Comparación Elementos

Tensión X

Momento Constante

N: numero de grados de libertad

El CST da tensión nula ya que cada elemento debe tener una única deformacíon! Entonces existe un empate.

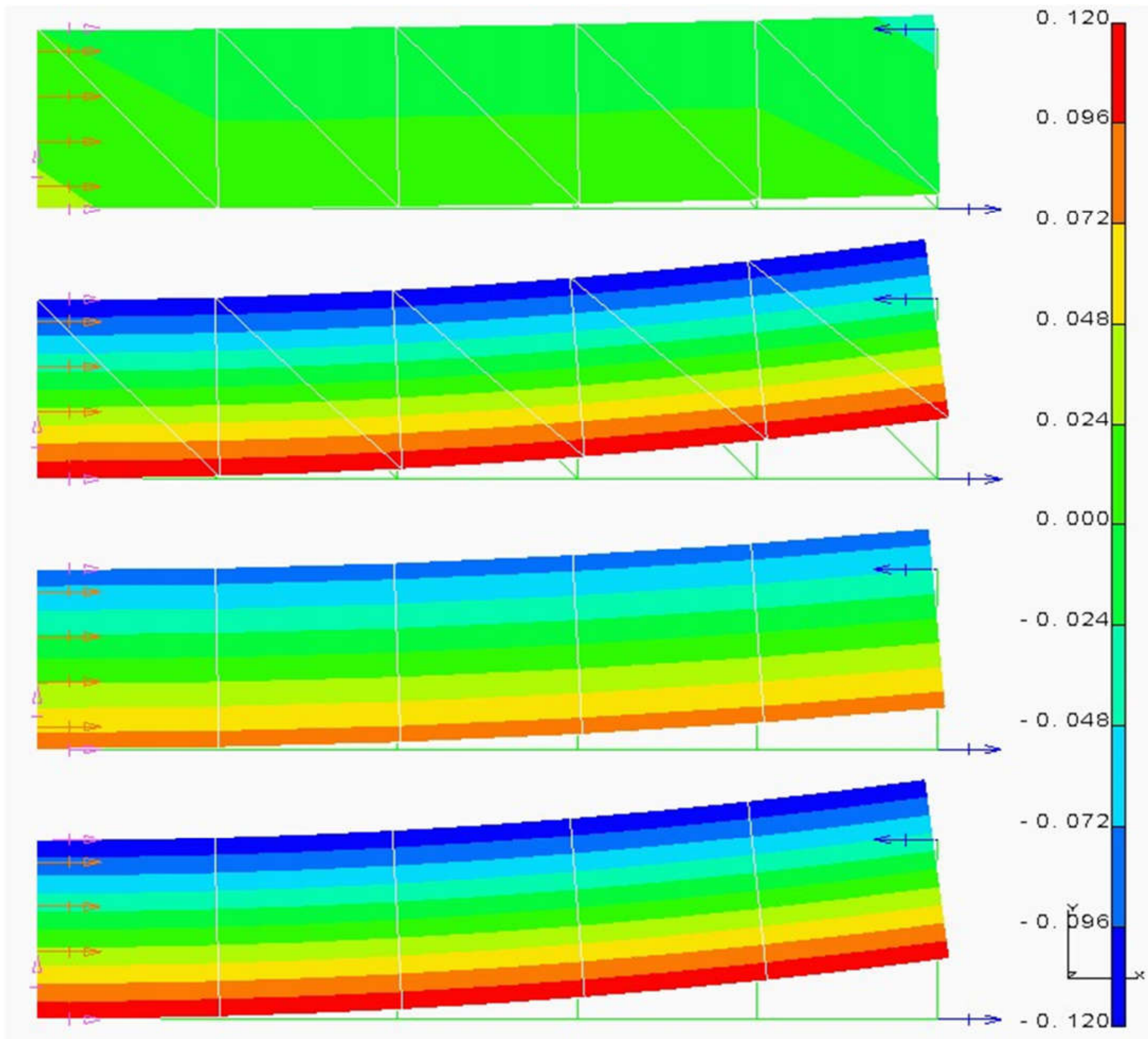
CST (N24)

LST (N66)

La viga no debería tener corte pero el elemento lleva parte de la energía a corte. No se curva lo que deberá

Q4 (N24)

Q8 (N56)



