The Macroeconomic Effects of Debt Relief Policies during Recessions *

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November 24, 2019

Abstract

I study the aggregate and microeconomic effects of debt relief programs during recessions. My model allows households to default on their mortgages and enter into foreclosure, default on unsecured debt and enter into bankruptcy, or both. The result is the first general equilibrium model with aggregate uncertainty, accommodating uninsurable income risk, unsecured debt, financial assets, mortgages, housing, bankruptcy, and foreclosure. The model successfully replicates the distribution of household wealth as well as key asset and debt components. Using this unique laboratory, I explore how one form of household debt forgiveness affects another, how households with differing asset positions are affected, as well as the consequences for aggregate series such as GDP and investment.

General equilibrium movements in house prices and interest rates play an important role in creating an interdependence between the portfolio adjustments of households and the long-term effects of a policy intervention. I find that a mortgage principal reduction program targeting loan-to-value ratios among highly leveraged borrowers delivers significant and persistent increases in aggregate consumption, investment, and output during a recession. It dampens the decline in house prices and stimulates capital accumulation, driving lower interest rates. The initial rise in house prices has lasting effects as it reduces subsequent foreclosures and effectively loosens financial constraints on households. Comparing mortgage forgiveness to a tax rebate, an untargeted transfer to all households, I find that the tax rebate is more effective in reducing bankruptcy, and the principal reduction is more effective in reducing foreclosure and supporting house prices. Both policies have similar overall effects on aggregate consumption, but their distributional effects are different.

*I indebted to my advisor, Aubhik Khan for his guidance and support. I am grateful to Julia Thomas and Kyle Dempsey for their comments and suggestions. I also thank Garth Baughman, Niklas Engbom, Makoto Nakajima, Ludwig Straub, Corina Boar, Kyle Herkenhoff and Ben Casner for helpful comments. I thank Rohan P. Shah and Ben Lidofsky for their generous help.

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1 Introduction

Since the Great Recession, there have been many studies on the relationship between household debt and the macroeconomy.[1] While many questions involving the recession remain unanswered, there is a growing consensus that high household leverage at the start of the recession exacerbated the severity of the downturn.

During a recession, and particularly during a recession that is associated with high leverage, debt relief programs can have a stabilizing effect. The thinking is that alleviating underwater borrowers' financial distress via mortgage forgiveness can prevent a rise in foreclosure and a fall in house prices.[2] Moreover, preventing large house price declines can prevent subsequent foreclosures, hence supporting future house prices. To date, however, there is little quantitative analysis of these channels, and thus of the macroeconomic effects of debt relief over the business cycle.

My goal is to assess the effects of debt relief programs during recessions quantitatively. I develop a dynamic stochastic general equilibrium model that goes a long way in capturing the rich heterogeneity in households' balance sheets. The model has three assets, which, I argue below, are the minimum necessary to study the effects of mortgage-related programs. Households may borrow or save in liquid assets, and can also hold illiquid assets in the form of houses and mortgages. Housing provides service flows, which households value alongside non-durable consumption, and incurs maintenance costs. As in Huggett (1993) and Aiyagari (1994), households face idiosyncratic risks and have limited ability to insure themselves due to incomplete financial markets. Both unsecured debt and mortgages are subject to default, and households may declare bankruptcy on unsecured debt and allow foreclosure on their mortgages.

The model differs from existing quantitative macroeconomic models with default in two ways. First, it includes aggregate business cycle risk alongside idiosyncratic income risk, while allowing default on unsecured and secured debt separately. These are essential elements for my analysis. To investigate the role of debt relief as a macroeconomic policy tool to mitigate the severity of downturns, the model must have aggregate risk. Further, it is crucial to allow bankruptcy and foreclosure to assess the effects of a debt forgiveness policy properly, since households' decisions regarding mortgage default can be influenced by their credit card positions, and vice versa, as happened over

[1] See, for example Mian et al. (2013), Mian and Sufi (2014), Guerrieri and Lorenzoni (2017), Jones et al. (2011) and Auclert et al. (2019), Verner and Gyongyosi (2018)

[2] Mian et al. (2015) show that foreclosures led to large declines in house prices, residential investment, and consumer demand from 2007 to 2009.

the Great Recession.[3]

One of the main experiments I consider is one-time mortgage forgiveness. If bankruptcy is not allowed, an underwater borrower has less ability to smooth consumption through unsecured credit, and of course, no ability to exit financial distress by using the bankruptcy system. As a result, the effect of mortgage forgiveness on foreclosure is likely to be overestimated. Furthermore, there would be no possibility of considering the indirect implications such a policy has on bankruptcy.

Second, unlike existing heterogeneous household models in this literature, all prices (interest rates, wages, house prices and loan rates) are determined endogenously in my model. This allows me to carefully consider the general equilibrium effects of debt relief programs, both for macroeconomic outcomes and the circumstances of individual households. How do the initial responses of households to a policy intervention impact house prices, wages and interest rates? How do these changes in prices feed back to household decisions and shape the paths of aggregate variables? Unlike existing papers in the literature, my model can answer these questions.

After calibrating my model to match a large set of cross-sectional features of the U.S.economy in the early 2010s, I evaluate its performance along untargeted dimensions. The model successfully reproduces the household wealth distribution, not only in terms of net worth but also in terms of key components: financial assets, housing wealth and mortgages. Additionally, the model captures key business cycle moments of the U.S. data.

I investigate the effects of debt relief programs by considering two policy experiments. First, I design a countercyclical intervention in which all households with loan-to-value (LTV) ratios above 95% have their LTV ratios reduced to this threshold via mortgage forgiveness.[4] This policy intervention affects about 26% of the population, and the average amount of debt forgiven is equivalent to roughly \$13,500 (2010 dollars). This one-time debt forgiveness program generates a large and persistent rise in both aggregate consumption and investment. Wages increase and interest rates fall in response to the rise in investment, while house prices rise. Examining consumption responses across households at the time of the intervention, I find that those with low net worth, low liquidity and high leverage respond most sharply to the principal reduc-

[3]For example, Calem et al. (2017) show that a nonpayment of mortgage expenses due to foreclosure process delay resulted in higher cure rates on delinquent credit cards and reduced credit card balances during the Great Recession.

[4]Kaplan et al. (2017) conduct a similar experiment. They study house price changes around the Great Recession in an equilibrium housing model. However, other prices (the risk-free rate and wages) in their model are exogenous, and they have no provision for bankruptcy.

tion. These households consume as much as 40% of the value transferred to them over the policy intervention. Since they are near hand-to-mouth, and mortgage forgiveness does not provide liquid assets for consumption, this large consumption response may seem puzzling. It turns out that these households increase consumption through refinancing. Approximately 23% of households that receive a principal reduction convert their increased home equity into liquid wealth.

While the consumption responses to the principal reduction are initially concentrated in the households that receive the reduction, the intervention affects all households over time through general equilibrium effects. While poor households increase consumption upon the intervention, relatively wealthy households with large houses and big mortgages invest their gains instead of consuming. As the capital stock increases, wages rise. This benefits all households. The higher capital stock also lowers interest rates, which benefits net borrowers and hurts net savers. These income effects build up over time, helping to support house prices.

The rise in house prices has lasting effects, as it reduces subsequent foreclosures and loosens financial constraints.[5] Lower foreclosure rates and eased borrowing limits imply a smaller drop in housing demand, which, in turn, prevents further declines in house prices. I find that the higher consumption and lower foreclosures, 7-8 years after the intervention, are mainly driven by households that sell their houses instead of defaulting, and by households that would have sold anyway, but are now able to sell at a higher price following the principal reduction policy.

In terms of distributional effects, the price changes induced by the intervention benefit indebted households that have small savings. They are now able to increase their consumption by selling at higher prices. In contrast, households that have large houses, substantial savings, and small debt actually reduce consumption. Higher house prices reduce disposable income for these households as maintenance costs rise. Further, the fall in interest rates lowers the return on their financial wealth.

These findings are inconsistent with Kaplan et al.'s (2017), who consider the same mortgage forgiveness program. They show that the program reduces foreclosure rates significantly but does not affect house prices or consumption. The existence of capital and equilibrium price movements lead to different policy outcomes. In Kaplan et al.'s (2017) model, only house prices are determined in equilibrium. In contrast, in my model, interest rates and wages are determined in the equilibrium as well as house prices. As explained above, the income effect from the higher capital stock helps to

[5] In the model, mortgages are constrained to not exceed 105% of the value of housing collateral. A rise in house prices eases this constraint.

support house prices, and higher house prices reduce subsequent foreclosures and loosen financial constraints, further preventing falls in house prices. In the absence of interest rate and wage responses, Kaplan et al.'s (2017) model delivers minimal subsequent responses in other aggregate variables.

I compare the effects of a principal reduction to the more familiar countercyclical policy of a tax rebate. For the sake of comparability, the total cost of the rebate is the same as that of the debt forgiveness program. An important difference between these policies is that the tax rebate is distributed equally to all households and thus provides liquidity evenly. In contrast, debt forgiveness applies only to a subset of households and does not directly increase liquidity. I find that the tax rebate is more effective in boosting consumption initially. As the rebate augments households' liquid wealth, it increases resources costlessly available for consumption. In addition, it distributes a larger amount of resources to households that have a high marginal propensity to consume (MPC).[6] The tax rebate is also more effective in reducing bankruptcy and unsecured borrowing through its effect on the income of poor households without a house or mortgage. In contrast, the mortgage debt reduction program is more effective in reducing foreclosure and supporting house prices, as it targets households that are likely to sell houses or foreclose and provides additional wealth for them. Surprisingly, the long-term effects of these interventions on consumption are similar. While the tax rebate initially causes a more substantial increase in consumption, the principal reduction yields higher aggregate consumption 6-7 years after the intervention due to the wealth effects from higher house prices.

I conclude by conducting two additional policy counterfactuals. First, I investigate the effects of lenient bankruptcy. Given that the bankruptcy system provides insurance to financially distressed households, it is plausible that the government can mitigate a fall in consumption and output during recessions by adjusting the bankruptcy system.[7] I assume that the government loosens the requirements for bankruptcy (e.g. bankruptcy filing costs) for 16 quarters so that households find it less prohibitive to use the system. As a result, the bankruptcy rate rises by nearly 100% and unsecured debt falls almost 40%. Consistent with the evidence in Dobbie and Song (2016), such intervention affects households' foreclosure decisions too. As households substitute bankruptcy for foreclosure, a fall in bankruptcy costs is as effective as a principal reduction in reducing foreclosure. It also has a lasting impact on investment, consump-

[6]My model replicates the observed heterogeneity in the consumption response to fiscal stimulus payments. See Misra and Surico (2014) and Kaplan and Violante (2014).

[7]See Auclert et al. (2019) for a related argument.

tion and house prices.

Second, I study the effects of a policy of reducing mortgage payments. Several empirical studies (e.g. Ganong and Noel (2018) and Indarte (2019)) have argured that mortgage principal reduction is less effective than mortgage payment reduction in reducing default rates. These results are not directly comparable to my results. However, in order to study payment reduction in my model environment, I adjust per-period repayment, so it does not exceed 31% of a household's labor income, for 16 quarters. By increasing eligible households' disposable income, aggregate consumption is slightly higher and foreclosure rates are slightly lower during the intervention periods compared to an economy without the intervention. However, after the intervention is over, foreclosure rates rise and house prices fall. Because of the slow repayment, eligible households' leverage remains high compared to the economy without the intervention. This higher leverage leads to a rise in foreclosure after the policy intervention. Rising foreclosure contributes to falling house prices, which have a knock on effect in raising foreclosure rates in subsequent periods.

In addition to examining the economy's dynamics over a business cycle, I also explore the role of default options and how they interact in stationary general equilibrium. Specifically, I investigate the welfare implications of default by comparing outcomes in my full model economy with bankruptcy and foreclosure to outcomes in two special case models: i) a model in which households cannot foreclose but can go bankrupt, and ii) a model in which only foreclosure is available to households. Considering the consumption-equivalent gains achieved in moving from each nested model economy to full economy, I find that the welfare gains from default provisions are unevenly distributed over households. As we might expect, those with low income and wealth value the bankruptcy option most, as they are more likely to use it. We might then also expect that the foreclosure option will be valued by households that have large mortgages and thus large houses. However, in fact, households with large houses and large mortgages do not value the foreclosure option. Again, this somewhat surprising result comes from general equilibrium price changes. Specifically, in the economy allowing foreclosure, the equilibrium housing price is higher than in an economy without that option because more low-wealth households are able to buy houses.[8] Therefore, purchasing the same house when foreclosure is available requires larger loans, and hence higher interest payments. They also have higher maintenance costs.

[8]The possibility of foreclosure enables households with low wealth to take on larger loans than otherwise to buy houses. When foreclosure is not available, the loan-to-value ratio is constrained to ensure repayment, preventing poorer households from entering the housing market.

Both effects reduce non-durable consumption, making the foreclosure option less desirable. A natural implication of this finding is that we must be careful about general equilibrium feedbacks in measuring the consequences of an economic policy involving mortgage debt, as they may lead to unanticipated effects.

Literature

This paper is related to multiple strands of the literature on household credit and default. First, my paper is related to recent work that estimates the effects of debt relief programs during the Great Recession. Agarwal et al. (2017) show that although the Home Affordable Mortgage Program (HAMP) participation rate was low, the program reduced foreclosure and increased spending. Ganong and Noel (2018) separate the impact of reducing mortgage balance and reducing short-term payments and show that while short-term payment reductions are effective in reducing defaults, mortgage balance reductions are not. Piskorski and Seru (2018) show that alleviating frictions affecting the pass-through of lower interest rates and debt relief (e.g. refinancing, loan renegotiation) could have reduced foreclosure rates and resulted in up to twice as fast a recovery of house prices, consumption, and employment. I add to this literature by using a structural model that explicitly accounts for the general equilibrium effects of such economic policies.

My paper is also related to work that studies debt relief as a macroprudential policy tool. Since the Great Recession, there have been many papers showing how high leverage of households can exacerbate an economic downturn.[9] Motivated by this literature, Korinek and Simsek (2016) show that policies that aim to reduce leverage can be welfare improving when an economy is in a liquidity trap. Auclert et al. (2019) find that debt forgiveness provided by the U.S. consumer bankruptcy system increases consumption and that this increased consumption helps to stabilize employment. Both papers assume an environment with nominal rigidities. I complement their work by showing that debt relief programs can have lasting impacts through household balance sheets. A policy intervention can support house prices, which loosens price-dependent financial constraints (LTV) and prevents subsequent foreclosures.

Regarding my model, my work is built on the literature on housing, mortgages, and foreclosure.[10] My model shares a similar set of available assets for households but also

[9] Mian et al. (2013), Mian and Sufi (2014), Guerrieri and Lorenzoni (2017), Jones et al. (2011), and Verner and Gyongyosi (2018)

[10] For example, Jeske et al. (2013), Corbae and Quintin (2015), Hatchondo et al. (2015), Chatterjee and Eyigungor (2015), Kaplan et al. (2017).

allows for default on unsecured debt.[11] To my knowledge, Mitman (2016) is the only existing paper that explores the interaction between bankruptcy and foreclosure. By allowing this interaction, he shows that his model can reconcile a *negative* correlation between the generosity of bankruptcy law and bankruptcy rates across U.S. states.[12] While my model abstracts from rich heterogeneity in the bankruptcy system across states, I study a model with aggregate uncertainty and quantify the role of debt relief policies during recessions. Using policy experiments, I show that bankruptcy and foreclosure can be substitutes. For example, when the government lowers bankruptcy costs, foreclosure rates fall as households substitute foreclosure for bankruptcy. I find that less punitive bankruptcy can be as effective as a policy that directly lower mortgages of highly leveraged households in reducing foreclosure.

The policy of mortgage principal reduction follows the seminal contribution of Kaplan et al. (2017) discussed above. As already mentioned, my paper focuses on business cycles in a general equilibrium model with capital. Nakajima and Ríos-Rull (2014) is the closest to my work in the consumer bankruptcy literature, as they study how access to credit affects the nature of business cycles in a model with aggregate uncertainty. I add housing, mortgages and foreclosure to their environment to study the effects of mortgage related policies.

The rest of the paper is organized as follows. In Section 2, I describe the model economy. Section 3 explains the calibration procedure and presents calibration results. I discuss the results in Section 4 and Section 5. Section 6 concludes.

2 Model

2.1 Overview

The economy consists of a continuum of infinitely-lived households, banks, nondurable goods producers and a government. Households are indexed by their holdings of liquid assets a, house h, mortgage b and their idiosyncratic labor productivity ε.

[11] A large body of work studies consumer bankruptcy. See, among others, Athreya (2002), Li and Sarte (2006), Livshits et al. (2007), Chatterjee et al. (2007), Nakajima and Ríos-Rull (2014).

[12] In a state with a generous exemption, homeowners are more likely to go bankrupt. Higher bankruptcy rates raise the equilibrium interest rate on unsecured borrowing, hence households substitute secured credit for unsecured credit by taking on more leveraged mortgages. Therefore in a state with a higher exemption, household portfolios are more heavily weighted toward secured debt. As a result, we observe lower bankruptcy rates and higher foreclosure rates in a state with a generous exemption.

Households are subject to uninsurable idiosyncratic shocks to their labor productivity, which they supply inelastically to competitive firms. Households can save or borrow in a financial asset whose return is determined in equilibrium. Households consume non-durable goods and service flows from their housing.

Housing services are derived from home ownership; however, households do not have to own a house. When a household buys a house, she can take out a mortgage to fund the purchase, which is subject to a loan-to-value (LTV) constraint at origination. The mortgage is a long-term debt, and mortgage borrowers are required to pay a fraction of a remaining principal and interest in each period. They can refinance or prepay their mortgage. Houses and mortgages are illiquid in the sense that costs are incurred when buying, selling, refinancing or prepaying.

Both types of debt (unsecured debt and mortgage) are defaultable. Defaulting on unsecured debt (bankruptcy) leads to utility loss and a bankruptcy record placed on a household's credit history, which excludes them from housing transactions and unsecured debt borrowing. When a household chooses to default on a mortgage (foreclosure), a bank takes over the house and liquidates it. A foreclosure also incurs utility loss and a foreclosure record on a household's credit history, which excludes the household from housing transactions. To summarise, households adjust their portfolio position substantially when they choose to i) buy or sell houses ii) refinance or prepay mortgages iii) default on unsecured debt iv) default on a mortgage v) default on both debts. When households do not make a substantial change to their portfolio, they choose how much to consume.

On the supply side, there are a large number of identical firms that produce output, which is consumed or invested in physical capital, using capital and labor through a constant return to scale technology. The supply of housing is fixed.

The financial sector is competitive. Banks price both types of debt reflecting household default risks. They expect zero profit for each loan. Aggregate uncertainty implies ex-post profits and losses which are absorbed by the government.[13] The government also collects tax from households and sets fiscal policy.

The aggregate states of the economy are g - the distribution of households over (a, b, ε, h) - and z, the aggregate total factor productivity. Time is continuous.

[13]See Ozkan et al. (2017) for a similar assumption.

2.2 Households

2.2.1 Household's Environment

Labor productivity Each household's labor productivity follows a Poisson process. With frequency λ_ε, households receive a labor productivity shock and draw a new productivity from a time-invariant distribution. These shocks arrive independently across households, and one household's productivity is independent of other households.

Liquid assets Households can save or borrow using a liquid asset a. When a is negative, it represents unsecured debt. Households may default on their debt. When a household chooses to default, the debt is forgiven. However, the borrower has a record of bankruptcy on her credit history. Further, she pays a utility cost, ξ_a. The possibility of default of households means that the price of unsecured borrowing depends on individual and aggregate states, $r_a(a, b, \varepsilon, h, g, z)$.

Iliquid assets A second asset, h is illiquid and households must pay an adjustment cost when they buy or sell it. The illiquid asset represents durable commodities such as houses, and it is chosen from a discrete set. This good provides utility flows to households and incurs maintenance costs. One component of these maintenance costs is a property tax, which households can use to reduce their income tax liability. I assume the total supply of houses is fixed to $\overline{H}$. Importantly, house prices, $p(g, z)$, are determined in equilibrium and vary as a function of the aggregate state.[14]

Secured debt House purchases can be funded using secured debt, b which is distinct from unsecured debt. Thus, similar to Mitman (2016) and differently from Kaplan et al. (2017) and Nakajima and Ríos-Rull (2014), I allow for both unsecured credit and mortgages.[15] This debt is refinancible, long-term, secured, and defaultable. Households can borrow b when they buy houses, using the housing as collateral. When choosing b, households are subject to a loan-to-value (LTV) constraint: the choice of b must be less than a fraction (γ) of the collateral value.

Mortgage loans are discounted by $q(a', b', \varepsilon, h, g, z)$ at the time of origination. The loan discount rate depends on the choice of b as well as individual and aggregate states,

[14] Thus the model differs from Kaplan et al. (2017) where the supply of houses varies over time. In my dynamic stochastic general equilibrium model, the fixed supply of houses facilitates tractability of rich heterogeneous agent model solved with aggregate uncertainty.

[15] This may be interpreted as a mortgage, and I will use mortgage and secured debt interchangeably.

which affect the possibility of default.

Households can refinance a mortgage loan by paying a fixed cost ξ_r. When refinancing, they first pay back the remaining balance of the current loan and then take out a new loan.[16] All mortgage debt interest rate is adjustable rate, so refinancing could be used to extract equity or prepay the remaining balance.

While a household is holding b, it is required to pay a loan interest rate, as well as a fraction of the principal at each instant.[17] Households pay a fraction of h value at each period: $\theta(b, ph) = \overline{\theta}ph/b$. Moreover, because b is long-term debt, there is no requirement that the size of this loan remains less than γ times the current value of non-financial asset, h. Thus the LTV limit is only imposed at origination. If the price of houses decreases, a household could find itself with negative equity. However, as long as the household pays off the required amount of the outstanding balance of the loan at the moment, $(\theta(b, ph) + r)b$, it is not forced to default or refinance.

When a household chooses to default, the remaining balance of the debt is forgiven, and a financial intermediary takes the house. I assume the financial intermediary suffers a loss when foreclosing, and the sale value is $(1 - \delta_h)ph$. The household will have a foreclosure recorded on their credit history, which excludes them from h transactions.[18] Households that enter foreclosure incur a utility cost, ξ_b, at that moment.

Bankruptcy and Foreclosure histories While a bankruptcy remains on a household's credit history, some fraction ξ_x of the household's labor income is garnished. Also, unsecured borrowing, refinancing a mortgage, new origination of mortgage, and purchasing a house are not allowed. However, bankrupt households' non-financial assets are fully protected and they can enter foreclosure if they have a mortgage. If a household chooses to do so, its remaining mortgage debt, b, will be forgiven but they will lose their house. When $h = 0$, there is no income garnishment.[19] I assume, for tractability, that the bankruptcy flag is removed stochastically with intensity λ_d.

A household that has defaulted on a mortgage is unable to purchase a new house while its credit history has a foreclosure record. Since they cannot buy a house, such households are excluded from taking new mortgages. However, they can still take on unsecured debt and choose to default on any such unsecured debt they already have.

[16]This implies that new loans are always subject to the same type of LTV limits, and that the discount rate continues to be a function of only the new mortgage.

[17]Loan interest rate is equal to return on saving plus a premium reflecting the unit cost of lending, $\iota(z)$.

[18]For simplicity, I assume that households cannot buy h even if a purchase is self-financed.

[19]All households with zero h and a bankruptcy flag are either the households who did not own a house before bankruptcy or lost the asset during a foreclosure.

[20] For tractability, I assume that the foreclosure flag is removed stochastically with intensity λ_f.

Preferences Households receive utility flow from consuming non-durable goods and from consuming a service flow from their houses. Their utility function is

$$u(c,h) = \frac{c^{1-\sigma}-1}{1-\sigma} + (\kappa h)^{\sigma_h}$$

The function u is strictly increasing and strictly concave in c and h.

2.2.2 Household Problem

Households with no flags

Households that do not currently have bankruptcy or foreclosure in their credit history are free to take on unsecured debt and enter into housing transactions. Such a household's problem is given by

$$v(a_t, b_t, \varepsilon_t, h_t) = \max_{\{c_t\},\tau} \mathbf{E_0} \int_0^{\tau} e^{-\rho t} u(c_t, h_t) dt + \mathbf{E_0} e^{-\rho\tau} v^*(a_\tau, b_\tau, \varepsilon_\tau, h_\tau) \tag{1}$$

$$\dot{a}_t = w_t \varepsilon_t + r_{at}(a,b,\varepsilon,h) a_t - (r_t + \theta(b, \overline{p}h)) b_t - c_t - T_t(b,\varepsilon,ph) - \xi_h p_t h_t \tag{2}$$

$$\dot{b}_t = \theta(b, \overline{p}h) b_t \tag{3}$$

$$(a_0, b_0, \varepsilon_0, h_0) = (a, b, \varepsilon, h) \tag{4}$$

Households choose non-durable consumption $\{c_t\}$ and their optimal stopping time τ. Stopping involves a household making a discrete choice that brings about a large shift in their asset position, or credit history when such a shift is desirable. For example, a household buys a house when it reaches a certain threshold in saving. Such a choice - buying a house - is an example of stopping. When a household chooses to stop, it does one of the following: i) h size adjustment ii) refinance, iii) default on a, iv) default on b or v) default on both a and b. In the absence of such a discrete choice, households that have secured debt repay it at a rate which is a fraction $\theta(b, \overline{p}h)$ of their house value, evaluated at the price $\overline{p}$. $T(\cdot)$ is a tax that depends on taxable income and $\xi_h ph$ is the maintenance cost of h.

