Firm Heterogeneity, Leverage and the Aftermath of the Pandemic

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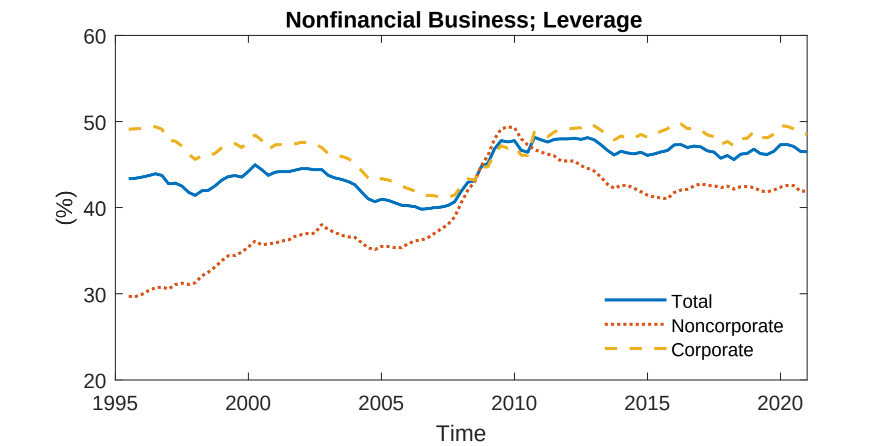
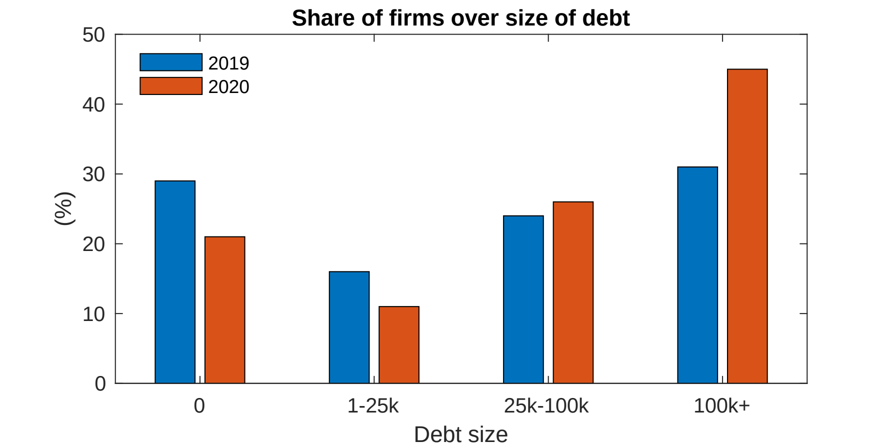
Covid-19 recession started when firm leverage was high



Data: Flow of funds

Concerns over firms' worsening balance sheets and a surge in default



Data: Small Business Credit Survey

Firm leverage and recovery

How does high leverage at the onset of a large recession affect the recovery?

Impact of business credit programs?

- reduce bankruptcies and support investment (short-run) vs debt overhang (long-run)

Overview

An incomplete markets model with entrepreneurship, entry and exit

- productivity, financial assets (short-term) and debt (long-term, illiquid)
- borrowers may default on debt

Replicate firm size, balance sheet distributions and default behavior

- also consistent with firm's growth dynamics

Quantitatively assess the effects of high leverage on recovery

- compare two economies: ordinary and high leverage
- consider policies that subsidize costs and defer debt obligations

Main findings

High leverage slows down a recovery

- when aggregate business leverage is 11 pp above average, a large negative TFP shock leads to a sharp increase in default
- the half life of GDP doubles
- a large increase in exits drives the slower recovery

Policy intervention speeds the recovery

- Subsidies reduce exits and thus distortions in production
- Debt payment deferrals raise leverage, dampen policy's effectiveness

Literature

Heterogeneous Firms and Financial Frictions

- Credit Shocks: Buera et al. (2015), Khan and Thomas (2013), Jo (2021)
- Default: Khan et al. (2019), Ottonello and Winberry (2020), Bustamante (2020)

