The Macroeconomic Effects of Debt Relief Policies during Recessions *

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November 16, 2019

Abstract

I study the aggregate and microeconomic effects of debt relief programs during recessions in a setting where households may default on their mortgages and enter into foreclosure, default on their credit card debt and enter into bankruptcy, neither, or both. Mine is the first dynamic stochastic general equilibrium model accommodating uninsurable income risk, unsecured debt or financial assets, mortgages, houses, bankruptcy, and foreclosure with aggregate risks. With this unique theoretical laboratory accounting for equilibrium price movements, I explore how one form of household debt forgiveness affects another, how households of differing asset positions are affected, as well as the consequences for aggregate series such as GDP and investment.

I find that a mortgage principal reduction program targeting loan-to-value ratios among highly leveraged borrowers delivers significant and persistent increases in aggregate consumption, investment, and output during a recession. The program not only reduces foreclosures but also, to a lesser extent, bankruptcy filings. It dampens the decline in house prices and stimulates capital accumulation, driving lower interest rates. The initial rise in house price has lasting effects by preventing subsequent foreclosures and loosening financial constraints; since a mortgage is subject to loan-to-value ratio limit constraint, which depends on a house price, the rise in house prices effectively ease the constraint. While the program initially benefits only households that receive the reduction, these equilibrium price implications spread the gains with time, especially to households with low net worth and high leverage.

*I indebted to my advisor, Aubhik Khan for his guidance and support. I am also grateful to Julia Thomas and Kyle Dempsey for their comments and suggestions. I thank Garth Baughman, Niklas Engbom, Mokoto Nakajima, Ludwig Straub, Corina Boar, Kyle Herkenhoff and Ben Casner for helpful comments. I thank Rohan P. Shah for his generous help.

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1. Introduction

Since the Great Recession, there have been many studies on the relationship between household debt and the macroeconomy.[1] While many questions surrounding this event remain unanswered, there is a growing consensus that high leverage at the start of the recessions exacerbated the severity of the downturn.

During a recession, and particularly during a recession that is associated with high leverage, debt relief programs can have a large effect. For example, alleviating underwater borrowers' financial distress via mortgage forgiveness can prevent a rise in foreclosure and a fall in house prices.[2] Moreover, preventing a drastic fall in house prices can prevent subsequent foreclosures hence further support house prices. To date, however, there is little evidence on the macroeconomic effects of debt relief over the business cycle.

My goal is to assess the effects of debt relief programs during recessions quantitatively. I develop a dynamic stochastic general equilibrium model that goes a long way in capturing the rich heterogeneity in households' balance sheets. The model has three assets to be able to study the effects of mortgage-related programs, as a mortgage is the largest fraction of households' debt. Households can have liquid assets, which they can use to borrow or save, and can also hold illiquid assets in the form of houses and mortgages. These heterogeneous households face idiosyncratic risks and have limited opportunities to insure themselves because of incomplete financial markets, as in Huggett (1993) and Aiyagari (1994). They value consumption and service flow from the housing. Both forms of debt are subject to default, so household's choices include bankruptcy on unsecured debt and foreclosure on secured debt.

[1] See, for example Mian, Rao, and Sufi (2013), Mian and Sufi (2014), Guerrieri and Lorenzoni (2017), Jones, Midrigan, and Philippon (2011) and Auclert, Dobbie, and Goldsmith-Pinkham (2019), Verner and Gyongyosi (2018)

[2] Mian, Sufi, and Trebbi (2015) show that foreclosures led to a large decline in house prices, residential investment, and consumer demand from 2007 to 2009.

The model differs from existing quantitative macro models with default in two ways. First, my model includes aggregate business cycle risk in addition to idiosyncratic income risk and allows defaults on unsecured and secured debt separately. These are essential elements to address my question. To investigate the role of debt relief as a macroeconomic policy tool to mitigate the severity of downturns, the model needs to have aggregate risks. It is crucial to allow interactions between the policy with existing debt forgiveness programs, consumer bankruptcy and foreclosure, to assess the effects of a policy properly. One of the main experiments is one-time mortgage forgiveness. If bankruptcy is not allowed, it hinders an underwater borrower's ability to smooth consumption by using unsecured credit or relieving financial distress by using the bankruptcy system. As a result, the effects of mortgage forgiveness can be overestimated.

Second, all prices - interest rates, wages, house prices and loan rates - are determined endogenously. My focus is to understand the general equilibrium effects of debt relief programs on macroeconomic outcomes as well as its impacts on individual households. How do the initial responses of households' to policy interventions impact prices? How do the changes in price feedback to households decision and shape paths of aggregate variables? Any answer to these questions would be incomplete without endogenous prices.

The model is calibrated to match key cross-sectional features of the U.S.economy in the early 2010s. The model successfully produces household wealth distribution, not only net worth but also its components; financial assets, housing wealth and mortgage. Additionally, the model captures the key cyclical properties of the U.S. data.

Before examining the dynamics of the economy over a business cycle, I explore the role of default options and how these options interact with the general equilibrium outcomes in the steady states. Specifically, I investigate the welfare implications of the default options by comparing i) the full model with bankruptcy and foreclosure, ii) a model in which households cannot foreclose but can go bankrupt iii) a model with the only foreclosure is available to households. The welfare gains of default options are unevenly distributed over households. Intuitively, those who have low income and wealth value the bankruptcy option most as they are more likely to use it. Similarly, we may expect that the foreclosure option will be used by households that have large mortgages (as well as a large house). However, households that have large houses and mortgages do not value the foreclosure option. This is due to general equilibrium effects. Specifically, in the economy with the possibility of foreclosure, the equilibrium housing price is higher than in an economy without a foreclosure option.[3] Therefore in order to households to afford the same size of house when foreclosure is available, households tend to have larger loans requiring a larger debt repayment. They also have higher maintenance costs. These higher costs reduce a household's budget for non-durable consumption, thereby making the foreclosure less desirable. This exercise illustrates that the effects of a policy will impact heterogeneous households differently, and the general equilibrium effects may have unanticipated implications.

I investigate the effects of debt relief programs using policy experiments. First, I design a policy intervention in which all households with loan-to-value ratios above 95% have their LTV ratios reduced to this threshold via mortgage forgiveness.[4] The policy intervention affects about 26% of the population and the average amount of debt forgiven is equivalent to roughly 13,500 2010 dollars. This one-time debt forgiveness program generates a large and persistent effect on aggregate variables. Investment and consumption both increase under the intervention.[5] Wages are higher and inter-

[3]The possibility of foreclosure enables households with low wealth to take on larger loans than otherwise to buy houses. When foreclosure is not available, the loan-to-value ratio is constrained to ensure repayment. This prevents poorer households from entering the housing market. Hence, the demand for housing rises when foreclosure is available, which, in turn, leads to an equilibrium price rise for housing.

[4]Kaplan, Mitman, and Violante (2017) do a similar experiment. To the comparison, Kaplan et al. (2017) study house prices rise and fall around the Great Recession in an equilibrium housing model. However, they assume other prices (risk free rates and wages) in the model are exogenous and abstracting from bankruptcy while allowing foreclosure.

[5]The aggregate consumption responsiveness in the period of intervention is 4%. Given that 64% of

est rates are lower thanks to the rise in investment, and house prices are also higher. Looking into consumption responses more closely at the time of the intervention, I find that households with low net worth, low liquidity and high leverage respond most to the principal reduction. Households consume as much as 40% of the value transferred to them by the intervention. Since they are near hand-to-mouth, and mortgage forgiveness does not provide liquidity, a significant consumption response may seem puzzling. It turns out that these households increased consumption using refinancing. Approximately 23% of households that received the principal reduction convert increased home equity into liquid wealth despite the refinancing cost and higher future interest rate payment.

While the consumption responses to the principal reduction are initially concentrated in the households who get the reduction, the intervention affects all households over time through general equilibrium effects. In particular, the initial rise in house price has lasting effects by preventing subsequent foreclosures and loosening financial constraints; since a mortgage is subject to loan-to-value ratio limit constraint, which depends on a house price, the rise in house prices effectively ease the constraint. The lower foreclosure rates and the eased borrowing limits imply less drop in housing demand, which, in turn, prevents further fall in house prices. I find that the higher levels of consumption and lower levels of foreclosure 7-8 years after the intervention are mainly driven by households that sell their houses instead of defaulting or by households that sell in both scenarios being able to sell at a higher price with the intervention. In terms of distributional effects, the price changes induced by the intervention benefit indebted households that have small savings. They are the households that were able to increase consumption by selling houses at higher prices. In contrast, households that have large houses, substantial savings, and small debt consume less due to the price changes. Higher house prices reduce their disposable income through

the population does not receive the principal reduction and hence does not adjust consumption, this is a sizable impact.

increased maintenance costs and lower interest rates lower the returns on their financial wealth.

Next, I compare the effects of the principal reduction against the effects of a tax rebate of the type often implemented during recessions. It can be useful to understand the effects of these two very different implementations to evaluate their cost effectiveness. For the purpose, the size of the tax rebate is set so the total cost of the rebate is the same cost of the debt forgiveness program. The difference is that the tax rebate is allocated equally to all households and provides liquidity to households. In contrast, debt forgiveness applies only to a subset of households and does not directly increase liquidity. I find that the tax rebate is more effective in boosting consumption. Partly it is because the rebate is added to households' liquid wealth, and it distributes relatively large amounts of money to high MPC households. The tax rebate is also more effective in reducing bankruptcy and unsecured borrowing by providing income to poor households without a house or mortgage. The principal reduction is more effective in reducing foreclosure and supporting house prices by targeting households that likely sell houses or foreclose and providing additional wealth for them. In both policy experiments, the responses are primarily driven by households that have low net worth and high leverage. A government that aims at boosting consumption, reducing foreclosure and supporting falling house prices should recognize that targeting highly leveraged households with a low net worth will yield stronger responses.

I conclude by conducting two additional policy counterfactuals. First, I investigate the effects of lenient bankruptcy. Given that the bankruptcy system provides insurance to financially distressed households, it is plausible that the government can mitigate a fall in consumption and output during recessions by adjusting leniency of the bankruptcy system. (see Auclert et al. (2019) for a related argument) I assume that the government loosens the requirements for bankruptcy (e.g. bankruptcy filing costs) so that more households can use the system. As a result, the bankruptcy rate rises by nearly 100% and unsecured debt falls almost 40%. Consistent with the evidence in Dobbie and Song (2016), lenient bankruptcy affects households' foreclosure decisions too. As households substitute foreclosure with bankruptcy, it is as effective as the principal reduction in reducing foreclosure. It also has lasting impacts on investment, consumption and house prices.

Second, I study the effects of mortgage payment reduction. A few empirical studies have shown (e.g. Ganong and Noel (2018) and Indarte (2019)) that a mortgage principal reduction is less effective than mortgage payment reduction in reducing default rates. While these results are not directly comparable to my results due to differences in identification assumptions, it is worth studying the effects of payment reduction in my model environment. I adjust per period repayment, so payment does not exceed 31% of a household's labor income for 16 quarters. By increasing eligible households' disposable income, aggregate consumption is slightly higher (0.01-0.07%) and foreclosure rates are slightly lower (0.01-0.05%) during the intervention periods. However, after the intervention is over, foreclosure rates rise and house prices fall. Because of the slow repayment, eligible households' leverage remains relatively higher than the economy without the intervention. The higher leverage leads to a rising foreclosure after the intervention. Rising foreclosure contributes to falling house prices, which induce more foreclosure in subsequent periods.

Literature

This paper is related to multiple strands of the literature on household credit and default. First, my paper is related to recent works that estimate the effects of debt relief programs during the Great Recession. Agarwal, Amromin, Ben-David, Chomsisengphet, Piskorski, and Seru (2017) show that although the HAMP participation rate was low, the program reduced foreclosure and increased spending. Ganong and Noel (2018) separate the impact of reducing mortgage balance and reducing short-term payments and show that while short-term payments are effective in reducing defaults, mortgage balance reductions are not. Piskorski and Seru (2018) show that alleviating frictions affecting the pass-through of lower interest rates and debt relief (e.g. refinancing, loan renegotiation) could have reduced the relative foreclosure rate by more than half and resulted in up to twice as fast recovery of house prices, consumption, and employment. I complement these works by using a structural model and explicitly accounting general equilibrium effects.

My paper is also related to papers that study debt relief as a macroprudential policy tool. There have been many works that show how high leverage of households can exacerbate an economic downturn since the Great Recession.[6] Motivated by this literature, Korinek and Simsek (2016) show that policies aim to reduce leverage can be welfare improving when an economy is in a liquidity trap. Auclert et al. (2019) show that debt forgiveness provided by the U.S. consumer bankruptcy system increases consumption and the increased consumption helps to stabilize employment. Both papers assume an environment with nominal rigidity. I complement these works by showing that debt relief programs can have lasting impacts through financial amplification. A policy intervention can support house price, which loosens price-dependent financial constraints (LTV) and prevents subsequent foreclosures.

In terms of modeling, my work is built on the literature on housing, mortgages, and foreclosure.[7] My model shares a similar set of available assets for households but also allows for default on unsecured debt.[8] To my knowledge, Mitman (2016) is the only paper that incorporates interactions between bankruptcy and foreclosure. He explores the importance of such interaction and assesses the effect of bankruptcy reform

[6]Mian et al. (2013), Mian and Sufi (2014), Guerrieri and Lorenzoni (2017), Jones et al. (2011), and Verner and Gyongyosi (2018)

[7]Jeske, Krueger, and Mitman (2013), Corbae and Quintin (2015), Hatchondo, Martinez, and Sánchez (2015), Chatterjee and Eyigungor (2015), Kaplan et al. (2017)

[8]A large body of works study consumer bankruptcy. See, among others, Athreya (2002), Li and Sarte (2006), Livshits, MacGee, and Tertilt (2007), Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007), Nakajima and Ríos-Rull (2014)

(Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA)). I improve on that model by adding aggregate risk and study the role of debt relief policies during recessions.

The rest of the paper is organized as follows. In Section 2, I describe the model economy. Section 3 explains the calibration procedure and presents calibration results. I discuss the results in section 4 and section 5. Section 6 concludes.

2. Model

The economy consists of a continuum of infinitely-lived households, banks, non-durable goods producers and a government. Households are indexed by their holdings of liquid assets a, illiquid assets h, secured debt b and their idiosyncratic labor productivity ε. There are a large number of identical firms in both the banking and durable goods sectors, so all markets are perfectly competitive. The government collects tax from households, absorbs gains and loss of financial intermediaries and sets fiscal policy. The aggregate state of the economy are g - a population distribution for the vector (a, b, ε, h) - and z, the aggregate total factor productivity. Time is continuous.

2.1. *Households*

2.1.1. *Household's Environment*

Labor productivity Each households labor productivity follows a Poisson process. With frequency λ_ε, households receive a labor productivity shock and draw a new productivity from a distribution. These shocks arrive independently for each household, so one household's productivity is independent of all the others'.

Liquid assets Households can save or borrow using a liquid asset, a. When a is negative households have an option to default. When a household chooses to default, the debt is forgiven but the household will have the bankruptcy record on her credit history and pays an utility cost, ξ_a. The possibility of default of households means that the price of unsecured borrowing, $r_a(a, b, \varepsilon, h, g, z)$, depends on individual and aggregate states.

Iliquid assets A second asset, h is illiquid in that households must pay an adjustment cost when they buy or sell it. The illiquid asset represents durable commodities such as houses and cars. The choice of h is from a discrete set. Households must choose their level of h off a finite grid of value. The durable good provides utility flows to households and incurs maintenance costs. One component of these maintenance costs is a property tax, which households can use to partially offset their other tax payments. I assume the total supply of this asset is fixed at $\overline{H}$. The price of this asset, $p(g, z)$, is determined in equilibrium.

Secured debt Durable goods purchases can be funded using secured debt, b which is distinct from unsecured debt. It can be interpreted as a mortgage and I will use mortgage and secured debt interchangeably. This debt is refinancible, long-term, secured, and defaultable. Households can borrow b when they buy illiquid assets, using their illiquid assets as collateral. When choosing b, households are constrained by a loan-to-value (LTV) ratio limit: the choice of b must be less than a fraction (γ) of the collateral value.

The loan is discounted by its discount rate $q(a', b', \varepsilon, h, g, z)$ at the time of origination. This loan discount rate depends on their choice of b as well as individual and aggregate states capturing default risk.

Households can refinance the loan by paying a fixed cost ξ_r. When refinancing the loan, households first pay back the remaining balance of the current loan and then take out a new loan. Therefore the maximum size of the loan is restricted to LTV limit and it will be discounted with $q(a', b', \varepsilon, h, g, z)$. All secured debt interest rate is adjustable rate, so refinancing would be used to extract equity or prepay the remaining balance.[9]

While a household is holding b, it is required to pay a loan interest rate, as well as a fraction of the principal at each instant.[10] However, because b is long-term debt, there is no requirement that the size of this loan remains less than γ times the current value of non-financial asset, h. If the price of the illiquid asset decreases, a household could find itself with negative equity. However, as long as the household pays off the required amount of the outstanding balance of the loan at the moment, $(\theta(b, ph) + r)b$, the household is not forced to default or refinance.[11]

When a household chooses to default, the remaining balance of the debt is forgiven and a financial intermediary takes over h and liquidates it. I assume the financial intermediary is not subject to any adjustment costs when selling h, but it has an inefficient technology such that the foreclosure sale value is $(1 - \delta_h)ph$. The household will have the foreclosure record on their credit history, which excludes them from h transaction.[12] The household also incurs a utility cost, ξ_b.

The bankruptcy flag While the bankruptcy record remains on a household's credit history, some fraction ξ_x of the household's labor income is garnished. Also, unsecured borrowing, refinancing or purchasing of h are not allowed. However, bankrupt households' non-financial assets are fully protected and they can enter foreclosure if they have b. If a household chooses to do so, the remaining b will be forgiven and their non-

[9] If secured interest rate is a fixed rate, households have incentives to refinance when interest rate falls, to take advantage of low rates.

[10] Loan interest rate is equal to return on saving plus a premium reflecting the unit cost of lending, $\iota(z)$

[11] Households pay a fraction of h value at each period. $\theta(b, ph) = \overline{\theta} ph/b$.

[12] For simplicity, I assume that households cannot buy h even if a purchase is self-financed.

financial asset will be confiscated. When $h = 0$, there is no income garnishment.[13]

For tractability, I assume the bankruptcy flag is removed stochastically with intensity λ_d.

The foreclosure flag A household is unable to purchase a house while its credit history is affected by a foreclosure flag. Since they cannot buy h such households are excluded from taking mortgages or refinancing. However, such households can still take on unsecured debt is allowed and households can choose to default on any such unsecured debt they already have. If these households go bankrupt, the foreclosure flag will be replaced by a bankruptcy flag. For tractability, I assume the foreclosure flag is removed stochastically with intensity λ_f.

Preferences Households receive utility flow from consuming non-durable goods and from consuming a service flow from their illiquid assets. Their utility function is

$$u(c,h) = \frac{c^{1-\sigma}-1}{1-\sigma} + (\kappa h)^{\sigma_h}$$

The function u is strictly increasing and strictly concave in c and h.

2.1.2. Household Problem

Households with no flags

When not subject to flags, a household's problem is given by

$$v(a_t,b_t,\varepsilon_t,h_t) = \max_{\{c_t\},\tau} \mathbf{E_0} \int_0^{\tau} e^{-\rho t} u(c_t,h_t)dt + \mathbf{E_0} e^{-\rho\tau} v^*(a_\tau,b_\tau,\varepsilon_\tau,h_\tau) \tag{1}$$

$$\dot{a}_t = w_t\varepsilon_t + r_{at}(a,b,\varepsilon,h)a_t - (r_t + \theta(b,\overline{p}h))b_t - c_t - T_t(b,\varepsilon,ph) - \xi_h p_t h_t \tag{2}$$

[13]All households with zero h and bankruptcy flag are either the households who did not own h before the bankruptcy or lost the asset due to the foreclosure.

$$\dot{b}_t = \theta(b, \overline{p}h)b_t \tag{3}$$

$$(a_0, b_0, \varepsilon_0, h_0) = (a, b, \varepsilon, h) \tag{4}$$

Households choose non-durable consumption $\{c_t\}$ and the optimal stopping time τ. Stopping means a households choose to make a choice that brings about a large shift in their asset position or credit history when such a shift is desirable. For example, a household starts to consider buying a house when it has some saving, and buys a house when it reaches a certain threshold in saving. Such a choice - buying a house - is an example of stopping. When a household stops, it does one of the following: i) h size adjustment ii) refinance, iii) default on a, iv) default on b, or v) default on a and b. Households that have secured debt repay it at a rate which is a fraction $\theta(b, \overline{p}h)$ of their house value that is evaluated with the price $\overline{p}$. $T(\cdot)$ is a tax that depends on taxable income and $\xi_h ph$ is a maintenance cost of h.

The Hamilton-Jacobi-Bellman (HJB) equation before stopping is,

$$\begin{aligned} \rho v(a,b,\varepsilon,h,g,z) = &\max_c u(c,h) + \partial_a v(a,b,\varepsilon,h,g,z)\dot{a} + \partial_b v(a,b,\varepsilon,h,g,z)\dot{b} \\ &+ \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v(a,b,\varepsilon_j,h,g,z) + \sum_{k=1}^{n_z} \lambda_{zz_k} v(a,b,\varepsilon,h,g,z_k) \\ &+ \int \frac{\delta v(a,b,\varepsilon,h,g,z)}{\delta g(a,b,\varepsilon,h)} \mathcal{K} g(a,b,\varepsilon,h) d[a \times b \times \varepsilon \times h] \end{aligned} \tag{5}$$

$$\dot{a} = w(g,z)\varepsilon + r_a(a,b,\varepsilon,h,g,z)a - (r(g,z) + \iota(z) + \theta(b,\overline{p}h))b - c - \xi_h p(g,z)h - T(b,\varepsilon,p(g,z)h)$$

$$\dot{b} = \theta(b,\overline{p}h)b$$

$$v(a,b,\varepsilon,h,g,z) \geq v^*(a,b,\varepsilon,h,g,z).$$

$\lambda_{\varepsilon\varepsilon_j}$ describes the labor productivity process.[14] Aggregate productivity follows stochas-

[14] $\lambda_{\varepsilon_i\varepsilon_j} < 0$ when $\varepsilon_i = \varepsilon_j$ which is the intensity of losing the current level of labor productivity. $\lambda_{\varepsilon_i\varepsilon_j} > 0$ when $\varepsilon_i \neq \varepsilon_j$, it is the intensity of jumping to ε_j from ε_i. $\sum_j \lambda_{\varepsilon_i\varepsilon_j} = 0 \quad \forall i$.

tic process described by λ_{zz_k}. $\mathcal{K}$ is a Kolmogorov Forward operator that operates on distributions g_t which evolves based on shocks and households decisions.[15]

$$\frac{dg_t(a,b,\varepsilon,h)}{dt} = \mathcal{K}g_t(a,b,\varepsilon,h)$$

Stopping values

The stopping value $v^*(a_\tau, b_\tau, \varepsilon, h_\tau, g, z)$ is the maximum of the following values.