The Hamilton-Jacobi-Bellman (HJB) equation prior to stopping is,

[20] If these households go bankrupt, their foreclosure flag will be replaced by a bankruptcy flag. Recall bankruptcy also prevents housing transactions.

$$
\begin{aligned}
\rho v(a,b,\varepsilon,h,g,z) = & \max_c u(c,h) + \partial_a v(a,b,\varepsilon,h,g,z)\dot{a} + \partial_b v(a,b,\varepsilon,h,g,z)\dot{b} \\
& + \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v(a,b,\varepsilon_j,h,g,z) + \sum_{k=1}^{n_z} \lambda_{zz_k} v(a,b,\varepsilon,h,g,z_k) \\
& + \int \frac{\delta v(a,b,\varepsilon,h,g,z)}{\delta g(a,b,\varepsilon,h)} \mathcal{K} g(a,b,\varepsilon,h) d[a \times b \times \varepsilon \times h]
\end{aligned} \tag{5}
$$

$$
\dot{a} = w(g,z)\varepsilon + r_a(a,b,\varepsilon,h,g,z)a - (r(g,z) + \iota(z) + \theta(b,\overline{p}h))b - c - \xi_h p(g,z)h - T(b,\varepsilon,p(g,z)h)
$$

$$
\dot{b} = \theta(b,\overline{p}h)b
$$

$$
v(a,b,\varepsilon,h,g,z) \geq v^*(a,b,\varepsilon,h,g,z).
$$

$\lambda_{\varepsilon\varepsilon_j}$ describes the labor productivity process.[21][22] Aggregate productivity follows a stochastic process described by λ_{zz_k}. $\mathcal{K}$ is a Kolmogorov Forward operator that operates on the distributions of households, g_t which evolves according to shocks and households' decisions.[23]

$$
\frac{dg_t(a,b,\varepsilon,h)}{dt} = \mathcal{K} g_t(a,b,\varepsilon,h)
$$

Stopping values

The stopping value $v^*(a_\tau, b_\tau, \varepsilon, h_\tau, g, z)$ is the maximum of the following values. These involve moving to a different house (i.e. adjusting the size of h), refinancing an existing mortgage, or defaulting on debt.

1. h size adjustment

$$
v^m(a_\tau, b_\tau, \varepsilon, h_\tau, g, z) = \max_{h',b'} v(a',b',\varepsilon,h',g,z)
$$

$$
a' = a_\tau - b_\tau + p(g,z)h_\tau - p(g,z)h' - \xi(p(g,z),h_\tau,h') - q(a',b',\varepsilon,h',g,z)b'
$$

$$
b' \leq \gamma p(g,z)h'
$$

[21]See Achdou et al. (2017) for details of continuous-time approach of solving heterogenous agent models.

[22]A shock intensity $\lambda_{\varepsilon_i\varepsilon_j}$ is negative when $\varepsilon_i = \varepsilon_j$, which is the intensity of losing the current level of labor productivity. $\lambda_{\varepsilon_i\varepsilon_j} > 0$ when $\varepsilon_i \neq \varepsilon_j$, it is the intensity of jumping to ε_j from ε_i. $\sum_j \lambda_{\varepsilon_i\varepsilon_j} = 0 \quad \forall i = 1,...,n_\varepsilon$.

[23]Ahn et al. (2018) describe the recursive formulation of an model with aggregate uncertainty using the Kolmogorov Forward operator.

When changing the size of its house, a household chooses the optimal size (h') and the amount of the secured loan (b'). The remaining balance that is attached to the current h has to be repaid. The transaction cost $\xi(\cdot)$ is given by

$$\xi(p(g,z), h_\tau, h') = p(g,z)\xi_0(h_\tau + h') + \xi_1 p(g,z)(|h_\tau - h'|)^2$$

where γ is the LTV limit as a proportion of collateral.

2. Refinancing b

$$v^r(a_\tau, b_\tau, \varepsilon, h_\tau, g, z) = \max_{b'} v(a', b', \varepsilon, h_\tau, g, z)$$

$$a' = a_\tau - b_\tau + q(a', b', \varepsilon, h, g, z)b' - \xi_r$$

$$b' \leq \gamma p(g,z)h_\tau$$

Households that hold mortgage b have the option to refinance by repaying the residual principal balance b_τ and a fixed cost ξ_r. They do not change their house size but simply originate a new loan, b', which is subject to the LTV limit, given by $b' \leq \gamma ph$.

3. Default on a

$$v^a(a_\tau, b_\tau, \varepsilon, h, g, z) = v^d(0, b, \varepsilon, h, g, z) - \xi_a$$

This is bankruptcy on unsecured borrowing, a. If a household chooses to default only on a, its house and mortgage are unaffected. The stopping value of default is evaluated with $v^d(a, b, \varepsilon, h, g, z)$ which is the value function for households with bankruptcy in their credit history, and ξ_a is the utility cost associated with default.

4. Default on b

$$v^b(a_\tau, b_\tau, \varepsilon, h, g, z) = v^f(a_\tau, \varepsilon, g, z) - \xi_b$$

This is foreclosure. If a household chooses to foreclose, her remaining debt b is forgiven, and a financial intermediary takes over the house. The value function of households with the foreclosure in their credit history is $v^f(a, \varepsilon, g, z)$ and ξ_b is the utility cost.

5. Default on both a and b

$$v^{ab}(a_\tau, b_\tau, \varepsilon, h, g, z) = v^d(0, 0, \varepsilon, 0, g, z) - \xi_a - \xi_b$$

When a household defaults on both types of debt, her debt held as a and b are forgiven, and a financial intermediary takes over h. Such households are not allowed to use any credit while they have a bankruptcy in their credit history.

The overall stopping value for a household is v^*, where

$$v^*(a_\tau, b_\tau, \varepsilon_\tau, h, g, z) = \max\{v^m, v^r, v^a, v^b, v^{ab}\}.$$

Thus households can choose among the available stopping options; adjusting h size, refinancing and default.

Households with bankruptcy flag
A household with the bankruptcy flag in their credit history solves the following problem.

$$v^d(a_t, b_t, \varepsilon_t, h_t) = \max_{\{c_t\},\tau} \mathbf{E_0} \int_0^\tau e^{-\rho t} u(c_t, h_t)dt + \mathbf{E_0} e^{-\rho\tau} v^{d*}(a_\tau, b_\tau, \varepsilon_\tau, h_\tau) \tag{6}$$

$$\dot{a}_t = (1 - \xi_x \mathbf{1}_{h>0})w_t\varepsilon_t + r_{at}(a, b, \varepsilon, h)a_t - (r_t + \theta(b, \overline{p}h))b_t - c_t - T_t(b, \varepsilon, ph) - \xi_h p_t h_t \tag{7}$$

$$\dot{b}_t = \theta(b, \overline{p}h)b_t \tag{8}$$

$$a_t \geq 0 \tag{9}$$

$$(a_0, b_0, \varepsilon_0, h_0) = (a, b, \varepsilon, h) \tag{10}$$

Such households also choose non-durable consumption $\{c_t\}$ and their optimal stopping time τ. However, only foreclosure and selling illiquid assets are available to them as stopping options. A fraction ξ_x of labor income is taken where a household owns a house after a bankruptcy filing. Bankrupt households cannot take on unsecured debt. The HJB equation before stopping is,

$$\begin{aligned}
\rho v^d(a, b, \varepsilon, h, g, z) = \max_c u(c, h) &+ \partial_a v^d(a, b, \varepsilon, h, g, z)\dot{a} + \partial_b v^d(a, b, \varepsilon, h, g, z)\dot{b} \\
&+ \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v^d(a, b, \varepsilon_j, h, g, z) + \sum_{k=1}^{n_z} \lambda_{zz_k} v^d(a, b, \varepsilon, h, g, z_k) \\
&+ \lambda_d(v(a, b, \varepsilon, h, g, z) - v^d(a, b, \varepsilon, h, g, z)) \\
&+ \int \frac{\delta v^d(a, b, \varepsilon, h, g, z)}{\delta g(a, b, \varepsilon, h)} \mathcal{K} g(a, b, \varepsilon, h) d[a \times b \times \varepsilon \times h]
\end{aligned}$$

$$\dot{a} = (1 - \xi_x \mathbf{1}_{h>0}) w(g,z)\varepsilon + r_a(a,b,\varepsilon,h,g,z)a - (r(g,z) + \iota(z) + \theta(b,\overline{p}h))b$$
$$- c - \xi_h p(g,z)h - T(b,\varepsilon,p(g,z)h)$$

$$\dot{b} = \theta(b,\overline{p}h)b$$

$$a \geq 0$$

$$v^d(a,b,\varepsilon,h,g,z) \geq v^{d*}(a,b,\varepsilon,h,g,z).$$

The stopping value is the maximum of defaulting on a or selling h.

1. Default on a

$$v^{da}(a_\tau,b_\tau,\varepsilon,h_\tau,g,z) = v^d(a_\tau,0,\varepsilon,0,g,z) - \xi_b$$

2. Sell h

$$v^{dm}(a_\tau,b_\tau,\varepsilon,h_\tau,g,z) = v^d(a,0,\varepsilon,0)$$

$$a = a_\tau - b_\tau + p(g,z)h_\tau - \xi(p(g,z),h_\tau,0)$$

$$v^{d*}(a,b,\varepsilon,h,g,z) = \max\{v^{da}, v^{dm}\}$$

Households with a foreclosure flag

Lastly, households with a foreclosure flag in their credit history solve the following problem.

$$v^f(a_t,\varepsilon_t) = \max_{\{c_t\},\tau} \mathbf{E_0} \int_0^\tau e^{-\rho t} u(c_t,0)dt + \mathbf{E_0} e^{-\rho\tau} v^{d*}(a_\tau,\varepsilon_\tau) \tag{11}$$

$$\dot{a}_t = w_t\varepsilon_t + r_{at}(a,\varepsilon)a_t - c_t - T_t(b,\varepsilon,ph) \tag{12}$$

$$(a_0,\varepsilon_0) = (a,\varepsilon) \tag{13}$$

These households choose non-durable consumption $\{c_t\}$ and the optimal stopping time τ. However, the only stopping option available to them is to default on a. The HJB equation before stopping is,

$$\rho v^f(a,\varepsilon,g,z) = \max_c u(c,0) + \partial_a v^f(a,\varepsilon,g,z)\dot{a} + + \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v^f(a,\varepsilon_j,g,z)$$
$$+ \lambda_f(v(a,0,\varepsilon,0,g,z) - v^f(a,\varepsilon,g,z)) + \sum_{k=1}^{n_z} \lambda_{zz_k} v^f(a,\varepsilon,g,z_k)$$
$$+ \int \frac{\delta v^f(a,\varepsilon,g,z)}{\delta g(a,b,\varepsilon,h)} \mathcal{K} g(a,b,\varepsilon,h) d[a \times b \times \varepsilon \times h]$$

$$\dot{a} = w(g,z)\varepsilon + r_a(a,\varepsilon,g,z)a - c - T(b,\varepsilon,p(g,z)h)$$

$$v^f(a,\varepsilon,g,z) \geq v^{f*}(a,\varepsilon,g,z).$$

The stopping value is,

$$v^{f*}(a_\tau,\varepsilon,g,z) = v^d(a_\tau,0,\varepsilon,0,g,z) - \xi_a.$$

The household's problem can be compactly written as an HJB variational inequality (HJBVI).

Households with no flags

$$\begin{aligned}
min[&\rho v(a,b,\varepsilon,h,g,z) - \max_c u(c,h) - \partial_a v(a,b,\varepsilon,h,g,z)\dot{a} - \partial_b v(a,b,\varepsilon,h,g,z)\dot{b} \\
&- \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v(a,b,\varepsilon_j,h,g,z) - \sum_{k=1}^{n_z} \lambda_{zz_k} v(a,b,\varepsilon,h,g,z_k) \\
&- \int \frac{\delta v(a,b,\varepsilon,h,g,z)}{\delta g(a,b,\varepsilon,h)} \mathcal{K} g(a,b,\varepsilon,h) d[a\times b\times\varepsilon\times h], \\
&v(a,b,\varepsilon,h,g,z) - v^*(a,b,\varepsilon,h,g,z)] = 0
\end{aligned}$$

Households with a bankruptcy flag

$$\begin{aligned}
min[&\rho v^d(a,b,\varepsilon,h,g,z) - \max_c u(c,h) - \partial_a v^d(a,b,\varepsilon,h,g,z)\dot{a} - \partial_b v^d(a,b,\varepsilon,h,g,z)\dot{b} \\
&- \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v^d(a,b,\varepsilon_j,h,g,z) - \lambda_d(v(a,b,\varepsilon,h,g,z) - v^d(a,b,\varepsilon,h,g,z)) \\
&- \sum_{k=1}^{n_z} \lambda_{zz_k} v^d(a,b,\varepsilon,h,g,z_k) - \int \frac{\delta v^d(a,b,\varepsilon,h,g,z)}{\delta g(a,b,\varepsilon,h)} \mathcal{K} g(a,b,\varepsilon,h) d[a\times b\times\varepsilon\times h], \\
&v^d(a,b,\varepsilon,h,g,z) - v^{d*}(a,b,\varepsilon,h,g,z)] = 0
\end{aligned}$$

Households with a foreclosure flag

$$\begin{aligned}
&min[\rho v^f(a,\varepsilon,g,z) - \max_c u(c,0) - \partial_a v^f(a,\varepsilon,g,z)\dot{a} - \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v^f(a,\varepsilon_j,g,z) \\
&\quad - \lambda_f(v(a,0,\varepsilon,0,g,z) - v^f(a,\varepsilon,g,z)) \\
&\quad - \sum_{k=1}^{n_z} \lambda_{zz_k} v^f(a,\varepsilon,g,z_k) - \int \frac{\delta v^f(a,\varepsilon,g,z)}{\delta g(a,b,\varepsilon,h)} \mathcal{K} g(a,b,\varepsilon,h) d[a\times b\times\varepsilon\times h], \\
&\quad v^f(a,\varepsilon,g,z) - v^{f*}(a,\varepsilon,g,z)] = 0
\end{aligned}$$

Let $\{C_t\}_{0\leq t\leq\tau}$ describe a households non-durable good consumption choice. The stopping time is τ. The households' buying and selling decisions, the size of their mortgage and house, when they buy or sell a house are, respectively, M,B_M,H_M. The refinancing decision and the size of secured debt when refinancing are Ref and B_R. The bankruptcy, foreclosure and default on both debt are D_a D_b and D_{ab}.

2.3 Financial intermediaries

There are risk neutral, competitive financial intermediaries. These banks issue short-term deposits and loans as well as mortgages to households. They also lend capital to firms. Due to the possibility of default, banks offer loan rates based on a household's portfolio and their persistent income, ε. Banks expect zero profits for each loan. However, although ex-ante profits are zero for each loan, ex-post returns can vary as a result of aggregate risk. I assume that the government absorbs any realized profits or losses using taxes or subsidies paid to intermediaries.

Unsecured debt As characterized in Bornstein (2018), the expected interest on lending in the region of no default ($D_a(a,b,\varepsilon,h,g,z) = 0$) is a return minus a default probablity. It can be written as

$$E[dr_a(a,b,\varepsilon,h,g,z)] = r_a(a,b,\varepsilon,h,g,z)dt - \lambda_z \sum_{z'} p_{zz'} \lambda_\varepsilon \sum_{\varepsilon'} p_{\varepsilon\varepsilon'} D_a(a,b,\varepsilon',h,g,z')dt$$

where $p_{\varepsilon\varepsilon'}$ is the probability of moving from ε to ε' conditional on receiving a labor productivity shock. In the default region ($D_a(a, b, \varepsilon, h, g, z) = 1$),

$$r_a(a, b, \varepsilon, h, g, z) = \infty. \tag{14}$$

The zero profit condition in the region of no default implies that the return $r_a(a, b, \varepsilon, h, g, z)$ should be equal to the risk free rate, $r(g, z)$.

$$r_a(a, b, \varepsilon, h, g, z) = r(g, z) + \lambda_z \sum_{z'} p_{zz'} \lambda_\varepsilon \sum_{\varepsilon'} p_{\varepsilon\varepsilon'} D_a(a, b, \varepsilon', h, g, z'). \tag{15}$$

Because no household defaults on a in the region where a is positive, $r_a(a, b, \varepsilon, h, g, z) = r(g, z)$ for savers.

Mortgages Borrowers pay an interest rate $r(g, z) + \iota(z)$ and a fraction $\theta(b, \bar{p}h))$ of the remaining balance b at each instant. Therefore, the flow income from a loan is $(r_t + \theta(b_t, \bar{p}h))b_t$. Banks discount the loan with an interest rate, $r_t + \theta_t$ as the loan matures at the rate θ_t. Recall that if a household defaults on its secured debt, the bank recovers the depreciated value of the house, $(1 - \delta_d)ph$.

Since the banks expect zero profit for each loan, the discounted value of the loan at origination has to be equal to its expected cash flow. The price of the loan in the non-default region is given by

$$q_0(a, b, \varepsilon, h, g, z)b_0 = \mathbb{E}\Big[\mathbb{E}_\tau \int_0^\tau e^{-\int_0^s (r_s + \iota_t + \theta_s)ds}(r_t + \iota_t + \theta_t)b_0 dt + e^{-\int_0^\tau r_s ds} b(a_\tau, b_\tau, \varepsilon_\tau, h, g, z)\Big]$$

The scrap value $b(a_\tau, b_\tau, \varepsilon_\tau, h, g, z)$ at the stopping point depends on a household's discrete choice. In the case of a foreclosure,

$$b(a_\tau, b_\tau, \varepsilon_\tau, h, g, z) = (1 - \delta_d)p(g, z)h.$$

When a household prepays the loan due to refinancing or a new house transaction, the scrap value is

$$b(a_\tau, b_\tau, \varepsilon_\tau, h, g, z) = e^{-\int_0^\tau \theta_s ds} b_0.$$

Applying the Feynman-Kac formula, the above equations can be written as the following partial differential equation.[24]

[24]The Feynman-Kac formula establishes a connection between a partial differential equation and

At $t \in [0, \tau)$,

$$
\begin{aligned}
(\theta(b, \overline{p}h) + r(g,z) + \iota(z))q(a,b,\varepsilon,h,g,z) = \theta(b,\overline{p}h) + r(g,z) + \iota(z) + q_a(a,b,\varepsilon,h,g,z)\dot{a} \\
+ q_b(a,b,\varepsilon,h,g,z)\dot{b} + \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} q(a,b,\varepsilon_j,h,g,z) + \sum_{k=1}^{n_z} \lambda_{zz_k} v(a,b,\varepsilon,h,g,z_k) \\
+ \int \frac{\delta v(a,b,\varepsilon,h,g,z)}{\delta g(a,b,\varepsilon,h)} \mathcal{K} g(a,b,\varepsilon,h) d[a \times b \times \varepsilon \times h],
\end{aligned}
\tag{16}
$$

$t = \tau$, in case of the foreclosure,

$$
q(a,b,\varepsilon,h,g,z) = \frac{(1-\delta_d)p(g,z)h}{b}, \tag{17}
$$

and $t = \tau$, in case of the prepayment,

$$
q(a,b,\varepsilon,h,g,z) = 1. \tag{18}
$$

2.4 Firms

There are identical, competitive firms that produce non-durable consumption goods using a constant return to scale technology.[25] Firms rent capital from the banks and employ labor to produce goods. The firms solve the following problem.

$$
\max_{k,\ell} z f(k,\ell) - (r(g,z) + \delta)k - w(g,z)\ell \tag{19}
$$

$$
f(k,\ell) = k^\alpha \ell^{1-\alpha}
$$

The production function $f(k,\ell)$ is $k^\alpha \ell^{1-\alpha}$ and the capital depreciation rate is δ. Firms' technology implies that the equilibrium interest rate is $r(g,z) = z\alpha \frac{\ell}{k}^{1-\alpha}$ and the equilibrium wage rate is $w(g,z) = z(1-\alpha)\frac{\ell}{k}^{-\alpha}$.

2.5 Government

The government collects taxes from households. Taxes are levied on labor income net of deductible costs. Households can deduct the interest paid on the mortgage and a part of the maintenance cost of their house from their taxable income. As already

stochastic processes. See Nuno and Thomas (2015), Kaplan et al. (2018) for similar usage of the Feynman-Kac formula.

[25] I assume the stock of durable goods is given to the economy.

noted, the government also absorbs realized profits or losses from the secured debt of banks through taxes or subsidies. The remaining revenue is spent on government consumption of non-durable goods, which are not valued by households.

2.6 Equilibrium

An equilibrium is a set of functions

$$(r_a, q, r, w, p, v, C, \tau, M, B_M, H_M, R, B^R, D_a, D_b, D_{ab}, L, K, G)$$

that satisfies the following:

1. Households optimize. Given prices $\{r_a, q, w, p\}$, v solves (1)-(4), v^d solves (6)-(10) and v^f solves (11)-(13). The associated policy functions are $C, \tau, M, H, B^m, R, B^r$, D_a, D_b, D_{ab}.
2. Firms maximize profits by solving (19) and L, K are the associated policy functions.
3. The unsecured debt price function r_a is determined by (14) and (15).
4. The secured debt price function q is determined by (16) - (18).
5. Capital market clears: $\int (a+b)g(a,b,\varepsilon,h)d[a \times b \times \varepsilon \times h] = k$.
6. Labor market clears: $\int \varepsilon g(a,b,\varepsilon,h)d[a \times b \times \varepsilon \times h] = \ell$.
7. Durable goods market clears: $\int hg(a,b,\varepsilon,h)d[a \times b \times \varepsilon \times h] = \overline{H}$.
8. The government budget constraint holds.
9. The Kolmogorov Forward Operator $\mathcal{K}$ that describes the change of density function g is generated by agents' optimal choices.

3 Mapping Model to Data

I choose model parameters to match key cross-sectional features of the U.S.economy in the early 2010s. To study the effects of debt relief programs, the model needs to match the distribution of assets and debt across households. Households' portfolios are a key determinant of default decisions. Moreover, a calibration of the stochastic process for labor earnings should be able to capture the earnings dynamics seen in the data as labor income shocks are the source of uninsurable risk driving household changes in assets and debt.

A subset of model parameters are assigned in advance of solving the model's stationary state. In addition, the earnings process is estimated outside of the model. Finally, 10 parameters are jointly calibrated in the steady state.[26] Table 8 lists calibrated parameters, and Table 3 reports targeted data moments and model moments. I describe targets with specific parameters, but since the parameters are jointly determined, this association is heuristic.

Earnings process I model the labor earnings process as a combination of two independent processes:

$$\varepsilon_{ij} = \varepsilon_i^p (1 + \varepsilon_j^t)$$

where each component follows a Poisson jump process. Jumps arrive at a Poisson rate λ^p for ε^p and λ^t for ε^t. Conditional on a jump, a new earnings state ε_k^p is drawn from a bounded Pareto distribution, and ε_l^t is drawn from the discrete set $\{-\chi, \chi\}$.

To pin down the levels of ε_i^p, it is necessary to set upper and lower bounds as well as the curvature parameter. I set these parameters to match the variance and the distribution of earnings. Specifically, I target earnings shares by quintile and the top 10%. I discretize ε_p to 4 points and choose this support to capture the 41.0, 28.0, 29.5 and 1.5 percent of the working population.[27] [28] Table 1 shows earnings distribution in the data and in the model. The model captures earnings shares by quintile and top 10% well.

I assume the probability of drawing a new value for ε_k^p depends on its level, ε_i^p. Therefore the intensity of jumping from i to k is given by $\lambda_{ik}^p = \lambda^p(f(\varepsilon_{k+1|i}) - f(\varepsilon_{k|i}))$, where $\lambda^p = \sum_{k=1}^{n_{\varepsilon^p}} \lambda_{ik}^p, \ \forall i$ and $f(\varepsilon_{k|i}) = \frac{1-(\underline{\varepsilon}/x_k)^{\eta_{\varepsilon_i}}}{1-(\underline{\varepsilon}/\overline{\varepsilon})^{\eta_{\varepsilon_i}}}$.[29]

I used a bounded Pareto distribution for $f(\varepsilon_{k|i})$. Given the discretized support, the shape parameters η_{ε_i} need to be estimated. To set these curvature values, the shock intensities λ^p and λ^t, the size of the shock χ and the probability of drawing a negative transitory component conditional on a jump in ε^t, I estimate the earnings process by Simulated Method of Moments to match the higher order moments of the earnings

[26]These are parameters specifying household preference ($\sigma, \sigma^d, \kappa, \rho$), various costs ($\xi_x, \xi_a, \xi_b, \xi_0, \xi_1$) and the tax function ($\tau_0$).