Entrepreneurship and Financial Frictions

- Wealth Inequality: Cagetti and De Nardi (2006)
- Development and TFP: Buera and Shin (2011), Buera et al. (2013)
- Risk and Productivity: Robinson (2021)

Pandemic

- Policy Alternatives: Eichenbaum et al. (2020), Glover et al. (2020)
- Firm Exit: Guerrieri et al. (2020)

Model

Model overview

- Time is continuous
- Agents: workers, entrepreneurs, banks and government
- Workers face income shocks (ε) and have a single asset (a)
- Entrepreneurs face productivity shocks (z), hold assets (a) and long-term debt (b)
    - DRS technology
    - may enter and exit
    - can default on debt
    - earn labor income when not operating a business

Worker

$$\rho v^w(a,\varepsilon) = \max_c u(c) + v_a^w(a,\varepsilon)\dot{a} + \sum \pi_{\varepsilon\varepsilon'} v^w(a,\varepsilon')$$
$$\dot{a} = \varepsilon w + ra - c - T(\cdot), \quad a \geq \underline{a}$$

- $T(\cdot)$ is tax or subsidy
- All value functions are functions of t

Entrepreneur

v^0 : inactive, can enter

v^2 : business owner with exiting and defaulting option.
exogenous exit shock and shock that gives them the borrowing option

v^1 : business owner with borrowing, exiting, and defaulting option.
exogenous exit shock

v^d : inactive, defaulted, cannot start a business or borrow.
exogenous shock that removes the default flag

Entrepreneur 1: business owner

A stopping time problem. Stopping options: adjusting b, exiting or defaulting

$$
\begin{aligned}
v^1(a_t, b_t, z_t) &= \max_{\{c_t, k_t, l_t\}_{t \geq 0}, \tau} \{E_0 \int_0^\tau u(c_t) + e^{-\rho\tau} v^{1*}(a_\tau, b_\tau, z_\tau)\} \\
\dot{a}_t &= z_t k_t^\alpha l_t^\nu - w_t l_t - (r_t + \delta) k_t + r_t a_t - c_t - (r_t + \theta) b_t - \xi(\cdot) - T(\cdot) \\
k_t &\leq \gamma a_t \\
\dot{b}_t &= -\theta b_t \\
(a_0, b_0, z_0) &= (a, b, z)
\end{aligned}
$$

- τ is a stopping time, z follows a stochastic process
- collateral constraint on capital rental
- $\xi(\cdot)$: cost of production, $T(\cdot)$: tax

Entrepreneur 1: business owner, not stopping

$$
\begin{aligned}
\rho v^1(a,b,z) &= \max_{c,k,l} u(c) + v_a^1(a,b,z)\dot{a} + v_b^1(a,b,z)\dot{b} \\
&\quad + \sum \pi_{zz'} v^1(a,b,z') - \lambda_{10}(v^1(a,b,z) - v^0(a-b,z)) \\
\dot{a} &= z k^{\alpha} l^{\nu} - wl - (r+\delta)k + ra - c - (r+\theta)b - \xi(\cdot) - T(\cdot) \\
k &\leq \gamma a \\
\dot{b} &= -\theta b \\
v^1(a,b,z) &\geq v^{1*}(a,b,z)
\end{aligned}
$$

- $\pi_{zz'}$ describes the productivity process
- $\lambda_{10}(v^1(a,b,z) - v^0(a-b,z))$ reflects the possibility of exiting where λ_{10} is the frequency of the exiting shock; need to pay off the debt when exiting

Entrepreneur 1: Stopping value: $\max[v^r, v^x, v^b]$

Adjusting debt

$$v^r(a_\tau, b_\tau, z) = \max_{a', b'} v^1(a', b', z)$$

$$a' = a_\tau - b_\tau + q(a', b', z)b' - \xi_b(b' - b), \quad b \leq b' \leq \underline{b}$$