1. h size adjustment

$$v^m(a_\tau, b_\tau, \varepsilon, h_\tau, g, z) = \max_{h',b'} v(a', b', \varepsilon, h', g, z)$$

$$a' = a_\tau - b_\tau + p(g,z)h_\tau - p(g,z)h' - \xi(p(g,z), h_\tau, h') - q(a', b', \varepsilon, h', g, z)b'$$

$$b' \leq \gamma p(g,z)h'$$

When changing the size of h, households choose the optimal size (h') and the amount of the secured loan (b'). The remaining balance that is attached to the current h has to be repaid. $\xi(\cdot)$ is the transaction cost and is given by

$$\xi(p(g,z), h_\tau, h') = p(g,z)\xi_0(h_\tau + h') + \xi_1 p(g,z)(|h_\tau - h'|)^2$$

where γ is the LTV limit as a proportion of collateral.

2. Refinancing b

$$v^r(a_\tau, b_\tau, \varepsilon, h_\tau, g, z) = \max_{b'} v(a', b', \varepsilon, h_\tau, g, z)$$

[15] Ahn, Kaplan, Moll, Winberry, and Wolf (2018) describe the recursive formulation of an model with an aggregate uncertainty using the Kolmogorov Forward operator.

$$a' = a_\tau - b_\tau + q(a', b', \varepsilon, h, g, z)b' - \xi_r$$

$$b' \leq \gamma p(g, z) h_\tau$$

Households that hold mortgage b have the option to refinance by repaying the residual principal balance b_τ and a fixed cost ξ_r. They do not change their house size but simply originate a new loan, b', which is subject to the LTV limit, given by $b' \leq \gamma ph$.

3. Default on a

$$v^a(a_\tau, b_\tau, \varepsilon, h, g, z) = v^d(0, b, \varepsilon, h, g, z) - \xi_a$$

This is bankruptcy on unsecured borrowing, a. If a household choose to default only on a, its house and mortgage are not affected by the decision. The stopping values of defaults are evaluated with $v^d(a, b, \varepsilon, h, g, z)$ which is a value of households with the bankruptcy flag in their credit history, and ξ_a is the utility cost associated with default.

4. Default on b

$$v^b(a_\tau, b_\tau, \varepsilon, h, g, z) = v^f(a_\tau, \varepsilon, g, z) - \xi_b$$

This is foreclosure. If a household choose to foreclose, her remaining debt b is forgiven and a financial intermediary takes over h. $v^f(a, \varepsilon, g, z)$ is a value of households with the foreclosure flag in their credit history. ξ_b is the utility cost.

5. Default on both a and b

$$v^{ab}(a_\tau, b_\tau, \varepsilon, h, g, z) = v^d(0, 0, \varepsilon, 0, g, z) - \xi_a - \xi_b$$

If a household choose to default on both types of debt, her remaining debt a and b are forgiven and a financial intermediary takes over h. Households are not allowed to use any type of credit while they have bankruptcy flag in their credit history without houses.

The stopping value v^* is

$$v^*(a_\tau, b_\tau, \varepsilon_\tau, h, g, z) = \max\{v^m, v^r, v^a, v^b, v^{ab}\}.$$

Thus households can choose among the available stopping options; adjusting h size, refinancing and defaults.

Households with bankruptcy flag

A household with the bankruptcy flag in their credit history solves the following problem.

$$v^d(a_t, b_t, \varepsilon_t, h_t) = \max_{\{c_t\},\tau} \mathbf{E_0} \int_0^\tau e^{-\rho t} u(c_t, h_t) dt + \mathbf{E_0} e^{-\rho\tau} v^{d*}(a_\tau, b_\tau, \varepsilon_\tau, h_\tau) \tag{6}$$

$$\dot{a}_t = (1 - \xi_x \mathbf{1}_{h>0}) w_t \varepsilon_t + r_{at}(a, b, \varepsilon, h) a_t - (r_t + \theta(b, \overline{p}h)) b_t - c_t - T_t(b, \varepsilon, ph) - \xi_h p_t h_t \tag{7}$$

$$\dot{b}_t = \theta(b, \overline{p}h) b_t \tag{8}$$

$$a_t \geq 0 \tag{9}$$

$$(a_0, b_0, \varepsilon_0, h_0) = (a, b, \varepsilon, h) \tag{10}$$

Such households also choose non-durable consumption $\{c_t\}$ and the optimal stopping time τ. However only foreclosure and selling illiquid assets are available as stopping options. A fraction ξ_x of labor income is taken away if households hold illiquid assets after the bankruptcy filing. Bankrupt households cannot take on unsecured debt.

The HJB equation before stopping is,

$$\rho v^d(a,b,\varepsilon,h,g,z) = \max_c u(c,h) + \partial_a v^d(a,b,\varepsilon,h,g,z)\dot{a} + \partial_b v^d(a,b,\varepsilon,h,g,z)\dot{b} + \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v^d(a,b,\varepsilon_j,h,g,z)$$

$$+ \lambda_d(v(a,b,\varepsilon,h,g,z) - v^d(a,b,\varepsilon,h,g,z)) + \sum_{k=1}^{n_z} \lambda_{zz_k} v^d(a,b,\varepsilon,h,g,z_k)$$

$$+ \int \frac{\delta v^d(a,b,\varepsilon,h,g,z)}{\delta g(a,b,\varepsilon,h)} \mathcal{K} g(a,b,\varepsilon,h) d[a \times b \times \varepsilon \times h]$$

$$\dot{a} = (1-\xi_x \mathbf{1}_{h>0}) w(g,z)\varepsilon + r_a(a,b,\varepsilon,h,g,z)a - (r(g,z)+\iota(z)+\theta(b,\overline{p}h))b - c - \xi_h p(g,z)h - T(b,\varepsilon,p(g,z)h)$$

$$\dot{b} = \theta(b,\overline{p}h)b$$

$$a \geq 0$$

$$v^d(a,b,\varepsilon,h,g,z) \geq v^{d*}(a,b,\varepsilon,h,g,z).$$

The stopping value is the maximum of

1. Default on a

$$v^{da}(a_\tau, b_\tau, \varepsilon, h_\tau, g, z) = v^d(a_\tau, 0, \varepsilon, 0, g, z) - \xi_b$$

2. Sell h

$$v^{dm}(a_\tau, b_\tau, \varepsilon, h_\tau, g, z) = v^d(a, 0, \varepsilon, 0)$$

$$a = a_\tau - b_\tau + p(g,z)h_\tau - \xi(p(g,z), h_\tau, 0)$$

$$v^{d*}(a,b,\varepsilon,h,g,z) = \max\{v^{da}, v^{dm}\}$$

Households with a foreclosure flag

Lastly, households with a foreclosure flag in their credit history solve the following problem.

$$v^f(a_t,\varepsilon_t) = \max_{\{c_t\},\tau} \mathbf{E_0} \int_0^\tau e^{-\rho t} u(c_t, 0)dt + \mathbf{E_0} e^{-\rho\tau} v^{d*}(a_\tau, \varepsilon_\tau) \tag{11}$$

$$\dot{a}_t = w_t \varepsilon_t + r_{at}(a,\varepsilon)a_t - c_t - T_t(b,\varepsilon,ph) \tag{12}$$

$$(a_0, \varepsilon_0) = (a, \varepsilon) \tag{13}$$

These households choose non-durable consumption $\{c_t\}$ and the optimal stopping time τ. However, the only stopping option is to default on a.

The HJB equation before stopping is,

$$\begin{aligned}\rho v^f(a,\varepsilon,g,z) = \max_c u(c,0) &+ \partial_a v^f(a,\varepsilon,g,z)\dot{a} + + \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v^f(a,\varepsilon_j,g,z) \\ &+ \lambda_f(v(a,0,\varepsilon,0,g,z) - v^f(a,\varepsilon,g,z)) + \sum_{k=1}^{n_z} \lambda_{zz_k} v^f(a,\varepsilon,g,z_k) \\ &+ \int \frac{\delta v^f(a,\varepsilon,g,z)}{\delta g(a,b,\varepsilon,h)} \mathcal{K} g(a,b,\varepsilon,h) d[a \times b \times \varepsilon \times h]\end{aligned}$$

$$\dot{a} = w(g,z)\varepsilon + r_a(a,\varepsilon,g,z)a - c - T(b,\varepsilon,p(g,z)h)$$

$$v^f(a,\varepsilon,g,z) \geq v^{f*}(a,\varepsilon,g,z).$$

The stopping value is,

$$v^{f*}(a_\tau,\varepsilon,g,z) = v^d(a_\tau,0,\varepsilon,0,g,z) - \xi_a$$

The households' problem can be compactly written as HJB variational inequality

(HJBVI).

Households with no flags

$$
\begin{aligned}
min[&\rho v(a,b,\varepsilon,h,g,z) - \max_c u(c,h) - \partial_a v(a,b,\varepsilon,h,g,z)\dot{a} - \partial_b v(a,b,\varepsilon,h,g,z)\dot{b} \\
&- \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v(a,b,\varepsilon_j,h,g,z) - \sum_{k=1}^{n_z} \lambda_{zz_k} v(a,b,\varepsilon,h,g,z_k) \\
&- \int \frac{\delta v(a,b,\varepsilon,h,g,z)}{\delta g(a,b,\varepsilon,h)} \mathcal{K} g(a,b,\varepsilon,h) d[a\times b\times\varepsilon\times h], \\
&v(a,b,\varepsilon,h,g,z) - v^*(a,b,\varepsilon,h,g,z)] = 0
\end{aligned}
$$

Households with a bankruptcy flag

$$
\begin{aligned}
min[&\rho v^d(a,b,\varepsilon,h,g,z) - \max_c u(c,h) - \partial_a v^d(a,b,\varepsilon,h,g,z)\dot{a} - \partial_b v^d(a,b,\varepsilon,h,g,z)\dot{b} \\
&- \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v^d(a,b,\varepsilon_j,h,g,z) - \lambda_d(v(a,b,\varepsilon,h,g,z) - v^d(a,b,\varepsilon,h,g,z)) \\
&- \sum_{k=1}^{n_z} \lambda_{zz_k} v^d(a,b,\varepsilon,h,g,z_k) - \int \frac{\delta v^d(a,b,\varepsilon,h,g,z)}{\delta g(a,b,\varepsilon,h)} \mathcal{K} g(a,b,\varepsilon,h) d[a\times b\times\varepsilon\times h], \\
&v^d(a,b,\varepsilon,h,g,z) - v^{d*}(a,b,\varepsilon,h,g,z)] = 0
\end{aligned}
$$

Households with a foreclosure flag

$$
\begin{aligned}
min[&\rho v^f(a,\varepsilon,g,z) - \max_c u(c,0) - \partial_a v^f(a,\varepsilon,g,z)\dot{a} - \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v^f(a,\varepsilon_j,g,z) \\
&- \lambda_f(v(a,0,\varepsilon,0,g,z) - v^f(a,\varepsilon,g,z)) \\
&- \sum_{k=1}^{n_z} \lambda_{zz_k} v^f(a,\varepsilon,g,z_k) - \int \frac{\delta v^f(a,\varepsilon,g,z)}{\delta g(a,b,\varepsilon,h)} \mathcal{K} g(a,b,\varepsilon,h) d[a\times b\times\varepsilon\times h], \\
&v^f(a,\varepsilon,g,z) - v^{f*}(a,\varepsilon,g,z)] = 0
\end{aligned}
$$

Let $\{C_t\}_{0\leq t\leq\tau}$ describe households non-durable good consumption choice. τ is the stopping time. M,B_M,$H_{_M}$ are the households' buying and selling decisions, size of secured debt and size of the house when the households buy or sell houses. Ref and B_R are the refinancing decision and the size of secured debt when refinancing. D_a D_b and D_{ab} show the bankruptcy, foreclosure and default on both debt.

2.2. *Financial intermediaries*

There are risk neutral, competitive financial intermediaries. Banks issues short-term deposits and issue loans and secured debt to households. They also lend capital to firms. Due to the possibility of default, banks offer loan rates based on a household's portfolio choices and their persistent state, ε. Banks expect zero profits for each loan. However, although ex-ante profits are zero for each loan, ex-post returns can be differ from zero due to aggregate risk. I assume that the government absorbs any realized profits or losses using taxes or subsidies to intermediaries.

Unsecured debt As characterized in Bornstein (2018), the expected interest on lending in the region of no default ($D_a(a, b, \varepsilon, h, g, z) = 0$) is a return minus a default probablity. It can be written as

$$E[dr_a(a, b, \varepsilon, h, g, z)] = r_a(a, b, \varepsilon, h, g, z)dt - \lambda_z \sum_{z'} p_{zz'} \lambda_\varepsilon \sum_{\varepsilon'} p_{\varepsilon\varepsilon'} D_a(a, b, \varepsilon', h, g, z')dt$$

where $p_{\varepsilon\varepsilon'}$ is the probability of moving from ε to ε' conditional on receiving a labor productivity shock. In the default region ($D_a(a, b, \varepsilon, h, g, z) = 1$),

$$r_a(a, b, \varepsilon, h, g, z) = \infty \tag{14}$$

The zero profit condition in the region of no default implies that the return $r_a(a, b, \varepsilon, h, g, z)$ should be equal to a risk free rate, $r(g, z)$.

$$r_a(a, b, \varepsilon, h, g, z) = r(g, z) + \lambda_z \sum_{z'} p_{zz'} \lambda_\varepsilon \sum_{\varepsilon'} p_{\varepsilon\varepsilon'} D_a(a, b, \varepsilon', h, g, z') \tag{15}$$

Because no household defaults on a in the region where a is positive, $r_a(a, b, \varepsilon, h, g, z) = r(g, z)$ for savers.

Secured debt Borrowers pay an interest rate $r(g, z) + \iota(z)$ and a fraction $\theta(b, \overline{p}h))$ of the remaining balance b at each instant. Therefore the flow income from a loan is $(r_t + \theta(b_t, \overline{p}h))b_t$. Banks discount the loan with a interest rate $r_t + \theta_t$ because the loan matures at rate θ_t. If the household defaults on the secured debt, the bank recovers the depreciated value of the house, $(1 - \delta_d)ph$.

Since the banks expect zero profit for each loan, the discounted value of the loan at the origination has to be equal to the expected cash flow from the loan. The price of the loan in the non-default region is given by

$$q_0(a, b, \varepsilon, h, g, z)b_0 = \mathbb{E}\Big[\mathbb{E}_\tau \int_0^\tau e^{-\int_0^s (r_s + \iota_t + \theta_s)ds}(r_t + \iota_t + \theta_t)b_0 dt + e^{-\int_0^\tau r_s ds} b(a_\tau, b_\tau, \varepsilon_\tau, h, g, z)\Big]$$

The scrap value of $b(a_\tau, b_\tau, \varepsilon_\tau, h, g, z)$ at the stopping point depends on the choice. In the case of default,

$$b(a_\tau, b_\tau, \varepsilon_\tau, h, g, z) = (1 - \delta_d)p(g, z)h.$$

When a household prepays the loan due to refinancing or a house transaction

$$b(a_\tau, b_\tau, \varepsilon_\tau, h, g, z) = e^{-\int_0^\tau \theta_s ds} b_0.$$

Applying the Feynman-Kac formula, the above equations can be written as the following partial differential equation.[16]

At $t \in [0, \tau)$

$$\begin{aligned}(\theta(b, \overline{p}h) + r(g, z) + \iota(z))q(a, b, \varepsilon, h, g, z) &= \theta(b, \overline{p}h) + r(g, z) + \iota(z) + q_a(a, b, \varepsilon, h, g, z)\dot{a} \\ &+ q_b(a, b, \varepsilon, h, g, z)\dot{b} + \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} q(a, b, \varepsilon_j, h, g, z) + \sum_{k=1}^{n_z} \lambda_{zz_k} v(a, b, \varepsilon, h, g, z_k) \\ &+ \int \frac{\delta v(a, b, \varepsilon, h, g, z)}{\delta g(a, b, \varepsilon, h)} \mathcal{K} g(a, b, \varepsilon, h) d[a \times b \times \varepsilon \times h]\end{aligned} \tag{16}$$

$t = \tau$, in case of the foreclosure,

$$q(a, b, \varepsilon, h, g, z) = \frac{(1 - \delta_d)p(g, z)h}{b}. \tag{17}$$

$t = \tau$, in case of the prepayment,

$$q(a, b, \varepsilon, h, g, z) = 1. \tag{18}$$

2.3. Firms

There are identical, competitive firms who produce non-durable consumption goods using a constant return to scale technology.[17] Firms rent capital from the banks and employ labor to produce goods. The firms solve the following problem.

[16] See Nuno and Thomas (2015), Kaplan, Moll, and Violante (2018) for similar usage.

[17] I assume the stock of durable good is given to the economy.

$$\max_{k,\ell} zf(k,\ell) - (r(g,z)+\delta)k - w(g,z)\ell \tag{19}$$

$$f(k,\ell) = k^{\alpha}\ell^{1-\alpha}$$

The production function $f(k,\ell)$ is $k^{\alpha}\ell^{1-\alpha}$ and the capital depreciation rate is δ. Firms' technology implies that the equilibrium interest rate is $r(g,z) = z\alpha\frac{\ell}{k}^{1-\alpha}$ and the equilibrium wage rate is $w(g,z) = z(1-\alpha)\frac{\ell}{k}^{-\alpha}$.

2.4. *Government*

The government collects tax from the households. Tax is levied on taxable income, which is labor income minus deductible costs. Households can deduct the interest paid on b and a part of the maintenance cost of h from their taxable income. The government also absorbs realized profits or losses from the secured debt of banks by adjusting taxes or subsidies to banks. The government spends the remaining revenue on services that are not valued.

2.5. *Equilibrium*

An equilibrium is a set of functions

$$(r_a, q, r, w, p, v, C, \tau, M, B_M, H_M, R, B^R, D_a, D_b, D_{ab}, L, K, G)$$

that satisfies the following:

1. Households optimize. Given prices $\{r_a, q, w, p\}$, v solves (1)-(4), v^d solves (6)-(10) and v^f solves (11)-(13). $C, \tau, M, H, B^m, R, B^r, D_a, D_b, D_{ab}$ are the associated policy functions.
2. Firms maximize profits by solving (19) and L, K are the associated policy functions.

3. The unsecured debt price function r_a is determined by (14) and (15).
4. The secured debt price function q is determined by (16) - (18).
5. Capital market clears: $\int (a+b)g(a,b,\varepsilon,h)d[a \times b \times \varepsilon \times h] = k$
6. Labor market clears: $\int \varepsilon g(a,b,\varepsilon,h)d[a \times b \times \varepsilon \times h] = \ell$
7. Durable goods market clears: $\int hg(a,b,\varepsilon,h)d[a \times b \times \varepsilon \times h] = \overline{H}$
8. The government budget holds.
9. The Kolmogorov Forward Operator $\mathcal{K}$ that describes the change of density function g is generated by agents' optimal choices given their states.

3. Mapping Model to Data

I choose model parameters to match key cross-sectional features of the U.S.economy in the early 2010's. To study the effects of debt relief programs, the model needs to match their distribution of assets and debt across households. Households' portfolios are the key determinants of default decisions along with other choices. Moreover, a calibration of the stochastic process for labor earnings should be able to capture the earnings dynamics seen in the data as labor income shocks are the source of uninsurable risk driving household adjustment of their asset and debt positions.

A subset of model parameters are assigned in advance of solving the model's stationary state. In additions, the earnings process is estimated outside of the model. Lastly, 11 parameters are jointly calibrated in the steady state.[18]

Earnings process I model the labor earnings process as a combination of two independent components:

$$\varepsilon_{ij} = \varepsilon_i^p(1+\varepsilon_j^t)$$

where each component follows a Poisson jump process. Jumps arrive at a Poisson rate

[18]These are parameters specifying household preference ($\sigma, \sigma^d, \kappa, \rho$), various costs ($\xi_r, \xi_x, \xi_a, \xi_b, \xi_0, \xi_1$) and the tax function ($\tau_0$).

λ^p for ε^p and λ^t for ε^t. Conditional on a jump, a new earnings state ε_k^p is drawn from a bounded Pareto distribution and ε_l^t is drawn from the discrete set $\{-\chi, \chi\}$.

To pin down the levels of ε_i^p, it is necessary to set upper and lower bound as well as a curvature. I set these parameters to match the variance of earnings and the distribution of earnings. Specifically, I target earnings shares by quintile and top 10%. I discretize ε_p to 4 points and the these values are set to represent the [41.0, 28.0, 29.5, 1.5] (%) of the working population respectively.[19] [20] Table 1 shows earnings distribution in the data and in the model. The model captures the overall earnings share by quintile well.

I assume the probability of drawing a new value for ε_k^p depends on its level, ε_i^p. Therefore the intensity of jumping from i to k is given by $\lambda_{ik}^p = \lambda^p(f(\varepsilon_{k+1|i}) - f(\varepsilon_{k|i}))$, where $\lambda^p = \sum_{k=1}^{n_{\varepsilon^p}} \lambda_{ik}^p, \ \forall i$ and $f(\varepsilon_{k|i}) = \frac{1-(\underline{\varepsilon}/x_k)^{\eta_{\varepsilon_i}}}{1-(\underline{\varepsilon}/\overline{\varepsilon})^{\eta_{\varepsilon_i}}}$.[21]

I used a bounded Pareto distribution for $f(\varepsilon_{k|i})$. Given the discretized support, the shape parameters η_{ε_i} need to be estimated. To set these curvature values, as well as the shock intensities λ^p and λ^t, the size of the shock χ and the probability of drawing $-\chi$ conditional on a jump in ε^t, I estimate the earnings process by Simulated Method of Moments to match the high-order moments of earnings growth rate distribution reported in Guvenen, Karahan, Ozkan, and Song (2015).[22][23] I simulate the model to

[19] The share from the 1st to the 2rd bin are chosen to represent the population with education attainment levels: less than a high school diploma or high school graduate, some college (the average over 1992 to 2013, BLS). The rest of the population has a bachelor's degree or higher, and I add one points to capture earnings concentration to the top.

[20] For example, ε_2 is a median value between x_1 and x_2 such that $f(x_1) = \frac{1-(\underline{\varepsilon}/x_1)^{\eta_\varepsilon}}{1-(\underline{\varepsilon}/\overline{\varepsilon})^{\eta_\varepsilon}} = 0.41$, $f(x_2) = \frac{1-(\underline{\varepsilon}/x_2)^{\eta_\varepsilon}}{1-(\underline{\varepsilon}/\overline{\varepsilon})^{\eta_\varepsilon}} = 0.41 + 0.28$ where $f(x_i)$ is the CDF of the bounded Pareto distribution.