Comparación Elementos

Corte XY

Momento Constante

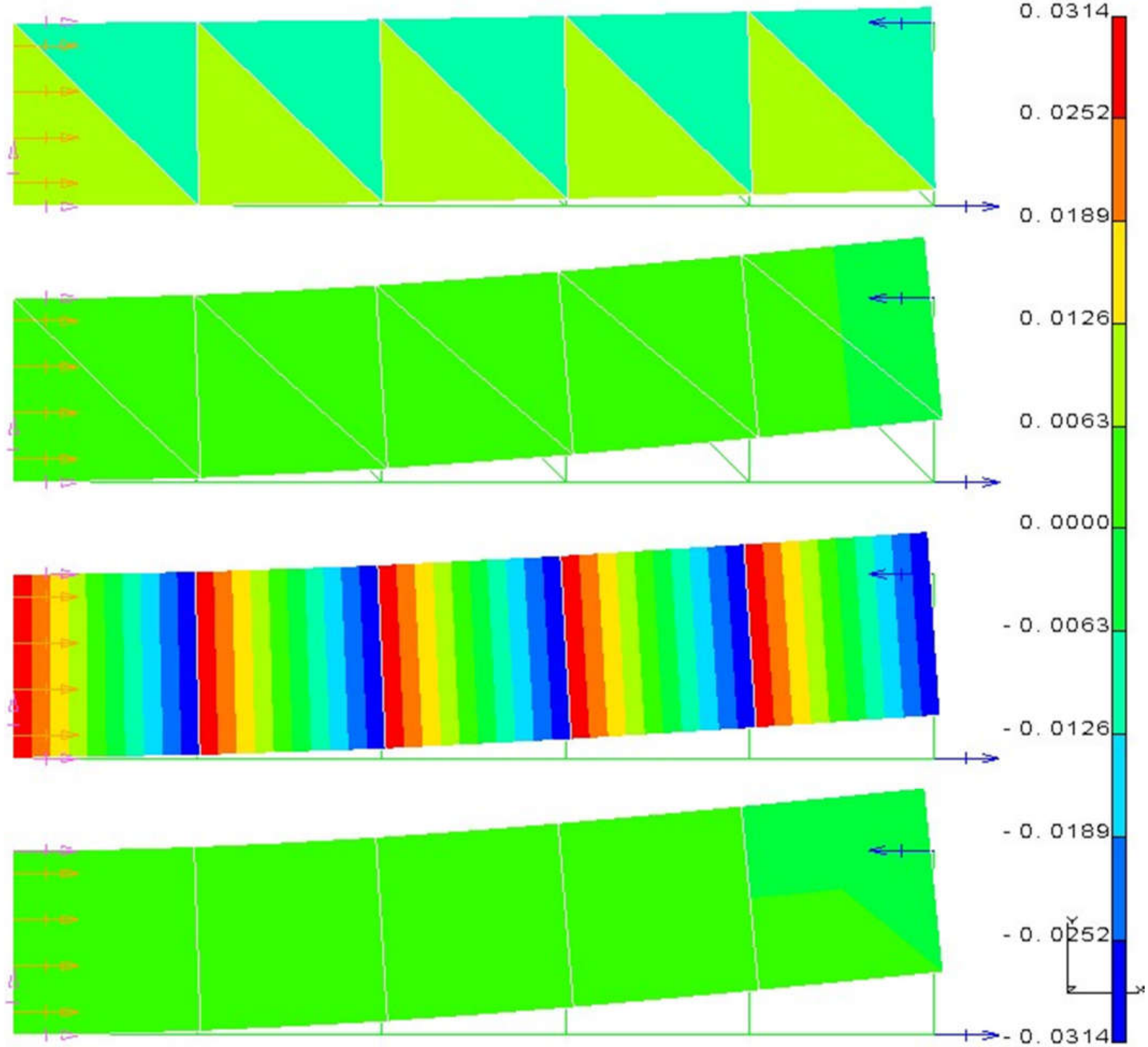
CST (N24)

LST (N66)

Aca se puede ver la energía que falta.

Q4 (N24)

Q8 (N56)



Comparación Elementos

Tensión X

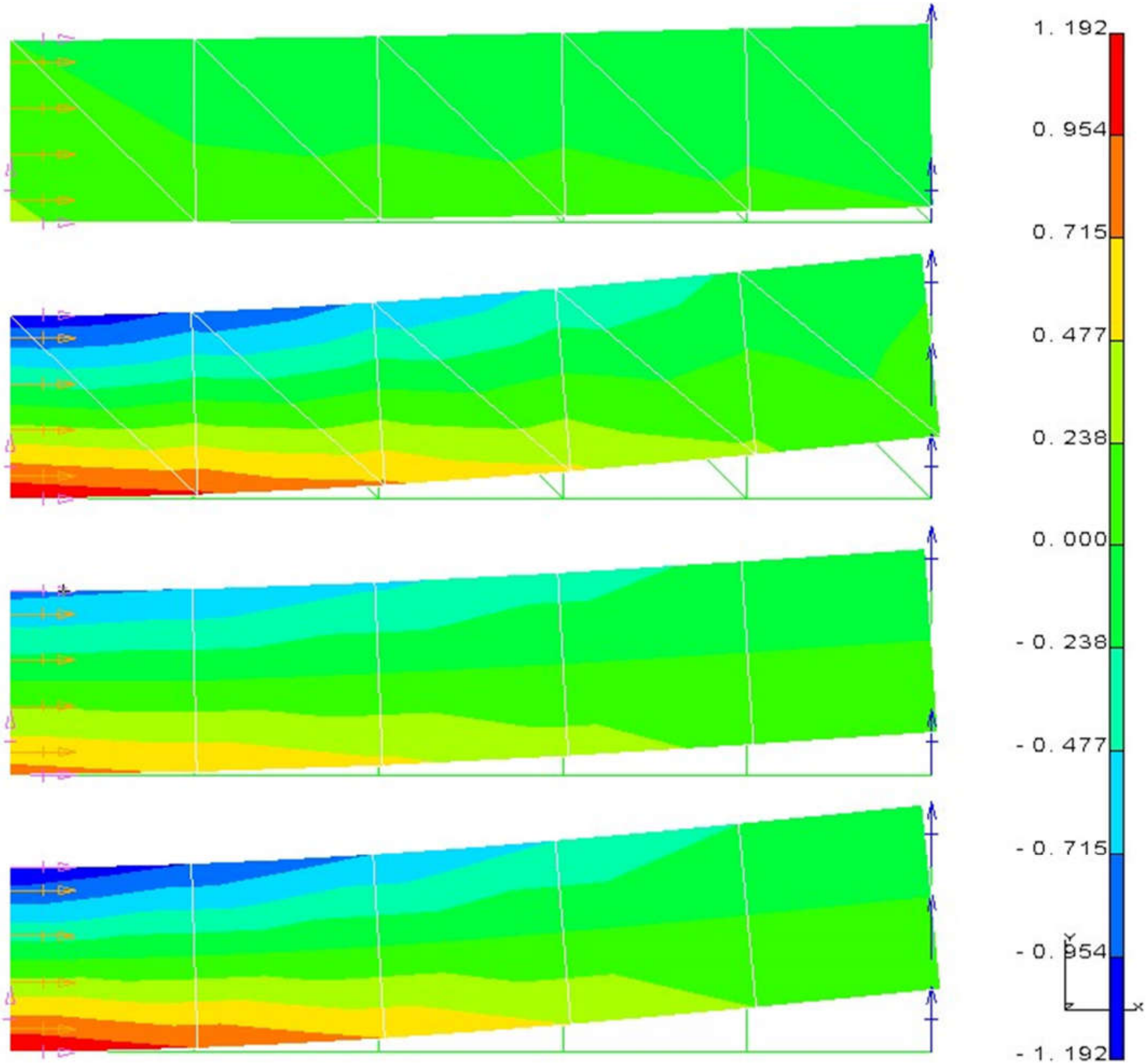
Momento Variable

CST (N24 / E=76.71%)

LST (N66 / E=3.57%)

Q4 (N24 / E=32.70%)

Q8 (N56 / E=0.65%)



