded policy interventions. It also provides additional discussion and results not included in Section 4. Lastly, Section E contains additional figures and tables.

A Firms, government and equilibrium

There are identical, competitive firms that produce non-durable consumption goods using a constant return to scale technology.[47] Firms rent capital from banks and employ labor to solve the following problem:

$$\max_{k,\ell} zf(k,\ell) - (r(\Omega)+\delta)k - w(\Omega)\ell. \tag{8}$$

The production function $f(k,\ell) = k^{\alpha}\ell^{1-\alpha}$ and the capital depreciation rate is δ. Let L and K represent the aggregate quantities of labor and capital. In equilibrium, firms' profit maximization implies that the equilibrium real interest rate satisfies $r(\Omega) = z\alpha \frac{L}{K}^{1-\alpha} - \delta$ while the wage rate is $w(\Omega) = z(1-\alpha)\frac{L}{K}^{-\alpha}$.

The government collects taxes from households, levied on the labor income net of deductions. Households can deduct the interest paid on their mortgage and property taxes. The government provides subsidy for mortgages, ι_m. As already noted, the government also absorbs realized profits and losses from unsecured and secured lending by banks arising from aggregate shocks, through taxes and subsidies. The remaining revenue is spent on government consumption of non-durable goods, which is not valued by households.

The government's budget constraint is

[47] I assume the stock of durable goods is given.

$$\int T(b, \varepsilon, p(\Omega)h)g(\omega)d\omega + \int (\iota_m b)|_{\iota_m<0} g(\omega)d\omega - G = 0, \tag{9}$$

where G is government consumption. In the definition of equilibrium in the following section, there is no government debt. However, when there is a policy intervention and the government issues bonds to finance debt relief or a tax rebate, the definition of equilibrium must be generalized. See Appendix D.1.

An equilibrium is a set of functions that satisfies the following:

1. Households solve their lifetime optimization problems. Given price functions $\{r_a, r, q, w, p\}$, v solves (1)–(3).
2. Firms maximize profits by solving (8).
3. The unsecured debt price function r_a is determined by (4) and (5).
4. The mortgage price function q is determined by (6)–(7).
5. Capital market clears: $\int (a - b)g(\omega)d\omega = K$.
6. Labor market clears: $\int \varepsilon g(\omega)d\omega = L$.
7. Housing market clears: $\int hg(\omega)d\omega = \overline{H}$.
8. The government budget constraint (9) holds.
9. The Kolmogorov forward operator, $\mathcal{K}$, that describes the change of density function g is generated by agents' optimal choices.

B Household problem reformulated

Household problems in Section 2 can be written as a HJB variational inequality (HJBVI). Households decide whether to continue or to stop and choose any of the stopping options listed above in Section 2. The value of the latter, which is the second choice below, is given by $v^*(\omega, \Omega)$ in (3), while the value of continuing, the first choice, is the HJB equation in (2):

$$
\min\left\{\rho v(\omega,\Omega) - \max_c u(c,h) - \partial_a v(\omega,\Omega)\dot{a} - \partial_b v(\omega,\Omega)\dot{b} - \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v(\omega^{\varepsilon_j},\Omega) \right.
$$

$$
- \lambda_d (v(\omega,\Omega|o=0) - v(\omega,\Omega|o=3))_{o=3} - \lambda_f (v(\omega,\Omega|o=0) - v(\omega,\Omega|o=2))_{o=2} \tag{10}
$$

$$
\left. - \sum_{k=1}^{n_z} \lambda_{zz_k} v(\omega,\Omega^{z_k}) - \int \frac{\delta v(\omega,\Omega)}{\delta g(\omega)} \mathcal{K} g(\omega) d\omega, \;\; v(\omega,\Omega) - v^*(\omega,\Omega) \right\} = 0.
$$

C Additional information on calibration

This section is organized as follows. Section C.1 explains the calibration of the earnings process. Next, Section C.2 describes the categorization of assets and debt used in computing calibration targets.

C.1 Earnings process

I model the labor earnings process as a combination of two independent processes:

$$
\varepsilon_{ij} = \varepsilon_i^p (1 + \varepsilon_j^t),
$$

where each component follows a Poisson jump process. Jumps arrive at Poisson rate λ^p for ε^p and λ^t for ε^t. Conditional on a jump in ε^p, a new earnings state ε_k^p is drawn from a bounded Pareto distribution; conversely, when ε^t has a jump, it is drawn from the discrete set $\{-\chi, \chi\}$. Kaplan et al. (2018) explain why this type of process is useful for matching high frequency earnings dynamics. The size and frequency of each shock determine the shape of the earnings distribution. Large, infrequent shocks are likely to generate a more leptokurtic distribution, and small, frequent shocks are likely to generate a platykurtic distribution. Kaplan et al. (2018) model the earnings process as a sum of two jump-drift processes, representing a persistent and a transitory component of the earnings process.

The distribution of ε_i^p is determined by a choice of the upper bound, the lower bound, and a curvature parameter. I choose these parameters to match the variance as well as several additional moments of the earnings distribution: the share of earnings over quintiles and the shares in the top 5–10, 1–5 and 1 percentiles. As there is little effect on the earnings distribution from ε^t, in matching these moments I start by discretizing ε^p. As a first step in this discretization, I create a set of 4 points that are not linearly spaced. Instead, the 4 points aim to capture the bottom 41.0, 69.0 and 98.0 percentiles of the earnings distribution and the remaining top 2.0%.

The first three probabilities represent population shares of three educational attainment levels. These educational levels are high school graduate and below, some college or a bachelor's degree, and higher (averaged over 1992 to 2013 as reported by the BLS). The last point to compute the share of earnings held by the top 1.5% is to capture concentration at the top of the distribution.

The set of 4 points is found as follows. Let $\underline{\varepsilon}^p$, $\overline{\varepsilon}^p$ and η_ε^p be the lower and upper bounds and a curvature parameter, respectively, of the bounded Pareto distribution. The first and second points, x_1 and x_2, solve $f(x_1) = \frac{1-(\underline{\varepsilon}^p/x_1)^{\eta_\varepsilon^p}}{1-(\underline{\varepsilon}^p/\overline{\varepsilon}^p)^{\eta_\varepsilon^p}} = 0.41$ and $f(x_2) = \frac{1-(\underline{\varepsilon}^p/x_2)^{\eta_\varepsilon^p}}{1-(\underline{\varepsilon}^p/\overline{\varepsilon}^p)^{\eta_\varepsilon^p}} = 0.69$, where $f(x_i)$ is the CDF of the bounded Pareto distribution.

The next step is to use these points to determine transition probabilities for a discretized grid for ε^p. As is conventional when discretizing a continuous distribution, the support is chosen so that the vector ε^p has ε_1^p as the midpoint between $\underline{\varepsilon}$ and x_1 and ε_2^p is a midpoint between x_1 and x_2 and so forth.

Having chosen the values for ε_i^p, the probability of drawing a new value upon the arrival of the income shock can be defined. In the following, I assume that the probability of drawing ε_k^p depends on the current level of productivity, ε_i^p. This requires choosing bounded Pareto distributions over ε_k^p for each ε_i^p. Each of these distributions is bounded by the same $\underline{\varepsilon}^p$, $\overline{\varepsilon}^p$ described above. They are distinguished by curvature parameters, $\eta_{\varepsilon_i^p}$, $i = 1, .., 4$. Given the discretized support, the shape parameters $\eta_{\varepsilon_i^p}$ need to be estimated.

We use the above distributions, alongside the points x_i described above, to construct conditional probabilities. Conditional on a jump, let the probability of a change from ε_i^p to ε_k^p be $f(\varepsilon_{k|i}) - f(\varepsilon_{k-1|i})$, where $f(\varepsilon_{k|i}) = \frac{1-(\underline{\varepsilon}^p/x_k)^{\eta_{\varepsilon_i}^p}}{1-(\underline{\varepsilon}^p/\overline{\varepsilon}^p)^{\eta_{\varepsilon_i^p}}}$. Recall λ^p is the intensity for the arrival of an ε^p shock. The intensity of jumping from i to k is $\lambda_{ik}^p = \lambda^p(f(\varepsilon_{k+1|i}) - f(\varepsilon_{k|i}))$.

To set the curvature values $\eta_{\varepsilon_i^p}$, $i = 1, .., 4$, the shock intensities λ^p and λ^t, the size of the shock χ, and the probability of drawing a negative transitory component conditional on arrival of shock in ε^t, I estimate the earnings process using the Simulated Method of Moments to match the higher order moments of the earnings growth rate distribution reported in Guvenen et al. (2015), using Social Security Administration (SSA) data from 1994 to 2013.

I simulate the discretized earnings process to compute corresponding moments. The panel size is 4,000 and the simulation length is 5,000. The 800 periods of each simulated series are discarded before computing statistics. Increasing the panel size or the number of periods has little effect on the results. Since the data moments are computed using annual earnings, I simulate at a higher frequency and aggregate the results into annual earnings.

To summarize, the number of parameters specifying the earnings process is 11 and the number of targets is 20. The parameters include the 3 parameters that shape the bounded Pareto distribution for ε^p: $\overline{\varepsilon}^p$, $\underline{\varepsilon}^p, \eta^p_\varepsilon$. In addition, the 4 parameters that set the probability of drawing a new value for ε^p are $\eta^p_{\varepsilon_i}, i \in [1, .., 4]$. Next, the 2 parameters that set shock intensity are λ^p and λ^t, and χ is the size of the ε_t shock. Finally, p^t is the probability of drawing negative value upon the arrival of an ε^t shock.

Note that as λ_{ik} affects the ergodic distribution of households over labor productivity, the cumulative population shares by ε_i^p could be different from the 41.0, 69.0, 98.0 and 100 percentiles I set above. Therefore, an additional restriction to the Simulated Method of Moments is that the abcissa x_i must be consistent with the educational attainment earnings shares provided at the start of this section. Hence, the objective function minimized includes the percentiles of the earnings distribution that are used to space the grid for ε^p. Beyond this, there are 20 targets listed in Tables 2 and 3 in Section 3 of the text.

The estimated process implies that a shock to ε^p arrives on average once every 21 years. Upon the arrival of a shock, one's income level jumps to a different level. Turning to the other labor productivity shock, a shock to ε^t arrives on average once every 0.9 years. The infrequent component of the shock, ε^p can be interpreted as the persistent component and ε^t as the transitory component. Households do not experience a large shock often, but income fluctuates around their persistent component through frequent shocks, ε^t.

C.2 Categorization of assets and debts

Mapping the model to the data requires categorizing assets held by US households into financial assets, non-financial assets and secured debt. I target the asset and debt distribution reported in the 2007 SCF. In the SCF data, net worth comprises assets and debt, and total assets are the sum of financial assets and non-financial assets. Financial assets include transaction accounts, certificates of deposit, money market funds, stocks, cash, quasi-liquid retirement accounts and other financial assets. Non-financial assets are predominantly the value of vehicles and houses (primary and non-primary residential property, non-residential real estate) and the value of business. Debt comprises debt secured by residential properties, credit card loans, and installment loans (e.g., student loans, vehicle loans). When mapping the model to the data, I exclude the value of a business and vehicles from non-financial assets because my model does not have such assets. For debt, I exclude installment loans, which include student loans and vehicle loans, for the same reason. Student loans are not short-term, unsecured debt nor are they secured by collateral or dischargeable in bankruptcy. After excluding installment loans, credit card loans are considered as unsecured debt, and the remaining components of debt are assigned to secured debt.