Exiting

$$v^x(a_\tau, b_\tau, z) = v^0(a_\tau - b_\tau, z)$$

Defaulting

$$v^b(a_\tau, b_\tau, z) = v^d(\chi a_\tau, z) - \xi_d$$

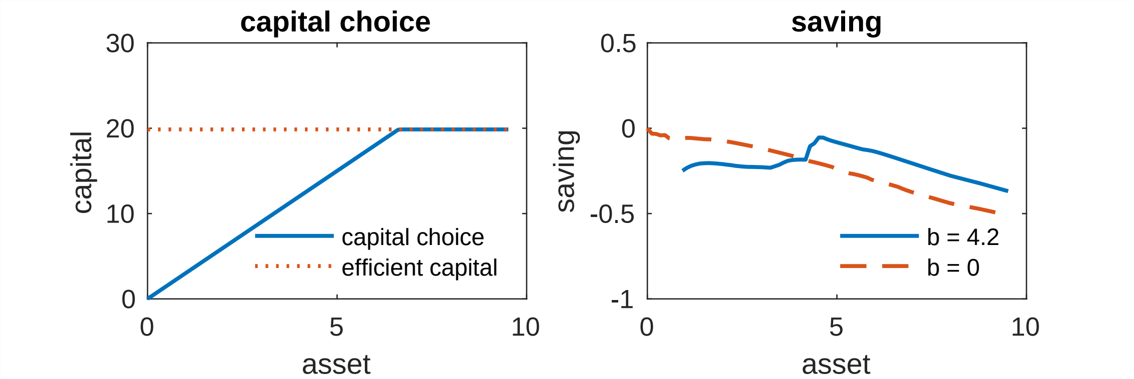
- ξ_b: refinancing cost. ξ_d: utility cost of defaulting cost

business owner w/o borrowing option | inactive | defaulted

Effects of aggregate leverage: Role of the distribution

- Entrepreneurs' capital are functions of their productivity and assets
    - collateral constrains renting efficient capital when assets are low
- Producers with more debt dissave faster (save slower) when assets are low
    - moving further below efficient level of capital
- but they dissave slower (save faster) when assets are high

Effects of aggregate leverage: Role of the distribution



Effects of aggregate leverage: Role of the distribution

- Given agg. leverage, greater dispersion increases inefficiency in capital allocation
- Greater dispersion implies
  - some producers have more assets than required
  - others have too little and produce below their efficient scale

  $\rightarrow$ *misallocation rises, aggregate TFP falls*

- **Effect of high leverage on recovery determined by the distribution of leverage**

Parameterization

Strategy

Focus on reproducing cross-sectional and dynamic features of US firms

- distribution of assets and debt across firms
- size distribution
- firm growth dynamics

Calibration

- values for a subset of parameters are assigned before solving the model
- 16 parameters are jointly calibrated in the steady state

Model fit

Moments	Data	Model	Source
Labor share (wn/y)	0.6	0.66	BLS
Exit rate	8.5%	10.1%	BDS
Bankruptcy rate	3.0%	3.4%	Dun & Brad Street
Share of small* firms without debt	30%	15.8%	SBCS
Share of small* firms with 100k+ debt	32%	73.8%	SBCS
Share of earnings in top 20%	63.5%	60.9%	SCF (2007)
Share of earnings in bottom 60%	15.8%	22.7%	SCF (2007)

Note: * the number employee is 500 or less. BDS data moments are means over 1984 - 2015 and SBCS moments are means over 2016-2019.

Model fit

Moments	Data	Model	Source
Top 10% employment share	69%	75.5%	Buera and Shin (2013)
Share of firms that hire 1-4	88.3%	87.3%	BDS
Share of firms that hire 5-19	9.8%	9.9%	BDS
Share of firms that hire 500+	0.4%	0.7%	BDS
Employment share of firms that hire 1-4	20.9%	21.8%	BDS
Relative size of age 6-10 firms	54.5%	97.6%	BDS
Relative size of age 0-5 firms	37.3%	40.8%	BDS
Exit rate of age 0-5 firms	54.8%	43.2%	BDS
Mean leverage	59.0%	57.0%	ORBIS
Aggregate leverage	36.6%	38.3%	Flow of Funds

Note: * the number employee is 500 or less. BDS data moments are means over 1984 - 2015. Flow of funds is 1954Q1 - 2021Q1.