[21] Since λ_{ik} affect the ergodic distribution and the population share by education attainment becomes a target.

[22] Kaplan et al. (2018) explains why this is proper to infer high frequency earnings dynamics. The key argument is that the size and frequency of the shock results the shape of the earnings distribution. Large, infrequent shocks are likely to generate more leptokurtic distribution and small, frequent shocks are likely to generate a platykurtic distribution. Kaplan et al. (2018) model the earnings process as a sum of two jump-drift processes, each represents a persistent and a transitory component of the earnings process.

[23] They use Social Security Administration (SSA) data from 1994 to 2013 to compute the moments.

compute the corresponding moments.[24] Since the data moments are computed using annual earnings, I simulated the model with a higher frequency and aggregate the result into annual earnings.

To summarize, the number of parameters specifying earnings process are 11 and the number of targets are 20.[25] The estimated process implies that the shock to ε^p arrives on average once every 21 years. Upon the arrival of shock, a state is likely to switch to adjacent states. Since the ε^p grid are not equi-distanced, the size of a shock depends on current state. In general, the size of the shock tends to be small when a labor productivity is low when the shock arrives. In contrast, a shock to ε^t arrives on average once every 0.9 years.

Infrequent component of labor income shock, ε^p can be interpreted as the persistent component of an income and ε^t as a transitory component. Households do not experience a large shock often, but income fluctuates around the level of their persistent component due to transitory shocks.

Table 2 shows moments from the data and the model. The estimated process does a fairly good job of generating moments that reproduce the data.

Table 1: Earnings distribution

	Variance	**Quintiles(%)**					**Top(%)**		
		1q	2q	3q	4q	5q	90-95	95-99	99-100
Data	0.92	-0.1	3.5	11.0	20.6	65.0	12.1	18.3	18.0
Model	0.93	4.3	5.7	6.8	21.1	62.0	13.4	15.2	16.1

Data: SCF (2010), Song, Price, Guvenen, Bloom, and Von Wachter (2018)

Assets and debt

[24]The panel size is 5000 and the simulation period is 6000. The 1st through the 800th periods simulated series are discarded when computing the statistics. Increasing the panel size or the number of periods has little effect on the results.

[25]3 parameters that shape a bounded Pareto distribution for ε^p: $\overline{\varepsilon}^p, \underline{\varepsilon}^p, \eta^p_\varepsilon$
4 parameters that set the probability of drawing a new value for ε^p: $\eta^p_{\varepsilon_i}, i \in [1, 2, 3, 4]$
2 parameters that set shock intensity: λ^p, λ^t
and χ is the size of a transitory shock and p^t is a probability of drawing negative transitory shock.

Table 2: Earnings dynamics

	Std.		**Skewness**		**Kurtosis**		$\mathbf{P}(\lvert\Delta y\rvert) < \mathbf{x}$*			$\mathbf{P}(\lvert\Delta y\rvert) \in [\ \underline{x}, \overline{x}\ ]$ *	
	1y	5y	1y	5y	1y	5y	x = 0.2	0.5	1.0	[0,0.25)	[0.25,1)
Data	0.51	0.78	-1.07	-1.25	14.93	9.51	0.67	0.83	0.93	0.31	0.16
Model	0.30	0.58	-0.08	-0.03	15.14	8.57	0.62	0.97	0.98	0.43	0.19

* $|\Delta y|$: Absolute log earnings change
Data: Guvenen et al. (2015)

Categorization of assets and debts

Mapping the model to the data of the U.S. economy requires categorizing assets held by U.S. households into financial assets, non-financial assets and secured debt.[26] I target the assets and debt distribution reported in the SCF 2010. In the SCF data, net worth is comprised of assets and debt, and the total assets are the sum of financial assets and non-financial assets. Financial assets include transaction accounts, certificate of deposits, money market funds, stocks, cash, quasi-liquid retirement accounts and other financial assets. Non-financial assets are predominantly the value of vehicles and houses (primary and non-primary residential property, non-residential real estate) and the value of business. Debt is comprised of debt secured by residential properties, credit card loans, installments loans (e.g., student loan, vehicle loans). When mapping the model to the data, I exclude the value of a business from non-financial assets because my model does not incorporate this type of asset. For the debt, I exclude student loans for the same reason. The student loan is not a short-term, unsecured debt but also is not secured by collateral nor dischargeable in bankruptcy. After excluding student loans, credit card loans are considered to be unsecured debt and the remaining debts are assigned to secured debt.[27]

[26]The model does not separate financial asset and unsecured debt.

[27]Table 10 and Table 11 in the appendix C shows that the portfolio composition by quintile excluding /including business assets and student loans. When excluding business assets and student loans, the shares of asset and debt in 2 to 4 quintiles are not very different. In the 5th quintile, the share of non-financial asset is lower, which implies that the large share of business assets is owned by the wealthiest group. The numbers in the 1st quintile rise. That is likely caused by the net worth of the group being

Non financial assets Maintenance costs of house, ξ_h, is set to 2%.[28][29] Two parameters in the house transaction cost function ξ_0 and ξ_1 are 0.02 and 0.05. The fixed stock of houses $\overline{H}$ is set to 4.5. It is jointly calibrated with other parameters to match household distribution of asset and debt.

Secured debt The loan-to-value ratio γ is set to 1.05 based on the fact that mortgages are available with zero down payment and home equity line of credit are available to households.[30][31] The amortization rate of mortgages, $\overline{\theta}$ is set to 0.03 which implies that the duration of a loan is approximately 30 years if a household fully finances the purchase of the house. The refinance cost ξ_r is set to 0.02 and it is jointly calibrated.

Bankruptcy and foreclosure The utility cost ξ_a, ξ_b and income garnishment rate ξ_x are set to be 29,2 and 0.2. These parameters are jointly calibrated but they predominatly affect the rate of bankruptcy and foreclosure. The bankruptcy rate target is 1.06%, which is constructed by using the number of Chapter 7 and Chapter 13 bankruptcy filings from the U.S. Bankruptcy Courts and the number of households from the U.S. Census (averaged over 2000-2017). The foreclosure rate target is 0.55% (Mortgage Banker's Association, the average rate in the U.S. during the late 1990s).

The intensities of removal of the bankruptcy and the foreclosure flag are set to match the average duration with the flags. After filing for Chapter 7 bankruptcy, households cannot file again for Chapter 7 for 6 years. Households that file for Chapter 13 bankruptcy enter into repayment plans that last for 3–5 years. Accordingly, I choose λ_d to 0.183 to match the average bankruptcy duration of 6 years. For foreclosure, Fair Issac reports that households' FICO scores can recover in as little as two

closer to zero due to the exclusion of student loans.

[28]Maintenance cost includes property tax, which is deductible.

[29]Base on Davis and Van Nieuwerburgh's (2015) survey, maintenance cost is 1-3%.

[30]The United States Department of Veterans Affairs and the United States Department of Agriculture guarantee purchase loans to 100%, and the Federal Housing Administration (FHA) insures purchase loans to 96.5%.

[31]In the 1989 - 2013 waves of SCF, the size of the secured debt exceeds the value of non-financial asset among the poorest 20% of the households.

years after a foreclosure (See Mitman (2016)). Hence λ_f is set to 0.693 to give an average duration of 2 years for the foreclosure flag. A depreciation rate of foreclosed houses, δ_d is 22% which is taken from Pennington-Cross (2006). He estimates the loss of value of a foreclosed property using a sample of real estate owned property.

Preference The discount rate ρ, the parameters in the utility function, σ, σ_h, κ are jointly calibrated to match households' asset and debt distribution. The curvatures and weight in the utility function mainly affect households asset and debt composition. ρ is set to 0.075. I set σ to 2.2, σ_h to 0.48 and the weight on durable consumption κ to 4.0. The targets and moments are reported in Table 3.

Production The production technology is constant returns to scale with capital share set as the residual of the labor share of output in Giandrea and Sprague (2017). They measure labor's share of output in the non-farm business sector from 1947 through 2016.[32] I use the long-term average (1989 to 2013) of labor share, 60.5%; thus, α is set to 0.395. I assume that the depreciation rate δ is 0.069. (See Khan and Thomas (2013).)

Government The income tax function $T(y) = y - \tau_0 y^{1-\tau_1}$ is taken from Heathcote, Storesletten, and Violante (2017) where y is a taxable income. Taxable income is labor income minus tax deductible interest payment of secured debt and property taxes. The taxable income is

$$y = w(g,z)\varepsilon - r(g,z)min(b,\bar{b}) - min(\tau_h p(g,z)h, \overline{\tau_h}).$$

The property tax rate τ_h is set to 1%, which is the median tax rate across US states (Tax Policy Center, Kaplan et al. (2017)). $\bar{b}$ is \$1,000,000 (Internal Revenue Service

[32] Sum of employee compensation and proprietors' labor compensation

Table 3: Targeted moments and model values

Moment	Data	Model	Data Source
Foreclosure rate	0.0055	0.0060	Mortgage Banker's Association
Bankruptcy rate	0.010	0.011	U.S. Courts, U.S. Census
Tax revenue/Output	0.16	0.16	CBO
p50 Non-financial asset/Asset	0.803	0.260	SCF (2010)
p70 Non-financial asset/Asset	0.945	0.507	SCF (2010)
p90 Non-financial asset/Asset	0.998	0.769	SCF (2013)
p50 Total debt payment/Income	0.112	0.071	SCF (2010)
p70 Total debt payment/Income	0.218	0.226	SCF (2010)
p90 Total debt payment/Income	0.399	0.497	SCF (2010)
p50 Debt/Asset	0.210	0.182	SCF (2010)
p70 Debt/Asset	0.524	0.497	SCF (2010)
p90 Debt/Asset	0.946	0.794	SCF (2010)
Household with secured debt	0.643	0.673	SCF (2010)

(IRS)) and $\overline{\tau_h}$ is $10,000.[33][34] (IRS)

A parameter τ_1 which determines the degree of progressivity of the tax system is 0.181, taken from Heathcote et al. (2017).[35] τ_0 is set to 0.6 to match the tax revenue-output ratio 16.7% (averaged over 2000-2014, Congressional Budget Office).

Aggregate shocks Aggregate productivity z follows a two state Poisson process, $z \in [z_1, z_2]$ with $z_2 > z_1$. The process jumps from state 1 to state 2 with intensity λ_1 and in the reverse direction with intensity λ_2. These two states represent recession (z_1) and expansion (z_2).

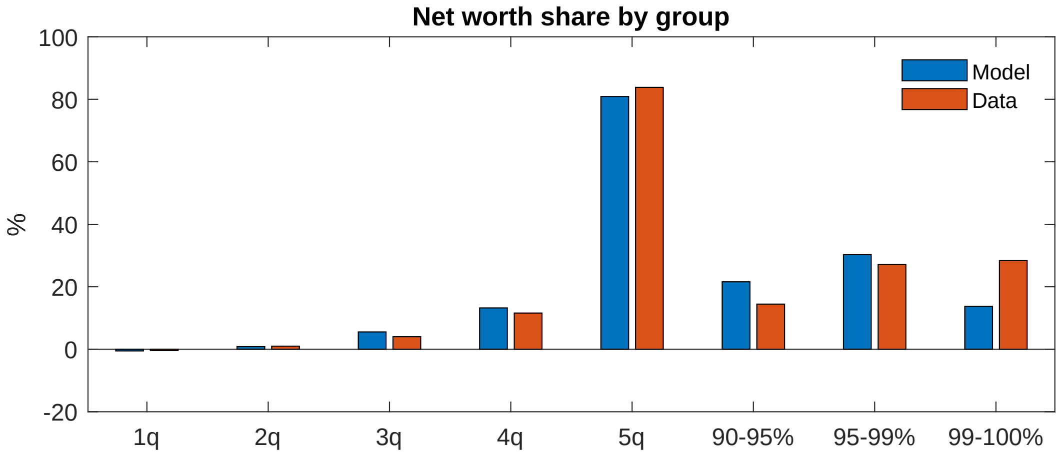
Th support of aggregate TFP, $[z1, z2]$ are [0.97, 1.03] to match the standard deviation of U.S. output. Table 5 shows the cyclical properties of the U.S. economy from the data and from the model. The shock intensities are 0.4041 which implies the average

[33]From 2018, deduction for home mortgage interest is up to the first $750,000 ($375,000 if married filing separately) of indebtedness. The limitation is $1,000,000 ($500,000 if married filing separately) of indebtedness if a household are deducting mortgage interest from indebtedness incurred on or before December 15, 2017. Since I target the early 2010's data, I apply the limit before 2017.

[34]This is the limitation on the deduction for state and local taxes. It includes general sales taxes, real estate taxes and personal property taxes.

[35]They estimate this parameter using the Panel Study of Income Dynamics (PSID) for survey years 2000, 2002, 2004, and 2006, in combination with the NBER's TAXSIM program.

Figure 1. Net worth distribution



Data: SCF (2010)

duration of a state is 3 years. I follow Nakajima and Ríos-Rull (2014) to pin down the number for the persistence of the aggregate shock.

Validation

I compute non-targeted moments to check the model's plausibility. First, the model matches the net worth distribution very well. (Figure 1) Moreover, my model allows me the further break down households assets. Figure 2 shows share of assets by net worth.[36] The distribution of non-financial assets, financial assets and secured debt from the model are reasonably close to the data. However, while unsecured debt is distributed evenly over quintiles, it is almost entirely held by the poor in the model.

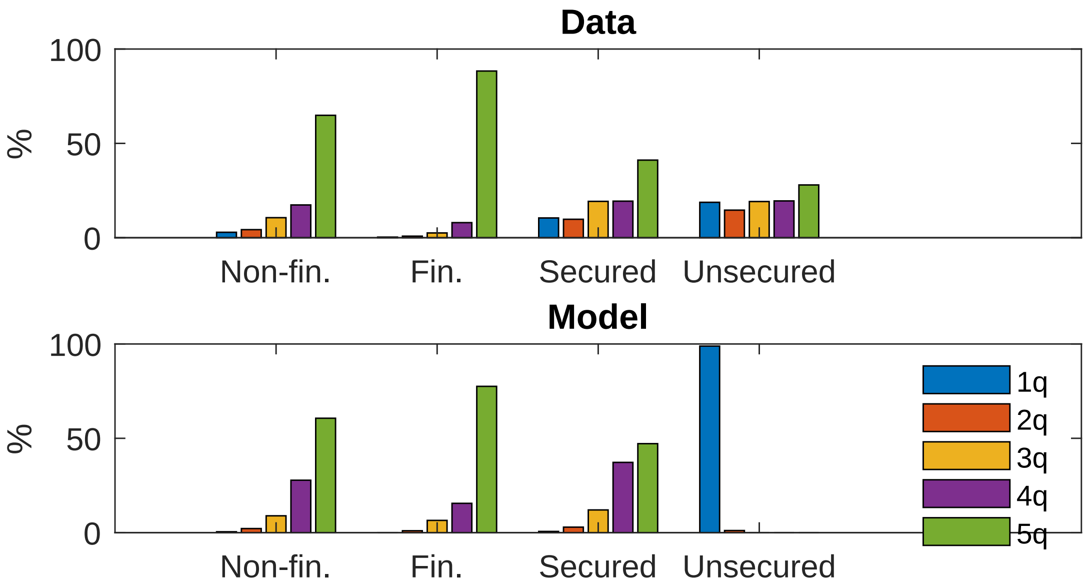
One reason is that despite it's rich asset structure, the households cannot have unsecured debt and liquid saving at the same time. As a result, given that the data indicate 9.6% of households have net negative financial assets, it becomes hard to match the distribution of financial assets and unsecured debt at the same time. Figure 3 shows the composition of assets across households by net worth.[37][38] Overall, house-

[36] More detailed tables are available in the appendix (Table 9).
[37] Net financial asset is total financial asset minus credit card debt.(SCF 2010)
[38] More detailed tables are available in the appendix (Table 10).

Table 4: Parameter values

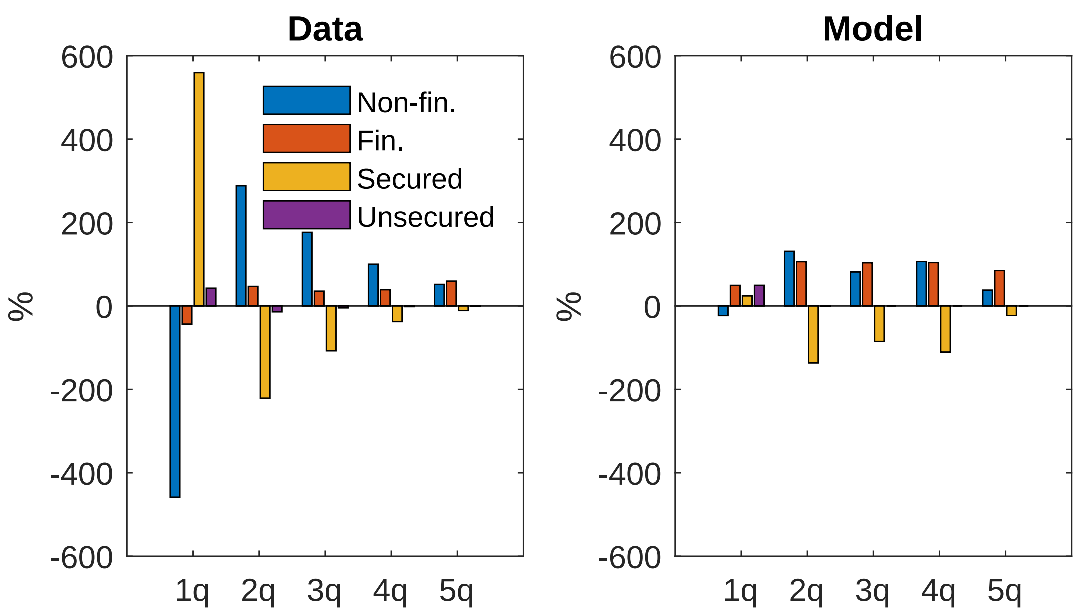
Parameter	Value	Internal	Description
Preferences and production			
ρ	0.075	Y	Discount rate
σ	2.2	Y	Curvature of the utility function
σ_h	0.48	Y	Curvature of the utility function
κ	4.0	Y	Weight on durable good
α	0.395	N	Capital share
δ	0.069	N	Depreciation rate
Tax			
τ_0	0.60	Y	Tax rate
τ_1	0.16	N	Tax progressivity
τ_h	0.01	N	Property tax rate
$\overline{b}$	1,000,000	N	Maximum indebtedness to deduct interest payment
$\overline{\tau_h}$	10,000	N	Maximum deduction on property tax
Labor productivity			
$\overline{\varepsilon}^p$	8.5	N	Upper bound of Pareto distribution
$\underline{\varepsilon}^p$	0.08	N	Lower bound of Pareto distribution
η^p_ε	1.526	N	Shape of Pareto distribution
$\eta_{\varepsilon^p_i}$	[1.9,1.5,1.3,0.6]	N	Shape of Pareto distribution
λ^p	0.048	N	Shock intensity
λ^t	1.260	N	Shock intensity
χ	0.239	N	Size of the ε^t shock
p^t	0.600	N	Probability of drawing negative ε^t
Assets and debts			
ξ_h	0.02	N	Depreciation rate of h
δ_h	0.07	N	h transaction cost
$\overline{H}_s$	2.02	N	Supply of durable good
γ	1.05	N	Loan-to-value ratio
$\overline{\theta}$	0.03	N	Amortization rate of b
ξ_r	0.02	Y	Refinancing cost
ξ_a	29	Y	Utility cost
ξ_b	2	Y	Utility cost
ξ_x	0.2	Y	Income garnishment rate
λ_d	0.1831	N	Removal of bankruptcy flag
λ_f	0.6929	N	Removal of foreclosure flag
δ_d	0.22	N	Depreciation due to foreclosure
Aggregate shock			
$[z_1, z_2]$	[0.9700, 1.0301]	N	Level of total productivity
$[\lambda_1, \lambda_2]$	[0.4041, 0.4041]	N	Shock intensity

Figure 2. Share of assets by net worth



Data: SCF (2010)

Figure 3. Asset composition by net worth



Data: SCF (2010)

Table 5: Cyclical properties

	Data		Model	
	std(%)	corr. with output	std(%)	corr. with output
Output	2.0	1.0	1.9	1.0
Consumption	1.8	0.9	0.4	0.9
Investment	8.0	0.8	8.7	0.6
Unsecured debt	5.2	0.8	0.2	0.0
Secured debt	5.1	0.5	1.6	0.6

Logs of the data are filtered using the H-P filter with a smoothing parameter of 100. Output: real GDP. Consumption: real private consumption expenditures; Investment: real gross domestic investment; Unsecured debt: Consumer credit (Flow of Funds), deflated by GDP deflator; Secured debt: Home Mortgages (Flow of Funds), deflated by GDP deflator.

holds' shares of non-financial assets in the model tend to be lower than the data but the model replicates the relatively high non-financial asset holdings for households in the 1-4th quintile as well as the relatively high financial share of the 5th quintile household.

To study the effects of government policies that effects households balance sheets during recessions, it is essential for any model to capture the cyclical properties of the economy. Table 5 compares shows the aggregate statistics from the U.S. data and from the model, and shows that the model captures the key properties of the data. In particular, both consumption and investment are strongly correlated with output, and the investment is more volatile than the consumption. Furthermore, secured credit is positively correlated with output and bankruptcy and foreclosure filings are countercyclical. The correlation between bankruptcy filings and output is -0.4, and the correlation between foreclosure filings and output is -0.3.

4. Steady state results

Prior to examining the dynamics of the economy over a business cycle, I explore the role of default options and how these options interact with the general equilibrium outcomes in the steady states. Specifically, I compare the following variation of my model i) the full model with bankruptcy and foreclosure (the benchmark model), ii) a model in which households cannot foreclose but can go bankrupt (Bankruptcy), iii) a model with only foreclosure is available to households (Foreclosure), and iv) a model without any default option (None).[39] Then I assess the value to households of being able to default by comparing the bench- mark economy to the alternative economies.

4.1. The role of default options on aggregate variables and the household distribution

The availability of default options affects aggregate variables as well as the distribution of asset across households. First, aggregate capital rises 1.2% when agents do not have default options. This is because the option to default acts as insurance against income risks, so the precautionary saving motive is stronger when it is not available. However, consumption is 2.2% lower in the economy without default options. Table 6 shows this result.

The marginal increase in saving is greatest for households close to the borrowing limit. Therefore when default is not an option, the households close to the borrowing limit have a higher share of wealth and, as a result, the wealth distribution becomes more equal. Table 7 shows that the share of net worth in the 5th quintile of wealthiest households falls with fewer default options.