D Stochastic results

D.1 Procyclical tax-rate economy

In Section 4, for policy interventions incurring explicit costs (mortgage principal reduction, tax rebate), the government finances its policy intervention by issuing bonds that are repaid by increasing taxes only during expansions. Therefore, when the government intervenes, households expect future taxes to rise in expansions. For tractability, I assume that households do not know exactly when the cost will be paid off but expect the procyclical tax rate regime will end with a 20% probability at any time. In this section, I present details for this procyclical tax-rate economy.

D.2 Household problem

As laid out in Section 2, a household may or may not have a bankruptcy or foreclosure flag and has different problems depending on these flags. However, all problems are stopping time problems and they can be compactly written as an HJBVI. There are two differences when tax is procyclical. First, the budget constraints become

$$\begin{aligned}\dot{a} =& w(\Omega)\varepsilon + (r_a(\omega,\Omega) + \iota(z)|_{a<0})a - (r(\Omega) + \iota_m + \theta)b \\ & - c - T(b,\varepsilon,p(\Omega)h,z) - \xi_h p(\Omega)h|_{o=0} - r_h h|_{o>0}.\end{aligned} \tag{11}$$

The only difference here is the tax function, which becomes $T(b,\varepsilon,p(\Omega)h,z)$ instead of $T(b,\varepsilon,p(\Omega)h)$ because the tax rate now varies with total factor productivity. Specifically, the tax function is $T(y,z) = y - \tau_0(z)y^{1-\tau_1}$. While τ_1 is unchanged at 0.181, τ_0 during expansions and recessions is 0.795 and τ_0 during severe recessions is 0.8. This implies approximately 2.5 percentage aggregate tax during expansions and recessions compared to the benchmark.

Second, households' problems become the following. Let $v^{tax}(\cdot)$ be the value function of the procyclical tax rate economy. The HJBVI for a household is shown below. Households decide whether to continue or to stop and choose any of the stopping options, which are computed the same way as in Equation (3). The value of continuing, the first choice, is the HJB equation as in Equation (2), with the modified budget constraint in Equation (11) and with the extra term $\lambda_{tax}(v(\omega,\Omega) - v^{tax}(\omega,\Omega))$ to account for the possibility of moving back to the economy without the procyclical tax rate. The HJBVI is

$$\begin{aligned}
&\min\Bigg\{\rho v^{tax}(\omega,\Omega)-\max_c u(c,h)-\partial_a v^{tax}(\omega,\Omega)\dot{a}-\partial_b v^{tax}(\omega,\Omega)\dot{b}\\
&-\sum_{j=1}^{n_\varepsilon}\lambda_{\varepsilon\varepsilon_j}v^{tax}(\omega^{\varepsilon_j},\emptyset-\lambda_d(v^{tax}(\omega,\Omega|o=0)-v^{tax}(\omega,\Omega|o=3))_{o=3}\\
&-\lambda_f(v^{tax}(\omega,\Omega|o=0)-v^{tax}(\omega,\Omega|o=2))_{o=2}-\sum_{k=1}^{n_z}\lambda_{zz_k}v^{tax}(\omega,\Omega^{z_k})\\
&-\int\frac{\delta v^{tax}(\omega,\Omega)}{\delta g(\omega)}\mathcal{K}g(\omega)d\omega-\lambda_{tax}(v(\omega,\Omega)-v^{tax}(\omega,\Omega)),\\
&v^{tax}(\omega,\Omega)-v^{tax*}(\omega,\Omega)\Bigg\}=0.
\end{aligned}$$

D.3 Financial intermediaries

Loan price functions for short-term debt and mortgages are determined by the zero expected profit condition of competitive banks, as in Section 2. The only difference is that there is a possibility of moving back to the benchmark economy, and the loan price functions take this possibility into account.

Unsecured debt Let $r_a^{tax}(\omega,\Omega)$ be the short-term loan price function. In the default region ($D_a^{tax}(\omega,\Omega)=1$), and we set

$$r_a^{tax}(\omega,\Omega)=\infty.$$

The zero profit condition in the region of no default implies that the return $r_a^{tax}(\omega,\Omega)$ should be equal to the risk free rate, $r(g,z)$,

$$r_a^{tax}(\omega,\Omega)=r(\Omega)+(1-p_{tax})(\lambda_z\sum_{z'}p_{zz'}\lambda_\varepsilon\sum_{\varepsilon'}p_{\varepsilon\varepsilon'}D_a^{tax}(\omega^{\varepsilon'},\Omega^{z'}))+p_{tax}D_a(\omega,\Omega).$$

Mortgages Since banks expect zero profit on each loan, the discounted value of the loan at origination has to equal its expected cash flow. The price of the loan in the no-default region is given by

$$q_0^{tax}(\omega,\Omega)b_0 = \mathbb{E}\Big[\mathbb{E}_\tau \int_0^\tau e^{-\int_0^s (r_s+\iota_m+\theta)ds}(r_t+\iota_m+\theta)b_0 dt + e^{-\int_0^\tau r_s ds} b(\omega_\tau,\Omega_\tau)\Big].$$

The scrap value $b(\omega_\tau,\Omega_\tau)$ at the stopping point depends on a household's discrete choice. In the case of a foreclosure, $b(\omega_\tau,\Omega_\tau) = (1-\delta_d)p(g,z)h$. When a household prepays its loan due to refinancing or a new house transaction, the scrap value is $e^{-\int_0^\tau \theta_s ds} b_0$.

Applying the Feynman-Kac formula, the above equations can be written as the following partial differential equation. At $t \in [0,\tau)$,

$$\begin{aligned}
(\theta + r(\Omega) + \iota_m)q^{tax}(\omega,\Omega) &= \theta + r(\Omega) + \iota_m + q_a^{tax}(\omega,\Omega)\dot{a} + q_b^{tax}(\omega,\Omega)\dot{b} \\
&+ \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} q^{tax}(\omega^{\varepsilon_j},\Omega) + \sum_{k=1}^{n_z} \lambda_{zz_k} q^{tax}(\omega,\Omega^{z_k}) + \int \frac{\delta q^{tax}(\omega,\Omega)}{\delta g(\omega)} \mathcal{K} g(\omega) d\omega \\
&+ \lambda_{tax}(q(\omega,\Omega) - q^{tax}(\omega,\Omega)).
\end{aligned}$$

When, at $t=\tau$, the stopping time decision is to foreclose,

$$q^{tax}(\omega,\Omega) = \frac{(1-\delta_d)p(\Omega)h}{b},$$

and, if instead, the stopping time decision involves prepayment, we have

$$q^{tax}(\omega,\Omega) = 1.$$

D.4 Government debt dynamics

When there is a policy intervention, the government issues bonds to finance debt relief or a tax rebate. This debt is repaid during expansions through higher tax rates. As before, government consumption is determined by pre-intervention tax rates. Given the tax function before the intervention, $T(b,\varepsilon,p(g,z)h)$, government spending, G, is given by

$$\int T(b,\varepsilon,p(\Omega)h)g(\omega)d\omega + \int (\iota_m b)|_{\iota_m<0}\, g(\omega)d\omega - G = 0.$$

Thus, there is no change in the government consumption function compared to the pre-intervention economy. However, now the government has debt, B, which it repays during expansions using extra tax revenue generated as the difference between the acylical tax function above and the procyclical function described in D.2. From the time the government intervenes to the beginning of an expansion, government debt evolves as

$$\dot{B}_t = r_t B_t.$$

Once the economy transitions to an expansion, the government starts to repay its debt and it evolves as

$$\dot{B}_t = r_t B_t - \int (T(b, \varepsilon, p(\Omega)h, z) - T(b, \varepsilon, p(\Omega)h)) g(\omega) d\omega,$$

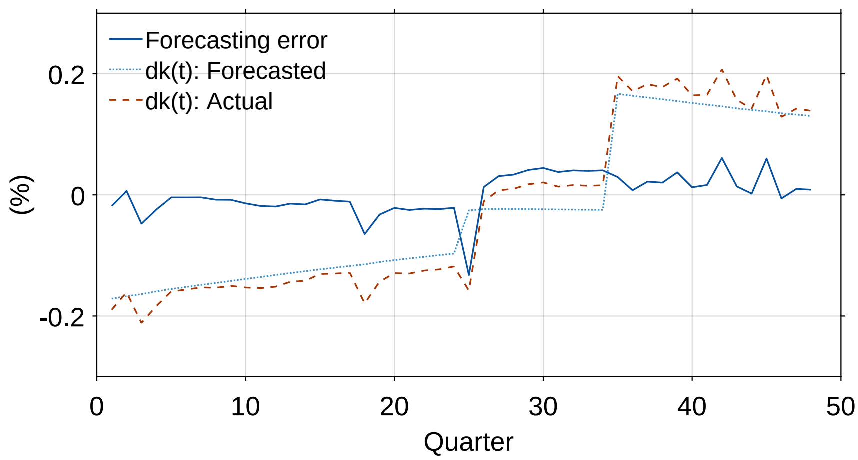
where $T(b, \varepsilon, p(\Omega)h, z)$ is the procyclical tax function. Therefore, the government uses the increase in its tax revenue to repay debt, leaving consumption as it would have been if there had been no change in tax rates for a given distribution of households.

D.5 Error in the forecasting function

The households' and financial intermediaries' problems described in this appendix involve a high-dimensional object in the state vector, the distribution of households. Solving their problems requires knowing how the distribution of households changes. The solution method used is the same as that applied for the model described in Section 2 of the main text. The algorithm is a version of that in Krusell and Smith (1998) and is described in Section A.3.

The solution algorithm uses forecasting rules, which are used to describe the change in the approximate aggregate state. I apply it to solve the economy described here as follows. First I solve the no-intervention model with an acyclical tax function, described in Section 2. The forecasts for this model are drawn from the stationary distribution of the stochastic economy. This is also the long-run model for the policy intervention economy, once debt is repaid and the tax policy returns to its original form. Until the post-intervention public debt is paid,

Figure 7: Forecasting error of the aggregate capital



Note: $dk(t)$ is the change of capital, $\dot{k}_t$. Forecasting error is $\dot{k}_t$(actual) - $\dot{k}_t$(forecasted).

taxes are procyclical and households expect to eventually transition to the long-run model. I solve this model using the forecast rules derived from an economy with a procylical tax policy but no policy intervention or government debt. This model has the same constant probability of permanently transitioning to the long-run model, in households' and firms' expectations, as the policy intervention model. Forecasting rules for this case are derived from a simulation of the economy before a transition.

Once the government issues debt, the aggregate capital has an additional term, B_t, and it becomes $K_t = \int (a-b) g_t d\omega - B_t$. Since there is no government debt in the simulation step, the forecast function for the procyclical tax economy will not fully capture the path of aggregate capital after the policy interventions. For instance, after the intervention and before the recession is over, aggregate capital in the intervention model falls faster than the forecasting function predicts as the government debt grows at the rate $\dot{B}_t = r_t B_t$. However, the forecasting function is estimated assuming $B_t = 0$. Likewise, once the economy transitions to an expansion, the aggregate capital stock grows faster than the speed that the forecasting function predicts because $\dot{B}_t$ is negative, thanks to the government's repayment.

Forecasting errors might mitigate policy effects on consumption and house price. Households' savings decisions involve expectations of future wages and interest rates, and these are functions of the future capital stock. Thus, if there is a downward bias in the forecasting function during expansions, it will lead to higher savings as a lower expected $\dot{k}$ implies a higher return on savings.