σ=1.2

Comparación Elementos

Corte XY

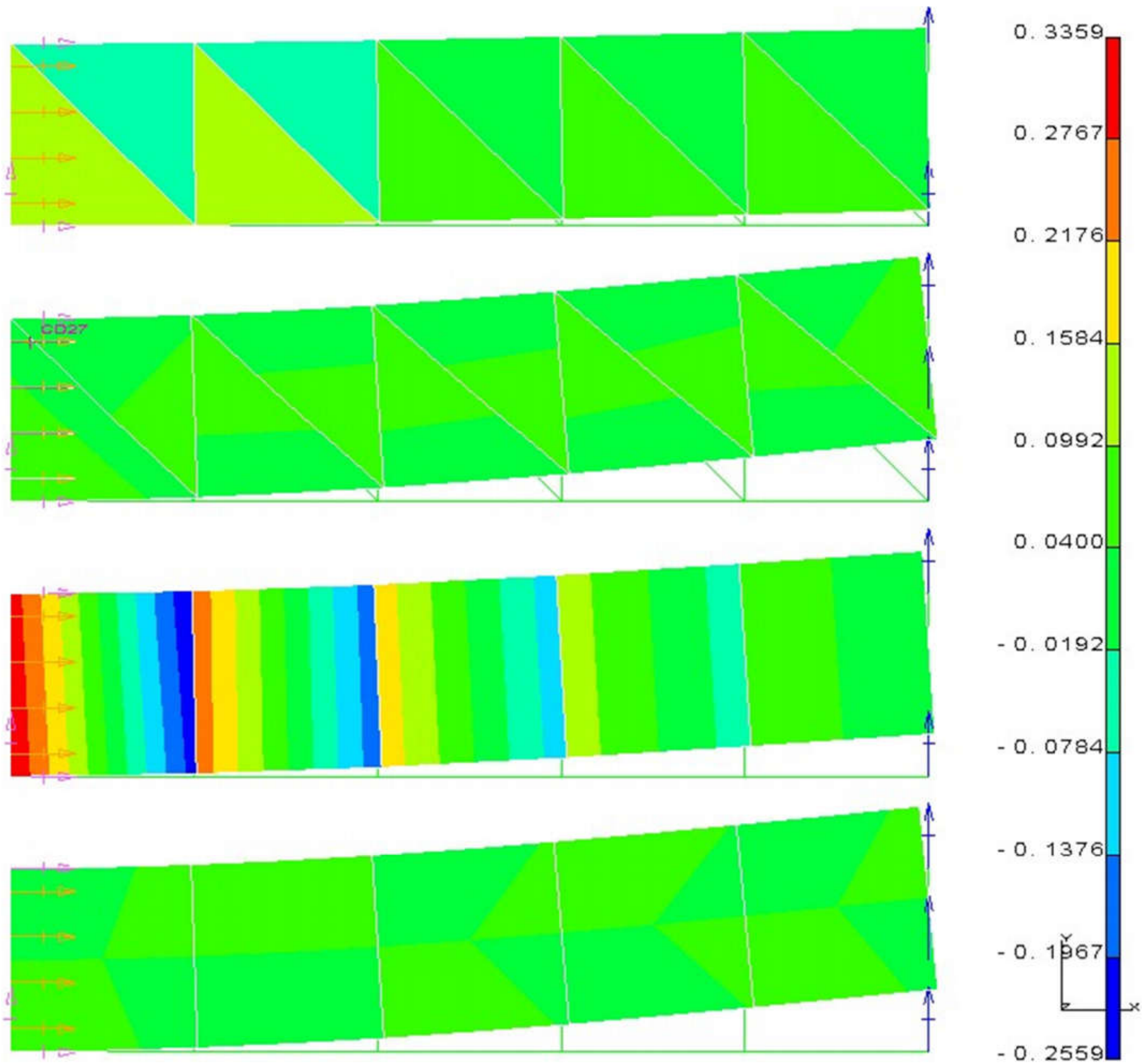
Momento Variable

CST (N24)

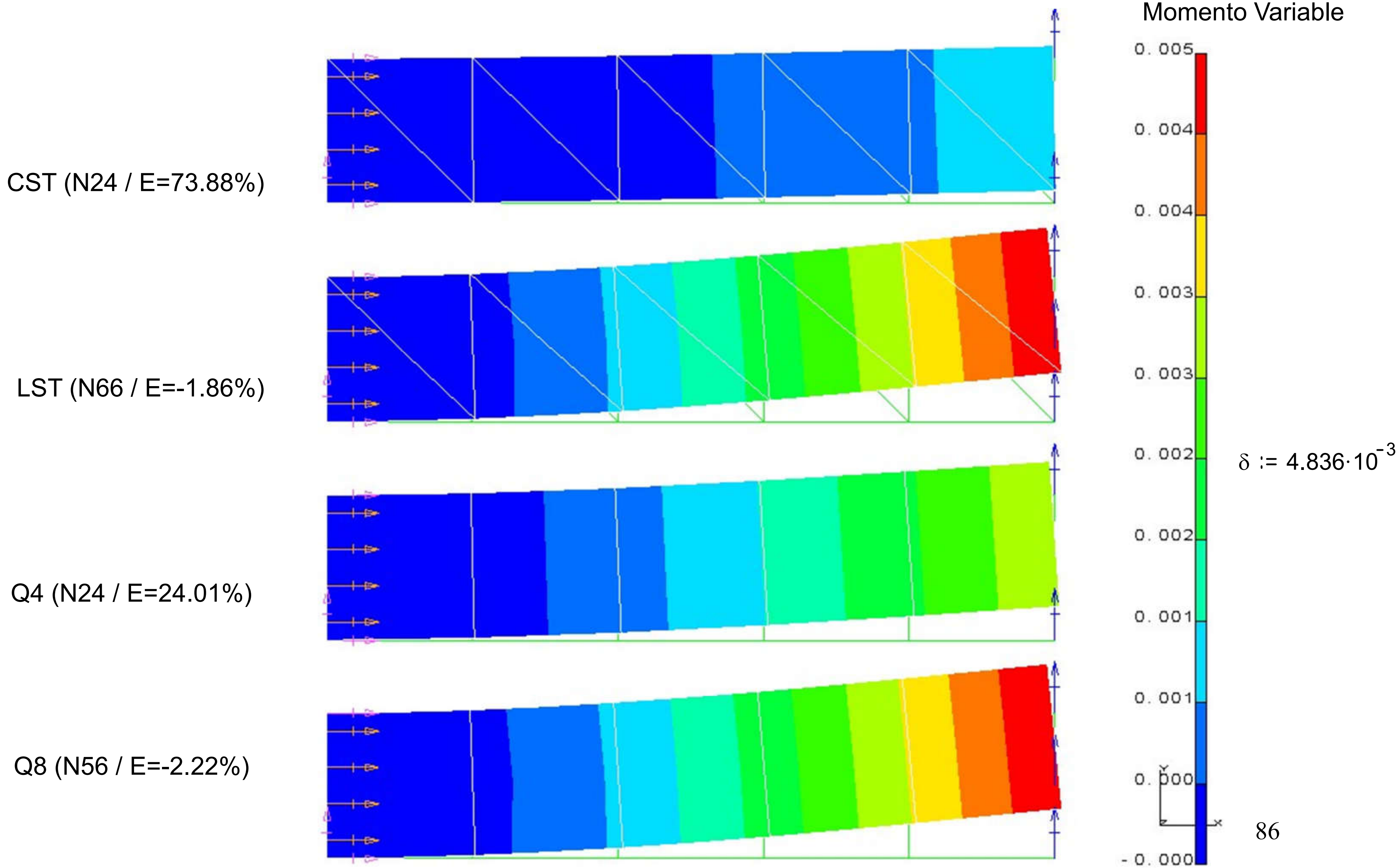
LST (N66)

