The Macroeconomic Effects of Debt Relief Policies during Recessions

Soyoung Lee *

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Abstract

I study debt relief as a stimulus policy. To do so, I develop a dynamic stochastic general equilibrium model that captures rich heterogeneity in households' balance sheets. A large-scale mortgage principal reduction can stimulate consumption in a recession, amplify a recovery, support house prices and lower foreclosures. General equilibrium has an important role in propagating the policy effects. Additionally, the nature of the intervention, in terms of its eligibility, liquidity and how it is financed, shapes the macroeconomic impact of the policy. This impact rests on how resources are redistributed across households that vary in their marginal propensities to consume. Lastly, the availability of bankruptcy on unsecured debt reduces precautionary savings and qualitatively changes the macroeconomic response to large-scale mortgage relief.

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*Bank of Canada. Email: soyoung.lee.n@gmail.com

1 Introduction

What are the effects of introducing debt relief programs during recessions? If a severe downturn is associated with high levels of household leverage, does reducing their debt burden help stabilize the economy? Since the Great Recession, a widely held view is that alleviating underwater borrowers' financial distress could have prevented the sharp rise in foreclosure and dampened the fall in house prices.[1] Moreover, preventing large initial declines in house prices might have reduced subsequent foreclosures, thereby supporting house prices at later dates, and household spending over time. To date, however, there is little quantitative analysis of debt relief as a stimulus policy.

My goal is to quantitatively assess the effects of mortgage relief programs using a dynamic stochastic general equilibrium model. The model is designed to reproduce the heterogeneity in households' assets and liabilities. I find that, in a recession that involves an unusual drop in house prices, a large-scale mortgage principal reduction can raise consumption and, eventually, output. Further, it dampens the fall in house prices and significantly lowers foreclosure. I show that general equilibrium responses in prices play an important role in propagating the effects of the policy intervention over time. Additionally, the nature of the intervention, in terms of eligibility, the liquidity of a transfer and how it is financed shapes the macroeconomics consequences of the policy. The magnitude of these effects rests on the extent to which debt relief policies redistribute resources across households that vary in their marginal propensities to consume (MPC).

I develop a model that allows me to study the effects of mortgage-related programs without shortcuts. Households may borrow or save in liquid assets, and can also hold illiquid assets and liabilities in the form of houses and mortgages. Housing provides service flows, which are valued alongside non-durable consumption. Households face idiosyncratic risks and have limited ability to insure themselves as markets are incomplete. Borrowers may declare bankruptcy on unsecured debt and allow foreclosure on their mortgages.

The model has three distinctive features. First, it includes aggregate business cycle risk in the form of shocks to total factor productivity. As we consider debt relief as a macroeconomic policy tool to mitigate the severity of downturns, the model must have aggregate uncertainty. Second, all prices (interest rates, wages, house prices,

[1]For example, see Eberly and Krishnamurthy (2014), Mian et al. (2015), Posner and Zingales (2009), and Agarwal et al. (2017).

and loan rates) are determined endogenously in my model. This allows me to carefully consider the general equilibrium responses to large scale debt relief programs, both for the macroeconomy and individual households. For all exercises, I show how the responses of households to a policy intervention impact house prices, wages, and interest rates, and how these changes in prices feedback to household decisions and shape the paths of aggregate variables. Third, the model allows default on unsecured and secured debt, separately. While Mitman (2016) and Li et al. (2011) show that there are significant interactions between these two types of debt, as well as their respective defaults, having both bankruptcy and foreclosure in a model is rare. I argue that when one tries to evaluate debt relief programs the model should contain both types of defaults. In their absence, the effects of a temporary debt relief program can be overstated.

The distributions of households over assets and liabilities change with the availability of default. For example, in the absence of short term credit and bankruptcy, households increase holdings of liquid assets for consumption smoothing. This, in turn, lowers housing demand, house prices, as well as the debt burden of households. In this way, allowing for bankruptcy lead to important changes in the effects of policy.

My model is calibrated to match a large set of relevant features of the U.S. economy. I then evaluate it along untargeted dimensions. The model successfully reproduces the household wealth distribution, not only for net worth but also for key components: financial assets, housing wealth and mortgages. It also captures business cycle moments of in the US data. Thus, I argue my model is an appropriate laboratory to study debt relief policies over the business cycle.

In a recession involving a large fall in house prices, I study an unanticipated intervention where households with loan-to-value (LTV) ratios above 95% have them reduced to this threshold via a one-time mortgage reduction. This is a large intervention that affects about 15% of the population and costs 5.6% of GDP. Crucially, the policy is debt financed and involves increases in future taxes. Nonetheless, I find that the mortgage reduction stimulates consumption in the recession and amplifies the recovery, while leaving its timing unchanged. Moreover, the policy significantly lowers foreclosure rates and supports house prices. These findings are differ from those in Kaplan et al. (2020) who consider the same mortgage forgiveness program. In their model, the program reduces foreclosure rates significantly but has little effect on house prices or consumption. Among other things, the presence of capital and bankruptcy, and equilibrium responses in wages and interest rates in my model help explain this difference. In addition, through detailed analysis and a series of counterfactual exercises, I draw four lessons.

First, general equilibrium effects matter. For consumption, general equilibrium effects explain about 40% of its rise after the principal reduction. As the policy is debt financed, there is no net transfer to the economy. Instead, the principal reduction redistributes resources from low MPC households to high MPC households, as highly indebted households tend to have high MPCs. This drives an initial rise in consumption and a fall in savings. Later, as the economy moves into an expansion, the government raises tax rates to repay debt. These payments are made to low MPC households holding government debt. They, in turn, increase their investment in physical capital. As the capital stock grows, wages rise. While the principal reduction directly benefits those who receive the transfer, higher wages benefit all households. These income effects build up over time, helping to support house prices and consumption beyond the direct effects of the mortgage principal reduction. Second, how the government transfers resources to households matters. I compare the effects of the mortgage debt relief with those of a tax rebate of the same size. Mortgage forgiveness targets highly indebted homeowners while tax rebates are not a targeted policy. The former provides illiquid assets by increasing home equity while the other provides liquid assets. These differences lead to different distributional and aggregate implications.

When looking at households' initial consumption responses, the distribution of MPCs match estimates of the effects of tax rebates; the MPC is larger for households with high levels of debt and low shares of liquid assets. In contrast, there is no monotone pattern between MPCs and liquidity or indebtedness following a principal reduction. As it provides illiquid assets, a large response in household consumption accompanies a costly adjustment in mortgage or housing. This changes the distribution of consumption responses compared to the tax rebate.

In the aggregate, the tax rebate is more effective than the principal reduction at boosting consumption because it directly increases households' liquid wealth, and distributes benefits to a larger group of high MPC households including those who do not own houses. Consumption initially rises by 1 percentage point more. In addition, the tax rebate is less effective in reducing foreclosure but more effective in lowering unsecured credit and bankruptcy.

Third, the efficacy of a policy is state-dependent. I consider several downturn episodes where house prices fell about eight percent. Across recessions, there is considerable variation in the share eligible for a mortgage reduction, which varies from 15.5 to 25.5%, and in the average amount forgiven, which ranges from $31,000 to $49,000 (2007 dollars). The effectiveness of a policy intervention, measured in terms of the fiscal multiplier, is higher when the average LTV of eligible households is relatively low. Hence interventions are more effective when the government acts early while households are relatively less indebted.

Fourth, a bankruptcy option changes the aggregate dynamics of the economy and its response to stimulus policies. To study the role of bankruptcy, I examine the response of an economy with only foreclosure. In such an economy, unsecured debt must be repayable following any income shock which sharply limits its use. For simplicity, I assume it is unavailable. Households now accumulate more liquid assets to self-insure against income risk. As a result, both the level and volatility of house prices fall and households are less indebted. This reduces the need for mortgage relief.

Not surprisingly, the dispersion in MPCs narrows as does the gap between the effects of targeted debt relief and untargeted tax rebates. Importantly, I find that now either intervention moves forward the timing of the recovery when compared to the model with bankruptcy.

Finally, I examine the effects of reducing mortgage payments. I assume that per period repayments of principal are halved for four years. This policy effectively extends the duration of loans by implying slower amortization of debt. While foreclosure rates fall with this policy, aggregate mortgage balances remain higher than the benchmark. The rise in households' disposable income initially leads to a slight increase in aggregate consumption. The slowdown in mortgage payments lowers savings available for investment, and capital falls. Eventually, larger interest payments from higher mortgage balances, and lower wages coming from decreases in capital, eventually depress consumption. This occurs even as mortgagors continue to pay a lower fraction of their balance. Overall, if we focus on the short-run effects of the reduced payment policy, it increases consumption. However, in the longer-run, general equilibrium effects dampen the stimulus effects of a payment reduction policy.

Literature

This work is part of the literature on household credit and default, in particular, to the branch that studies debt relief as a stimulus policy tool. Since the Great Recession, a large body of work has shown that the high leverage of households can exacerbate an economic downturn or slow down a recovery.[2]. Following these findings, a set of papers have estimated the effectiveness of mortgage debt relief programs. For exam-

[2]Examples include Mian et al. (2013), Mian and Sufi (2014), Guerrieri and Lorenzoni (2017), Jones et al. (2011), and Verner and Gyongyosi (2018).

ple, Agarwal et al. (2017) show that although participation rates were low, the Home Affordable Mortgage Program (HAMP) reduced foreclosure and increased spending. Further examining HAMP, Ganong and Noel (2018) found that principal reduction was less effective than payment reduction in reducing default and increasing consumption.

The principal reduction I consider is chosen to be comparable to Kaplan et al. (2020). In contrast to HAMP, it affects a larger population and provides more substantial debt reduction that, critically, does not leave borrowers underwater. Thus Ganong and Noel's (2018) findings are not inconsistent with mine. Empirical evidence supporting the effectiveness of larger principal reductions is in Cespedes et al. (2021). Examining cramdowns that discharged the underwater portion of mortgages during Chapter 13 bankruptcy proceedings between 1989 and 1993, they find that foreclosure rates fell. Lastly, Piskorski and Seru (2021) show that alleviating frictions affecting the pass-through of lower interest rates and debt relief (e.g. refinancing, loan renegotiation) could have reduced foreclosure rates and resulted in up to twice as fast recovery in house prices, consumption, and employment during the Great Recession.

Two papers, both assuming nominal rigidities, are most closely related to my work. Examining unsecured loans, Auclert et al. (2019) find that debt forgiveness provided by the U.S. consumer bankruptcy system increases consumption and this increased consumption helps to stabilize employment. Gete and Zecchetto (2018) argue that non-recourse mortgages in the US contributed to the faster recovery of the US compared to Europe, where most countries only have recourse mortgages. I complement their work by showing that debt relief programs, over and beyond existing bankruptcy and foreclosure arrangements, can have a lasting impact on the economy.

My work is broadly related to research on fiscal policy with heterogeneous agents and incomplete markets. Oh and Reis (2012) show that increases in transfers, over the Great Recession, exceeded that in goods and services. In a model with incomplete markets, Heathcote (2005) studies the effects of temporary tax changes and finds large real effects. Brinca et al. (2016) find that the effects of fiscal policy are sensitive to the fraction of the population with binding credit constraints as well as the level of average wealth. My results are consistent with the findings from these papers. The stimulus policies I study are effectively transfers and have large real effects. These effects depend on the distribution of households over assets and liabilities.

Several recent papers study the role of households' balance sheets in determining their consumption spending. Both Berger et al. (2018) and Guren et al. (2021) explore consumption responses to changes in housing wealth. Focusing on the illiquidity of housing, Boar et al. (2021) study how it affects consumption smoothing. Cloyne et al.

(2020) show that households with mortgages adjust their consumption the most to interest rate changes, thus, driving the aggregate consumption responses. Similarly, I find that the responses in non-durable consumption, to changes in policy, depend on households' portfolio positions and their decisions to adjust illiquid assets and liabilities. Indeed, in the short-term consumption may fall when households increase their stock of housing or prepay their mortgage. Over the longer term, the illiquidity of housing is evident as households exhibit large consumption changes when they transact in the housing market. In particular, as debt relief policies lead to changes in house prices, housing transactions drive larger responses in consumption.

The model developed here is related to existing quantitative analysis of housing, mortgages, and foreclosure.[3] My model contains the same elements while also allowing for default on unsecured debt.[4] To my knowledge, Mitman (2016) is the only other paper exploring the interaction between bankruptcy and foreclosure. By allowing for both types of default, he can reconcile the negative correlation between the generosity of bankruptcy laws and bankruptcy rates, across U.S. states. While my model abstracts from rich heterogeneity in the bankruptcy system across states, I introduce aggregate uncertainty to quantify the role of debt relief policies during recessions. I show that bankruptcy and foreclosure can be substitutes. Further, I find that bankruptcy affects households' self-insurance motives and thus the distribution of household leverage.

The design of mortgage principal reduction follows the seminal contribution of Kaplan et al. (2020) discussed above. Although my model shares many elements with theirs, the introduction of capital and bankruptcy, and equilibrium movements in wages and interest rates, lead to much larger aggregate effects of a debt relief program in my framework.

The rest of the paper is organized as follows. In Section 2, I describe the model economy. Section 3 explains the calibration and examines untargeted moments. I discuss steady state results in Section 4 and business cycle results in Section 5. Section 6 concludes.

[3]See, for example, Jeske et al. (2013), Corbae and Quintin (2015), Hatchondo et al. (2015), Chatterjee and Eyigungor (2015), Kaplan et al. (2020), and Garriga and Hedlund (2020).

[4]In this sense, I integrate models with secured debt and models with consumer credit and bankruptcy. Examples of the latter include Athreya (2002), Li and Sarte (2006), Livshits et al. (2007), Chatterjee et al. (2007), and Nakajima and Ríos-Rull (2019).

2 Model

2.1 Overview

The economy consists of a continuum of infinitely-lived households, banks, non-durable goods producers and a government. Households are indexed by their holdings of liquid assets a, house h, mortgage b, labor productivity ε, and their credit history o. They are subject to uninsurable idiosyncratic shocks to their labor productivity, which they supply inelastically to competitive firms. Households can save or borrow in a financial asset whose return is determined in equilibrium. They consume non-durable goods and service flows from their housing.

Housing services are derived from home ownership; however, households do not have to own a house. When a household buys a house, it can take out a mortgage to fund the purchase. Mortgages are long-term debt and subject to a loan-to-value (LTV) constraint at origination. Time is continuous and mortgage borrowers are required to pay a fraction of remaining principal and interest at every instant. Households can refinance or prepay their mortgage. Houses and mortgages are illiquid in the sense that costs are incurred when buying, selling, refinancing or prepaying.

Both types of debt (unsecured debt and mortgage) are defaultable. Defaulting on unsecured debt (bankruptcy) leads to a utility loss and a record of bankruptcy is placed in the household's credit history. This excludes them from housing transactions and unsecured debt borrowing. When a household chooses to default on a mortgage (foreclose), a bank takes over the house and liquidates it. A foreclosure also incurs a utility loss and a record of foreclosure is entered in the household's credit history, excluding them from housing transactions. Fixed costs and indivisibilities lead to discrete choices. Households' assets and liabilities have a large adjustment when they choose to i) buy or sell houses ii) refinance or prepay mortgages iii) default on unsecured debt iv) default on a mortgage v) default on both debts. When households do not make a substantial change to their portfolio, they choose how much to consume.

There are a large number of identical firms that produce using capital and labor and constant returns to scale technology. Their output is consumed or invested in physical capital. The supply of housing is fixed.

The financial sector is competitive. Banks price both types of debt taking into account households' default risk. Perfect competition leads to zero expected profit on each loan. The government collects taxes from households which funds transfers alongside government consumption.

The aggregate states of the economy are g - the distribution of households over

$(a, b, \varepsilon, h, o)$ - and z, aggregate total factor productivity. Below I suppress the credit history indicator, o, where its value is unambiguous. For example, I drop o when I describe a problem or function for households with a specific credit history.

2.2 Households

Below, after describing household earnings, I describe their assets and liabilities, followed by a description of bankruptcy and foreclosure.

2.2.1 Environment

Labor productivity Each household's labor productivity follows a Poisson process. With frequency λ_ε, households receive a shock and draw new productivity from a time-invariant distribution. These shocks arrive independently across households.

Liquid assets Households can save or borrow using a liquid asset a. When a is negative, it represents unsecured debt. Households may default on their debt. When a household chooses to default, the debt is erased. However, the borrower has a record of bankruptcy in her credit history. Further, she pays a utility cost, ξ_a. The possibility of default by households means that the price of unsecured borrowing will depend on individual and aggregate states as these determine the probability of repayment. Borrowing costs include a unit cost of lending, $\iota(z)$.

Houses A house, h, is chosen from a discrete set and buying or selling a house involves adjustment costs. Houses provide utility flows to households and incur maintenance costs. One component of these maintenance costs is a property tax, which is tax deductible. As housing is a discrete choice, it is an illiquid asset that households adjust infrequently. I assume the total supply of houses is fixed to $\overline{H}$. Importantly, house prices, $p(g, z)$, are determined in equilibrium and vary as a function of the aggregate state. Households do not need to hold this asset. When they do not have h, they do not pay maintenance costs but receive utility flow from $\underline{h} < min\{h\}$.[5]

[5]Given the complexity of the model, I abstract from a rental market while still allowing households to not to own houses. While Favilukis et al. (2017), Kaplan et al. (2020), and Greenwald and Guren (2021) show modeling a rental market is crucial to explain effects of credit conditions on house prices, abstracting from it is unlikely to affect my results. First, in my model, aggregate dynamics are not driven by credit shocks. Second, as noted, households do not have to own houses.

Mortgages House purchases can be funded using mortgages, b. These are distinct from unsecured debt. Thus, similarly to Mitman (2016) and differently from Kaplan et al. (2020) and Nakajima and Ríos-Rull (2019), I allow for both unsecured credit and mortgages. This debt is refinanceable, long-term, secured, and defaultable. Households can borrow b when they purchase a house, using it as collateral. When choosing b', households are subject to a loan-to-value (LTV) constraint: the choice of b must be less than a fraction (γ) of the value of collateral.

Mortgage loans are discounted by $q(a', b', \varepsilon, h, g, z)$ at the time of origination. That is, a household that takes debt with a face value b', receives a loan $q(a', b', \varepsilon, h, g, z)b'$ where $q(\cdot)$ reflects the probability of payment. Notice the loan discount rate depends not only on the choice of b' but on the full individual state and the aggregate states as these determine the probability of default.

Households can refinance a mortgage loan by paying a fixed cost ξ_r. When refinancing, they first pay the remaining balance on the current loan and then take out a new loan.[6] All mortgage debt interest is adjustable rate. Thus refinancing is only used to extract equity or prepay the outstanding balance. If mortgages had involved fixed rates, households would have an incentive to refinance when interest rates fall. While a household is holding a mortgage, it pays the loan interest rate as well as a fraction of the principal at each instant. The mortgage interest rate is equal to the return on saving plus a premium reflecting the unit cost of lending, $\iota(z)$.[7]

The fraction of the principal that is paid each instant is $\theta(b, \overline{p}h)b$ where $\theta(b, \overline{p}h) = \overline{\theta}\overline{p}h/b$ and $\overline{p}$ represents the long-run average price of housing. Actual mortgages involve the borrower paying a fraction of the original loan at the time of mortgage origination. However, this requires the introduction of an additional state variable for the original loan size. Commonly, the housing literature avoids this problem by assuming that households pay a fraction of their outstanding mortgage. To better capture the nature of conventional mortgages, I assume that this fraction of the mortgage rises over time. This implies that a household pays a constant fraction of the value of the house, valued using a long-run average price that is different from the actual purchase price, p.

As b is long-term debt, there is no requirement that the size of this loan remain less than γ times the current value of a house. Thus the LTV limit only applies at origination. If the price of houses decreases, a household could find itself with negative equity. However, as long as the household pays off the required amount of the

[6] This implies that new loans are always subject to the same type of LTV limits and loan discounting.

[7] Allowing this cost of lending to vary with the aggregate shock is important to reproduce the procyclicality of credit.

outstanding balance of the loan at the moment, $(\theta(b, \overline{p}h) + r + \iota(z))b$, it is not forced to default or refinance.

When a household chooses to default, the remaining balance of debt is forgiven, and a financial intermediary takes the house. I assume the financial intermediary suffers a loss when foreclosing, and the sale value is $(1 - \delta_h)ph$. The household will have a foreclosure recorded on their credit history, which excludes them from buying houses.[8] Households that enter foreclosure incur a utility cost, ξ_b, at that moment.

Bankruptcy and foreclosure histories While a bankruptcy remains on a household's credit history, unsecured borrowing, refinancing a mortgage, new origination of a mortgage, and purchasing a house are not allowed. I assume, for tractability, that the bankruptcy flag is removed stochastically with intensity λ_d. Bankrupt households' non-financial assets are fully protected and they can enter foreclosure if they have a mortgage.

A household that has defaulted on a mortgage is unable to purchase a new house while its credit history has a foreclosure record. Since they cannot buy a house, such households are excluded from taking new mortgages. However, they can still take on unsecured debt and choose to default on any such unsecured debt they already have. If these households go bankrupt, their foreclosure flag will be replaced by a bankruptcy flag. For tractability, I assume that the foreclosure flag is removed stochastically with intensity λ_f.

2.2.2 Household Problem

Households receive flow utility from consuming non-durable goods and from consuming the service flow from their houses. Their utility function, $u(c, h)$ is strictly increasing and strictly concave in c and h.

Households with no flags

Households that do not currently have bankruptcy or foreclosure in their credit history are free to take on unsecured debt and enter into housing transactions. Recalling that a is savings or unsecured debt, b is the mortgage, ε is earnings and h is the house size, such a household's problem is given by (1) - (4),

$$v(a_t, b_t, \varepsilon_t, h_t) = \max_{\{c_t\}, \tau} \mathbf{E_0} \int_0^{\tau} e^{-\rho t} u(c_t, h_t) dt + \mathbf{E_0} e^{-\rho \tau} v^*(a_\tau, b_\tau, \varepsilon_\tau, h_\tau) \tag{1}$$

[8] For simplicity, I assume that households cannot buy houses even if the purchase is self-financed.

$$\dot{a}_t = w_t\varepsilon_t + r_{at}(a,b,\varepsilon,h,o)a_t - (r_t + \theta(b,\overline{p}h))b_t - c_t - T_t(b,\varepsilon,ph) - \xi_h p_t h_t \tag{2}$$

$$\dot{b}_t = -\theta(b,\overline{p}h)b_t \tag{3}$$

$$(a_0,b_0,\varepsilon_0,h_0) = (a,b,\varepsilon,h). \tag{4}$$

Households choose non-durable consumption $\{c_t\}$ and their optimal stopping time τ. Stopping involves a household making a discrete choice that brings about a large shift in their asset position, or credit history. For example, a household buys a house when it reaches a certain threshold in saving. When a household chooses to stop, they do one of the following: i) buy or sell a house ii) refinance their mortgage, iii) default on unsecured debt, iv) default on their mortgage or v) default on both unsecured debt and mortgage. In the absence of such a discrete choice, households that have mortgages repay them at the rate $\theta(b,\overline{p}h)b$. A household's income tax is given by the function, $T(\cdot)$. Taxable income is a function of earnings, mortgage interest payments and the property tax. Homeowners also need to pay the maintenance cost of their houses, $\xi_h ph$.

The Hamilton-Jacobi-Bellman (HJB) equation prior to stopping is,

$$\begin{aligned}\rho v(a,b,\varepsilon,h,g,z) = &\max_c u(c,h) + \partial_a v(a,b,\varepsilon,h,g,z)\dot{a} + \partial_b v(a,b,\varepsilon,h,g,z)\dot{b} \\ &+ \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v(a,b,\varepsilon_j,h,g,z) + \sum_{k=1}^{n_z} \lambda_{zz_k} v(a,b,\varepsilon,h,g,z_k) \\ &+ \int \frac{\delta v(a,b,\varepsilon,h,g,z)}{\delta g(a,b,\varepsilon,h,o)} \mathcal{K} g(a,b,\varepsilon,h,o) d[a\times b\times\varepsilon\times h\times o]\end{aligned} \tag{5}$$

$$\begin{aligned}\dot{a} = &w(g,z)\varepsilon + (r_a(a,b,\varepsilon,h,o,g,z) + \iota(z)|_{a<0})a - (r(g,z) + \iota(z) + \theta(b,\overline{p}h))b \\ &- c - \xi_h p(g,z)h - T(b,\varepsilon,p(g,z)h)\end{aligned}$$

$$\dot{b} = -\theta(b,\overline{p}h)b$$

$$v(a,b,\varepsilon,h,g,z) \geq v^*(a,b,\varepsilon,h,g,z).$$

Above, $\lambda_{\varepsilon\varepsilon_j}$ describes the labor productivity process.[9] Aggregate productivity follows a stochastic process described by λ_{zz_k}. $\mathcal{K}$ is a Kolmogorov Forward operator that operates on the distributions of households, g_t, which evolves according to shocks and

[9]A shock intensity $\lambda_{\varepsilon_i\varepsilon_j}$ is negative when $\varepsilon_i = \varepsilon_j$, which is the intensity of losing the current level of labor productivity. $\lambda_{\varepsilon_i\varepsilon_j} > 0$ when $\varepsilon_i \neq \varepsilon_j$, it is the intensity of jumping to ε_j from ε_i. $\sum_j \lambda_{\varepsilon_i\varepsilon_j} = 0 \quad \forall i = 1,...,n_\varepsilon$.

households' decisions.[10]

$$\frac{dg_t(a,b,\varepsilon,h,o)}{dt} = \mathcal{K} g_t(a,b,\varepsilon,h,o).$$

Stopping values

The stopping value $v^*(a_\tau, b_\tau, \varepsilon, h_\tau, g, z)$ is the maximum of the values listed below. These involve moving to a different house (adjusting the size of h), refinancing an existing mortgage, or defaulting on debt.

1. Buying or selling a house,

$$v^m(a_\tau, b_\tau, \varepsilon, h_\tau, g, z) = \max_{h',b'} v(a', b', \varepsilon, h', g, z)$$

$$a' = a_\tau - b_\tau + p(g,z)h_\tau - p(g,z)h' - \xi(p(g,z), h_\tau, h') + q(a', b', \varepsilon, h', g, z)b'$$

$$b' \leq \gamma p(g,z)h'.$$

In the second constraint, γ is the LTV limit as a proportion of collateral. When changing the size of its house, a household chooses the optimal size (h') and the amount of the mortgage (b'). The remaining balance that is attached to the current house has to be repaid. The transaction cost $\xi(\cdot)$ is given by

$$\xi(p(g,z), h_\tau, h') = p(g,z)\xi_0 \frac{(h_\tau + h')}{2}.$$

2. Refinancing a mortgage

$$v^r(a_\tau, b_\tau, \varepsilon, h_\tau, g, z) = \max_{b'} v(a', b', \varepsilon, h_\tau, g, z)$$

$$a' = a_\tau - b_\tau + q(a', b', \varepsilon, h, g, z)b' - \xi_r$$

$$b' \leq \gamma p(g,z)h_\tau.$$

Households that hold a mortgage have the option to refinance by repaying the re-

[10] Ahn et al. (2018) describe the recursive formulation of an model with aggregate uncertainty using the Kolmogorov Forward operator.

maining balance b_τ and a fixed cost ξ_r. They do not change their house size but simply originate a new loan, b', which is subject to the LTV limit, given by $b' \leq \gamma p(g, z)h$.