Any such error is not likely to significantly change the results because the errors are small. Figure 7 shows the error. At $t = 24$ the economy transitions to an expansion. As explained above, the forecasting errors are negative during the severe recession and positive during the recovery. The largest negative error of 0.13% in the first period of recovery, but most forecasting errors are close to zero.

D.6 Mortgage relief policies in the Great Recession

During the Great Recession, the US government intervened in mortgage markets through household debt relief policies. In particular, the government introduced principal reduction modifications in 2010 in the Home Affordable Modification Program (HAMP). This was a response to growing concerns that debt levels, not just debt repayments, were causing high foreclosure rates. Under this modification, mortgage borrowers' principals were forgiven until their new monthly payment fell below 31% of income or the LTV ratio dropped to 115%, whichever came first. Although participation rates were perceived to be low, Agarwal et al. (2017) show that the program was associated with reduced rates of foreclosure, consumer debt delinquencies and house price declines. I complement studies estimating causal relations, as in Agarwal et al. (2017), by assessing the effects in aggregate quantities from resulting changes in the distribution of households and prices.

E Additional tables and figures

Table 9: Mortgage and unsecured debt payment to income

Moment	**Data**	**Model**	**Source**
50th pctl mortgage payment to income	0.00	0.00	SCF (2007)
70th pctl mortgage payment to income	0.15	0.21	SCF (2007)
90th pctl mortgage payment to income	0.29	0.47	SCF (2007)
50th pctl unsecured debt payment to income	0.00	0.00	SCF (2007)
70th pctl unsecured debt payment to income	0.01	0.00	SCF (2007)
90th pctl unsecured debt payment to income	0.05	0.02	SCF (2007)

Table 10: Portfolio composition

	Asset		**Debt**	
	Non-financial	Financial	Secured	Unsecured
Data				
Q1	-524.62	-112.52	581.57	155.57
Q2	257.43	52.01	-196.31	-13.14
Q3	147.40	36.29	-80.25	-3.44
Q4	98.34	39.16	-36.08	-1.42
Q5	54.03	56.56	-10.038	-0.21
Model				
Q1	117.65	107.44	-123.37	-1.72
Q2	70.33	100.66	-70.80	-0.20
Q3	91.38	81.62	-49.44	-23.56
Q4	86.26	53.29	-37.72	-1.83
Q5	52.30	68.68	-20.97	-0.01

Note: Average portfolio composition by net worth quintiles. Non-financial assets include “Housing and cars” and “Business and non-financial assets” in the SCF. Here, business assets and vehicles are excluded from non-financial assets. Installment loans are excluded. After excluding installment loans, credit card loans are considered as unsecured debt and the remaining compositions of debt are assigned to secured debt.
Data: SCF (2007)

Table 11: Cyclical properties

	Benchmark		Foreclosure only	
	std (%)	corr. w/ Y	std (%)	corr. w/ Y
Output	1.8	1.0	1.8	1.0
Consumption	0.6	0.8	0.4	0.8
Investment	4.3	0.8	5.4	0.7
Unsecured debt	3.0	0.0	-	-
Mortgage	3.9	0.7	2.7	0.7
Bankruptcy	7.7	0.1	-	-
Foreclosure	1.3	-0.7	3.5	0.1
House price	2.6	0.7	2.5	0.6

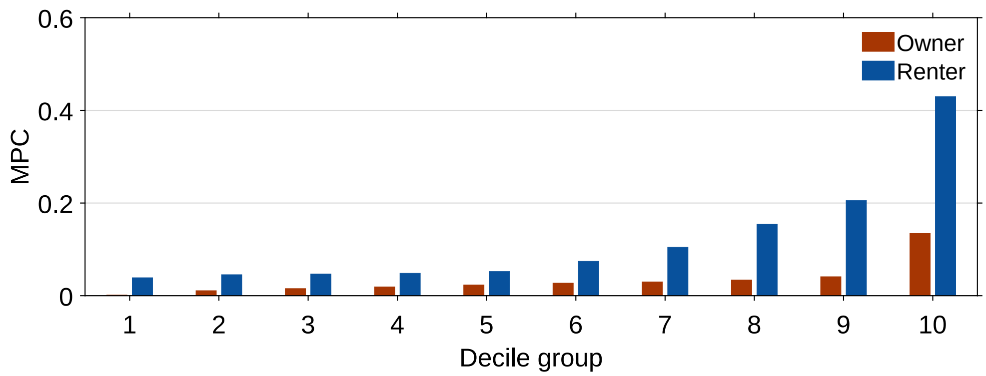
Note: "Bankruptcy" only refers to an economy in which households cannot foreclose but can go bankrupt, and "Foreclosure" only refers to an economy with only foreclosures. None refers to an economy without any option to default. Y in "corr. w/ Y" is output.

Table 12: Output and consumption multipliers

		Output	Consumption
	LTV target: 0.95, mid tax hikes	0.07	-0.01
	LTV target: 0.9, mid tax hikes	0.02	-0.02
Principal reduction	LTV target: 0.85, mid tax hikes	0.00	-0.02
	LTV target: 0.95, low tax hikes	0.03	-0.03
	LTV target: 0.95, high tax hikes	0.21	-0.06
	LTV target: 0.95, mid tax hikes	0.08	-0.01
	LTV target: 0.9, mid tax hikes	0.01	-0.00
Tax rebate	LTV target: 0.85, mid tax hikes	-0.01	-0.00
	LTV target: 0.95, low tax hikes	0.04	-0.00
	LTV target: 0.95, high tax hikes	0.20	-0.03

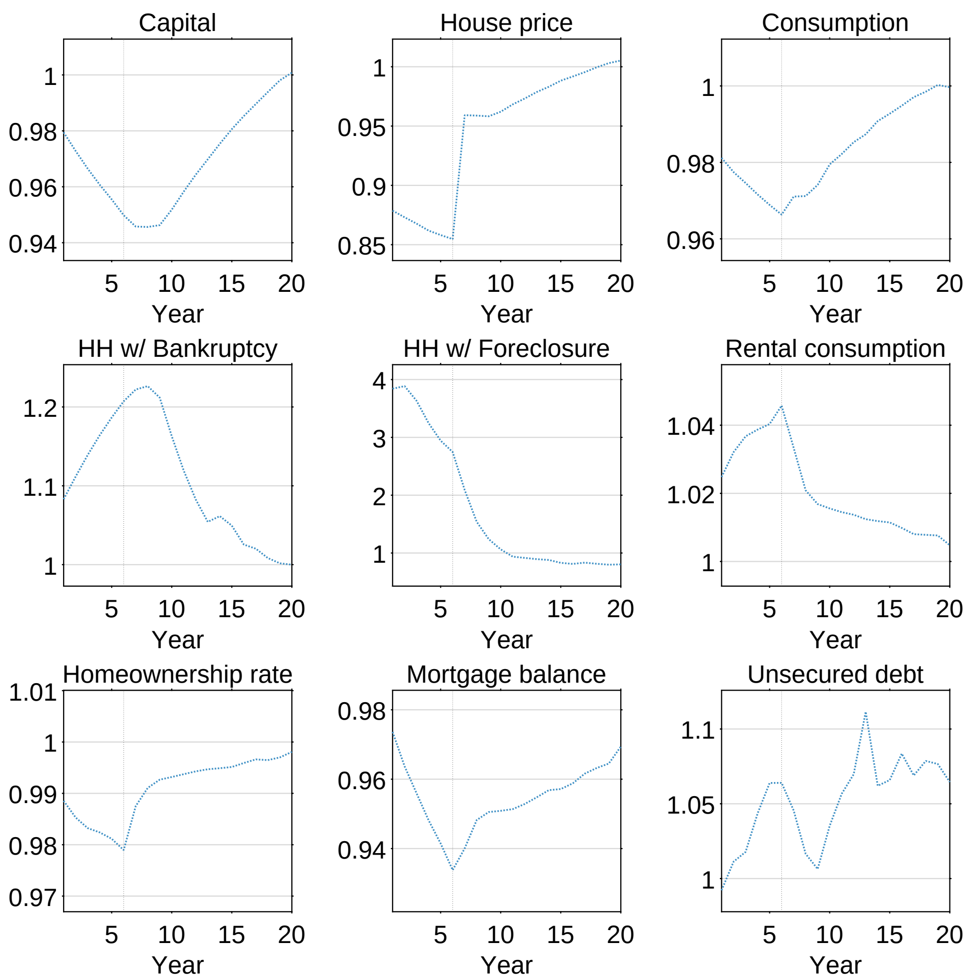
Note: The cumulative fiscal multiplier through time T as the discounted cumulative change in output over government spending, that is, $M_T = \frac{\int_{t=0}^{T} e^{-r_s t}(y_t^p - y_t^b)dt}{\int_{t=0}^{T} e^{-r_s t} G_t dt}$ where y_t^p and y_t^b are outputs or consumptions of the policy economy and the benchmark economy, G_t is the cost of the intervention, and r_s is the average interest rate over time. Consumption is sum of non-durable consumption and housing consumption $(\xi_h p(\Omega) h|_{o=0} + r_h h|_{o>0})$ and T is 24 years. LTV target determines the size of the policy.

Figure 8: Average MPC by decile



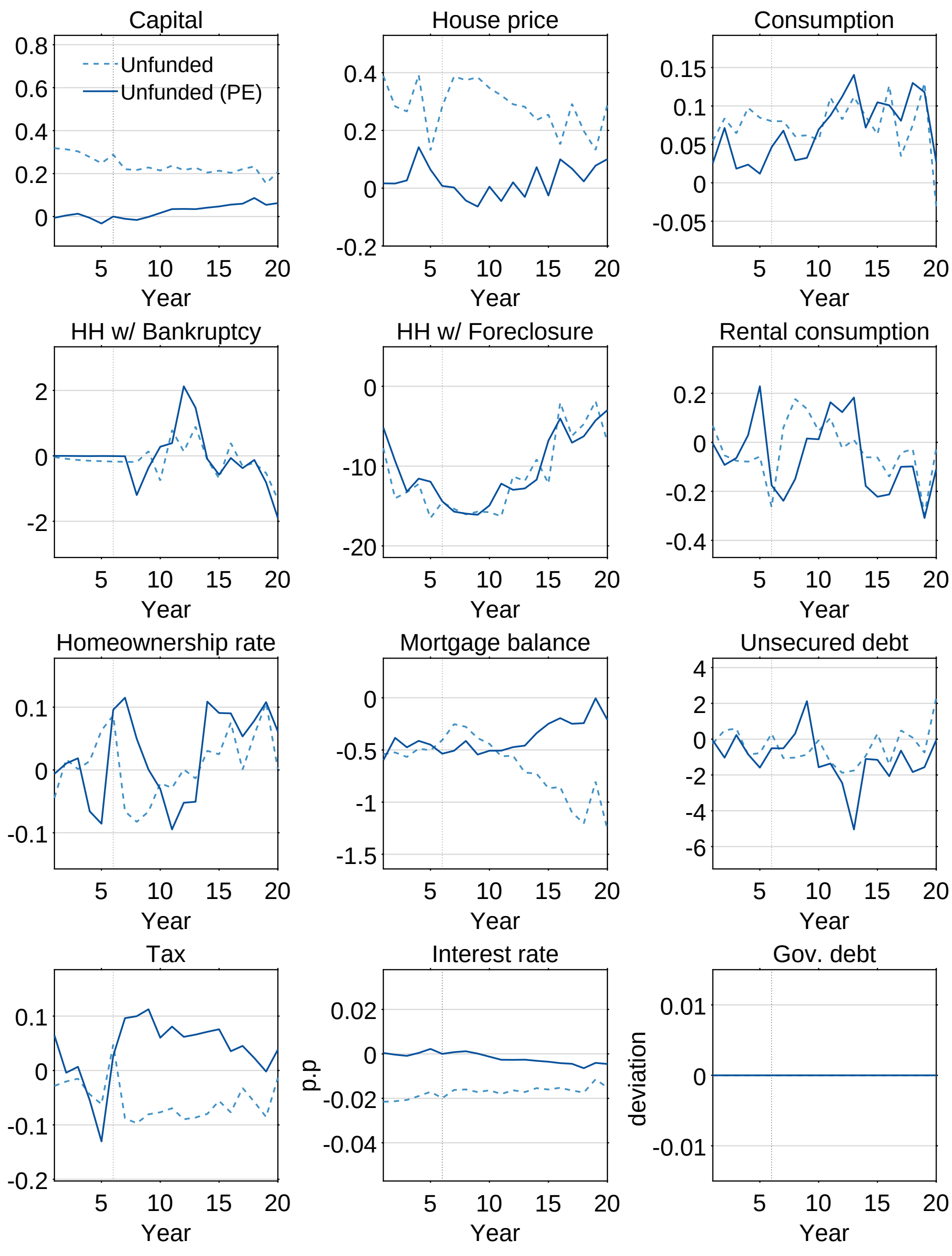
Note: Each bar shows average MPCs of homeowners or renters in a decile group.