Results

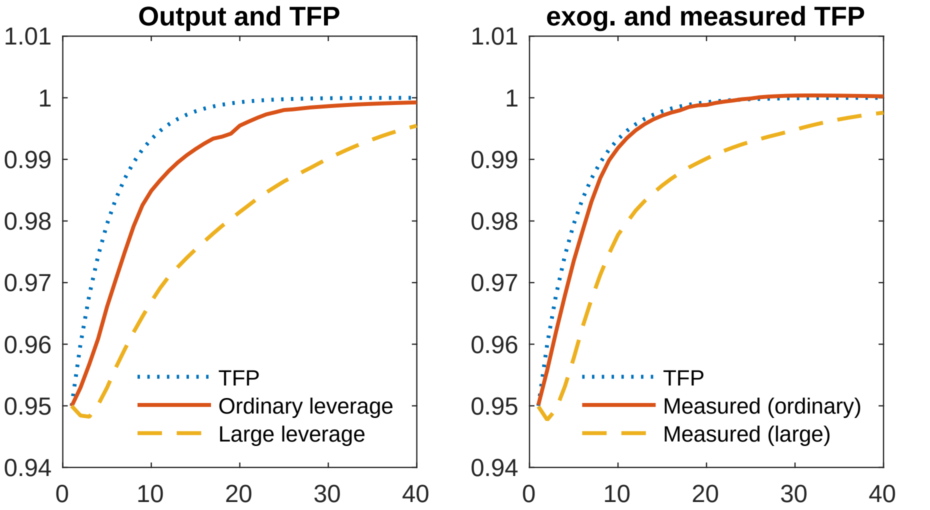
Role of high leverage on recovery

Compute impulse responses to TFP shock

	Model	Data	
Ordinary leverage	38%	37%	(avg. over 54Q1-21Q1)
High leverage	49%	47%	(as of 19Q4)

- Compare **ordinary and high leverage cases**
  - increased workers' savings offset rise in entrepreneurs' debt
  - no change in net assets

Higher leverage slows the recovery

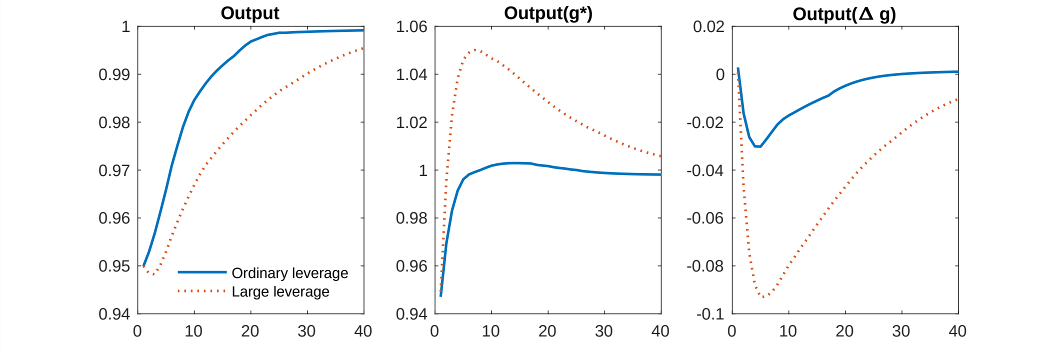


Higher leverage slows the recovery

Beyond aggregate capital, changes in its distribution shape the recovery

- half life of TFP is 4 periods
- half life of GDP with ordinary leverage is 7 periods
  - gap between measured and exog. TFP is small
  - amplification is mainly from aggregate capital
- High leverage economy, the half life of GDP rises to 15 years
  - large gap between measured and exog. TFP
  - changes in distribution from rise in default propagate the shock
    – extensive: smaller number of producers
    – intensive: rise in misallocation