[27]The shares from the 1st to the 2rd bin are chosen to represent the population with education attainment levels: less than a high school diploma or high school graduate, some college (the average over 1992 to 2013, BLS). The rest of the population has a bachelor's degree or higher, and I add one point to capture earnings concentration at the top.

[28]For example, ε_2 is a median value between x_1 and x_2 such that $f(x_1) = \frac{1-(\underline{\varepsilon}/x_1)^{\eta_\varepsilon}}{1-(\underline{\varepsilon}/\overline{\varepsilon})^{\eta_\varepsilon}} = 0.41$ and $f(x_2) = \frac{1-(\underline{\varepsilon}/x_2)^{\eta_\varepsilon}}{1-(\underline{\varepsilon}/\overline{\varepsilon})^{\eta_\varepsilon}} = 0.41 + 0.28$, where $f(x_i)$ is the CDF of the bounded Pareto distribution.

[29]Since λ_{ik} affects the ergodic distribution of households over labor productivity, the population share by education attainment becomes a target.

Table 1: Earnings distribution

	Variance	Quintiles(%)					Top(%)		
		1q	2q	3q	4q	5q	90-95	95-99	99-100
Data	0.92	-0.1	3.5	11.0	20.6	65.0	12.1	18.3	18.0
Model	0.93	4.3	5.7	6.8	21.1	62.0	13.4	15.2	16.1

Data: Song et al. (2018), SCF (2010)

growth rate distribution reported in Guvenen et al. (2015).[30] [31] I simulate the model to compute the corresponding moments.[32] Since the data moments are computed using annual earnings, I simulate the model at a higher frequency and aggregate the result into annual earnings.

To summarize, the number of parameters specifying the earnings process is 11 and the number of targets is 20.[33] The estimated process implies that a shock to ε^p arrives on average once every 21 years. Upon the arrival of shock, the state is likely to switch to an adjacent one. Since the ε^p grid points are not equi-distant, the size of a shock depends on the current state. In general, the size of a shock tends to be small when labor productivity is low. Turning to the other labor productivity shock, a shock to ε^t arrives on average once every 0.9 years.

The infrequent component of labor income shock, ε^p can be interpreted as the persistent component and ε^t as the transitory component. Households do not experience a large shock often, but income fluctuates around their persistent component through transitory shocks.

Table 2 shows moments from the data and the model. The estimated process generates moments that reproduce the data well.

[30]Kaplan et al. (2018) explains why this is appropriate for inferring high frequency earnings dynamics. The key argument is that the size and frequency of the shock determine the shape of the earnings distribution. Large, infrequent shocks are likely to generate a more leptokurtic distribution and small, frequent shocks are likely to generate a platykurtic distribution. Kaplan et al. (2018) model the earnings process as a sum of two jump-drift processes, representing a persistent and a transitory component of the earnings process.

[31]They use Social Security Administration (SSA) data from 1994 to 2013 to compute their moments.

[32]The panel size is 5000 and the simulation length is 6000. The 800 periods of each simulated series are discarded when computing the statistics. Increasing the panel size or the number of periods has little effect on the results.

[33]The 3 parameters that shape the bounded Pareto distribution for ε^p are $\overline{\varepsilon}^p$, $\underline{\varepsilon}^p, \eta^p_{\varepsilon}$. The 4 parameters that set the probability of drawing a new value for ε^p are $\eta^p_{\varepsilon_i}, i \in [1, .., 4]$. The 2 parameters that set shock intensity are λ^p, λ^t and χ is the size of a transitory shock. Finally, p^t is a probability of drawing negative transitory shock.

Table 2: Earnings dynamics

	Std.		**Skewness**		**Kurtosis**		**P($\lvert\Delta y\rvert$) < x***			**P($\lvert\Delta y\rvert$) $\in$ [$\underline{x}, \overline{x}$] ***	
	1y	5y	1y	5y	1y	5y	x = 0.2	0.5	1.0	[0,0.25)	[0.25,1)
Data	0.51	0.78	-1.07	-1.25	14.93	9.51	0.67	0.83	0.93	0.31	0.16
Model	0.30	0.58	-0.08	-0.03	15.14	8.57	0.62	0.97	0.98	0.43	0.19

* $|\Delta y|$: Absolute log earnings change. Data: Guvenen et al. (2015)

Assets and debt

Categorization of assets and debts Mapping the model to the data requires categorizing assets held by U.S. households into financial assets, non-financial assets and secured debt.[34] I target the asset and debt distribution reported in the 2010 SCF. In the SCF data, net worth is comprised of assets and debt, and total assets are the sum of financial assets and non-financial assets. Financial assets include transaction accounts, certificates of deposit, money market funds, stocks, cash, quasi-liquid retirement accounts and other financial assets. Non-financial assets are predominantly the value of vehicles and houses (primary and non-primary residential property, non-residential real estate) and the value of business. Debt is comprised of debt secured by residential properties, credit card loans, installment loans (e.g., student loan, vehicle loans). When mapping the model to the data, I exclude the value of a business from non-financial assets because my model does not have such assets. For debt, I exclude student loans for the same reason. Student loans are not short-term, unsecured debt nor are they secured by collateral or dischargeable in bankruptcy. After excluding student loans, credit card loans are considered as unsecured debt, and the remaining components of debt are assigned to secured debt.[35]

Non financial assets The survey by Davis and Van Nieuwerburgh's (2015) finds maintenance costs are between 1 and 3% of the value of a house, so I set maintenance costs, ξ_h to 2%. The two parameters for the housing transactions cost function, ξ_0 and ξ_1 are 0.02 and 0.05. The fixed stock of houses $\overline{H}$ is set to 4.5. These three parameters are jointly calibrated to match the share of non-financial asset to total asset ratios, which

[34]The model does not separate financial assets and unsecured debt.

[35]Table 10 and Table 11 shows the portfolio composition by quintiles excluding and including business assets and student loans. When excluding business assets and student loans, the shares of assets and debt in 2nd to 4th quintiles are not very different. In the 5th quintile, the share of non-financial assets is lower, which implies that a large share of business assets are owned by the wealthiest households. The share of the 1st quintile rises. This is likely caused by the net worth of the group being closer to zero due to the exclusion of student loans.

is reported in Table 3.

Mortgages The loan-to-value ratio γ is set to 1.05 based on the fact that mortgages are available with zero down payment and home equity lines of credit are available to households.[36][37] The amortization rate of mortgages, $\overline{\theta}$ is set to 0.03, which implies that the duration of a loan is approximately 30 years if a household fully finances the purchase of the house. The refinance cost ξ_r is set to the equivalent of \$2,500 (2010 dollars) in the model accounting for the sum of application, loan origination, attorney, insurance and inspection fees.[38]

Bankruptcy and foreclosure The utility cost ξ_a, ξ_b and the income garnishment rate ξ_x are calibrated to match the bankruptcy rate, the foreclosure rate and the percent of households with secured debt. The bankruptcy rate target is 1.06%, which is constructed using the number of Chapter 7 and Chapter 13 bankruptcy filings from the U.S. Bankruptcy Courts over the number of households from the U.S. Census (averaged over 2000-2017). The foreclosure rate target is 0.55%.[39]

The intensities at which the bankruptcy and the foreclosure flags are removed are set to match the following. After filing for Chapter 7 bankruptcy, households cannot file again for 6 years. Households that file for Chapter 13 bankruptcy enter into repayment plans that last for 3–5 years. Accordingly, I choose λ_d to 0.183 to match an average bankruptcy duration of 6 years. For foreclosure, Fair Issac reports that households' FICO scores can recover in as little as 2 years (see Mitman (2016)). Hence λ_f is set to 0.693 to give an average duration of 2 years for the foreclosure flag. The depreciation rate when foreclosing, δ_d is 22%, which is taken from Pennington-Cross (2006).[40]

Preferences The curvatures and weight of the utility function mainly affect households asset and debt composition. The discount rate ρ and the parameters of the utility function, σ, σ_h, κ are jointly calibrated to match total debt to asset ratios and debt payment to income ratios across households. I set ρ to 0.075, σ to 2.2, σ_h to 0.48 and the weight on durable consumption κ to 4.0.

[36]The United States Department of Veterans Affairs and the United States Department of Agriculture guarantee purchase loans to 100%, and the Federal Housing Administration (FHA) insures purchase loans to 96.5%.

[37]In the 1989 - 2013 waves of the SCF, the size of secured debt exceeded the value of non-financial assets among the poorest 20% of households.

[38]See `https://www.federalreserve.gov/pubs/refinancings/default.htm`

[39]This is the average rate in the U.S. during the late 1990s. (Mortgage Banker's Association)

[40]Pennington-Cross (2006) estimates the loss of value of a foreclosed property using a sample of real estate owned property.

Table 3: Targeted moments and model values

Moment	Data	Model	Data Source
Foreclosure rate	0.0055	0.0060	Mortgage Banker's Association
Bankruptcy rate	0.010	0.011	U.S. Courts, U.S. Census
Tax revenue to output	0.16	0.16	CBO
70th percentile non-fin. to total assets	0.945	0.507	SCF (2010)
90th percentile non-fin. to total assets	0.998	0.769	SCF (2010)
50th percentile debt payment to income	0.112	0.071	SCF (2010)
70th percentile debt payment to income	0.218	0.226	SCF (2010)
90th percentile debt payment to income	0.399	0.497	SCF (2010)
50th percentile debt to asset	0.210	0.182	SCF (2010)
70th percentile debt to asset	0.524	0.497	SCF (2010)
90th percentile debt to asset	0.946	0.794	SCF (2010)
Households with secured debt	0.643	0.673	SCF (2010)

Note: Non-financial assets are the sum of housing, car, and other non-financial assets. Business assets are excluded from non-financial assets. Total assets are the sum of financial and non-financial assets. Debt is the sum of collateralized and uncollateralized debt, excluding student loans.

Production The production technology is constant returns to scale with the capital share set as the residual of the labor share of output as measured in Giandrea and Sprague (2017). They calculate the labor share of output in the non-farm business sector from 1947 through 2016.[41] I use the average value of labor share between 1989 and 2013, 60.5%; thus, α is 0.395. I assume that the depreciation rate δ is 0.069 (see Khan and Thomas (2013)).

Government The income tax function $T(y) = y - \tau_0 y^{1-\tau_1}$ is taken from Heathcote et al. (2017) where y is a taxable income. Taxable income is labor income minus the tax deductible interest payments on mortgages and property taxes. Taxable income is

$$y = w(g,z)\varepsilon - r(g,z)min(b,\bar{b}) - min(\tau_h p(g,z)h, \overline{\tau_h}).$$

The property tax rate τ_h is set to 1%, which is the median tax rate across US states.[42]

[41]Labor share of output is sum of employee compensation and proprietors' labor compensation.
[42]See Kaplan et al. (2017) who references data from the Tax Policy Center.

Table 4: Parameter values

Parameter	Value	Internal	Description
Preferences and production			
ρ	0.075	Y	Discount rate
σ	2.2	Y	Curvature of the utility function
σ_h	0.48	Y	Curvature of the utility function
κ	4.0	Y	Weight on durable good
α	0.395	N	Capital share
δ	0.069	N	Depreciation rate
Tax			
τ_0	0.60	Y	Tax rate
τ_1	0.16	N	Tax progressivity
τ_h	0.01	N	Property tax rate
$\overline{b}$	1,000,000	N	Maximum debt to deduct interest payments
$\overline{\tau_h}$	10,000	N	Maximum deduction on property tax
Labor productivity			
$\overline{\varepsilon}^p$	8.5	N	Upper bound of Pareto distribution
$\underline{\varepsilon}^p$	0.08	N	Lower bound of Pareto distribution
η_{ε}^p	1.526	N	Shape of Pareto distribution
$\eta_{\varepsilon_i^p}$	[1.9,1.5,1.3,0.6]	N	Shape of Pareto distribution
λ^p	0.048	N	Shock intensity
λ^t	1.260	N	Shock intensity
χ	0.239	N	Size of the ε^t shock
p^t	0.600	N	Probability of drawing negative ε^t
Assets and debts			
ξ_h	0.02	N	Depreciation rate of h
δ_h	0.07	N	h transaction cost
$\overline{H}_s$	2.02	N	Supply of durable good
γ	1.05	N	Loan-to-value ratio
$\overline{\theta}$	0.03	N	Amortization rate of b
ξ_r	0.02	N	Refinancing cost
ξ_a	29	Y	Utility cost
ξ_b	2	Y	Utility cost
ξ_x	0.2	Y	Income garnishment rate
λ_d	0.1831	N	Removal of bankruptcy flag
λ_f	0.6929	N	Removal of foreclosure flag
δ_d	0.22	N	Depreciation due to foreclosure
Aggregate shock			
$[z_1, z_2]$	[0.9700, 1.0301]	N	Level of total productivity
$[\lambda_1, \lambda_2]$	[0.4041, 0.4041]	N	Shock intensity

According to Internal Revenue Service (IRS), $\bar{b}$ is $1,000,000 and $\overline{\tau_h}$ is $10,000.[43][44]

The parameter τ_1, determining the degree of progressivity of the tax system, is 0.181 as in Heathcote et al. (2017).[45] Next, τ_0 is set to 0.6 to match the tax revenue-output ratio 16.7%.[46]

Aggregate shocks Aggregate productivity z follows a two-state Poisson process, $z \in [z_1, z_2]$ with $z_2 > z_1$. The process jumps from state 1 to state 2 with intensity λ_1 and in the reverse direction with intensity λ_2. These two states represent a recession (z_1) and an expansion (z_2).

Th support of aggregate TFP, $[z1, z2]$ is [0.97, 1.03] to match the standard deviation of U.S. output. Table 5 shows the cyclical properties of the U.S. economy from the data and the model. The shock intensities are 0.4041, which implies the average duration of a state is 3 years. I choose the average duration following Nakajima and Ríos-Rull (2014).

Validation I compute non-targeted moments to check the model's plausibility. First, the model matches the net worth distribution very well as shown in Figure 1. Moreover, my model allows me to further break-down households' assets. Figure 2 shows the share of assets by net worth.[47] The distribution of non-financial assets, financial assets and secured debt from the model are reasonably close to the data. However, although unsecured debt is distributed evenly over quintiles, it is almost entirely held by the poor in the model. Despite its rich asset structure, households cannot have unsecured debt and liquid savings at the same time. As a result, given that the data indicates that 9.6% of households have net negative financial assets, it is hard to match the distribution of financial assets and unsecured debt at the same time.

Figure 3 shows the composition of assets across households by net worth.[48] Overall, households' shares of non-financial assets in the model tend to be lower than the data. However, the model replicates the relatively high non-financial asset holdings

[43] From 2018, the deduction for home mortgage interest is up to the first $750,000 ($375,000 if married filing separately) of indebtedness. The limitation is $1,000,000 ($500,000 if married filing separately) of indebtedness if a household is deducting mortgage interest from indebtedness incurred on or before December 15, 2017. Since I target the early 2010's data, I apply the limit before 2017.

[44] This is the limit on the deduction for state and local taxes. It includes general sales taxes, real estate taxes and personal property taxes.

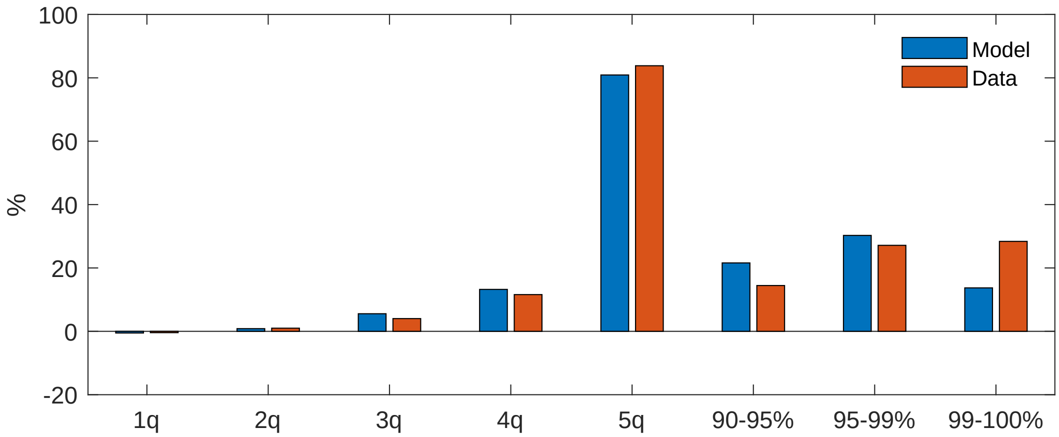
[45] They estimate this parameter using the Panel Study of Income Dynamics (PSID) for survey years 2000, 2002, 2004, and 2006, in combination with the NBER's TAXSIM program.

[46] I use the value of tax revenue-output ratio (Congressional Budget Office) between 2000 and 2014.

[47] Detailed tables for Figure 2 and Figure 3 are available in the appendix C. (Table 9 and Table 10)

[48] Net financial asset is the total financial asset minus credit card debt. (SCF 2010)

Figure 1: Net worth distribution



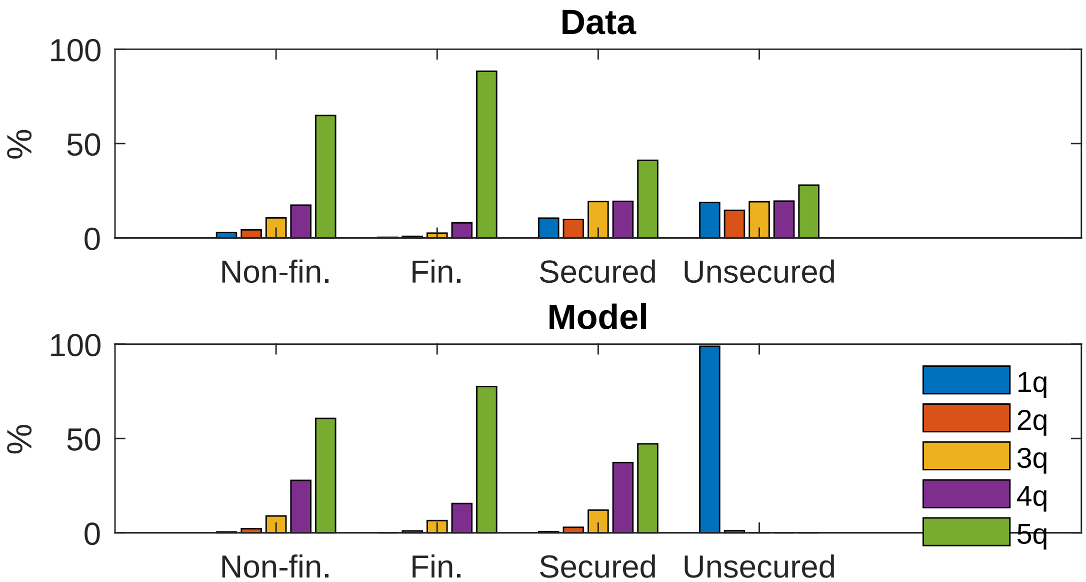
Note: Each bar shows a net worth share held by quintiles or the top 10% of households. Business assets and student loans are excluded. Data: SCF(2010)

for households in the 1-4th quintile as well as the relatively high financial share of 5th quintiles of households.

To study the effects of government policies that affect households balance sheets during recessions, it is essential for any model to capture the cyclical properties of the economy. Table 5 compares the aggregate statistics from the U.S. data and from my model. It shows that the model captures key properties of the data. In particular, both consumption and investment are strongly correlated with output, and the investment is more volatile than consumption. Furthermore, secured credit is positively correlated with output and bankruptcy and foreclosure filings are counter-cyclical. However, the volatility of aggregate variables in the model is smaller than the data. Also, unsecured debt is acyclical while it is procyclical in the data.[49]

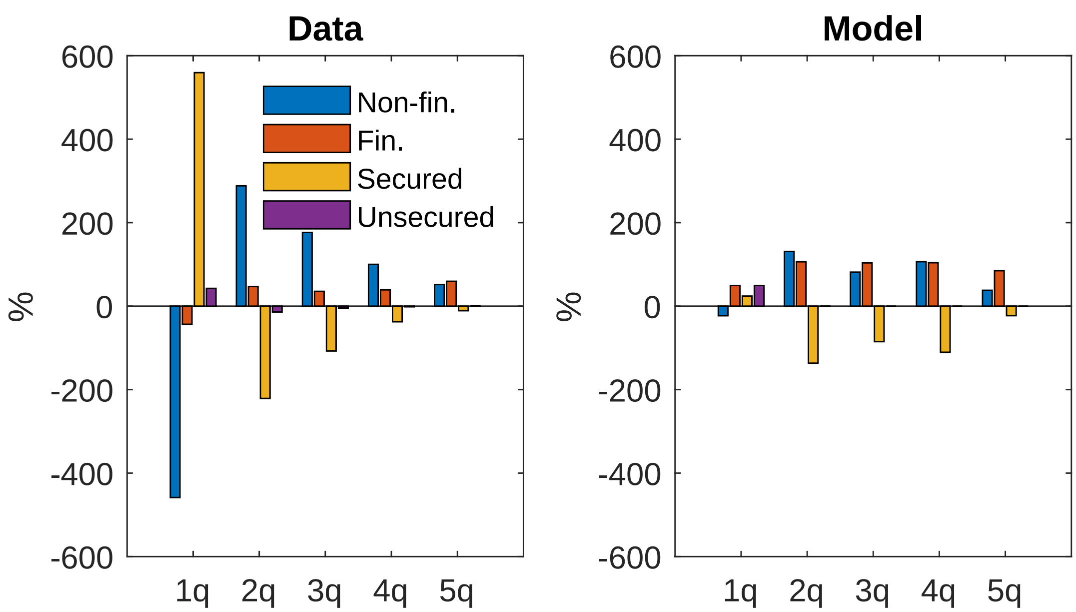
[49] Nakajima and Ríos-Rull (2014) shows that the procyclicality of unsecured credit is hard to generate in the standard theory of unsecured credit. They also show that the standard model can reconcile the procyclicality of unsecured credit if there are counter-cyclical earnings risks. The current version of the model does not have counter-cyclical earnings risks. I expect explicit modeling of such risks will help to match the procyclicality of unsecured credit and increase the volatility of credits. A future version of this paper will address this issue.

Figure 2: Share of assets by net worth



Note: Each bar shows a share of assets or debt held by households in a net worth quintile. Business assets are excluded from non-financial assets. Student loans are excluded from debt. After excluding student loans, credit card loans are unsecured debt, and the remaining components of debt are secured debt. Data: SCF(2010)

Figure 3: Assets and debt composition by net worth



Note: Each bar shows an average share of assets or debt held by households in a net worth quintile. Business assets are excluded from non-financial assets. Student loans are excluded from debt. After excluding student loans, credit card loans are unsecured debt, and the remaining components of debt are secured debt. Data: SCF(2010)

Table 5: Cyclical properties

	Data		Model	
	std(%)	corr. with output	std(%)	corr. with output
Output	2.1	1.0	1.7	1.0
Consumption	1.6	0.9	0.5	0.8
Investment	8.0	0.8	5.2	0.7
Unsecured debt	5.5	0.4	3.7	0.0
Mortgage	5.4	0.4	3.4	0.2

Logs of the data are filtered using the H-P filter with a smoothing parameter of 100. Output, consumption and investment data combine information from NIPA tables 1.1.5 and 2.3.5. from 1963 to 2014. Output: real GDP minus net export. Consumption: real private consumption expenditures minus housing service; Investment: real gross domestic investment; Unsecured debt: Consumer credit (Flow of Funds), deflated by GDP deflator; Secured debt: Home Mortgages (Flow of Funds), deflated by GDP deflator.

4 Steady state results

Prior to examining the dynamics of the economy over the business cycle, I explore the role of defaulting on unsecured debt and foreclosing on a mortgage and how these options interact through general equilibrium in the steady state. Specifically, I compare the following variations of my model i) the full model with bankruptcy and foreclosure (the benchmark model), ii) a model in which households cannot foreclose but can go bankrupt (Bankruptcy), iii) a model with only foreclosure is available to households (Foreclosure), and iv) a model without any option to default (None).[50] I assess the value to households of having an option to default by comparing the benchmark economy to these alternative economies.

4.1 The effect of default on aggregate variables and the distribution of household

The possibility of default affects aggregate variables as well as the distribution of assets held by households. First, aggregate capital rises 1.2% when agents do not have the option to default. This is because the option to default acts as insurance against income risk so the precautionary saving motive is stronger when it is not available. However, consumption is 2.2% lower in the economy without default. Table 6 shows this result.

[50] An exogenous borrowing constraint is set to 0 when there is no bankruptcy option and the loan to value ratio is set to 0.88 when there is no foreclosure option to prevent the consumption set becoming empty.

Table 6: Aggregate variables

	Liquid saving	Secured debt	Capital	h price	Consumption	Output
Bankruptcy	-7.08	-17.29	0.88	-2.72	-1.43	0.35
Foreclosure	0.25	0.98	0.32	-0.01	-0.61	0.13
None	-7.25	-16.92	1.18	-2.51	-2.20	0.46

Note: Bankruptcy refers to an economy in which households cannot foreclose but can go bankrupt, and Foreclosure refers to an economy with only foreclosure. None refers to an economy without any option to default. All numbers are percentage deviation from the benchmark economy.