Q4 (N24)

Q8 (N56)



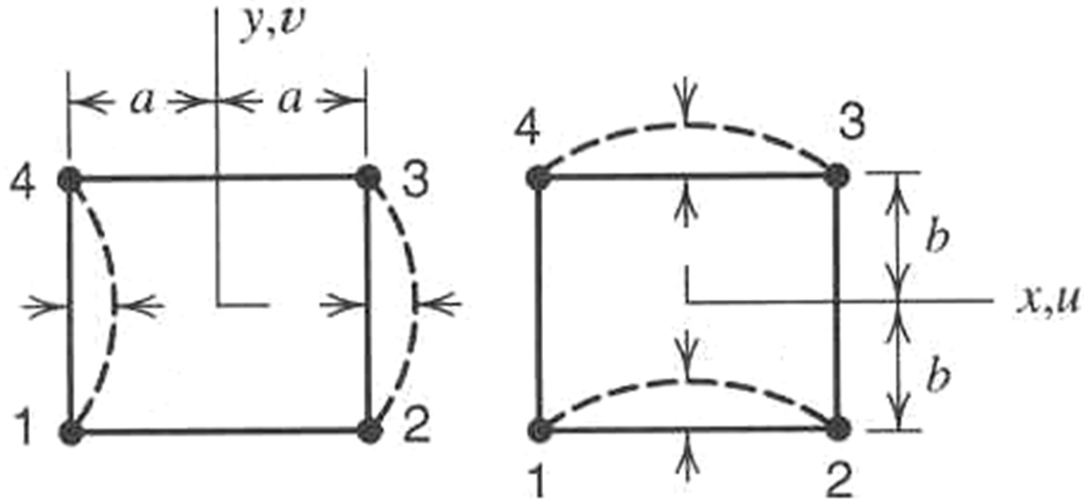
Comparación Elementos



Elementos Rectangulares

Deformación Constante – Incompatible Q6

Seda y nu son "grados de libertad jerárquicos" ya que no se encuentran sobre un nodo.



$$\xi = \frac{x}{a}; \quad \eta = \frac{y}{b}$$

$$u = a_1 + a_2 x + a_3 y + a_4 xy + d_1\left(1-\xi^2\right) + d_2\left(1-\eta^2\right) = N_1 u_1 + \cdots + N_4 u_4 + d_1\left(1-\xi^2\right) + d_2\left(1-\eta^2\right)$$

$$v = a_5 + a_6 x + a_7 y + a_8 xy + d_3\left(1-\xi^2\right) + d_4\left(1-\eta^2\right) = N_1 v_1 + \cdots + N_4 v_4 + d_3\left(1-\xi^2\right) + d_4\left(1-\eta^2\right)$$

Condensación

$$\begin{bmatrix} \begin{bmatrix} K_{nn} \\ \text{Viejos} \end{bmatrix} & \begin{bmatrix} K_{ni} \\ \text{Viejos con nuevos} \end{bmatrix} \\ \begin{bmatrix} K_{in} \\ \text{Nuevos con viejos} \end{bmatrix} & \begin{bmatrix} K_{ii} \\ \text{Nuevos} \end{bmatrix} \end{bmatrix} \begin{Bmatrix} u_1 \\ \vdots \\ v_4 \\ d_1 \\ \vdots \\ d_4 \end{Bmatrix} = \begin{Bmatrix} F_{x1} \\ \vdots \\ F_{y4} \\ 0 \\ \vdots \\ 0 \end{Bmatrix}$$

$$\left[K_{nn}\right] \begin{Bmatrix} u_1 \\ \vdots \\ v_4 \end{Bmatrix} + \left[K_{ni}\right] \begin{Bmatrix} d_1 \\ \vdots \\ d_4 \end{Bmatrix} = \begin{Bmatrix} F_{x1} \\ \vdots \\ F_{y4} \end{Bmatrix}$$

$$\left[K_{in}\right] \begin{Bmatrix} u_1 \\ \vdots \\ v_4 \end{Bmatrix} + \left[K_{ii}\right] \begin{Bmatrix} d_1 \\ \vdots \\ d_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ \vdots \\ 0 \end{Bmatrix} \rightarrow \begin{Bmatrix} d_1 \\ \vdots \\ d_4 \end{Bmatrix} = -\left[K_{ii}\right]^{-1}\left[K_{in}\right] \begin{Bmatrix} u_1 \\ \vdots \\ v_4 \end{Bmatrix}$$

Siempre que tengo grados de libertad que no se pueden mover, se puede condensar la matriz de rigidez.
Q6 condensado tiene los mismos grados de libertad que Q4