These results imply that the dynamics of aggregate variables could be affected by the possibility of defaulting in two ways. First, the possibility of default allows households to reduce debt burdens when they receive income shocks, thereby affecting portfolio adjustment. In particular, demand for liquid and illiquid asset will vary as the number of households who choose to default varies and this affects prices which drive

[39]An exogenous borrowing constraint is set to 0 when there is no bankruptcy option and the loan to value ratio is set to 0.88 when there is no foreclosure option to prevent the consumption set becoming empty.

Table 6: Aggregate variables (Deviation from the benchmark (%))

	Liquid saving	Secured debt	Capital	h price	Consumption	Output
Bankruptcy	-7.08	-17.29	0.88	-2.72	-1.43	0.35
Foreclosure	0.25	0.98	0.32	-0.01	-0.61	0.13
None	-7.25	-16.92	1.18	-2.51	-2.20	0.46

Table 7: Net worth share by quintile

	Benchmark	**Bankruptcy**	**Foreclosure**	**None**
1q	-0.53	-0.54	0.11	0.11
2q	0.85	0.77	1.29	1.21
3q	5.55	5.64	5.65	5.69
4q	13.23	14.47	12.96	14.06
5q	80.90	79.66	79.98	78.94

to other changes in aggregates.

Second, the possibility of default leads to an equilibrium with lower levels of capital and higher leverage as shown in Table 6. Therefore the state of the economy with and without default will be different at the start of any recessions, and this difference will affect the propagation of the shocks.

4.2. Who values default options?

I assess the value to households of being able to default by comparing the benchmark economy to the alternative economies. The value of bankruptcy and foreclosure

Table 8: Debt/Asset

	Data	**Benchmark**	**Bankruptcy**	**Foreclosure**	**None**
p50	0.21	0.18	0.16	0.25	0.22
p70	0.52	0.50	0.43	0.54	0.46
p90	0.95	0.79	0.69	0.83	0.72

option is calculated as the consumption-equivalent gain obtained from moving from one of the alternative economy to the benchmark economy. As shown in section 2 the value of non-stopping household is:

$$\rho v(a,b,\varepsilon,h) = \max_c u(c,h) + \partial_a v(a,b,\varepsilon,h)\dot{a} + \partial_b v(a,b,\varepsilon,h)\dot{b} + \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v(a,b,\varepsilon_j,h)$$

For ease of the notation, let $\partial_a v(a,b,\varepsilon,h)\dot{a} + \partial_b v(a,b,\varepsilon,h)\dot{b} + \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v(a,b,\varepsilon_j,h) = \mathbb{A}v$. In addition, let x be the amount that makes the value of the economy with bankruptcy only/foreclosure only/no default options indifferent to the value of the benchmark economy.

$$\rho v_{benchmark} = \rho v_i + x = u(c_i,h) + \mathbb{A}v_i + x = u(c_i + c_i^*, h) + \mathbb{A}v$$

where c^* is the consumption equivalent gain and $i = \{\text{bankruptcy,foreclosure,none}\}$.[40] [41]

Figure 4 presents the consumption compensation that is required to make households indifferent between the benchmark economy and the bankruptcy only economy. It illustrates the value of the foreclosure option. Likewise, Figure 5 shows the consumption compensation required for moving from the benchmark economy to one with foreclosure. It calculates the value of bankruptcy. Each bar in the figures represents the average consumption equivalent gain of households who have a specific house size and level of labor productivity.

These figures show that the benefit of default options is unevenly distributed. Intu-

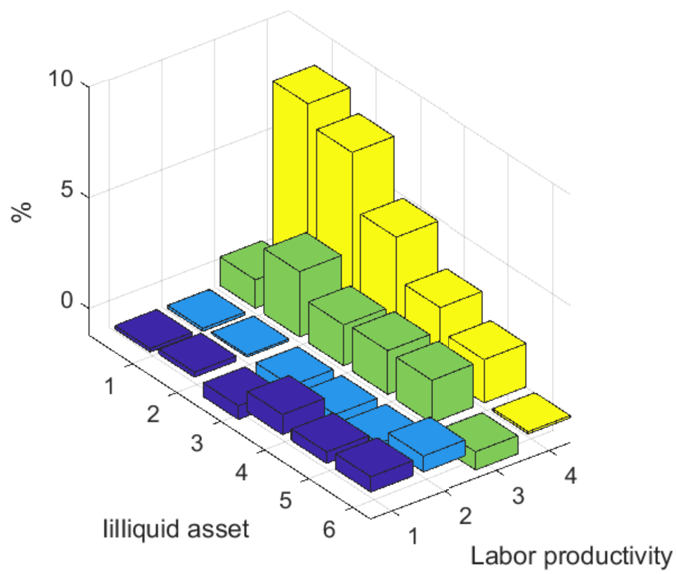
[40]A percertage deviation of consumption-equivalent gain can be computed as below.

$$\frac{c_i^*}{c_i} = \frac{[(1-s)(\rho v_{benchmark} - \rho v_i + u(c_i))]^{\frac{1}{1-s}}}{c_i} - 1$$

See appendix B for derivation.

[41]Since the economy without a foreclosure option has a lower loan-to-value limit and the equilibrium h price varies, the consumption equivalent gain is compared on $(a, \frac{b}{ph}, \varepsilon, h)$ instead of (a,b,ε,h).

Figure 4. Value of foreclosure



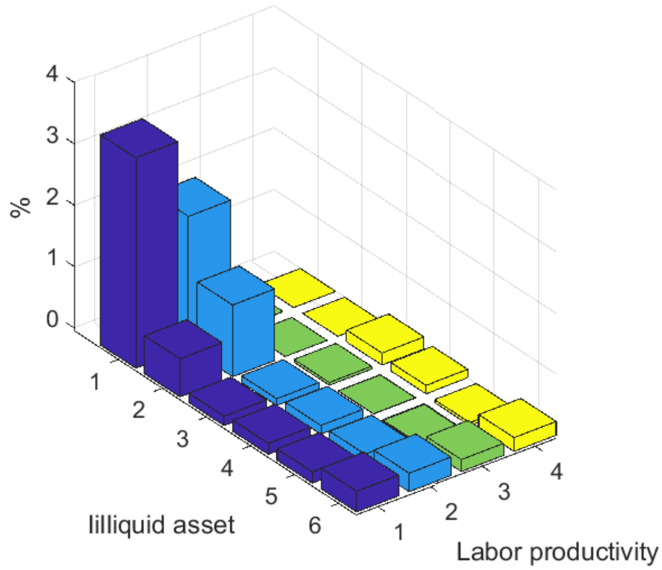
Note: Average consumption equivalence gain at given persistent component of labor productivity level and illiquid asset size.

itively, those who have low income and low wealth would value the bankruptcy option the most, and the results confirm this intuition. Figure 5 shows that those who have low income and low illiquid assets gain the most from the bankruptcy. These households not only have small house but also tend to have little liquid wealth, (Figure 24) therefore they are likely to use the option.

The foreclosure option is mainly valued by high income households. However, as house size increases the average gain decreases. This may seem counter-intuitive: As the house size increase, secured debt tends to increase, and the ability to walk away from a large loan would be more valuable. Figure 22 and Figure 23 shows that indeed households with large houses tend to have more debt.

Then why do more indebted households value foreclosure less? This follows from the difference in house price. The possibility of foreclosure enables households with low wealth to take on larger loans than otherwise to buy houses. The demand for

Figure 5. Value of bankruptcy



Note: Average consumption equivalence gain at given persistent component of labor productivity level and illiquid asset size.

housing rises which, in turn, leads to an equilibrium price rise for housing. Therefore to afford the same size of house when the foreclosure is available, households need to bear higher maintenance costs. Also, the size of the loan is larger at the same level of loan-to-value ratio and this requires larger debt repayment. The higher maintenance costs and debt repayments reduce a household's resource available to purchase non-durable consumption. This makes the foreclosure option less desirable.

5. Effects of debt relief programs in recessions

My model captures the distribution of households as well as the cyclical properties in the data. This enables me to investigate the effects of debt relief programs during recessions. In this section I conduct an analysis of such programs using a series of policy experiments.

During the Great recession, the U.S. government intervened in mortgage markets through household debt relief policies to support falling house price and to slow rising delinquencies.[42] These policies provided incentives to financial intermediaries and households to restructure their debt contracts. One such program was principal reduction, which forgives a fraction of a mortgage borrower's remaining balance.[43] While participation rates were perceived to be low, Agarwal et al. (2017) show that the program was associated with with reduced rates of foreclosure, consumer debt delinquencies and house price declines. While they provide important evidence from the microeconomic data, the estimated causal relations cannot be interpreted to accurately reflect macroeconomic responses to the program, since they do not account for the accompanying price changes. This model provides a useful laboratory for evaluating the general equilibrium effects of such policies.

I design a policy intervention in which all households with loan-to-value (LTV) ratios above 95% at the time of the intervention have a fraction of their secured debt forgiven so that their LTV ratio becomes 95%. I assume that the policy intervention is unanticipated and does not imply increased tax rates.[44]

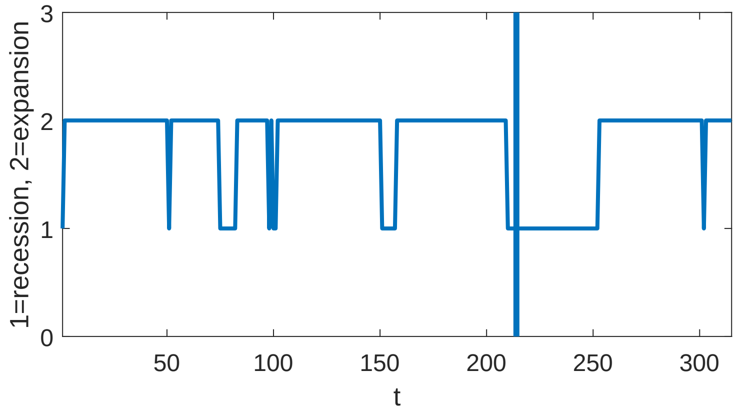
I compare the effects of the targeted mortgage debt relief program with those of a tax rebate. Tax rebates are a commonly used stimulus package and were approved by the U.S. Congress in the last two recessions of 2001 and 2007–2009. For comparability, I also model the rebate as an unexpected intervention, and I set its overall size to match the total cost of the debt forgiveness program. In this case, each household re-

[42] For example, Home Affordable Modification Program (HAMP), the Principal Reduction Alternative (PRA), the Home Affordable Foreclosure Alternatives (HAFA) and the Home Affordable Refinance Program (HARP).

[43] In particular, the government introduced principal reduction modifications in 2010 in Home Affordable Modification Program (HAMP). This was a response to growing concern that debt levels, not just debt repayments, caused high foreclosure rates. Under this modification, mortgage borrowers principal was forgiven until the new monthly payment achieves the 31% of income or their LTV ratio hits 115%, whichever comes first.

[44] Before and after the shock, the simulation is based on on forecasting functions estimated in an environment without the policy intervention.

Figure 6. Total factor productivity



ceives a lump sum transfer equivalent to 3,530 U.S. 2010 dollars.[45] The critical difference between the two programs is that the tax rebate is distributed equally, providing liquidity to all households, whereas debt forgiveness applies only to some households and does not directly affect liquidity.[46]

I begin with the principal reduction program, first presenting its effects on aggregate variables and then analyzing in more detail the consumption responses among different segments of the population. I next contrast these results with those from the across-the-board tax rebate. Finally, I reconsider the consequences of the mortgage forgiveness program when intervention is late, thereby illustrating the state-dependent nature of its effects, and close by discussing the results under two alternative debt relief programs. Throughout all exercises, my graphs show the responses of variables from the intervention period in terms of deviations from their counterparts in an economy without policy intervention.

5.1. *Mortgage forgiveness*

Figure 6 shows the sequence of aggregate shocks surrounding this policy evaluation, and the timing of the intervention. Following a relatively long expansion, the government intervenes promptly with the principal reduction program at t = 214, just after the start of a recession. In the model, this program affects approximately 26% of households, and the average size of the mortgage principal reduction for eligible households corresponds to roughly about 13,500 U.S. 2010 dollars.

5.1.1. *Aggregate responses*

Figure 7 shows the aggregate response of output, capital, consumption and house price responses after the policy intervention. The principal reduction is successful at increasing consumption and capital. The rise in capital leads to higher output. The aggregate marginal propensity to consume (MPC) measured as the ratio of the increase in aggregate consumption with the intervention (versus without) relative to the total debt forgiven, is 4%.

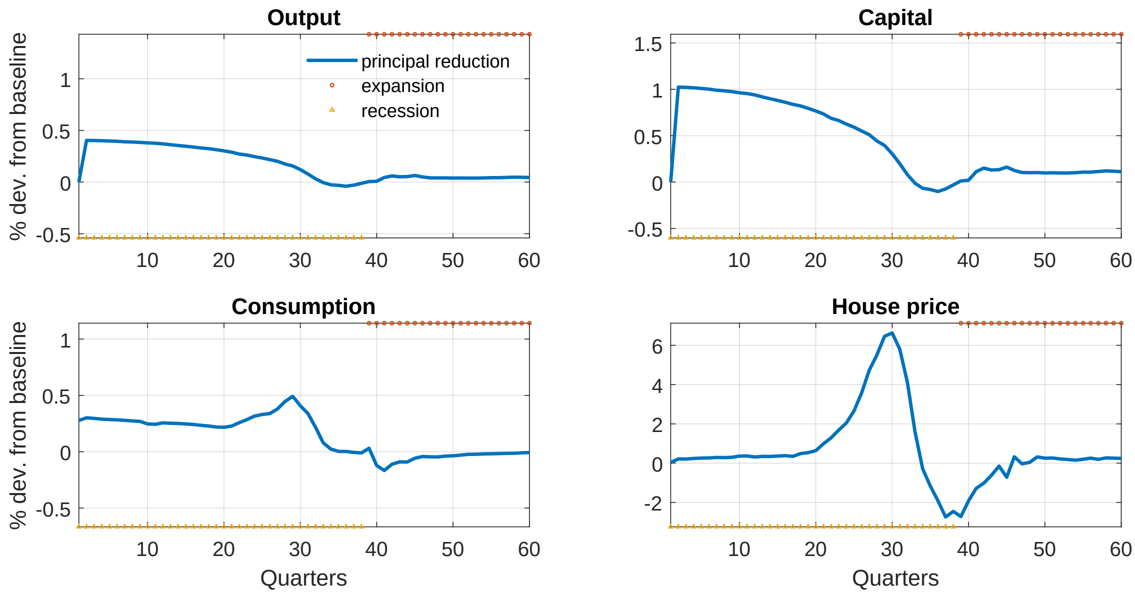
Before the interest rate and house price begin adjusting to the intervention, households not targeted by the principal reduction respond very little.[47] By contrast, the total response across eligible households is substantial, since these households are 26% of the population. Home equity gains arising from forgiven debt may drive an increase in wealth, and wealth effects could raise households' consumption. However, since the proceeds of the principal reduction cannot be used to purchase goods and services, it is not obvious why nondurable consumption responds to the extent it does. I will return to this point in the next sub-section.

[45]The Economic Stimulus Act of 2008 consisted of a 100 billion dollar program that sent tax rebates to approximately 130 million US tax filers. Single individuals received \$300–\$600 and couples received \$600–\$1,200. In addition, eligible households received \$300 per child.

[46]There is an indirect liquidity effect of the principal reduction program. Since households who have secured debt pay interest rate on their loan, a reduction in the outstanding of the loan increases liquidity by reducing interest rate payments.

[47]House price increases by 0.2% at the time so the changes in decisions are negligible.

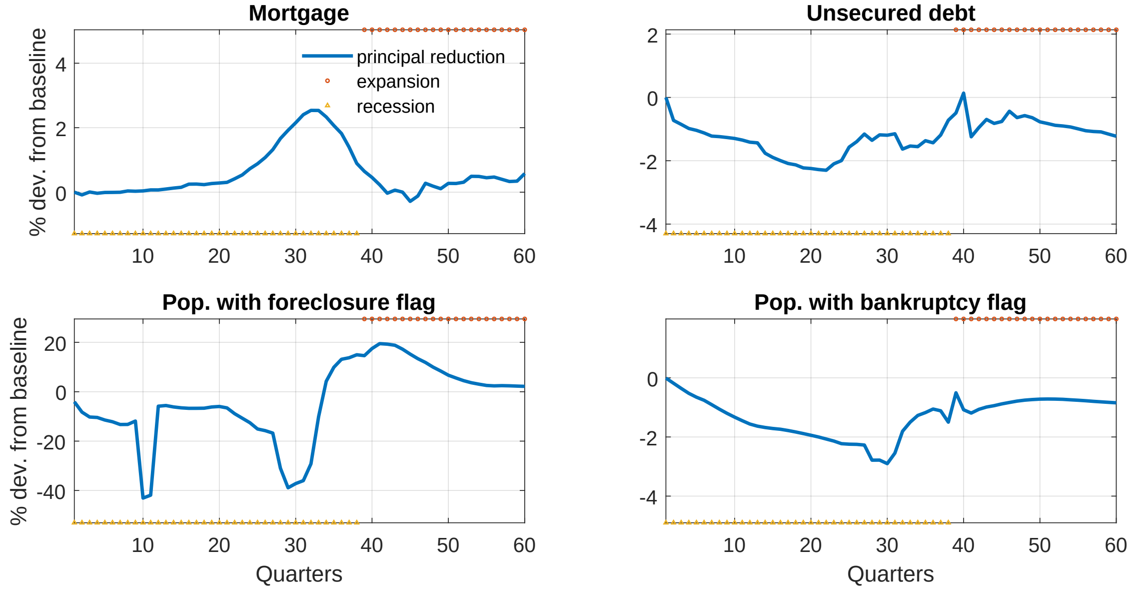
Figure 7. Response of aggregate variables



The effects are also very persistent. In particular, capital is higher than the benchmark economy with no policy intervention after 100 quarters, and the impact on consumption persists for at least a decade. The principal reduction also mitigates the fall in house prices the recession otherwise implies. While initially modest, the program's implications for house prices grow over time, as an increasing number of foreclosures are prevented. After 30 quarters, the price of a house is approximately 7% higher than it would be without the intervention.

Figure 8 shows responses of credits and defaults. By reducing the number of financially distressed households, the intervention significantly reduces foreclosures. This result is consistent with the negative relationship between the amount of negative equity and mortgage default rates that are shown in Haughwout, Okah, and Tracy (2009) and Gerardi, Herkenhoff, Ohanian, and Willen (2017). Since foreclosures increase the stock of houses for sale, a reduction in foreclosure rates dampens the fall in house prices. Although the policy forgives a fraction of targeted households' mortgages, total mortgage debt rises over roughly ten periods following the intervention.

Figure 8. Response of credits and defaults



Two factors contribute to this. First, because the intervention induces a rise in aggregate capital, future interest rate payments on any given sized loan fall, making it less costly to hold a mortgage. Second, since LTV ratio limit depends on house prices, higher house prices enable households to take more mortgage. Notice, in the right-hand panels, how the program targeted to relieve the burden of secured debt spills over to unsecured debt. Although the principal reduction policy only applies to mortgages directly, it reduces both the stock of credit card debt and bankruptcy rates.

As mentioned above, Kaplan et al. (2017) do a similar experiment. In their model, the main driver of house price movement is a change in agents' beliefs, and the housing wealth effect is the key driver of consumption dynamics. While my environment shares several features in common with theirs, differences in the structure of my housing market, the presence of capital, and equilibrium interest rate movements together imply different policy outcomes. In the Kaplan et al.'s (2017) model, only the house price is determined in equilibrium, and agents form expectations of future house prices based on the current price alongside current labor productivity, credit conditions and aggregate uncertainty about the future demand for housing services. Their forecasting functions are estimated in an environment without policy intervention and do not include the capital stock as an argument. Given the sizeable aggregate capital response to the principal reduction and the strong consumption reaction to the corresponding decline in interest rates in my model, it is not surprising that the mortgage relief policy affects not just foreclosures but also house prices and consumption. In the absence of such price responses, the Kaplan et al.'s (2017) model delivers no subsequent responses in other aggregate variables.

In my model, capital, labor and the housing market clear. While agents in my model also form their expectations over future prices without considering the possibility of government intervention, the increase in the capital stock does have an impact on agents expectations as well as their decisions. Therefore prices respond and this leads to subsequent general equilibrium feedback. It is worth noting that the fixed housing stock assumption also contributes to a large house price response. If my model included a construction sector so that housing supply could respond to prices as in Kaplan et al. (2017), then the housing price response would be smaller than that seen here.

5.1.2. A closer look at the non-durable consumption response

The lower left panel of Figure 7 shows aggregate consumption response to the principal reduction. In this section, I analyse how the policy intervention affects different segments of the population.[48]

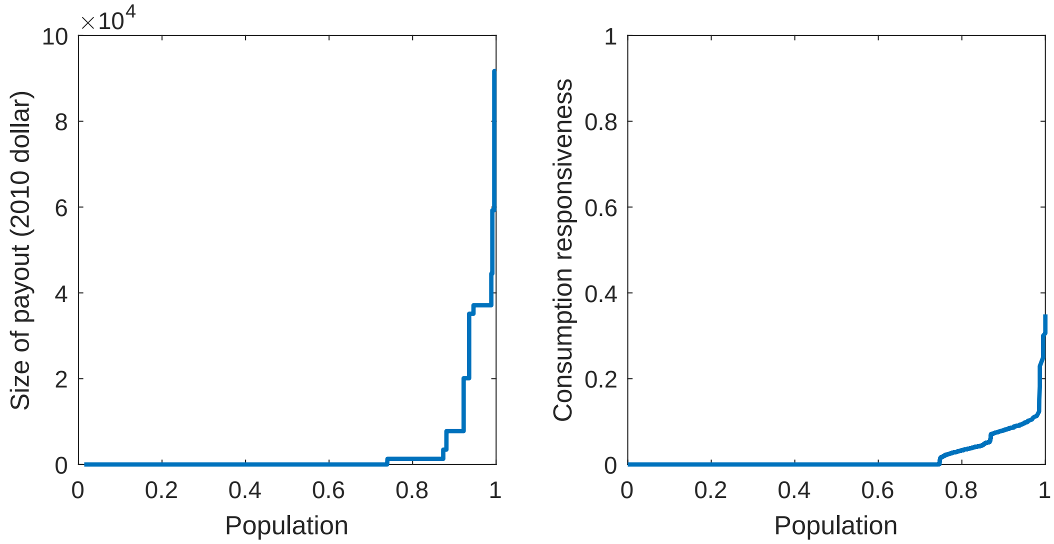
Figure 9 shows cumulative distribution functions of principal reduction size and MPC at the intervention period over population (normalized to 1).[49] [50] These distri-

[48]Most results in this section are generated from a simulated panel. The sample size is 21,357.

[49]Individual MPCs are computed in the same way the aggregate MPC was computed in the previous section.