3. Bankruptcy

$$v^a(a_\tau, b_\tau, \varepsilon, h, g, z) = v^d(0, b, \varepsilon, h, g, z) - \xi_a.$$

This option involves defaulting on unsecured borrowing, a. If a household chooses to default only on a, its house and mortgage are unaffected. The stopping value for default is evaluated using $v^d(a, b, \varepsilon, h, g, z)$ which is the value function for households with bankruptcy in their credit history; and ξ_a is the utility cost associated with default.

4. Foreclosure

$$v^b(a_\tau, b_\tau, \varepsilon, h, g, z) = v^f(a_\tau, \varepsilon, g, z) - \xi_b.$$

This is foreclosure. If a household chooses to foreclose, its remaining mortgage is erased, and the financial intermediary takes over the house. The value function of households with foreclosure in their credit history is $v^f(a, \varepsilon, g, z)$ and ξ_b is the utility cost of foreclosing.

5. Defaulting on both mortgage and unsecured debt

$$v^{ab}(a_\tau, b_\tau, \varepsilon, h, g, z) = v^d(0, 0, \varepsilon, 0, g, z) - \xi_a - \xi_b.$$

Recall the assumption a household that both forecloses and declares bankruptcy, receives a bankruptcy flag in its credit history. This implies the value of defaulting both types of debts is given by the function $v^d(\cdot)$. When a household defaults on both types of debt, its debts held as a and b are erased, and a financial intermediary takes over h. Such households are not allowed to use any credit while they have a bankruptcy in their credit history.

The overall stopping value for a household is v^*, where

$$v^*(a_\tau, b_\tau, \varepsilon_\tau, h, g, z) = \max\{v^m, v^r, v^a, v^b, v^{ab}\}. \tag{6}$$

Thus households can choose among the available stopping options; buying or selling a house, refinancing mortgage, bankruptcy on a, foreclosure on b, and both.

Households with a bankruptcy flag

A household with a bankruptcy flag in their credit history solves the following problem,

$$v^d(a_t, b_t, \varepsilon_t, h_t) = \max_{\{c_t\},\tau} \mathbf{E_0} \int_0^\tau e^{-\rho t} u(c_t, h_t) dt + \mathbf{E_0} e^{-\rho\tau} v^{d*}(a_\tau, b_\tau, \varepsilon_\tau, h_\tau) \tag{7}$$

$$\dot{a}_t = w_t \varepsilon_t + r_t a_t - (r_t + \theta(b, \overline{p}h)) b_t - c_t - T_t(b, \varepsilon, ph) - \xi_h p_t h_t \tag{8}$$

$$\dot{b}_t = -\theta(b, \overline{p}h) b_t \tag{9}$$

$$a_t \geq 0 \tag{10}$$

$$(a_0, b_0, \varepsilon_0, h_0) = (a, b, \varepsilon, h). \tag{11}$$

Such households also choose non-durable consumption $\{c_t\}$ and their optimal stopping time τ. However, only foreclosure and selling houses are available to them as stopping options. Bankrupt households cannot take on unsecured debt. The HJB equation before stopping is,

$$\begin{aligned} \rho v^d(a, b, \varepsilon, h, g, z) = &\max_c u(c, h) + \partial_a v^d(a, b, \varepsilon, h, g, z)\dot{a} + \partial_b v^d(a, b, \varepsilon, h, g, z)\dot{b} \\ &+ \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v^d(a, b, \varepsilon_j, h, g, z) + \sum_{k=1}^{n_z} \lambda_{zz_k} v^d(a, b, \varepsilon, h, g, z_k) \\ &+ \lambda_d (v(a, b, \varepsilon, h, g, z) - v^d(a, b, \varepsilon, h, g, z)) \\ &+ \int \frac{\delta v^d(a, b, \varepsilon, h, g, z)}{\delta g(a, b, \varepsilon, h, o)} \mathcal{K} g(a, b, \varepsilon, h, o) d[a \times b \times \varepsilon \times h \times o] \end{aligned} \tag{12}$$

$$\begin{aligned} \dot{a} = &w(g, z)\varepsilon + r(g, z)a - (r(g, z) + \iota(z) + \theta(b, \overline{p}h))b \\ &- c - \xi_h p(g, z)h - T(b, \varepsilon, p(g, z)h) \end{aligned}$$

$$\dot{b} = -\theta(b, \overline{p}h)b$$

$$a \geq 0$$

$$v^d(a, b, \varepsilon, h, g, z) \geq v^{d*}(a, b, \varepsilon, h, g, z).$$

Recall the assumption that households with a bankruptcy flag cannot buy a house. This implies that their stopping value is the maximum of either defaulting on a mortgage or selling a house.

1. Defaulting on a mortgage

$$v^{da}(a_\tau, b_\tau, \varepsilon, h_\tau, g, z) = v^d(a_\tau, 0, \varepsilon, 0, g, z) - \xi_b$$

2. Selling a house

$$v^{dm}(a_\tau, b_\tau, \varepsilon, h_\tau, g, z) = v^d(a, 0, \varepsilon, 0, g, z)$$

$$a = a_\tau - b_\tau + p(g,z)h_\tau - \xi(p(g,z), h_\tau, 0)$$

$$v^{d*}(a, b, \varepsilon, h, g, z) = \max\{v^{da}, v^{dm}\} \tag{13}$$

Households with a foreclosure flag

Lastly, households with a foreclosure flag in their credit history solve the following problem,

$$v^f(a_t, \varepsilon_t) = \max_{\{c_t\},\tau} \mathbf{E_0} \int_0^\tau e^{-\rho t} u(c_t, \underline{h}) dt + \mathbf{E_0} e^{-\rho\tau} v^{d*}(a_\tau, \varepsilon_\tau) \tag{14}$$

$$\dot{a}_t = w_t \varepsilon_t + r_{at}(a, 0, \varepsilon, 0, o) a_t - c_t - T_t(b, \varepsilon, ph) \tag{15}$$

$$(a_0, \varepsilon_0) = (a, \varepsilon). \tag{16}$$

These households choose non-durable consumption $\{c_t\}$ and the optimal stopping time τ. However, the only stopping option available to them is to default on a. The HJB equation before stopping is listed below,

$$\begin{aligned}
\rho v^f(a, \varepsilon, g, z) = \max_c u(c, \underline{h}) + \partial_a v^f(a, \varepsilon, g, z)\dot{a} + \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v^f(a, \varepsilon_j, g, z) \\
+ \lambda_f (v(a, 0, \varepsilon, 0, g, z) - v^f(a, \varepsilon, g, z)) + \sum_{k=1}^{n_z} \lambda_{zz_k} v^f(a, \varepsilon, g, z_k) \\
+ \int \frac{\delta v^f(a, \varepsilon, g, z)}{\delta g(a, b, \varepsilon, h, o)} \mathcal{K} g(a, b, \varepsilon, h, o) d[a \times b \times \varepsilon \times h \times o]
\end{aligned} \tag{17}$$

$$\dot{a} = w(g,z)\varepsilon + (r_a(a, 0, \varepsilon, 0, o, g, z) + \iota(z)|_{a<0})a - c - T(b, \varepsilon, p(g,z)h)$$

$$v^f(a, \varepsilon, g, z) \geq v^{f*}(a, \varepsilon, g, z).$$

The stopping value is,

$$v^{f*}(a_\tau, \varepsilon, g, z) = v^d(0, 0, \varepsilon, 0, g, z) - \xi_a. \tag{18}$$

The household's problem can be compactly written as an HJB variational inequality (HJBVI). See Appendix A for details.

Let $\{C_t\}_{0\le t\le \tau}$ describe a households decision rule for its consumption of non-durable goods with a stopping time of τ. When a household stops and buys or sells a house, its decision rule for buying and selling is M. The decision rules for the size of its mortgage and house are B_M and H_M, respectively. The refinancing decision and the size of secured debt when refinancing are given by the functions R and B_R. Decision rules for bankruptcy, foreclosure and defaulting on both types of debt are D_a D_b and D_{ab}.

2.3 Financial intermediaries

Financial intermediaries are perfectly competitive and risk neutral. These banks issue short-term deposits and loans, as well as mortgages, to households. They also lend capital to firms. The possibility of default leads banks to offer loan rates based on a household's portfolio and income. Banks' loans have expected profits equal to zero. While individual loans may generate a profit or a loss, ex post, in the absence of aggregate shocks, banks would have zero profit on their total portfolio. However, there will be systematic profits and losses as a result of aggregate risk. I assume that the government absorbs any realized profits or losses using taxes or subsidies paid to intermediaries.[11]

Unsecured debt As shown in Bornstein (2018), the expected interest on lending in the region of no default ($D_a(a, b, \varepsilon, h, o, g, z) = 0$) is a return minus a default probability. In our context, the expected return to an unsecured loan is the interest payment net of the expected loss from the borrower moving into the default region. Over an interval of duration dt, we then have

$$E[dr_a(a, b, \varepsilon, h, o, g, z)] = r_a(a, b, \varepsilon, h, o, g, z)dt - \lambda_z \sum_{z'} p_{zz'} \lambda_\varepsilon \sum_{\varepsilon'} p_{\varepsilon\varepsilon'} D_a(a, b, \varepsilon', h, o, g, z')dt.$$

where $p_{\varepsilon\varepsilon'}$ is the probability of moving from ε to ε' conditional on receiving a labor

[11]See Ozkan et al. (2017) for a similar assumption.

productivity shock. In the default region ($D_a(a, b, \varepsilon, h, o, g, z) = 1$), and we set

$$r_a(a, b, \varepsilon, h, o, g, z) = \infty. \tag{19}$$

The zero profit condition in the region of no default implies that the return $r_a(a, b, \varepsilon, h, o, g, z)$ should be equal to the risk free rate, $r(g, z)$,

$$r_a(a, b, \varepsilon, h, o, g, z) = r(g, z) + \lambda_z \sum_{z'} p_{zz'} \lambda_\varepsilon \sum_{\varepsilon'} p_{\varepsilon\varepsilon'} D_a(a, b, \varepsilon', h, o, g, z'). \tag{20}$$

When considering savings, because no household defaults on a in the region where a is positive, $r_a(a, b, \varepsilon, h, o, g, z) = r(g, z)$.

Mortgages As explained above, households holding a mortgage pay an interest rate $r(g, z) + \iota(z)$ and a fraction $\theta(b, \overline{p}h))$ of the remaining balance b at each instant. Therefore, the flow income from a loan is $(r_t + \theta(b_t, \overline{p}h))b_t$. Banks discount the loan with an interest rate, $r_t + \theta_t$ as the loan matures at the rate θ_t. Recall that if a household defaults on its secured debt, the bank recovers the depreciated value of the house, $(1 - \delta_d)ph$.

Since banks expect zero profit on each loan, the discounted value of the loan at origination has to be equal to its expected cash flow. The price of the loan in the non-default region is given by

$$q_0(a, b, \varepsilon, h, g, z)b_0 = \mathbb{E}\Big[\mathbb{E}_\tau \int_0^\tau e^{-\int_0^s (r_s + \iota_t + \theta_s)ds}(r_t + \iota_t + \theta_t)b_0 dt + e^{-\int_0^\tau r_s ds} b(a_\tau, b_\tau, \varepsilon_\tau, h, g, z)\Big].$$

The scrap value $b(a_\tau, b_\tau, \varepsilon_\tau, h, g, z)$ at the stopping point depends on a household's discrete choice. In the case of foreclosure,

$$b(a_\tau, b_\tau, \varepsilon_\tau, h, g, z) = (1 - \delta_d)p(g, z)h.$$

When a household prepays the loan due to refinancing or a new house transaction, the scrap value is

$$b(a_\tau, b_\tau, \varepsilon_\tau, h, g, z) = e^{-\int_0^\tau \theta_s ds} b_0.$$

Applying the Feynman-Kac formula, the above equations can be written as the following partial differential equation.[12] At $t \in [0, \tau)$,

$$(\theta(b, \overline{p}h) + r(g, z) + \iota(z))q(a, b, \varepsilon, h, g, z) = \theta(b, \overline{p}h) + r(g, z) + \iota(z) + q_a(a, b, \varepsilon, h, g, z)\dot{a} \\ + q_b(a, b, \varepsilon, h, g, z)\dot{b} + \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} q(a, b, \varepsilon_j, h, g, z) + \sum_{k=1}^{n_z} \lambda_{zz_k} q(a, b, \varepsilon, h, g, z_k) \\ + \int \frac{\delta q(a, b, \varepsilon, h, g, z)}{\delta g(a, b, \varepsilon, h, o)} \mathcal{K} g(a, b, \varepsilon, h, o) d[a \times b \times \varepsilon \times h \times o]. \tag{21}$$

When, at $t = \tau$, the stopping time decision is to foreclose,

$$q(a, b, \varepsilon, h, g, z) = \frac{(1 - \delta_d)p(g, z)h}{b}, \tag{22}$$

and, if instead, the stopping time decision involves prepayment, we have

$$q(a, b, \varepsilon, h, g, z) = 1. \tag{23}$$

2.4 Firms

There are identical, competitive firms that produce non-durable consumption goods using a constant return to scale technology.[13] Firms rent capital from banks and employ labor to solve the following problem,

$$\max_{k, \ell} z f(k, \ell) - (r(g, z) + \delta)k - w(g, z)\ell \tag{24}$$

The production function $f(k, \ell) = k^\alpha \ell^{1-\alpha}$ and the capital depreciation rate is δ. Let L and K represent the aggregate quantities of labor and capital. In equilibrium, firms' profit maximization implies that the equilibrium real interest rate satisfies $r(g, z) = z\alpha \frac{L}{K}^{1-\alpha} - \delta$ while the wage rate is $w(g, z) = z(1 - \alpha)\frac{L}{K}^{-\alpha}$.

2.5 Government

The government collects taxes from households, levied on labor income net of deductions. Households can deduct the interest paid on their mortgage and property taxes. As already noted, the government also absorbs realized profits or losses from

[12] The Feynman-Kac formula establishes a connection between partial differential equations and stochastic processes. See Nuno and Thomas (2015) and Kaplan et al. (2018) for a similar usage of the Feynman-Kac formula.

[13] I assume the stock of durable goods is given.

unsecured and secured lending by banks, arising from aggregate shocks, through taxes or subsidies. The remaining revenue is spent on government consumption of non-durable goods, which is not valued by households.

The government's budget constraint is

$$\int T(b,\varepsilon,p(g,z)h)g(a,b,\varepsilon,h,o)d[a\times b\times\varepsilon\times h\times o]-G=0 \tag{25}$$

where G is government consumption. In the definition of equilibrium in the following section, there is no government debt. However, when there is a policy intervention and the government issues bonds to finance debt relief or a tax rebate, the definition of equilibrium must be generalized. See Appendix B.

2.6 Equilibrium

An equilibrium is a set of functions

$$(r_a,q,r,w,p,v,C,\tau,M,B_M,H_M,R,B_R,D_a,D_b,D_{ab},L,K,G)$$

that satisfies the following:

1. Households solve their lifetime optimization problems. Given price functions $\{r_a,r,q,w,p\}$, v solves (1)-(4), v^d solves (7)-(11) and v^f solves (14)-(16). The associated policy functions are $C,\tau,M,B_M,H_M,R,B_R,D_a,D_b,D_{ab}$.
2. Firms maximize profits by solving (24) and L,K are the associated policy functions.
3. The unsecured debt price function r_a is determined by (19) and (20).
4. The mortgage price function q is determined by (21) - (23).
5. Capital market clears: $\int(a-b)g(a,b,\varepsilon,h,o)d[a\times b\times\varepsilon\times h\times o]=K$.
6. Labor market clears: $\int\varepsilon g(a,b,\varepsilon,h,o)d[a\times b\times\varepsilon\times h\times o]=L$.
7. Housing market clears: $\int hg(a,b,\varepsilon,h,o)d[a\times b\times\varepsilon\times h\times o]=\overline{H}$.
8. The government budget constraint (25) holds.
9. The Kolmogorov Forward Operator $\mathcal{K}$ that describes the change of density function g is generated by agents' optimal choices.

2.7 Computation of equilibrium

The aggregate state of the model contains distribution of households and is high dimensional. The solution algorithm I use is based on the finite difference method in

Achdou et al. (2022) with several notable differences. First, there are multiple stopping choices including two types of default, the buying and selling of houses, refinancing and prepayment. Second, the model solution is nonlinear in both the individual and aggregate state vectors. Importantly, I do not assume certainty equivalence but allow for aggregate uncertainty. I solve stochastic equilibrium following the approach in Krusell and Smith (1998). Appendix C provides more details on the computation.

3 Mapping Model to Data

I choose model parameters to match key cross-sectional features of the U.S.economy in the early 2010s. A quantitative study of debt relief programs requires reproducing the distribution of assets and liabilities across households. Moreover, as changes in income drive changes in households' balance sheets, and labor income shocks are the source of uninsurable risk, the calibration of the stochastic process for labor earnings must be consistent with the data.

A subset of parameters is assigned in advance of solving the model's stationary state. In addition, the earnings process is estimated independently, outside of the model. Finally, 8 parameters are jointly calibrated in the steady state. These are parameters specifying household preference (σ, κ, ρ), default costs ($\xi_a, \xi_b,$), the grid of house sizes (h, $\underline{h}$) , and the tax function (τ_0). Table 4 lists calibrated parameters, and Table 3 reports targeted data moments and model moments. I associate targets with specific parameters, but this correspondence is only suggestive as the parameters are jointly determined.

Earnings process I model the labor earnings process as a combination of two independent processes:

$$\varepsilon_{ij} = \varepsilon_i^p (1 + \varepsilon_j^t)$$

In calibrating this earnings process, I target both cross-sectional inequality and individual risk. The former involves the earnings distribution described in Table 1 and the latter uses moments from the distribution of earnings growth listed in Table 2.

I choose 11 parameters targeting 20 moments. Detailed information about calibration of earnings process can be found in Appendix D.1. The calibrated process implies that a shock to ε^p arrives on average once every 21 years. Upon the arrival of a shock, one's income level jumps to a different level. Turning to the other labor productivity shock, a shock to ε^t arrives on average once every 0.9 years. The infrequent component

Table 1: Earnings distribution

	Variance	Quintiles(%)					Top(%)		
		1q	2q	3q	4q	5q	90-95	95-99	99-100
Data	0.92	-0.1	4.2	11.7	20.8	63.5	11.7	16.6	18.7
Model	0.93	5.6	6.7	7.0	20.3	60.0	10.4	15.9	15.6

Data: Song et al. (2018), SCF (2007)

of labor income shock, ε^p can be interpreted as the persistent component and ε^t as the transitory component. Households do not experience a large shock often, but income fluctuates around their persistent component through frequent shocks, ε^t.

Table 1 and Table 2 shows the moments targetted in the data and the corresponding moments generated by my model's earnings process. The estimated process generates moments that reproduce the data well.

Table 2: Earnings dynamics

	Std.		Skewness		Kurtosis		$\mathbf{P}(\lvert\Delta y\rvert) < \mathbf{x}$*			$\mathbf{P}(\lvert\Delta y\rvert) \in [\ \underline{x}, \overline{x}\ ]$ *	
	1y	5y	1y	5y	1y	5y	x = 0.2	0.5	1.0	[0,0.25)	[0.25,1)
Data	0.51	0.78	-1.07	-1.25	14.93	9.51	0.67	0.83	0.93	0.31	0.16
Model	0.30	0.58	-0.08	-0.03	15.14	8.57	0.62	0.97	0.98	0.43	0.19

* $|\Delta y|$: Absolute log earnings change. Data: Guvenen et al. (2015)

Assets and debt

Houses The survey by Davis and Van Nieuwerburgh (2015) finds maintenance costs are between 1% and 3% of the value of a house, so I set maintenance costs, ξ_h, to 2%. The parameter for the housing transactions cost function, ξ_0 is 0.07 to match the average transaction cost of 7% in Delcoure and Miller (2002).[14] The fixed stock of houses $\overline{H}$ is set to the value of housing wealth in SCF (2007). Normalizing this by average labor income, the corresponding ratio in the model implies $\overline{H}$ is 1.4.[15]

Mortgages The loan-to-value ratio γ is set to 1.05 based on the observations that mortgages are available with zero down payment and home equity lines of credit are avail-

[14] Other estimates are similar to 7%. Smith et al. (1988) estimate the transaction costs of changing houses, and their estimate is approximately 8-10%. Martin (2003) shows that costs of of buying a new home is 7 - 11% of the purchase price of a house.

[15] To convert the 2007 dollars to model consistent values, I use average labor income in SCF (2007) and the corresponding value in the model.

able to households.[16] This is comparable to the two values, 0.95 and 1.1, in Kaplan et al. (2020). The amortization rate of mortgages, $\overline{\theta}$ is set to 0.025, which implies that the duration of a loan is approximately 40 years if a household fully finances the purchase of the house. The refinance cost ξ_r is set to the equivalent of $1,500 (2007 dollars) in the model accounting for the sum of application, loan origination, attorney, insurance and inspection fees.[17]

Bankruptcy and foreclosure The utility costs ξ_a and ξ_b are calibrated to match the bankruptcy and foreclosure rates. The bankruptcy rate target is 1.06%, which is constructed using the number of Chapter 7 and Chapter 13 bankruptcy filings from the U.S. Bankruptcy Courts over the number of households from the U.S. Census (averaged over 2000-2017). The foreclosure rate target is 0.55%. This is the average rate in the U.S. during the late 1990s. (Mortgage Banker's Association)

The intensities at which the bankruptcy and the foreclosure flags are removed are set to match the following. After filing for Chapter 7 bankruptcy, households cannot file again for 6 years. Households that file for Chapter 13 bankruptcy enter into repayment plans that last for 3–5 years. Accordingly, I choose λ_d to 0.183 to match an average bankruptcy duration of 6 years. For foreclosure, Fair Issac reports that households' FICO scores can recover in as little as 2 years (see Mitman (2016)). Hence λ_f is set to 0.693 to give an average duration of 2 years for the foreclosure flag. The depreciation rate when foreclosing, δ_d is 22%, and taken from Pennington-Cross (2006). Pennington-Cross (2006) estimates the loss of value of a foreclosed property using a sample of real estate owned property.

Preferences Households receive utility flow from consuming non-durable goods and from consuming a service flow from their houses. Their utility function is

$$u(c,h) = \frac{c^{1-\sigma}-1}{1-\sigma} + \kappa log(h + \underline{h}).$$

The function u is strictly increasing and strictly concave in c and h. The parameter $\underline{h}$ allows for households without houses, and its value is chosen to match the share of

[16] An LTV limit above 1 is consistent with the following observations. First, 100% LTV loans are available. For example, the United States Department of Veterans Affairs and the United States Department of Agriculture guarantee purchase loans to 100%, and the Federal Housing Administration (FHA) insures purchase loans to 96.5%. Further, in data, there are households with negative home equity. In the 1989 - 2013 waves of the SCF, the size of secured debt exceeded the value of non-financial assets among the poorest 20% of households.

[17] See `https://www.federalreserve.gov/pubs/refinancings/default.htm` for more information about refinancing costs.

Table 3: Targeted moments and model values

Moment	Data	Model	Data Source
Foreclosure rate	0.0055	0.0048	Mortgage Banker's Association
Bankruptcy rate	0.010	0.0056	U.S. Courts, U.S. Census
Tax revenue to output	0.16	0.17	CBO
30th percentile non-fin. assets to assets	0.31	0.26	SCF (2007)
50th percentile non-fin. assets to assets	0.68	0.42	SCF (2007)
70th percentile non-fin. assets to assets	0.87	0.72	SCF (2007)
30th percentile debt payment to income	0.00	0.00	SCF (2007)
50th percentile debt payment to income	0.12	0.10	SCF (2007)
70th percentile debt payment to income	0.22	0.23	SCF (2007)
Net worth/Income	4.72	4.86	SCF (2007)
Share of homeowners	0.68	0.67	SCF (2007)
Households with mortgage	0.49	0.60	SCF (2007)

Note: Business assets and vehicles are excluded from non-financial assets. More details on the categorization of assets and debt can be found in appendix D.2.

homeowners. The curvature parameter σ and weight κ primarily affect households' asset and debt composition. They, along with the discount rate ρ are jointly calibrated to match the total debt to asset ratio and the debt payment to income ratio across households. I set ρ to 0.086, σ to 2.0 and the weight on durable consumption κ to 1.0.

Production The production technology is constant returns to scale. The capital share set as the residual of the labor share of output and the labor share is measured in Giandrea and Sprague (2017). They calculate the labor share of output in the non-farm business sector from 1947 through 2016.[18] I use the average value of labor share between 1989 and 2013, 60.5%; thus, α is 0.395. I assume that the depreciation rate δ is 0.069 (see Khan and Thomas (2013)).

Government The income tax function $T(y) = y - \tau_0 y^{1-\tau_1}$ is taken from Heathcote et al. (2017) where y is taxable income. Taxable income is labor income minus the tax deductible interest payments on mortgages and property taxes. Taxable income is

$$y = w(g,z)\varepsilon - r(g,z)\min(b,\bar{b}) - \min(\tau_h p(g,z)h, \overline{\tau_h}).$$

The property tax rate τ_h is set to 1%, which is the median tax rate across US states.[19]

[18]Labor share of output is sum of employee compensation and proprietors' labor compensation.
[19]See Kaplan et al. (2020) who references data from the Tax Policy Center.

Table 4: Parameter values

Parameter	Value	Internal	Description
Preferences and production			
ρ	0.086	Y	Discount rate
σ	2.0	Y	Curvature of the utility function
κ	1.0	Y	Weight on durable good
α	0.395	N	Capital share
δ	0.069	N	Depreciation rate
Tax			
τ_0	0.58	Y	Tax rate
τ_1	0.181	N	Tax progressivity
τ_h	0.01	N	Property tax rate
$\overline{b}$	1,000,000	N	Maximum debt to deduct interest payments
$\overline{\tau_h}$	10,000	N	Maximum deduction on property tax
Labor productivity			
$\overline{\varepsilon}^p$	8.5	N	Upper bound of Pareto distribution
$\underline{\varepsilon}^p$	0.08	N	Lower bound of Pareto distribution
η^p_ε	1.526	N	Shape of Pareto distribution
$\eta_{\varepsilon^p_i}$	[1.9,1.5,1.3,0.6]	N	Shape of Pareto distribution
λ^p	0.048	N	Shock intensity
λ^t	1.260	N	Shock intensity
χ	0.239	N	Size of the ε^t shock
p^t	0.600	N	Probability of drawing negative ε^t
Assets and debts			
ξ_h	0.02	N	Depreciation rate of h
δ_h	0.07	N	h transaction cost
$\overline{H}_s$	1.4	N	Supply of durable good
h	[0.0, 0.2, 1.7, 3.1, 4.9,8.5]	Y	House sizes
$\underline{h}$	0.13	Y	Input to utility of non-homeowners
γ	1.05	N	Loan-to-value ratio
$\overline{\theta}$	0.025	N	Amortization rate of b
ξ_r	0.01	N	Refinancing cost
ξ_a	0	Y	Utility cost
ξ_b	1.3	Y	Utility cost
λ_d	0.183	N	Removal of bankruptcy flag
λ_f	0.693	N	Removal of foreclosure flag
δ_d	0.22	N	Depreciation due to foreclosure
Aggregate shock			
$[z_1, z_2]$	[0.96, 1.02]	Y	Level of total productivity
$[\lambda_1, \lambda_2]$	[0.364, 0.149]	N	Shock intensity
$[\iota(z_1), \iota(z_2)]$	[0.011, 0.0]	Y	loan rate premium

The Internal Revenue Service's rules imply that the maximum size of mortgage ($\bar{b}$) for interest rate payment deduction is $1,000,000 and maximum value of tax deductible property tax ($\overline{\tau_h}$) is $10,000.[20,21]

The parameter τ_1, determining the degree of progressivity of the tax system, is 0.181 as in Heathcote et al. (2017).[22] Next, τ_0 is set to 0.585 to match the tax revenue-output ratio 16.7%.[23]

Aggregate shocks Aggregate productivity z follows a two-state Poisson process, $z \in [z_1, z_2]$ with $z_2 > z_1$. The process jumps from state 1 to state 2 with intensity λ_1 and in the reverse direction with intensity λ_2. These two states represent a recession (z_1) and an expansion (z_2).