Elementos Rectangulares

Deformación Constante – Incompatible Q6

$$\begin{Bmatrix} d_1 \\ \vdots \\ d_4 \end{Bmatrix} = -[K_{ii}]^{-1}[K_{in}]\begin{Bmatrix} u_1 \\ \vdots \\ v_4 \end{Bmatrix} \rightarrow [K_{nn}]\begin{Bmatrix} u_1 \\ \vdots \\ v_4 \end{Bmatrix} - [K_{ni}][K_{ii}]^{-1}[K_{in}]\begin{Bmatrix} u_1 \\ \vdots \\ v_4 \end{Bmatrix} = \Big(\underbrace{[K_{nn}] - [K_{ni}][K_{ii}]^{-1}[K_{in}]}_{[K]}\Big)\begin{Bmatrix} u_1 \\ \vdots \\ v_4 \end{Bmatrix} = \begin{Bmatrix} F_{x1} \\ F_{y1} \\ \vdots \\ F_{y4} \end{Bmatrix}$$

Deformaciones

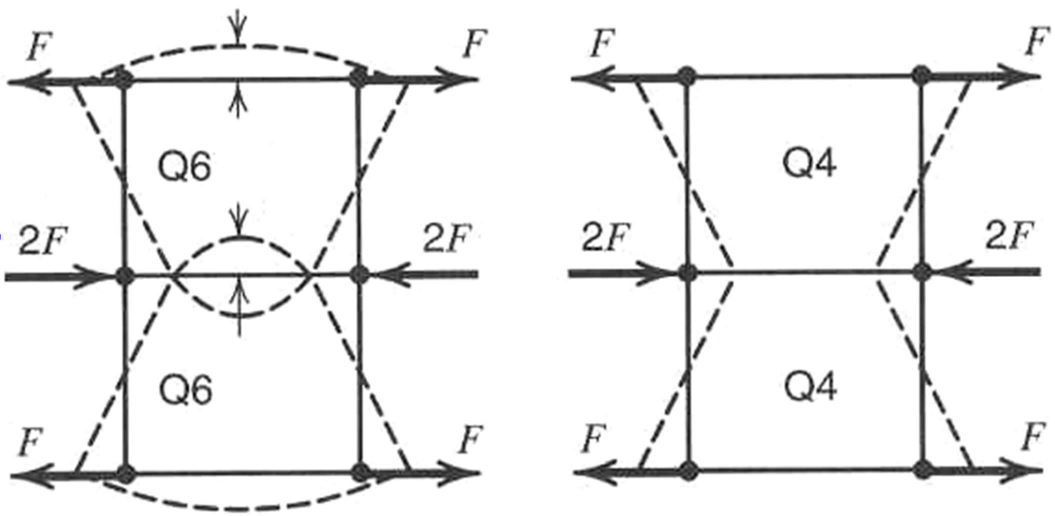
$$\varepsilon_x = \frac{\partial u}{\partial x} = a_2 + a_4 y - \frac{2x}{a^2} d_1 \quad ; \quad \varepsilon_y = \frac{\partial v}{\partial y} = a_7 + a_8 x - \frac{2y}{b^2} d_4$$

$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = a_3 + a_6 + a_4 x + a_8 y - \frac{2x}{a^2} d_3 - \frac{2y}{b^2} d_2$$

Estos nuevos coeficientes eliminan lso términos del corte espúreo.

Compatibilidad

Deja de haber compatibilidad en la continuidad de las deformaciones, ahora los elementos, aunque esten unidos en los nodos, se rompen por las lineas.



Elementos Rectangulares

Deformación Constante – Incompatible Q6

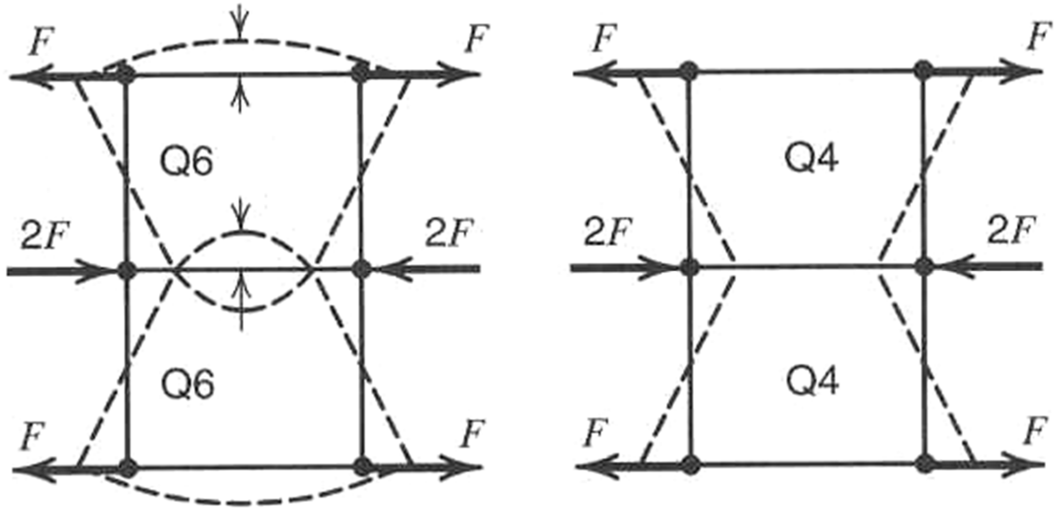
$$\begin{Bmatrix} d_1 \\ \vdots \\ d_4 \end{Bmatrix} = -[K_{ii}]^{-1}[K_{in}]\begin{Bmatrix} u_1 \\ \vdots \\ v_4 \end{Bmatrix} \rightarrow [K_{nn}]\begin{Bmatrix} u_1 \\ \vdots \\ v_4 \end{Bmatrix} - [K_{ni}][K_{ii}]^{-1}[K_{in}]\begin{Bmatrix} u_1 \\ \vdots \\ v_4 \end{Bmatrix} = \left(\underbrace{[K_{nn}] - [K_{ni}][K_{ii}]^{-1}[K_{in}]}_{[K]} \right)\begin{Bmatrix} u_1 \\ \vdots \\ v_4 \end{Bmatrix} = \begin{Bmatrix} F_{x1} \\ F_{y1} \\ \vdots \\ F_{y4} \end{Bmatrix}$$