[50]Negative MPCs are dropped. Roughly 0.2% of households consumed less than the households who did not receive the principal reduction. These are the households that would have chosen to default if

Figure 9. Principal reduction size and consumption responsiveness



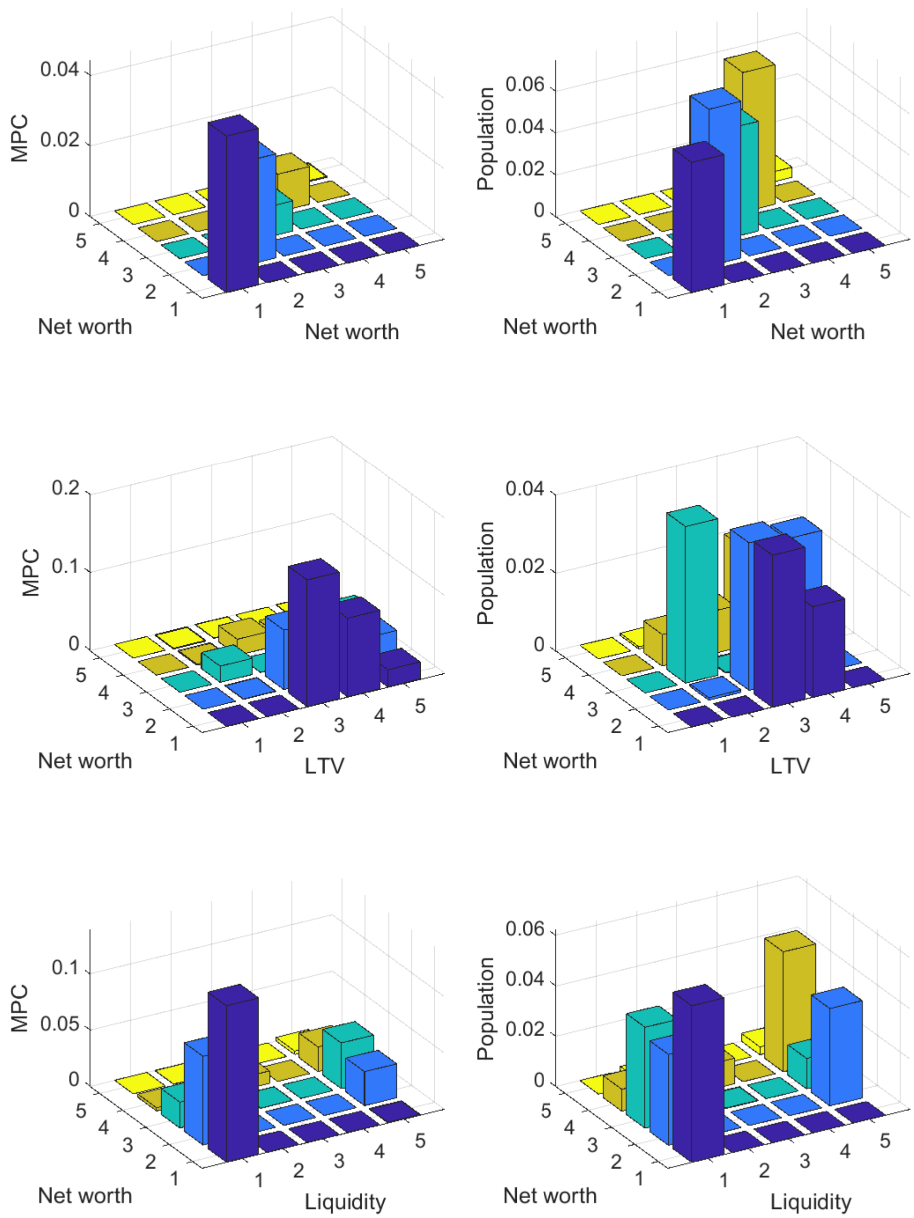
butions are computed using simulated panels. As mentioned, the average size of the principal reduction is approximately 13,500 2010 dollars but the amount each household receives varies greatly. While the aggregate MPC is 4%, the MPC distribution ranges from zero to nearly 40%. Most households not receiving the reduction do not respond because prices stay close to the no-intervention counterparts.

Who responds most? Having seen the dispersed MPC distribution, it is natural to investigate the common characteristics of households that show high responsiveness or low responsiveness. While there has been some work looking at heterogeneous MPCs using tax rebates as natural experiments, we cannot assume that the consumption response to the principal reduction will show a similar pattern.[51] To my knowl-

they had not received the principal reduction. While these households keep their houses and consume housing services, their non-durable consumption are lower than the households that default.

[51]Misra and Surico (2014) allow the propensity to consume of the 2001 and 2008 tax rebate to vary across household groups using quantile regressions. They find that almost half of households did not adjust their consumption and 20% of households with low income spent a small but significant amount. Households that show high MPC held high levels of mortgage debt. Kaplan and Violante (2014) study the heterogeneity in MPC using a two assets (a low-return liquid asset and a high-return illiquid asset) model. They show that many households in the model are 'wealthy hand-to-mouth' (holding little liquid wealth despite holding a sizable amount of illiquid assets) and these households display large propensities to consume out of additional transitory income such as tax rebate.

Figure 10. MPC and population



Note: These bars plot MPC and population over household net worth, LTV ratio and liquidity quintile. MPC is $\frac{\text{c(policy) - c(no policy)}}{\text{principal reduction size}}$ and liquidity is a ratio of liquid saving over total asset (saving plus house value). For MPC, each bar is an average MPC of households that are in corresponding quintile. Only households receiving the principal reduction are counted in the population.

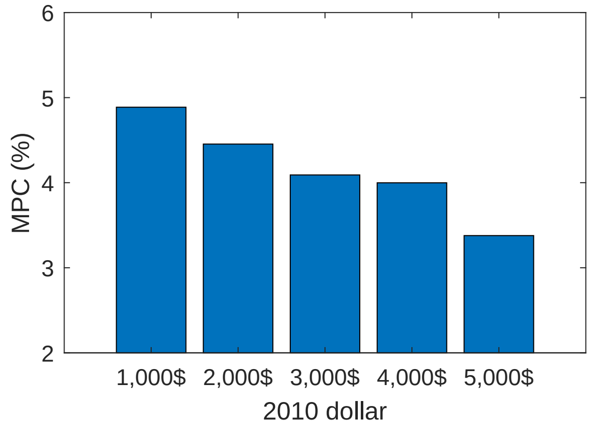
edge, heterogeneous consumption responses to mortgage principal reduction have not been estimated.[52]

Figure 10 shows MPC and population over households ordered by net worth, LTV ratio and liquidity quintile. Liquidity is a ratio of liquid saving over net worth. For MPC, each bar is an average MPC of households in the corresponding quintile. This average includes only households receiving the principal reduction. The upper left panel shows that high LTV households are equally distributed over the 1st - 4th quintiles and MPC falls with as net worth. The bottom right panel shows that households eligible for principal reduction are concentrated in the 1st or 5th quintile based on liquidity. Of these two groups, of households with low liquid wealth show stronger consumption responses. Sorting households by MPC, I find characteristics of high MPC households are consistent with the observations seen in Figure 10. Specifically, top 10% of households tend to be in the 1st-2nd quintiles of net worth and liquid saving, 2nd-3rd quintiles of housing wealth, income and mortgage size.

How do they increase consumption? Figure 10 reveals households with low net worth and low liquidity respond most to the principal reduction. Given that these households are nearly hand-to-mouth, gains in home equity cannot fully explain the large consumption responses. It turns out that these households increase their consumption using refinancing. Approximately 23% of households eligible for principal reduction refinance their mortgage. Two factors contribute to this. First, some of these households were near the borrowing limit (LTV 105%), so refinancing was not available for them. Second, refinancing costs decrease. As explained in section 2, refinancing requires to pay the fixed refinance fee (ξ_r) and repay the remaining mortgage

[52] Ganong and Noel (2018) estimated consumption responses to principal reduction using Administrative data on HAMP participants and consumer credit bureau records but they do not focus on heterogeneity. At the aggregate level, they find principal reduction has no significant impact on borrowers' consumption or default rate which is inconsistent with my result. They compare two groups that both received payment reduction, and the treatment group also received a principal reduction. Moreover, the principal reduction only reduced LTV ratio to 115%, which means borrowers were still underwater after the reduction. Therefore these results are not directly comparable.

Figure 11. MPC over refinancing cost

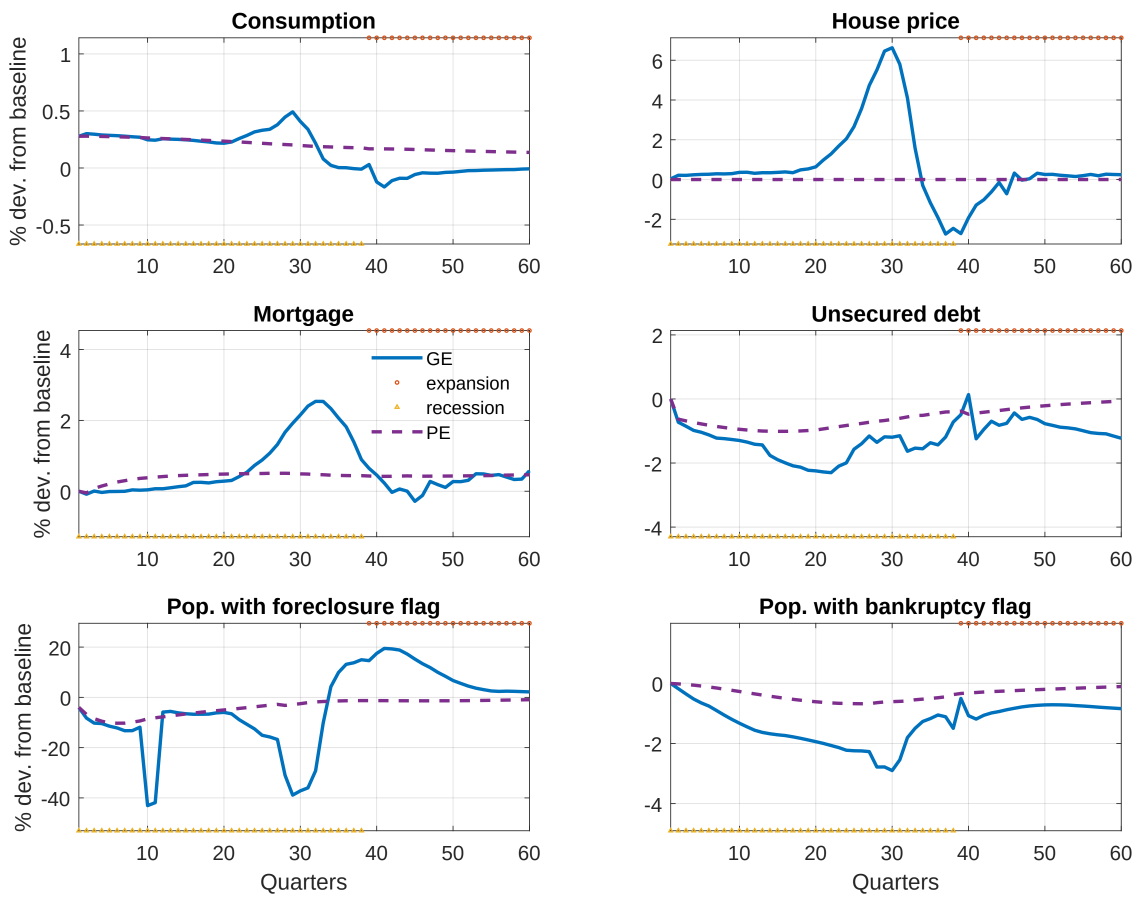


Note: Aggregate MPC over refinance costs. In each case, households received unchanging principal reduction; only refinancing costs are different. The refinancing cost is set to 2,500$ in the baseline calibrated model.

balance first. Besides, a new mortgage is subjected to discount due to default risks. As principal reduction reduce default risks of these households, they can refinance with the lower discount rate, which implies lower total refinancing costs. Figure 11 shows that consumption responds less as the fixed refinancing fee increases.

General equilibrium effects In the periods after to an intervention, households will consume differently compared to households in an economy without the intervention, even if the two households have the same portfolio and income. These differences arise from price responses following the intervention. As Figure 7 shows, the mortgage forgiveness program lead to a rise in aggregate capital. As a result, the interest rate is lower and the wage rate is higher over periods following a policy intervention. Although higher wages benefit all households, the effects of low interest rates are more complex. These benefit households with substantial debt and little liquid savings by reducing their interest payments. Conversely, households with substantial savings are made worse off, since they receive a lower return on their liquid assets. Therefore the overall effect depends on the distribution of households over liquid saving and housing debt. The intervention also keeps house prices higher than the baseline econ-

Figure 12. Responses of aggregate variables: Partial equilibrium



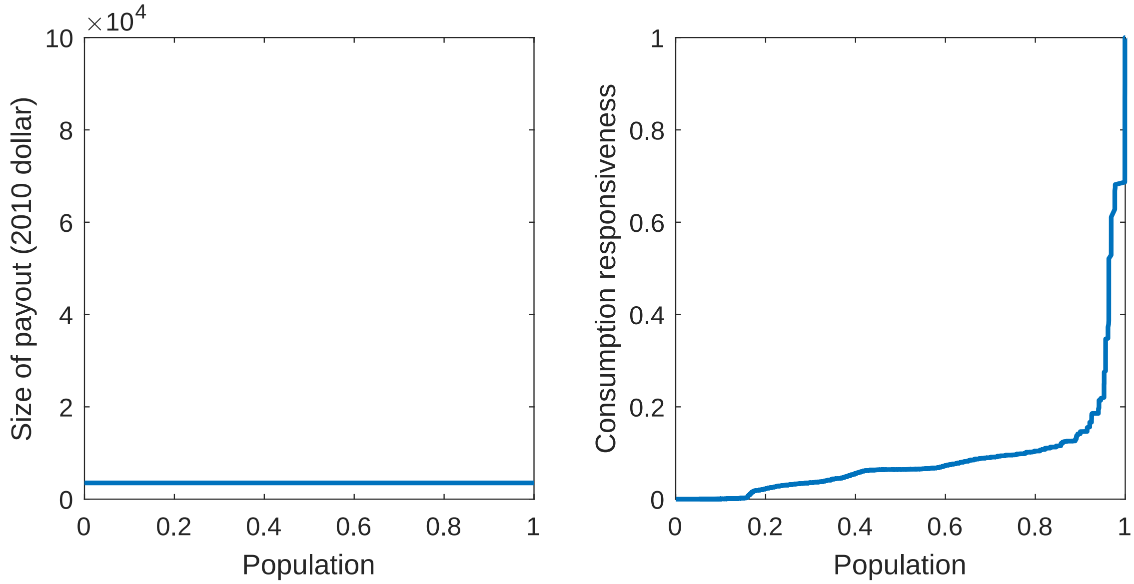
Note: PE responses are computed using price paths (interest rate, wage and house price) from the economy without the policy intervention. GE are the responses under the market clearing prices.

omy for almost 8 years, and the effects here are also complicated in that they benefit sellers at the expense of buyers. For homeowners that do not buy or sell, raised house prices lower disposable income through increases in maintenance costs and property taxes.

To quantify the general equilibrium effects of the principal reduction, I feed in the price paths (interest rate, wage and house price) from the no-intervention baseline and re-compute aggregate responses to the program. Figure 12 shows the responses of aggregate variables with the market clearing prices (GE) and with the baseline no-policy prices (PE). Overall, aggregate responses are smoother and more muted without price changes mentioned above. In general equilibrium, the house price response increases and reaches its peak after about 30 quarters after the intervention, and consumption and mortgage debt co-move with it. The rise in consumption is mainly driven by households that sell their houses rather than defaulting or by households which sell in both scenarios being able to sell at a higher price with the intervention.

Households are disparately affected by the policy's implied price changes depending on their portfolio positions and decisions. Here, the increased wages and house prices and lower interest rates tend to reduce the gains of the wealthy and benefit indebted households. Comparing cumulative consumption for 5 years after the intervention under GE and PE, households that consume more under GE are in the 1st - 2nd quintiles of net worth and high leverage at the intervention period. These households can benefit from the price changes by selling their houses at higher prices and paying lower interest rates on their debt. In contrast, households consuming less under GE have large houses, large savings, and small debt. The equilibrium price changes hurts them, because higher house prices reduce their disposable income through increased maintenance costs and lower interest rates lower the returns on their financial wealth.

Figure 13. Tax rebate size and consumption responsiveness



5.2. *Tax rebate*

In this section, I consider the effects of a lump sum tax rebate selected for comparability with the mortgage forgiveness program studied above. As there, I assume that the policy intervention is unanticipated. All households receive the same transfer, and the total cost is set to match the total cost of the principal reduction. Defining MPC in the same fashion as in the principal reduction exercise, notice from Figure 13 that the MPC distribution is more dispersed compared to that following the targeted mortgage forgiveness program. This is not surprising considering that the rebate is given to all households and is an income that households can use without paying any cost. The characteristics of households exhibiting high MPCs are similar to those under the principal reduction (low net worth, high LTV, and low liquidity). However, the tax rebate program includes households that do not own a house, and among those households, those with little liquid wealth (their only wealth) also have high MPCs.[53]

[53] In Figure 26, this group appears in 2nd quintile of net worth and LTV and in 5th quintile of liquidity. Although they have low wealth, since liquidity is defined as liquid wealth over total wealth, lack of house make them classified as high liquidity group.

Figure 14. Response of aggregate variables

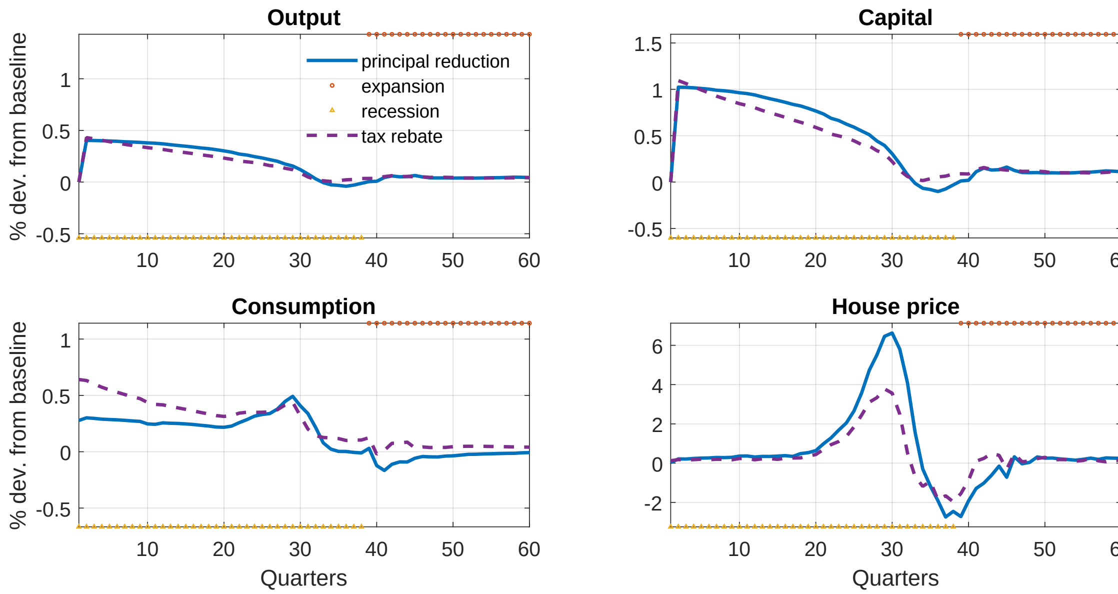
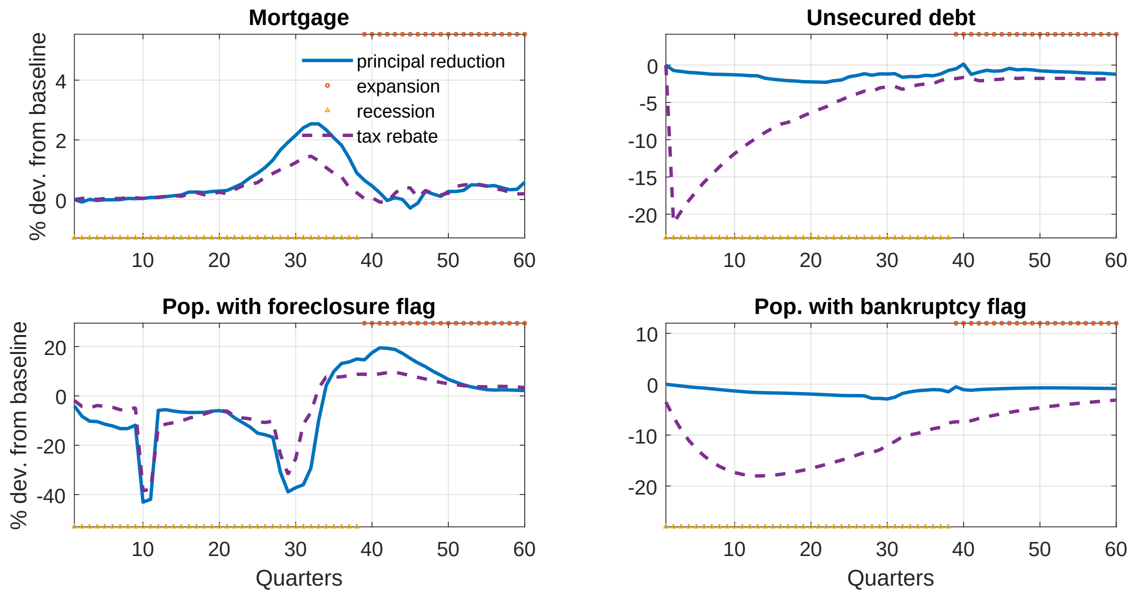


Figure 14 and Figure 15 show the responses of aggregate variables to the tax rebate alongside responses to the principal reduction. The tax rebate is more effective than the principal reduction at boosting consumption, partly because the rebate is added directly to households' liquid wealth, and partly because it distributes a higher fraction of benefits to high MPC households. As seen in Figure 9, the sizes of principal reduction benefits vary greatly across households. Since households with low net worth tend to have small houses and small mortgages, the reduction amount they receive is also small. Therefore, while the consumption responsiveness of each such household is high, their overall contribution to aggregate consumption is low.[54]

The tax rebate is also more effective in reducing bankruptcy and unsecured credit

[54] When such households receive a higher fraction of benefits, aggregate consumption responses to the principal reduction rise. To see this, I compute responses under two alternative specifications of the principal reduction program. First, I add an eligibility criteria; requiring that households be below 60th percentile in the net worth distribution. Second, I maintain the eligibility criteria of the original program, but even the benefits across them, assuming that all eligible households receive an equal reduction. In each case, I set the total cost to match the cost of the original specification, and the alternative plan delivers a larger reduction amount to poor households. Figure 27 and Figure 28 in Appendix C show the responses to these alternative interventions. Both deliver initial consumption responses close to that under the tax rebate, and both are more effective than the baseline principal reduction in reducing foreclosure and bankruptcy.