Figure 9: Aggregate variables, without a policy intervention



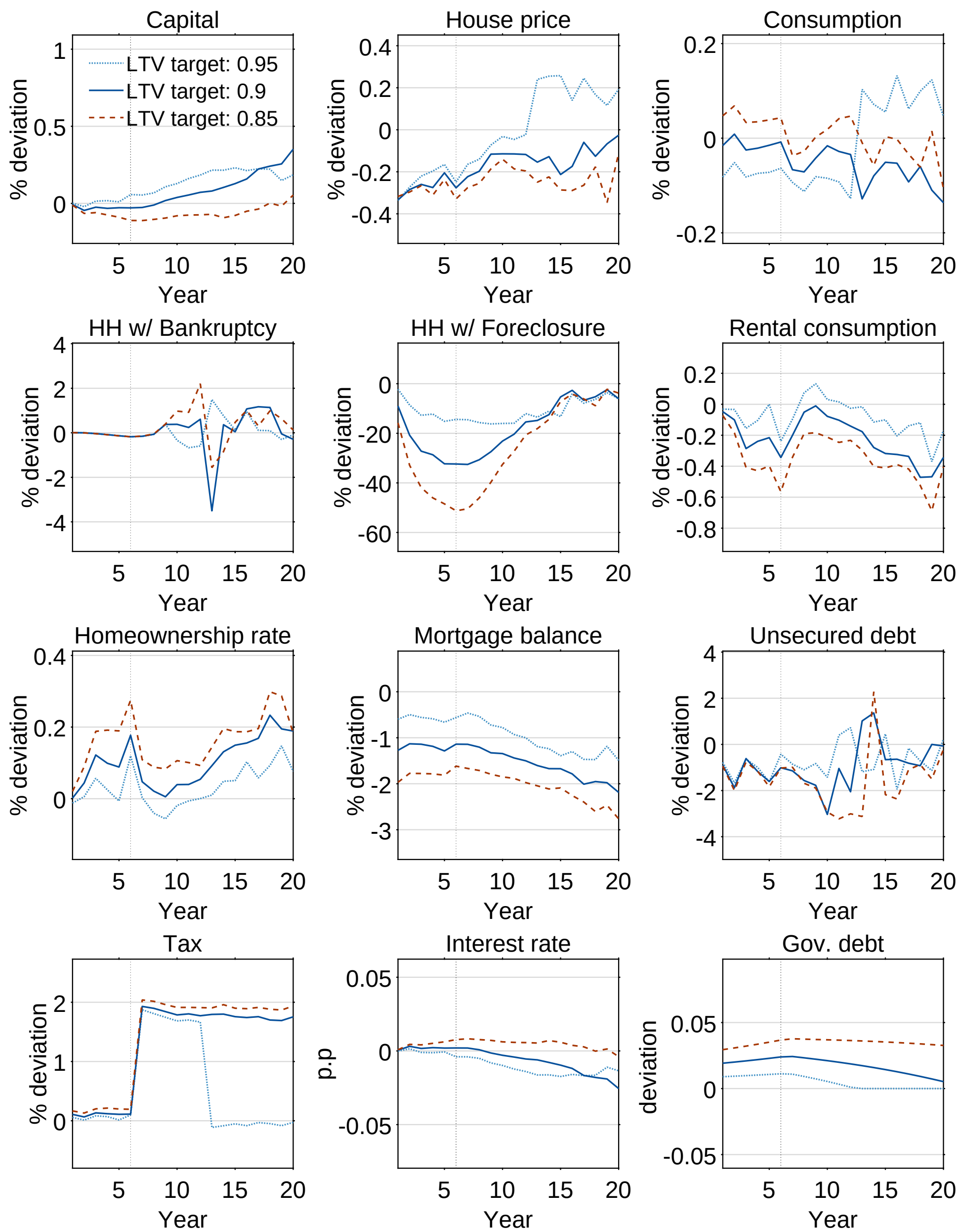
Note: Aggregate variable movements in the baseline economy without policy intervention. All series are normalized to the long-run average of the simulated series.

Figure 10: The effects of unfunded principal reduction



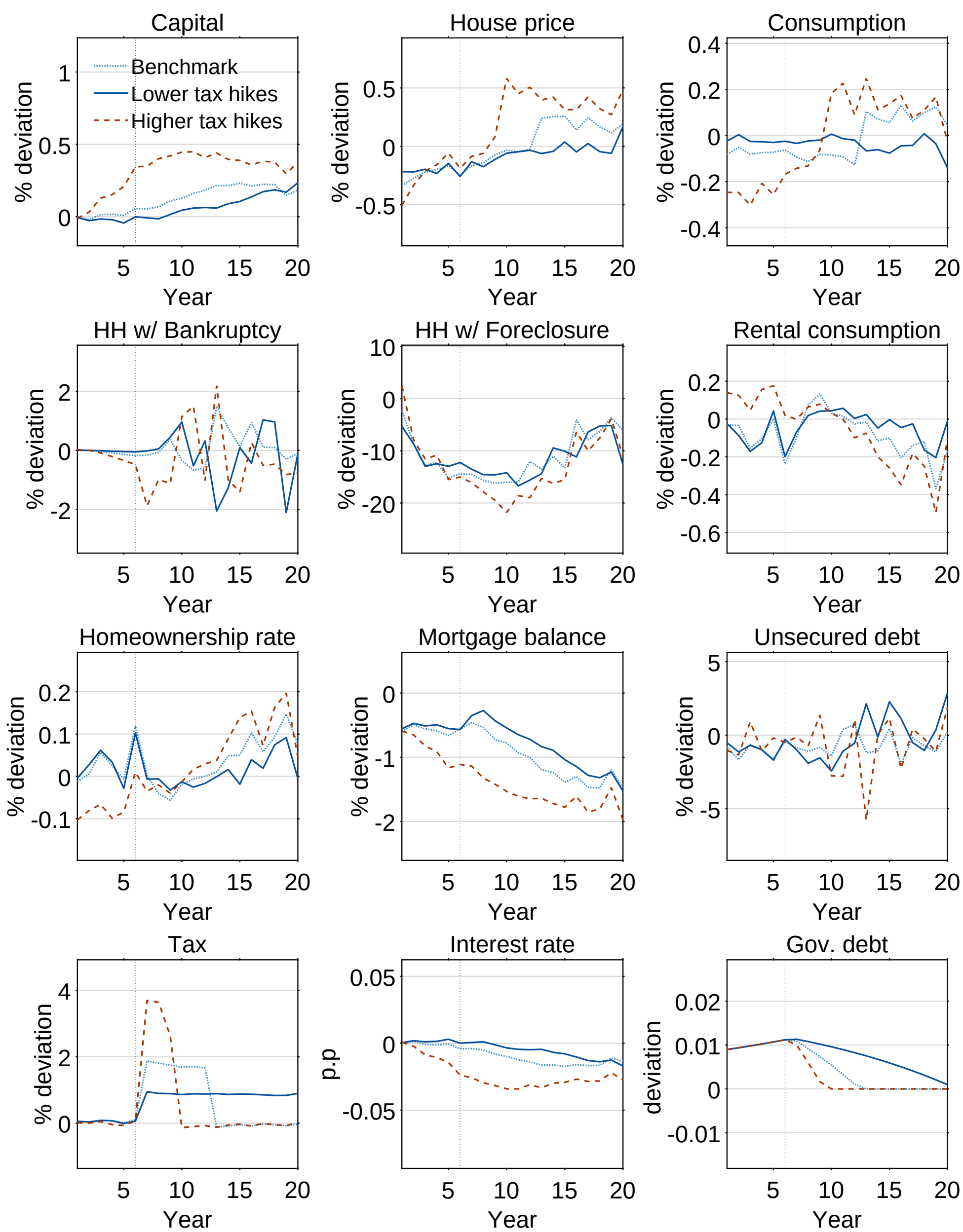
Note: Aggregate series are shown as percentage deviations, percentage point changes, or differences relative to the corresponding series in the no-policy intervention economy. The dashed lines indicate responses to unfunded principal reduction, and the solid lines indicate unfunded principal reduction without the changes in interest rates and wages.