The support of aggregate TFP, $[z1, z2]$ is $[0.96, 1.02]$ to match the standard deviation of U.S. output given the shock intensities. These are $\lambda_2 = 0.1498$ and $\lambda_1 = 0.3636$. This implies the average duration of an expansion is 26.7 quarters and the average duration of a recession is 11 quarters. I choose these durations following Nakajima and Ríos-Rull (2019). The loan rate premium in recessions is set to 1.1% to replicate the procyclicality of credits. Table 6 reports the cyclical properties of the U.S. economy from the data and the model.

Validation I compute non-targeted moments to confirm model's validity. First, the model captures the household net worth distribution very well as shown in Figure 1. Moreover, my model allows me to further break-down households' net worth. Table 5 shows the distribution of assets and debt. The share held by each net worth quintile of households of non-financial assets, financial assets and secured debt from the model is reasonably close to the data.[24] However, unsecured debt is almost entirely held by the middle class in the model while it is evenly distributed over net worth quintiles in the data. Despite its rich asset structure, households in the model cannot have unsecured debt and liquid savings at the same time. As a result, given that the data indicates that 9.6% of households have net negative financial assets, it is hard to match

[20]When considering $\bar{b}$, in 2018, the deduction for home mortgage interest was limited to the first $750,000 ($375,000 if married filing separately) of debt. The limit was $1,000,000 ($500,000 if married filing separately) if a household was deducting mortgage interest incurred on or before December 15, 2017. Since I target 2007 data, I apply the limit before 2017.

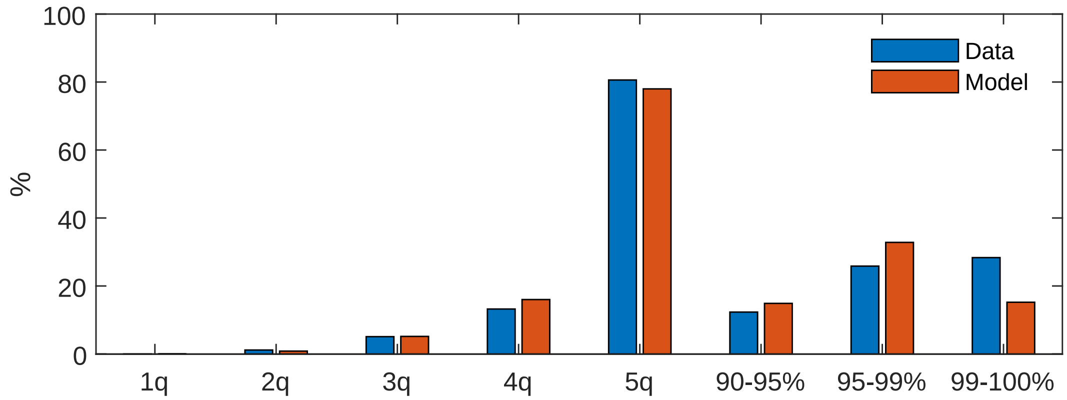
[21]The parameter ($\overline{\tau_h}$) matches the limit on deductions of state and local taxes. These include general sales taxes, real estate taxes and personal property taxes.

[22]They estimate this parameter using the Panel Study of Income Dynamics (PSID) for survey years 2000, 2002, 2004, and 2006, in combination with the NBER's TAXSIM program.

[23]I use the value of tax revenue-output ratio (Congressional Budget Office) between 2000 and 2014.

[24]Appendix D.2 provides detailed information about categorization of assets and debts.

Figure 1: Net worth distribution



Note: Each bar shows a net worth share held by quintiles or the top 10% of households. Business assets and student loans are excluded. Data: SCF(2007)

the distribution of financial assets and unsecured debt at the same time. Table 13 in Appendix H shows the composition of assets across households by net worth. The model replicates the relatively high non-financial asset holdings for households in the 1-3th quintile as well as the negative correlation between the share of non-financial assets and wealth.

Second, the model generates rich heterogeneity in the MPC from a changes in liquid assets. This is consistent with evidence in Misra and Surico (2014) and Parker et al. (2013). The average MPC is 11% and it varies over income, liquid assets, mortgages, and housing. For example, households with a share of liquid assets over net worth less than 10% have a MPC of 27% while those with a share of liquid assets over net worth higher than 50% have a MPC of 8%.

Third, the model captures the cyclical properties of the economy. Table 6 compares the aggregate statistics from the U.S. data and my model. It shows that the model captures key properties of the data. In particular, both consumption and investment are strongly correlated with output, and the investment is more volatile than consumption. Furthermore, secured credit is positively correlated with output.

House prices are procyclical in the model as in the data and the volatility of house prices is also consistent with the data. It is useful to mention that the model is not calibrated to generate an episode like the Great Recession nor to capture house price volatility. Therefore not every recession causes large house price falls. House price decreases significantly when a recession is exceptionally long, or when households are financially distressed at the beginning of a recession.

Table 5: Share of assets and debt

		Asset		Debt	
		Non-financial	Financial	Secured	Unsecured
Data	Q1	0.59	0.16	2.22	14.71
	Q2	4.42	1.13	11.40	18.92
	Q3	11.14	3.47	20.52	21.83
	Q4	19.29	9.72	23.94	23.41
	Q5	64.58	85.53	41.97	21.13
Model	Q1	0.01	0.00	0.01	0.01
	Q2	2.07	1.04	3.12	0.55
	Q3	9.36	6.22	13.94	6.41
	Q4	23.32	14.80	23.53	89.94
	Q5	65.24	77.94	59.40	3.10

Note: Share of assets and debt by net worth quintiles. Data: SCF(2007)

Table 6: Cyclical properties

	Data		Model	
	std(%)	corr. with output	std(%)	corr. with output
Output	1.8	1.0	1.8	1.0
Consumption	1.3	0.9	0.6	0.9
Investment	6.9	0.8	5.3	0.9
Unsecured debt	5.8	0.4	15.7	0.1
Mortgage	4.4	0.2	3.4	0.8
Bankruptcy	11.1	0.1	13.5	-0.4
Foreclosure	-	-	12.0	-0.6
House price	4.3	0.2	3.6	0.8

Logs of the data are filtered using the HP filter with a smoothing parameter of 100. Output, consumption and investment data combine information from NIPA tables 1.1.5 and 2.3.5. from 1963 to 2007. Output: real GDP minus net export. Consumption: real private consumption expenditures minus housing service; Investment: real gross domestic investment; Unsecured debt: Consumer credit (Flow of Funds), deflated by GDP deflator; Secured debt: Home Mortgages (Flow of Funds), deflated by GDP deflator. House price: Freddie Mac US house price index, divided by durable good GDP deflator (NIPA table 1.1.4, line 4). Unlike other data, house price statistics are computed using 1975-2007 data. Standard deviation and correlation with output of bankruptcy is taken from Nakajima and Ríos-Rull (2019). Number of bankruptcies (from U.S. courts) is normalized by the number of households (from Census) using 1995-2004 data. The model series are simulated quarterly and aggregated to annual frequency to compute the statistics.

4 Steady state results

Bankruptcy and foreclosure exist to offer partial consumption insurance to households who are unable to pay their debt, as such, additional debt relief may be unnecessary in ordinary times. However, when one tries to evaluate extraordinary debt relief programs during crises, the model needs to have these options. In their absence, the effects of a temporary debt relief program can be overstated. Another compelling reason to include bankruptcy and foreclosure in a model is that their existence alters households' saving and borrowing decisions. Distributions of households will not be the same with and without these default options and this distribution is a key determinant of policy effects.

In this section, I explore the role of defaulting on unsecured debt and foreclosing on mortgages in shaping the distribution of assets and liabilities across households, and how these options interact in the stationary equilibrium. Specifically, I compare the following variations of my model i) the full model with bankruptcy and foreclosure (the benchmark model), ii) a model in which households cannot foreclose but can go bankrupt (Bankruptcy only), iii) a model where only foreclosure is available to households (Foreclosure only), and iv) a model without any option to default (None). An exogenous borrowing constraint is set to 0 when there is no bankruptcy option and the loan to value ratio is set to 0.85 when there is no foreclosure option to prevent the consumption set becoming empty. I assess the value to households of having an option to default by comparing the benchmark economy to these alternative economies.

4.1 The effect of default on aggregate variables and the distribution of household

The possibility of default affects aggregate variables as well as the distribution of assets held by households. First, aggregate capital rises 3.1% when agents do not have the option to default on either unsecured debt or mortgages (None). This is because the option to default acts as insurance against income risk so the precautionary saving motive is stronger when it is not available. However, consumption is 0.9% lower in the economy without default. Table 7 shows this result.

Starting with the Bankruptcy only case, the first row of Table 7 shows that house prices rise. In the absence of foreclosure, mortgages become cheaper but the LTV limit is lower. This creates tension for the total effect on housing demand. Mortgages are cheaper, increasing the demand for houses. On the other hand, borrowers cannot

Table 7: Aggregate variables

	Liquid saving	Secured debt	Capital	h price	Consumption
Bankruptcy only (%)	-12.9	-35.4	0.7	6.3	1.7
Foreclosure only (%)	-1.5	-4.4	0.8	-14.7	-1.1
None (%)	-7.2	-23.9	3.1	-17.7	-0.9

Table 8: Net worth share by quintiles (%)

	Benchmark	**Bankruptcy only**	**Foreclosure only**	**None**
1Q	0.0	-0.2	0.0	0.0
2Q	0.8	1.7	0.6	0.5
3Q	5.2	8.3	4.7	4.0
4Q	16.0	18.0	14.1	12.4
5Q	78.0	72.2	80.6	83.1

Table 9: Debt payment/Income (%)

	Benchmark	**Bankruptcy only**	**Foreclosure only**	**None**
50th percentile	9.8	7.1	10.2	8.5
70th percentile	23.2	12.9	19.9	16.2
90th percentile	38.2	37.0	35.9	31.6

Note: Bankruptcy only refers to an economy in which households cannot foreclose but can go bankrupt, and Foreclosure only refers to an economy with only foreclosure. None refers to an economy without any option to default.

finance as much of the total cost of purchases, reducing the demand for houses. We see that the first effect dominates and house prices rise.

In the absence of bankruptcy (the Foreclosure only case), house prices are lower (Table 7). Without bankruptcy, households prefer to have more liquid assets to insure themselves as they no longer have access to short-term credit. Therefore, housing demand is lower and equilibrium house prices are lower. Households of all incomes have a higher desire to save but the increase in savings is larger for high-income households, raising the share of wealth held by the richest 20%. (Table 8)

These results imply that the dynamics of aggregate variables could be affected by the options to default. Default allows households to reduce their debt burden when they receive income shocks, thereby affecting portfolio choices. Economies with more default options tend to have lower capital and higher leverage (Table 7 and 9). Over the business cycle, the demand for liquid and illiquid assets changes with the number of households who choose to default. As we will see below in Section 5, this changes the path of aggregate quantities and alters the propagation of shocks.

4.2 Who values the default option?

I assess the value to households of being able to default by comparing the benchmark economy to the alternative economies specified above. The value of bankruptcy and foreclosure options are calculated as the non-durable consumption-equivalent gain obtained in moving from one of the alternative economies to the benchmark economy. The welfare measure is

$$\begin{aligned} \mathcal{W}_i &= \int g_i(a,b,\varepsilon,h,o) x_i \, d[a \times b \times \varepsilon \times h \times o] \\ x_i &= \frac{[(1-\sigma)(\rho v_{benchmark} - \rho v_i + u(c_i))]^{\frac{1}{1-\sigma}}}{c_i} - 1 \end{aligned} \tag{26}$$

where x_i is a consumption equivalent gain and i is one of { bankruptcy only, foreclosure only, none }. This derivation is in Appendix E and the welfare is computed using household without a default flag.

The average welfare measure $\mathcal{W}_i$ is 0.4%, 0.5%, and 0.9% respectively for the bankruptcy only, foreclosure only and no default (none) economies. Not surprisingly, the welfare gain is the largest when both options are available and between them, households value the bankruptcy option slightly more than the foreclosure option.

Figure 2: Value of bankruptcy

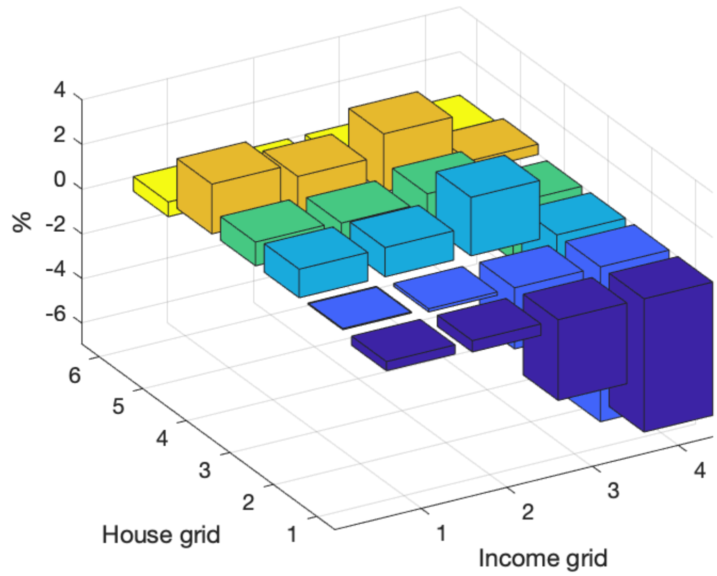
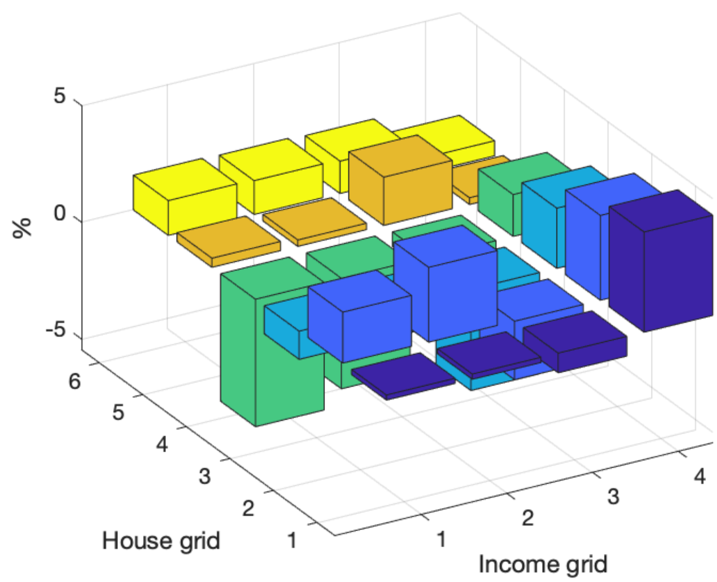


Figure 3: Value of foreclosure



Note: Consumption equivalent gain in moving from the bankruptcy only to the benchmark economy. Each bar shows an average consumption equivalence gain at given labor productivity and house size. The figure plots over only one set of persistent components of labor productivity with a low level of the transitory component. The distribution of gains over the same level of a persistent component but with a different transitory component is similar.

Figures 2 and 3 show the consumption compensation required when moving from the benchmark economy to one with either foreclosure only or bankruptcy only, hence they illustrate the value of the bankruptcy and foreclosure options, respectively. Each bar in the figures represents the average consumption equivalent gain needed to compensate households with a specific house size and level of labor productivity. At each house size and labor income, households are distributed over liquid savings and mortgages. While we sum over these conditional distributions, welfare gains and losses vary with savings and mortgage holdings as well as income and house size.[25]

The figures show that the benefit of default is unevenly distributed. Figure 2 studies the foreclosure only economy without bankruptcy. All else equal, the value of bankruptcy depends on a household's probability of using the bankruptcy option. Those who have low liquid wealth are more likely to declare bankruptcy and, therefore, value the option the most. However, households who are unlikely to use the bankruptcy option are also affected by its presence because it loss changes equilibrium prices. As house prices are higher in the benchmark economy (see Table 7), households who own large houses tend to dislike the bankruptcy option. Higher house prices imply higher costs of buying and maintaining houses, reduce resources available for non-durable consumption. Figure 2 shows that high-income households dislike the bankruptcy option since these households tend to have large amounts of savings and either own or want to buy large houses. Most welfare gains are shared across households with mid-size houses and low incomes, as these households tend to have small savings.

Likewise, the value of foreclosure depends on the probability of foreclosure and the changes it induces in equilibrium prices. Figure 3 studies the bankruptcy only economy without foreclosure. It shows that the foreclosure option is valued by households with high income and those with large houses. Both groups benefit from the lower house prices that exist in the benchmark economy as shown in Table 7. Households with low income and mid-size houses (house sizes 3 and 4) dislike the option as they are likely to sell and would prefer higher house prices. Households with low income and small houses (house size 2) are likely to use the foreclosure option, therefore they value the option.

In summary, we see large differences not only in equilibrium quantities but also in prices, across models with different default options. In turn, these price differences

[25] Labor productivity is composed of persistent and transitory components. The figure plots over only one set of persistent components with a low level of the transitory component. The distribution of gains with the other value for the transitory component is similar.

determine the value of default options. In the model with aggregate uncertainty, the availability of walking away from debt will shape the level and volatility of capital, output, and house prices. This may affect the need for policy interventions as well as the effects of such policies.

5 Effects of debt relief programs in recessions

Above, I have established that my model is consistent with salient empirical regularities characterizing the distribution of households (Section 3). Furthermore, my model generates business cycles that resemble the data along important margins. First, the volatility of output, consumption, and investment, as well as the pro-cyclicality of credit are consistent with the data. Second, the volatility of house prices and the correlation between house prices and output are consistent with the data (Section 3, Table 6). These results make the model useful for the analysis of debt relief programs during recessions. In this section, I analyze such programs using a series of policy experiments.

For the policy experiments below, I chose a recession where, in the absence of intervention, house prices fall more than 8% from their peak.[26] Using this no policy scenario as the benchmark economy, I compare it to an alternative where the government intervenes when it observes house prices having fallen by 5%. In this intervention, all households with loan-to-value (LTV) ratios above 95% are eligible for a mortgage principal reduction. These households receive partial mortgage debt forgiveness and their LTV ratios fall to 95%. I assume that this intervention is unanticipated.[27]

The mortgage debt relief studied here, larger than policies that were implemented during the Great Recession, is chosen to allow comparability with the principal reduction examined by Kaplan et al. (2020). (Appendix F discusses mortgage relief policies in the Great Recession.) This program affects 15.7% of households. The average size of

[26] As total factor productivity shock is calibrated to capture salient features of the US business cycle, most model recessions do not involve such large house price drops. The model is designed to study the effects of debt-related policies, not to explain the origins of housing crises. While, in the data, house prices declines may lead recessions, falls in income cause house price declines in this model. The ordering of a fall in house prices and income is unlikely to be important to the results that follow, since interventions happen when both are relatively low. Furthermore, a house price drop before a recession can be generated by a shock to preferences or credit conditions.

[27] Before and after the shock, the simulation is based on forecasting functions estimated in an environment without the policy intervention. Thus these policies are unanticipated. This is intended to capture the unusual nature of the Great Recession, whose severity was unexpected by most policy makers and market participants.

the mortgage principal reduction for eligible households corresponds to roughly about \$38,000 (2007 dollars) and the total cost is 5.6% of GDP.

The government covers the cost of the policy by issuing debt which is repaid by increasing taxes over subsequent expansions. Therefore, when the government intervenes, households expect future taxes to rise. Once the debt is repaid, the government returns to the pre-intervention tax regime. For tractability, I assume that households do not know exactly when the cost will be paid off but expect the procyclical tax regime will end with a 20% probability at any time. A full characterization of the procyclical tax rate economy can be found in Appendix B. Whether the policy is funded or not, and how it is funded, is critical in shaping the effects of the policy intervention. I show this by first reporting the results of an unfunded debt relief program.

I compare the effects of the targeted mortgage debt relief program with those of a tax rebate, a cash transfer of equal amount to all households. While the tax rebate is not a direct debt relief policy, it is a common stimulus policy. There are important differences between these two policies. First, mortgage forgiveness targets highly indebted homeowners while tax rebates are not a targeted policy. Second, mortgage forgiveness provides illiquid assets (increases in home equity) but the tax rebate provides liquid assets.[28] For comparability, I set the size of the tax rebate to match the cost of the mortgage reduction. This implies each household receives a lump sum transfer equivalent to \$5,900 (2007 dollars).

In Section 5.1 and 5.2, I show the effects of each policy on aggregate variables and then analyze consumption responses, and welfare, among different segments of the population. Next, in Section 5.3, I examine the state-dependent effects of mortgage forgiveness using recessions that differ in their initial distributions of households. Thereafter, to highlight the importance of allowing for bankruptcy, I examine the principal reduction in an economy with only a foreclosure option in Section 5.4. Finally, I examine an alternative program that reduces mortgage payments in Section 5.5.

[28]There is an indirect liquidity effect of the principal reduction program. Since households who have mortgages pay interest on their loans, a reduction in the outstanding loan increases liquidity by reducing interest rate payments.

Figure 4: Response of aggregate variables

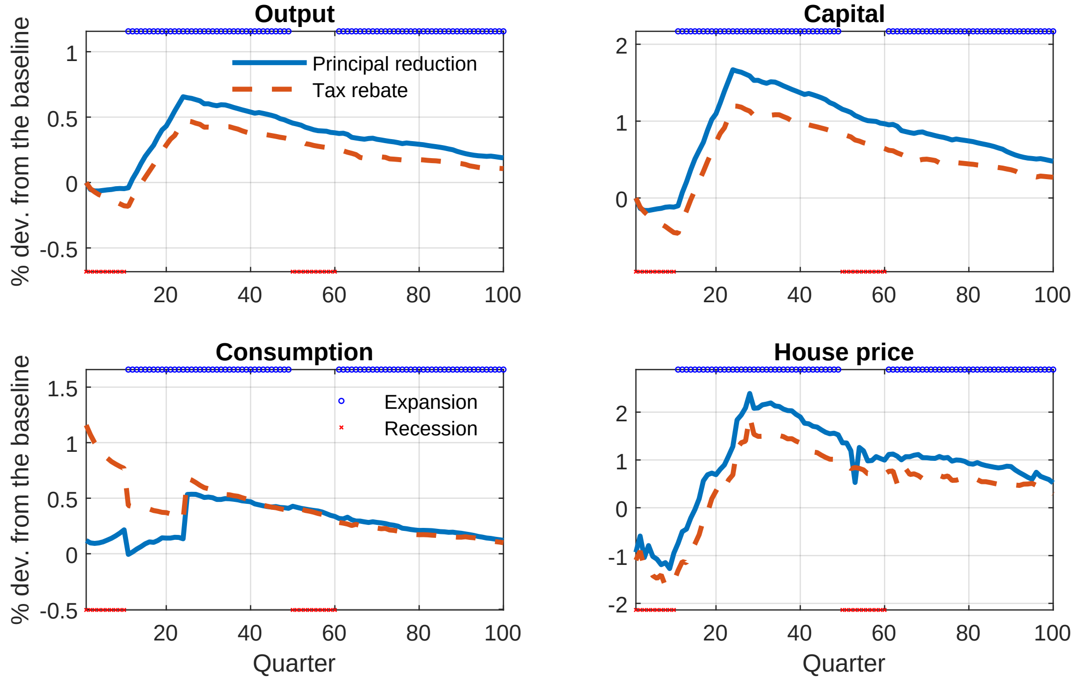
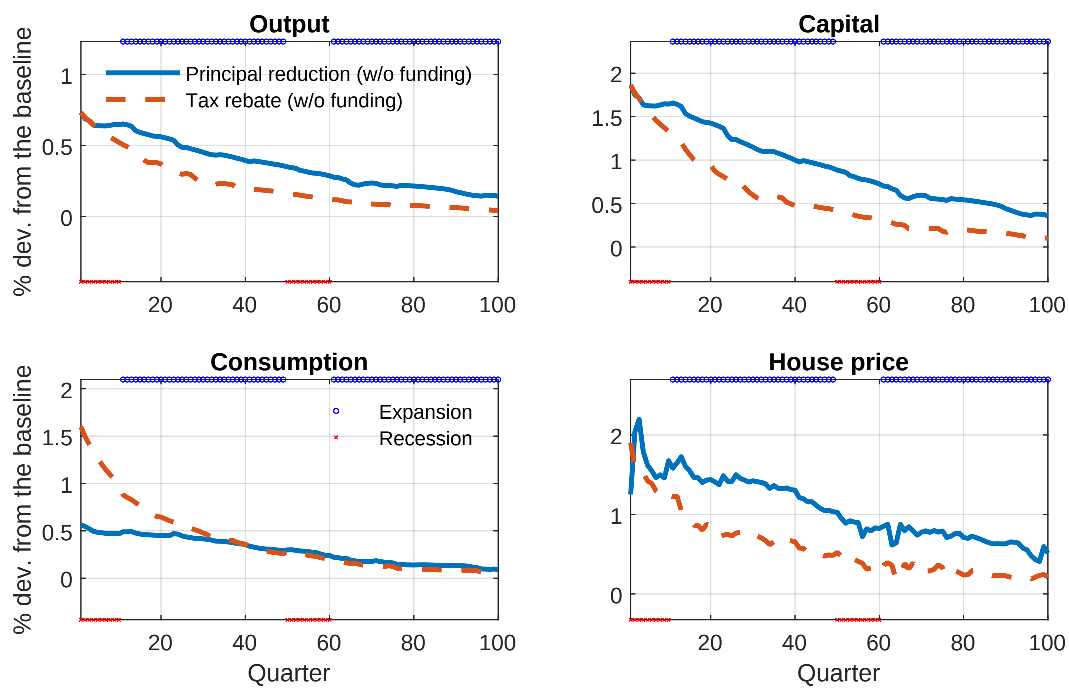


Figure 5: Response of aggregate variables (without funding)



Note: Aggregate series are shown as percent deviations from the corresponding series in the economy without policy intervention.

5.1 Mortgage forgiveness versus tax rebate

5.1.1 Aggregate responses

Figures 4 - 7 show an economy initially in a recession, transitioning to an expansion in the 12th quarter. Throughout all exercises, figures show variables as *percent deviations from the corresponding value in the economy without policy intervention*.[29] I begin by examining the response in the model economy to an unfunded mortgage principal reduction as studied by Kaplan et al. (2020). As will be seen below, when the government borrows to fund a debt relief program, there is a large change in the timing of its impact. Studying the unfunded case allows me to highlight this effect.