Table 7: Net worth share by quintiles (%)

	Benchmark	**Bankruptcy**	**Foreclosure**	**None**
1q	-0.53	-0.54	0.11	0.11
2q	0.85	0.77	1.29	1.21
3q	5.55	5.64	5.65	5.69
4q	13.23	14.47	12.96	14.06
5q	80.90	79.66	79.98	78.94

Note: Bankruptcy refers to an economy in which households cannot foreclose but can go bankrupt, and Foreclosure refers to an economy with only foreclosure. None refers to an economy without any option to default.

The marginal increase in saving is greatest for households close to the borrowing limit. Therefore when default is not an option, these households have a higher share of wealth. As a result, the wealth distribution becomes less unequal. Table 7 shows that the share of net worth held by the poorest 40% of households increases, while that of the richest 20% falls, with fewer default options.

These results imply that the dynamics of aggregate variables could be affected by the possibility of default in two ways. First, default allows households to reduce their debt burden when they receive income shocks, thereby affecting portfolio adjustment. In particular, the demand for liquid and illiquid asset will vary as the number of households who choose to default varies and this affects prices, which lead to additional changes in aggregate.

Second, the possibility of default leads to an equilibrium with lower levels of capital and higher leverage, as shown in Table 6. It follows that the distribution of assets and debt, with and without default, will be different at the start of any recessions. Such differences may affect the propagation of shocks.

Table 8: Debt/Asset

	Data	Benchmark	Bankruptcy	Foreclosure	None
50th percentile	0.21	0.18	0.16	0.25	0.22
70th percentile	0.52	0.50	0.43	0.54	0.46
90th percentile	0.95	0.79	0.69	0.83	0.72

Note: Bankruptcy refers to an economy in which households cannot foreclose but can go bankrupt, and Foreclosure refers to an economy with only foreclosure. None refers to an economy without any option to default.

4.2 Who values default options?

I assess the value to households of being able to default by comparing the benchmark economy to the alternative economies specified above. The value of bankruptcy and foreclosure options are calculated as the consumption-equivalent gain obtained in moving from one of the alternative economies to the benchmark economy. As shown in section 2, the value of a non-stopping household is:

$$\rho v(a,b,\varepsilon,h) = \max_c u(c,h) + \partial_a v(a,b,\varepsilon,h)\dot{a} + \partial_b v(a,b,\varepsilon,h)\dot{b} + \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v(a,b,\varepsilon_j,h)$$

For ease of notation, let $\mathbb{A}v = \partial_a v(a,b,\varepsilon,h)\dot{a} + \partial_b v(a,b,\varepsilon,h)\dot{b} + \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v(a,b,\varepsilon_j,h)$. In addition, let x be the amount that makes the value of benchmark economy the same as the value of the alternative economies in each scenario.

$$\rho v_{benchmark} = \rho v_i + x = u(c_i,h) + \mathbb{A}v_i + x = u(c_i + c_i^*, h) + \mathbb{A}v$$

where c^* is the consumption equivalent gain and $i = \{\text{bankruptcy,foreclosure,none}\}$.[51] [52]

Figure 4 shows the consumption compensation required for moving from the benchmark economy to one with foreclosure. It illustrates the value of bankruptcy. Likewise, Figure 5 presents the consumption compensation that is required to make households indifferent between the benchmark economy and the bankruptcy only economy. It il-

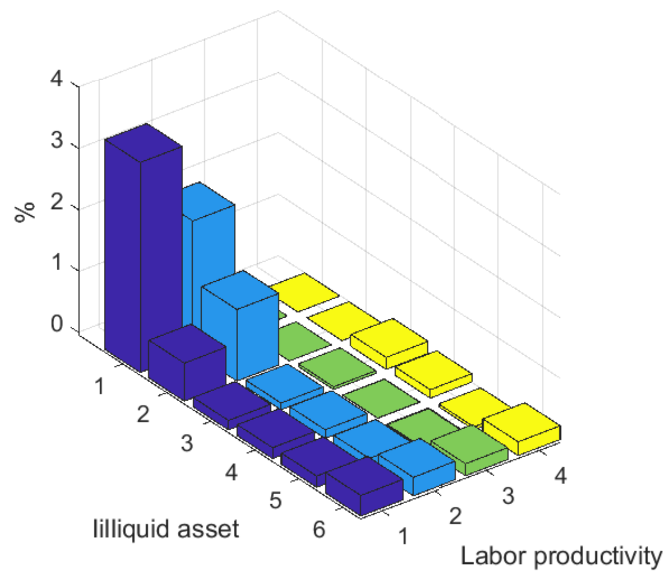
[51] A percentage deviation of consumption-equivalent gain can be computed as below.

$$\frac{c_i^*}{c_i} = \frac{[(1-s)(\rho v_{benchmark} - \rho v_i + u(c_i))]^{\frac{1}{1-s}}}{c_i} - 1$$

See appendix B for a derivation.

[52] Since the economy without a foreclosure option has a lower loan-to-value limit and the equilibrium h price varies, the consumption equivalent gain is compared with $(a, \frac{b}{ph}, \varepsilon, h)$ instead of (a,b,ε,h).

Figure 4: Value of bankruptcy



Note: Consumption equivalent gain from the benchmark and the foreclosure only economy. Each bar shows an average consumption equivalence gain at a given labor productivity and illiquid asset holdings. The figure plots over only one set of persistent components of labor productivity with a low level of the transitory component. The distribution of gains over the same level of a persistent component but with a different transitory component is similar.

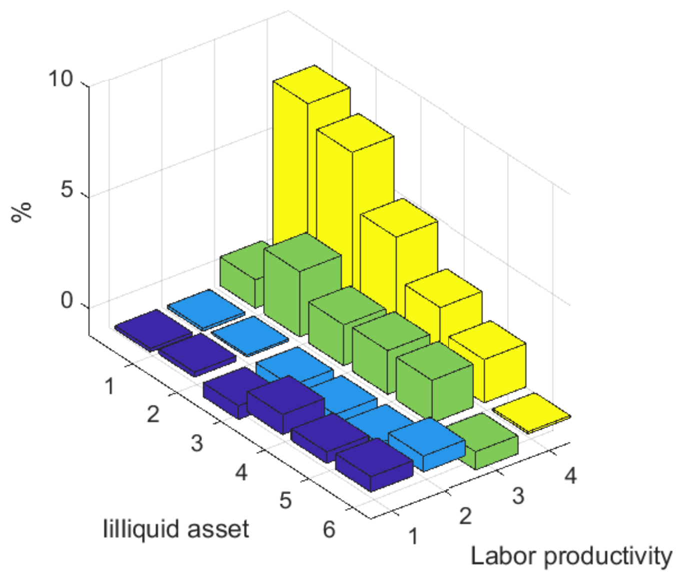
lustrates the value of the foreclosure option. Each bar in the figures represents the average consumption equivalent gain of households who have a specific house size and level of labor productivity.[53]

These figures show that the benefit of default is unevenly distributed. Intuitively, those who have low income and low wealth value the bankruptcy option the most, and the results confirm this intuition. Figure 4 shows that those who have low income and low illiquid assets gain the most from bankruptcy. These households not only have small houses but also tend to have little liquid wealth, (Figure 24) therefore, they are likely to use the option.

The foreclosure option is mainly valued by high income households. However, as house size increases the average gain decreases. This may seem counter-intuitive: as house size increases, secured debt tends to increase as well, and the ability to walk

[53] Labor productivity is composed of persistent and transitory components. The figure plots over only one set of persistent components with a low level of the transitory component. The distribution of gains over the same level of persistent labor productivity component but with a different transitory component is similar.

Figure 5: Value of foreclosure



Note: Consumption equivalent gain from the benchmark and the bankruptcy only economy. Each bar shows an average consumption equivalence gain at a given labor productivity and illiquid asset holdings. The figure plots over only one set of persistent components of labor productivity with a low level of the transitory component. The distribution of gains over the same level of a persistent component but with a different transitory component is similar.

away from a large loan is more valuable. Figures 22 & 23 show that indeed households with large houses tend to have more debt. Then why do more indebted households value foreclosure less? This follows from a house price effect that arises from foreclosure. The possibility of foreclosure enables households with low wealth to take on larger loans than otherwise and buy houses. The demand for housing rises, which, in turn, leads to an equilibrium price increase. Therefore to afford the same size of house when foreclosure is available, households need to bear higher maintenance costs. As loan size increases at each loan-to-value ratio, debt payments rise. Higher maintenance costs and debt repayments in the economy with foreclosure, reduce resources available for non-durable consumption. This makes the foreclosure option less desirable.

5 Effects of debt relief programs in recessions

Above, I have established that my model is consistent with salient empirical regularities characterizing the distribution of households. Furthermore, my model generates business cycles that resemble the data long important margins. These results make useful for the analysis of debt relief programs during recessions. In this section, I analyze such programs using a series of policy experiments.

During the Great Recession, the US government intervened in mortgage markets through household debt relief policies to support falling house prices and to slow the rising number of delinquencies.[54] These policies provided incentives to financial intermediaries and households to restructure their debt contracts. One such program was a principal reduction which forgave a fraction of a mortgage borrower's remaining balance.[55] Although participation rates were perceived to be low, Agarwal et al. (2017) show that the program was associated with reduced rates of foreclosure, consumer debt delinquencies and house price declines. While they provide important evidence from the microeconomic data, the estimated causal relations do not offer an aggregate estimate of the macroeconomic response of the economy to such a program. My model allows me to account for any resulting changes in the distribution of households and accompanying prices for which they do not account.

I design a policy intervention in which all households with loan-to-value (LTV) ratios above 95% at the time of the intervention, have a fraction of their mortgage debt forgiven. Eligible households see their LTV ratio fall to 95%. I assume that the policy intervention is unanticipated, moreover, it is not accompanied by increases in taxes.[56][57]

I compare the effects of the targeted mortgage debt relief program with those of a tax rebate. Tax rebates were stimulus packages approved by the US Congress in

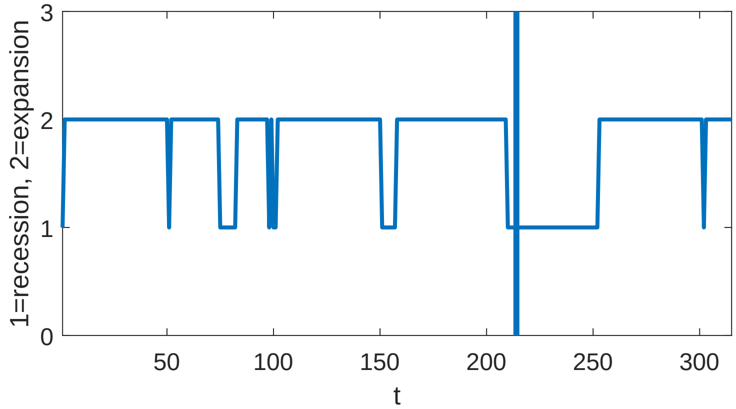
[54]For example, through programs such as the Home Affordable Modification Program (HAMP), the Principal Reduction Alternative (PRA), the Home Affordable Foreclosure Alternatives (HAFA) and the Home Affordable Refinance Program (HARP).

[55]In particular, the government introduced principal reduction modifications in 2010 in HAMP. This was a response to growing concerns that debt levels, not just debt repayments, were causing high foreclosure rates. Under this modification, mortgage borrowers' principal was forgiven until the new monthly payment fell below 31% of income or LTV ratio dropped to 115%, whichever came first. See Ganong and Noel (2018) for details of the modification.

[56]This allows comparability to a similar exercise in the work of Kaplan et al. (2017). Further, the costs of the program are partly evident from comparison to the unfunded tax rebate examined below.

[57]Before and after the shock, the simulation is based on forecasting functions estimated in an environment without the policy intervention. Thus these policies were unanticipated. This is intended to capture the unusual nature of the Great Recession, whose severity was unexpected by most policy makers and market participants.

Figure 6: Total factor productivity



Note: The sequence of aggregate shocks surrounding policy evaluations and the timing of the intervention. The government intervenes at t = 214.

the last two recessions of 2001 and 2007–2009. For comparability, I also model the rebate as an unexpected intervention, and I set its overall size to match the total cost of the debt forgiveness program. In the case of the rebate, each household receives a lump sum transfer equivalent to $3,530 (2010 dollars).[58] The critical difference between the two programs is that the tax rebate is distributed equally, providing liquid income to all households. In contrast, debt forgiveness directly affects only some eligible households. Furthermore, the increased housing equity of those households is illiquid, given the transaction costs.[59]

I begin with the principal reduction program, first presenting its effects on aggregate variables and then analyzing in detail the consumption responses among different segments of the population. Afterward, I contrast these results with those from the across-the-board tax rebate. I reconsider the consequences of the mortgage forgiveness program when the policy intervention comes late, thereby illustrating the state-dependent nature of its effects. Finally, I close by discussing the results under two alternative debt relief programs.

5.1 Mortgage forgiveness

Figure 6 shows the sequence of aggregate shocks surrounding this policy evaluation, and the timing of the intervention. Following a relatively long expansion, the government intervenes promptly with a mortgage principal reduction program at t = 214, just after the start of a recession. In the model, this program affects approximately 26% of households, and the average size of the mortgage principal reduction for eligible households corresponds to roughly about \$13,500 (2010 dollars).

5.1.1 Aggregate responses

Figure 7 shows the aggregate responses of output, capital, consumption and house prices after the policy intervention.[60] The principal reduction is successful at increasing consumption and the capital stock. The rise in capital leads to higher output in subsequent periods. The aggregate marginal propensity to consume (MPC), measured as the ratio of the increase in aggregate consumption with the intervention (versus without) relative to the total debt forgiven, is 4%.

Initially, before interest rates and house prices begin adjusting to the policy intervention, ineligible households respond very little.[61] By contrast, the total response across eligible households is substantial. Home equity gains arising from forgiven debt for such households, lead to an increase in their wealth. However, the wealth effect on household consumption is unclear. Transaction costs imply that the proceeds of a mortgage reduction cannot be used costlessly to purchase goods and services. Thus it is not evident that nondurable consumption responds to the extent it does. I will return to this point in the next sub-section.

The effects of the policy are also very persistent. In particular, capital is higher than the benchmark economy with no policy intervention even 100 quarters later, and the impact on consumption persists for more than 8 years. The principal reduction also mitigates the fall in house prices that otherwise occur in the recession. Recall all

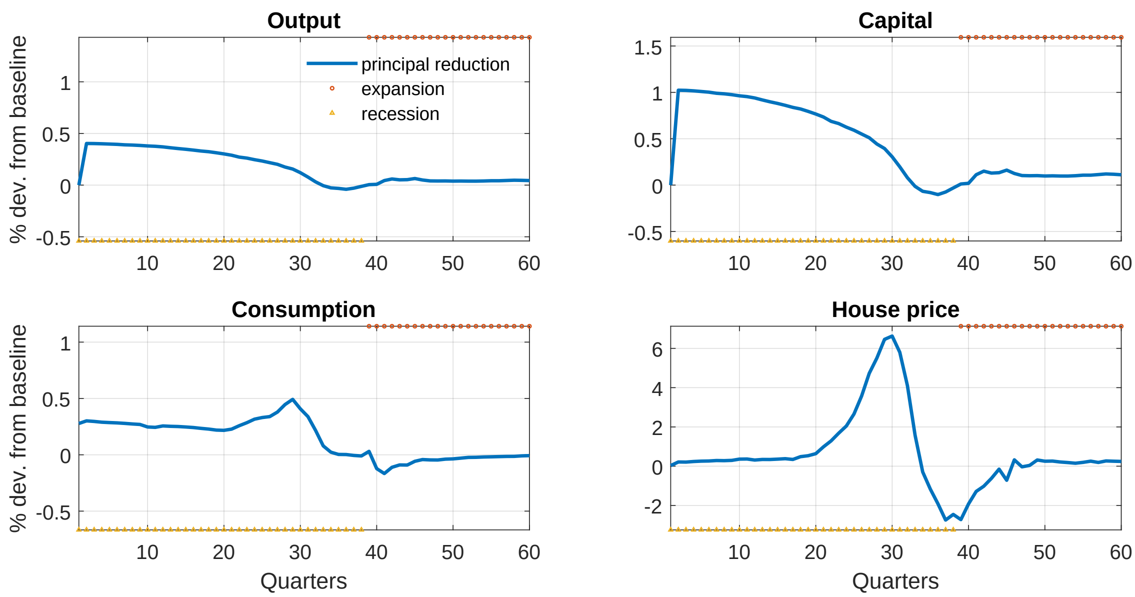
[58] The Economic Stimulus Act of 2008 consisted of a 100 billion dollar program that sent tax rebates to approximately 130 million US tax filers. Single individuals received \$300–\$600 and couples received \$600–\$1,200. Besides, eligible households received \$300 per child. See Parker et al. (2013) for details.

[59] There is an indirect liquidity effect of the principal reduction program. Since households who have secured debt pay interest rates on their loans, a reduction in the outstanding of the loan increases liquidity by reducing interest rate payments.

[60] Throughout all exercises, my graphs show the responses of variables in terms of their deviations from the corresponding values in the economy without a policy intervention. Aggregate variables movements in the baseline economy without a policy intervention can be found in Appendix C, Figure 26 & 27.

[61] House prices increase by 0.2% at the time the mortgage reduction policy is implemented. This leads to negligible changes in decision rules for ineligible households.

Figure 7: Response of aggregate variables



Note: Aggregate variable movements in terms of their deviations from the corresponding values in the economy without policy intervention.

quantities are presented as a percentage deviation from a baseline economy experiencing the same recession without any policy. While initially modest, the program's implications for house prices grow over time as an increasing number of foreclosures are prevented. After 30 quarters, the price of a house is approximately 7% higher than it would be without the intervention.

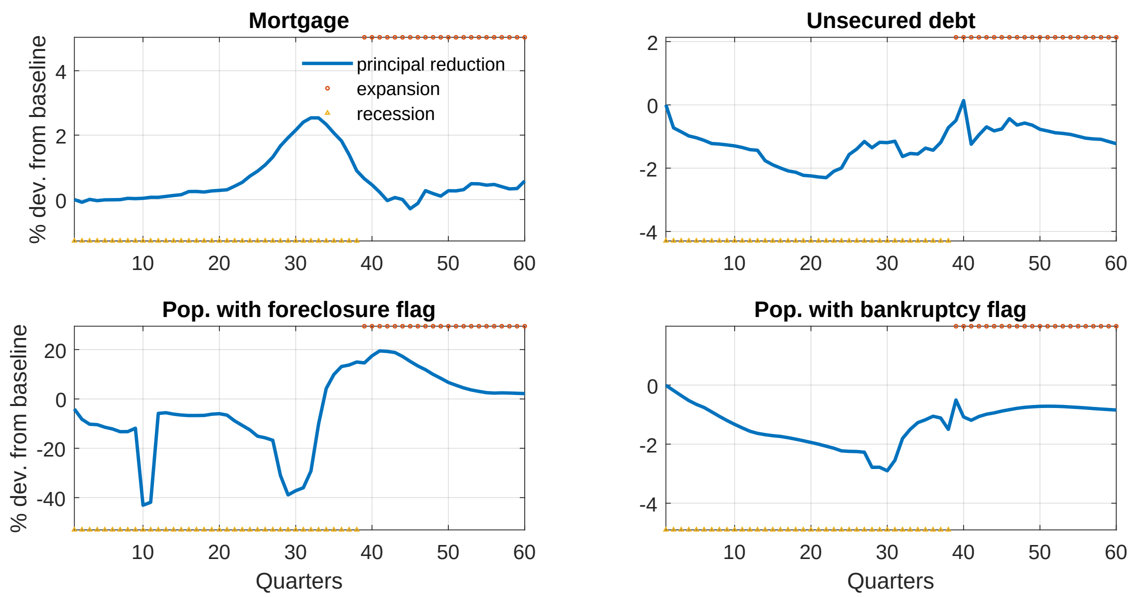
Figure 8 shows the response of credit and default rates. By reducing the number of financially distressed households, the intervention significantly reduces foreclosures.[62] [63] Since foreclosures increase the stock of houses for sale, a reduction in foreclosure rates dampens the fall in equilibrium house prices. Although the policy forgives a fraction of targeted households' mortgages, total mortgage debt actually rises for roughly 10 years following the intervention.

Two factors contribute to this rise in mortgages. First, because the intervention induces a rise in aggregate capital, future interest rate payments on any given sized loan

[62]House prices fell 18.7% during the Great Recession, based on the All-Transactions House Price Index for the United States between their peak (4th quarter of 2006) and trough (2nd quarter of 2012). (US Federal Housing Finance Agency) In the model, house prices fall by 12% from the intervention period to the 31st quarter after the intervention.

[63]This result is consistent with the negative relationship between the amount of negative equity and mortgage default rates in Haughwout et al. (2009) and Gerardi et al. (2017).

Figure 8: Response of credits and defaults



Note: Aggregate variable movements in terms of their deviations from the corresponding values in the economy without policy intervention.

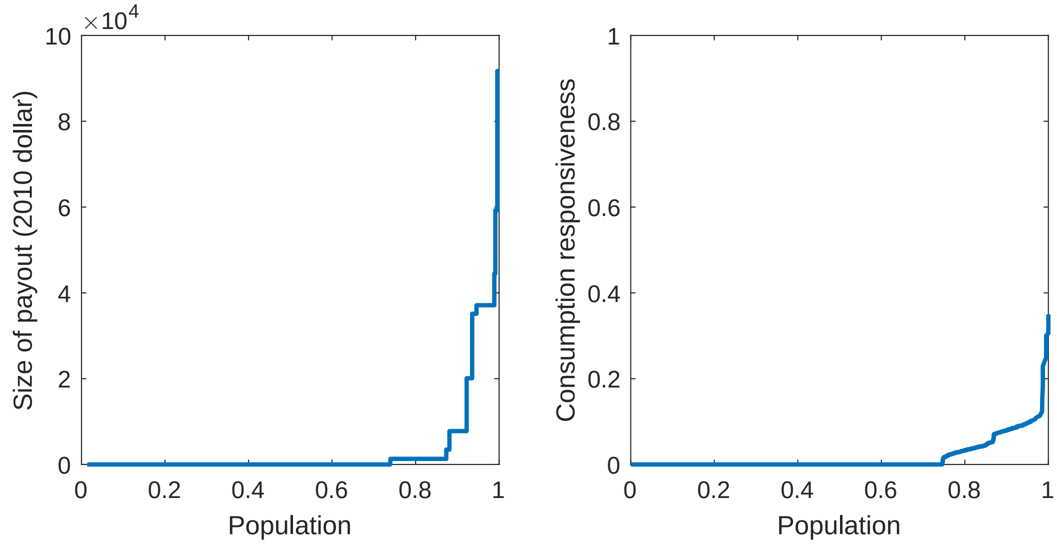
fall, making it less costly to hold a mortgage. Second, since a mortgage loan-to-value constraint is relaxed by rising house prices, higher house prices enable households to take on larger mortgages. Notice, in the right-hand panels of Figure 8, how the program targeted to relieve the burden of secured debt spills over to unsecured debt. Although the principal reduction policy only applies to mortgages directly, it reduces both the stock of credit card debt and bankruptcy rates.

As discussed in Section 1, Kaplan et al. (2017) perform a similar experiment. They also consider a policy that forgives a fraction of mortgages so no household has a LTV ratio higher than 95%.[64] However, the results are very different. They find a mortgage forgiveness program would not have prevented the sharp drop in house prices and aggregate expenditures, but would have significantly dampened the rise in foreclosures. While my household environment shares several features in common with theirs, the presence of capital, and equilibrium interest rate and wage movements together imply different policy outcomes.

In the Kaplan et al.'s (2017) model, only house prices are determined in equilib-

[64]The timing and the scale of the policy are also similar. They assume the policy was implemented in a timely manner (two years into the bust), and it affects over 1/4 of homeowners with mortgages.

Figure 9: Principal reduction size and consumption responsiveness



Note: Consumption responsiveness is the difference in consumption between the economies with and without the principal reduction program to the reduction size.

rium. In contrast, in my model, interest rates and wages are determined in equilibrium, as well as house prices. As seen above, capital rises upon the intervention. This increases wages which benefit all households. It also implies lower interest rates which benefit net borrowers at the expense of net savers. These income effects from the changes in aggregate capital help to support house prices. Higher house prices reduce subsequent foreclosures and loosen financial constraints, preventing further falls in house prices. In the absence of interest rate and wage responses, Kaplan et al.'s (2017) model delivers minimal subsequent responses in other aggregate variables.