Figure 11: The effects of the principal reduction



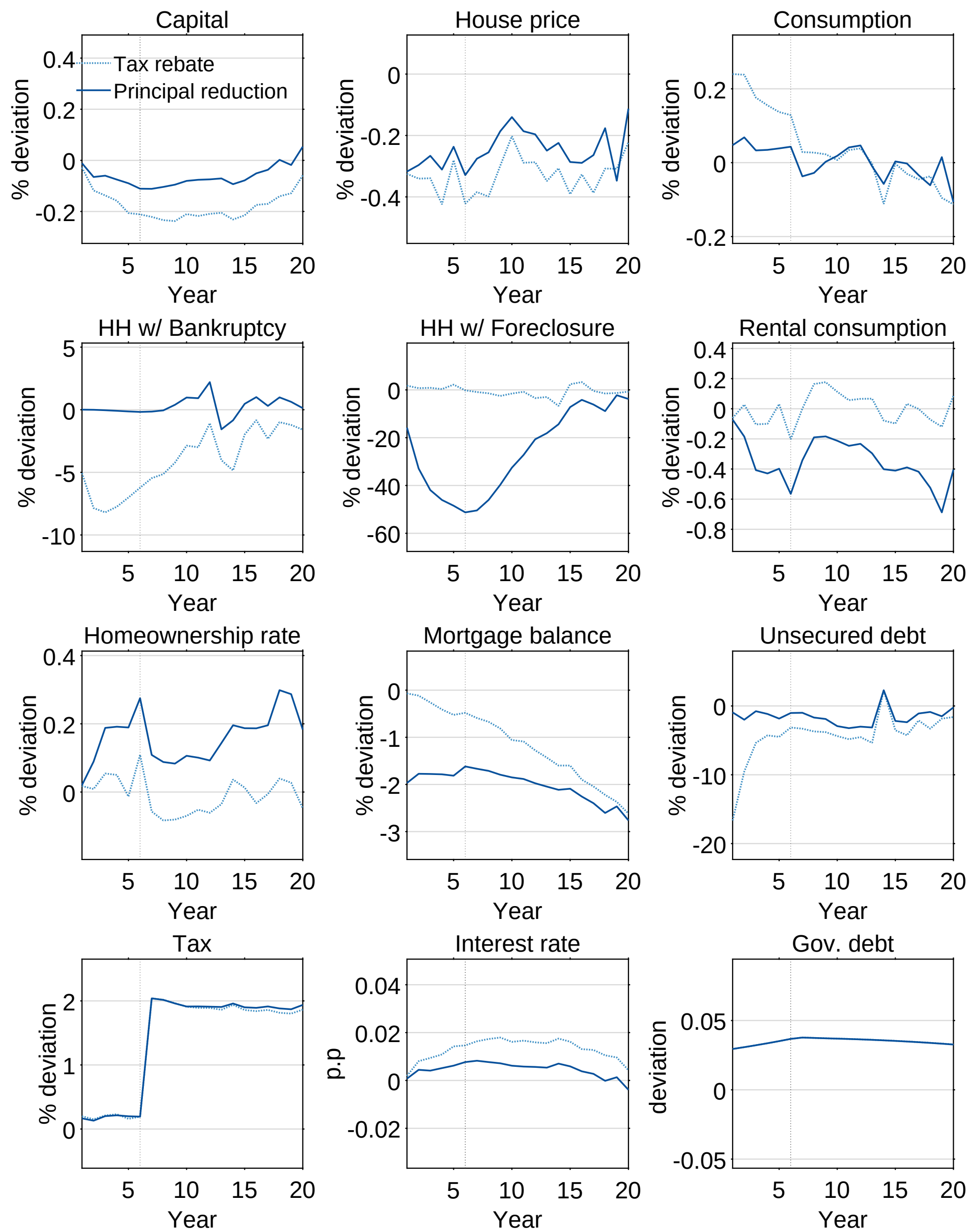
Note: Aggregate series are shown as percentage deviations, percentage point changes, or differences relative to the corresponding series in the no-policy intervention economy. Each line indicates the responses of the policy that reduces mortgages of households with LTV higher than 95/90/80% so that their LTVs become 95/90/80% at the time of the intervention.

Figure 12: The effects of the principal reduction



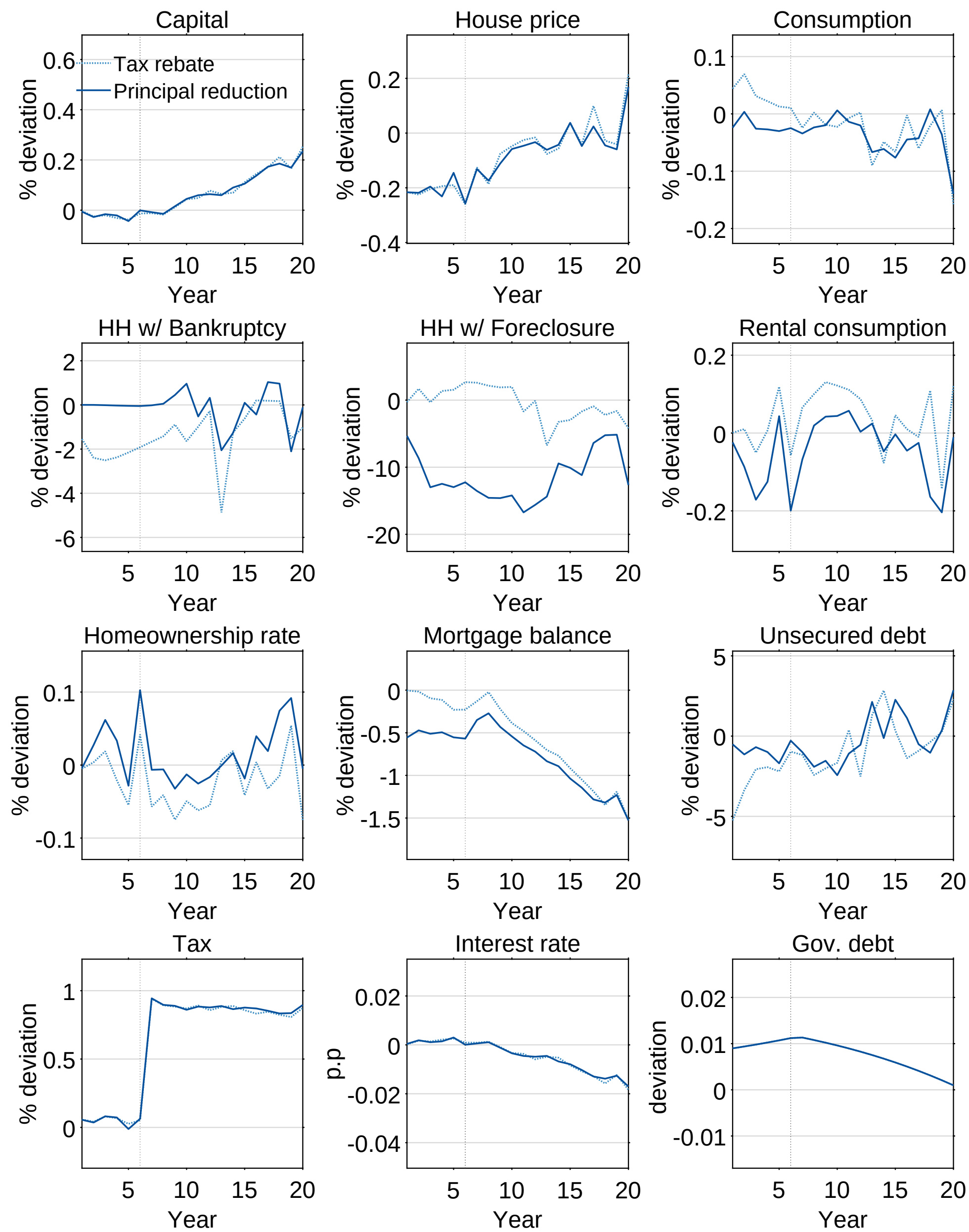
Note: Aggregate series are shown as percentage deviations, percentage point changes, or differences relative to the corresponding series in the no-policy intervention economy. Each line indicates the responses of the policy that reduces mortgages of households with the different size of tax hikes.

Figure 13: Tax rebate vs principal reduction, LTV target = 0.85



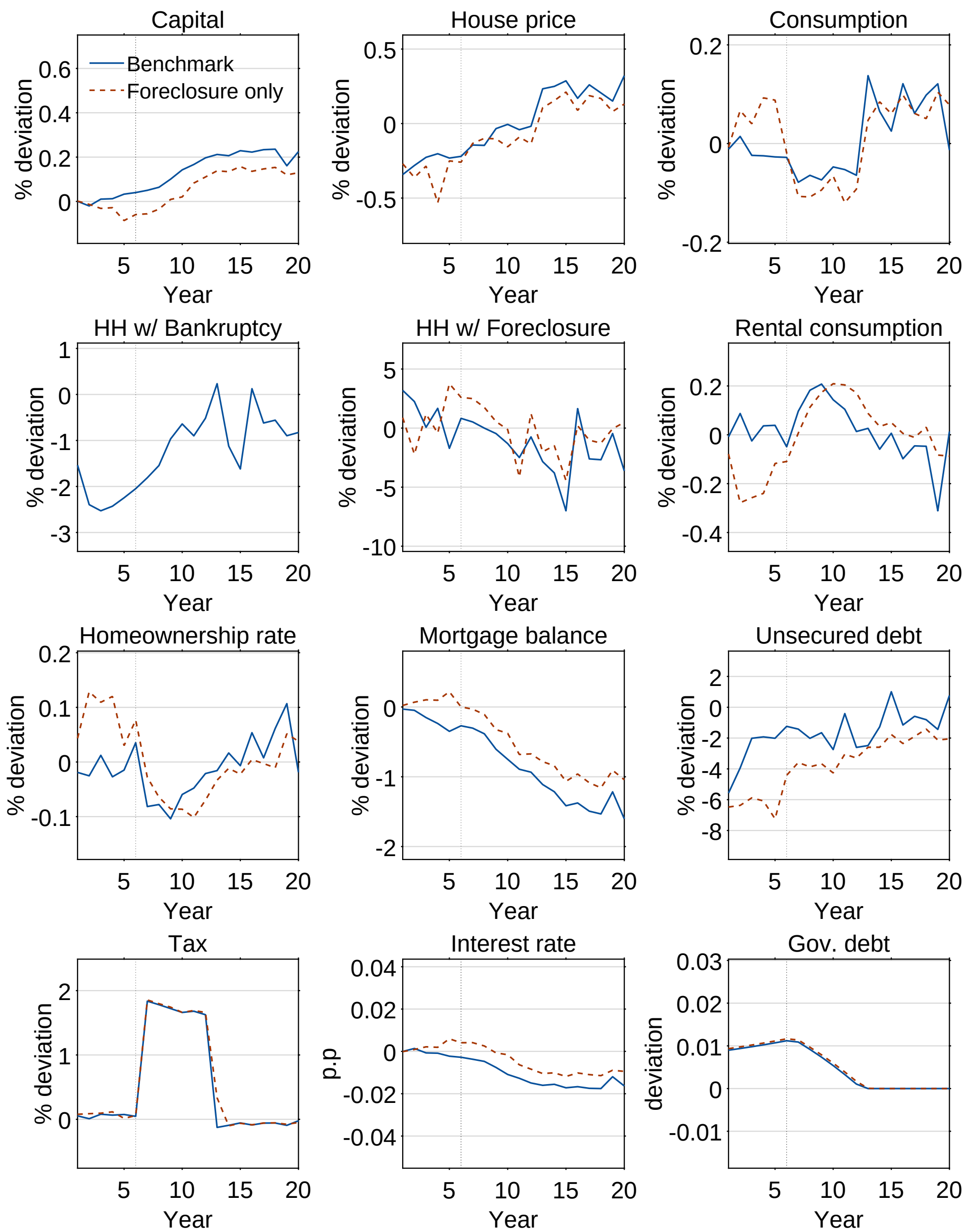
Note: Aggregate series are shown as percentage deviations, percentage point changes, or differences relative to the corresponding series in the no-policy intervention economy. These lines show the responses of aggregate variables when the principal reduction reduces mortgages with households LTV higher 0.85 so that their LTVs become 0.85. The size of the intervention is 3.9% of the GDP.