Figure 5 shows that a one-time mortgage reduction has an immediate, strong and persistent stimulus effect on the economy. As the eligibility for the policy depends solely on a household's LTV, the recipients of the mortgage reduction vary in their net worth. Some are households with relatively large savings, however more have low savings. These highly indebted households have high MPCs.

Among the recipients, households with high MPC increase their consumption and aggregate consumption rises by 0.5%. The policy intervention increases the illiquid assets – home equity. As interest payments fall, income available for consumption rises. However most of a program recipient's consumption response comes when they refinance their mortgages, withdrawing home equity and increasing liquid assets. As refinancing happens gradually, this mutes the initial response of aggregate consumption and adds persistence.

Financial assets, in the model with an unfunded policy intervention, are allocated to financing mortgages and investment in physical capital. The reduction in aggregate mortgage debt increases investment and physical capital rises. This leads to a protracted increase in GDP and real wages. Higher wages and lower LTV ratios reduce foreclosures (Figure 7). The fall in foreclosures reduces the supply of available houses, and the rise in wealth increases the demand for houses. In equilibrium, house prices rise.

I now study the exact same debt relief program but assume that it must be fully funded. As above, the policy intervention is at $t = 0$ when the economy is in a recession. Now, however, it is financed by issuing government debt. The proceeds are immediately spent on mortgage principal reduction. All households know that at the onset of a recovery taxes will rise.

[29] Aggregate variables in levels, for the baseline economy are shown in Appendix H, Figure 17.

The private sector balance sheet sees no net change following the intervention. As the fall in mortgage debt is offset by a rise in government debt, total resources available to the economy do not change. Nonetheless, the reduction in mortgage debt increases the net worth of heavily indebted households. In effect, the mortgage principal reduction tends to be a redistribution from low MPC households to heavily indebted households with higher MPCs. This drives a rise in aggregate consumption and a fall in savings and therefore capital. These changes are partly offset by households' expectations of increases in future taxes. The fall in future disposable income implies a rise in current savings as households attempt to smooth their consumption. This dampens the rise in aggregate consumption. At the same time, as low MPC households increase savings, the effect of the policy on capital is small. As a result, Figure 4 show that, before the economy transitions into recovery, capital is only slightly lower than the benchmark.

At the onset of a recovery, the government increases taxes and uses the additional revenue to begin repaying its debt. This debt is held by wealthier households with lower MPCs and high savings rates. As these households adjust their portfolios away from government bonds and into capital, aggregate capital rises faster than in the no policy benchmark (see Figure 4).[30] This drives a more rapid recovery in GDP and consumption than seen in the absence of policy.

An unintended consequence of the principal reduction policy is that it further lowers house prices during the recession, compared to the no policy benchmark. The anticipated rise in future taxes, and resulting increase in liquid savings, reduces the demand for housing.[31] However, once the economy begins a recovery, house prices begin to climb with capital and output. As these series rise above the benchmark, so do house prices.

Summarising the effects on aggregate quantities and house prices, the anticipated rise in future taxes that accompanies a funded debt relief policy shifts the timing of responses to the stimulus. There is less effect on consumption and output during the recession, but growth in these series is amplified over the recovery. In this sense, the debt relief policy strengthens a recovery without changing its timing. This is a result of assuming that the government will begin to repay its debt, redistributing resources

[30]Appendix B.4 discusses the accuracy of forecast rules over the debt policy.

[31]More generally, an anticipated rise in future taxes leads to lower house prices at any given aggregate state. For example, at the period the government intervenes, the equilibrium house price would have been 1.4% lower, with procyclical taxes but without the principal reduction. Instead, house prices fall 0.9% because those who receive the principal reduction foreclose less often reducing the supply of houses for sale.

to low MPC households that invest in physical capital, only once a recovery begins.

Startlingly, while the the stimulus effects of debt relief are delayed until the recovery, the impact of funding the debt relief program has almost no effect on mortgages and unsecured debt. Figure 6 shows the path of credit and default under the funded program and Figure 7 show their path for the unfunded case. The funded policy sees a larger fall in foreclosures and almost identical paths for mortgages, unsecured debt and bankruptcy. While program recipients vary in their liquid assets, changes in aggregate credit following the policy interventions are the result of decisions by highly indebted households with little liquid assets. Having high MPCs, these households respond little to future tax changes, hence these variables are largely unchanged by funding.

By reducing the number of financially distressed households, the interventions significantly reduce foreclosures.[32] Forgiving a fraction of eligible households' mortgages, this program reduces mortgage debt relative to the benchmark economy. Thereafter, mortgages rise over roughly 4 years in both the funded and unfunded cases while remaining below their level in the benchmark economy. Program recipients with low liquid assets were high MPC households. Now having equity in their homes, they gradually refinance to increase consumption spending. Eventually the aggregate stock returns to its benchmark level.

Thereafter, two factors contribute to further increases in mortgages. First, as the intervention induces an eventual rise in aggregate capital, compared to the benchmark, future interest rate payments on any given sized loan will lower, making it less costly to hold a mortgage. Second, since mortgage LTV constraints are relaxed by rising house prices, households are able to take on larger mortgages.

Lastly, unsecured credit and bankruptcy rise following the mortgage principal reduction. While many households with high MPCs who receive a principal reduction, go on to increase their mortgages and withdraw home equity, a smaller set uses unsecured debt to further reduce their principal. This provides them with more home equity to withdraw at the cost of higher unsecured debt. However, these households tend to strategically default on their unsecured debt. This behavior is most pronounced among households with low income, mid-size houses, and low savings. While the percentage deviation of unsecured debt is large, its size is small compared to mortgages.

[32] This result is consistent with the negative relationship between the amount of negative equity and mortgage default rates in Haughwout et al. (2009) and Gerardi et al. (2017).

Figure 6: Response of aggregate variables

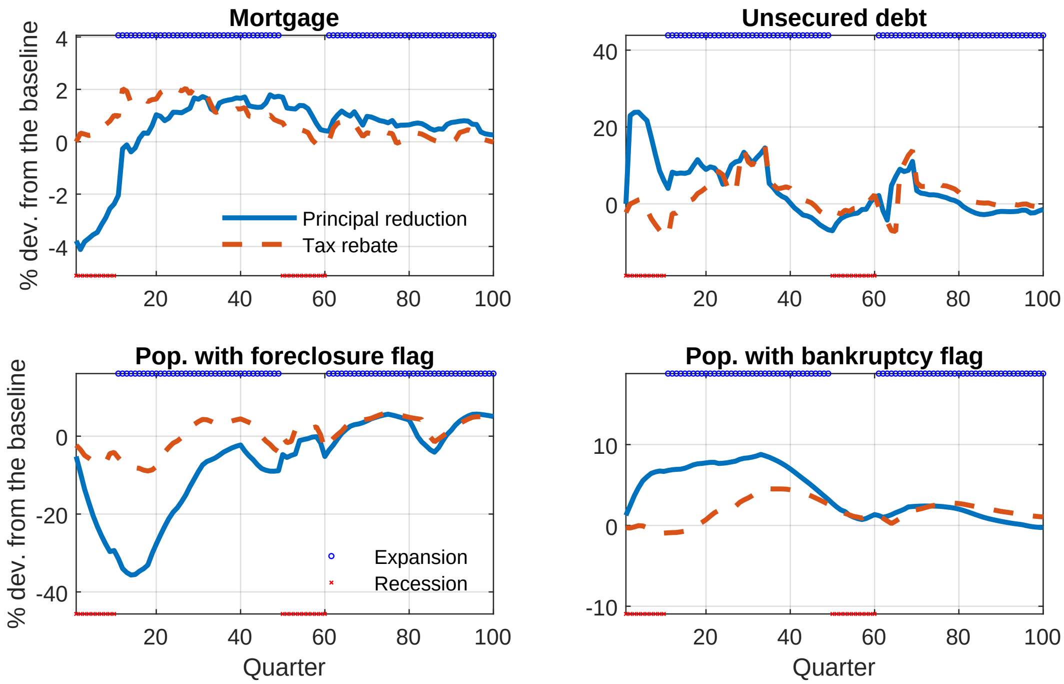
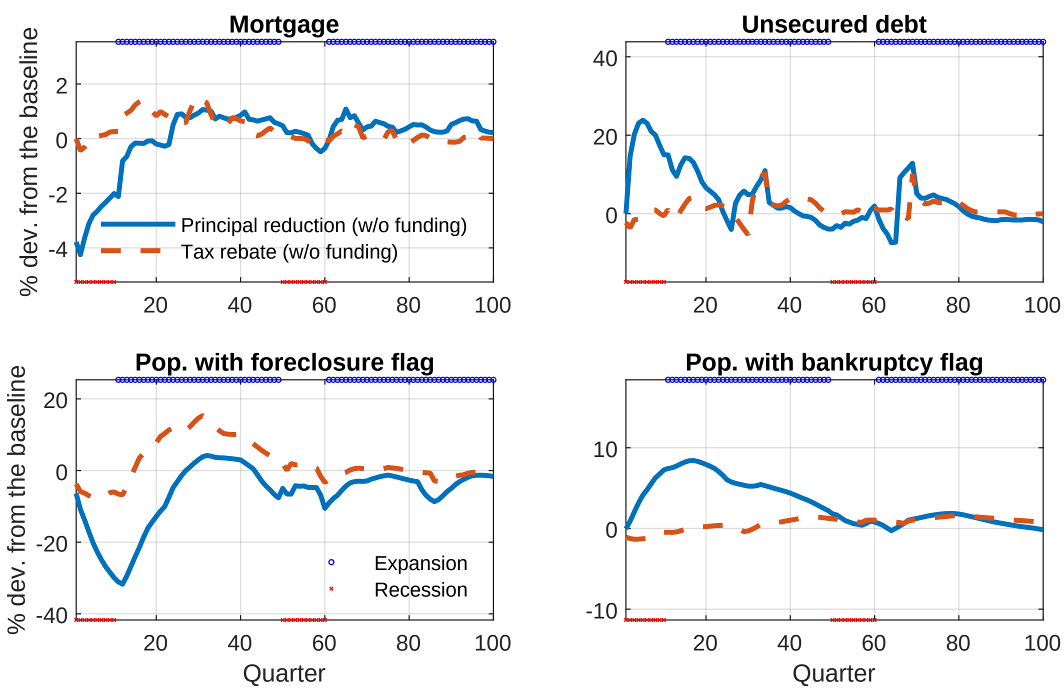


Figure 7: Response of aggregate variables (without funding)



Note: Aggregate series are shown as percent deviations from the corresponding series in the economy without policy intervention.

Comparison to Kaplan et al. (2020) As discussed in Section 1, Kaplan et al. (2020) perform a similar experiment. They also consider a policy that forgives a fraction of mortgages, leaving no household with an LTV ratio higher than 95%. The timing and the scale of the policy are also similar to mine. They assume the policy is implemented in a timely manner (two years into the bust), and it affects over a quarter of homeowners with mortgages. However, their results are very different to my results. They find a mortgage forgiveness program would not have prevented a sharp drop in house prices and aggregate expenditures, but would have significantly dampened the rise in foreclosures. While my household environment shares several features in common with theirs, there are several important differences that lead to different policy outcomes. These include i) the presence of capital, ii) equilibrium wages and interest rates, iii) differences in the preferences for housing, and iv) the availability of bankruptcy.

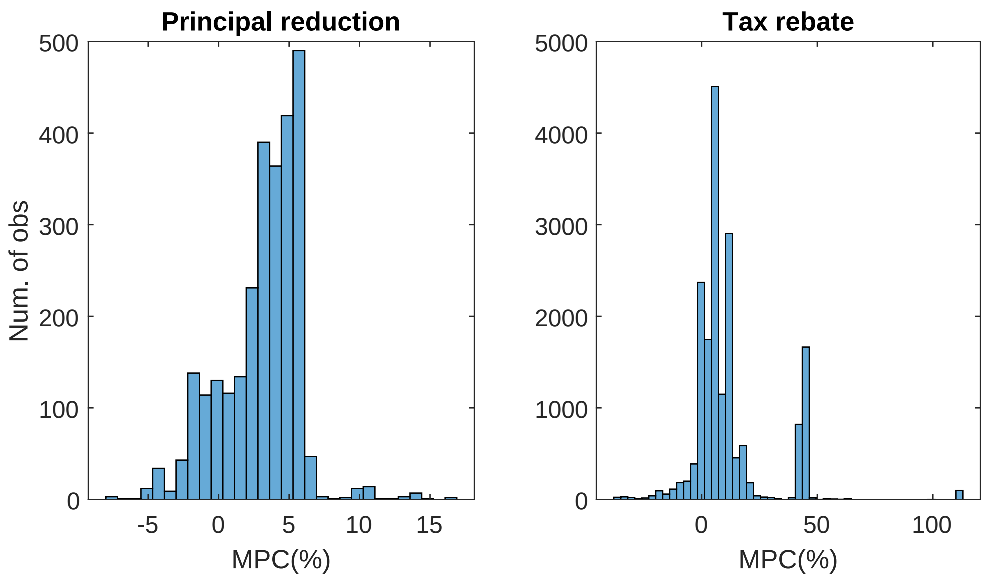
As Kaplan et al. (2020) examine an unfunded program, I focus on comparing their results to the effects of the unfunded principal reduction shown in Figures 5 & 7. Both models predict that the principal reduction leads to a large fall in foreclosure. In my model, there is also a sharp rise in house prices. Turning to real quantities, in contrast to their results, we see a large and persistent increase in aggregate consumption.

In the Kaplan et al. (2020) model, households' taste for housing varies over time and changes in taste account for most of their house price dynamics. Leverage has a limited role in affecting house prices. In addition, only house prices are determined in equilibrium. In contrast, in my model, interest rates and wages are also determined in equilibrium. Figure 5 show that, with the intervention, capital is persistently higher than the benchmark economy. The resulting higher wage benefits all households and the associated fall in interest rates helps net borrowers at the expense of net savers. Overall, the income and substitution effect arising from changes in aggregate capital help to support consumption and house prices. Higher house prices reduce subsequent foreclosures and loosen financial constraints, preventing further decline in house prices. In the absence of interest rate and wage responses, Kaplan et al.'s (2020) model delivers small responses in aggregate variables.

Comparison to a tax rebate We now compare the mortgage principal reduction program to a tax rebate. This transfers funds equally across households, directly adding to their liquid wealth. While the total cost of the tax rebate is the same as the principal reduction, the *equal* distribution of *liquid* wealth leads to different aggregate dynamics. Whether or not the policy is funded (Figures 4 & 6 versus Figures 5 & 7), it is more effective than the principal reduction at boosting consumption. This is because the rebate applies to households with high MPCs that would not have been eligible for the principal reduction (for example, households without houses and mortgages).

In the funded example shown in Figure 4, the large increase in consumption implies that capital falls faster compared to the principal reduction until $t = 14$, thereafter growing more slowly. This leads to output and house prices following similar paths. The tax rebate is less effective in reducing foreclosure and supporting house prices because it does not target households who are likely to sell their houses or foreclose. Poor households without a mortgage are most likely to use unsecured credit and enter bankruptcy. The transfer to these households leads to lower levels of unsecured credit for approximately 20 quarters when compared to the economy with the debt relief program (see Figure 6). Bankruptcy rates are lower for twice as long.

Figure 8: MPC distribution



Note: I simulate a sample of 18,000 households. MPCs are defined as is $\frac{\text{c(policy) - c(no policy)}}{\text{transfer size}}$. The number of observations for the principal reduction is smaller because it only applies to eligible households. Half percent of observations from left and right tails are dropped.

5.1.2 A closer look at non-durable consumption responses

Having explored aggregate responses in consumption, I now explore differences in households' consumption responses.[33] Two main results arise. First, my model reproduces results from empirical work that uses tax rebates as natural experiments. The MPC is larger for households with high levels of debt and low shares of liquid

[33]Most results in this section are generated from a simulated panel. The sample size is 18,000.

assets. Second, this clear relation disappears when we consider mortgage principal reduction. As explained below, this follows from the impact of housing and mortgage transactions on consumption responses to a mortgage principal reduction.

Figure 8 shows the distributions of the MPC after each policy intervention. Each MPC is a difference between consumption in the intervention economy and the benchmark economy, over the size of transfer, at the time of intervention. The number of observations seen for the principal reduction is less as it only applies to eligible households. The MPC distributions are very dispersed following both policies, but more so after the tax rebate.

One feature of these results worth discussing is the significant number of negative MPCs after both policies. Partly, these arise from housing or mortgage adjustments. For example, some households move to larger houses after receiving the transfer and, as a result, spend less on non-durable consumption than in the benchmark. The remainder arise from the negative wealth effect of the expected future tax rise. Importantly, such negative MPCs are not at odds with existing empirical and theoretical evidence. Using the US 2001 and 2008 economic stimulus payments, Misra and Surico (2014) show that a substantial share of estimated MPCs were at or below zero. Kaplan and Violante (2014) show that, following a tax rebate, some households reduce consumption as they transfer an amount that exceeds the rebate into their illiquid asset account in their model with transactions costs of adjusting illiquid assets.

Who responds most?

Figures 9 and 10 show the mean and standard deviation of MPCs by quintiles. For example, the first bar in the upper left panel is the average MPC, to a policy intervention, of the bottom 20% of the net worth distribution.[34] In these figures, liquidity is the share of the liquid assets over net worth and indebtedness is total debt over net worth.

Existing studies measuring the MPC from cash handouts, find that liquidity and indebtedness are important determinants of the differences in MPCs. (For example, see Misra and Surico (2014) and Fagereng et al. (2021).) Figure 10 shows that the distributions of MPCs to a tax rebate are consistent with these findings. As in Misra and Surico (2014), the MPC is larger for households with high levels of debt and low shares of liquid assets.

[34] After the mortgage reduction, the MPC is computed using a sample of eligible households. However, consumption of ineligible households may also differ from the benchmark because of differences in prices and tax rates. In Appendix H, Figures 19 and 20 show the distribution of consumption deviation across all households.

Figure 9: MPC distribution after the principal reduction

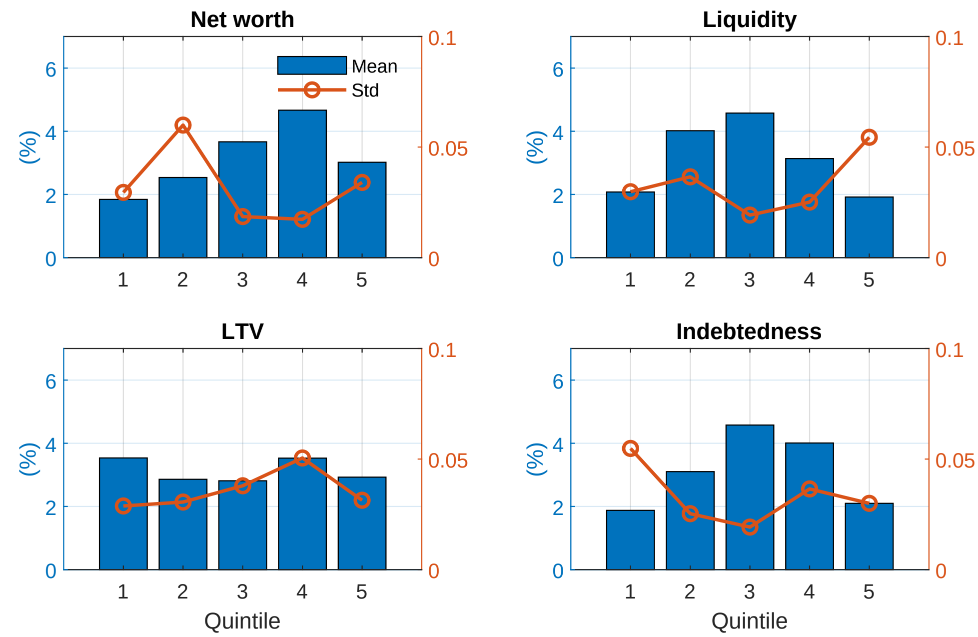
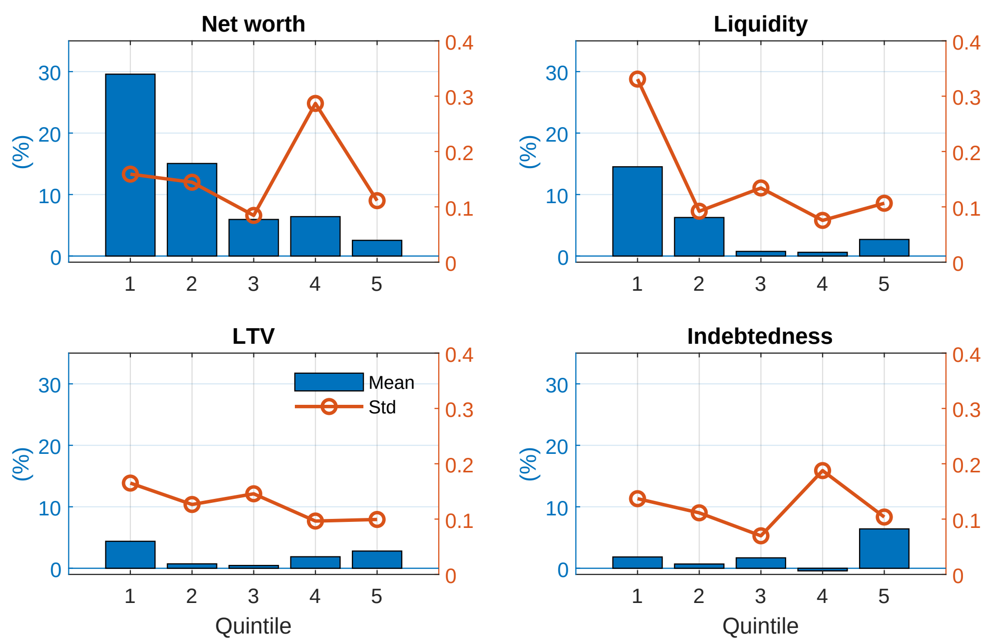


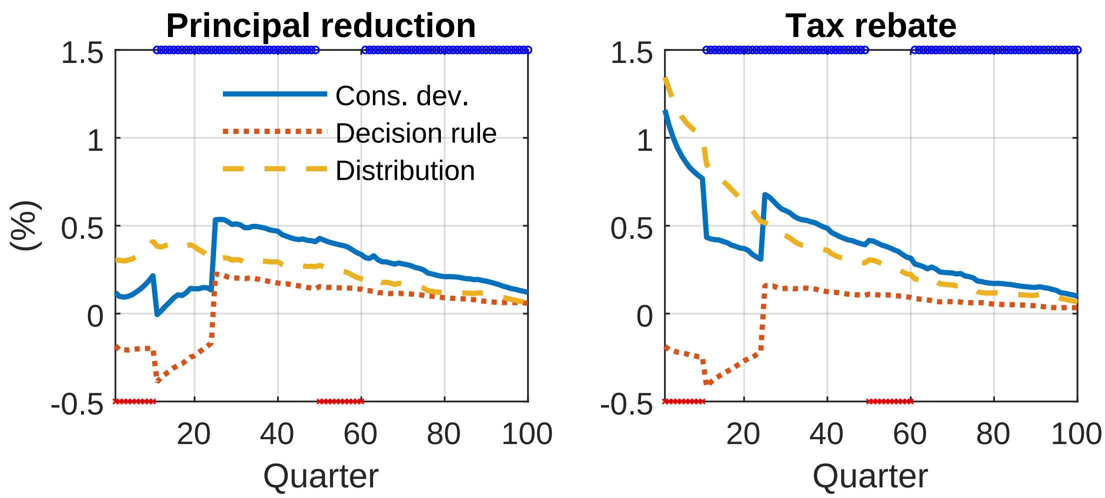
Figure 10: MPC distribution after the tax rebate



Note: I simulate a sample of 18,000 households. MPCs are defined as is $\frac{\text{c(policy) - c(no policy)}}{\text{transfer size}}$. Liquidity is a ratio of liquid assets over net worth. Indebtedness is total debt over net worth. In the panel showing liquidity, households without houses are dropped. For the LTV panel, households without mortgages are dropped. For the indebtedness panel, households without debt are dropped. Each bar is the average MPC of households, and the line shows the standard deviation of MPCs within a quintile.

In contrast, there is no monotone pattern between MPCs and liquidity or indebtedness after the principal reduction. As debt relief provides illiquid assets, funds are not immediately available for households to spend. In general, a large response in non-durable consumption occurs after a discrete choice, such as refinancing. This changes the distribution of consumption responses compared to the tax rebate. For example, in the upper right panel of Figure 9, the MPC is highest for those in the middle of the distribution of liquidity. The principal reduction makes downsizing feasible for previously underwater households. As some of these households downsize, they achieve large increases in consumption and reach a median ratio to liquid assets to net worth.

Figure 11: Consumption response decomposition



Note: Counterfactual consumption is computed as follows. 'Decision rule' is computed using the consumption decision rule of the policy economy and the distribution of the benchmark economy. 'Distribution' is computed using the consumption decision rule of the benchmark economy and the distribution of the policy economy.

Long-term consumption responses While we have seen the initial responses in households' consumption, Figure 4 shows aggregate consumption remains higher than in the benchmark for many periods. The response in consumption is, in part, the result of changes in wealth that shift the distribution of households. Also, it is the result of changes in households' decisions, at each wealth level, respond to changes in the tax regime and general equilibrium price movements. House prices and loan rates respond immediately to the policy intervention, while, given the initially small effect on aggregate capital, wages and interest rates see little change at first. Twelve quarters after the intervention, as the economy enters a recovery, the government increases tax rates to reduce public debt. As public debt falls, private investment rises and aggregate capital starts to rise compared to its benchmark. As a result, the equilibrium risk-free real interest rate falls and the wage rises. Higher wages benefit all households while lower interest rates benefit net borrowers. The intervention also boosts house prices in the recovery, benefiting sellers at the expense of buyers. For homeowners that do not buy or sell, higher house prices lower disposable income as they increase property taxes and maintenance costs.

I decompose the consumption response into 'Decision rule' and 'Distribution' in Figure 11, to see how general equilibrium and changes in wealth determine consumption responses. Let $c_t^p(a, b, \varepsilon, h, o)$ and $g_t^p(a, b, \varepsilon, h, o)$ be consumption decision rules and the distribution of households, with the policy interventions, and $c_t^b(a, b, \varepsilon, h, o)$ and $g_t^b(a, b, \varepsilon, h, o)$ be those in the benchmark economy. The decomposition is as follows.