5.1.2 A closer look at non-durable consumption responses

The lower left panel of Figure 7 shows the aggregate consumption response to the mortgage principal reduction relative to the baseline economy without a policy. In this section, I analyze how the policy intervention affects different segments of the population.[65]

Figure 9 shows the cumulative distribution functions over the population (normalized to 1) of the principal reduction size and marginal propensity to consume (MPC) at

[65]Most results in this section are generated from a simulated panel. The sample size is 21,357.

the intervention period.[66][67] These distributions are computed using simulated panels. The average size of the principal reduction is approximately $13,500 (2010 dollars), but the amount each household receives varies greatly. Although the aggregate MPC is 4%, the MPC distribution ranges from zero to nearly 40%. Most households not receiving the reduction do not respond because prices are close to their values in the no-intervention baseline economy.

Who responds most? Having seen the dispersed MPC distribution, it is natural to investigate the common characteristics of households that show high or low consumption responsiveness to the principal reduction policy. While there has been some work looking at heterogeneous MPCs using tax rebates as natural experiments, the consumption response to the mortgage principal reduction hinges on eligibility, thus is likely to differ.[68] To my knowledge, the heterogeneity of consumption responses to a mortgage principal reduction program have not been examined before.[69]

Figure 10 shows the marginal propensity to consume and the population of households over net worth, LTV ratios and liquidity quintiles. Liquidity is a ratio of liquid saving over net worth. For the MPC, each bar is the average MPC of households in the corresponding quintile. This average includes only households receiving the principal reduction. The upper left panel shows that high LTV households are equally distributed over the 1st - 4th quintiles and that MPC falls with net worth. The bottom right panel shows that households eligible for the principal reduction are concentrated

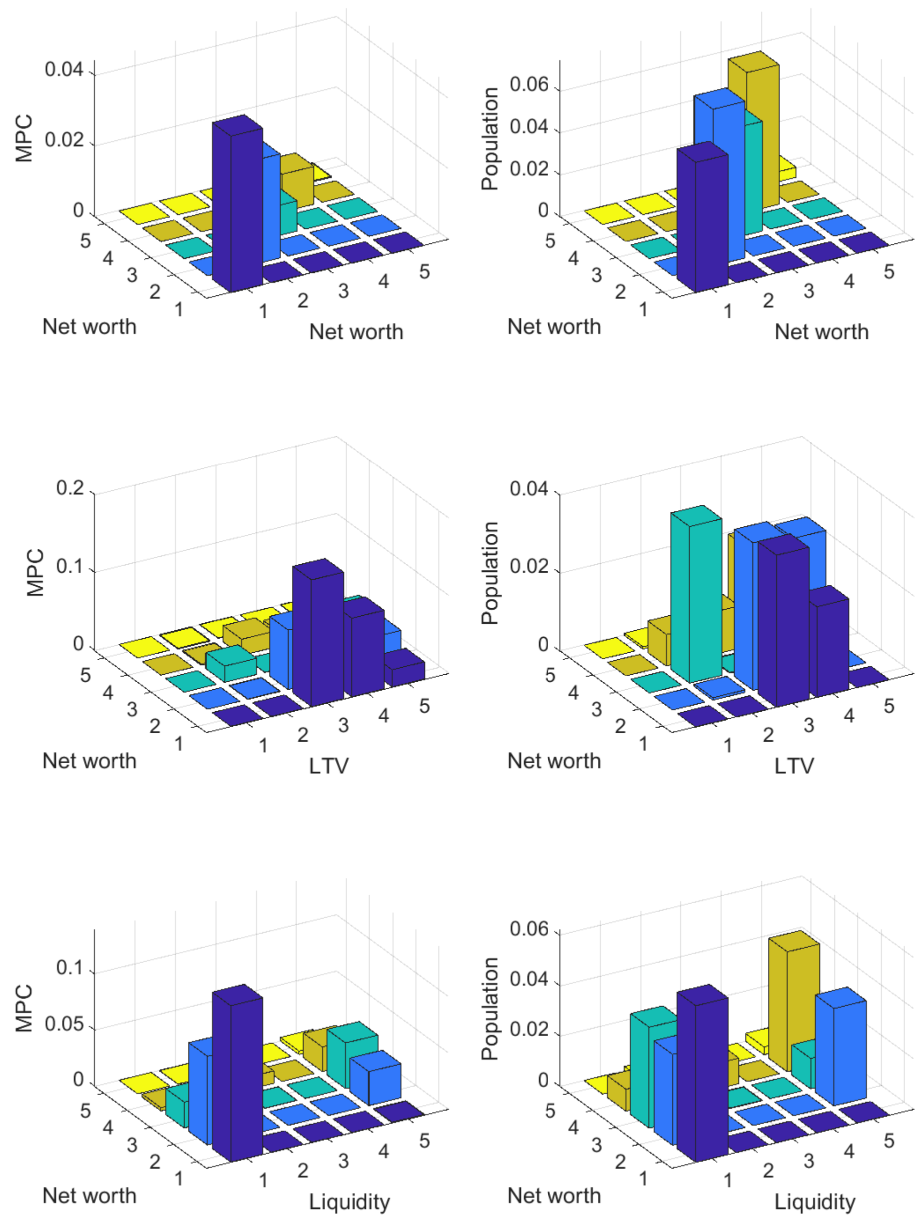
[66] Individual MPCs are computed in the same way the aggregate MPC was computed in the previous section.

[67] Negative MPCs are dropped. Roughly 0.2% of households consumed less than the households who did not receive the principal reduction. These are households that would have chosen to default if they had not received the principal reduction. While these households keep their houses and consume housing services, their non-durable consumption is lower than households that default.

[68] Misra and Surico (2014) allow the propensity to consume of the 2001 and 2008 tax rebate to vary across household groups using quantile regressions. They find that almost half of households did not adjust their consumption and, 20% of households with low income spent a small but significant amount. Households that show high MPC held high levels of mortgage debt. Kaplan and Violante (2014) study the heterogeneity in MPC using a two assets (a low-return liquid asset and a high-return illiquid asset) model. They show that many households in the model are 'wealthy hand-to-mouth' (holding little liquid wealth despite holding a sizable amount of illiquid assets) and these households display large propensities to consume out of additional transitory income such as tax rebate.

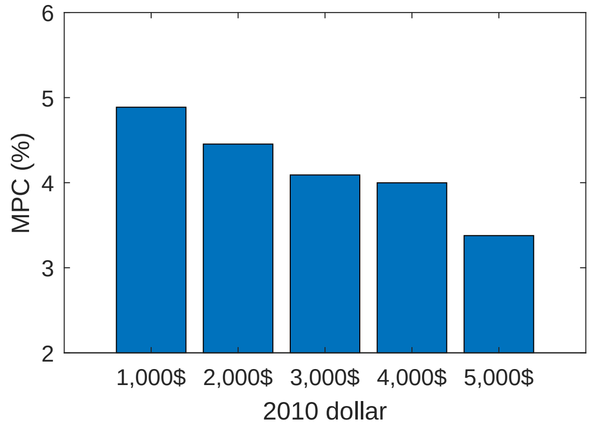
[69] Ganong and Noel (2018) estimated consumption responses to principal reduction using Administrative data on HAMP participants and consumer credit bureau records, but they do not focus on heterogeneity. At the aggregate level, they find that principal reduction has no significant impact on borrowers' consumption or default rate, which is inconsistent with my result. They compare two groups that both received payment reduction, and the treatment group also received a principal reduction. Moreover, the principal reduction only reduced the LTV ratio to 115%, which means borrowers were still underwater after the reduction. Therefore these results are not directly comparable.

Figure 10: MPC and population



Note: These bars plot the MPC and population of households over net worth, the LTV ratio and liquidity quintiles. The MPC is $\frac{\text{c(policy) - c(no policy)}}{\text{principal reduction size}}$ and liquidity is a ratio of liquid savings over total assets (saving plus house value). For the MPC, each bar is the average MPC of households that are in the quintile. Only households receiving the principal reduction are counted in the population measures reported.

Figure 11: MPC over refinancing cost



Note: The figure shows aggregate MPC over refinance costs (ξ_r). In each case, households receive the same principal reduction; only refinancing costs change. This refinancing cost is 2,500$ in the baseline calibrated model.

in the 1st or 5th quintiles of distribution of liquidity. Of these two groups, households with low liquid wealth show stronger consumption responses. Sorting households by MPC, I find that the characteristics of high MPC households are consistent with the observations seen in Figure 10. Specifically, the top 10% of households in terms of MPC response to the policy tend to be in the 1st-2nd quintiles of net worth and liquid saving. In terms of housing wealth, income and mortgage size, these households are mostly in the 2nd-3rd quintiles.

How do they increase consumption? Figure 10 shows that households with low net worth and low liquidity respond most to the principal reduction. Given that these households are nearly hand-to-mouth, gains in home equity cannot immediately explain the large consumption responses.[70] This is because, in the absence of adjustment to their liquid resources, these households lack the ability to increase their consumption. It turns out that these households increase their consumption using refinancing. Approximately 23% of households eligible for a principal reduction refinance their mortgage. Two factors contribute to this. First, some of these households were near the borrowing limit (LTV 105%), so refinancing was not available to them. Second, refinancing costs decrease following the policy. As explained in Section 2, refinancing requires the payment of a fixed refinance fee (ξ_r) and the repayment of the remaining mortgage balance first. In addition, a new mortgage involves a discount due to default risk. As a principal reduction reduces the default risks of households, they

[70]Hand-to-mouth are households that hold little or no wealth.

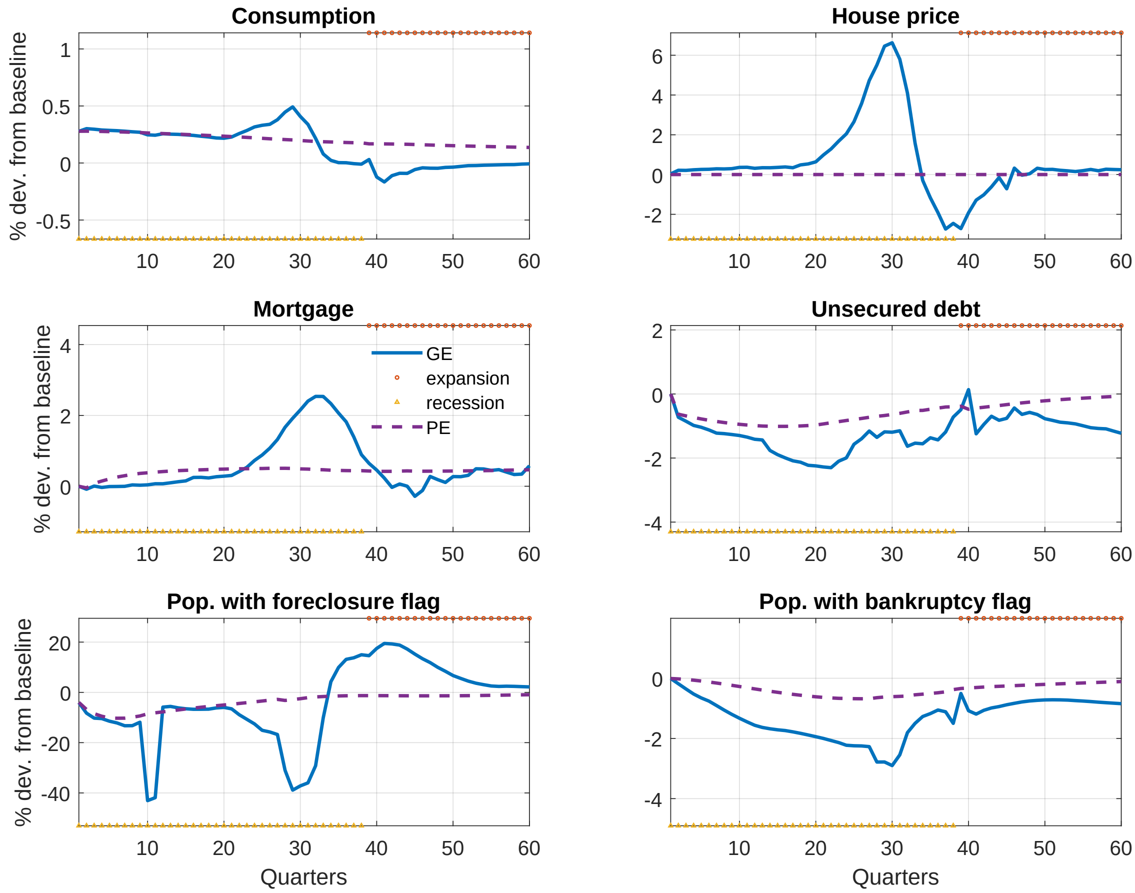
can refinance at lower discount rates, which implies lower total refinancing costs. In other words, the debt reduction reduces the loan rates at which eligible households can refinance. Figure 11 shows that aggregate consumption responds less as the fixed refinancing fee increases.

General equilibrium effects In the periods after the mortgage reduction policy, households consume differently compared to households in an economy without the intervention, even when they have the same initial portfolio and income. These differences arise from responses in prices following the new policy. As Figure 7 shows, the mortgage forgiveness program leads to a rise in aggregate capital. As a result, the equilibrium risk-free real interest rate falls and the wage rate rises. Although higher wages benefit all households, the effects of low interest rates are more nuanced. They benefit households with substantial debt and little liquid savings by reducing their interest payments. Conversely, households with substantial financial wealth are worse off; as they now earn a lower return on their liquid assets. Therefore the overall effect depends on the distribution of households over liquid savings and housing debt. The intervention also keeps house prices higher than the baseline economy for almost 8 years and the effects here are also complex, benefiting sellers at the expense of buyers. For homeowners that do not buy or sell, raised house prices lower disposable income through increases in maintenance costs and property taxes.

To quantify the general equilibrium effects of the mortgage principal reduction, I feed in the price paths (for interest rates, wages and house prices) from the no-intervention baseline and re-compute aggregate responses to the program. Figure 12 shows the responses of aggregate variables with market clearing prices (GE) and using baseline economy prices (PE). Overall, aggregate responses are smoother and more muted without the price changes discussed above. In general equilibrium, house prices increase and reach their peak after about 30 quarters from the start of the intervention, and consumption and mortgage debt co-move with it. The rise in consumption is mainly driven by households that sell their houses rather than defaulting, and by households who would sell anyway but now sell at a higher price.

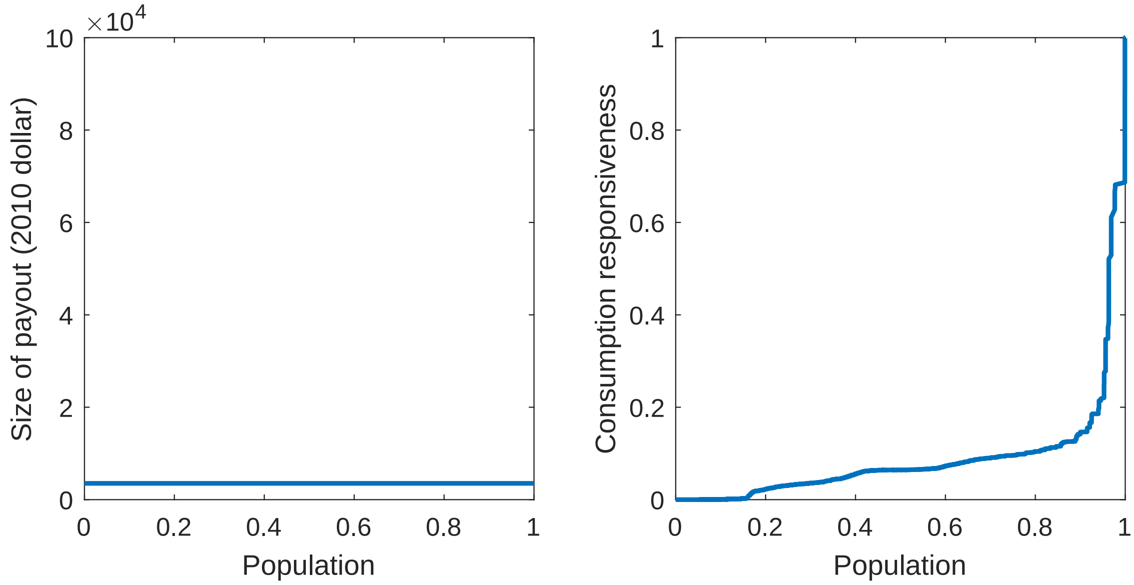
As already mentioned, households are affected by the policy's implied price changes differently depending on their portfolios. Here, the increase in wages and house prices, and lower interest rates tend to reduce the gains of wealthy households and benefit indebted households. Comparing cumulative consumption for 5 years after the intervention under GE and PE, households that consume more under GE are in the 1st - 2nd quintiles of the net worth distribution and have high leverage. These households can benefit from price changes by selling their houses at higher prices and paying

Figure 12: Responses of aggregate variables: Price effects



Note: Aggregate variable movements in terms of their deviations from the corresponding values in the economy without policy intervention. PE: The responses are computed using price paths (interest rate, wage and house price) from the economy without the policy intervention. GE: The responses under the market clearing prices.

Figure 13: Tax rebate size and consumption responsiveness



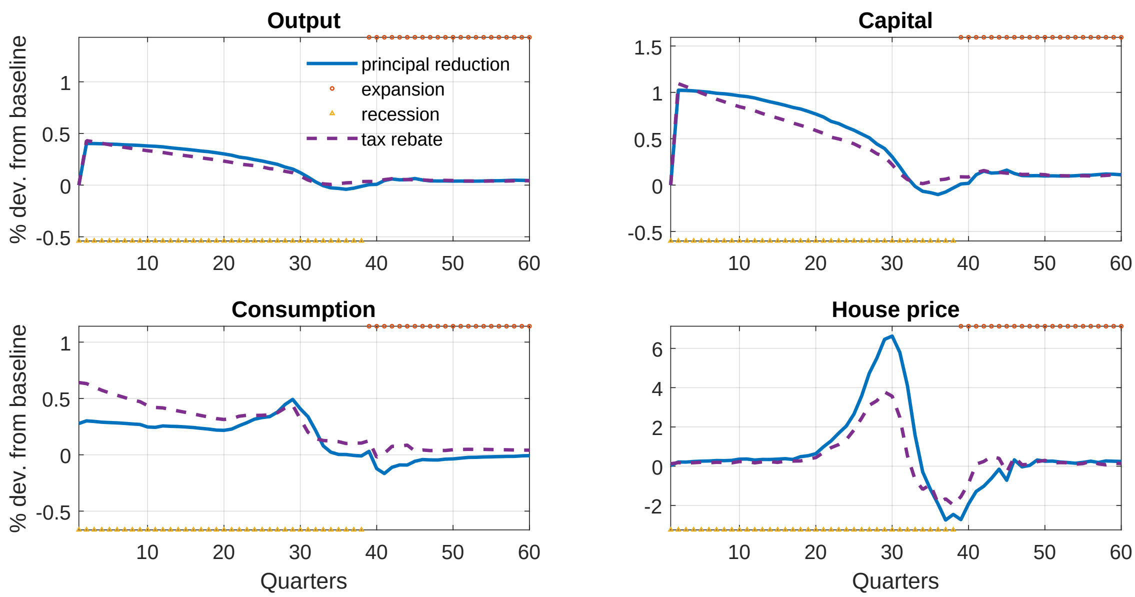
Note: Consumption responsiveness is the difference in consumption between the economies with and without the tax rebate to the rebate size.

lower interest rates on their debt. In contrast, households consuming less under GE have large houses, high savings, and small debt. Equilibrium price changes hurt them because higher house prices reduce their disposable income through increased maintenance costs and reduce interest rates lower the returns on their financial wealth.

5.2 Tax rebate

In this section, I consider the effects of a lump-sum tax rebate selected for comparability with the mortgage forgiveness program studied above. I assume that the policy intervention is unanticipated. All households receive the same transfer, and the total cost is set to match the total cost of the principal reduction. Defining MPC in the same fashion as in the principal reduction exercise, Figure 13 shows that the MPC distribution is more dispersed compared to that following the targeted mortgage forgiveness program. This is not surprising considering that the rebate is given to all households, and rebates directly increase liquid resources available for consumption spending. The characteristics of households exhibiting high MPCs are similar to those under the principal reduction (low net worth, high LTV, and low liquidity). However, the tax rebate program includes households that do not own a house and, among these,

Figure 14: Response of aggregate variables



Note: Aggregate variable movements in terms of their deviations from the corresponding values in the economy without policy intervention.

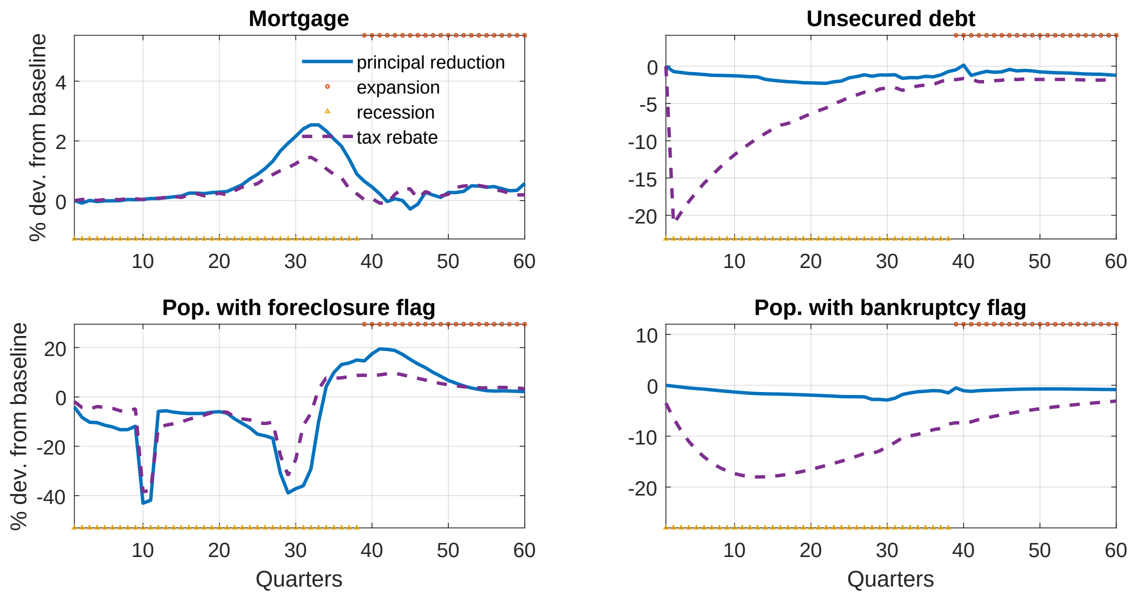
those with little liquid wealth (their only wealth) have high MPCs.[71]

Figures 14 and 15 show the responses of aggregate variables to the tax rebate alongside responses following the principal reduction program. The tax rebate is more effective than the principal reduction at boosting consumption, partly because the rebate directly adds to households' liquid wealth and partly because it distributes a higher fraction of benefits to high MPC households. As shown in Figure 9, the sizes of principal reduction benefits varied greatly across households. Since households with low net worth tend to have small houses and small mortgages, the reduction they received was also small. Therefore, although the consumption responsiveness of each such household is large, their overall contribution to aggregate consumption is small.[72]

[71] In Figure 28, this group appears in 2nd quintile of net worth and LTV and in 5th quintile of liquidity. Although they have low wealth, since liquidity is defined as liquid wealth over total wealth, the lack of a house makes them appear as a high liquidity group.

[72] When such households receive a higher fraction of benefits, aggregate consumption responses to the principal reduction increases. To see this, I compute responses under two alternative specifications of the principal reduction program. First, I add an eligibility criterion, requiring that households be below the 60th percentile in the net worth distribution. Second, I maintain the eligibility criteria of the original program, but make the benefit the same for all eligible households across them. In each case, I set the total program spending to match the cost of the original policy. These alternative plans deliver a larger reduction amount to poor households. Figure 29 and Figure 30 in Appendix C show

Figure 15: Response of credits and defaults



Note: Aggregate variable movements in terms of their deviations from the corresponding values in the economy without policy intervention.

The tax rebate is also more effective at reducing bankruptcy and unsecured credit by providing income to poor households lacking a house or mortgage since these are the households most likely to use unsecured borrowing and bankruptcy. Conversely, the mortgage principal reduction program is more effective in reducing foreclosure and supporting house prices, because it targets households likely to sell their houses or enter foreclose and provides them additional wealth.

Beyond the Wealthy Hand-to-Mouth A wide body of work has studied consumption responses to a tax rebate. In particular, using the U.S. tax rebate episodes of 2001 and 2008, empirical studies find that households spend approximately 25% of rebates on non-durables in the quarter that they are received, in contrast to what would be suggested by the permanent income hypothesis.[73] Kaplan and Violante (2014) develop a structural model that can replicate these high estimated MPCs in response to a tax rebate. The key insight of their theory is that many households choose to be

the responses these alternative interventions. Both deliver initial consumption responses close to that under the tax rebate, and both are more effective than the baseline principal reduction in reducing foreclosure and bankruptcy.

[73] See Parker et al. (2013), Johnson et al. (2006), Misra and Surico (2014),Sahm et al. (2010), Agarwal et al. (2007) and Shapiro and Slemrod (2009).

‘wealthy hand-to-mouth’: they hold a small amount of low-return liquid assets (e.g., cash, checking account) while owning a sizable portfolio of high-return illiquid assets (e.g., housing, retirement account) that requires transactions costs to adjust. Since they are hand-to-mouth, when they receive a windfall gain such as tax rebate, they consume a significant amount of it.