Figure 14: Tax rebate vs principal reduction, lower tax hikes



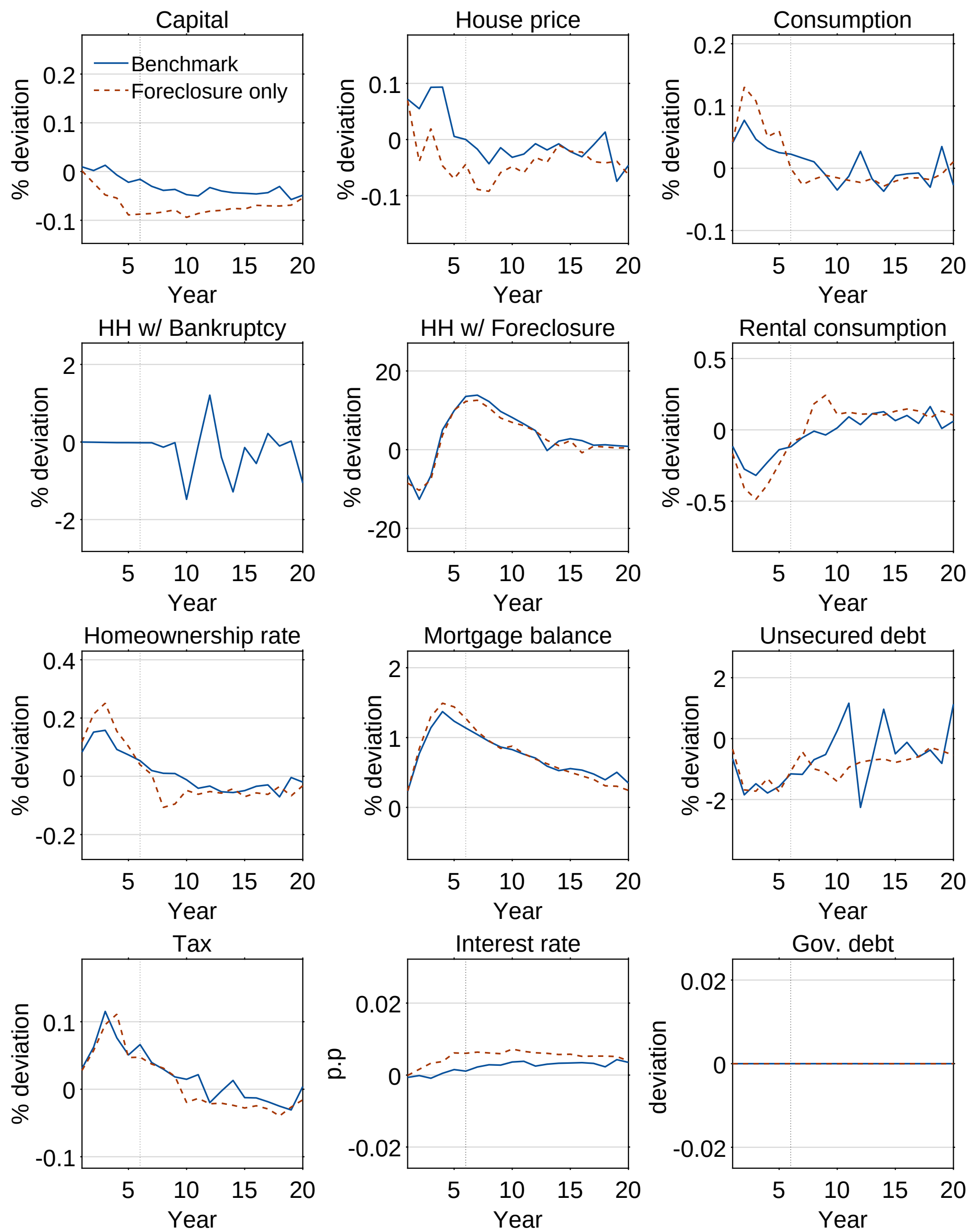
Note: Aggregate series are shown as percentage deviations, percentage point changes, or differences relative to the corresponding series in the no-policy intervention economy. These lines show the responses of aggregate variables when the households expect more moderate tax hikes compared to the benchmark. Aggregate tax rises about 2.0% in the benchmark and rises about 1% in this implementation.

Figure 15: Effects of tax rebate



Note: Aggregate series are shown as percentage deviations, percentage point changes, or differences relative to the corresponding series in the no-policy intervention economy. The solid lines indicate responses to the tax rebate in the model with both bankruptcy and foreclosure option, and the dashed lines indicate responses to the tax rebate in the model with foreclosure option only.

Figure 16: Effects of payment reduction



Note: Aggregate series are shown as percentage deviations, percentage point changes, or differences relative to the corresponding series in the no-policy intervention economy. The solid lines indicate responses to the mortgage payment reduction in the model with both bankruptcy and foreclosure option, and the dashed lines indicate responses to the mortgage payment reduction in the model with foreclosure option only. Households with PTI higher than 50% are eligible fore this policy.

Technical Appendix

A Numerical Solution Method

This section describes the computational method used to solve the model and a measure of accuracy. The existence and uniqueness of the viscosity solution of an HJB equation are shown by Crandall and Lions (1983). The existence of the viscosity solution of an HJBVI equation when a value function is not always differentiable, which corresponds to the problem in the paper, was proven in Øksendal and Sulem (2005) (See chapter 9, theorem 9.8).

The solution algorithm is based on the finite difference method in Achdou et al. (2022) with several important differences. First, there are multiple stopping choices including two types of default, the buying and selling of houses and refinancing and prepayment. Additionally, the model solution is nonlinear in both the individual and aggregate state vectors. Since the aggregate state vector is high-dimensional, I use state-space approximation, following the approach in Krusell and Smith (1998).

A.1 HJBVI as a Linear Complementarity Problem (LCP)

To solve the stopping time problems, for example the three HBJVI problems described in Appendix B and those in D.2, I transform each into a linear complementarity problem.[48] The HJBVI equation can be written as

$$\min\Big[\rho\mathbf{v} - \mathbf{u} - \mathbf{A}\mathbf{v},\ \ \mathbf{v} - \mathbf{v}^*\Big] = 0, \tag{12}$$

where $\mathbf{A}$ summarizes changes caused by decisions and shocks. Below I describe how to construct $\mathbf{A}$. Equation (12) implies

$$\Big(\mathbf{v} - \mathbf{v}^*\Big)'\Big(\rho\mathbf{v} - \mathbf{u} - \mathbf{A}\mathbf{v}\Big) = 0 \tag{13}$$

[48] See `http://www.princeton.edu/ moll/HACTproject/option-simple.pdf` for an example.

$$\rho \mathbf{v} - \mathbf{u} - \mathbf{A}\mathbf{v} \geq \mathbf{0}$$

$$\mathbf{v} \geq \mathbf{v}^*.$$

Let $\mathbf{z} = \mathbf{v} - \mathbf{v}^*$, $\mathbf{B} = \rho\mathbf{I} - \mathbf{A}$ and $\mathbf{q} = -\mathbf{u} + \mathbf{B}\mathbf{v}^*$. Then (13) is equivalent to (14).

$$\mathbf{z}'\big(\mathbf{B}\mathbf{z} + \mathbf{q}\big) = 0 \tag{14}$$

$$\mathbf{B}\mathbf{z} + \mathbf{q} \geq 0$$

$$\mathbf{z} \geq \mathbf{0}.$$

This is the standard form for a linear complementarity problem, and several numerical solvers are available.[49]

Construction of A I solve for optimal decisions over a discretized grid of the state space by iterating on the value function, v. Let Equation (13) be a matrix representation of Equation (10), where $\mathbf{v} = [v_1, v_2, .., v_N]$, $\mathbf{u} = [u_1, u_2, .., u_N]$ and N is the number of points in the value function. Thus, $\mathbf{A}$ describes $\dot{a}$, $\dot{b}$, and the shocks that households are exposed to. Here, I first describe the construction of $\mathbf{A}$ excluding terms related to the aggregate states (g, z), and Section A.3 contains a description of how to solve the model with aggregate uncertainty. Also, I describe the problem of households without default flags, $o = 0$, but the method applies to problems of households with default flags. As $\dot{b}$ is given by $\theta(b, ph)b$ and the earnings process is exogenously set, below I explain how to solve $\dot{a}$.

I choose a number of grid points $(n_a, n_b, n_\varepsilon, n_h)$ for the corresponding variables (a, b, ε, h). Let v_{ijkp} be the value function of a household without a bankruptcy or foreclosure flag, with liquid assets a_i, mortgage b_j, labor productivity ε_k and house h_p. The derivative with respect

[49] I use the LCP solver at `https://www.mathworks.com/matlabcentral/fileexchange/20952-lcp-mcp-solver-newton-based`.

to a, v^a_{ijkp} is approximated with either a forward or a backward first difference:

$$v^a_{ijkp} \approx \frac{v_{i+1jkp} - v_{ijkp}}{\Delta a} = v^{a,F}_{ijkp} \text{ or } v^a_{ijkp} \approx \frac{v_{ijkp} - v_{i-1jkp}}{\Delta a} = v^{a,B}_{ijkp}.$$

Likewise, the derivative with respect to b, v^b_{ijkp} can be approximated with a forward or backward difference.

Applying this method to the first argument in Equation (10), we have

$$\begin{aligned} \rho v_{ijkp} = u(c_{ijkp}, h_p) + v^a_{ijkp}\dot{a}_{ijkp} + v^b_{ijkp}\dot{b}_{ijkp} + \sum_{\varepsilon'_k} \lambda(\varepsilon_k, \varepsilon_{k'})(v_{ijk'p} - v_{ijkp}) \\ \forall\ i = \{1, .., n_a\}, j = \{1, .., n_b\}, k = \{1, .., n_\varepsilon\}, p = \{1, .., n_h\}, \end{aligned} \tag{15}$$

where

$$\dot{a}_{ijkp} = w\varepsilon_k + r_{a,ijkp}a_i - (r+\theta)b_j - c_{ijkp} - T(b_j, ph_p, \varepsilon_k) - \xi_h p h_p|_{o=0} - r_h h_p|_{o>0},$$

$$\dot{b}_{ijkp} = -\theta b_j.$$

The household choice of non-durable consumption can be solved from the FOC:

$$\begin{aligned} u^c(c_{ijkp}, h_p) - v^a_{ijkp} = 0, \\ c_{ijkp} = (v^a_{ijkp})^{\frac{1}{-\sigma}}. \end{aligned}$$

As the derivatives of the value function have two forms, forward and backward, c_{ijkp} is either $(v^{a,F}_{ijkp})^{\frac{1}{-\sigma}}$ or $(v^{a,B}_{ijkp})^{\frac{1}{-\sigma}}$.

To find the drift, $\dot{a}$, it is necessary to select which derivative to use. I follow Achdou et al.'s (2022) upwind scheme. The key idea is to use a forward derivative when the drift is positive and a backward derivative when it is negative. To ease notation, for variable x, let $x^+ = max(x, 0)$ and $x^- = min(x, 0)$. Also, let x^F be the value computed using a forward derivative and x^B be the value derived from the backward derivative. With this notation, savings, $s(=\dot{a})$, can be computed as below:

$$\begin{aligned} s^{c,F}_{ijkp} &= w\varepsilon_k + r_{a,ijkp}a_i - (r+\theta)b_j - c^F_{ijkp} - T(b_j, ph_p, \varepsilon_k) - \xi_h p h_p|_{o=0} - r_h h_p|_{o>0} \\ s^{c,B}_{ijkp} &= w\varepsilon_k + r_{a,ijkp}a_i - (r+\theta)b_j - c^B_{ijkp} - T(b_j, ph_p, \varepsilon_k) - \xi_h p h_p|_{o=0} - r_h h_p|_{o>0}. \end{aligned} \tag{16}$$

The upwind scheme can be applied to all of the points in the state space except for the points at the boundaries. Clearly, only one of the forward or backward derivatives can be approximated using a finite difference at the boundaries. The way this is handled for an exogenous borrowing constraint in a one-asset model is explained in Achdou et al. (2022). In my model, there are three assets. Across the two loans, mortgages are secured but there may be an endogenous borrowing limit with respect to a as there is an option to default. Over levels of a where a household would default, there cannot be lending and an endogenous bound is imposed such that $\dot{a} \geq 0$. I explain the details of how I handle boundaries in the rest of this section.