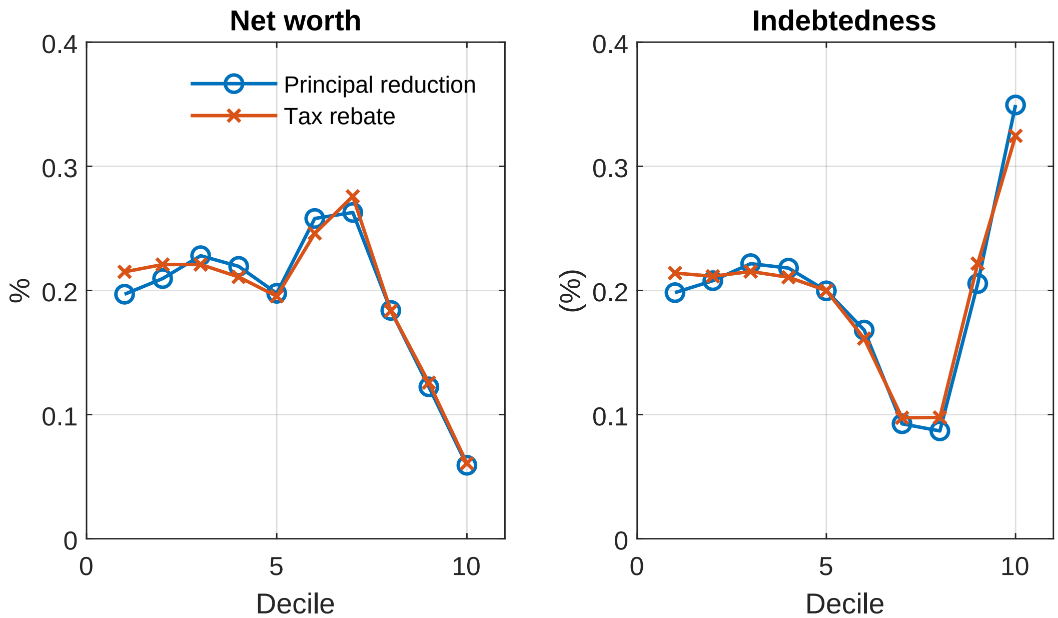
$$
\begin{aligned}
\Delta C_t &= \frac{\int c_t^p(s) g_t^p(s)\, d\mathcal{S} - \int c_t^b(s) g_t^b(s)\, d\mathcal{S}}{\int c_t^b(s) g_t^b(s)\, d\mathcal{S}} \\
&= \underbrace{\frac{(c_t^p(s) - c_t^b(s)) g_t^p(s) d\mathcal{S}}{\int c_t^b(s) g_t^b(s)\, d\mathcal{S}}}_{\text{Decision rule}} + \underbrace{\frac{(g_t^p(s) - g_t^b(s)) c_t^b(s) d\mathcal{S}}{\int c_t^b(s) g_t^b(s)\, d\mathcal{S}}}_{\text{Distribution}}
\end{aligned}
$$

where $s = (a, b, \varepsilon, h, o)$ and $\mathcal{S} = [a \times b \times \varepsilon \times h \times o]$. ΔC is the consumption deviation from the benchmark and the first term above, labelled 'Decision rule' captures general equilibrium effects. The second term, 'Distribution', show the effect of changes in the distribution of households, initially the result of changes in wealth brought about by the policy interventions. In both the case of the principal reduction and the tax rebate, the distributional effect drives all of the rise in consumption until the debt from a policy intervention in paid off at $t = 25$.

At the policy intervention, future taxes are expected to rise. This happens at $t = 11$ as the economy enters an expansion. The 'Decision rule' effect on aggregate consumption is dominated by a strong negative wealth effect of higher taxes which dampens the 'Distribution' effect. This becomes stronger when taxes actually rise, and disappears when they return to normal. At that point general equilibrium increases in wage leads to 'Decision rule' boosting consumption.

The distribution effect is stronger with the tax rebate driving a larger consumption response and lower capital. As seen in Figure 4, capital rises by more after the principal reduction and this implies larger general equilibrium effects on consumption. The general equilibrium effects explain about 40% and 20% of consumption rise, after the principal reduction and the tax rebate respectively.[35]

Figure 12: Consumption equivalent gain distribution



Note: Consumption equivalent gain over net worth and indebtedness deciles. Indebtedness is the sum of mortgages and unsecured credit over net worth, and the plotted numbers are average consumption equivalent gains in each decile.

5.2 Fiscal multiplier and welfare

In this section, I assess the effectiveness of each funded policy using two common measures. The first one is the fiscal multiplier. As is standard in the literature, I compute the cumulative fiscal multiplier through time T as the discounted cumulative change in output over government spending. That is,

$$M_T = \frac{\int_{t=0}^{T} e^{-r_s t}(y_t^p - y_t^b)dt}{\int_{t=0}^{T} e^{-r_s t} G_t dt}$$

where y_t^p and y_t^b are output of the policy economy and the benchmark economy, G_t is the cost of the intervention, and r_s is the average interest rate over time following

[35] Appendix G provides additional results showing how non-durable consumption, at the household level, responds to housing stock.

Mountford and Uhlig (2009).

The fiscal multiplier of the principal reduction and tax rebate policies are 0.78 and 0.46, respectively. The higher multiplier for the mortgage reduction policy stems from its stronger effect on capital accumulation discussed above. Conversely, the fiscal multiplier on consumption is larger for the tax rebate (0.58) than the principal reduction (0.33).

The second measure of policy effectiveness is household welfare. I report the consumption equivalent gain as in Equation 4.2, where now $i \in \{$principal reduction, tax rebate$\}$, when the government intervenes.
While not true for all households, there is a positive average welfare gain in each decline of the net worth and indebtedness distributions. Across both policies, the average consumption equivalent gain is 0.2%. Figure 12 shows, over deciles of net worth and indebtedness, who values the policy interventions more (or less). The principal reduction and the tax rebate show similar distributions of consumption equivalent gains; both policies are less favored by rich households and more favored by highly indebted households.

5.3 State dependent policy effectiveness

Table 10: Ranges for relevant variables

	Principal reduction	**Tax rebate**
Eligible households (%)	[15.5, 25.5]	-
Average transfer amount/household ($)	[31,000, 49,000]	[5,900, 7,900]
Total cost/GDP (%)	[5.8, 7.7]	-
Fiscal multiplier	[0.67, 0.78]	[0.35, 0.46]
Average consumption equivalent gain (%)	[0.18, 0.20]	[0.18, 0.20]

Note: Ranges in each row are derived from economies with different initial distributions.

The efficacy of a policy intervention can be state-dependent; the economy's response may vary with the distribution of households over assets and liabilities.[36] In this section, I study the state dependence of policy interventions. I choose eight recessionary episodes that are associated with a large house price drop, and assume that the government intervenes when it observes a 5% fall in the house price as in Section 5.1.1.

[36]For example, Beraja et al. (2018) and Eichenbaum et al. (2018) show that the effects of monetary policy are state dependent.

As each episode follows a different history, their distributions of households are different when the government intervenes. Table 10 shows the range of variables summarising these distributions. For example, eligible households for mortgage reduction - those who have LTVs higher than 95% - vary from 15.5% to 25.5%, and the average forgiven amount per household ranges from $31,000 to $49,000 (2007 dollars).

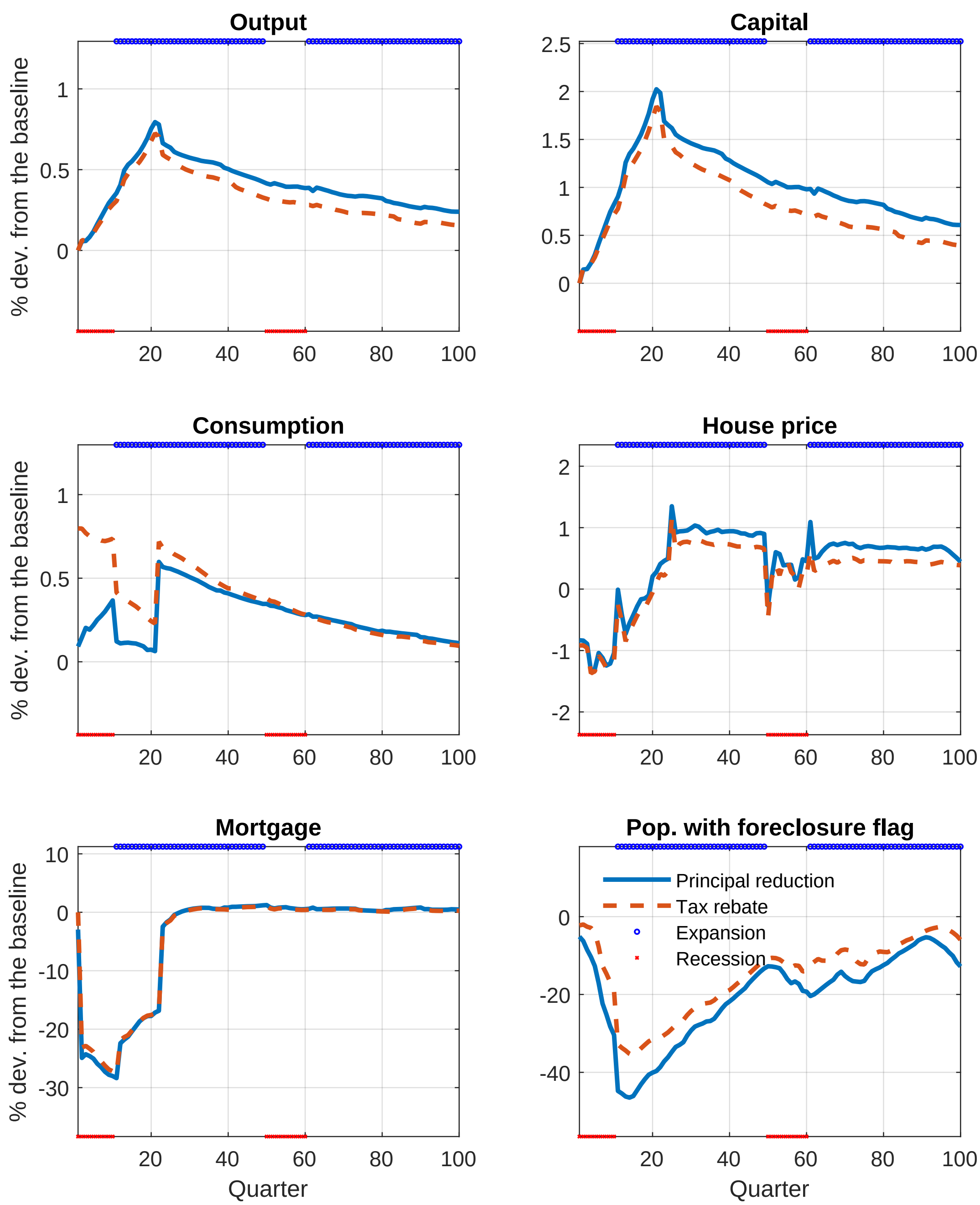
Figures 21 and 22 in Appendix H show the aggregate responses to the principal reduction and tax rebate policies across these different economies. For comparability, I set the path of total factor productivity, from the intervention date onwards, to be the same across all economies. While all series are qualitatively similar, they show considerable quantitative dispersion. The solid lines in Figures 21 and 22 correspond to the series in Figures 4 and 6. We now see that this economy, studied above, represents one of the smaller interventions in terms of its cost and share of eligible households. Despite its relatively small effect on aggregate quantities, this case yields the largest fall in foreclosures. Table 10 shows the range of fiscal multipliers and average consumption equivalent gains over all initial conditions. Both are negatively correlated with the policy's cost and the average LTV of eligible households. This implies that an intervention is more effective if the government intervenes early when households are relatively less indebted.

5.4 Role of bankruptcy

Bankruptcy – default on unsecured debt – can affect mortgages, house price dynamics and thus the effects of mortgage relief policies, because households' choices of assets and liabilities depend on its availability as seen in Section 4. Here, I investigate the role of bankruptcy in determining the effects of debt relief and tax rebate policies by studying the response of an economy with foreclosure but without bankruptcy. I find that the intervention drives an immediate recovery in capital and output, moving forward the timing of the recovery when compared to the model with bankruptcy. The presence of bankruptcy qualitatively changes the response of debt relief.

This difference arises from the unavailability of unsecured credit. Consequently, households' precautionary savings in liquid financial assets increases. This leads to less dispersed MPCs and smaller MPCs. As a result, a policy intervention has less effect on households' consumption. Instead, anticipating future tax increases, they further increase savings after either a principal reduction or a tax rebate.

Figure 13: Response of aggregate variables



Note: Aggregate series are shown as percent deviations from the corresponding series in the economy without policy intervention.

The lack of bankruptcy affects aggregate dynamics. In this model, while the volatility of output, consumption and investment are similar, (see Table 6 in Section 3 and Table 15 in Appendix H) the variability of house prices and mortgage are not. The standard deviation of house prices and mortgages are 49.2% and 20.5% lower than in the model with both bankruptcy and foreclosure. Moreover, the average house price is 11.9% lower and the average fraction of people with a foreclosure flag is 28.6% lower. As households now hold more liquid assets, housing demand is lower and the volatility of house prices and the use of foreclosure are also lower.

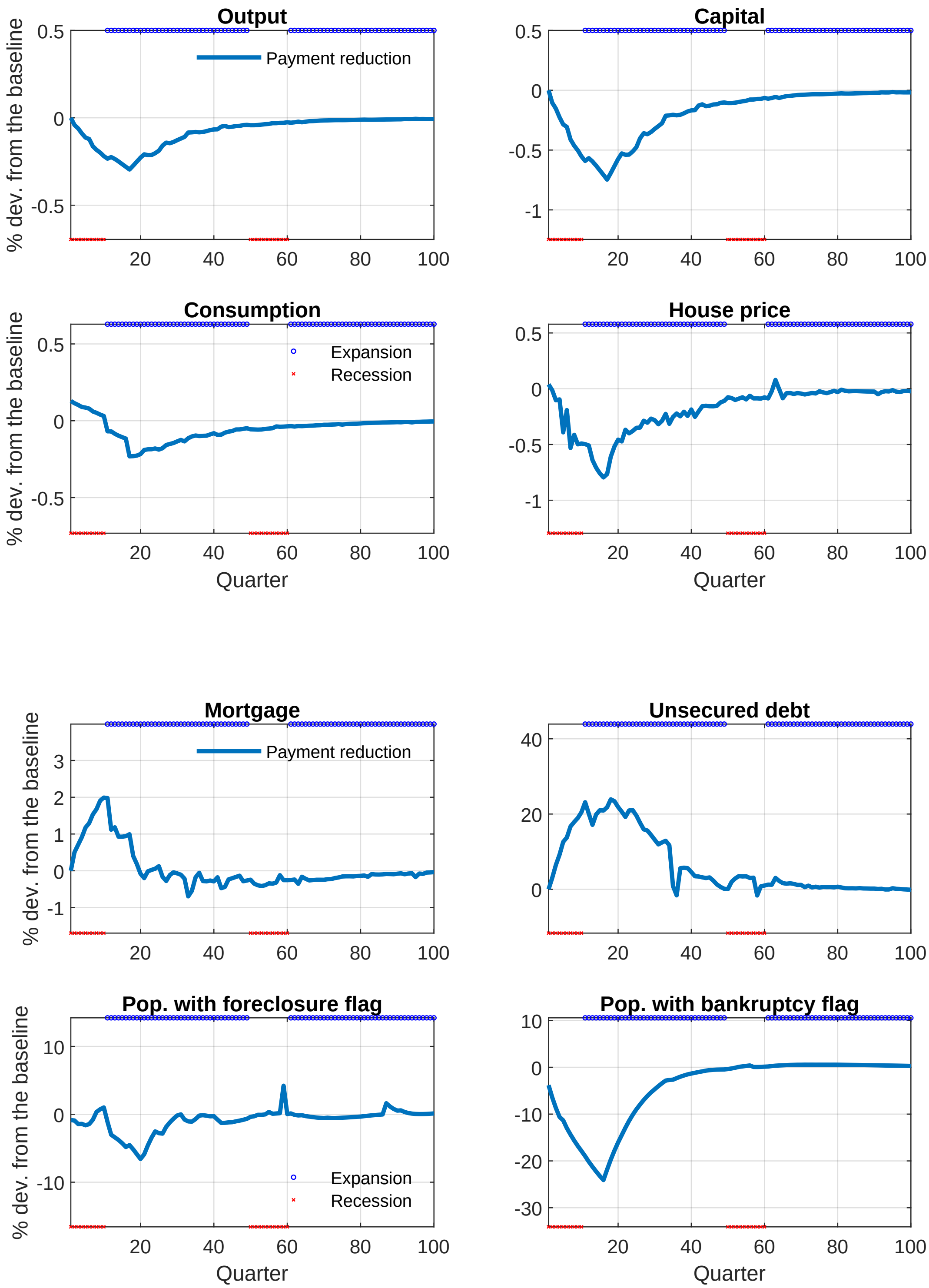
Lower house price volatility implies that the policy intervention criterion used above is less commonly met. Nonetheless, I assume that the government intervenes at the same date as in Section 5.1 and uses the same policy specification. At the time of the intervention, the share of eligible households (13.3%) is 2.4 percentage points lower and the average size of mortgage reduction ($35,000 (2007 dollars)) is 7.8% lower than the benchmark case.

Figure 13 shows the responses of aggregate variables. As explained, and in contrast to Figure 4, the intervention now raises capital and output during a recession. In Section 5.1, the recipients of the principal reduction tended to have high MPCs, which led to higher consumption, lower savings and lower investment. Here, households with relatively high LTVs tend to have enough savings, given the absence of short-term credit, and their MPCs are not particularly high.

The mortgage response is also different. At $t = 1$, mortgages are 3% lower than in the no-intervention case because of the principal reduction. Immediately after, it falls by more than 20% because many households who receive the principal reduction prepay. This also happens in Section 5.1, because a subset of households who receive the principal reduction have large savings. When bankruptcy is not available, the share of such households is larger and more households prepay. Thanks to this rise in prepayment, aggregate mortgage balances are significantly lower for about 5 years and this substantially reduces foreclosure. Lower mortgage balance lower households' interest payments, allowing them to increase consumption and investment. The drops in consumption at the beginning of the recovery ($t = 11$) are caused by higher tax rates to repay the cost of the policy, as in Section 5.1.

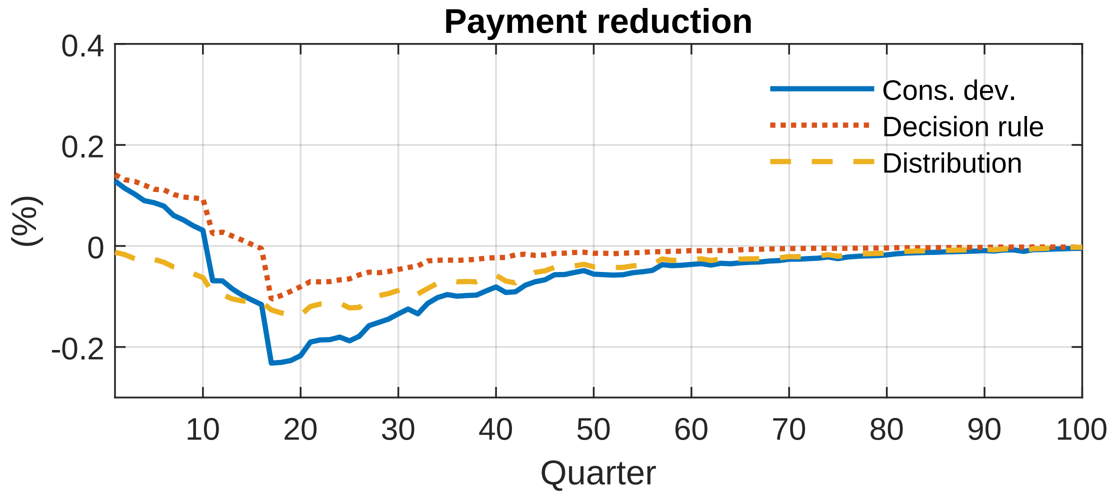
The responses of the tax rebate are close to those of the principal reduction, except for larger initial responses in consumption. This is the result of differences in the liquidity of the transfers associated with tax rebates and mortgage principal reductions. As discussed, here, the dispersion in MPCs seen in previous sections is reduced. This narrows the gap between the effects of targeted debt relief policies and untargeted tax rebates.

Figure 14: Response of aggregate variables



Note: Aggregate series are shown as percent deviations from the corresponding series in the economy without policy intervention.

Figure 15: Consumption response decomposition



Note: 'Decision rule' is computed using the consumption decision rule of the policy economy and the distribution of the benchmark economy. 'Distribution' is computed using the consumption decision rule of the benchmark economy and the distribution of the policy economy.

5.5 Mortgage payment reduction

Instead of mortgage forgiveness, I consider a mortgage payment reduction. As there's no actual reduction in debt, the policy does not require government funding and the tax regime does not change upon implementation.

Several papers compare policies that provide wealth (principal reduction) against those providing liquidity (payment reduction). Both Ganong and Noel (2018), who examine mortgages, and Indarte (2019), who studies bankruptcy, find that debt relief is less effective at reducing default than payment reductions. In contrast, examining cramdowns that discharged the underwater portion of mortgages during Chapter 13 bankruptcy, Cespedes et al. (2021) find that foreclosure rates did fall. Due to the differences in implementation, these findings are not comparable to the results in my work.[37] However, they still suggest a potentially important role for payment reductions, and I explore that now.

As I will show, if we focus on the short-run effects of the reduced payment policy, it is effective in increasing consumption and in reducing foreclosure. However, when we also consider the long-run, general equilibrium effects, a payment reduction is not as

[37] Ganong and Noel (2018) use HAMP as a natural experiment. In contrast to cramdowns in Cespedes et al. (2021), and the mortgage reduction policy I study, HAMP reduced mortgages to 115% of house value leaving households still underwater. Moreover, the share of households who modified their mortgages via HAMP was far lower than the 15% who receive the mortgage reductions in my experiments.

effective as a mortgage principal reduction policy.

In my model, mortgage borrowers are required to repay a fraction of their debt at every period. The size of repayment is set to make the average duration of a loan 40 years if a household fully finances a house. For this policy experiment, I assume that per period repayments of principal are reduced by half for 16 quarters. This policy effectively extends the duration of a loan by allowing slower amortization of debt.

Figure 14 shows how the economy responds to the payment reduction. As it increases households' disposable income, aggregate consumption rises for 10 quarters. Over the same period, a slowdown in mortgage payments lowers savings available for investment, thus capital falls. During the first 10 quarters, household leverage rises compared to the economy without the intervention. Larger interest payments due to higher mortgage balances, and lower wages coming from lower capital, depress consumption even as mortgagors continue to pay a lower fraction of their balance. Figure 15 shows the consumption decomposition introduced in Section 5.1.2. We see that both general equilibrium effects (in Decision rule) and higher household leverage (in Distribution) contribute to a persistent decline in aggregate consumption. Additionally, lower capital depresses output and house prices for an extended period.

6 Concluding remarks

I have quantitatively assessed the effects of debt relief programs during recessions. While a growing empirical literature studies effects of such policies on foreclosure and household consumption, there is little equilibrium analysis of the dynamic response in the aggregate economy to large-scale debt relief policies.

To understand the effects of debt relief programs better, I build a model with financial assets, unsecured debt, housing, and mortgages as well as the option to default on both forms of borrowing. The model successfully replicates the household distribution for overall net worth, as well as liquid assets, mortgages, and housing. My model captures key business cycle moments while reproducing households' default behavior.

I show that a mortgage principal reduction can stimulate consumption in a recession and amplify a recovery without changing its timing. The one-time intervention has persistent effects on aggregate output as well as bankruptcy, foreclosure and house prices. The intervention is welfare improving, in particular for households with low wealth and large debt. I compare debt relief to a tax rebate. The latter is an untargeted, liquid income transfer to all households. I find that the liquidity of transfers, and the distribution of recipients' MPCs, play a large role in determining the response in macroeconomic variables.

The qualitative response in the economy hinges on whether mortgage principal reduction and tax rebates are funded or unfunded. In the latter case, the lack of anticipated increases in future taxes leads to far stronger responses in consumption. Allowing for unsecured credit and bankruptcy implies very different aggregate effects of mortgage debt relief policies. The absence of bankruptcy increases households' stock of liquid assets, thereby reducing MPCs and the consumption response to stimulus. Lastly, policy intervention is more effective if a government intervenes early in a severe recession, before household leverage deteriorates.

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Appendix For Online Publication

This Appendix is organized as follows. Section A presents the household problem in an alternative form. Section B presents the procyclical tax-rate economy that is used in experiments with funded policy interventions. Section C describes the numerical solution method. Section D describes the estimation of the earnings process and categorization of assets and debt used in computing calibration targets. Section E derives the consumption equivalent gain described in section 4.2. Sections F and G provide additional discussion and results not included in Section 5. Lastly, Section H contains additional figures and tables.

A Alternative representation of household problem

Household problems in Section 2 can be written as a HJB variational inequality (HJBVI).

Households with no flags The HJBVI for a household that has neither bankruptcy nor foreclosure in its credit history is shown below. Such households decide whether to continue, or stop and choose any of the stopping options listed above in section 2. The value of the latter, which is the second choice below, is given by $v^*(a, b, \varepsilon, h, g, z)$ in (6), while the value of continuing, the first choice, is the HJB equation in (5),

$$\begin{aligned}
\min\Bigg\{&\rho v(a,b,\varepsilon,h,g,z) - \max_c u(c,h) - \partial_a v(a,b,\varepsilon,h,g,z)\dot{a} - \partial_b v(a,b,\varepsilon,h,g,z)\dot{b} \\
&- \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v(a,b,\varepsilon_j,h,g,z) - \sum_{k=1}^{n_z} \lambda_{zz_k} v(a,b,\varepsilon,h,g,z_k) \\
&- \int \frac{\delta v(a,b,\varepsilon,h,g,z)}{\delta g(a,b,\varepsilon,h,o)} \mathcal{K} g(a,b,\varepsilon,h,o) d[a \times b \times \varepsilon \times h \times o], \\
&v(a,b,\varepsilon,h,g,z) - v^*(a,b,\varepsilon,h,g,z)\Bigg\} = 0.
\end{aligned} \tag{27}$$

Households with a bankruptcy flag

A household that has bankruptcy in its credit history has the following HJBVI equation. The value of continuing, the first option in the discrete choice below, is described in (12) while the stopping value, the second option, is defined in (13),

$$
\begin{aligned}
\min\Bigg\{ & \rho v^d(a,b,\varepsilon,h,g,z) - \max_c u(c,h) - \partial_a v^d(a,b,\varepsilon,h,g,z)\dot{a} - \partial_b v^d(a,b,\varepsilon,h,g,z)\dot{b} \\
& - \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v^d(a,b,\varepsilon_j,h,g,z) - \lambda_d(v(a,b,\varepsilon,h,g,z) - v^d(a,b,\varepsilon,h,g,z)) \\
& - \sum_{k=1}^{n_z} \lambda_{zz_k} v^d(a,b,\varepsilon,h,g,z_k) - \int \frac{\delta v^d(a,b,\varepsilon,h,g,z)}{\delta g(a,b,\varepsilon,h,o)} \mathcal{K} g(a,b,\varepsilon,h,o) d[a\times b\times\varepsilon\times h\times o], \\
& v^d(a,b,\varepsilon,h,g,z) - v^{d*}(a,b,\varepsilon,h,g,z)\Bigg\} = 0.
\end{aligned}
\tag{28}
$$

Households with a foreclosure flag

The last HJBVI equation describes the problem of a household that has foreclosure in its credit history. Such a household has a value of not stopping, the first option in the discrete choice below, described in (17) while the stopping value is defined in (18),

$$
\begin{aligned}
\min\Bigg\{ & \rho v^f(a,\varepsilon,g,z) - \max_c u(c,\underline{h}) - \partial_a v^f(a,\varepsilon,g,z)\dot{a} - \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v^f(a,\varepsilon_j,g,z) \\
& - \lambda_f(v(a,0,\varepsilon,0,g,z) - v^f(a,\varepsilon,g,z)) \\
& - \sum_{k=1}^{n_z} \lambda_{zz_k} v^f(a,\varepsilon,g,z_k) - \int \frac{\delta v^f(a,\varepsilon,g,z)}{\delta g(a,b,\varepsilon,h,o)} \mathcal{K} g(a,b,\varepsilon,h,o) d[a\times b\times\varepsilon\times h\times o], \\
& v^f(a,\varepsilon,g,z) - v^{f*}(a,\varepsilon,g,z)\Bigg\} = 0.
\end{aligned}
\tag{29}
$$

B Procyclical tax-rate economy

In Section 5, for policy interventions incurring explicit costs (mortgage principal reduction, tax rebate), the government finances its policy intervention by issuing bonds which are repaid by increasing taxes only during expansions. Therefore, when the government intervenes, households expect future taxes to rise in expansions. For tractability, I assume that households do not know exactly when the cost will be paid off but expect the procyclical tax rate regime will end with a 20% probability at any time. In this section, I present details for this procyclical tax-rate economy.