I can further decompose household portfolios in my model to investigate whether indebtedness affects the consumption responses to income shocks. As Misra and Surico (2014) show, homeownership without mortgages does not vary across consumption responsiveness, but mortgage level tends to rise as MPC rises. Conditioning on hand-to-mouth status, can indebtedness explain consumption responsiveness better? Indebtedness may affect a household’s consumption decision because it affects disposable income. Highly indebted households have a lower discretionary income to spend due to debt repayment obligations compared to households that have the same net worth but no debt. Moreover, high leverage introduces more risk in the return wealth as house prices fluctuate over time.

I find that leverage does affect a household’s marginal propensity to consume. The aggregate MPC to a tax rebate is 9% while that of hand-to-mouth homeowners is 14%. Moreover, the MPC of mortgage borrowers is 15%, while that of households that own houses outright is 8%.[74] Furthermore, Figure 31 shows that MPCs rise as leverage increases. The figure plots average MPCs over household quintiles for several elements of the household distribution, restricting attention to hand-to-mouth homeowners.

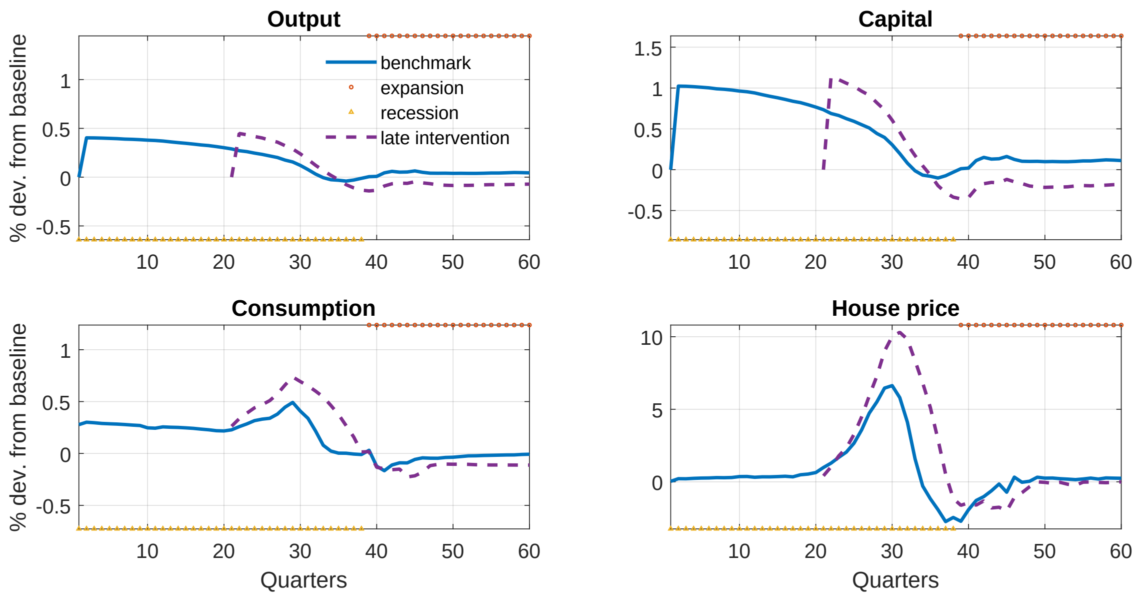
5.3 Does mortgage forgiveness timing matter?

The efficacy of a policy can be state-dependent.[75] In the previous analysis I assume that the government intervened promptly. Does late intervention bring a different outcome? To investigate this, I assume that the government intervenes five years later than in the benchmark case. For comparability to the benchmark specification, I choose households that have LTV higher than 95% but adjust reduction amounts to match the total cost at the total cost of the benchmark case. At the time of the intervention, leverage is 1% point higher, 1.2% more households are eligible for principal reduction, and 1.4% more households have unsecured borrowing. Figure 16 and Figure 17 plot the responses of aggregate variables to the late intervention. At

[74] I classify households with liquid savings less than 50% of their labor income as hand-to-mouth households.

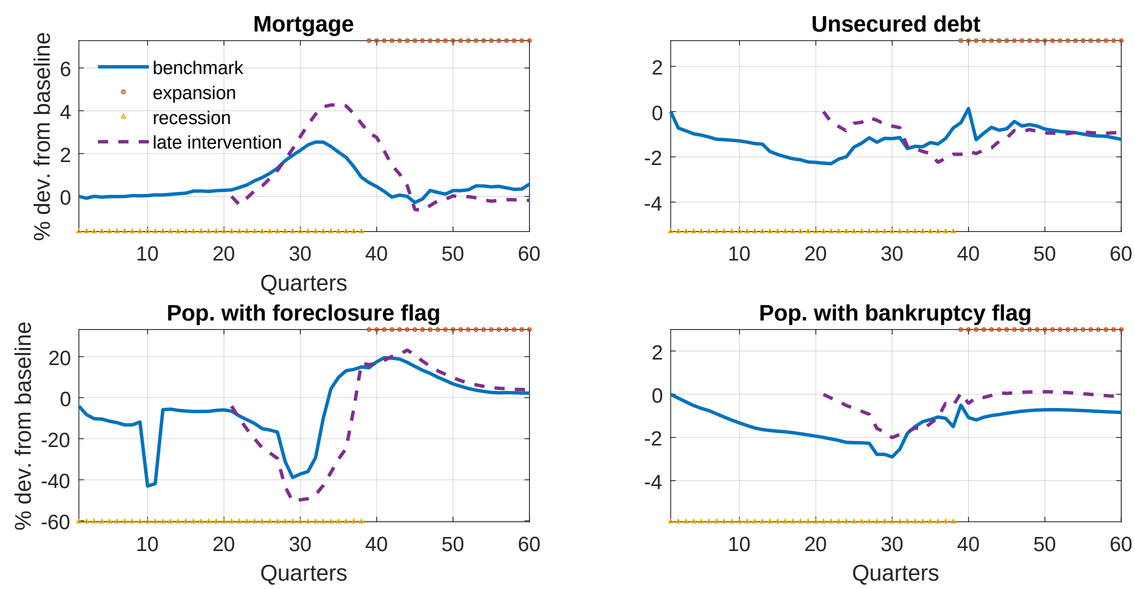
[75] Beraja et al. (2018) and Eichenbaum et al. (2018) show that the effects of monetary policy are state dependent. My analysis suggests state dependence for policies that directly affect household leverage.

Figure 16: Response of aggregate variables



Note: Aggregate variable movements in terms of their deviations from the corresponding values in the economy without policy intervention.

Figure 17: Response of credits and defaults



Note: Aggregate variable movements in terms of their deviations from the corresponding values in the economy without policy intervention.

first, the consumption responsiveness is slightly lower (10% lower), but house prices react strongly, which leads to a more significant response in consumption, mortgages and foreclosures. This may be the result of the most financially distressed households already defaulted or sold houses, such that downward pressure to house prices could have been lower at the time. While the short-run responses are greater than the responses of early intervention, the effects die out faster as the aggregate shock changes and there is an expansion. The long-run effect of the policy, measured using cumulative consumption, is larger when the government intervenes earlier rather than later.[76]

The series of policy experiments studied here suggests that a mortgage principal reduction policy has a sizable impact in the short-run by directly reducing debt for eligible households, and has an impact in the long-run through the price effects of the policy. In particular, these responses are primarily driven by households with low net worth and high leverage. A government that aims to reduce foreclosure and support falling house prices should recognize that targeting highly leveraged households with low net worth will yield higher responses.

5.4 Other debt relief programs

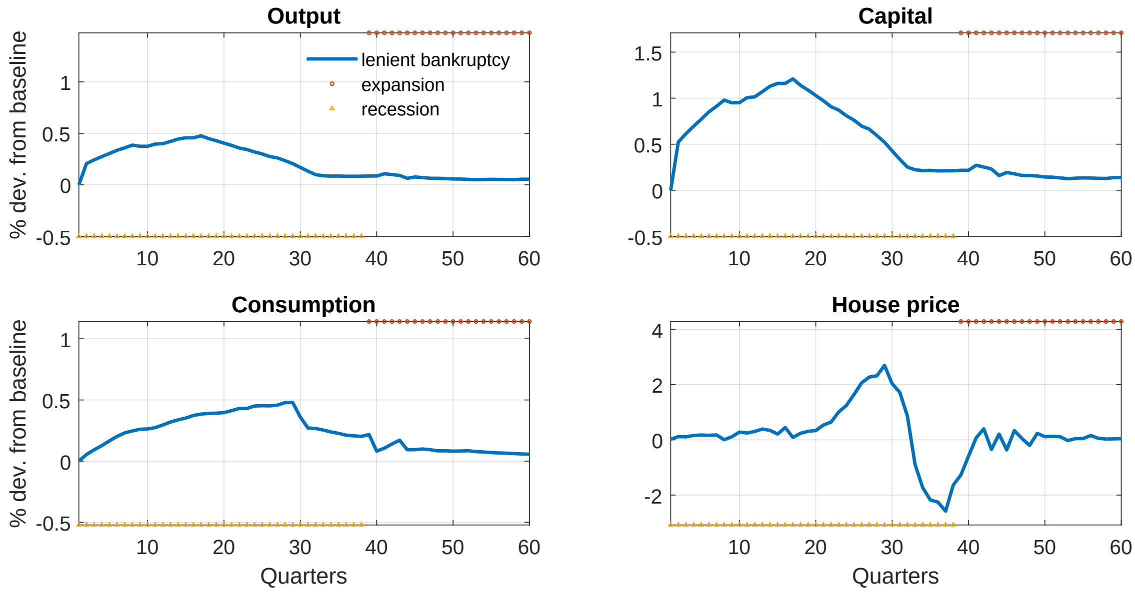
In addition to mortgage forgiveness, I consider several other policy alternatives. I study the effects of reducing the cost of declaring bankruptcy and also mortgage payment. As in section 5.1 & 5.2, I assume that both policies are unexpected. While these policies do not require government funding explicitly, financial intermediaries' losses increase when bankruptcy or foreclosures rise as a result changes in policy. I assume that the government absorbs such losses.

5.4.1 Easier bankruptcy

Consumer bankruptcy is one of the largest social insurance programs in the U.S. Indeed, Dobbie and Song (2016) show that Chapter 13 protection increases annual earnings, decreases mortality, and decreases foreclosure rates. Given its positive impacts on financially distressed households, I explore whether the government can mitigate a fall in consumption and output during a recession by reducing the penalties

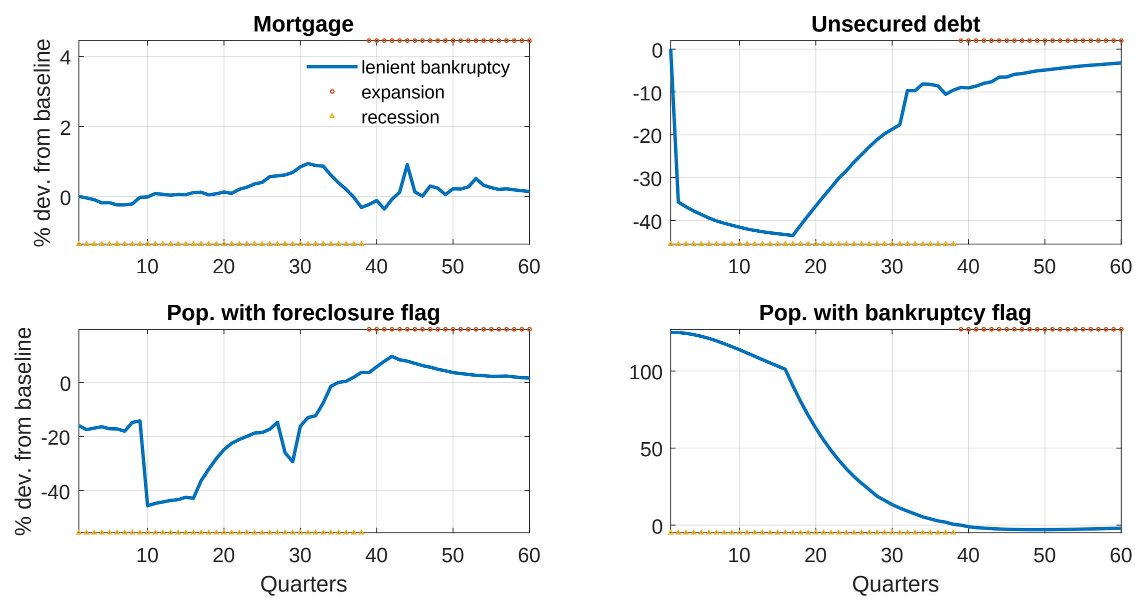
[76]The cumulative consumption over 30 quarters after the intervention is 0.3% higher than the cumulative consumption without the intervention in case of early intervention. It is 0.2% higher after the late intervention.

Figure 18: Response of aggregate variables



Note: Aggregate variable movements in terms of their deviations from the corresponding values in the economy without policy intervention.

Figure 19: Response of credits and defaults



Note: Aggregate variable movements in terms of their deviations from the corresponding values in the economy without policy intervention.

for filing for bankruptcy.[77]

I assume that the government loosens the requirements (e.g. bankruptcy filing costs) for bankruptcy so that more households can use the system. Specifically, I cut the utility cost ξ_a by half for 16 quarters. Figure 18 and Figure 19 show the impact of this policy. As intended, more households go bankrupt and more unsecured debt is forgiven as a result (the two right panels of Figure 19). If I consider the amount of forgiven unsecured debt due to the intervention as a cost of the policy, it is approximately 54% of the total cost of the principal reduction.

Consistent with the evidence in Dobbie and Song (2016), easier bankruptcy also affects households' foreclosure decisions. As households substitute bankruptcy for foreclosure, this policy becomes as effective as the principal reduction policy in reducing foreclosure. Capital increases partly because more debt is forgiven compared to the economy without the intervention and partly because unsecured borrowing is not allowed for households that have declared bankruptcy. Consumption increases gradually as unsecured debt is discharged. This reduces interest payments on debt. There are also positive income effects as capital increases. House price responses are similar, but the magnitude is smaller than the principal reduction.

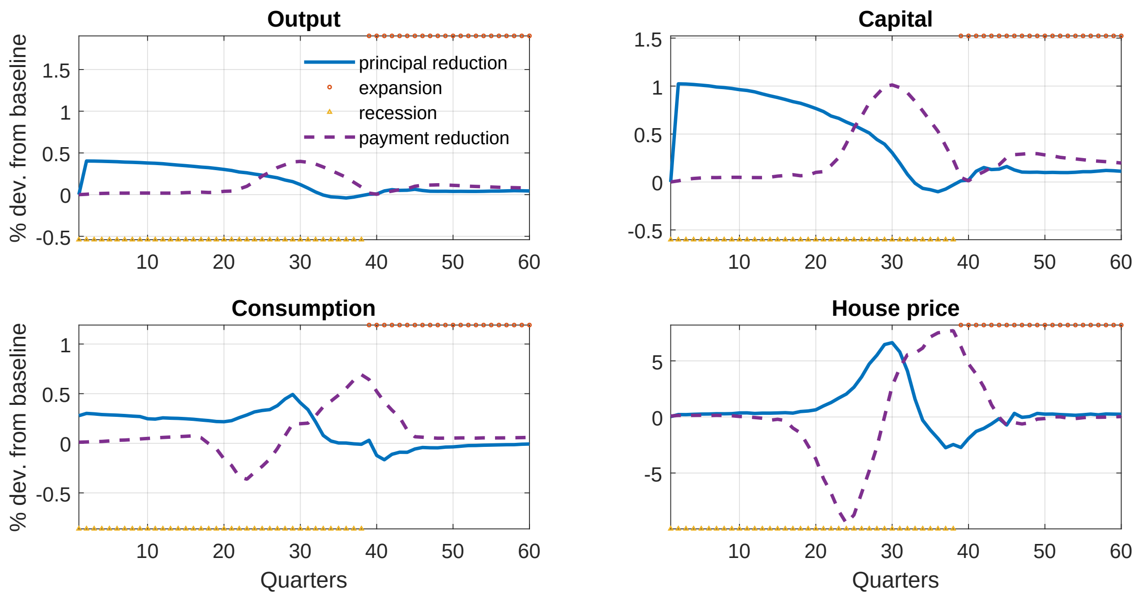
5.4.2 Mortgage payment reduction

As mentioned above in Section 5.1.2, Ganong and Noel (2018) show that principal reduction programs are not effective at reducing foreclosure, but mortgage repayment reductions are. Indarte (2019) also finds that bankruptcy filings respond weakly to changes in the generosity of bankruptcy (asset exemption) but is strongly discouraged by reductions in minimum debt payments. This suggests that providing wealth (principal reduction) will be less effective than providing liquidity (payment reduction) in reducing default rates. While these results are not directly comparable to the results in my work, it is worth studying the effects of a payment reduction in my model environment.

Mortgage borrowers are required to repay a fraction of their debt at every period. The size of repayment is set to make the average duration of a loan 30 years when a household fully finances a house. As a result, the per period payment depends only on a house value. For this policy experiment, I assume that per period repayments cannot exceed 31% of household labor income for 16 quarters. This policy effectively

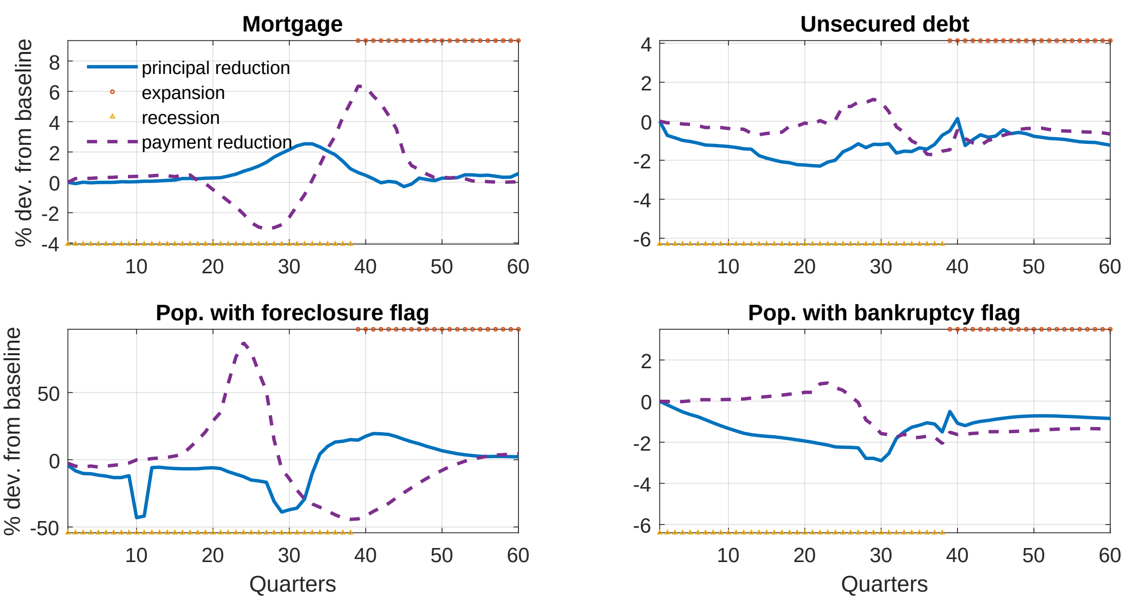
[77] Auclert et al. (2019) show that states with more generous bankruptcy exemptions had significantly smaller declines in non-tradable employment and more substantial increases in unsecured debt write-downs compared to states with less generous exemptions.

Figure 20: Response of aggregate variables



Note: Aggregate variable movements in terms of their deviations from the corresponding values in the economy without policy intervention.

Figure 21: Response of credits and defaults



Note: Aggregate variable movements in terms of their deviations from the corresponding values in the economy without policy intervention.

extends the duration of a loan by allowing slower amortization of debt.

Figures 20 and 21 show the responses following payment reduction. By increasing eligible households' disposable income, aggregate consumption is slightly higher during the intervention period. Foreclosure rates are also slightly lower during the same period. However, after the intervention is over, foreclosure rates rise and house prices fall. Due to the slowdown in repayment, eligible households remain slightly more leveraged than in an economy without the intervention. This higher leverage leads to rising foreclosure around the 20th quarter. Rising foreclosure contributes to falling house prices, which induces more foreclosure.

If we focus on the short-run effects of the reduced payment policy, it is effective in increasing consumption and in reducing foreclosure. However, when we also consider the long-run, general equilibrium effects, a payment reduction is not as effective as a mortgage principal reduction policy.

6 Concluding remarks

I have quantitatively assessed the effects of debt relief programs during recessions. I build a model with financial assets, unsecured debt, housing and mortgages as well as the options to default on both types of borrowing. The model successfully replicates the household wealth distribution, for net worth as well as its components, and default rates.

Although only a few households choose to default, I find the possibility of default affects both their distribution and aggregate variables significantly. In the steady state, not allowing bankruptcy leads to an equilibrium with higher mortgages as households shift their credit. This shows that the state of the economy with or without default will be different at the start of any recessions, and this difference will affect the propagation of shocks as well as the assessment of policy.

I show that a one-time mortgage forgiveness program significantly lowers foreclosure rates and supports house prices. Debt forgiveness also generates a significant and persistent rise in investment and consumption. Compared to a tax rebate, an untargeted, liquid income transfer to all households, the initial consumption response from the mortgage forgiveness program is smaller. However, its medium-term consumption response is larger, so the overall effects in the consumption of these two policies are similar. In terms of distributional effects, the price changes caused by a mortgage principal reduction policy tend to benefits poor households and hurts wealthy households.

A fruitful direction for future work would be to integrate the rich household portfolio and default studied here with nominal rigidities. Both policymakers and researchers have suggested that financial distress resulting from high leverage and falling house prices, forced households to reduce spending in the Great Recession, and that this led to a reduction in employment. Introducing nominal rigidities allows us to explore the extent to which mortgage debt relief programs, in episodes with low nominal interest rates, feedback to employment and output through their effect on demand.

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This Appendix is organized as follows. Section A describes the numerical solution methodology. Section B derives the consumption equivalent gain described in section 4.2. Section C contains additional figures and tables.

A Computation

A.1 Viscosity solution for HJBVI equation

The existence and uniqueness of the viscosity solution of an HJB equation are shown by Crandall and Lions (1983). One might question the existence and uniqueness of the viscosity solution of an HJBVI equation. The existence of the viscosity solution of an HJBVI equation when a value function is not always differentiable, which corresponds to the problem in the paper, was proven in Øksendal and Sulem (2005).[78]

A.2 Numerical Solution Methodology

The solution algorithm is based on a finite difference method using the upwind scheme in Achdou et al. (2017) with several extensions: multiple stopping choices (two types of default, buying and selling of the illiquid asset, refinancing) and aggregate uncertainty. I describe the details of the solution method in this section. First, I describe the solution methodology with the model without aggregate uncertainty. Section A.2.4 contains a description of how to solve the model with aggregate uncertainty.

A.2.1 Solving HJBVI as a Linear Complementarity Problem (LCP)

To solve the stopping time problem, I transformed the problem into a linear complementarity problem.[79]

The problem can be written as

$$min[\rho v - u - \mathbf{A}v, \;\; v - v^*] = 0$$

where A summarizes changes caused by decisions and shocks.[80]

$$(v - v^*)'(\rho v - u - \mathbf{A}v) = 0$$

[78] See chapter 9, theorem 9.8.

[79] I referred `http://www.princeton.edu/ moll/HACTproject/option-simple.pdf.`

[80] The next section describes how to construct A.

$$\rho v - u - \mathbf{A}v \geq 0$$

$$v \geq v^*$$

Let $z = v - v^*$, $\mathbf{B} = \rho\mathbf{I} - \mathbf{A}$ and $q = -u + \mathbf{B}v^*$. Then

$$(v - v^*)'(\rho v - u - \mathbf{A}v) = 0$$

$$\rho v - u - \mathbf{A}v \geq 0$$

$$v \geq v^*$$

and

$$z'(\mathbf{B}z + q) = 0$$

$$\mathbf{B}z + q \geq 0$$

$$z \geq 0$$

are equivalent. This is the standard form of linear complementarity problem and several solvers are available.[81]

A.2.2 Solving HJB equation with a finite difference method

I solve the problem on the discretized grid of the state space. I choose the number of grid $(n_a, n_b, n_\varepsilon, n_h)$ for the corresponding variables, (a, b, ε, h). Let v_{ijkp} be the value function of a household without the bankruptcy or the foreclosure flag, with an liquid asset a_i, an secured debt b_j, labor productivity ε_k and illiquid asset h_p. The derivative with respect to a, v^a_{ijkp} is approximated with either a forward or a backward difference approximation.

$$v^a_{ijkp} \approx \frac{v_{i+1jkp} - v_{ijkp}}{\Delta a} = v^{a,F}_{ijkp} \text{ or } v^a_{ijkp} \approx \frac{v_{ijkp} - v_{i-1jkp}}{\Delta a} = v^{a,B}_{ijkp}$$

Likewise, the derivative with respect to b, v^b_{ijkp} can be approximated with forward or backward numerical derivation.