I follow Bornstein (2018), who shows how to solve the problem of an endogenous borrowing limit with respect to a. For each $(b_j, \varepsilon_k, h_p)$, $\forall\, j = \{1, .., n_b\}, k = \{1, .., n_\varepsilon\}, p = \{1, .., n_h\}$, assume that we know the level of unsecured debt, $\underline{a}(b_j, \varepsilon_k, h_p)$, where a household chooses to default when a is below $a_{Djkp} = \underline{a}(b_j, \varepsilon_k, h_p)$. The value function below a_{Djkp} becomes flat over a, and the backward derivative does not exist at $(D, b_j, \varepsilon_k, h_p)$. I impose the endogenous borrowing constraint $\underline{a}(b_j, \varepsilon_k, h_p)$; below this point, I restrict consumption to equal income. One potential issue is consumption at the endogenous borrowing limit (or below the limit) can be negative. If this is the case, I assign a very low value to ensure that these are default points. Note that a will not fall below $\underline{a}(b_j, \varepsilon_k, h_p)$ as the household would have defaulted beforehand. Without loss of generality I set savings to be zero at these points.

Finally, we must know the set of default points, $i \leq D$, at each $(b_j, \varepsilon_k, h_p)$ to find $\underline{a}(b_j, \varepsilon_k, h_p)$. As I solve the value function iteratively, I used the default points identified by the last iteration to set $\underline{a}(b_j, \varepsilon_k, h_p)$. Stopping points including default points are given by LCP solution algorithms, as will be explained at the end of this subsection. Effectively, the endogenous borrowing limit is implemented at each iteration of the solution algorithm as follows. At iteration n, let D^A be a set of points where a household will default on a or on both a and b. If a point $[a_i, b_j, \varepsilon_k, h_p] \in D^A$, set $s^{c,F}_{ijkp} = s^{c,B}_{ijkp} = 0$ in Equation (17). These savings functions are the endogenous component of $\mathbf{A}$ in (13), which is used to solve v, as described next.

Before describing how to build $\mathbf{A}$, one last issue remains. At a_{n_a}, a forward derivative cannot be computed. However, if I set a_{n_a} large enough, savings at this point will be negative and the forward derivative will not be needed.

I now describe matrix $\mathbf{A}$ in (13). Using (16) in (15), the system of equations for the value function can be written as

$$\begin{aligned}\rho v_{ijkp} &= u(c_{ijkp}, h_p) + x^a_{ijkp} v_{i-1jkp} + y_{ijkp} v_{ijkp} + z^a_{ijkp} v_{i+1jkp} \\ &\quad + x^b_{ijkp} v^{n+1}_{ij-1kp} + z^b_{ijkp} v_{ij+1kp} + \sum_{k'} \lambda_{kk'} v_{ijk'p},\end{aligned} \tag{17}$$

$$x^a_{ijkp} = -\frac{(s^{c,B})^-}{\Delta a}, \qquad x^b_{ijkp} = -\frac{(\theta b)^-}{\Delta b},$$

$$y_{ijkp} = -\frac{(s^{c,F})^+}{\Delta a} + \frac{(s^{c,B})^-}{\Delta a} - \frac{(\theta b)^+}{\Delta b} + \frac{(\theta b)^-}{\Delta b} + \lambda_{kk},$$

$$z^a_{ijkp} = \frac{(s^{c,F})^+}{\Delta a}, \qquad z^b_{ijkp} = \frac{(\theta b)^+}{\Delta b}.$$

There are $n_a \times n_b \times n_\varepsilon \times n_h$ linear equations (17), one for each grid point. The system of equations can be written in matrix notation:

$$\rho \mathbf{v} = \mathbf{u} + \mathbf{A}\mathbf{v}. \tag{18}$$

Matrix $\mathbf{A}$ can be constructed step by step. First, I define a submatrix for a given level of housing. For each h_p, $\mathbf{A}_p, p \in \{1, 2, ..., n_h\}$ can be written as

$$\mathbf{A}_p = \begin{bmatrix} A_{11|p} & A_{12|p} & .. & A_{1n_\varepsilon|p} \\ A_{21|p} & A_{22|p} & .. & A_{2n_{\varepsilon|p}} \\ \vdots & \ddots & & \vdots \\ A_{n_\varepsilon 1|p} & A_{n_\varepsilon 2|p} & .. & A_{n_\varepsilon n_\varepsilon|p} \end{bmatrix},$$

where $A_{kk|p}$ is a matrix that is composed of x^a_{ijkp}, x^b_{ijkp}, y_{ijkp}, z^a_{ijkp} and z^b_{ijkp}, $k \in \{1, 2, ..., n_\varepsilon\}$. For example,

$$A_{11|2} = \begin{bmatrix} y_{1112} & z^a_{1112} & 0 & .. & 0 & z^b_{1112} & 0 & .. & 0 & 0 & 0 & .. & & 0 \\ x^a_{2112} & y_{2112} & z^a_{2112} & 0 & .. & 0 & z^b_{2112} & 0 & ... & 0 & 0 & 0 & .. & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & .. & 0 & x^a_{n_a 112} & y^a_{n_a 112} & 0 & .. & 0 & z^b_{n_a 112} & & 0 & ... & 0 & 0 \\ x^b_{1212} & .. & & & 0 & y_{1212} & z^a_{1212} & 0 & .. & 0 & z^b_{1212} & 0 & & 0 \\ \vdots & x^b_{2212} & \ddots & \ddots & \ddots & x^a_{2212} & y_{2212} & z^a_{2212} & \ddots & \ddots & \ddots & z^b_{2212} & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & .. & & & & & & x^a_{n_a n_b 12} & y^a_{n_a n_b 12} & 0 & .. & & 0 & z^b_{n_a n_b 12} \end{bmatrix}$$

and when $k \neq l$, $A_{kl|p}$ is a diagonal matrix with diagonal terms λ_{kl}. Using $\mathbf{A_p}$, $\mathbf{A}$ is a block diagonal matrix composed of $\mathbf{A}_1$,..,$\mathbf{A}_{n_h}$,

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & 0 & .. & & 0 \\ 0 & \mathbf{A}_2 & 0 & .. & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & .. & & 0 & \mathbf{A}_{n_h} \end{bmatrix}.$$

These steps have described how to create the system of equations for a household described by (10) with neither bankruptcy nor foreclosure in its credit record. There are similar systems of linear equations describing a renter, a household with a bankruptcy flag or foreclosure flag.

The systems of equations in (18) are converted into the form in (14), then solved iteratively using an LCP solver. Let $\mathbf{z} = \mathbf{v}^{n+1} - \mathbf{v}^{*,n}$, $\mathbf{B} = \frac{1}{\Delta} - \rho - \mathbf{A}$ and $\mathbf{q} = \mathbf{B}\mathbf{v}^{*,n} - \frac{\mathbf{v}^n}{\Delta} - \mathbf{u}^n$. Then

$$(\mathbf{v}^{n+1} - \mathbf{v}^{*,n})(\frac{\mathbf{v}^{n+1} - \mathbf{v}^n}{\Delta} - \rho\mathbf{v}^{n+1} - \mathbf{u}^n - \mathbf{A}\mathbf{v}^{n+1}) = 0$$

$$\frac{\mathbf{v}^{n+1} - \mathbf{v}^n}{\Delta} - \rho\mathbf{v}^{n+1} - \mathbf{u}^n - \mathbf{A}\mathbf{v}^{n+1} \geq 0$$

$$\mathbf{v}^{n+1} - \mathbf{v}^{*,n} \geq 0$$

is equivalent to

$$\mathbf{z}'(\mathbf{B}\mathbf{z} + \mathbf{q}) = 0$$

$$\mathbf{B}\mathbf{z} + \mathbf{q} \geq 0$$

$$\mathbf{z} \geq \mathbf{0},$$

which is the LCP problem from (14). As mentioned, I describe the value function solution algorithm below. The parameter Δ determines the speed of updating.

- Guess the value functions $\mathbf{v}^n$.
- Using the guessed value functions, construct $\mathbf{A}$.
- Compute $\mathbf{v}^{*,n}$ by solving Equation (3).
- Let $\mathbf{z} = \mathbf{v}^{n+1} - \mathbf{v}^{*,n}$, $\mathbf{B} = \frac{1}{\Delta} - \rho - \mathbf{A}$ and $\mathbf{q} = \mathbf{B}\mathbf{v}^{*,n} - \frac{\mathbf{v}^n}{\Delta} - \mathbf{u}^n$; solve the LCP.
- Using the solution to the LCP, set $\mathbf{v}^{n+1} = \mathbf{z} + \mathbf{v}^{*,n}$.
- With the updated value function $\mathbf{v}^{n+1}$, if $\max|\mathbf{v}^{n+1} - \mathbf{v}^n|$ is not small enough, return to the second step.

Above, $\mathbf{z}$ is the solution provided by the LCP solver, and, where $\mathbf{z} = 0$, households find it optimal to stop. Recall that a region of default on unsecured debt is necessary to construct $\mathbf{A}$ for the next iteration. This is found from $\mathbf{z}$ as follows. In every iteration, D^A is the set of points that satisfy $\{\mathbf{z} = 0\} \cap \{\mathbf{v}^{*,n} = \mathbf{v}^{a,n}\}$. Finally, with many discrete choices, I find using a large value of Δ (for example, larger than 10) often leads to unstable updates.

A.2 Kolmogorov forward equation

After solving the value functions, I need to solve for the household density over their assets, mortgages, houses, labor productivity and default flags. In this section, I describe how to solve the density function.

Without stopping decisions, the Kolmogorov forward equation is

$$\partial_t g_{ijkp,t} = -\partial_a s^a_{ijkp} g_{ijkp,t} - \partial_b s^b_{ijkp} g_{ijkp,t} + \sum_{k'} \lambda_{k'k} g_{ijk'p,t},$$

where s^x_{ijkp} is shorthand notation for x decision rule at $(a_i, b_j, \varepsilon_k, h_p)$, $x \in \{a, b\}$ and $g_{ijkp,t}$ is a density function at time t. In my model, I need to account for i) movements due to housing transactions, refinancing, bankruptcy and foreclosure, ii) flows between a state without the default flag and a state with the bankruptcy flag and iii) flows between a state without the default flag and a state with the foreclosure flag.

The mathematical formulation of Kolmogorov forward equations with stopping choices is not straightforward.[50] Flows due to stopping decisions can be treated with the *intervention matrix*, M. First, let g_i be the i^{th} element of the density function where $i \in \{1, ...N\}$ and N is the total number of grid points:[51]

$$M_{i,j} = \begin{cases} 1 & \text{if } i \in I \text{ and } i = j \\ 1 & \text{if } i \notin I \text{ and } j^*(i) = j \\ 0 & \text{otherwise,} \end{cases}$$

where I is the non-stopping region and $j^*(i)$ is the target point of point i. A target point is a point arrived at as a result of a stopping choice such as buying a house. For example, if a household with $(a_i, b_k, \varepsilon_j, h_p)$ decides to buy a house and ends up having $(a_{i1}, b_{k1}, \varepsilon_j, h_{p1})$ as

[50] See "Liquid and Illiquid Assets with Fixed Adjustment Costs" by Greg Kaplan, Peter Maxted and Benjamin Moll at `https://benjaminmoll.com/wp-content/uploads/2020/06/liquid_illiquid_numerical.pdf`

[51] $N = n_a \times n_b \times n_\varepsilon \times n_h + n_a \times n_\varepsilon \times n_h \times 3$. $n_a \times n_\varepsilon \times n_h$ is the number of points for renters without default flags, renters with foreclosure flags, and renters with bankruptcy flags.

a result of the transaction, the latter is the target point.