Figure 15. Response of credits and defaults



by providing income to poor households lacking a house or mortgage, since these are the households most likely to use unsecured borrowing and the bankruptcy option. Conversely, principal reduction is more effective in reducing foreclosure and supporting house prices, because it targets households likely to sell their houses or enter foreclose and provides them additional wealth.

Beyond the Wealthy Hand-to-Mouth A wide body of work has studied consumption response to a tax rebate. In particular, using the U.S. tax rebate episodes of 2001 and 2008, empirical studies find that households spend approximately 25% of rebates on non-durables in the quarter that they are received, counter to what would be suggested by the permanent income hypothesis. [55] Kaplan and Violante (2014) develop a structural model that can replicate high MPCs in response to a tax rebate. The key insight of their theory is that many households choose to be 'wealthy hand-to-mouth': they hold few low-return liquid assets(e.g., cash, checking account) while owning a

[55]See Parker, Souleles, Johnson, and McClelland (2013), Johnson, Parker, and Souleles (2006), Misra and Surico (2014),Sahm, Shapiro, and Slemrod (2010), Agarwal, Liu, and Souleles (2007) and Shapiro and Slemrod (2009).

sizable portfolio of high-return illiquid assets (e.g., housing, retirement account) that requires transaction cost to adjust. Since they are hand-to-mouth, when they receive a windfall gain such as tax rebate, they consume a significant amount of it.

I can further decompose the household portfolio to investigate whether indebtedness affects consumption responses to income shocks. As Misra and Surico (2014) show, homeownership without mortgage does not vary across consumption responsiveness, but mortgage level tends to rise as MPC rises. Conditioning on hand-to-mouth status, can indebtedness explain consumption responsiveness better? Indebtedness may affect a household's consumption decision because it affects disposable income. Highly indebted households have a lower income to spend due to debt repayment compare to households that have the same net worth but no debt. Also, high leverage is riskier because house prices fluctuate.

The answer to this question is yes. The aggregate MPC to tax rebate is 9% and hand-to-mouth homeowners' MPC is 14%. Moreover, MPC of mortgage borrowers is 15%, but MPC of households that own houses outright is 8% among these homeowners.[56] Moreover, Figure 29 shows that MPC rises as leverage rises. The figure plots average MPCs over households' state quintiles for several household states, restricting attention to hand-to-mouth homeowners.

5.3. Does mortgage forgiveness timing matter?

The efficacy of a policy can be state-dependent and the state varies over time.[57] As seen in Figure 6, I assume that the government intervenes promptly. Does late intervention bring a different outcome? To see this, I assume that the government intervenes five years later than the benchmark case. To make it compare to the benchmark

[56] I classified households with liquid saving less than 50% of their labor income as hand-to-mouth households.

[57] Beraja, Fuster, Hurst, and Vavra (2018), Eichenbaum, Rebelo, and Wong (2018) show that effects of monetary policy are state dependent.

Figure 16. Response of aggregate variables

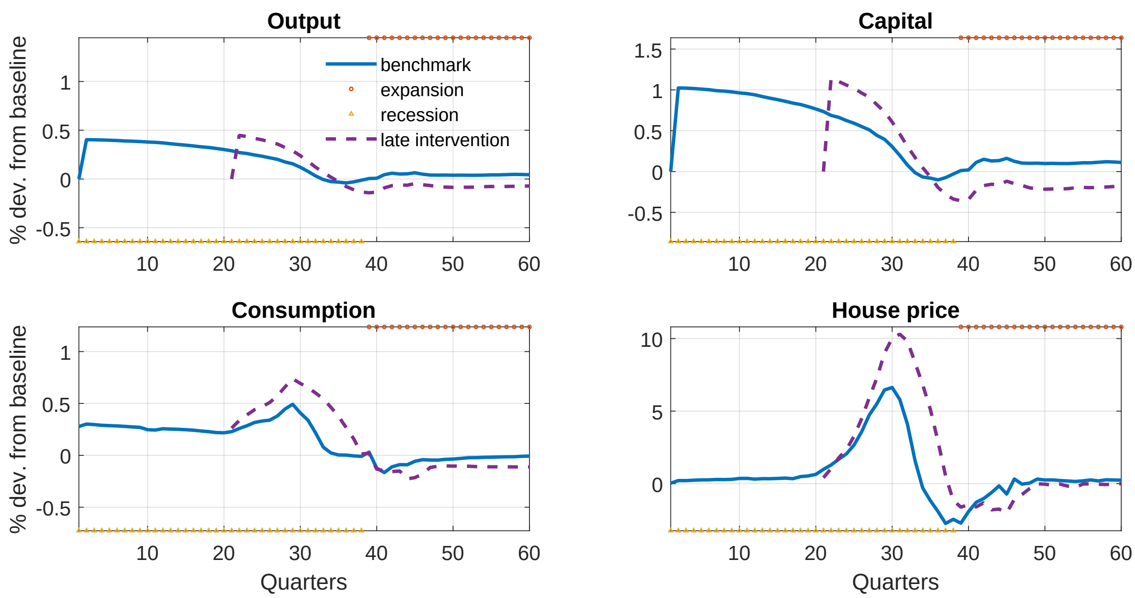
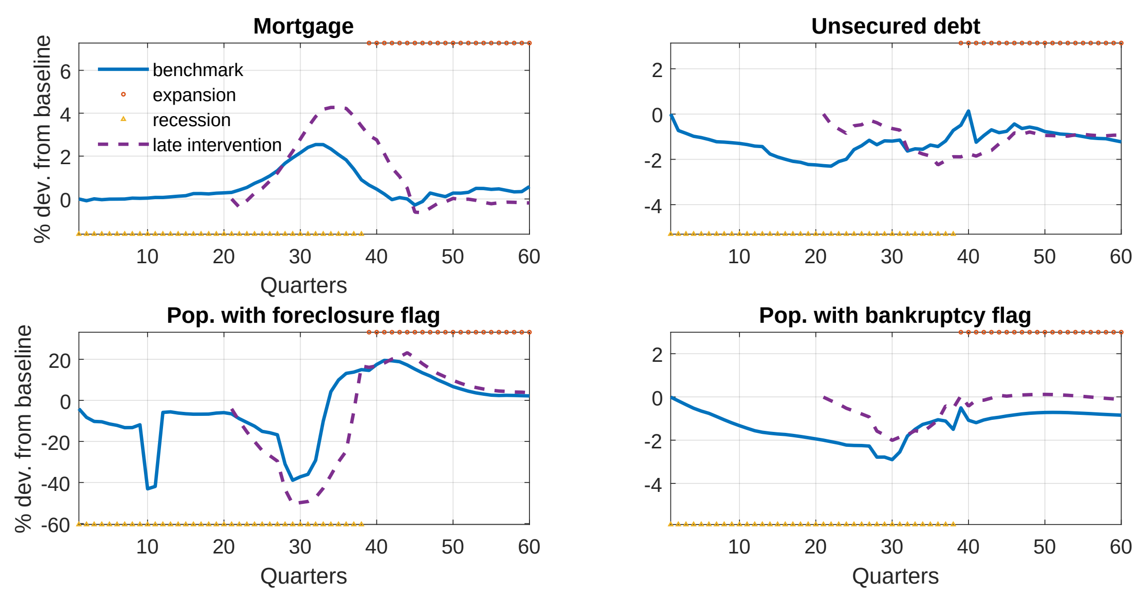


Figure 17. Response of credits and defaults



specification, I choose households that have LTV higher than 95% but adjust reduction amounts to match the total cost at the total cost of the benchmark case. At the time of the intervention, leverage is 1%p higher, 1.2% more households are eligible for principal reduction, and 1.4% more households have unsecured borrowing. Figure 16 and Figure 17 plot the responses of aggregate variables to the late intervention. At first, the responsiveness is slightly lower (10%), but house prices react strongly, which leads to more significant responses in consumption, mortgage and foreclosure. One possibility is that the most financially distressed households already defaulted or sold houses, downward pressure to house prices could have been lower at the time. While the short-run responses are greater than the responses of early intervention, the effects die out fast as the state switch to an expansion. A long-run effect of the policy, which is measured by cumulative consumption, is larger when the government intervenes earlier than later.[58]

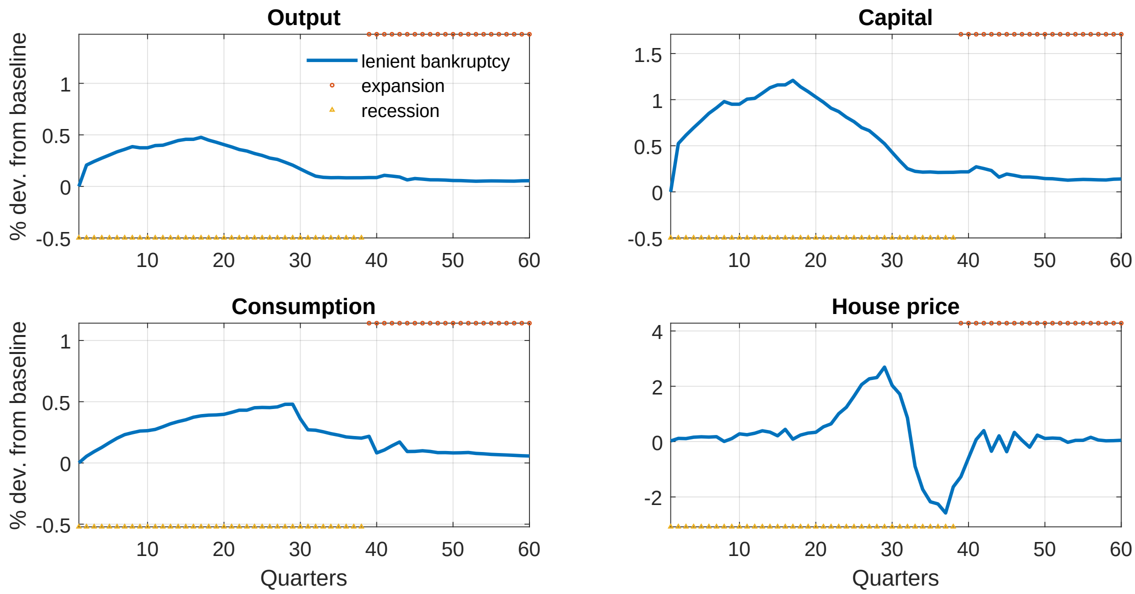
The lessons from a series of policy experiments are that a principal reduction has a sizable impact in the short-run by directly reducing mortgage for eligible households and has an impact in the long-run through price reactions to the policy. In particular, these responses are primarily driven by households that have low net worth and high leverage. A government that aims at reducing foreclosure and supporting falling house prices should recognize that targeting highly leveraged households with a low net worth will yield stronger responses.

5.4. Other debt relief programs

Besides mortgage forgiveness, there can be several policy alternatives. I study the effects of lenient bankruptcy and mortgage payment reduction in this section. I

[58]The cumulative consumption over 30 quarters after the intervention is 0.3% higher than the cumulative consumption without the intervention in case of early intervention. It is 0.2% higher after the late intervention.

Figure 18. Response of aggregate variables



assume that both policies are unexpected interventions. While these policies do not require government funding explicitly, financial intermediaries' losses will increase if bankruptcy or foreclosure rises as a result of the interventions. I assume that the government absorbs such losses.

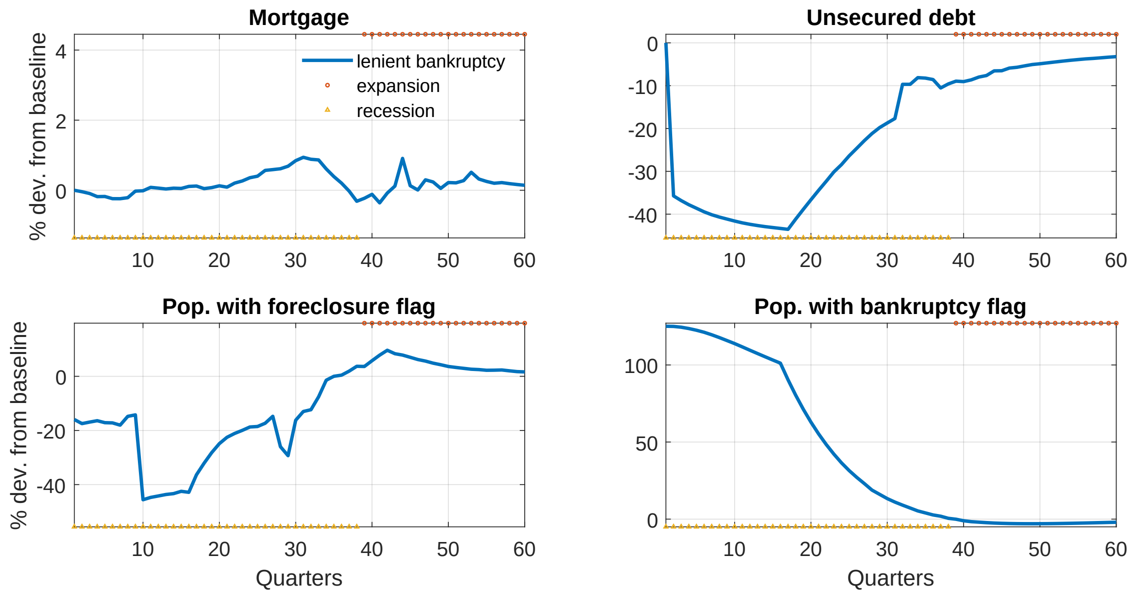
5.4.1. Lenient bankruptcy

Consumer bankruptcy is one of the largest social insurance programs in the U.S. Indeed, Dobbie and Song (2016) show that Chapter 13 protection increases annual earnings, decreases mortality, and decreases foreclosure rates. Given its positive impacts on financially distressed households, it is plausible that the government can mitigate a fall in consumption and output during recessions by reducing the penalties from filing for bankruptcy.[59]

I assume that the government loosens the requirements (e.g. bankruptcy filing

[59] Auclert et al. (2019) show that states with more generous bankruptcy exemptions had significantly smaller declines in non-tradable employment and more substantial increases in unsecured debt write-downs compared to states with less generous exemptions.

Figure 19. Response of credits and defaults



costs) for bankruptcy so that more households can use the system. Specifically, I cut the utility cost ξ_a by half for 16 quarters. Figure 18 and Figure 19 show the impacts of the policy. As intended, more households go bankrupt and more unsecured debt is forgiven as a result (two right panels of Figure 19). If I consider the amount of forgiven unsecured debt due to the intervention as a cost of the policy, it is approximately 54% of the total cost of the principal reduction.

Consistent with the evidence in Dobbie and Song (2016), lenient bankruptcy affects households' foreclosure decisions too. As households substitute foreclosure with bankruptcy, it is as effective as the principal reduction in reducing foreclosure. Capital increases over the intervention periods partly because more debt is forgiven compared to the economy without the intervention and partly because unsecured borrowing is not allowed to households with the bankruptcy flag. Consumption increases gradually by discharging unsecured debt and interest payment on the debt as well as positive income effects from the higher capital. House price responses are similar, but the magnitude is smaller than the responses to the principal reduction.

Figure 20. Response of aggregate variables

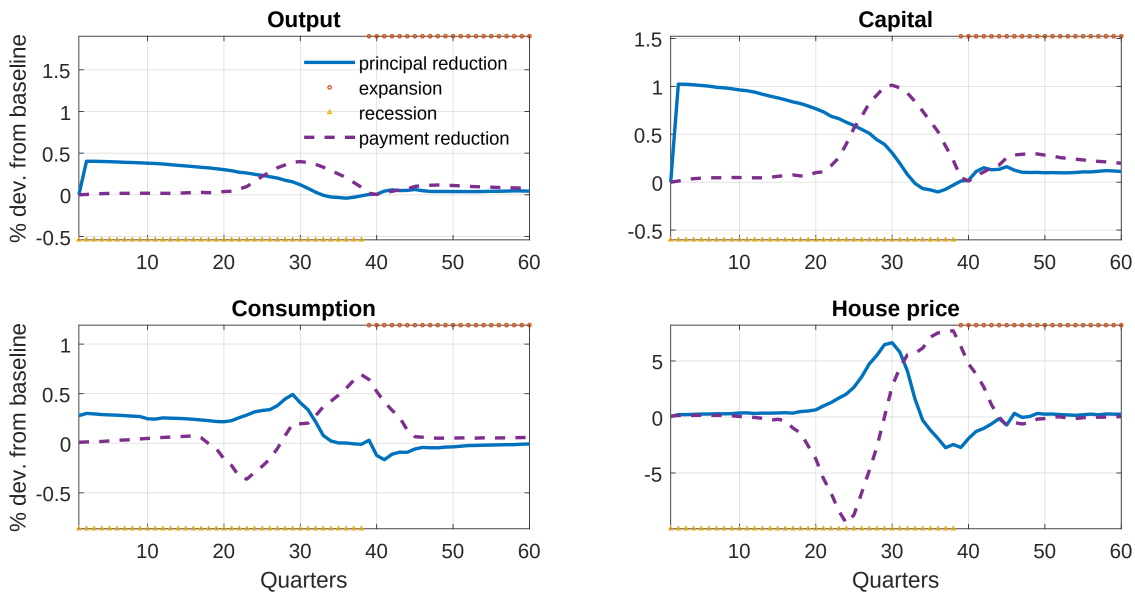
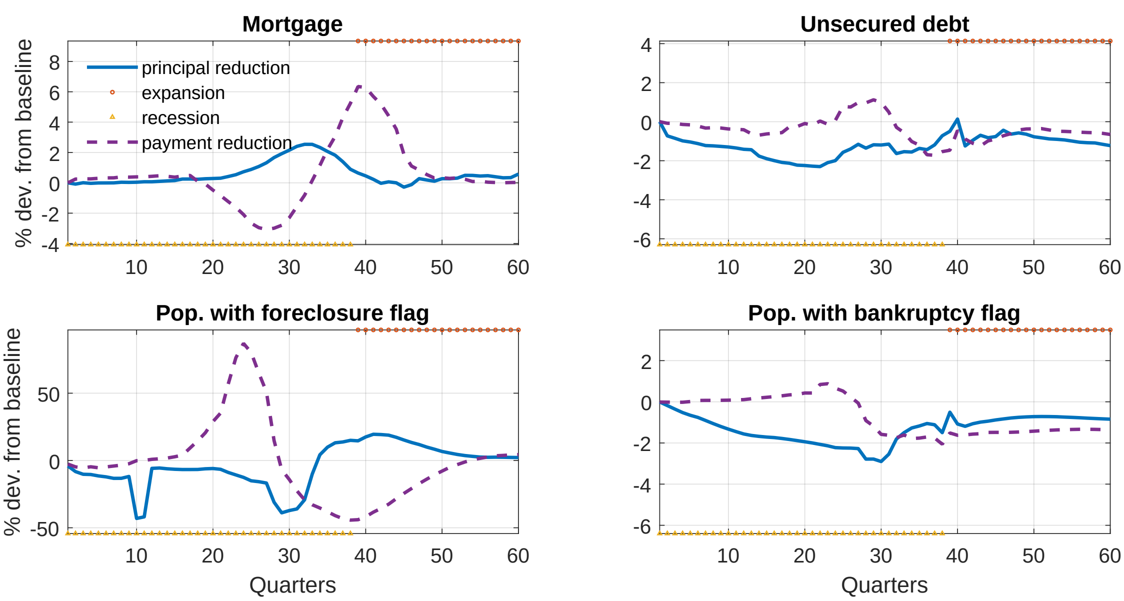


Figure 21. Response of credits and defaults



5.4.2. *Mortgage payment reduction*

As mentioned above, Ganong and Noel (2018) show that principal reduction programs are not effective at reducing foreclosure, but mortgage repayment reductions are. Indarte (2019) also finds that bankruptcy filings respond weakly to changes in the generosity of bankruptcy (asset exemption) but is strongly discouraged by reductions in minimum debt payments. This result implies that providing wealth (principal reduction) will be less effective than providing liquidity (payment reduction) in reducing default rates. While these results are not directly comparable to the results in this work,it is worth studying the effects of payment reduction in my model environment.

Mortgage borrowers are required to repay a fraction of their debt at every period. A size of repayment is set to make an average duration of a loan 30 years if a household fully finances a purchasing of a house. Therefore per period payment depends only on house value. I assume that a per period repayment cannot exceed 31% of a household's labor income for 16 quarters. This intervention effectively extends the duration of a loan by allowing the slow amortization of debt.

Figure 20 and 21 show the responses of the payment reduction. By increasing eligible households' disposable income, aggregate consumption is slightly higher during the intervention periods. Foreclosure rates are also slightly lower during the same periods. However, after the intervention is over, foreclosure rates rise and house prices fall. Because of the slow repayment, eligible households remain slightly more leveraged than in an economy without the intervention. The higher leverage leads to rising foreclosure around the 20th quarters. Rising foreclosure contributes to falling house prices, which induce more foreclosure in subsequent periods.

If we focus on the short-run effects of the policy, we can conclude that it is effective in increasing consumption and in reducing foreclosure. However, if we also consider the long-run, general equilibrium effects, a payment reduction is not more effective than a principal reduction.

6. Concluding remarks

I have quantitatively assessed the effects of debt relief programs during recessions. I build a model with financial assets and unsecured debt, housing and mortgages as well as options to default on both types of borrowing. I then show that a one-time mortgage forgiveness program significantly lowers the foreclosure rate. Debt forgiveness also generates a significant and persistent rise in investment and consumption. Although the policy benefits households who have high loan-to-value ratios initially, the gains are distributed evenly over time as the program dampens the fall in house prices and affects real interest rates.

A fruitful direction for future work would be to integrate the rich household portfolio and default studied here with nominal rigidities. In such an extension, the effects of debt relief programs feed back to employment and output through their effect on demand. Moreover, the model would be able to speak to the literature on the role of household leverage in propagating the Great recession. Both policymakers and researchers have suggested that financial distress resulting from high leverage and falling house prices forced households to reduce spending and led to a reduction in employment.

Appendix A. Computation

A.1. *Viscosity solution for HJBVI equation*

The existence and uniqueness of the viscosity solution of a HJB equation is shown by Crandall and Lions (1983). One might question the existence and uniqueness of the viscosity solution of a HJBVI equation. The existence of the viscosity solution of a HJBVI equation when value function is not always differentiable, which correspond to the problem in the paper, proved in Øksendal and Sulem (2005).[60]

A.2. *Numerical Solution Methodology*

The solution algorithm is based on a finite difference method using upwind scheme in Achdou, Han, Lasry, Lions, and Moll (2017) but with several extensions: Multiple stopping choices - two types of defaults, buying and selling of the illiquid asset, refinancing - and aggregate uncertainty. I describe the details of the solution method in this section. First I describe the solution methodology with the model without the aggregate uncertainty. Section A.2.4 explains how to solve the model with the aggregate uncertainty.