B.1 Household problem

As laid out in Section 2, a household may or may not have a bankruptcy or foreclosure flag and have different problems depending on the flags. However, all problems are stopping time problems and they can be compactly written as an HJBVI. There are two differences when tax is procyclical. First, the budget constraints becomes

$$\begin{aligned}\dot{a} &= w(g,z)\varepsilon + r_a(a,b,\varepsilon,h,o,g,z)a - (r(g,z) + \iota(z) + \theta(b,\overline{p}h))b \\ &\quad - c - \xi_h p(g,z)h - T(b,\varepsilon,p(g,z)h,z) \\ \dot{b} &= -\theta(b,\overline{p}h)b.\end{aligned} \tag{30}$$

The only difference here is the tax function which becomes $T(b,\varepsilon,p(g,z)h,z)$, instead of $T(b,\varepsilon,p(g,z)h)$, because the tax rate now varies with total factor productivity. Specifically, the tax function is $T(y,z) = y - \tau_0(z)y^{1-\tau_1}$. While τ_1 is unchanged at 0.181, τ_0 during expansions is 0.57, and τ_0 during recessions is 0.58. This implies approximately 0.5 - 1.3 percentage point higher tax rates during expansions compared to recessions.

Second, the households' problems becomes the following. Let $v^{tax}(\cdot)$ is the value function of the procyclical tax rate economy. The HJBVI for a household that has neither bankruptcy nor foreclosure in its credit history is shown below. Such households decide whether to continue or stop and choose any of the stopping options, which are computed the same way as in equation (6). The value of continuing, the first choice, is the HJB equation as in equation (5), with the modified budget constraint in equation (30) and with the extra term, $\lambda_{tax}(v(a,b,\varepsilon,h,g,z) - v^{tax}(a,b,\varepsilon,h,g,z))$ to account for the possibility of moving back to the economy without the procyclical tax rate. The HJBVI is

$$\begin{aligned}\min\Bigg\{&\rho v^{tax}(a,b,\varepsilon,h,g,z) - \max_c u(c,h) - \partial_a v^{tax}(a,b,\varepsilon,h,g,z)\dot{a} - \partial_b v^{tax}(a,b,\varepsilon,h,g,z)\dot{b} \\ &- \sum_{j=1}^{n_\varepsilon}\lambda_{\varepsilon\varepsilon_j}v^{tax}(a,b,\varepsilon_j,h,g,z) - \sum_{k=1}^{n_z}\lambda_{zz_k}v^{tax}(a,b,\varepsilon,h,g,z_k) \\ &- \int \frac{\delta v^{tax}(a,b,\varepsilon,h,g,z)}{\delta g(a,b,\varepsilon,h,o)}\mathcal{K}g(a,b,\varepsilon,h,o)d[a\times b\times\varepsilon\times h\times o] \\ &- \lambda_{tax}(v(a,b,\varepsilon,h,g,z) - v^{tax}(a,b,\varepsilon,h,g,z)), \\ &v^{tax}(a,b,\varepsilon,h,g,z) - v^{tax*}(a,b,\varepsilon,h,g,z)\Bigg\} = 0.\end{aligned}$$

The modifications of the problems for households with a bankruptcy flag and households with a foreclosure flag are the same. Let $v^{tax,d}(\cdot)$ and $v^{tax,f}(\cdot)$ represent the value functions for these households. The value of continuing is computed the same way as in the equation (12) and (17) with the extra term $\lambda_{tax}(v^d(a,b,\varepsilon,h,g,z)-v^{tax,d}(a,b,\varepsilon,h,g,z))$ and $\lambda_{tax}(v^f(a,0,\varepsilon,0,g,z)-v^{tax,f}(a,\varepsilon,g,z))$ and with the modified budget constraints in (30). The stopping values are defined in the same way as (13) and (18). A household that has bankruptcy in its credit history has the following HJBVI equation,

$$
\begin{aligned}
\min\Bigg\{&\rho v^{tax,d}(a,b,\varepsilon,h,g,z) - \max_c u(c,h) - \partial_a v^{tax,d}(a,b,\varepsilon,h,g,z)\dot{a} - \partial_b v^{tax,d}(a,b,\varepsilon,h,g,z)\dot{b} \\
&- \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v^{tax,d}(a,b,\varepsilon_j,h,g,z) - \lambda_d(v^{tax}(a,b,\varepsilon,h,g,z) - v^{tax,d}(a,b,\varepsilon,h,g,z)) \\
&- \sum_{k=1}^{n_z} \lambda_{zz_k} v^{tax,d}(a,b,\varepsilon,h,g,z_k) - \int \frac{\delta v^{tax,d}(a,b,\varepsilon,h,g,z)}{\delta g(a,b,\varepsilon,h,o)} \mathcal{K}g(a,b,\varepsilon,h,o)d[a\times b\times\varepsilon\times h\times o] \\
&- \lambda_{tax}(v^d(a,b,\varepsilon,h,g,z) - v^{tax,d}(a,b,\varepsilon,h,g,z)), \\
&v^{tax,d}(a,b,\varepsilon,h,g,z) - v^{tax,d*}(a,b,\varepsilon,h,g,z)\Bigg\} = 0.
\end{aligned}
$$

The last HJBVI equation describes the problem of a household that has foreclosure in its credit history,

$$
\begin{aligned}
\min\Bigg\{&\rho v^{tax,f}(a,\varepsilon,g,z) - \max_c u(c,\underline{h}) - \partial_a v^{tax,f}(a,\varepsilon,g,z)\dot{a} - \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v^{tax,f}(a,\varepsilon_j,g,z) \\
&- \lambda_f(v^{tax}(a,0,\varepsilon,0,g,z) - v^{tax,f}(a,\varepsilon,g,z)) \\
&- \sum_{k=1}^{n_z} \lambda_{zz_k} v^{tax,f}(a,\varepsilon,g,z_k) - \int \frac{\delta v^{tax,f}(a,\varepsilon,g,z)}{\delta g(a,b,\varepsilon,h,o)} \mathcal{K}g(a,b,\varepsilon,h,o)d[a\times b\times\varepsilon\times h\times o] \\
&- \lambda_{tax}(v^f(a,0,\varepsilon,0,g,z) - v^{tax,f}(a,\varepsilon,g,z)), \\
&v^{tax,f}(a,\varepsilon,g,z) - v^{tax,f*}(a,\varepsilon,g,z)\Bigg\} = 0.
\end{aligned}
$$

B.2 Financial intermediaries

Loan price functions for short-term debt and mortgages are determined by zero expected profit condition of competitive banks as in Section 2. The only difference is that there is a possibility of moving back to the benchmark economy, and the loan price functions takes into account this possibility.

Unsecured debt Let $r_a^{tax}(a,b,\varepsilon,h,o,g,z)$ be the short-term loan price function. In the default region ($D_a^{tax}(a,b,\varepsilon,h,o,g,z)=1$), and we set

$$r_a^{tax}(a,b,\varepsilon,h,o,g,z)=\infty.$$

The zero profit condition in the region of no default implies that the return $r_a^{tax}(a,b,\varepsilon,h,o,g,z)$ should be equal to the risk free rate, $r(g,z)$,

$$\begin{aligned} r_a^{tax}(a,b,\varepsilon,h,o,g,z) = r(g,z) + (1-p_{tax})(\lambda_z \sum_{z'} p_{zz'} \lambda_\varepsilon \sum_{\varepsilon'} p_{\varepsilon\varepsilon'} D_a^{tax}(a,b,\varepsilon',h,o,g,z')) \\ + p_{tax} D_a(a,b,\varepsilon,h,o,g,z). \end{aligned}$$

Mortgages Since banks expect zero profit on each loan, the discounted value of the loan at origination has to equal to its expected cash flow. The price of the loan in the non-default region is given by

$$q_0^{tax}(a,b,\varepsilon,h,g,z)b_0 = \mathbb{E}\Big[\mathbb{E}_\tau \int_0^\tau e^{-\int_0^s (r_s+\iota_t+\theta_s)ds}(r_t+\iota_t+\theta_t)b_0 dt + e^{-\int_0^\tau r_s ds} b(a_\tau,b_\tau,\varepsilon_\tau,h,g,z)\Big].$$

The scrap value $b(a_\tau,b_\tau,\varepsilon_\tau,h,g,z)$ at the stopping point depends on a household's discrete choice. In the case of a foreclosure,

$$b(a_\tau,b_\tau,\varepsilon_\tau,h,g,z) = (1-\delta_d)p(g,z)h.$$

When a household prepays the loan due to refinancing or a new house transaction, the scrap value is

$$b(a_\tau,b_\tau,\varepsilon_\tau,h,g,z) = e^{-\int_0^\tau \theta_s ds} b_0.$$

Applying the Feynman-Kac formula, the above equations can be written as the following partial differential equation. At $t \in [0, \tau)$,

$$
\begin{aligned}
(\theta(b, \overline{p}h) + r(g, z) + \iota(z))q^{tax}(a, b, \varepsilon, h, g, z) = \theta(b, \overline{p}h) + r(g, z) + \iota(z) + q_a^{tax}(a, b, \varepsilon, h, g, z)\dot{a} \\
+ q_b^{tax}(a, b, \varepsilon, h, g, z)\dot{b} + \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} q^{tax}(a, b, \varepsilon_j, h, g, z) + \sum_{k=1}^{n_z} \lambda_{zz_k} q^{tax}(a, b, \varepsilon, h, g, z_k) \\
+ \int \frac{\delta q^{tax}(a, b, \varepsilon, h, g, z)}{\delta g(a, b, \varepsilon, h, o)} \mathcal{K} g(a, b, \varepsilon, h, o) d[a \times b \times \varepsilon \times h \times o] \\
+ \lambda_{tax}(q(a, b, \varepsilon, h, g, z) - q^{tax}(a, b, \varepsilon, h, g, z)).
\end{aligned}
$$

When, at $t = \tau$, the stopping time decision is to foreclose,

$$q^{tax}(a, b, \varepsilon, h, g, z) = \frac{(1 - \delta_d)p(g, z)h}{b},$$

and, if instead, the stopping time decision involves prepayment, we have

$$q^{tax}(a, b, \varepsilon, h, g, z) = 1.$$

B.3 Government debt dynamics

When there is a policy intervention, the government issues bonds to finance debt relief or a tax rebate. This debt is repaid during expansions through higher tax rates. As before, government consumption is determined by pre-intervention tax rates. Given the tax function before the intervention, $T(b, \varepsilon, p(g, z)h)$, government spending, G, is given by

$$\int T(b, \varepsilon, p(g, z)h) g(a, b, \varepsilon, h, o) d[a \times b \times \varepsilon \times h \times o] - G = 0.$$

Thus there is no change in the government consumption function compared to the pre-intervention economy. However now the government has debt, B, which it repays during expansions using extra tax revenue generated as the difference between the acylical tax function above and the procyclical function described in B.1. From the time the government intervenes to the beginning of an expansion, government debt evolves as

$$\dot{B}_t = r_t B_t.$$

Once the economy transitions to an expansion the government starts to repay its debt and it evolves as

$$\dot{B}_t = r_t B_t - \int (T(b, \varepsilon, p(g, z)h, z) - T(b, \varepsilon, p(g, z)h))g(a, b, \varepsilon, h, o)d[a \times b \times \varepsilon \times h \times o]$$

where $T(b, \varepsilon, p(g, z)h, z)$ is the procyclical tax function. Therefore, the government uses the increase in its tax revenue to repay debt, leaving consumption as it would have been if there had been no change in tax rates, for a given distribution of households.

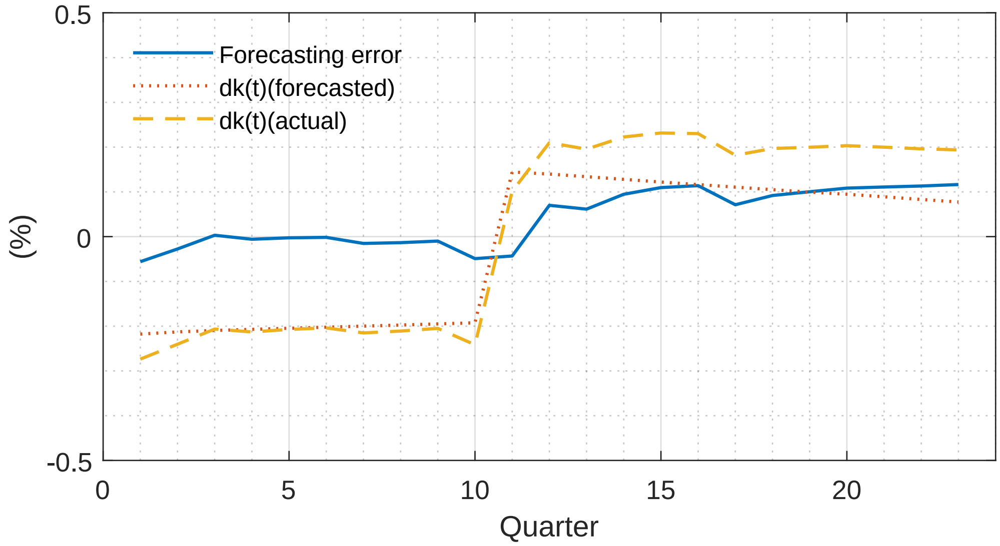
B.4 Errors in the forecasting function

The households' and financial intermediaries problems described in this appendix involve a high-dimensional object in the state vector, the distribution of households. Solving their problems requires knowing how the distribution of households changes. The solution method used is the same as that applied for the model described in section 2 of the main text. The algorithm is a version of that in Krusell and Smith (1998) and described in Section C.3.

The solution algorithm uses forecasting rules which are used to describe the change in the approximate aggregate state. I apply it to solve the economy described here as follows. First I solve the no-intervention model with an acyclical tax function, described in 2. The forecasts for this model are drawn from the stationary distribution of the stochastic economy. This is also the long-run model for the policy intervention economy, once debt is repaid and the tax policy returns to its original form. Until the post-intervention public debt is paid, taxes are procyclical and households expect to eventually transition to the long-run model. I solve this model using the forecast rules derived from an economy with a procylical tax policy but no policy intervention or government debt. This model has the same constant probability of permanently transitioning to the long-run model, in households' and firms' expectations, as the policy intervention model. Forecasting rules for this case are derived from a simulation of the economy before a transition.

Once the government issues debt, the aggregate capital has an additional term, B_t, and it becomes $K_t = \int (a - b) g_t d[a \times b \times \varepsilon \times h \times o] - B_t$. Since there is no government debt in the simulation step, the forecast function for the procyclical tax economy will not fully capture the path of aggregate capital after the policy interventions. For instance, after the intervention, and before the recession is over, aggregate capital in the intervention model falls faster than the forecasting function predicts as the government debt grows at the rate $\dot{B}_t = r_t B_t$. However, the forecasting function is estimated assuming $B_t = 0$. Likewise, once the economy transitions to an expansion, the aggregate capital

Figure 16: Forecasting error of the aggregate capital



Note: $dk(t)$ is the change of capital, $\dot{k}_t$. Forecasting error is $\dot{k}_t$(actual) - $\dot{k}_t$(forecasted).

stock grows faster than the speed that the forecasting function predicts, because $\dot{B}_t$ is negative thanks to the government's repayment.

Forecasting errors might mitigate policy effects on consumption and house price. Households' savings decision involve expectations of future wages and interest rates and these are functions of the future capital stock. Thus, if there is a downward bias in the forecasting function during expansions, it will lead to higher savings as a lower expected $\dot{k}$ implies a higher return on savings.

Any such error is not likely to significantly change the results because the errors are small. Figure 16 shows the error. At $t = 12$ the economy transitions to an expansion. As explained above, the forecasting errors are negative during the recession and the errors are positive during the expansion. We see that the largest positive error is slightly above 0.1%. This is comparable to the maximum and minimum errors of the benchmark model which are 0.09% and -0.13%. This model has no policy intervention or procyclical tax, and therefore no related source of possible bias in its forecast errors.

C Numerical Solution Method

This section describes the computational method used to solve the model and a measure of accuracy. The existence and uniqueness of the viscosity solution of an HJB equation are shown by Crandall and Lions (1983). The existence of the viscosity solution of an HJBVI equation when a value function is not always differentiable, which corresponds to the problem in the paper, was proven in Øksendal and Sulem (2005) (See chapter 9, theorem 9.8).

The solution algorithm is based on the finite difference method in Achdou et al. (2022) with several important differences. First, there are multiple stopping choices including two types of default, the buying and selling of houses, refinancing and prepayment. Additionally, the model solution is nonlinear in both the individual and aggregate state vectors. Since the aggregate state vector is high-dimensional, I use state-space approximation following the approach in Krusell and Smith (1998).

C.1 HJBVI as a Linear Complementarity Problem (LCP)

To solve the stopping time problems, for example the three HBJVI problems described in Appendix A and those in B.1, I transform each into a linear complementarity problem.[38]. Each of the HJBVI equations can be written as

$$\min\Big[\rho\mathbf{v} - \mathbf{u} - \mathbf{A}\mathbf{v},\ \ \mathbf{v} - \mathbf{v}^*\Big] = 0 \tag{31}$$

where A summarizes changes caused by decisions and shocks. Below I describes how to construct A. Equation (31) implies

$$\Big(\mathbf{v} - \mathbf{v}^*\Big)'\Big(\rho\mathbf{v} - \mathbf{u} - \mathbf{A}\mathbf{v}\Big) = 0 \tag{32}$$

$$\rho\mathbf{v} - \mathbf{u} - \mathbf{A}\mathbf{v} \geq \mathbf{0}$$

$$\mathbf{v} \geq \mathbf{v}^*.$$

Let $\mathbf{z} = \mathbf{v} - \mathbf{v}^*$, $\mathbf{B} = \rho\mathbf{I} - \mathbf{A}$ and $\mathbf{q} = -\mathbf{u} + \mathbf{B}\mathbf{v}^*$. Then (32) is equivalent to (33).

$$\mathbf{z}'\big(\mathbf{B}\mathbf{z} + \mathbf{q}\big) = 0 \tag{33}$$

$$\mathbf{B}\mathbf{z} + \mathbf{q} \geq 0$$

$$\mathbf{z} \geq \mathbf{0}$$

This is the standard form for a linear complementarity problem and several numerical solvers are available.[39]

[38]See `http://www.princeton.edu/ moll/HACTproject/option-simple.pdf` for an example.

[39]I use the LCP solver at `https://www.mathworks.com/matlabcentral/fileexchange/20952-lcp-mcp`

Construction of $\mathbf{A}$ I solve for optimal decisions over a discretized grid of the state space, by iterating on the value function, v. Let Equation (32) be a matrix representation of equation (27) where $\mathbf{v} = [v_1, v_2, .., v_N]$, $\mathbf{u} = [u_1, u_2, .., u_N]$ and N is the number of points in the value function. Thus $\mathbf{A}$ describes $\dot{a}$, $\dot{b}$ and shocks which households are exposed to. Here, I first describe the construction of $\mathbf{A}$ excluding terms related to the aggregate states (g, z) and Section C.3 contains a description of how to solve the model with aggregate uncertainty. As $\dot{b}$ is given by $\theta(b, ph)b$ and the earnings process is exogenously set, below I explain how to solve $\dot{a}$.

I choose a number of grid points $(n_a, n_b, n_\varepsilon, n_h)$ for the corresponding variables, (a, b, ε, h). Let v_{ijkp} be the value function of a household without the bankruptcy or foreclosure flag, with liquid assets a_i, mortgage b_j, labor productivity ε_k and house h_p. The derivative with respect to a, v^a_{ijkp} is approximated with either a forward or a backward first difference.

$$v^a_{ijkp} \approx \frac{v_{i+1jkp} - v_{ijkp}}{\Delta a} = v^{a,F}_{ijkp} \text{ or } v^a_{ijkp} \approx \frac{v_{ijkp} - v_{i-1jkp}}{\Delta a} = v^{a,B}_{ijkp}$$

Likewise, the derivative with respect to b, v^b_{ijkp} can be approximated with forward or backward difference.

Applying this method to the first argument in Equation (27), we have

$$\begin{aligned} \rho v_{ijkp} = u(c_{ijkp}, h_p) + v^a_{ijkp}\dot{a}_{ijkp} + v^b_{ijkp}\dot{b}_{ijkp} + \sum_{\varepsilon'_k} \lambda(\varepsilon_k, \varepsilon_{k'})(v_{ijk'p} - v_{ijkp}) \\ \forall\ i = \{1, .., n_a\}, j = \{1, .., n_b\}, k = \{1, .., n_\varepsilon\}, p = \{1, .., n_h\}, \end{aligned} \tag{34}$$

where

$$\dot{a}_{ijkp} = w\varepsilon_k + r_{a,ijkp}a_i - (r + \theta(b_j, ph_p))b_j - c_{ijkp} - T(b_j, ph_p, \varepsilon_k) - \xi_h ph_p,$$

$$\dot{b}_{ijkp} = -\theta(b_j, ph_p)b_j.$$

The household choice of non-durable consumption can be solved from the FOC,

$$\begin{aligned} u^c(c_{ijkp}, h_p) - v^a_{ijkp} = 0, \\ c_{ijkp} = (v^a_{ijkp})^{\frac{1}{-\sigma}}. \end{aligned}$$

As the derivatives of the value function have two forms, forward and backward, c_{ijkp}

-solver-newton-based.

is either $(v^{a,F}_{ijkp})^{\frac{1}{-\sigma}}$ or $(v^{a,B}_{ijkp})^{\frac{1}{-\sigma}}$.

To find the drift, $\dot{a}$, it is necessary to select which derivative to use. I follow Achdou et al.'s (2022) upwind scheme. The key idea is to use a forward derivative when the drift is positive and use a backward derivative when it is negative. To ease notation, for a variable x, let $x^+ = max(x, 0)$ and $x^- = min(x, 0)$. Also, let x^F be the value computed using a forward derivative and x^B be the value derived from the backward derivative. With this notation, savings, $s(= \dot{a})$, can be computed as below:

$$
\begin{aligned}
s^{c,F}_{ijkp} &= w\varepsilon_k + r_{a,ijkp}a_i - (r + \theta(b_j, ph_p))b_j - c^F_{ijkp} - T(b_j, ph_p, \varepsilon_k) - \xi_h ph_p \\
s^{c,B}_{ijkp} &= w\varepsilon_k + r_{a,ijkp}a_i - (r + \theta(b_j, ph_p))b_j - c^B_{ijkp} - T(b_j, ph_p, \varepsilon_k) - \xi_h ph_p.
\end{aligned} \tag{35}
$$

The upwind scheme can be applied to all of the points in the state space except for the points at the boundaries. Clearly, only one of the forward or backward derivatives can be approximated using a finite difference at the boundaries. The way this is handled for an exogenous borrowing constraints in a one asset model is explained in Achdou et al. (2022). In my model, there are three assets. Across the two loans, mortgages are secured but there may be an endogenous borrowing limit with respect to a as there is an option to default. Over levels of a where a households would default, there cannot be lending and an endogenous bound is imposed such that $\dot{a} \geq 0$. I explain the details of how I handle boundaries in the rest of this section.

I follow Bornstein (2018) who shows how to solve the problem of an endogenous borrowing limit with respect to a. For each $(b_j, \varepsilon_k, h_p)$, $\forall\, j = \{1, .., n_b\}, k = \{1, .., n_\varepsilon\}, p = \{1, .., n_h\}$, assume that we know the level of unsecured debt, $\underline{a}(b_j, \varepsilon_k, h_p)$ where a household chooses to default when a is below $a_{Djkp} = \underline{a}(b_j, \varepsilon_k, h_p)$. The value function below a_{Djkp} becomes flat over a and the backward derivative does not exist at $(D, b_j, \varepsilon_k, h_p)$. I impose the endogenous borrowing constraint $\underline{a}(b_j, \varepsilon_k, h_p)$; below this point, I restrict consumption to equal income. One potential issue is consumption at the endogenous borrowing limit (or below the limit) can be negative. If this is the case, I assign a very low value to ensure that these are default points. Note that a will not fall below $\underline{a}(b_j, \varepsilon_k, h_p)$ as the household would have defaulted beforehand. Without loss of generality I set savings to be zero at these points.

Finally, we must know the set of default points, $i \leq D$ at each $(b_j, \varepsilon_k, h_p)$ to find $\underline{a}(b_j, \varepsilon_k, h_p)$. As I solve the value function iteratively, I used the default points identified by the last iteration to set $\underline{a}(b_j, \varepsilon_k, h_p)$. Stopping points including default points are given by LCP solution algorithms as will be explained at the end of this subsection.

Effectively, the endogenous borrowing limit is implemented at each iteration of the solutin algorithm as follows. At iteration n, let D^A be a set of points that where a household will default on a or a and b both. If a point $[a_i, b_j, \varepsilon_k, h_p] \in D^A$, set $s^{c,F}_{ijkp} = s^{c,B}_{ijkp} = 0$ in Equation (36). These savings functions determine are the endogenous component of A in (32) which is used to solve v as described next.

Before describing how to build A, one last issue is remains. At a_{n_a}, a forward derivative cannot be computed. However, if I set a_{n_a} large enough, savings at this point will be negative and the forward derivative will not be needed.