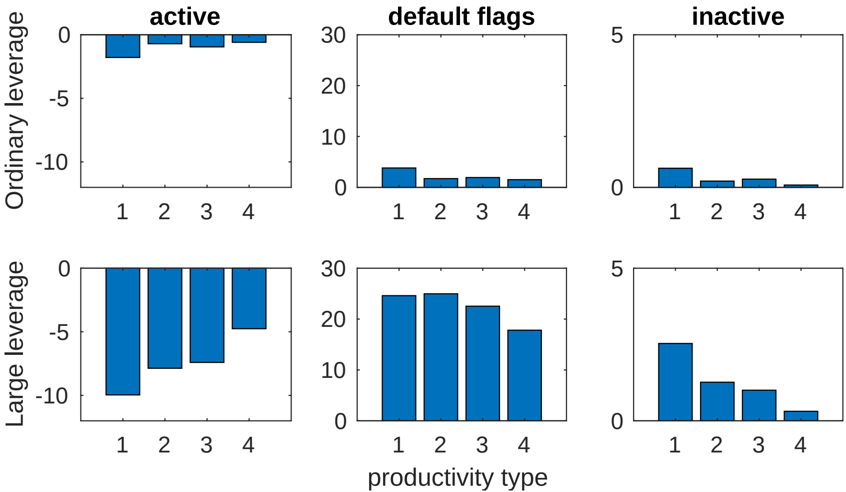
Decomposing the total effects: prices vs distribution



$$y_t = A_t \int z_i k_{it}^{\alpha} l_{it}^{\nu} g_{it} di = A_t \int z_i k_{it}^{\alpha} l_{it}^{\nu} (g_i^* + \Delta g_{it}) di$$

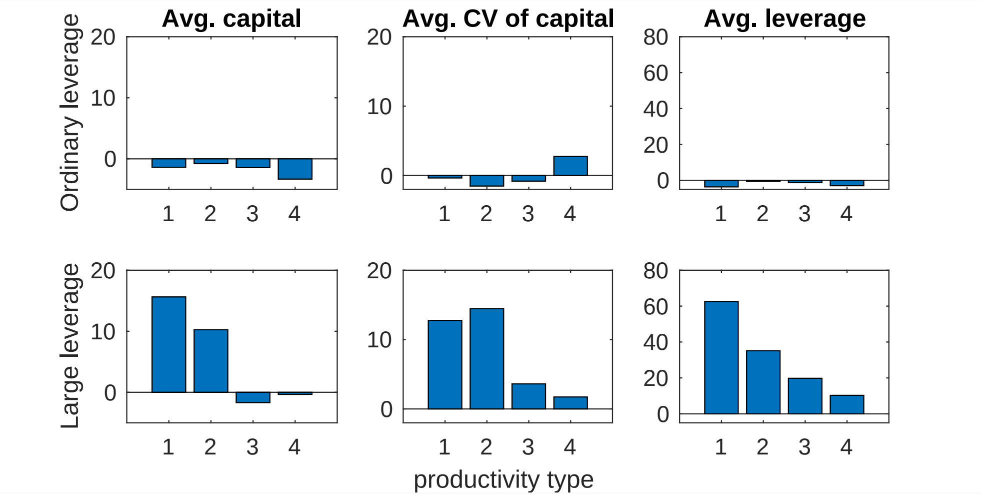
$$= A_t \int z_i k_{it}^{\alpha} l_{it}^{\nu} g_i^* di + A_t \int z_i k_{it}^{\alpha} l_{it}^{\nu} \Delta g_{it} di$$