A.2.1. *Solving HJBVI as a Linear Complementarity Problem (LCP)*

To solve the stopping time problem, I transformed the problem as a linear complementarity problem.[61]

The problem can be written as

$$min[\rho v - u - \mathbf{A}v, \;\; v - v^*] = 0$$

[60]Chapter 9, Theorem 9.8.

[61]I referred `http://www.princeton.edu/ moll/HACTproject/option-simple.pdf`.

where **A** summarizes changes caused by decisions and shocks.[62]

$$(v - v^*)'(\rho v - u - \mathbf{A}v) = 0$$

$$\rho v - u - \mathbf{A}v \geq 0$$

$$v \geq v^*$$

Let $z = v - v^*$, $\mathbf{B} = \rho\mathbf{I} - \mathbf{A}$ and $q = -u + \mathbf{B}v^*$. Then

$$(v - v^*)'(\rho v - u - \mathbf{A}v) = 0$$

$$\rho v - u - \mathbf{A}v \geq 0$$

$$v \geq v^*$$

and

$$z'(\mathbf{B}z + q) = 0$$

$$\mathbf{B}z + q \geq 0$$

$$z \geq 0$$

are equivalent. This is the standard form of linear complementarity problem and several solvers are available.[63]

[62]The next section describes how to construct A.

[63]I used `https://www.mathworks.com/matlabcentral/fileexchange/20952-lcp-mcp-solver-newton-based`.

A.2.2. *Solving HJB equation with a finite difference method*

I solve the problem on the discretized grid of the state space. I choose the number of grids $(n_a, n_b, n_\varepsilon, n_h)$ for the corresponding variables, (a, b, ε, h). Let v_{ijkp} be the value function of a household without the bankruptcy or the foreclosure flag, with an liquid asset a_i, an secured debt b_j, labor productivity ε_k and illiquid asset h_p. The derivative with respect to a, v^a_{ijkp} is approximated with either a forward or a backward difference approximation.

$$v^a_{ijkp} \approx \frac{v_{i+1jkp} - v_{ijkp}}{\Delta a} = v^{a,F}_{ijkp} \text{ or } v^a_{ijkp} \approx \frac{v_{ijkp} - v_{i-1jkp}}{\Delta a} = v^{a,B}_{ijkp}$$

Likewise, the derivative with respect to b, v^b_{ijkp} can be approximated with forward or backward numerical derivation.

The equation (5) is approximated

$$\rho v_{ijkp} = u(c_{ijkp}, h_p) + v^a_{ijkp}\dot{a}_{ijkp} + v^b_{ijkp}\dot{b}_{ijkp} + \sum_{\varepsilon'_k} \lambda(\varepsilon_k, \varepsilon_{k'})(v_{ijk'p} - v_{ijkp})$$
$$\forall\, i = \{1, .., n_a\}, j = \{1, .., n_b\}, k = \{1, .., n_\varepsilon\}, p = \{1, .., n_h\}$$

where

$$\dot{a}_{ijkp} = w\varepsilon_k + r_{a,ijkp}a_i - (r + \theta(b_j, h_p))b_j p h_p - c_{ijkp} - T(b_j, ph_p, \varepsilon_k) - \xi_h p h_p$$

$$\dot{b}_{ijkp} = \theta(b_j, h_p)b_j$$

Household choice of non-durable consumption can be solved from the FOC.

$$u^c(c_{ijkp}, h_p) - v^a_{ijkp} = 0$$
$$c_{ijkp} = (v^a_{ijkp})^{\frac{1}{-\sigma}}$$

As the derivatives of value function has two forms, forward and backward, c_{ijkp} is either $(v^{a,F}_{ijkp})^{\frac{1}{-\sigma}}$ or $(v^{a,B}_{ijkp})^{\frac{1}{-\sigma}}$.

Upwinding and Boundaries To find drifts $\dot{a}$, it is necessary to select which derivative to use.[64] I follow Achdou et al.'s (2017) upwind scheme. The key idea is using a forward derivative when a drift is positive and use a backward derivative when a drift is negative. To ease the notation, let $x^+ = max(x,0)$ and $x^- = min(x,0)$. Also let x^F be the computed value using a forward derivative and x^B be the computed value using a backward derivative. With these notations, saving can be computed as below:

$$s^{c,F}_{ijkp} = w\varepsilon_k + r_{a,ijkp}a_i - (r+\theta(b_j,h_p))b_j ph_p - c^F_{ijkp} - T(b_j, ph_p, \varepsilon_k) - \xi_h ph_p$$
$$s^{c,B}_{ijkp} = w\varepsilon_k + r_{a,ijkp}a_i - (r+\theta(b_j,h_p))b_j ph_p - c^B_{ijkp} - T(b_j, ph_p, \varepsilon_k) - \xi_h ph_p$$

Finally, let v^n be the value function at the nth iteration. Then an upwind finite difference approximation is given by[65]

$$\begin{aligned}\frac{v^{n+1}_{ijkp} - v^n_{ijkp}}{\Delta} = u(c_{ijkp}, h_p) &+ \frac{v^{n+1}_{i+1jkp} - v^{n+1}_{ijkp}}{\Delta a}(s^{c,F})^+ + \frac{v^{n+1}_{ijkp} - v^{n+1}_{i-1jkp}}{\Delta a}(s^{c,B})^- \\ &+ \frac{v^{n+1}_{ij+1kp} - v^{n+1}_{ijkp}}{\Delta b}\theta(b_j,h_p)b_j^+ + \frac{v^{n+1}_{ijkp} - v^{n+1}_{ij-1kp}}{\Delta b}\theta(b_j,h_p)b_j^- \\ &+ \sum_{\varepsilon_k'} \lambda(\varepsilon_k, \varepsilon_{k'})(v^{n+1}_{ijk'p} - v^{n+1}_{ijkp}) - \rho v^{n+1}_{ijkp}\end{aligned} \tag{20}$$

This scheme satisfies the Barles-Souganidis monotonicity condition: Let S the right hand side of the equation (20). S is non-decreasing in v_{i+1jkp}, v_{i-1jkp}, v_{ij+1kp}, v_{ij-1kp}, v_{ijk+1p} and v_{ijk-1p}.

The upwind scheme can be applied to the points in the state space except for the points on the boundaries. Clearly, only forward or backward derivative can be computed on the boundaries. The way of handling the exogenous borrowing constraints

[64] $\dot{b}$ is a fraction of b, there is no need to solve it.
[65] I use implicit method to update a value function. See Achdou et al. (2017) for details.

in a one asset model is well explained in Achdou et al. (2017). In my model, there are three asset, and there is an exogenous borrowing limit for b but the limit can be endogenous with respect to a thanks to the option to default. I explain the details of handling boundaries in the rest of the section.

I referred Bornstein (2018) to take care of endogenous borrowing limit with respect to a. For each $(b_j, \varepsilon_k, h_p)$, $\forall\, j = \{1, .., n_b\}, k = \{1, .., n_\varepsilon\}, p = \{1, .., n_h\}$, if a household choose to default on a below $\underline{a}(b_j, \varepsilon_k, h_p)$, a value is approximated

$$\begin{aligned}\frac{v^{n+1}_{ijkp} - v^n_{ijkp}}{\Delta} + \rho v^{n+1}_{ijkp} =& u(c^n_{ijkp}, h_p) + v^{a,n+1}_{ijkp}\dot{a}_{ijkp} + v^{m,n+1}_{ijkp}\dot{b}_{ijkp} \\ &+ \sum_{\varepsilon'_k} \lambda(\varepsilon_k, \varepsilon_{k'})(v^{n+1}_{ijk'p} - v^{n+1}_{ijkp}) \quad \text{for } a_i > \underline{a}^n(b_j, \varepsilon_k, h_p)\end{aligned}$$

$$v^{n+1}_{ijkp} = v^{n+1}_{\underline{a}^n jkp} \quad \text{for } a_i \leq \underline{a}^n(b_j, \varepsilon_k, h_p)$$

Therefore, backward derivatives does not exist below $\underline{a}(b_j, \varepsilon_k, h_p)$. I impose the endogenous borrowing constraint as $u^c(c_{\underline{a}^n jkp}) = v^{a,B}_{\underline{a}^n jkp}$ where $c_{\underline{a}^n jkp}$ is $w\varepsilon_k + r_{a,ijkp}\underline{a}(b_j, \varepsilon_k, h_p) - (r + \theta(b_j, h_p))b_j p h_p - T(\cdot) - \xi_h p h_p$ when it is binding.

At a_{n_a}, a forward derivative cannot be computed. However, if I set high enough value for a_{n_a} saving at the point will be negative and the forward derivative will not be used.

A drift of b is a fraction of b and is always positive. Its value is not computed using derivative of the value function and there is no need to determine the upwind scheme for this variable.

Equation (20) can be written as below:

$$\begin{aligned}\frac{v^{n+1}_{ijkp} - v^n_{ijkp}}{\Delta} + \rho v^{n+1}_{ijkp} =& u(c_{ijkp}, h_p) + x^a_{ijkp} v^{n+1}_{i-1jkp} + y_{ijkp} v^{n+1}_{ijkp} + z^a_{ijkp} v^{n+1}_{i+1jkp} \\ &+ x^b_{ijkp} v^{n+1}_{ij-1kp} + z^b_{ijkp} v^{n+1}_{ij+1kp} + \sum_{k'} \lambda_{kk'} v^{n+1}_{ijk'p}\end{aligned} \tag{21}$$

$$x^a_{ijkp} = -\frac{(s^{c,B})^-}{\Delta a} \qquad x^b_{ijkp} = -\frac{(\theta(b,h)b)^-}{\Delta b}$$

$$y_{ijkp} = -\frac{(s^{c,F})^+}{\Delta a} + \frac{(s^{c,B})^-}{\Delta a} - \frac{(\theta(b,h)b)^+}{\Delta b} + \frac{(\theta(b,h)b)^-}{\Delta b} + \lambda_{kk}$$

$$z^a_{ijkp} = \frac{(s^{c,F})^+}{\Delta a} \qquad z^b_{ijkp} = \frac{(\theta(b,h)b)^+}{\Delta b}$$

We can write the equation (21) for each points. The $n_a \times n_b \times n_\varepsilon \times n_h$ linear equations can be written in a matrix notation:

$$\frac{\mathbf{v}^{n+1} - \mathbf{v}^n}{\Delta} + \rho \mathbf{v}^{n+1} = \mathbf{u}^n + \mathbb{A}\mathbf{v}^{n+1} \tag{22}$$

Making the matrix $\mathbb{A}$ requires several steps. First, let's focus on a fraction for each level of h.

For each h, $\mathbb{A}_p, p \in \{1, 2, ..., n_h\}$ can be written $\mathbb{A}_p = \begin{bmatrix} A_{11|p} & A_{12|p} & .. & A_{1n_\varepsilon|p} \\ A_{21|p} & A_{22|p} & .. & A_{2n_\varepsilon|p} \\ \vdots & \ddots & & \vdots \\ A_{n_\varepsilon 1|p} & A_{n_\varepsilon 2|p} & .. & A_{n_\varepsilon n_\varepsilon|p} \end{bmatrix}$ where $A_{kk|p}$ is a matrix that is composed of x^a_{ijkp}, x^b_{ijkp},y_{ijkp}, z^a_{ijkp} and z^b_{ijkp}, $k \in \{1, 2, ..., n_\varepsilon\}$. For example,

$$A_{11|2} = \begin{bmatrix}
y_{1112} & z^a_{1112} & 0 & .. & 0 & z^b_{1112} & 0 & .. & 0 & 0 & 0 & .. & & 0 \\
x^a_{2112} & y_{2112} & z^a_{2112} & 0 & .. & 0 & z^b_{2112} & 0 & ... & 0 & 0 & 0 & .. & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\
0 & .. & 0 & x^a_{n_a112} & y^a_{n_a112} & 0 & .. & 0 & z^b_{n_a112} & & 0 & ... & 0 & 0 \\
x^b_{1212} & .. & & & 0 & y_{1212} & z^a_{1212} & 0 & .. & 0 & z^b_{1212} & 0 & & 0 \\
\vdots & x^b_{2212} & \ddots & \ddots & \ddots & x^a_{2212} & y_{2212} & z^a_{2212} & \ddots & \ddots & \ddots & z^b_{2212} & \ddots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\
0 & .. & & & & & & x^a_{n_an_b12} & y^a_{n_an_b12} & 0 & .. & & 0 & z^b_{n_an_b12}
\end{bmatrix}$$

and when $k \neq l$, $A_{kl|p}$ is a diagonal matrix where diagonal terms are λ_{kl}. Then $\mathbb{A}$ is a block diagonal matrix which is composed of $\mathbb{A}_1$,..,$\mathbb{A}_{n_h}$.

$$\mathbb{A} = \begin{bmatrix} \mathbb{A}_1 & 0 & .. & & 0 \\ 0 & \mathbb{A}_2 & 0 & .. & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & .. & & 0 & \mathbb{A}_{n_h} \end{bmatrix}$$

By the same fashion, linear equations for the value with the bankruptcy or the foreclosure flag can be written as

$$\frac{\mathbf{v}^{d,n+1} - \mathbf{v}^{d,n}}{\Delta} + \rho \mathbf{v}^{d,n+1} = \mathbf{u}^{d,n} + \mathbb{A}^d \mathbf{v}^{d,n+1} \tag{23}$$

$$\frac{\mathbf{v}^{f,n+1} - \mathbf{v}^{f,n}}{\Delta} + \rho \mathbf{v}^{f,n+1} = \mathbf{u}^{f,n} + \mathbb{A}^f \mathbf{v}^{f,n+1} \tag{24}$$

Because $a \geq 0$ with the bankruptcy flag, values are not defined below $a = 0$. Therefore the number of equation is $n_{a+} \times n_b \times n_\varepsilon \times n_h$ where n_{a+} is the number of a grid where $a \geq 0$. In the state with the foreclosure flag values are defined where $h = 0$, $b = 0$ so the number of equation is $n_a \times n_\varepsilon$.

Let $z = \mathbf{v}^{n+1} - \mathbf{v}^{*,n}$, $\mathbf{B} = \frac{1}{\Delta} - \rho - \mathbb{A}$ and $q = \mathbf{B}\mathbf{v}^{*,n} - \frac{\mathbf{v}^n}{\Delta} - u^n$. Then

$$(\mathbf{v}^{n+1} - \mathbf{v}^{*,n})(\frac{\mathbf{v}^{n+1} - \mathbf{v}^n}{\Delta} - \rho \mathbf{v}^{n+1} - \mathbf{u}^n - \mathbb{A}\mathbf{v}^{n+1}) = 0$$

$$\frac{\mathbf{v}^{n+1} - \mathbf{v}^n}{\Delta} - \rho \mathbf{v}^{n+1} - \mathbf{u}^n - \mathbb{A}\mathbf{v}^{n+1} \geq 0$$

$$\mathbf{v}^{n+1} - \mathbf{v}^{*,n} \geq 0$$

and

$$z'(\mathbf{B}z + q) = 0$$

$$\mathbf{B}z + q \geq 0$$

$$z \geq 0$$

are equivalent.

A.2.3. Kolmogorov Forward equation

Without the stopping decisions, the Kolmogorov Forward equation is

$$\partial_t g_{ijkp,t} = -\partial_a s^a_{ijkp} g_{ijkp,t} - \partial_b s^b_{ijkp} g_{ijkp,t} + \sum_{k'} \lambda_{k'k} g_{ijk'p,t}$$

where s^x_{ijkp} is shorthand notation for x decision rule at $(a_i, b_j, \varepsilon_k, h_p)$, $x \in \{a, b\}$ and $g_{ijkp,t}$ is a density function at time t. In my model, i) shifts due to h transaction, refinancing, bankruptcy and foreclosure ii) flows between a state without the flags and a state with the bankruptcy flag iii) flows between a state without the flags and a state with the foreclosure flag are need to be taken care of.

The mathematical formulation of Kolmogorov Forward equations due to h transaction, refinancing, bankruptcy and foreclosure is not straightforward.[66] However, there is a way of handling the stopping decisions in computation. Flows due to stopping decisions can be treated with the 'intervention matrix', M.[67]

First, let g_i be the ith element of the density function where $i \in \{1, ...n\}$ and n is the total number of grid points.[68]

$$M_{i,j} = \begin{cases} 1 & \text{if } i \in I \text{ and } i = j \\ 1 & \text{if } i \notin I \text{ and } j^*(i) = j \\ 0 & \text{otherwise} \end{cases}$$

[66] 'Liquid and Illiquid Assets with Fixed Adjustment Costs' by Greg Kaplan, Peter Maxted and Benjamin Moll
http://www.princeton.edu/ moll/HACTproject/liquid-illiquid-numerical.pdf

[67] I followed the approach in http://www.princeton.edu/ moll/HACTproject/liquid-illiquid-numerical.pdf.

[68] $n = n_a \times n_b \times n_\varepsilon \times n_h \times 2 + n_a \times n_\varepsilon$. $n_a \times n_b \times n_\varepsilon \times n_h$ is multiplied by 2 because there are points with and without bankruptcy flag. $n_a \times n_\varepsilon$ is the number of points in the state with the foreclosure flag.

where I is the non-stopping region and $j^*(i)$ is the target point of point i.

The flow from a state with the bankruptcy/foreclosure flag to a state without the bankruptcy flag is a shock and can be expressed as below. Let nd means 'non-default', and d means 'default'.

$$\partial_t g^{nd}_{ijkp,t} = -\partial_a s^{a,nd}_{ijkp} g^{nd}_{ijkp,t} - \partial_b s^{b,nd}_{ijkp} g^{nd}_{ijkp,t} + \sum_{k'} \lambda_{k'k} g^{nd}_{ijk'p,t} + \lambda_l g^{d}_{ijkp,t}$$

$$\partial_t g^{d}_{ijkp,t} = -\partial_a s^{a,d}_{ijkp} g^{d}_{ijkp,t} - \partial_b s^{b,d}_{ijkp} g^{d}_{ijkp,t} + \sum_{k'} \lambda_{k'k} g^{d}_{ijk'p,t} - \lambda_l g^{d}_{ijkp,t}$$

where $\lambda_l = \lambda_d$ for the bankruptcy flag state and $\lambda_l = \lambda_f$ for the foreclosure flag state. These flows can be treated with the matrix A^d and A^f.

$$A^d_{i,j} = \begin{cases} \lambda_d & \text{if } 1 \leq j \leq n_1 \text{ and } i = j + n_1 \\ -\lambda_d & \text{if } n_1 + 1 \leq j \leq n_1 \times 2 \text{ and } i = j \\ 0 & \text{otherwise} \end{cases}$$

$$A^f_{i,j} = \begin{cases} \lambda_f & \text{if } 1 \leq j \leq n_a \times n_\varepsilon \text{ and } i = j + n_1 \times 2 \\ -\lambda_f & \text{if } n_1 \times 2 + 1 \leq j \leq n \text{ and } i = j \\ 0 & \text{otherwise} \end{cases}$$

where $n_1 = n_a \times n_b \times n_\varepsilon \times n_h$. Size of A^d and A^f are $n \times n$ and I stack points from the state without the flags, the bankruptcy flag state and to the foreclosure flag.[69] Lastly, define B as below:

$$B = \mathcal{A} + A^d + A^f$$

[69] n_1 is the number of points in the state without the flags and $n_1 \times 2$ is sum of the number of points of the no flag state and the bankruptcy flag state.

where $\mathcal{A}$ be a block diagonal matrix which is composed of $\mathbb{A}$, $\mathbb{A}^d$ and $\mathbb{A}^f$.

$$\mathcal{A} = \begin{bmatrix} \mathbb{A} & 0 & 0 \\ 0 & \mathbb{A}^d & 0 \\ 0 & 0 & \mathbb{A}^f \end{bmatrix}$$

Given M and B, the density function can be solved by iterating below two steps until g converges.

$$g^{n+\frac{1}{2}} = M^T g^n$$

$$\frac{g^{n+1} - g^{n+\frac{1}{2}}}{\Delta t} = (BM)^T g^{n+1}$$

A.2.4. Stochastic model

Households problems have an argument that has an infinite dimension, g. To make the computation feasible, the distribution needs to be approximated. I assume the households only track a finite set of moments of g_t to form their expectations as in Krusell and Smith (1998).[70] Specifically, I assume that the households keep track of the aggregate capital, k and illiquid asset price.

I redefine the problem on the approximated aggregate states, (k, p_{-t}, z) where $p_{-t} = pt - dp_{-t}dt$. For example, equation (5) can be written as

$$\begin{aligned}
\rho v(a, b, \varepsilon, h, k, p_{-t}, z) = & \max_c u(c, h) + \partial_a v(a, b, \varepsilon, h, k, p_{-t}, z)\dot{a} + \partial_b v(a, b, \varepsilon, h, k, p_{-t}, z)\dot{b} \\
& + \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v(a, b, \varepsilon_j, h, k, p_{-t}, z) + \sum_{k=1}^{n_z} \lambda_{zz_k} v(a, b, \varepsilon, h, k, p_{-t}, z_k) \\
& + \partial_k v(a, b, \varepsilon, h, k, p_{-t}, z)\dot{k}.
\end{aligned}$$

[70] Ahn et al. (2018) develop a method of solving continuous time heterogeneous agent models with aggregate uncertainty based on linearization and dimension reduction. Fernández-Villaverde, Hurtado, and Nuno (2019) also presents a method to solve such models. They assume the households only track a finite set of moments of the distribution to form their expectations as well, but use tools from machine learning to estimate the perceived law of motion of the households.

$$\dot{k}_t = \frac{\mathbb{E}[dk_t; k_t, z_t]}{dt} = f^k(k_t, p_{-t}, z_t) \quad p_t = f^p(k_t, p_{-t}, z_t)$$

The last term in the equation (5) which captures the evolution of the distribution is replaced with $\partial_k v(a, b, \varepsilon, h, k, z)\dot{k}$. I assume f as a log linear form.

$$dln(k_t)dt = \beta_z^0 + \beta_z^1 ln(p_t) + (\beta_z^2 - 1)ln(k_t)$$

On a approximated aggregated state, $p(k, p_{-t}, z)$ is necessary to solve the model. I also assume the log linear form to estimate the h price.

$$dln(p_t)dt = \phi_z^0 + (\phi_z^1 - 1)ln(p_t) + \phi_z^2 ln(k_t + dln(k_t)dt)$$

Solution algorithm

1. Guess parameters of the forecasting functions. With the forecasting functions, $\dot{k}$ and h price over (k, p_{-t}, z) can be forecasted. Also, interest rate and wage over (k, p_{-t}, z) can be computed using the firm's marginal conditions.
2. Solve the value function.
   - Guess the loan price functions, $r_a(a, b, \varepsilon, h, k, p_{-t}, z)$ and $q(a, b, \varepsilon, h, k, p_{-t}, z)$.
   - Guess the value function, $v^0(a, b, \varepsilon, h, k, p_{-t}, z)$, $v^{0,d}(a, b, \varepsilon, h, k, p_{-t}, z)$ and $v^{0,f}(a, \varepsilon, k, p_{-t}, z)$.
   - Update the value functions and the loan price schedules until they converge.
   - Save decision rules.
3. Simulate the model for n periods. Simulation gives the sequence of aggregate variables $\{z_t, k_t, p_t\}_{t=1}^n$.