[81] I used `https://www.mathworks.com/matlabcentral/fileexchange/20952-lcp-mcp-solver-newton-based`.

The equation (5) is approximated

$$\rho v_{ijkp} = u(c_{ijkp}, h_p) + v^a_{ijkp}\dot{a}_{ijkp} + v^b_{ijkp}\dot{b}_{ijkp} + \sum_{\varepsilon_k^{'}} \lambda(\varepsilon_k, \varepsilon_{k'})(v_{ijk'p} - v_{ijkp})$$
$$\forall\, i = \{1, .., n_a\}, j = \{1, .., n_b\}, k = \{1, .., n_\varepsilon\}, p = \{1, .., n_h\}$$

where

$$\dot{a}_{ijkp} = w\varepsilon_k + r_{a,ijkp}a_i - (r + \theta(b_j, h_p))b_j ph_p - c_{ijkp} - T(b_j, ph_p, \varepsilon_k) - \xi_h ph_p$$

$$\dot{b}_{ijkp} = \theta(b_j, h_p)b_j$$

The household choice of non-durable consumption can be solved from the FOC.

$$u^c(c_{ijkp}, h_p) - v^a_{ijkp} = 0$$
$$c_{ijkp} = (v^a_{ijkp})^{\frac{1}{-\sigma}}$$

As the derivatives of the value function have two forms, forward and backward, c_{ijkp} is either $(v^{a,F}_{ijkp})^{\frac{1}{-\sigma}}$ or $(v^{a,B}_{ijkp})^{\frac{1}{-\sigma}}$.

Upwinding and Boundaries To find drifts $\dot{a}$, it is necessary to select which derivative to use.[82] I follow Achdou et al.'s (2017) upwind scheme. The key idea is to use a forward derivative when drift is positive and use a backward derivative when drift is negative. To ease notation, let $x^+ = max(x, 0)$ and $x^- = min(x, 0)$. Also, let x^F be the computed value using a forward derivative and x^B be the computed value using a backward derivative. With these notations, saving can be computed as below:

$$s^{c,F}_{ijkp} = w\varepsilon_k + r_{a,ijkp}a_i - (r + \theta(b_j, h_p))b_j ph_p - c^F_{ijkp} - T(b_j, ph_p, \varepsilon_k) - \xi_h ph_p$$
$$s^{c,B}_{ijkp} = w\varepsilon_k + r_{a,ijkp}a_i - (r + \theta(b_j, h_p))b_j ph_p - c^B_{ijkp} - T(b_j, ph_p, \varepsilon_k) - \xi_h ph_p$$

Finally, let v^n be the value function at the nth iteration. Then an upwind finite differ-

[82] $\dot{b}$ is a fraction of b; there is no need to solve it.

ence approximation is given by[83]

$$
\begin{aligned}
\frac{v_{ijkp}^{n+1}-v_{ijkp}^{n}}{\Delta} = u(c_{ijkp},h_p) &+ \frac{v_{i+1jkp}^{n+1}-v_{ijkp}^{n+1}}{\Delta a}(s^{c,F})^{+} + \frac{v_{ijkp}^{n+1}-v_{i-1jkp}^{n+1}}{\Delta a}(s^{c,B})^{-} \\
&+ \frac{v_{ij+1kp}^{n+1}-v_{ijkp}^{n+1}}{\Delta b}\theta(b_j,h_p)b_j^{+} + \frac{v_{ijkp}^{n+1}-v_{ij-1kp}^{n+1}}{\Delta b}\theta(b_j,h_p)b_j^{-} \\
&+ \sum_{\varepsilon_k^{'}} \lambda(\varepsilon_k,\varepsilon_{k'})(v_{ijk'p}^{n+1}-v_{ijkp}^{n+1}) - \rho v_{ijkp}^{n+1}
\end{aligned} \tag{20}
$$

This scheme satisfies the Barles-Souganidis monotonicity condition: Let S the right hand side of the equation (20). S is non-decreasing in v_{i+1jkp}, v_{i-1jkp}, v_{ij+1kp}, v_{ij-1kp}, v_{ijk+1p} and v_{ijk-1p}.

The upwind scheme can be applied to all of the points in the state space except for the points on the boundaries. Clearly, only one of the forward or backward derivatives can be computed on the boundaries. The way of handling the exogenous borrowing constraints in a one asset model is well explained in Achdou et al. (2017). In my model, there are three assets, and there is an exogenous borrowing limit for b but the limit can be endogenous with respect to a due to the option to default. I explain the details of handling boundaries in the rest of the section.

I refer to Bornstein (2018) to account for the endogenous borrowing limit with respect to a. For each (b_j,ε_k,h_p), $\forall\, j=\{1,..,n_b\}, k=\{1,..,n_\varepsilon\}, p=\{1,..,n_h\}$, if a household chooses to default on a below $\underline{a}(b_j,\varepsilon_k,h_p)$, a value is approximated by

$$
\begin{aligned}
\frac{v_{ijkp}^{n+1}-v_{ijkp}^{n}}{\Delta} + \rho v_{ijkp}^{n+1} =& u(c_{ijkp}^{n},h_p) + v_{ijkp}^{a,n+1}\dot{a}_{ijkp} + v_{ijkp}^{m,n+1}\dot{b}_{ijkp} \\
&+ \sum_{\varepsilon_k^{'}} \lambda(\varepsilon_k,\varepsilon_{k'})(v_{ijk'p}^{n+1}-v_{ijkp}^{n+1}) \quad \text{for } a_i > \underline{a}^n(b_j,\varepsilon_k,h_p)
\end{aligned}
$$

$$
v_{ijkp}^{n+1} = v_{\underline{a}^n jkp}^{n+1} \quad \text{for } a_i \le \underline{a}^n(b_j,\varepsilon_k,h_p)
$$

Therefore, the backward derivative does not exist below $\underline{a}(b_j,\varepsilon_k,h_p)$. I impose the endogenous borrowing constraint as $u^c(c_{\underline{a}^n jkp}) = v_{\underline{a}^n jkp}^{a,B}$ where $c_{\underline{a}^n jkp}$ is $w\varepsilon_k + r_{a,ijkp}\underline{a}(b_j,\varepsilon_k,h_p) - (r+\theta(b_j,h_p))b_j p h_p - T(\cdot) - \xi_h p h_p$ when it is binding.

At a_{n_a}, a forward derivative cannot be computed. However, if I set a high enough value for a_{n_a} saving at that point will be negative and the forward derivative will not be used.

[83] I use implicit method to update a value function. See Achdou et al. (2017) for details.

A drift of b is a fraction of b and is always positive. Its value is not computed using the derivative of the value function, and there is no need to determine the upwind scheme for this variable.

Equation (20) can be written as:

$$\begin{aligned}\frac{v^{n+1}_{ijkp}-v^{n}_{ijkp}}{\Delta}+\rho v^{n+1}_{ijkp} &= u(c_{ijkp},h_p)+x^a_{ijkp}v^{n+1}_{i-1jkp}+y_{ijkp}v^{n+1}_{ijkp}+z^a_{ijkp}v^{n+1}_{i+1jkp} \\ &+x^b_{ijkp}v^{n+1}_{ij-1kp}+z^b_{ijkp}v^{n+1}_{ij+1kp}+\sum_{k'}\lambda_{kk'}v^{n+1}_{ijk'p}\end{aligned} \tag{21}$$

$$x^a_{ijkp}=-\frac{(s^{c,B})^-}{\Delta a} \qquad x^b_{ijkp}=-\frac{(\theta(b,h)b)^-}{\Delta b}$$
$$y_{ijkp}=-\frac{(s^{c,F})^+}{\Delta a}+\frac{(s^{c,B})^-}{\Delta a}-\frac{(\theta(b,h)b)^+}{\Delta b}+\frac{(\theta(b,h)b)^-}{\Delta b}+\lambda_{kk}$$
$$z^a_{ijkp}=\frac{(s^{c,F})^+}{\Delta a} \qquad z^b_{ijkp}=\frac{(\theta(b,h)b)^+}{\Delta b}$$

We can write equation (21) for each grid point. The $n_a\times n_b\times n_\varepsilon\times n_h$ linear equations can be written in a matrix notation:

$$\frac{\mathbf{v}^{n+1}-\mathbf{v}^n}{\Delta}+\rho\mathbf{v}^{n+1}=\mathbf{u}^n+\mathbb{A}\mathbf{v}^{n+1} \tag{22}$$

Making the matrix $\mathbb{A}$ requires several steps. First, focus on a fraction for each level of h.

For each h, $\mathbb{A}_p, p\in\{1,2,...,n_h\}$ can be written $\mathbb{A}_p=\begin{bmatrix}A_{11|p} & A_{12|p} & .. & A_{1n_\varepsilon|p}\\ A_{21|p} & A_{22|p} & .. & A_{2n_\varepsilon|p}\\ \vdots & \ddots & & \vdots \\ A_{n_\varepsilon 1|p} & A_{n_\varepsilon 2|p} & .. & A_{n_\varepsilon n_\varepsilon|p}\end{bmatrix}$
where $A_{kk|p}$ is a matrix that is composed of x^a_{ijkp}, x^b_{ijkp},y_{ijkp}, z^a_{ijkp} and z^b_{ijkp}, $k\in\{1,2,...,n_\varepsilon\}$. For example,

$$A_{11|2} = \begin{bmatrix} y_{1112} & z^a_{1112} & 0 & .. & 0 & z^b_{1112} & 0 & .. & 0 & 0 & 0 & .. & & 0 \\ x^a_{2112} & y_{2112} & z^a_{2112} & 0 & .. & 0 & z^b_{2112} & 0 & ... & 0 & 0 & 0 & .. & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & .. & 0 & x^a_{n_a112} & y^a_{n_a112} & 0 & .. & 0 & z^b_{n_a112} & & 0 & ... & 0 & 0 \\ x^b_{1212} & .. & & & 0 & y_{1212} & z^a_{1212} & 0 & .. & 0 & z^b_{1212} & 0 & & 0 \\ \vdots & x^b_{2212} & \ddots & \ddots & \ddots & x^a_{2212} & y_{2212} & z^a_{2212} & \ddots & \ddots & \ddots & z^b_{2212} & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & .. & & & & & & x^a_{n_an_b12} & y^a_{n_an_b12} & 0 & .. & & 0 & z^b_{n_an_b12} \end{bmatrix}$$

and when $k \neq l$, $A_{kl|p}$ is a diagonal matrix where diagonal terms are λ_{kl}. Then $\mathbb{A}$ is a block diagonal matrix which is composed of $\mathbb{A}_1$,..,$\mathbb{A}_{n_h}$.

$$\mathbb{A} = \begin{bmatrix} \mathbb{A}_1 & 0 & .. & & 0 \\ 0 & \mathbb{A}_2 & 0 & .. & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & .. & & 0 & \mathbb{A}_{n_h} \end{bmatrix}$$

In the same fashion, linear equations for the value with the bankruptcy flag or the foreclosure flag can be written as

$$\frac{\mathbf{v}^{d,n+1} - \mathbf{v}^{d,n}}{\Delta} + \rho \mathbf{v}^{d,n+1} = \mathbf{u}^{d,n} + \mathbb{A}^d \mathbf{v}^{d,n+1} \tag{23}$$

$$\frac{\mathbf{v}^{f,n+1} - \mathbf{v}^{f,n}}{\Delta} + \rho \mathbf{v}^{f,n+1} = \mathbf{u}^{f,n} + \mathbb{A}^f \mathbf{v}^{f,n+1} \tag{24}$$

Because $a \geq 0$ with the bankruptcy flag, values are not defined below $a = 0$. Therefore the number of equations is $n_{a+} \times n_b \times n_\varepsilon \times n_h$ where n_{a+} is the number of a grid where $a \geq 0$. In the state with the foreclosure flag, values are defined where $h = 0$ and $b = 0$ so the number of equations is $n_a \times n_\varepsilon$.

Let $z = \mathbf{v}^{n+1} - \mathbf{v}^{*,n}$, $\mathbf{B} = \frac{1}{\Delta} - \rho - \mathbb{A}$ and $q = \mathbf{B}\mathbf{v}^{*,n} - \frac{\mathbf{v}^n}{\Delta} - u^n$. Then

$$(\mathbf{v}^{n+1} - \mathbf{v}^{*,n})(\frac{\mathbf{v}^{n+1} - \mathbf{v}^n}{\Delta} - \rho \mathbf{v}^{n+1} - \mathbf{u}^n - \mathbb{A}\mathbf{v}^{n+1}) = 0$$

$$\frac{\mathbf{v}^{n+1} - \mathbf{v}^n}{\Delta} - \rho \mathbf{v}^{n+1} - \mathbf{u}^n - \mathbb{A}\mathbf{v}^{n+1} \geq 0$$

$$\mathbf{v}^{n+1} - \mathbf{v}^{*,n} \geq 0$$

and

$$z'(\mathbf{B}z + q) = 0$$

$$\mathbf{B}z + q \geq 0$$

$$z \geq 0$$

are equivalent.

A.2.3 Kolmogorov Forward equation

Without stopping decisions, the Kolmogorov Forward equation is

$$\partial_t g_{ijkp,t} = -\partial_a s^a_{ijkp} g_{ijkp,t} - \partial_b s^b_{ijkp} g_{ijkp,t} + \sum_{k'} \lambda_{k'k} g_{ijk'p,t}$$

where s^x_{ijkp} is shorthand notation for x decision rule at $(a_i, b_j, \varepsilon_k, h_p)$, $x \in \{a, b\}$ and $g_{ijkp,t}$ is a density function at time t. In my model i) shifts due to a h transaction, refinancing, bankruptcy and foreclosure ii) flows between a state without the flags and a state with the bankruptcy flag and iii) flows between a state without the flags and a state with the foreclosure flag need to be accounted for.

The mathematical formulation of Kolmogorov Forward equations due to h transaction, refinancing, bankruptcy and foreclosure is not straightforward.[84] However, there is a way of handling stopping decisions in computation. Flows due to stopping decisions can be treated with the 'intervention matrix', M.[85]

First, let g_i be the ith element of the density function where $i \in \{1, ...n\}$ and n is the total number of grid points.[86]

$$M_{i,j} = \begin{cases} 1 & \text{if } i \in I \text{ and } i = j \\ 1 & \text{if } i \notin I \text{ and } j^*(i) = j \\ 0 & \text{otherwise} \end{cases}$$

where I is the non-stopping region and $j^*(i)$ is the target point of point i.

[84] 'Liquid and Illiquid Assets with Fixed Adjustment Costs' by Greg Kaplan, Peter Maxted and Benjamin Moll
http://www.princeton.edu/ moll/HACTproject/liquid-illiquid-numerical.pdf

[85] I followed the approach in http://www.princeton.edu/ moll/HACTproject/liquid-illiquid-numerical.pdf.

[86] $n = n_a \times n_b \times n_\varepsilon \times n_h \times 2 + n_a \times n_\varepsilon$. $n_a \times n_b \times n_\varepsilon \times n_h$ is multiplied by 2 because there are points with and without bankruptcy flag. $n_a \times n_\varepsilon$ is the number of points in the state with the foreclosure flag.

The flow from a state with the bankruptcy and foreclosure flag to a state without the bankruptcy flag is a shock and can be expressed as below. Let nd represent 'non-default', and d represent 'default'.

$$\partial_t g^{nd}_{ijkp,t} = -\partial_a s^{a,nd}_{ijkp} g^{nd}_{ijkp,t} - \partial_b s^{b,nd}_{ijkp} g^{nd}_{ijkp,t} + \sum_{k'} \lambda_{k'k} g^{nd}_{ijk'p,t} + \lambda_l g^{d}_{ijkp,t}$$

$$\partial_t g^{d}_{ijkp,t} = -\partial_a s^{a,d}_{ijkp} g^{d}_{ijkp,t} - \partial_b s^{b,d}_{ijkp} g^{d}_{ijkp,t} + \sum_{k'} \lambda_{k'k} g^{d}_{ijk'p,t} - \lambda_l g^{d}_{ijkp,t}$$

where $\lambda_l = \lambda_d$ for the bankruptcy flag state and $\lambda_l = \lambda_f$ for the foreclosure flag state. These flows can be treated with the matrix A^d and A^f.

$$A^d_{i,j} = \begin{cases} \lambda_d & \text{if } 1 \le j \le n_1 \text{ and } i = j + n_1 \\ -\lambda_d & \text{if } n_1 + 1 \le j \le n_1 \times 2 \text{ and } i = j \\ 0 & \text{otherwise} \end{cases}$$

$$A^f_{i,j} = \begin{cases} \lambda_f & \text{if } 1 \le j \le n_a \times n_\varepsilon \text{ and } i = j + n_1 \times 2 \\ -\lambda_f & \text{if } n_1 \times 2 + 1 \le j \le n \text{ and } i = j \\ 0 & \text{otherwise} \end{cases}$$

where $n_1 = n_a \times n_b \times n_\varepsilon \times n_h$. The sizes of A^d and A^f are $n \times n$ and I stack points from the state without the flags, points from the state with the bankruptcy flag state and points from the state with the foreclosure flag.[87] Finally, define B as below:

$$B = \mathcal{A} + A^d + A^f$$

where $\mathcal{A}$ be a block diagonal matrix which is composed of $\mathbb{A}$, $\mathbb{A}^d$ and $\mathbb{A}^f$.

$$\mathcal{A} = \begin{bmatrix} \mathbb{A} & 0 & 0 \\ 0 & \mathbb{A}^d & 0 \\ 0 & 0 & \mathbb{A}^f \end{bmatrix}$$

Given M and B, the density function can be solved by iterating the following two steps until g converges.

$$g^{n+\frac{1}{2}} = M^T g^n$$

[87] n_1 is the number of points in the state without the flags and $n_1 \times 2$ is sum of the number of points of the no flag state and the bankruptcy flag state.

$$\frac{g^{n+1} - g^{n+\frac{1}{2}}}{\Delta t} = (BM)^T g^{n+1}$$

A.2.4 Stochastic model

The household problem contains an argument that has an infinite dimension, g, the distribution of over (a, b, ε, h). To make the computation feasible, a distribution needs to be approximated. I assume the households only track a finite set of moments of g_t to form their expectations as in Krusell and Smith (1998).[88] Specifically, I assume that the households keep track of the aggregate capital, k and house prices.

I redefine the problem on the approximated aggregate states, (k, p_{-t}, z) where $p_{-t} = pt - dp_{-t}dt$. For example, equation (5) can be written as

$$\begin{aligned}
\rho v(a,b,\varepsilon,h,k,p_{-t},z) = & \max_c u(c,h) + \partial_a v(a,b,\varepsilon,h,k,p_{-t},z)\dot{a} + \partial_b v(a,b,\varepsilon,h,k,p_{-t},z)\dot{b} \\
& + \sum_{j=1}^{n_\varepsilon} \lambda_{\varepsilon\varepsilon_j} v(a,b,\varepsilon_j,h,k,p_{-t},z) + \sum_{k=1}^{n_z} \lambda_{zz_k} v(a,b,\varepsilon,h,k,p_{-t},z_k) \\
& + \partial_k v(a,b,\varepsilon,h,k,p_{-t},z)\dot{k}.
\end{aligned}$$

$$\dot{k}_t = \frac{\mathbb{E}[dk_t; k_t, z_t]}{dt} = f^k(k_t, p_{-t}, z_t) \qquad p_t = f^p(k_t, p_{-t}, z_t)$$

The last term in the equation (5) which captures the evolution of the distribution is replaced with $\partial_k v(a,b,\varepsilon,h,k,z)\dot{k}$. I assume f as a log linear form.

$$dln(k_t)dt = \beta_z^0 + \beta_z^1 ln(p_t) + (\beta_z^2 - 1)ln(k_t)$$

On a approximated aggregated state, $p(k, p_{-t}, z)$ is necessary to solve the model. I also assume a log linear form to estimate house prices.

$$dln(p_t)dt = \phi_z^0 + (\phi_z^1 - 1)ln(p_t) + \phi_z^2 ln(k_t + dln(k_t)dt)$$

[88] Ahn et al. (2018) develop a method of solving continuous time heterogeneous agent models with aggregate uncertainty based on linearization and dimension reduction. Fernández-Villaverde et al. (2019) also presents a method to solve such models. They assume the households only track a finite set of moments of the distribution to form their expectations as well, but use tools from machine learning to estimate the perceived law of motion of the households.

Solution algorithm

1. Guess parameters of the forecasting functions. With the forecasting functions, $\dot{k}$ and h price over (k, p_{-t}, z) can be forecasted. Also, interest rate and wage over (k, p_{-t}, z) can be computed using the firm's marginal conditions.
2. Solve the value function.
   - Guess the loan price functions, $r_a(a, b, \varepsilon, h, k, p_{-t}, z)$ and $q(a, b, \varepsilon, h, k, p_{-t}, z)$.
   - Guess the value function, $v^0(a, b, \varepsilon, h, k, p_{-t}, z)$, $v^{0,d}(a, b, \varepsilon, h, k, p_{-t}, z)$ and $v^{0,f}(a, \varepsilon, k, p_{-t}, z)$.
   - Update the value functions and the loan price schedules until they converge.
   - Save decision rules.
3. Simulate the model for n periods. Simulation gives the sequence of aggregate variables $\{z_t, k_t, p_t\}_{t=1}^n$.
   - Guess the initial distribution. The distribution in the steady state can be a good initial distribution.
   - At the beginning of each period, (k, z) are known. The risk free rate and wage can be computed.
   - Compute the loan price functions. To compute $r_a(a_t, b_t, \varepsilon_t, h_t, k_t, z_t)$, interpolate the default decisions that are obtained from the step 2. Using the default decisions and the risk free rate, $r_a(a_t, b_t, \varepsilon_t, h_t, k_t, z_t)$ can be computed using the equation (15). To compute $q(a_t, b_t, \varepsilon_t, h_t, k_t, z_t)$, interpolate the $q(a, b, \varepsilon, h, k, z)$ that are obtained from step 2 over k.
   - Guess the h price.
   - With the wage, the loan price schedules and the h price, solve the household problem.
   - Compute the h demand. If the aggregate demand is not close enough to the supply, adjust h price to clear h market.
   - Once the h market is cleared, move to the next period
4. Using the sequence of aggregate variables $\{z_t, k_t, p_t\}_{t=1}^n$, update the forecasting function.
5. Check convergence of the simulated aggregate variables. If the distance between the $\{k_t\}_{t=1}^n$ from the current iteraion and the previous iteration is less than the tolerance level, an approximate recursive equilibrium is obtained. Otherwise, go back to step 2 with the updated forecasting functions.