I now describe the matrix A in (32). Using (35) in (34), the system of equations for the value function can be written as:

$$\begin{aligned}\rho v_{ijkp} &= u(c_{ijkp}, h_p) + x^a_{ijkp} v_{i-1jkp} + y_{ijkp} v_{ijkp} + z^a_{ijkp} v_{i+1jkp} \\ &\quad + x^b_{ijkp} v^{n+1}_{ij-1kp} + z^b_{ijkp} v_{ij+1kp} + \sum_{k'} \lambda_{kk'} v_{ijk'p},\end{aligned} \tag{36}$$

$$\begin{aligned} x^a_{ijkp} &= -\frac{(s^{c,B})^-}{\Delta a}, \qquad x^b_{ijkp} = -\frac{(\theta(b,ph)b)^-}{\Delta b}, \\ y_{ijkp} &= -\frac{(s^{c,F})^+}{\Delta a} + \frac{(s^{c,B})^-}{\Delta a} - \frac{(\theta(b,ph)b)^+}{\Delta b} + \frac{(\theta(b,ph)b)^-}{\Delta b} + \lambda_{kk}, \\ z^a_{ijkp} &= \frac{(s^{c,F})^+}{\Delta a}, \qquad z^b_{ijkp} = \frac{(\theta(b,ph)b)^+}{\Delta b}. \end{aligned}$$

There are $n_a \times n_b \times n_\varepsilon \times n_h$ linear equations (36), one for each grid point. The system of equations can be written in matrix notation:

$$\rho \mathbf{v} = \mathbf{u} + \mathbf{A}\mathbf{v}. \tag{37}$$

The matrix A can be constructed step by step. First, I define a submatrix for a given level of housing. For each h_p, $\mathbf{A}_p, p \in \{1, 2, ..., n_h\}$, can be written as

$$\mathbf{A}_p = \begin{bmatrix} A_{11|p} & A_{12|p} & .. & A_{1n_\varepsilon|p} \\ A_{21|p} & A_{22|p} & .. & A_{2n_{\varepsilon|p}} \\ \vdots & \ddots & & \vdots \\ A_{n_\varepsilon 1|p} & A_{n_\varepsilon 2|p} & .. & A_{n_\varepsilon n_\varepsilon|p} \end{bmatrix}$$

where $A_{kk|p}$ is a matrix that is composed of x^a_{ijkp}, x^b_{ijkp}, y_{ijkp}, z^a_{ijkp} and z^b_{ijkp}, $k \in \{1, 2, ..., n_\varepsilon\}$. For example,

$$A_{11|2} = \begin{bmatrix}
y_{1112} & z^a_{1112} & 0 & .. & 0 & z^b_{1112} & 0 & .. & 0 & 0 & 0 & .. & & 0 \\
x^a_{2112} & y_{2112} & z^a_{2112} & 0 & .. & 0 & z^b_{2112} & 0 & ... & 0 & 0 & 0 & .. & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\
0 & .. & 0 & x^a_{n_a112} & y^a_{n_a112} & 0 & .. & 0 & z^b_{n_a112} & & 0 & ... & 0 & 0 \\
x^b_{1212} & .. & & & 0 & y_{1212} & z^a_{1212} & 0 & .. & 0 & z^b_{1212} & 0 & & 0 \\
\vdots & x^b_{2212} & \ddots & \ddots & \ddots & x^a_{2212} & y_{2212} & z^a_{2212} & \ddots & \ddots & \ddots & z^b_{2212} & \ddots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\
0 & .. & & & & & & x^a_{n_an_b12} & y^a_{n_an_b12} & 0 & .. & & 0 & z^b_{n_an_b12}
\end{bmatrix}$$

and when $k \neq l$, $A_{kl|p}$ is a diagonal matrix with diagonal terms λ_{kl}. Using $\mathbf{A_p}$, $\mathbf{A}$ is a block diagonal matrix composed of $\mathbf{A}_1$,..,$\mathbf{A}_{n_h}$,

$$\mathbf{A} = \begin{bmatrix}
\mathbf{A}_1 & 0 & .. & & 0 \\
0 & \mathbf{A}_2 & 0 & .. & 0 \\
0 & \ddots & \ddots & \ddots & 0 \\
0 & .. & & 0 & \mathbf{A}_{n_h}
\end{bmatrix}.$$

These steps have described how to create the system of equations in for a household described by (27) with neither bankruptcy nor foreclosure in its credit record. In the same fashion, a systemt of linear equations describing the household with the bankruptcy flag (28), or with a foreclosure flag (29), can be written as

$$\rho \mathbf{v}^d = \mathbf{u}^d + \mathbf{A}^d \mathbf{v}^d, \tag{38}$$

$$\rho \mathbf{v}^f = \mathbf{u}^f + \mathbf{A}^f \mathbf{v}^f. \tag{39}$$

Because $a \geq 0$ with the bankruptcy flag, values are not defined below $a = 0$. Therefore the number of equations is $n_{a+} \times n_b \times n_\varepsilon \times n_h$ where n_{a+} is the number of a grid where $a \geq 0$. When the household state includes a foreclosure flag, values are defined where $h = 0$ and $b = 0$, and the number of equations is $n_a \times n_\varepsilon$.

The systems of equations in (37), (39) and (39) are converted into the form in (33), then solved iteratively using a LCP solver. Let $\mathbf{z} = \mathbf{v}^{n+1} - \mathbf{v}^{*,n}$, $\mathbf{B} = \frac{1}{\Delta} - \rho - \mathbf{A}$ and $\mathbf{q} = \mathbf{B}\mathbf{v}^{*,n} - \frac{\mathbf{v}^n}{\Delta} - \mathbf{u}^n$. Then

$$(\mathbf{v}^{n+1} - \mathbf{v}^{*,n})(\frac{\mathbf{v}^{n+1} - \mathbf{v}^n}{\Delta} - \rho \mathbf{v}^{n+1} - \mathbf{u}^n - \mathbf{A}\mathbf{v}^{n+1}) = 0$$

$$\frac{\mathbf{v}^{n+1}-\mathbf{v}^{n}}{\Delta}-\rho\mathbf{v}^{n+1}-\mathbf{u}^{n}-\mathbf{A}\mathbf{v}^{n+1}\geq 0$$

$$\mathbf{v}^{n+1}-\mathbf{v}^{*,n}\geq 0$$

is equivalent to

$$\mathbf{z}'(\mathbf{B}\mathbf{z}+\mathbf{q})=0$$

$$\mathbf{B}\mathbf{z}+\mathbf{q}\geq 0$$

$$\mathbf{z}\geq \mathbf{0}$$

which is the LCP problem from (33). As mentioned, I describe the value function solution algorithm below. The parameter Δ determines the speed of updating.

- Guess the value functions $\mathbf{v}^n$
- Using the guessed value functions, construct $\mathbf{A}$
- Compute $\mathbf{v}^{*,n}$ by solving Equation (6) which is $v^*(a_\tau, b_\tau, \varepsilon_\tau, h, g, z) = \max\{v^m, v^r, v^a, v^b, v^{ab}\}$.
- Let $\mathbf{z}=\mathbf{v}^{n+1}-\mathbf{v}^{*,n}$, $\mathbf{B}=\frac{1}{\Delta}-\rho-\mathbf{A}$ and $\mathbf{q}=\mathbf{B}\mathbf{v}^{*,n}-\frac{\mathbf{v}^n}{\Delta}-\mathbf{u}^n$ and solve the LCP
- From the solution to the LCP, $\mathbf{v}^{n+1}=\mathbf{z}+\mathbf{v}^{*,n}$
- With updated value function $\mathbf{v}^{n+1}$, if $\max|v^{n+1}-v^n|$ is not small enough, return to the second step.

Above, $\mathbf{z}$ is the solution provided by the LCP solver, and, where $\mathbf{z}=0$, households find it optimal to stop. Recall that the region of default on unsecured debt is necessary to construct $\mathbf{A}$ for the next iteration. This is found from $\mathbf{z}$ as follows. In every iteration, D^A are the set of points that satisfy $\{\mathbf{z}=0\}\cap\{\mathbf{v}^{*,n}=\mathbf{v}^{a,n}\}\cup\{\mathbf{v}^{*,n}=\mathbf{v}^{ab,n}\}$. Finally, with many discrete choices, I find using a large value of Δ (for example, larger than 10) often leads to unstable updates.

$\mathbf{v}^d$ and $\mathbf{v}^f$ can be solved using the same algorithm.

C.2 Kolmogorov Forward equation

After solving the value functions, I need to solve for the household density over their assets, mortgages, houses, labor productivity and default flags. In this section, I describe how to solve the density function.

Without stopping decisions, the Kolmogorov Forward equation is

$$\partial_t g_{ijkp,t} = -\partial_a s^a_{ijkp} g_{ijkp,t} - \partial_b s^b_{ijkp} g_{ijkp,t} + \sum_{k'} \lambda_{k'k} g_{ijk'p,t}$$

where s^x_{ijkp} is shorthand notation for x decision rule at $(a_i, b_j, \varepsilon_k, h_p)$, $x \in \{a, b\}$ and $g_{ijkp,t}$ is a density function at time t. In my model i) movements due to housing transactions, refinancing, bankruptcy and foreclosure, ii) flows between a state without the default flag and a state with the bankruptcy flag and iii) flows between a state without the default flag and a state with the foreclosure flag need to be accounted for.

The mathematical formulation of Kolmogorov Forward equations with stopping choices is not straightforward.[40] However, there is a way of handling stopping decisions in computation. I follow the approach in a note by Kaplan et al.[41] Flows due to stopping decisions can be treated with the *intervention matrix*, M. First, let g_i be the i^{th} element of the density function where $i \in \{1, ...N\}$ and N is the total number of grid points,[42]

$$M_{i,j} = \begin{cases} 1 & \text{if } i \in I \text{ and } i = j \\ 1 & \text{if } i \notin I \text{ and } j^*(i) = j \\ 0 & \text{otherwise,} \end{cases}$$

where I is the non-stopping region and $j^*(i)$ is the target point of point i. A target point is a point arrived at as a the result of a stopping choice such as buying a house. For example, if a household with $(a_i, b_k, \varepsilon_j, h_p)$ decides to buy a house, and ends up having $(a_{i1}, b_{k1}, \varepsilon_j, h_{p1})$ as a result of the transaction, the latter is the target point.

The flow from a state with the bankruptcy and foreclosure flags to a state without the bankruptcy flag is a shock and can be expressed as below. Let nd represent 'non-default', and d represent 'default',

$$\partial_t g^{nd}_{ijkp,t} = -\partial_a s^{a,nd}_{ijkp} g^{nd}_{ijkp,t} - \partial_b s^{b,nd}_{ijkp} g^{nd}_{ijkp,t} + \sum_{k'} \lambda_{k'k} g^{nd}_{ijk'p,t} + \lambda_l g^d_{ijkp,t},$$

$$\partial_t g^{d}_{ijkp,t} = -\partial_a s^{a,d}_{ijkp} g^{d}_{ijkp,t} - \partial_b s^{b,d}_{ijkp} g^{d}_{ijkp,t} + \sum_{k'} \lambda_{k'k} g^{d}_{ijk'p,t} - \lambda_l g^d_{ijkp,t},$$

where $\lambda_l = \lambda_d$ for the bankruptcy flag state and $\lambda_l = \lambda_f$ for the foreclosure flag state.

[40] See 'Liquid and Illiquid Assets with Fixed Adjustment Costs' by Greg Kaplan, Peter Maxted and Benjamin Moll at `http://www.princeton.edu/ moll/HACTproject/liquid-illiquid-numerical.pdf`

[41] The note can be found at `http://www.princeton.edu/ moll/HACTproject/liquid-illiquid-numerical.pdf`.

[42] $N = n_a \times n_b \times n_\varepsilon \times n_h \times 2 + n_a \times n_\varepsilon$. $n_a \times n_b \times n_\varepsilon \times n_h$ is multiplied by 2 because there are points with and without bankruptcy flag. $n_a \times n_\varepsilon$ is the number of points in the state with the foreclosure flag.

These flows can be treated with the matrices A^d and A^f,

$$\mathcal{A}^d_{i,j} = \begin{cases} \lambda_d & \text{if } 1 \leq j \leq n_1 \text{ and } i = j + n_1 \\ -\lambda_d & \text{if } n_1 + 1 \leq j \leq n_1 \times 2 \text{ and } i = j \\ 0 & \text{otherwise,} \end{cases}$$

$$\mathcal{A}^f_{i,j} = \begin{cases} \lambda_f & \text{if } 1 \leq j \leq n_a \times n_\varepsilon \text{ and } i = j + n_1 \times 2 \\ -\lambda_f & \text{if } n_1 \times 2 + 1 \leq j \leq N \text{ and } i = j \\ 0 & \text{otherwise.} \end{cases}$$

where $n_1 = n_a \times n_b \times n_\varepsilon \times n_h$. The sizes of A^d and A^f are $N \times N$ and I stack points from the state without the flag, points from the state with the bankruptcy flag state and points from the state with the foreclosure flag.[43] Finally, define B as below:

$$B = \mathcal{A} + \mathcal{A}^d + \mathcal{A}^f$$

where $\mathcal{A}$ be a block diagonal matrix which is composed of $\mathbf{A}$, $\mathbf{A}^d$ and $\mathbf{A}^f$,

$$\mathcal{A} = \begin{bmatrix} \mathbf{A} & 0 & 0 \\ 0 & \mathbf{A}^d & 0 \\ 0 & 0 & \mathbf{A}^f \end{bmatrix}.$$

Given M and B, the density function can be solved by iterating the following two steps until g converges,

$$g^{n+\frac{1}{2}} = M^T g^n,$$
$$\frac{g^{n+1} - g^{n+\frac{1}{2}}}{\Delta t} = (BM)^T g^{n+1}.$$

C.3 Stochastic model

The aggregate state contains an argument that has an infinite dimension, g, the distribution of households over $(a, b, \varepsilon, h, o)$. To make the computation feasible, this distribution needs to be approximated. I assume the households only use a finite set of moments from g to form their expectations as in Krusell and Smith (1998).[44]

[43] n_1 is the number of points in the state without the flags and $n_1 \times 2$ is sum of the number of points of the no flag state and the bankruptcy flag state.

[44] Ahn et al. (2018) develop a method of solving continuous time heterogeneous agent models with aggregate uncertainty based on linearization and dimension reduction. Fernández-Villaverde et al. (2019) also present a method to solve such models. They assume the households only track a finite set

Specifically, I assume that the households keep track of the aggregate capital stock, k.

I redefine the problem using the approximate state, (k, z). For example, equation (5) can be written as

$$\rho v(a, b, \varepsilon, h, k, z) = \max_c u(c, h) + \partial_a v(a, b, \varepsilon, h, k, z)\dot{a} + \partial_b v(a, b, \varepsilon, h, k, z)\dot{b}$$
$$+ \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v(a, b, \varepsilon_j, h, k, z) + \sum_{k=1}^{n_z} \lambda_{zz_k} v(a, b, \varepsilon, h, k, z_k)$$
$$+ \partial_k v(a, b, \varepsilon, h, k, z)\dot{k}.$$
$$\dot{k}_t = \frac{\mathbb{E}[dk_t; k_t, z_t]}{dt} = f^k(k_t, z_t).$$

The last term in equation (5) which captures the evolution of the distribution is replaced with $\partial_k v(a, b, \varepsilon, h, k, z)\dot{k}$. I assume f has a log-linear form,

$$dlog(k_t)dt = \beta_z^0 + (\beta_z^1 - 1)log(k_t).$$

On an approximated aggregated state, equilibrium wage rates and interest rates are marginal productivity of labor and capital. Since the working hour is not a choice, the aggregate labor is fixed. Therefore computing wage rates and interest rates at a given capital is trivial. On top of wage rates and interest rates, house price $p(k, z)$ is necessary to solve the model. I also assume a log-linear form to estimate house prices,

$$log(p_t) = \phi_z^0 + \phi_z^1 log(k_t).$$

In Section C.1, I described the computational steps in the absence of an aggregate state, (g, z). Having approximated the high-dimensional object g with k, now I include terms related to the aggregate state. Like other variables, I discretize k and n_k is the number of grid points for k. From Section C.1, the only part needing to be changed is the construction of the matrix A, which describes $\dot{a}$, $\dot{b}$ and the shocks. Consider the A in Section C.1 as a A at a given (k_i, z_j), A^{ij}. Then A should be replaced with $\mathbb{A}_a + \mathbb{A}_k + \mathbb{A}_z$ where

of moments of the distribution to form their expectations as well, but use tools from machine learning to estimate the perceived law of motion of the households.

$$\mathbb{A}_a = \begin{bmatrix} \mathbf{A}^{11} & & & & & & \\ & \ddots & & & & & \\ & & \mathbf{A}^{n_k1} & & & & \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ & & & & \mathbf{A}^{1n_z} & & \\ & & & & & \ddots & \\ & & & & & & \mathbf{A}^{n_kn_z} \end{bmatrix}$$

$$\mathbb{A}_k = \begin{bmatrix} x_{11}\mathbf{I} & x^F_{11}\mathbf{I} & & & & & \\ x^B_{21}\mathbf{I} & x_{21}\mathbf{I} & x^F_{21}\mathbf{I} & & & & \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ & & & & & \ddots & \\ & & & & & x^B_{n_kn_z}\mathbf{I} & x_{n_kn_z}\mathbf{I} \end{bmatrix}$$

$$\mathbb{A}_z = \begin{bmatrix} -\lambda_{z1}\mathbf{II} & \lambda_{z1}\mathbf{II} \\ \lambda_{z2}\mathbf{II} & -\lambda_{z2}\mathbf{II} \end{bmatrix}$$

and $-\frac{\dot{k}^+_{ij}}{\Delta k} + \frac{\dot{k}^-_{ij}}{\Delta k} = x_{ij}$,$-\frac{\dot{k}^-_{ij}}{\Delta k} = x^B_{ij}$, $\frac{\dot{k}^+_{ij}}{\Delta k} = x^F_{ij}$, $\mathbf{I}$ is $N \times N$ identity matrix, and $\mathbf{II}$ is $Nn_k \times Nn_k$ indentity matrix. I use $n_z = 2$ and $n_k = 10$.

Solution algorithm

1. Guess parameters of the forecasting functions. With the forecasting functions, $\dot{k}$ and the house price over (k, z) are known. Also, interest rates and wages over (k, z) can be computed using the firm's marginal conditions.
2. Solve the value function.
   - Guess the loan price functions, $r_a(a, b, \varepsilon, h, o, k, z)$ and $q(a, b, \varepsilon, h, k, z)$.
   - Guess the value function, $v^0(a, b, \varepsilon, h, k, z)$, $v^{0,d}(a, b, \varepsilon, h, k, z)$ and $v^{0,f}(a, \varepsilon, k, z)$.
   - Update the value functions and the loan price schedules until they converge.
   - Save decision rules.
3. Simulate the model for n periods. Simulation gives the sequence of aggregate variables $\{z_t, k_t, p_t\}_{t=1}^n$.
   - Guess the initial distribution. The distribution in the steady state can be used as a good initial distribution.

- At the beginning of each period, (k, z) are known. The risk free rate and wage can be computed.
- Compute the loan price functions. To compute $r_a(a_t, b_t, \varepsilon_t, h_t, o_t, k_t, z_t)$, interpolate the default decisions that are obtained from the step 2. Using the default decisions and the risk free rate, $r_a(a_t, b_t, \varepsilon_t, h_t, o_t, k_t, z_t)$ can be computed using the equation (20). To compute $q(a_t, b_t, \varepsilon_t, h_t, k_t, z_t)$, interpolate $q(a, b, \varepsilon, h, k, z)$ obtained from step 2, over k.
- Guess the house price.
- With the wage, the loan price schedules and the house price, solve the household problem.
- Compute the aggregate demand for housing. If the aggregate demand is not close enough to the supply, adjust the house price to clear the housing market.
- Once the housing market is cleared, move to the next period.

4. Using the sequence of aggregate variables $\{z_t, k_t, p_t\}_{t=1}^n$, update the forecasting functions.
5. Check convergence of the simulated aggregate variables. If the distance between the $\{k_t\}_{t=1}^n$ from the current iteration and the previous iteration is less than a tolerance level, an approximate recursive equilibrium is obtained. Otherwise, go back to step 2 with the updated forecasting functions.

Predictive power of the forecasting functions

Table 11: R^2 of forecasting functions

	Benchmark		Foreclosure only	
	Exp.	Rec.	Exp.	Rec.
k'	0.999	0.999	0.999	0.999
p	0.988	0.972	0.966	0.948
	Procyclical tax-rate			
k'	0.999	0.999	0.999	0.999
p	0.984	0.968	0.954	0.938

Note: Based on a simulation of 4,000 periods.

Table 11 shows the R^2 of the forecasting functions. Since forecasting functions are conditional on the aggregate state, there are R^2s for both expansions and recessions. The R^2s are high for k. Capital moves slowly and this makes the forecasting function very accurate for any version of the model. R^2s for house prices are slightly lower and they are lower during recessions than expansions. This is probably because housing choices are discrete.

D Additional information on calibration

This section is organized as follows. Section D.1 explains the calibration of earnings process. Next, Section D.2 describes the categorization of assets and debt used in computing calibration targets.

D.1 Earnings process

I model the labor earnings process as a combination of two independent processes:

$$\varepsilon_{ij} = \varepsilon_i^p(1 + \varepsilon_j^t)$$

where each component follows a Poisson jump process. Jumps arrive at a Poisson rate λ^p for ε^p and λ^t for ε^t. Conditional on a jump in ε^p, a new earnings state ε_k^p is drawn from a bounded Pareto distribution; conversely, when ε^t has a jump, it is drawn from the discrete set $\{-\chi, \chi\}$. Kaplan et al. (2018) explain why this type of process is useful for matching high frequency earnings dynamics. The size and frequency of each shock determine the shape of the earnings distribution. Large, infrequent shocks are likely to generate a more leptokurtic distribution and small, frequent shocks are likely to generate a platykurtic distribution. Kaplan et al. (2018) model the earnings process as a sum of two jump-drift processes, representing a persistent and a transitory component of the earnings process.

The distribution of ε_i^p is determined by a choice of upper and lower bound and a curvature parameter. I choose these parameters to match the variance as well as several additional moments of the earnings distribution: the share of earnings over quintiles, and the shares in the top 5-10, 1-5 and 1 percentiles. As there is little effect on the earnings distribution from ε^t, in matching these moments I start by discretizing ε^p. As a first step in this discretization, I create a set of 4 points that are not linearly spaced. Instead, the 4 points aims to capture the bottom 41.0, 69.0 and 98.5 percentiles of the earnings distribution, and the remaining top 1.5%.

The first three probabilities represent population shares of three educational attainment levels. These educational levels are high school graduate and below, some college or bachelor's degree, and higher (averaged over 1992 to 2013 as reported by the BLS). The last point to compute the share of earnings held by the top 1.5% is to capture concentration at the top of the distribution.

The set of 4 points is found as follows. Let $\underline{\varepsilon}^p$, $\overline{\varepsilon}^p$ and η_ε^p be the lower and upper bounds, and a curvature parameter, of the bounded Pareto distribution. The first and second points, x_1 and x_2, solve $f(x_1) = \frac{1-(\underline{\varepsilon}^p/x_1)^{\eta_\varepsilon^p}}{1-(\underline{\varepsilon}^p/\overline{\varepsilon}^p)^{\eta_\varepsilon^p}} = 0.41$ and $f(x_2) = \frac{1-(\underline{\varepsilon}^p/x_2)^{\eta_\varepsilon^p}}{1-(\underline{\varepsilon}^p/\overline{\varepsilon}^p)^{\eta_\varepsilon^p}} = 0.69$ where $f(x_i)$ is the CDF of the bounded Pareto distribution.

The next step is to use these points to determine transition probabilities for a discretized grid for ε^p. As is conventional when discretizing a continuous distribution, the support is chosen so that the vector ε^p has ε_1^p as the midpoint between $\underline{\varepsilon}$, x_1, and ε_2^p is a midpoint between x_1 and x_2 and so forth.

Having chosen the values for ε_i^p, the probability of drawing a new value upon arrival of income shock can be defined. In the following, I assume that the probability of drawing ε_k^p depends on the current level of productivity, ε_i^p. This requires choosing bounded Pareto distributions over ε_k^p for each ε_i^p. Each of these distributions is bounded by the same $\underline{\varepsilon}^p$, $\overline{\varepsilon}^p$ described above. They are distinguished by curvature parameters, $\eta_{\varepsilon_i^p}$, $i = 1,..,4$. Given the discretized support, the shape parameters $\eta_{\varepsilon_i^p}$ need to be estimated.

We use the above distributions, alongside the points x_i described above, to construct conditional probabilities. Conditional on a jump, let the probability of a change from ε_i^p to ε_k^p be $f(\varepsilon_{k|i}) - f(\varepsilon_{k-1|i})$ where $f(\varepsilon_{k|i}) = \frac{1-(\underline{\varepsilon}^p/x_k)^{\eta_{\varepsilon_i^p}}}{1-(\underline{\varepsilon}^p/\overline{\varepsilon}^p)^{\eta_{\varepsilon_i^p}}}$. Recall λ^p is the intensity for the arrival of a ε^p shock. The intensity of jumping from i to k is $\lambda_{ik}^p = \lambda^p(f(\varepsilon_{k+1|i}) - f(\varepsilon_{k|i}))$.

To set the curvature values $\eta_{\varepsilon_i^p}$, $i = 1,..,4$, the shock intensities λ^p and λ^t, the size of the shock χ and the probability of drawing a negative transitory component conditional on arrival of shock in ε^t, I estimate the earnings process by Simulated Method of Moments to match the higher order moments of the earnings growth rate distribution reported in Guvenen et al. (2015) using Social Security Administration (SSA) data from 1994 to 2013.

I simulate the discretized earnings process to compute corresponding moments. The panel size is 5,000 and the simulation length is 6,000. The 800 periods of each simulated series are discarded before computing statistics. Increasing the panel size or the number of periods has little effect on the results. Since the data moments are computed using annual earnings, I simulate at a higher frequency and aggregate the results into annual earnings.

To summarize, the number of parameters specifying the earnings process is 11 and the number of targets is 20. The parameters include the 3 parameters that shape the bounded Pareto distribution for ε^p are $\overline{\varepsilon}^p$, $\underline{\varepsilon}^p, \eta_\varepsilon^p$. In addition, there are 4 parameters that set the probability of drawing a new value for ε^p are $\eta^p_{\varepsilon_i}, i \in [1, .., 4]$. Nest, we have the 2 parameters that set shock intensity are λ^p, λ^t and χ is the size of ε_t shock. Finally, p^t is a probability of drawing negative value upon an arrival of ε^t shock.

Note that as λ_{ik} affects the ergodic distribution of households over labor productivity, the cumulative population shares by ε^p_i could be different from the 41.0, 69.0, 98.5 and 100 percentiles I set above. Therefore an additional restriction to the Simulated Method of Moments is that the abcissa x_i must be consistent with the educational attainment earnings shares stated at the start of this section. Hence the objective function minimized includes the percentiles of the earnings distribution which are used to space the grid for ε^p. Beyond this, there are 20 targets listed in Tables 1 and 2 in Section 3 of the text.

The estimated process implies that a shock to ε^p arrives on average once every 21 years. Upon the arrival of a shock, one's income level jumps to a different level. Turning to the other labor productivity shock, a shock to ε^t arrives on average once every 0.9 years. The infrequent component of labor income shock, ε^p can be interpreted as the persistent component and ε^t as the transitory component. Households do not experience a large shock often, but income fluctuates around their persistent component through frequent shocks, ε^t.

D.2 Categorization of assets and debts

Mapping the model to the data requires categorizing assets held by U.S. households into financial assets, non-financial assets and secured debt. I target the asset and debt distribution reported in the 2007 SCF. In the SCF data, net worth is comprised of assets and debt, and total assets are the sum of financial assets and non-financial assets. Financial assets include transaction accounts, certificates of deposit, money market funds, stocks, cash, quasi-liquid retirement accounts and other financial assets. Non-financial assets are predominantly the value of vehicles and houses (primary and non-primary residential property, non-residential real estate) and the value of business. Debt is comprised of debt secured by residential properties, credit card loans, installment loans (e.g., student loans, vehicle loans). When mapping the model to the data, I exclude the value of a business and vehicles from non-financial assets because my model does not have such assets. For debt, I exclude installment loans which include student loans and vehicle loans for the same reason. Student loans are not short-term, unsecured debt nor are they secured by collateral or dischargeable in bankruptcy. After excluding installment loans, credit card loans are considered as unsecured debt, and the remaining components of debt are assigned to secured debt. Table 13 and Table 14 shows the portfolio composition by quintiles excluding and including business assets, vehicles and installment loans. When excluding them, the shares of assets and debt in the 2nd to 4th quintiles are not very different. In the 5th quintile, the share of non-financial assets is lower, which implies that a large share of business assets is owned by the wealthiest households. The numbers of the 1st quintile are higher when excluding business assets, vehicles and installment loans. This is likely caused by the net worth of the group being closer to zero due to the exclusion of installment loans.