- Guess the initial distribution. The distribution in the steady state can be a good initial distribution.
- At the beginning of each period, (k, z) are known. The risk free rate and wage can be computed.
- Compute the loan price functions. To compute $r_a(a_t, b_t, \varepsilon_t, h_t, k_t, z_t)$, interpolate the default decisions that are obtained from the step 2. Using the default decisions and the risk free rate, $r_a(a_t, b_t, \varepsilon_t, h_t, k_t, z_t)$ can be computed using the equation (15). To compute the $q(a_t, b_t, \varepsilon_t, h_t, k_t, z_t)$, interpolate the $q(a, b, \varepsilon, h, k, z)$ that are obtained from the step 2 over k.
- Guess the h price.
- With the wage, the loan price schedules and the h price, solve the household problem.
- Compute the h demand. If the aggregate demand is not close enough to the supply, adjust h price to clear h market.
- Once the h market is cleared, move to the next period

4. Using the sequence of aggregate variables $\{z_t, k_t, p_t\}_{t=1}^{n}$, update the forecasting function.
5. Check convergence of the simulated aggregate variables. If the distance between the $\{k_t\}_{t=1}^{n}$ from the current iteraion and the previous iteration is less than the tolerance level, an approximate recursive equilibrium is obtained. Otherwise, go back to step 2 with the updated forecasting functions.

Appendix B. Consumption equivalent gain of default options

$$\begin{aligned}
\rho v_{benchmark} &= \rho v_i + x \\
&= u(c_i, h) + \mathbb{A} v_i + x \\
&= u(c_i + c_i^*, h) + \mathbb{A} v \\
&= u(c_i + c_i^*) + u(h) + \mathbb{A} v + u(c_{i)} - u(c_i) \\
&= u(c_i + c_i^*) + \rho v_i - u(c_i)
\end{aligned}$$

Since the utility function is separable in c and h, $u(c_i + c_i^*, h) = u(c_i + c_i^*) + u(h)$.

$$\rho v_{benchmark} - \rho v_i + u(c_i) = u(c_i + c_i^*)$$

$$((1-s)(\rho v_{benchmark} - \rho v_i + u(c_i)))^{\frac{1}{1-s}} = c_i + c_i^*$$

$$\frac{((1-s)(\rho v_{benchmark} - \rho v_i + u(c_i)))^{\frac{1}{1-s}}}{c_i} - 1 = \frac{c_i^*}{c_i}$$

Appendix C. Additional tables and figures

Table 9: Share of asset and debt by quintile

	Asset		Debt	
	Non-financial	Financial	Secured	Unsecured
Data				
Q1	2.88	0.32	-10.47	-18.77
Q2	4.27	0.82	-9.77	-14.62
Q3	10.62	2.52	-19.27	-19.17
Q4	17.36	7.99	-19.38	-19.50
Q5	64.88	88.34	-41.13	-27.96
Model				
Q1	1.77	-0.72	3.36	96.81
Q2	4.91	1.85	9.31	0.23
Q3	10.87	7.34	20.14	0.10
Q4	23.21	17.67	30.07	1.78
Q5	59.24	73.85	37.12	1.09

Data: SCF (2010)

Table 10: Portfolio composition

	Asset		Debt	
	Non-financial	Financial	Secured	Unsecured
Data				
Q1	-458.48	-43.45	559.34	42.59
Q2	288.18	46.91	-221.05	-14.04
Q3	176.53	35.47	-107.46	-4.54
Q4	100.13	38.96	-37.49	-1.60
Q5	51.76	59.57	-11.01	-0.32
Model				
Q1	-81.87	44.60	90.06	47.21
Q2	245.44	123.93	-269.25	-0.12
Q3	120.18	108.68	-128.85	-0.01
Q4	78.81	80.32	-59.06	-0.06
Q5	43.36	72.37	-15.72	-0.01

Data: SCF (2010)

Table 11: Portfolio composition (not excluding businsess and student loan)

	Asset		Debt	
	Non-financial	Financial	Secured	Unsecured
Q1	-258.70	-28.70	364.90	22.50
Q2	298.30	52.10	-237.40	-13.00
Q3	177.20	35.40	-107.60	-5.00
Q4	98.30	37.40	-34.00	-1.70
Q5	63.50	45.90	-8.90	-0.50

Data: SCF (2010)

Figure 22. The model with bankruptcy option only. Distribution ($\varepsilon = 3, h = 2$)

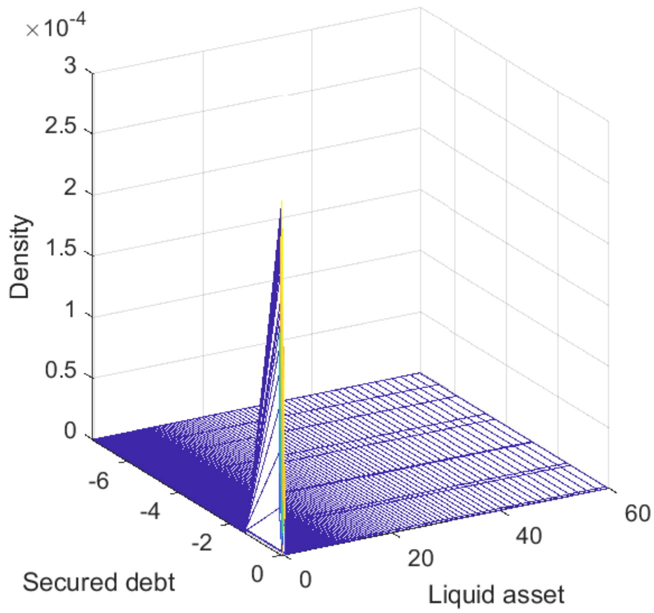


Figure 23. The model with bankruptcy option only. Distribution ($\varepsilon = 3, h = 6$)

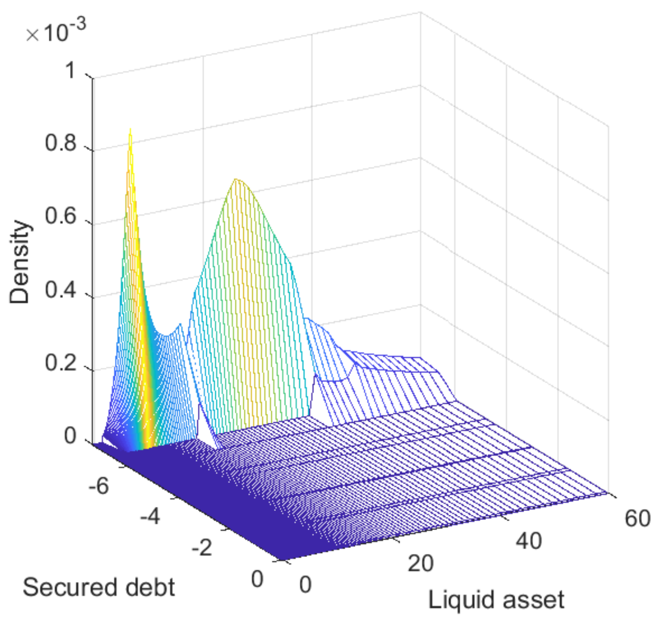


Figure 24. The model with foreclosure option only. Distribution ($\varepsilon = 2$,$h = 2$)

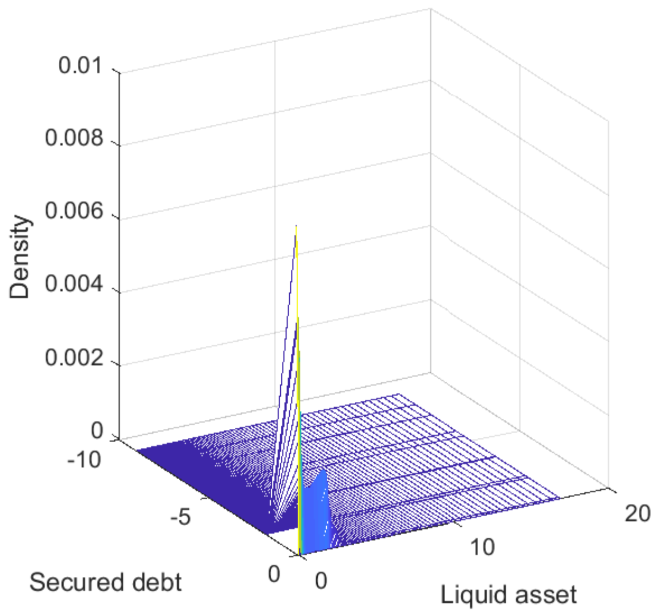


Figure 25. The model with foreclosure option only. Distribution ($\varepsilon = 3$,$h = 5$)

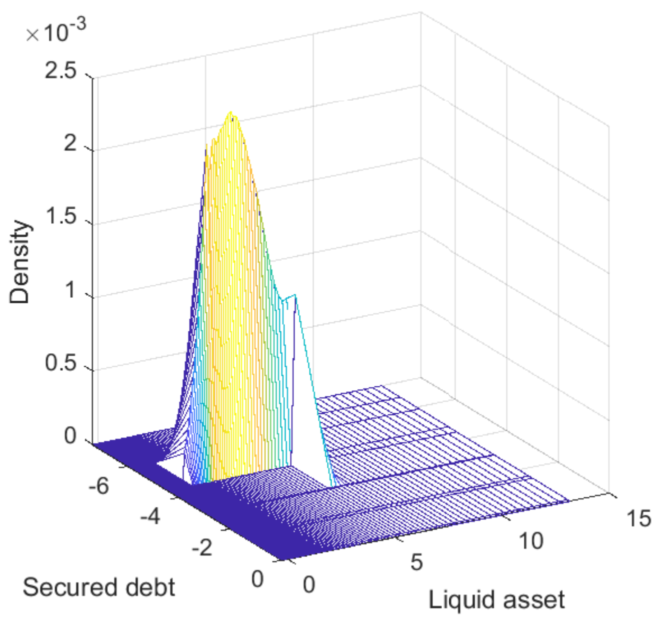


Figure 26. MPC and population (Tax rebate)

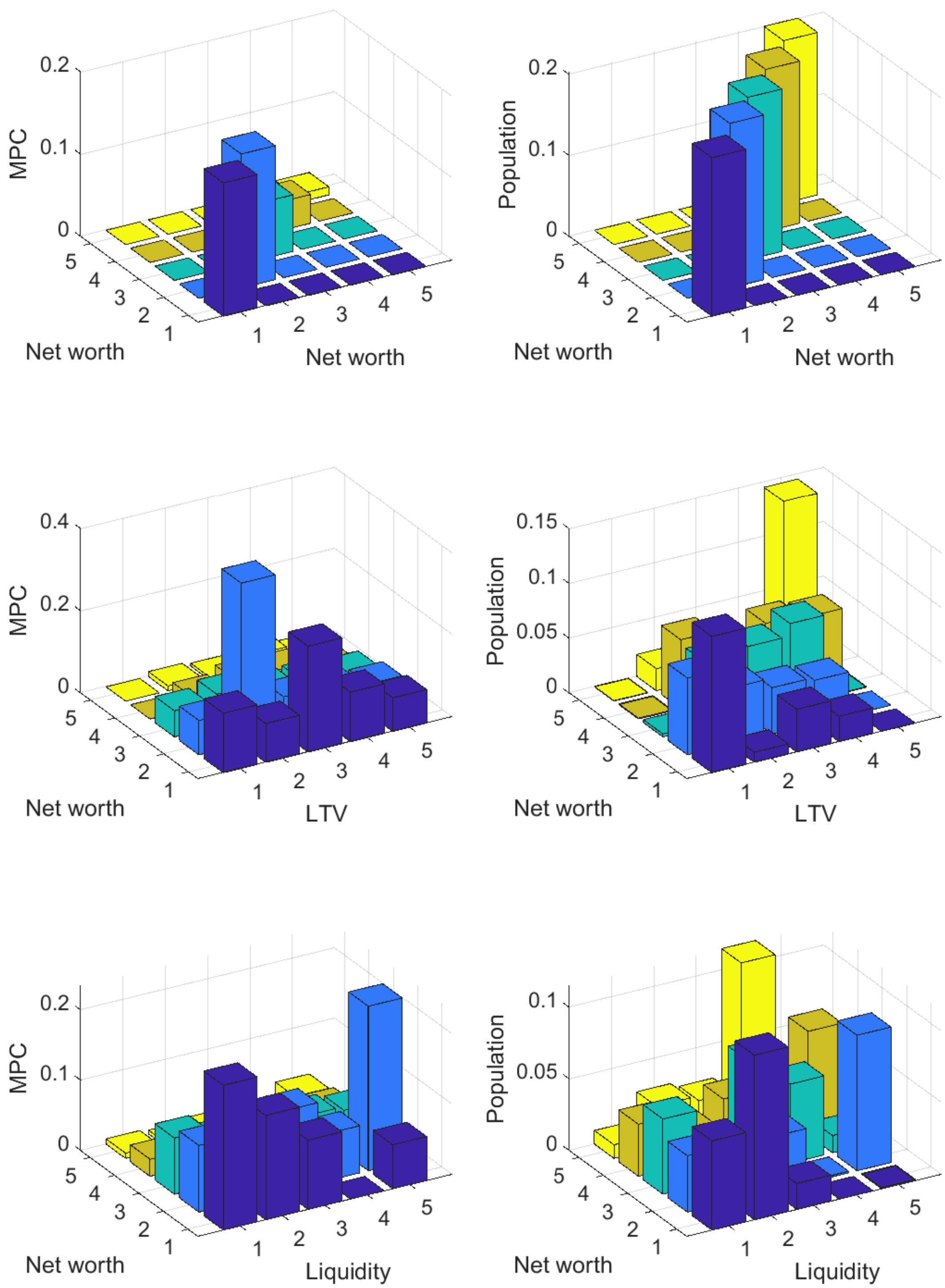


Figure 27. Response of the aggregate variables

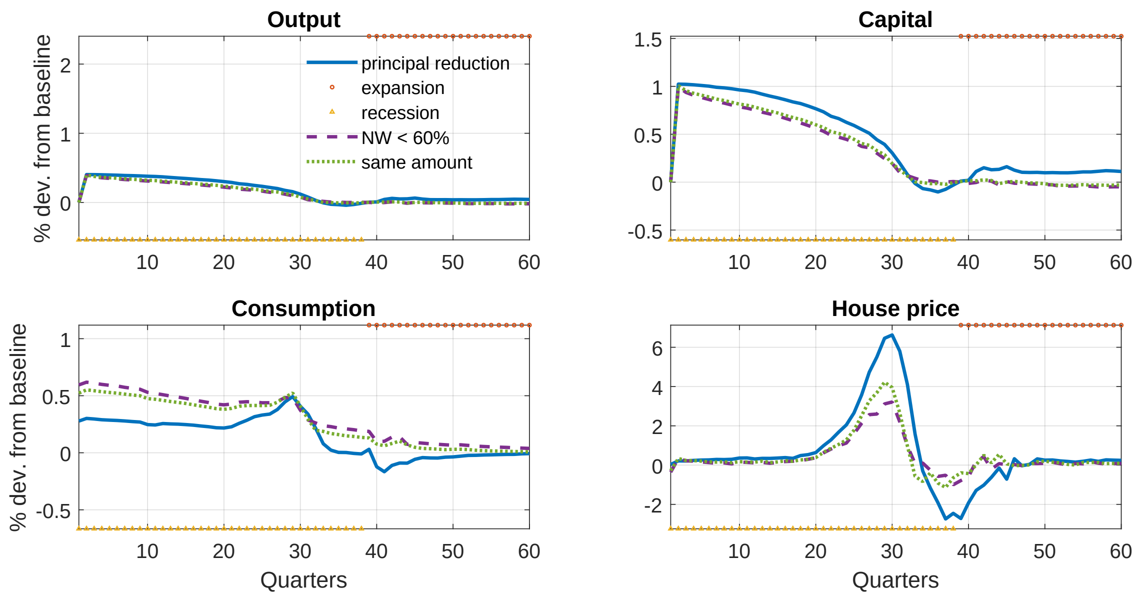


Figure 28. Response of credits and defaults

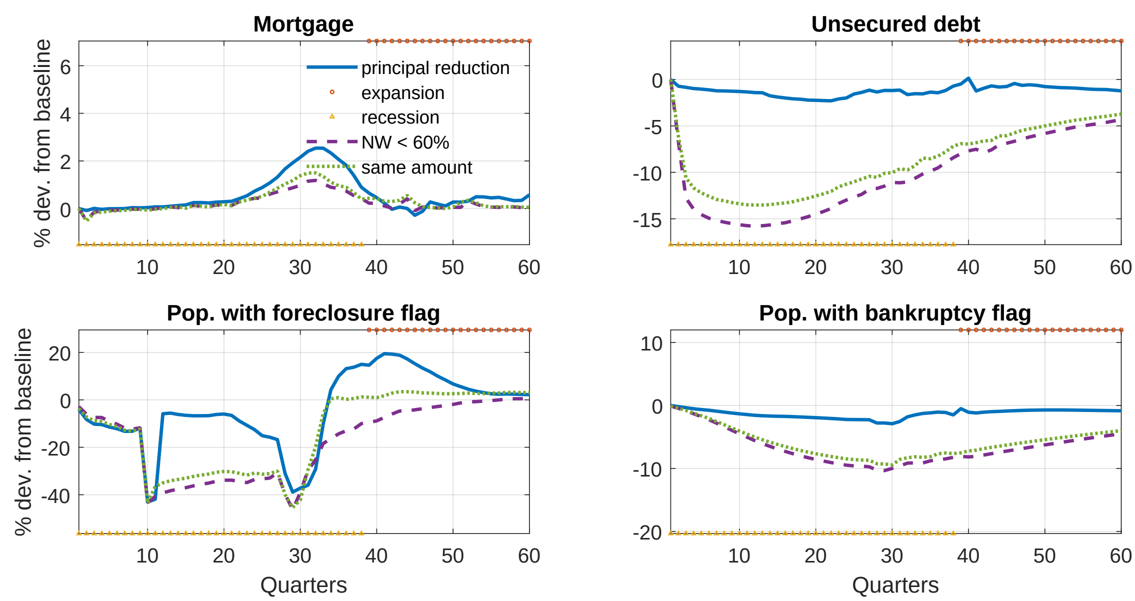
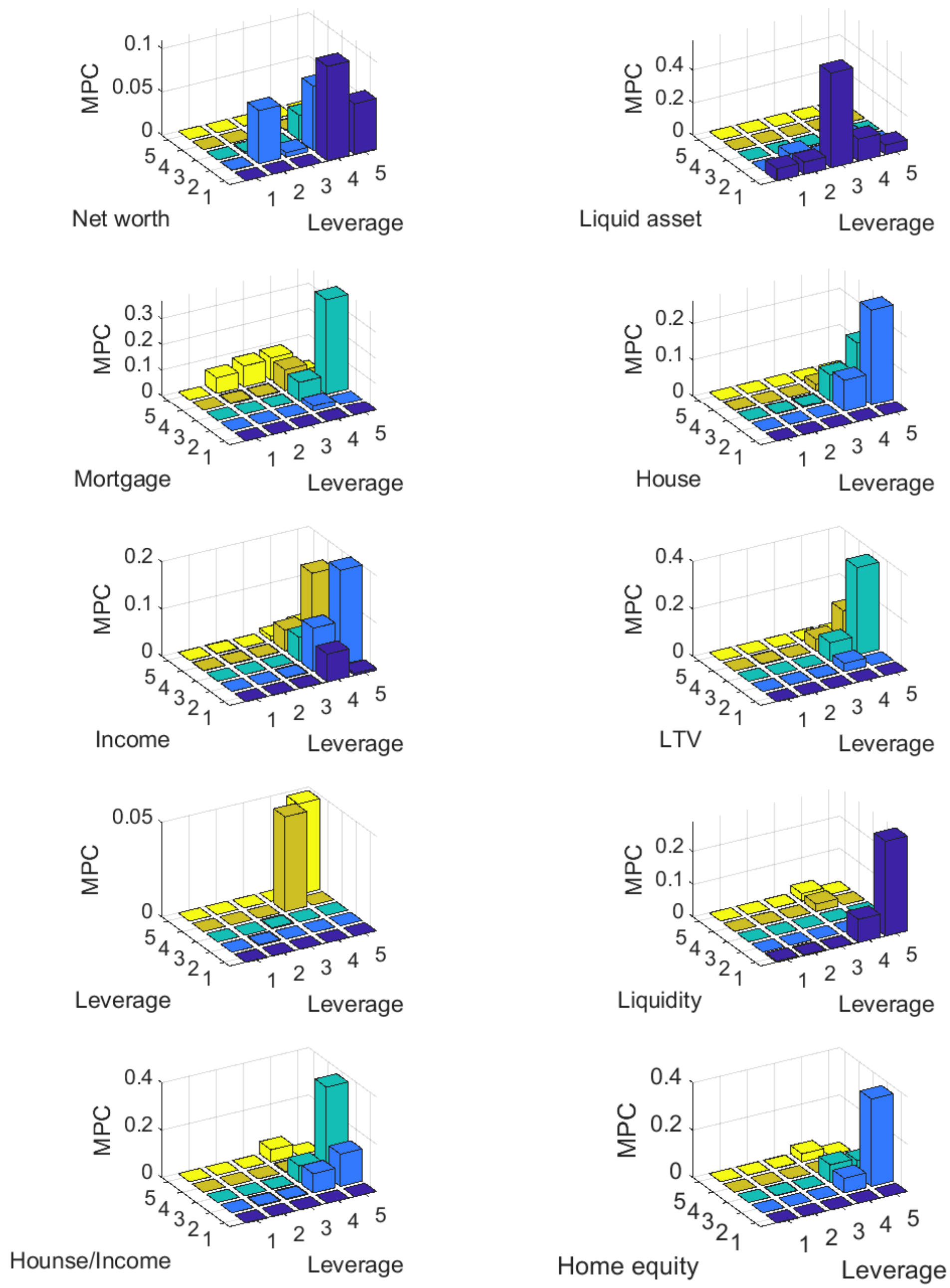


Figure 29. Consumption responsiveness



Note: These bars plot MPC over various households' states qunitile. Each bar is an average MPC of households that are in corresponding quintiles. Only hand-to-mouth households with house are counted. MPC is $\frac{\text{c(policy) - c(no policy)}}{\text{tax rebate size}}$. Liquidity is a ratio of liquid saving over total asset (saving plus house value). Leverage is total debt(unsecured borrowing plus mortgage) to total asset.

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