Exit & default drives slow recovery



% deviation from steady state; average of first 10 periods

High leverage increases distortions



% deviation from steady state; average of first 10 periods

Role of policy to recovery

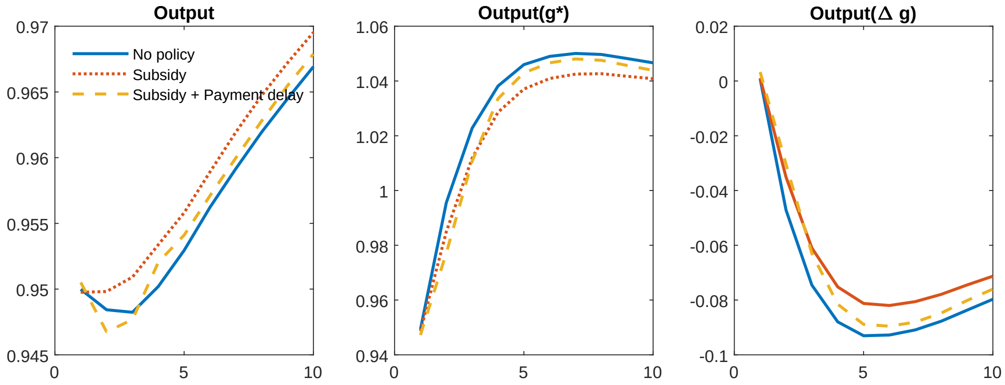
Business Credit Programs in response to Covid-19 shock

- Corporate Credit Facilities
- Main Street Lending Program
- Paycheck Protection Program
  – total volume (20-21) $791bn, amount forgiven is $441bn

Compare **ordinary and policy intervention**

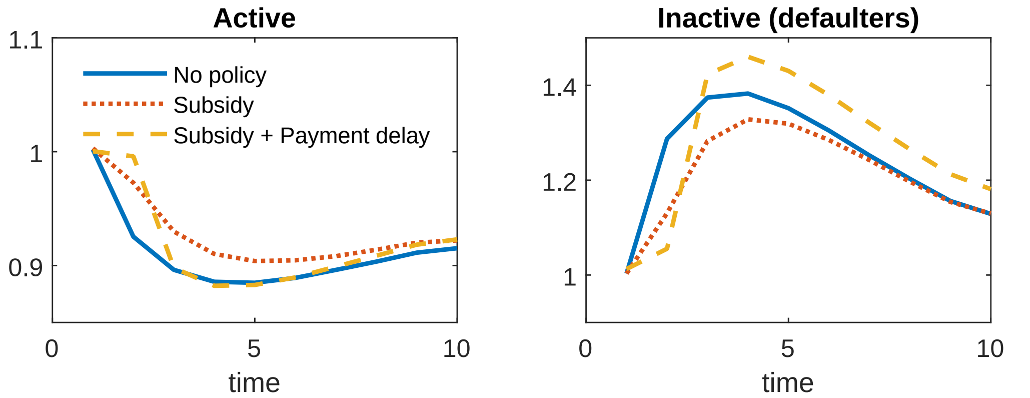
- the fiscal authority subsidizes production costs for 2 periods
  – total amount of subsidy: 0.2% of GDP ($\approx$ forgiven amount of PPP)
- over same period, businesses are able to defer debt payments

Policy intervention speed the recovery



- policy dampens drop in wages and interest rates, reducing its effectiveness prices

Subsidy helps producers stay in business



Conclusion

Heterogeneous producers, long-term debt, and default option

- Effects of high leverage in a recovery depends on its distributions
- Large recessions are propagated by high leverage
    - doubles the half life of recovery
- Changes in the producer distribution drives down aggregate TFP
- Policies subsidizing costs leads to faster recoveries
- Deferring debt payment dampens the effects of PPP

Appendix

Entrepreneur 2: business owner, not stopping

Business owners without borrowing option

$$
\begin{aligned}
\rho v^2(a,b,z) &= \max_{c,k,l} u(c) + v_a^2(a,b,z)\dot{a} + v_b^2(a,b,z)\dot{b} + \sum \pi_{zz'} v^1(a,b,z') \\
&\quad - \lambda_{10}(v^1(a,b,z) - v^0(a-b,z)) - \lambda_{21}(v^2(a,b,z) - v^1(a,b,z)) \\
\dot{a} &= zk^\alpha l^\nu - wl - (r+\delta)k + ra - c - (r+\theta)b - \xi(\cdot) - T(\cdot), \quad k \le \gamma a \\
\dot{b} &= -\theta b \\
v^2(a,b,z) &\ge v^{2*}(a,b,z)
\end{aligned}
$$