E Consumption equivalent gain of the default option

In this section, I derive the consumption equivalent gain described in section 4.2. The consumption equivalent gain is used to assess the value to households of being able to default. Let v_i be the value function of an alternative economy and let x be the non-durable consumption difference that makes the value of the benchmark economy indifferent to the value of the economy with bankruptcy only or foreclosure only or without any default.

As shown in section 2, the value of a non-stopping household is:

$$\rho v(a,b,\varepsilon,h) = \max_c u(c,h) + \partial_a v(a,b,\varepsilon,h)\dot{a} + \partial_b v(a,b,\varepsilon,h)\dot{b} + \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v(a,b,\varepsilon_j,h).$$

For ease of notation, let $\mathbb{A}v = \partial_a v(a,b,\varepsilon,h)\dot{a} + \partial_b v(a,b,\varepsilon,h)\dot{b} + \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v(a,b,\varepsilon_j,h)$.

$$\begin{aligned}
\rho v_{benchmark} &= u((1+x_i)c_i,h) + \mathbb{A}v_i \\
&= u((1+x_i)c_i) + u(h) + \mathbb{A}v_i + u(c_i) - u(c_i) \\
&= u((1+x_i)c_i) + \rho v_i - u(c_i)
\end{aligned}$$

Since the utility function is separable in c and h, $u((1+x_i)c_i,h) = u((1+x_i)c_i) + u(h)$.

$$\rho v_{benchmark} - \rho v_i + u(c_i) = u((1+x_i)c_i)$$

$$((1-s)(\rho v_{benchmark} - \rho v_i + u(c_i)))^{\frac{1}{1-s}} = (1+x_i)c_i$$

$$\frac{((1-s)(\rho v_{benchmark} - \rho v_i + u(c_i)))^{\frac{1}{1-s}}}{c_i} - 1 = x_i$$

F Mortgage relief policies during the Great Recession

During the Great Recession, the US government intervened in mortgage markets through household debt relief policies. In particular, the government introduced principal reduction modifications in 2010 in Home Affordable Modification Program (HAMP). This was a response to growing concerns that debt levels, not just debt repayments, were causing high foreclosure rates. Under this modification, mortgage borrowers' principal was forgiven until the new monthly payment fell below 31% of income or LTV ratio dropped to 115%, whichever came first. Although participation rates were perceived to be low, Agarwal et al. (2017) show that the program was associated with reduced rates of foreclosure, consumer debt delinquencies and house price declines. I complement studies estimating causal relations as in Agarwal et al. (2017) by assessing the effects in aggregate quantities from resulting changes in the distribution of households and prices.

G Cumulative effects on non-durable consumption

Table 12: Cumulative effects on non-durable consumption

	Principal reduction		**Tax rebate**
	control group	treatment group	
Less housing (%)	1.2	2.1	1.3
	(4,949)	(995)	(4,658)
Same housing (%)	-0.1	2.2	1.0
	(8,868)	(796)	(10,026)
More housing (%)	2.0	4.5	1.9
	(1,434)	(958)	(3,316)

Note: Average of consumption deviation from the benchmark of corresponding groups. Average of 100 quarters. Control group are households who do not receive the principal reduction, and treatment group are those who receive the principal reduction. The numbers in parenthesis is the number of observations.

At an individual level, a large difference in non-durable consumption responses between the policy and the benchmark economies is often related to differences in housing consumption. For example, households who sold their houses at a price higher than the benchmark, raise their consumption by more. Table 12 summarizes cumulative effects on nondurable consumption by housing consumption (average housing consumption over 100 quarters). Households who receive the principal reduction (treatment group) always consume more. Households who do not receive the reduction (control group) consume 0.1% less than they do in the benchmark economy, when they own the same sized house in both. Surprisingly, consumption rises by more for households that consume more housing, both after the principal reduction and the tax rebate. This is driven by households who buy houses early at lower prices (before $t = 20$), in the intervention economies, and households who sell houses later at higher prices.

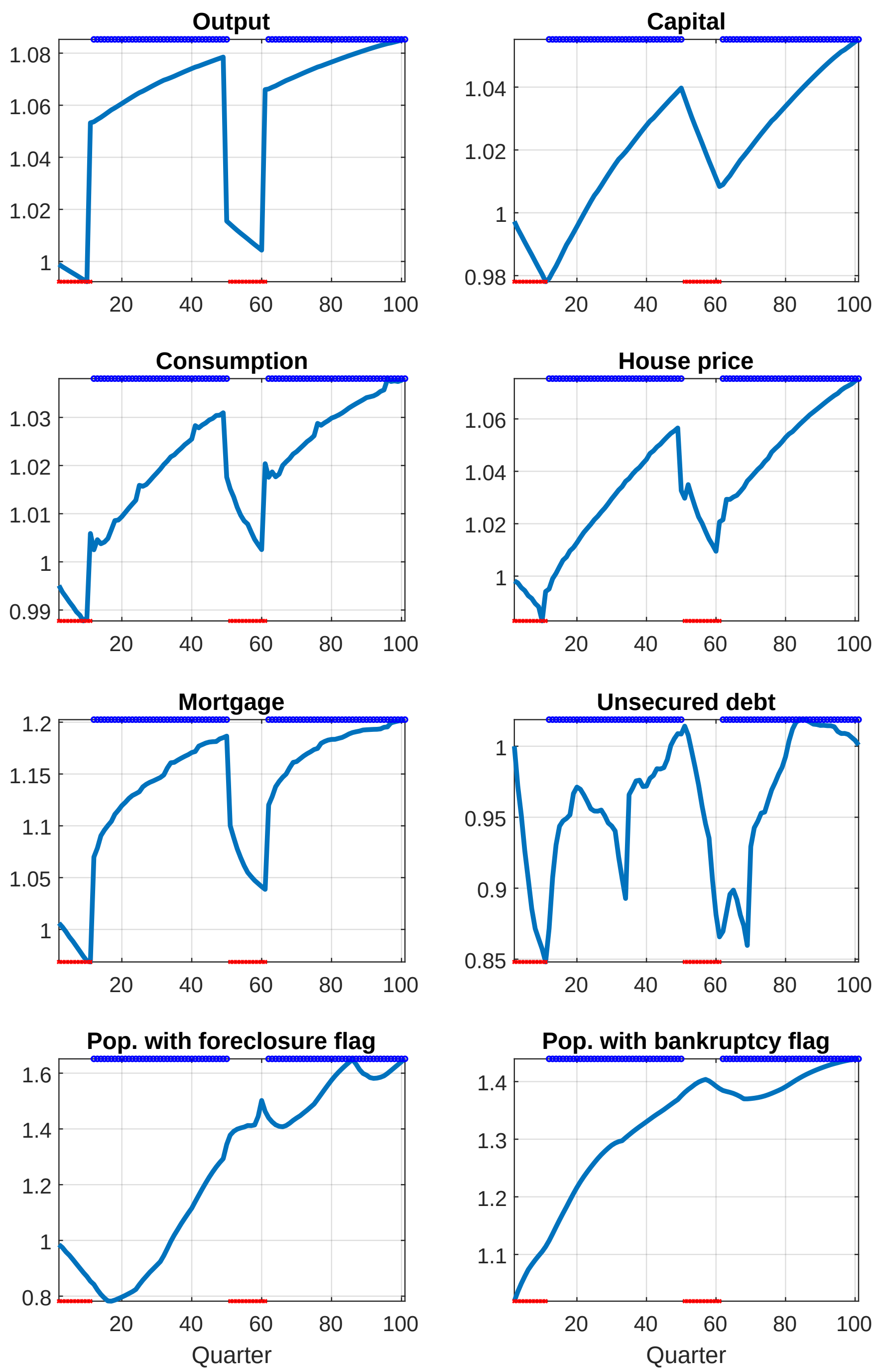
H Additional tables and figures

Table 13: Portfolio composition

	Asset		**Debt**	
	Non-financial	Financial	Secured	Unsecured
Data				
Q1	-524.62	-112.52	581.57	155.57
Q2	257.43	52.01	-196.31	-13.14
Q3	147.40	36.29	-80.25	-3.44
Q4	98.34	39.16	-36.08	-1.42
Q5	54.03	56.56	-10.038	-0.21
Model				
Q1	-2394.12	-29.02	2503.66	19.48
Q2	118.38	105.04	-123.05	-0.37
Q3	87.99	103.12	-90.40	-0.71
Q4	72.57	81.22	-50.49	-3.30
Q5	40.35	85.01	-25.34	-0.02

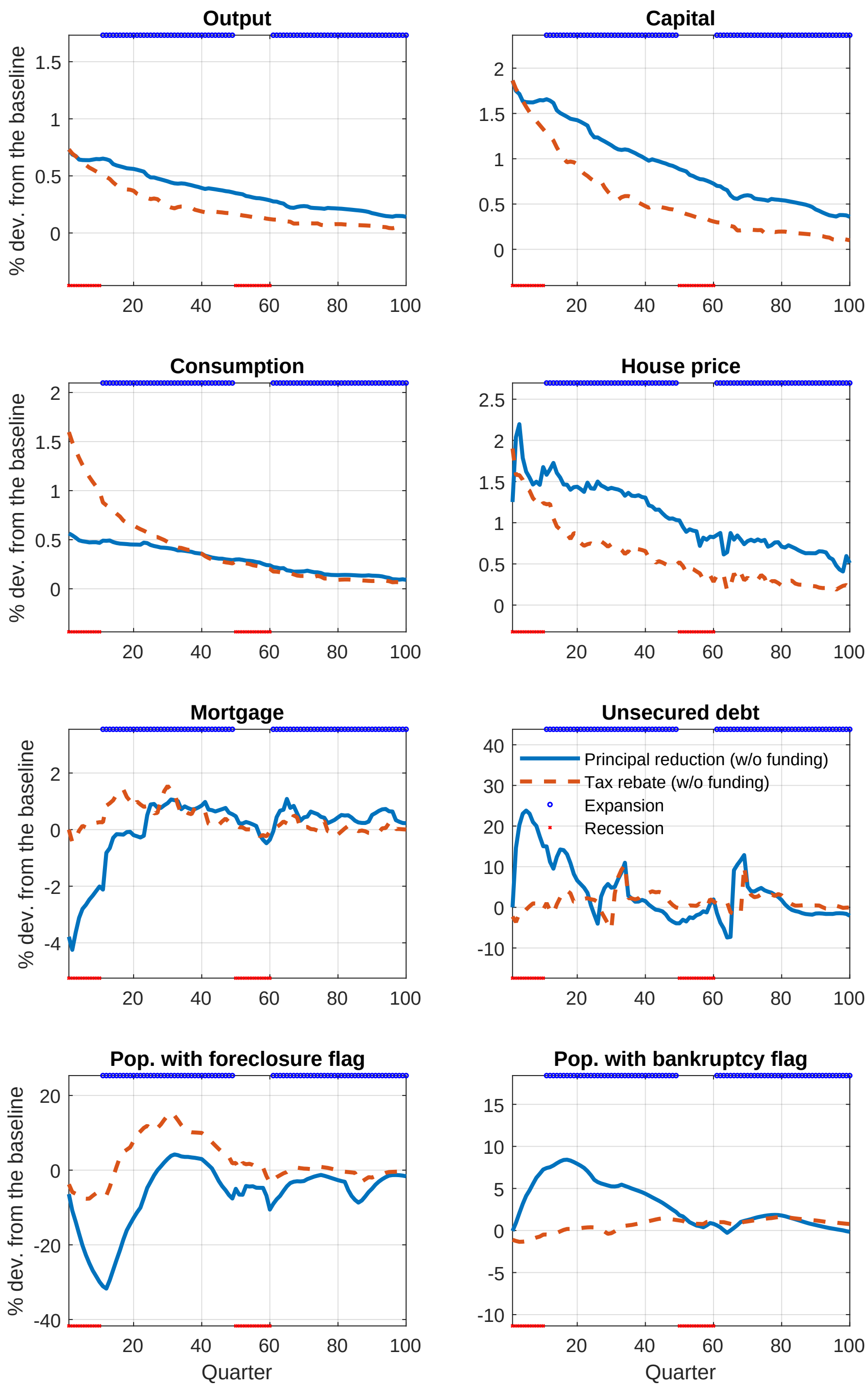
Note: Average portfolio composition by net worth quintiles. Non-financial asset includes 'Housing and cars', 'Business and non-financial assets' in the SCF. Here, business assets and vehicles are excluded from non-financial assets. Installment loans are excluded. After excluding installment loans, credit card loans are considered as unsecured debt and the remaining compositions of debt are assigned to secured debt. Data: SCF(2007)

Figure 17: Aggregate variables



Note: Aggregate variable movements in the baseline economy without policy intervention. All series are normalized to 1 in the first period.

Figure 18: Aggregate responses without funding



Note: Policy intervention with funding means that the government pays the intervention by borrowing at the time of intervention and pays the cost by raising taxes during expansions. Therefore households expect a higher tax during expansions. The intervention without funding does not involve the government borrowing nor shifts in households' expectations.

Figure 19: Consumption deviation from benchmark after principal reduction

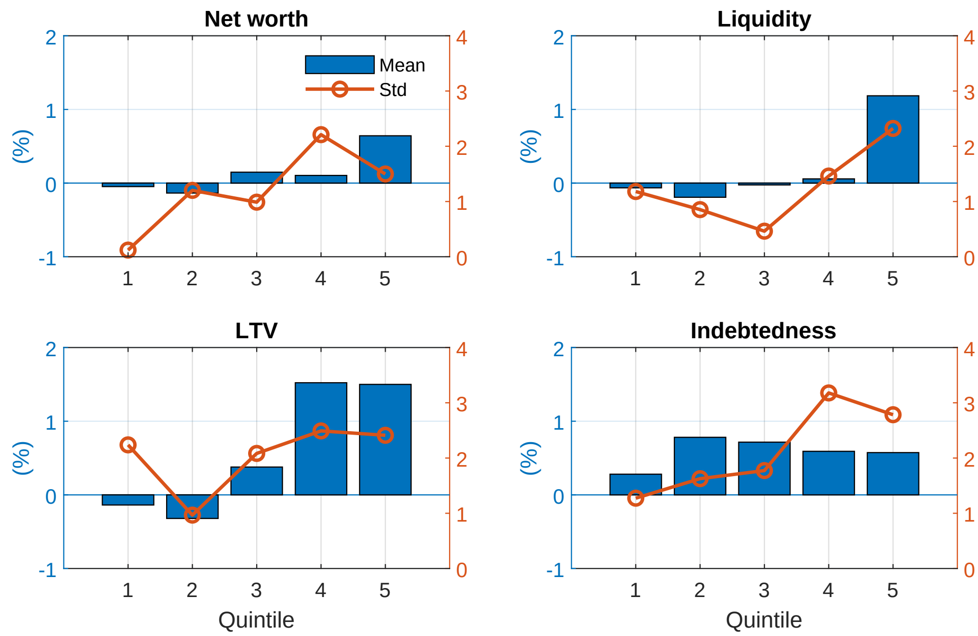
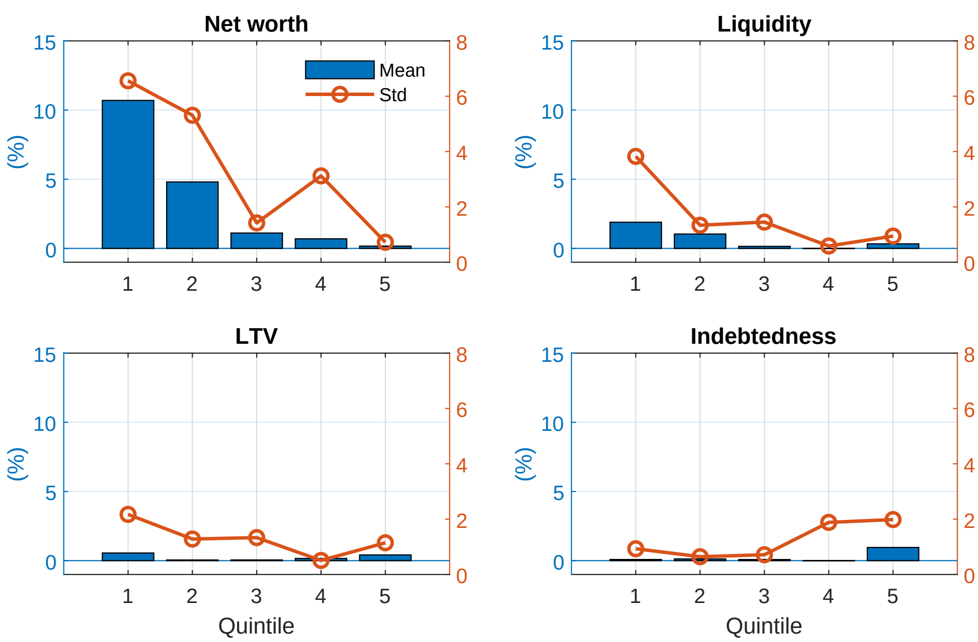
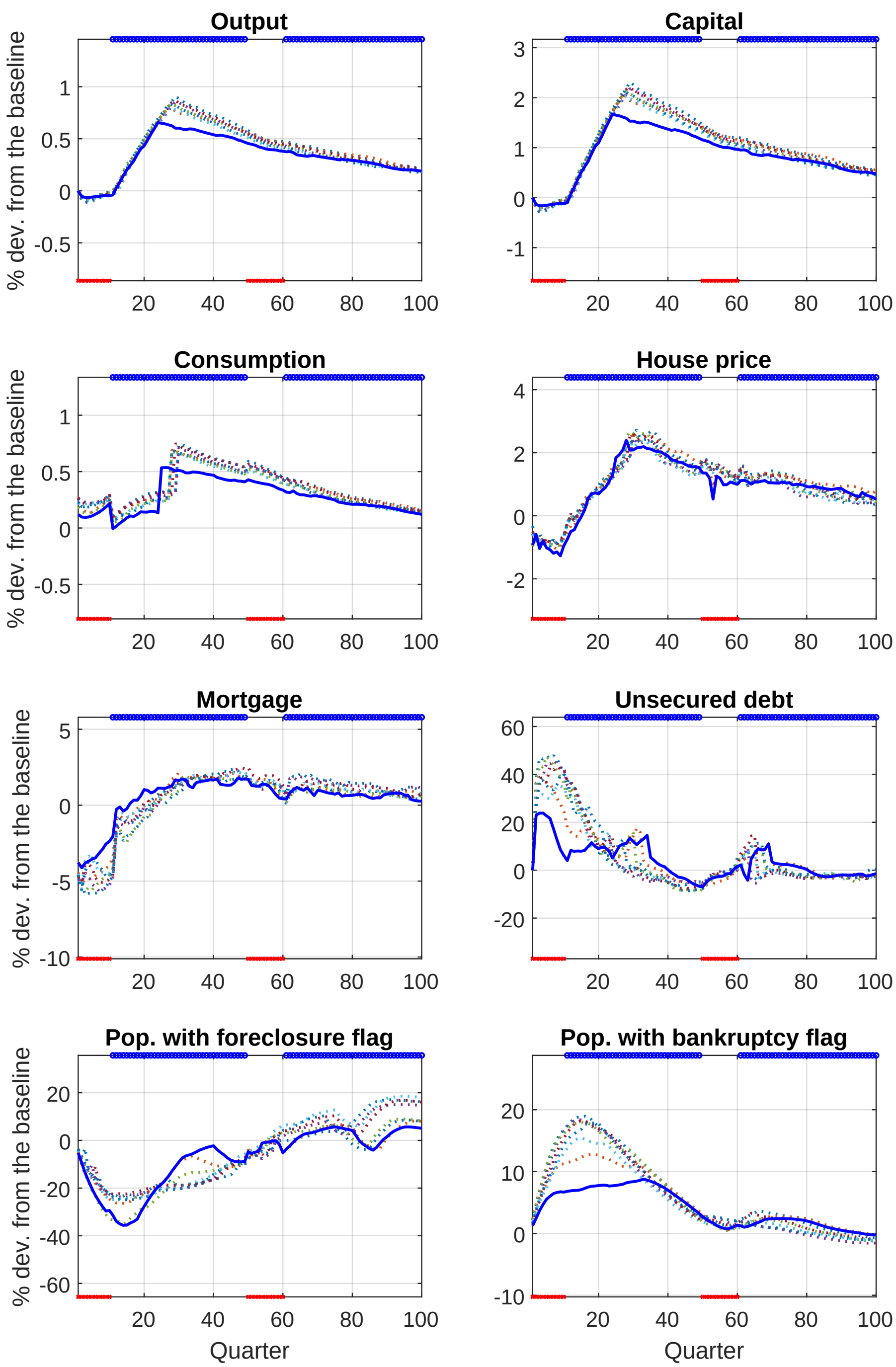


Figure 20: Consumption deviation from benchmark after tax rebate



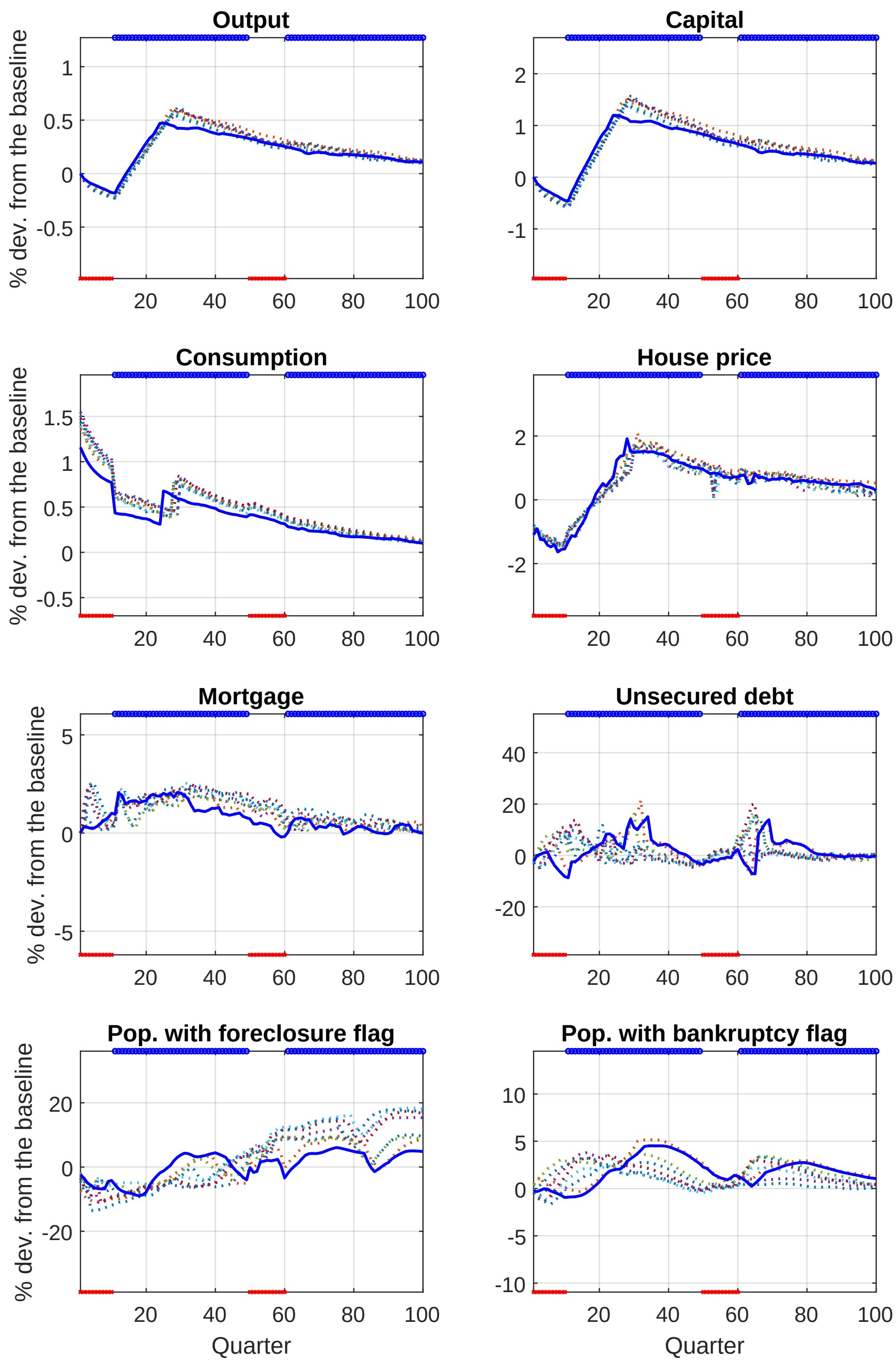
Note: The consumption deviation is computed using simulated data and the sample size is 18,000. The consumption deviation is $\frac{\text{c(policy) - c(no policy)}}{\text{c(no policy)}}$. Liquidity is a ratio of liquid savings over net worth. Indebtedness is total debt over net worth. For Liquidity panel, households without houses are dropped. For LTV panel, households without mortgages are dropped. For Indebtedness panel, households without debt are dropped. Each bar is the average consumption deviations of households and the line shows the standard deviation of consumption deviations that are in the quintile.

Figure 21: Aggregate variables: Principal reduction



Note: Dashed lines show responses of aggregate variables with different initial distributions. The solid line shows the series in the Figure 4 and 6.

Figure 22: Aggregate variables: Tax rebate



Note: Dashed lines show responses of aggregate variables with different initial distributions. The solid line shows the series in the Figure 4 and 6.

Table 14: Portfolio composition

	Asset		Debt	
	Non-financial	Financial	Secured	Unsecured
Q1	-377.09	-56.47	487.11	46.45
Q2	244.48	40.57	-174.85	-10.19
Q3	147.58	32.55	-77.05	-3.07
Q4	98.23	37.48	-34.48	-1.23
Q5	67.45	40.79	-8.07	-0.16

Note: Average portfolio composition by net worth quintiles. Credit card loans are considered as unsecured debt and the remaining compositions of debt are assigned to secured debt. Data: SCF(2007)

Table 15: Cyclical properties

	Bankruptcy only		**Foreclosure only**		**None**	
	std(%)	corr. with Y	std(%)	corr. with Y	std(%)	corr. with Y
Output	1.8	1.0	1.8	1.0	1.8	1.0
Consumption	0.6	0.9	0.5	0.8	0.5	0.8
Investment	5.2	0.9	5.8	0.9	5.6	0.9
Unsecured debt	71.0	0.0	-	-	-	-
Mortgage	3.0	0.7	2.7	0.8	2.5	0.7
Bankruptcy	4.4	0.6	-	-	-	-
Foreclosure	-	-	17.0	0.4	-	-
House price	2.9	0.7	1.8	0.7	1.6	0.7

Bankruptcy only refers to an economy in which households cannot foreclose but can go bankrupt, and Foreclosure only refers to an economy with only foreclosure. None refers to an economy without any option to default. Y in ‘corr. with Y’ is output.