B Consumption equivalent gain of default options

In this Appendix, I derive the consumption equivalent gain described in section 4.2. The consumption equivalent gain is used to assess the value to households of being able to default. Let v_i is the value function of an alternative economy and let x be the amount that makes the value of benchmark economy indifferent to the value of the economy with bankruptcy only or foreclosure only or without any default.

$$\begin{aligned}
\rho v_{benchmark} &= \rho v_i + x \\
&= u(c_i, h) + \mathbb{A}v_i + x \\
&= u(c_i + c_i^*, h) + \mathbb{A}v \\
&= u(c_i + c_i^*) + u(h) + \mathbb{A}v + u(c_{i)} - u(c_i) \\
&= u(c_i + c_i^*) + \rho v_i - u(c_i)
\end{aligned}$$

Since the utility function is separable in c and h, $u(c_i + c_i^*, h) = u(c_i + c_i^*) + u(h)$.

$$\rho v_{benchmark} - \rho v_i + u(c_i) = u(c_i + c_i^*)$$

$$((1-s)(\rho v_{benchmark} - \rho v_i + u(c_i)))^{\frac{1}{1-s}} = c_i + c_i^*$$

$$\frac{((1-s)(\rho v_{benchmark} - \rho v_i + u(c_i)))^{\frac{1}{1-s}}}{c_i} - 1 = \frac{c_i^*}{c_i}$$

C Additional tables and figures

Table 9: Share of assets and debt

	Asset		Debt	
	Non-financial	Financial	Secured	Unsecured
Data				
Q1	2.88	0.32	10.47	18.77
Q2	4.27	0.82	9.77	14.62
Q3	10.62	2.52	19.27	19.17
Q4	17.36	7.99	19.38	19.50
Q5	64.88	88.34	41.13	27.96
Model				
Q1	1.77	-0.72	3.36	96.81
Q2	4.91	1.85	9.31	0.23
Q3	10.87	7.34	20.14	0.10
Q4	23.21	17.67	30.07	1.78
Q5	59.24	73.85	37.12	1.09

Note: Share of assets and debt by net worth quintiles. Data: SCF(2010)

Table 10: Portfolio composition

	Asset		Debt	
	Non-financial	Financial	Secured	Unsecured
Data				
Q1	-458.48	-43.45	559.34	42.59
Q2	288.18	46.91	-221.05	-14.04
Q3	176.53	35.47	-107.46	-4.54
Q4	100.13	38.96	-37.49	-1.60
Q5	51.76	59.57	-11.01	-0.32
Model				
Q1	-81.87	44.60	90.06	47.21
Q2	245.44	123.93	-269.25	-0.12
Q3	120.18	108.68	-128.85	-0.01
Q4	78.81	80.32	-59.06	-0.06
Q5	43.36	72.37	-15.72	-0.01

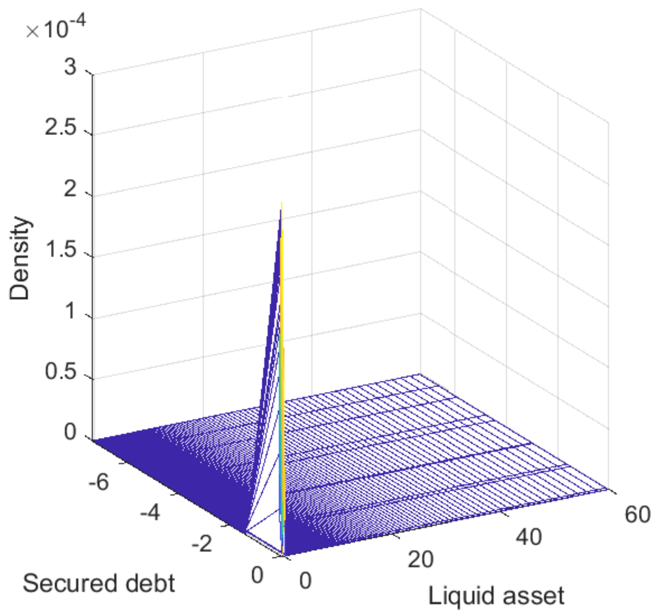
Note: Average portfolio composition by net worth quintiles. Non-financial asset includes 'Housing and cars', 'Business and non-financial assets' in the SCF. Business assets are excluded from non-financial assets. Student loans are excluded. After excluding student loans, credit card loans are considered as unsecured debt and the remaining compositions of debt are assigned to secured debt. Data: SCF(2010)

Table 11: Portfolio composition

	Asset		Debt	
	Non-financial	Financial	Secured	Unsecured
Q1	-258.70	-28.70	364.90	22.50
Q2	298.30	52.10	-237.40	-13.00
Q3	177.20	35.40	-107.60	-5.00
Q4	98.30	37.40	-34.00	-1.70
Q5	63.50	45.90	-8.90	-0.50

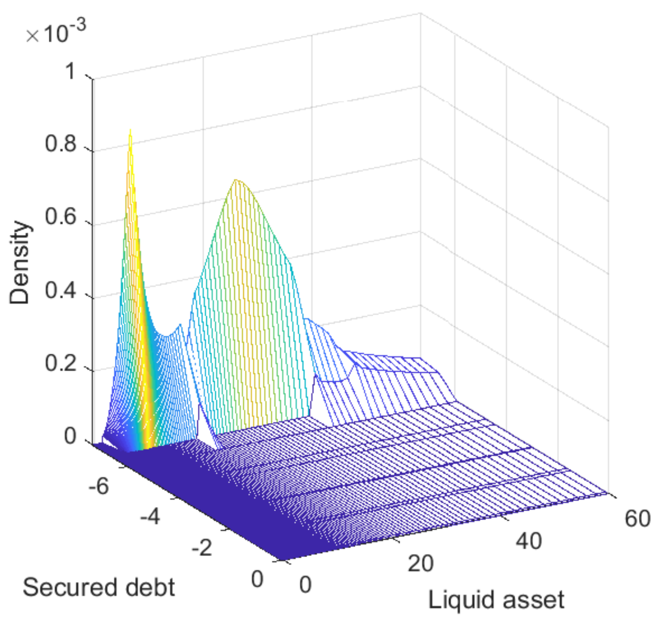
Note: Average portfolio composition by net worth quintiles. Non-financial asset includes 'Housing and cars', 'Business and non-financial assets' in SCF. Data: SCF(2010)

Figure 22: Household distribution



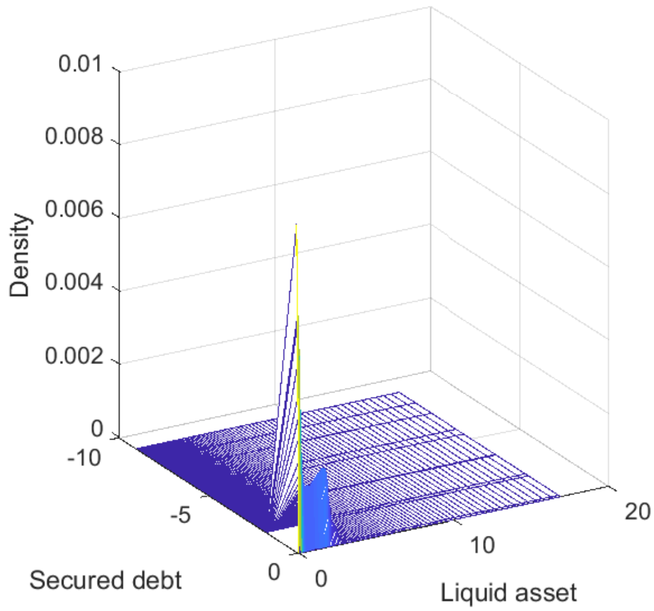
Note: A stationary distribution of households with specific labor productivity and house (ε_3, h_2) from the model with only bankruptcy.

Figure 23: Household distribution



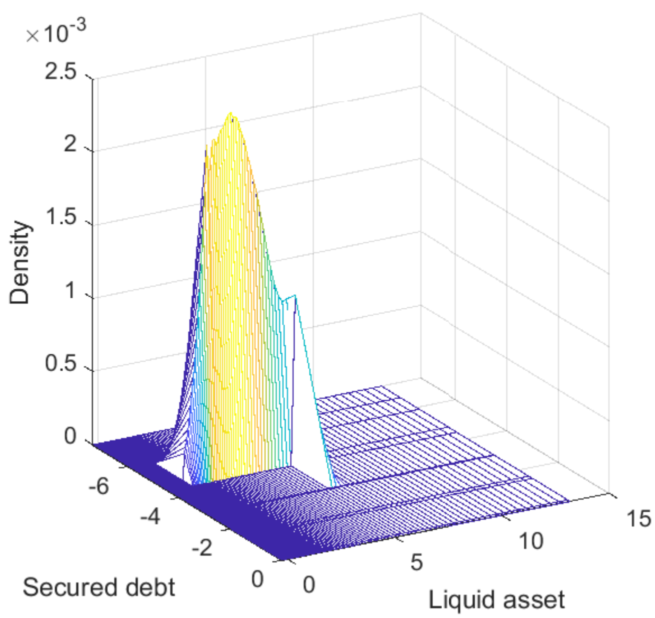
Note: A stationary distribution of households with specific labor productivity and house (ε_3, h_6) from the model with only bankruptcy.

Figure 24: Household distribution



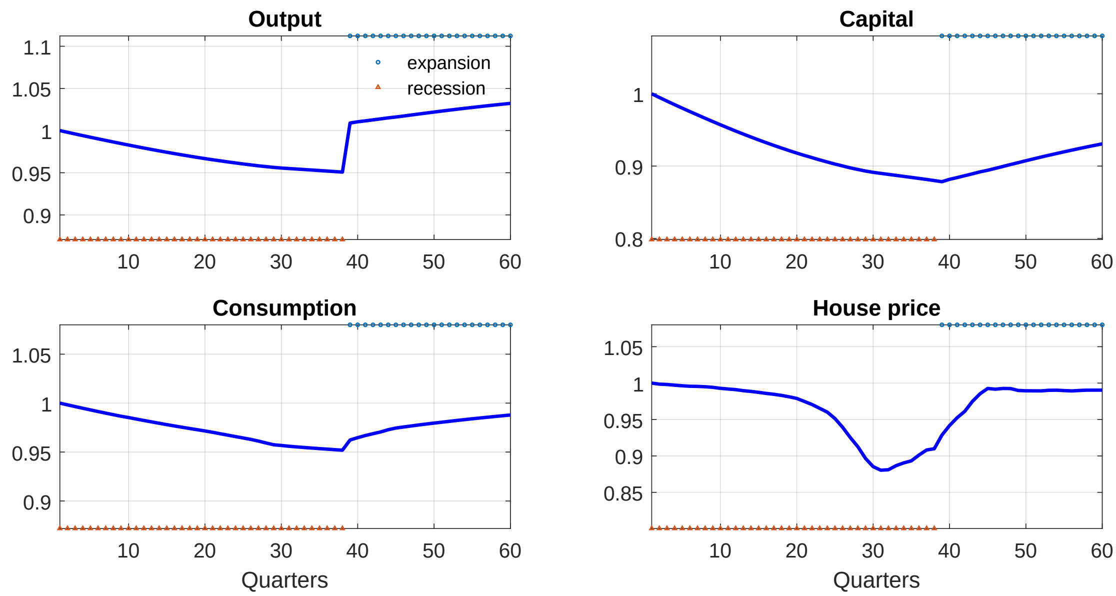
Note: A stationary distribution of households with specific labor productivity and house (ε_2, h_2) from the model with only foreclosure.

Figure 25: Household distribution



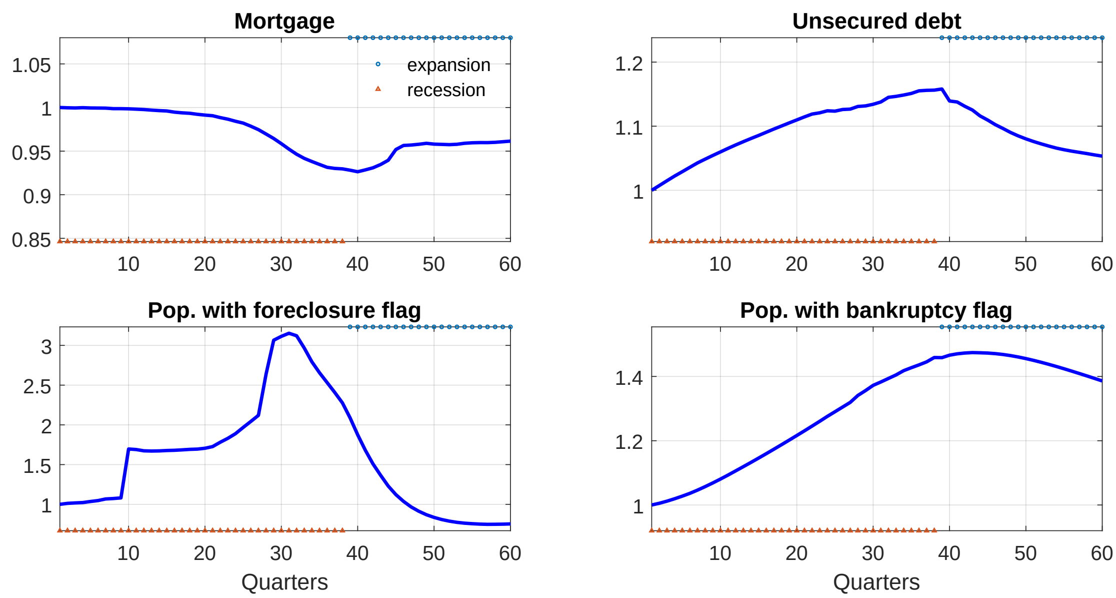
Note: A stationary distribution of households with specific labor productivity and house (ε_3, h_5) from the model with only foreclosure.

Figure 26: Aggregate variables



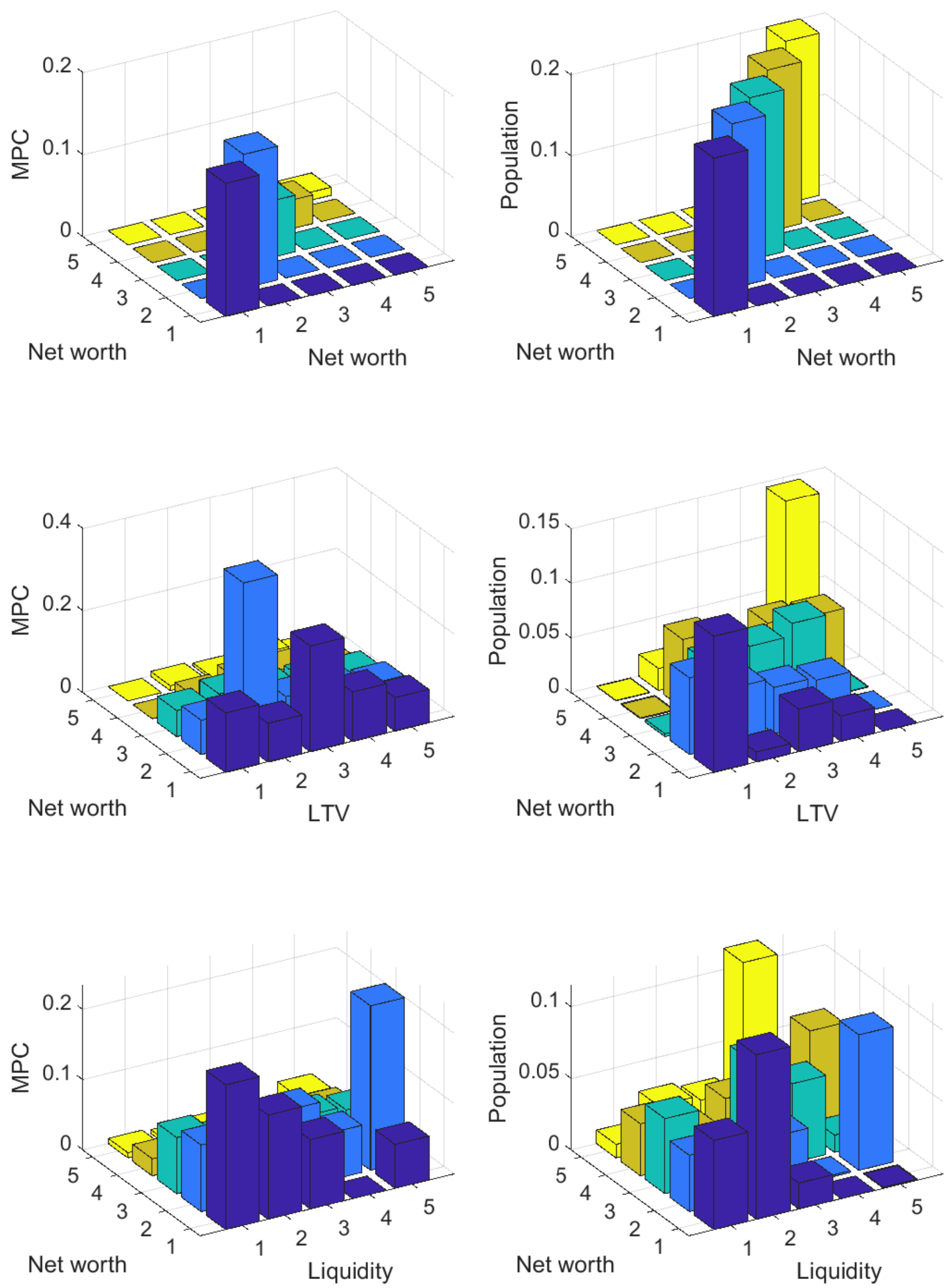
Note: Aggregate variable movements in the baseline economy without policy intervention. All series are normalized to 1 in the first period.

Figure 27: Aggregate variables



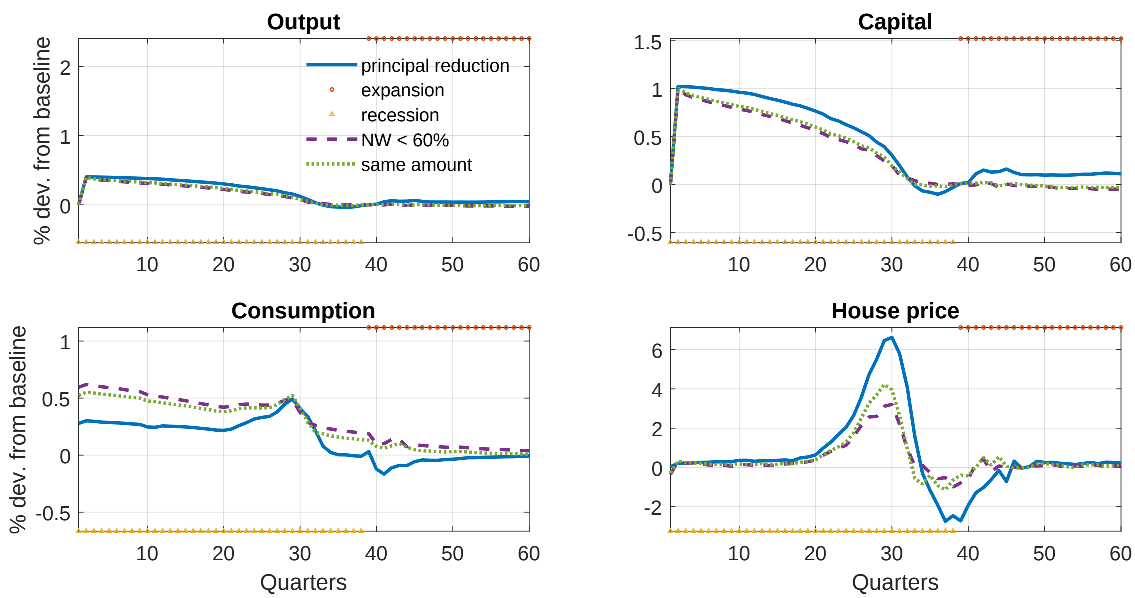
Note: Aggregate variable movements in the baseline economy without policy intervention. All series are normalized to 1 in the first period.

Figure 28: MPC and population (Tax rebate)



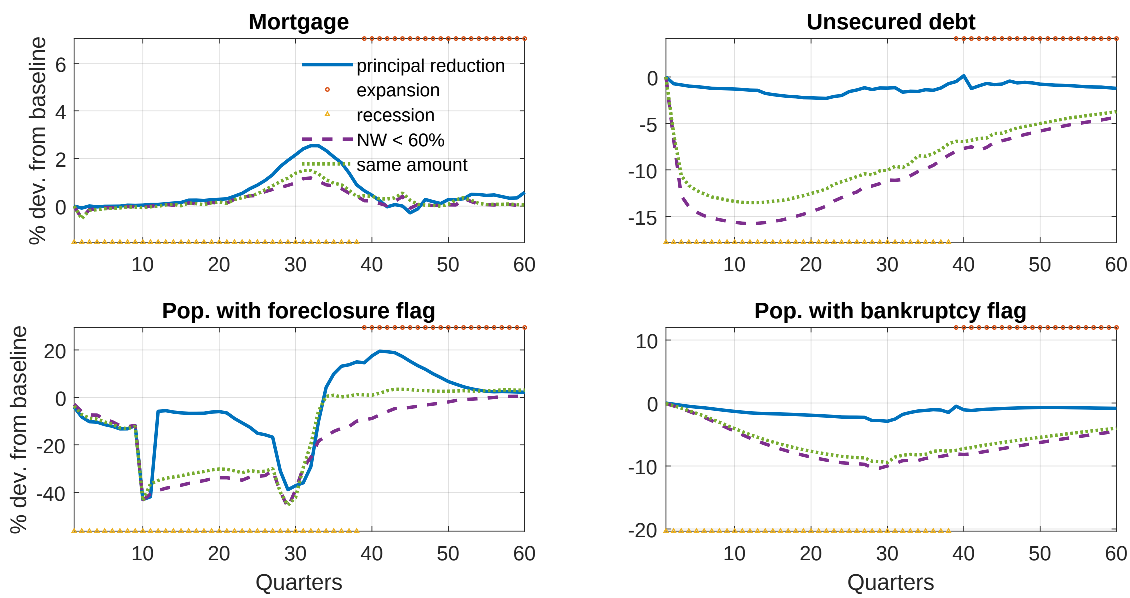
Note: These bars plot MPC and population over household net worth, LTV ratio and liquidity quintiles. MPC is $\frac{\text{c(policy) - c(no policy)}}{\text{principal reduction size}}$ and liquidity is a ratio of liquid saving over total assets (liquid saving plus house value). For the MPC, each bar is an average MPC of households that are in corresponding quintiles.

Figure 29: Response of the aggregate variables



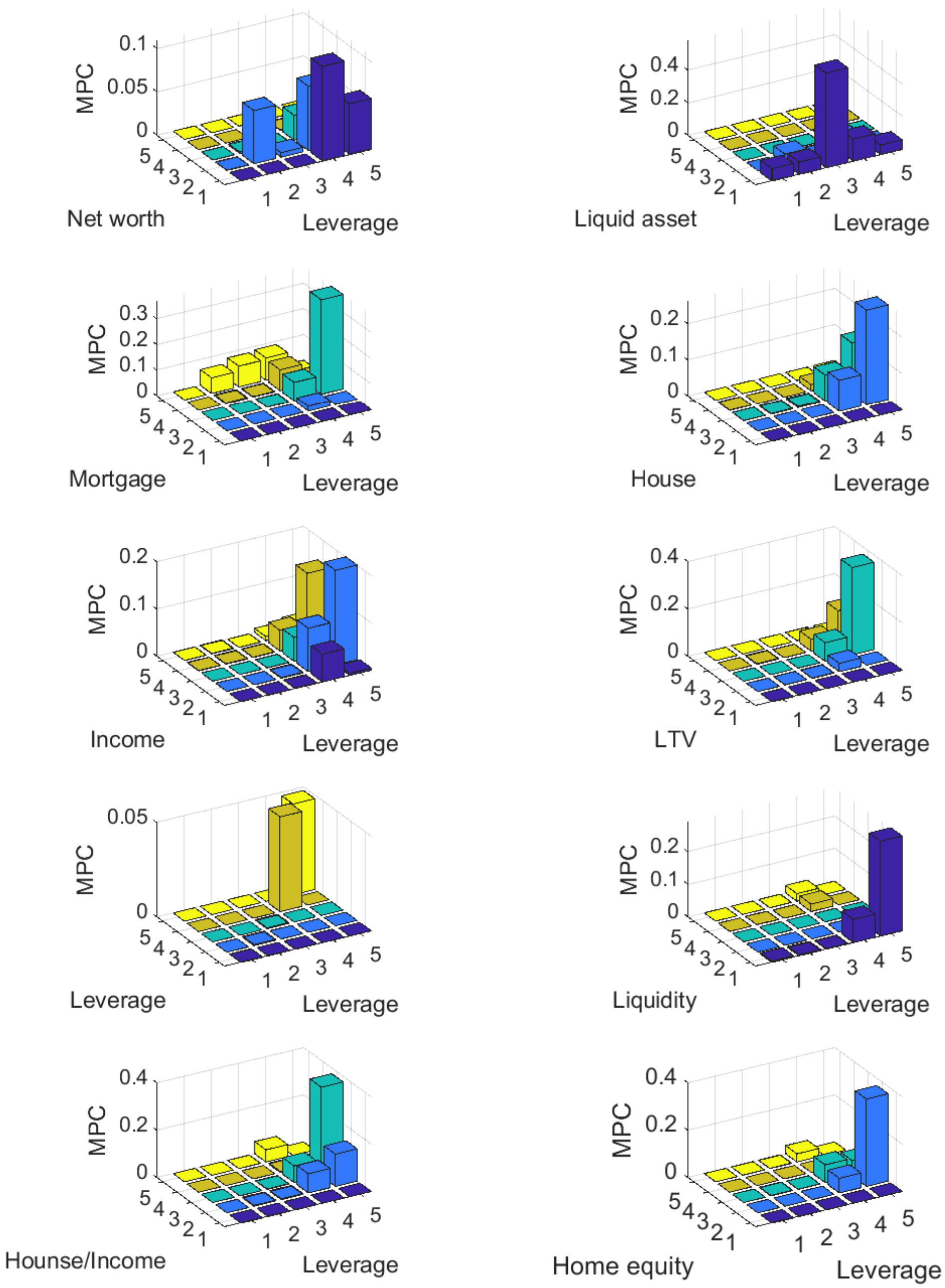
Note: Response of the aggregate variables to the principal reduction. principal reduction: Households with LTV ratio higher than 95% are eligible. NW<60%: Households with LTV ratio higher 95% and net worth less than 60th percentile are eligible. same amount: Households with LTV ratio higher than 95% are eligible. Instead of forgiving a fraction of mortgages to LTV ratio fall to 95%, all eligible households receive the same amount of reduction.

Figure 30: Response of credits and defaults



Note: Response of the aggregate variables to the principal reduction. principal reduction: Households with LTV ratio higher than 95% are eligible. NW<60%: Households with LTV ratio higher 95% and net worth less than 60th percentile are eligible. same amount: Households with LTV ratio higher than 95% are eligible. Instead of forgiving a fraction of mortgages to LTV ratio fall to 95%, all eligible households receive the same amount of reduction.

Figure 31: Consumption responsiveness



Note: These bars plot MPC over various households' states quintiles. Each bar is an average MPC of households that are in corresponding quintiles. Only hand-to-mouth households with houses are counted. MPC is $\frac{\text{c(policy) - c(no policy)}}{\text{tax rebate size}}$. Liquidity is a ratio of liquid saving over total assets (liquid saving plus house value). Leverage is total debt(unsecured borrowing plus mortgage) to total assets.