Deformaciones

$$\varepsilon_x = \frac{\partial u}{\partial x} = a_2 + a_4 y - \frac{2x}{a^2} d_1 \quad ; \quad \varepsilon_y = \frac{\partial v}{\partial y} = a_7 + a_8 x - \frac{2y}{b^2} d_4$$

$$\gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = a_3 + a_6 + a_4 x + a_8 y - \frac{2x}{a^2} d_3 - \frac{2y}{b^2} d_2$$

Compatibilidad

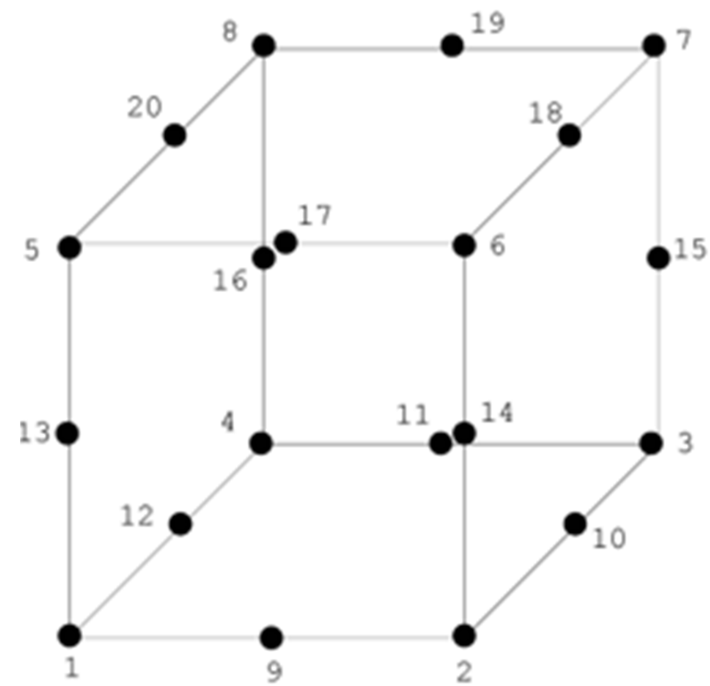


Elementos 3D

Funciones de Forma

$$u=\sum_i N_i u_i \quad v=\sum_i N_i v_i \quad w=\sum_i N_i w_i$$

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix}=\begin{bmatrix} N_1 & 0 & 0 & \dots & N_n & 0 & 0 \\ 0 & N_1 & 0 & \cdots & 0 & N_n & 0 \\ 0 & 0 & N_1 & \cdots & 0 & 0 & N_n \end{bmatrix}$$



Sólo hay 1 matriz constitutiva

Ecuación Constitutiva (Ingeniería)

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix}=\begin{bmatrix} A & B & B & 0 & 0 & 0 \\ B & A & B & 0 & 0 & 0 \\ B & B & A & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix}\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix}-\frac{E\alpha\Delta T}{1-2\upsilon}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A=\frac{(1-\upsilon)E}{(1+\upsilon)(1-2\upsilon)} \quad B=\frac{\upsilon E}{(1+\upsilon)(1-2\upsilon)} \quad G=\frac{E}{2(1+\upsilon)}$$