- $\lambda_{20}(v^2(a,b,z) - v^1(a,b,z))$ reflects the possibility of getting borrowing option where λ_{21} is the frequency of the shock

back

Entrepreneur 2: business owner, stopping values

Exiting

$$v^x(a_\tau, b_\tau, z) = v^0(a - b, z)$$

Defaulting

$$v^b(a_\tau, b_\tau, z) = v^d(\chi a_\tau, z) - \xi_d$$

back

Entrepreneur: inactive

A stopping time problem. A stopping option: Entering

$$v^0(a_t, z_t) = \max_{\{c_t\}_{t\geq 0}, \tau} \{E_0 \int_0^\tau u(c_t) + e^{-\rho\tau} v^{0*}(a_\tau, z_\tau)\}$$

$$\dot{a}_t = \psi z_t w_t + r_t a_t - c_t - T(\cdot)$$

$$(a_0, z_0) = (a, z)$$

- given a schedule of the entering cost ($\kappa(z, r, w)$) choose to enter or not
- can borrow up to $\gamma_\kappa \kappa(z, r, w)$

back

Entrepreneur: inactive, not stopping

The HJB equation prior to stopping is,

$$
\begin{aligned}
\rho v^0(a,z) &= \max_c u(c) + v_a^0(a,z)\dot{a} + \sum \pi_{zz'} v^0(a,z') \\
\dot{a} &= \psi z w + ra - c - T(\cdot) \\
v^0(a,z) &\geq v^{0*}(a,z)
\end{aligned}
$$

back

Entrepreneur: inactive, stopping values

Entering

$$v^e(a_\tau, z) = \max_{a', b'} v^2(a', b', z)$$

$$a' = a_\tau + q(a', b', z)b' - \xi_b b' - \kappa(z, r, w)$$

$$b' \leq \gamma_\kappa \kappa(z, r, w)$$

back

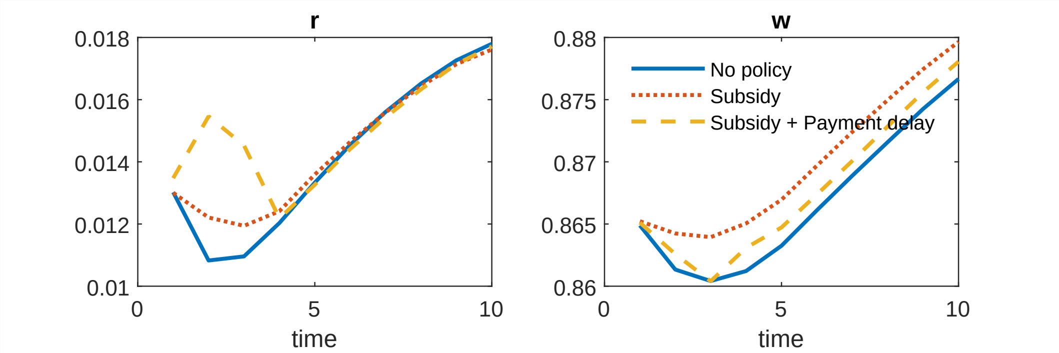
Entrepreneur: defaulted

$$\rho v^d(a,z) = \max_c u(c) + v^d_a(a,z)\dot{a} + \sum \pi_{zz'} v^d(a,z') - \lambda_d(v^d(a,z) - v^0(a,z))$$
$$\dot{a} = \psi_d z w + ra - c - T(\cdot)$$

- Default flag is removed after $1/\lambda_d$ on average

back

Policy economy experiences a more rapid recovery in output



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