In footnote 11, I mention that utility costs of discrete choices costs are included not only for calibration purposes but also for computation. Without them, I find that some points where households choose to arrive as a result of discrete choices (target points), $(a', b', \varepsilon, h', o')$, can coincide with the points where households choose to leave by making discrete choices (stopping points), $(a_\tau, b_\tau, \varepsilon, h_\tau, o_\tau)$. These points are problematic as they may lead to negative mass in a distribution. Such points can be eliminated by adjusting the size of utility costs.

This is common issue when solving stopping time problems, as we discretize continuous variables (liquid assets, mortgages) and solve problems on grids. This issue is also addressed in the codes for 'Liquid and Illiquid Assets with Fixed Adjustment Costs' by Greg Kaplan, Peter Maxted, and Benjamin Moll, available at `https://benjaminmoll.com/wp-content/uploads/2020/06/liquid_illiquid_numerical.pdf`.

The flow from a state with bankruptcy and foreclosure flags to a state without a bankruptcy flag is a shock and can be expressed as below. Let nd represent "non-default" and d represent "default":

$$\partial_t g^{nd}_{ijkp,t} = -\partial_a s^{a,nd}_{ijkp} g^{nd}_{ijkp,t} - \partial_b s^{b,nd}_{ijkp} g^{nd}_{ijkp,t} + \sum_{k'} \lambda_{k'k} g^{nd}_{ijk'p,t} + \lambda_l g^{d}_{ijkp,t},$$

$$\partial_t g^{d}_{ijkp,t} = -\partial_a s^{a,d}_{ijkp} g^{d}_{ijkp,t} - \partial_b s^{b,d}_{ijkp} g^{d}_{ijkp,t} + \sum_{k'} \lambda_{k'k} g^{d}_{ijk'p,t} - \lambda_l g^{d}_{ijkp,t},$$

where $\lambda_l = \lambda_d$ for the bankruptcy flag state and $\lambda_l = \lambda_f$ for the foreclosure flag state. These flows can be treated with matrices $\mathcal{A}^d$ and $\mathcal{A}^f$.

$\mathcal{A}^d_{i,j} = -\lambda_d$ where i's are locations of households with bankruptcy flags and $i = j$. $\mathcal{A}^d_{i,j} = \lambda_d$ where i's are locations of households with bankruptcy flags and j's are the destination once bankruptcy flags are removed. Otherwise, $\mathcal{A}^d_{i,j} = 0$. $\mathcal{A}^f_{i,j}$ can be filled similarly. The sizes of $\mathcal{A}^d$ and $\mathcal{A}^f$ are $N \times N$, and I stack points from the state without default flags (homeowners then renters), points from the state with the foreclosure flag and points from the state with the bankruptcy flag. Finally, define B as below:

$$B = \mathcal{A} + \mathcal{A}^d + \mathcal{A}^f,$$

where $\mathcal{A}$ is a block diagonal matrix that is composed of $\mathbf{A}$, $\mathbf{A}^d$ and $\mathbf{A}^f$, which summarize $\dot{a}$, $\dot{b}$, and shock to ε.

$$\mathcal{A} = \begin{bmatrix} \mathbf{A} & 0 & 0 \\ 0 & \mathbf{A}^d & 0 \\ 0 & 0 & \mathbf{A}^f \end{bmatrix}.$$

Given M and B, the density function can be solved by iterating over the following two steps until g converges:

$$g^{n+\frac{1}{2}} = M^T g^n,$$

$$\frac{g^{n+1} - g^{n+\frac{1}{2}}}{\Delta t} = (BM)^T g^{n+1}.$$

A.3 Stochastic model

The aggregate state contains an argument that has an infinite dimension, g, the distribution of households over $\omega = (a, b, \varepsilon, h, o)$. To make the computation feasible, this distribution needs to be approximated. I assume the households only use a finite set of moments from g to form their expectations, as in Krusell and Smith (1998).[52] Specifically, I assume that the households keep track of the aggregate capital stock, k.

I redefine the problem using the approximate state, (k, z). For example, Equation (2) can

[52] Ahn et al. (2018) develop a method of solving continuous time heterogeneous agent models with aggregate uncertainty based on linearization and dimension reduction. Fernández-Villaverde et al. (2023) also present a method to solve such models. They assume the households only track a finite set of moments of the distribution to form their expectations as well, but use tools from machine learning to estimate the perceived law of motion of the households.

be written as

$$
\begin{aligned}
\rho v(\omega, k, z) = \max_{c} u(c,h) &+ \partial_a v(\omega,k,z)\dot{a} + \partial_b v(\omega,k,z)\dot{b} + \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v(\omega^{\varepsilon_j}, k, z) \\
&+ \sum_{k=1}^{n_z} \lambda_{zz_k} v(\omega,k,z_k) + \lambda_d (v(\omega,k,z|o=0) - v(\omega,k,z|o=3))_{o=3} \\
&+ \lambda_f (v(\omega,k,z|o=0) - v(\omega,k,z|o=2))_{o=2} + \partial_k v(\omega,k,z)\dot{k}, \\
\dot{k}_t = \frac{\mathbb{E}[dk_t; k_t, z_t]}{dt} &= f^k(k_t, z_t).
\end{aligned}
$$

The last term in Equation (2), which captures the evolution of the distribution, is replaced with $\partial_k v(a,b,\varepsilon,h,k,z)\dot{k}$. I assume f has a log-linear form:

$$
dlog(k_t)dt = \beta_z^0 + (\beta_z^1 - 1)log(k_t).
$$

Given the approximate aggregated state, equilibrium wage rates and interest rates are marginal productivities of labor and capital. Since hours worked is not a choice, aggregate labor is fixed. Therefore, computing wage rates and interest rates at a given capital is trivial. However, in addition to wage rates and interest rates, house prices, $p(k,z)$, are necessary to solve the model. I also assume a log-linear form to estimate house prices:

$$
log(p_t) = \phi_z^0 + \phi_z^1 log(k_t).
$$

In Section A.1, I described the computational steps in the absence of an aggregate state (g,z). Having approximated the high-dimensional object g with k, now I include terms related to the aggregate state. Like other variables, I discretize k, and n_k is the number of grid points for k. From Section A.1, the only part needing to be changed is the construction of matrix $\mathbf{A}$, which describes $\dot{a}$, $\dot{b}$ and the shocks. Consider the $\mathbf{A}$ in Section A.1 as an $\mathbf{A}$ at a given (k_i, z_j), $\mathbf{A}^{ij}$. Then $\mathbf{A}$ should be replaced with $\mathbb{A}_a + \mathbb{A}_k + \mathbb{A}_z$, where

$$\mathbb{A}_a = \begin{bmatrix} \mathbf{A}^{11} & & & & & & \\ & \ddots & & & & & \\ & & \mathbf{A}^{n_k 1} & & & & \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ & & & & \mathbf{A}^{1 n_z} & & \\ & & & & & \ddots & \\ & & & & & & \mathbf{A}^{n_k n_z} \end{bmatrix}$$

$$\mathbb{A}_k = \begin{bmatrix} x_{11}\mathbf{I} & x^F_{11}\mathbf{I} & & & & & \\ x^B_{21}\mathbf{I} & x_{21}\mathbf{I} & x^F_{21}\mathbf{I} & & & & \\ & & & & & & \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ & & & & & & \\ & & & & & \ddots & \\ & & & & & x^B_{n_k n_z}\mathbf{I} & x_{n_k n_z}\mathbf{I} \end{bmatrix}$$

$$\mathbb{A}_z = \begin{bmatrix} -\lambda_{11}\mathbf{II} & \lambda_{12}\mathbf{II} & \lambda_{13}\mathbf{II} \\ \lambda_{21}\mathbf{II} & -\lambda_{22}\mathbf{II} & \lambda_{23}\mathbf{II} \\ \lambda_{31}\mathbf{II} & \lambda_{32}\mathbf{II} & -\lambda_{33}\mathbf{II} \end{bmatrix}$$

and $-\frac{\dot{k}^+_{ij}}{\Delta k} + \frac{\dot{k}^-_{ij}}{\Delta k} = x_{ij}$,$-\frac{\dot{k}^-_{ij}}{\Delta k} = x^B_{ij}$, $\frac{\dot{k}^+_{ij}}{\Delta k} = x^F_{ij}$, $\mathbf{I}$ is $N \times N$ identity matrix, and $\mathbf{II}$ is $Nn_k \times Nn_k$ identity matrix. I use $n_z = 3$ and $n_k = 10$.

Solution algorithm

1. Guess parameters of the forecasting functions. With the forecasting functions, $\dot{k}$ and the house price over (k, z) are known. Also, interest rates and wages over (k, z) can be computed using the firm's marginal conditions.
2. Solve the value function.

- Guess the loan price functions, $r_a(\omega, k, z)$ and $q(\omega, k, z)$.
- Guess the value function, $v^0(\cdot)$.
- Update the value functions and the loan price schedules until they converge.
- Save decision rules.

3. Simulate the model for n periods. Simulation gives the sequence of aggregate variables $\{z_t, k_t, p_t\}_{t=1}^n$.

   - Guess the initial distribution. The distribution in the steady state can be used as a good initial distribution.
   - At the beginning of each period, (k, z) are known. The risk free rate and wage can be computed.
   - Compute the loan price functions. To compute $r_a(\omega_t, k_t, z_t)$, interpolate the default decisions that are obtained from step 2. Using the default decisions and the risk free rate, $r_a(\omega_t, k_t, z_t)$ can be computed using Equation (5). To compute $q(\omega_t, k_t, z_t)$, interpolate $q(\omega, k, z)$ obtained from step 2, over k.
   - Guess the house price.
   - With the wage, the loan price schedules and the house price, solve the household problem.
   - Compute the aggregate demand for housing. If the aggregate demand is not close enough to the supply, adjust the house price to clear the housing market.
   - Once the housing market is cleared, move to the next period.

4. Using the sequence of aggregate variables $\{z_t, k_t, p_t\}_{t=1}^n$, update the forecasting functions.
5. Check the convergence of the simulated aggregate variables. If the distance between the $\{k_t\}_{t=1}^n$ from the current iteration and the previous iteration is less than a tolerance level, an approximate recursive equilibrium has been found. Otherwise, go back to step 2 with the updated forecasting functions.

Predictive power of the forecasting functions

Table 13 shows the R^2 of the forecasting functions. Since forecasting functions are

Table 13: R^2 of forecasting functions

	Benchmark			Foreclosure only		
Flat-tax	Exp.	Rec.	Severe Rec.	Exp.	Rec.	Severe Rec.
k'	0.999	0.999	0.999	0.999	0.999	0.999
p	0.996	0.994	0.993	0.995	0.994	0.995
Procyclical-tax	Exp.	Rec.	Severe Rec.	Exp.	Rec.	Severe Rec.
k'	0.999	0.999	0.999	0.999	0.999	0.999
p	0.995	0.991	0.992	0.995	0.990	0.991

Note: Based on a simulation of 2,400 periods.

conditional on the aggregate state, there are R^2s for both expansions, recessions, and severe recessions. The R^2s are high for k'. Capital moves slowly and this makes the forecasting function very accurate for any version of the model. R^2s for house prices are slightly lower and they are lower during recessions than expansions. This is probably because housing choices are discrete.