Matriz de Rigidez

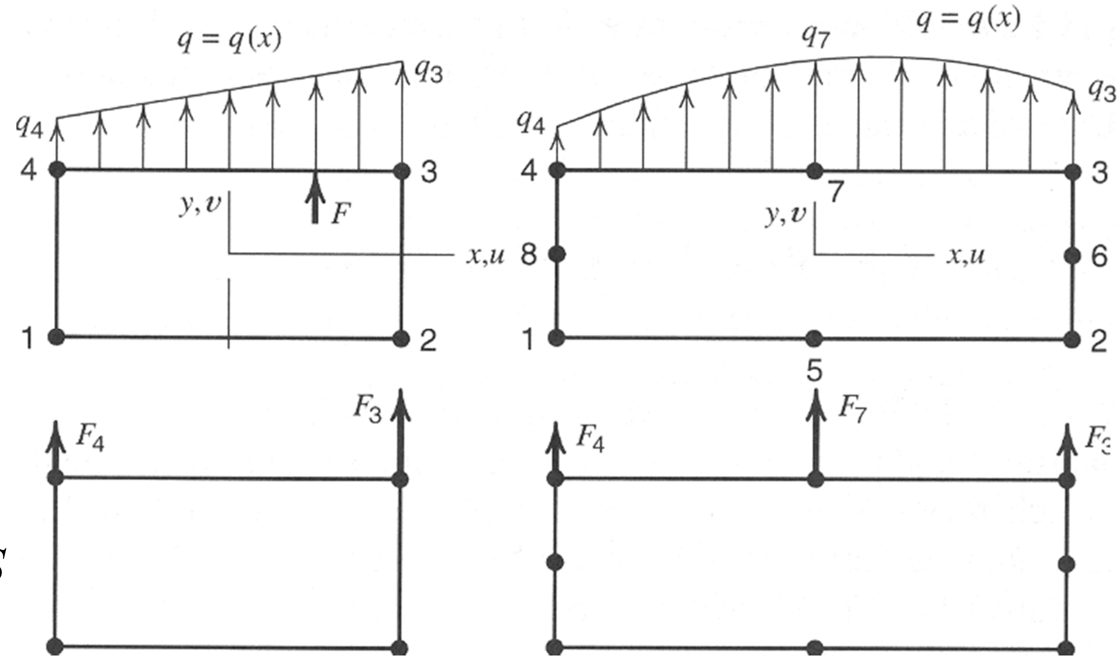
$$\varepsilon_{6x1}=B_{6x3N}d_{3Nx1}$$

$$k_{3Nx3N}=\iiint_\Omega B_{3Nx6}{}^T E_{6x6} B_{6x3N}\, dx\, dy\, dz$$

Aplicación de Cargas

Energéticamente Equivalente

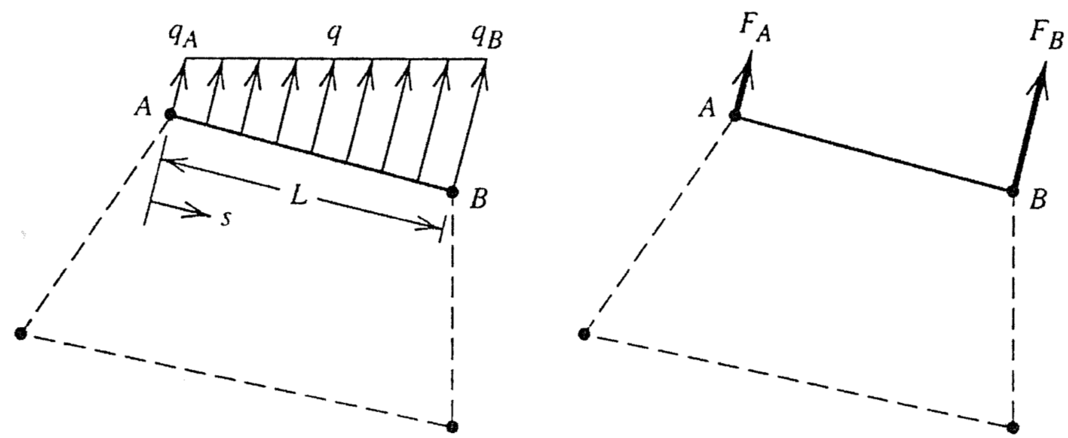
$$W=\{d\}^T\{r_e\}=\int_\Omega\{u\}^T\{F\}dV+\int_S\{u\}^T\{\Phi\}dS$$



$$\int_S\{u\}^T\{\Phi\}dS=\{d\}^T\int_S[N]^T\{\Phi\}dS\Rightarrow$$

$$\{d\}^T\{r_e\}=\{d\}^T\int_S[N]^T\{\Phi\}dS\;\therefore\;\{r_e\}=\int_S[N]^T\{\Phi\}dS$$

$$\{\Phi\}=[N]\{q\}\rightarrow\{r_e\}_\Phi=\int_S[N]^T[N]dS\{q\}$$

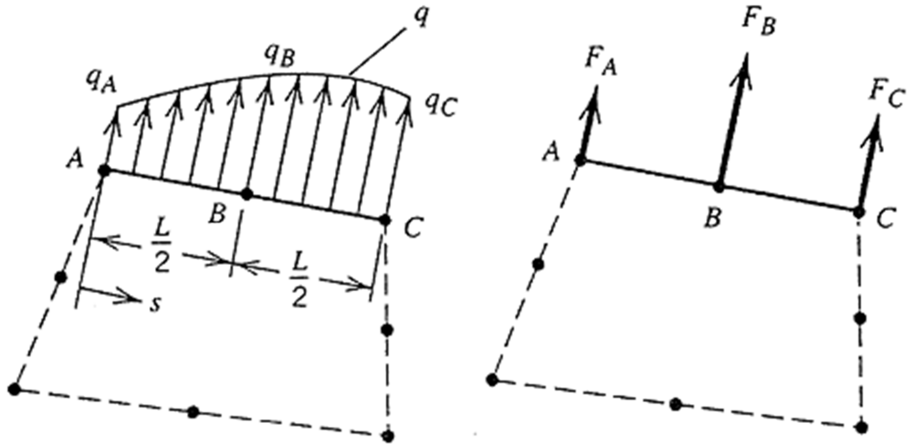


$$\{r_e\}_\Phi=\int_L\begin{bmatrix}\dfrac{L-x}{L} & \dfrac{x}{L}\end{bmatrix}^T\begin{bmatrix}\dfrac{L-x}{L} & \dfrac{x}{L}\end{bmatrix}dL\begin{Bmatrix}q_A\\q_B\end{Bmatrix}$$

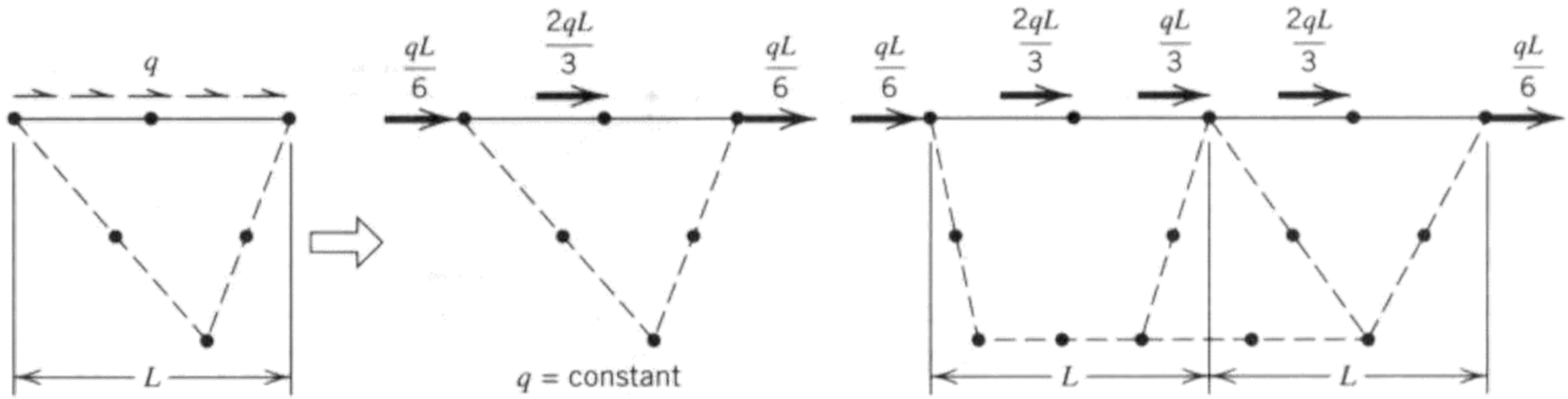
$$\{r_e\}_\Phi=\frac{1}{L^2}\int_L\begin{bmatrix}x^2-2xL+L^2 & xL-x^2\\ xL-x^2 & x^2\end{bmatrix}dL\begin{Bmatrix}q_A\\q_B\end{Bmatrix}$$

$$\{r_e\}_\Phi=\frac{L}{6}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}\begin{Bmatrix}q_A\\q_B\end{Bmatrix}\rightarrow\qquad q_1=q_2=q\rightarrow\{r_e\}_\Phi=\begin{Bmatrix}F_A\\F_B\end{Bmatrix}=\frac{qL}{2}\begin{Bmatrix}1\\1\end{Bmatrix}$$

Aplicación de Cargas



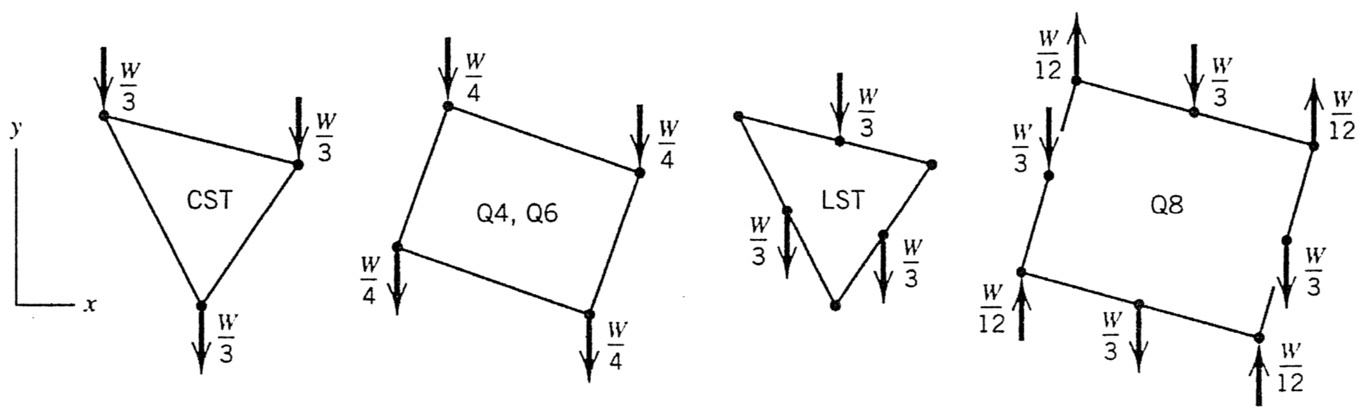
$$\begin{Bmatrix} F_A \\ F_B \\ F_C \end{Bmatrix} = \frac{L}{30}\begin{bmatrix} 4 & 2 & -1 \\ 2 & 16 & 2 \\ -1 & 2 & 4 \end{bmatrix}\begin{Bmatrix} q_A \\ q_B \\ q_C \end{Bmatrix}$$



Cargas de Volumen

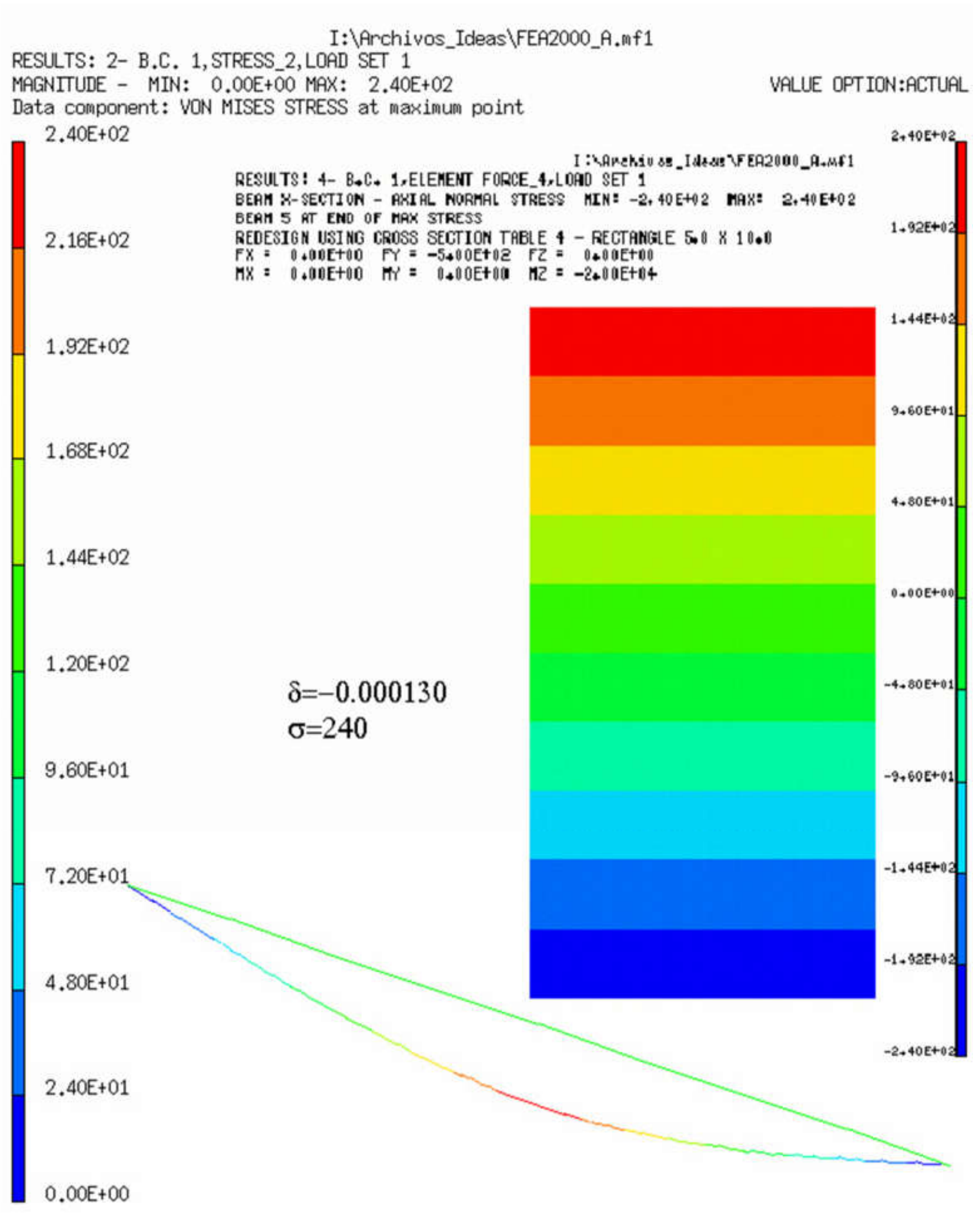
$$\{F\} = [N]\{f\}$$

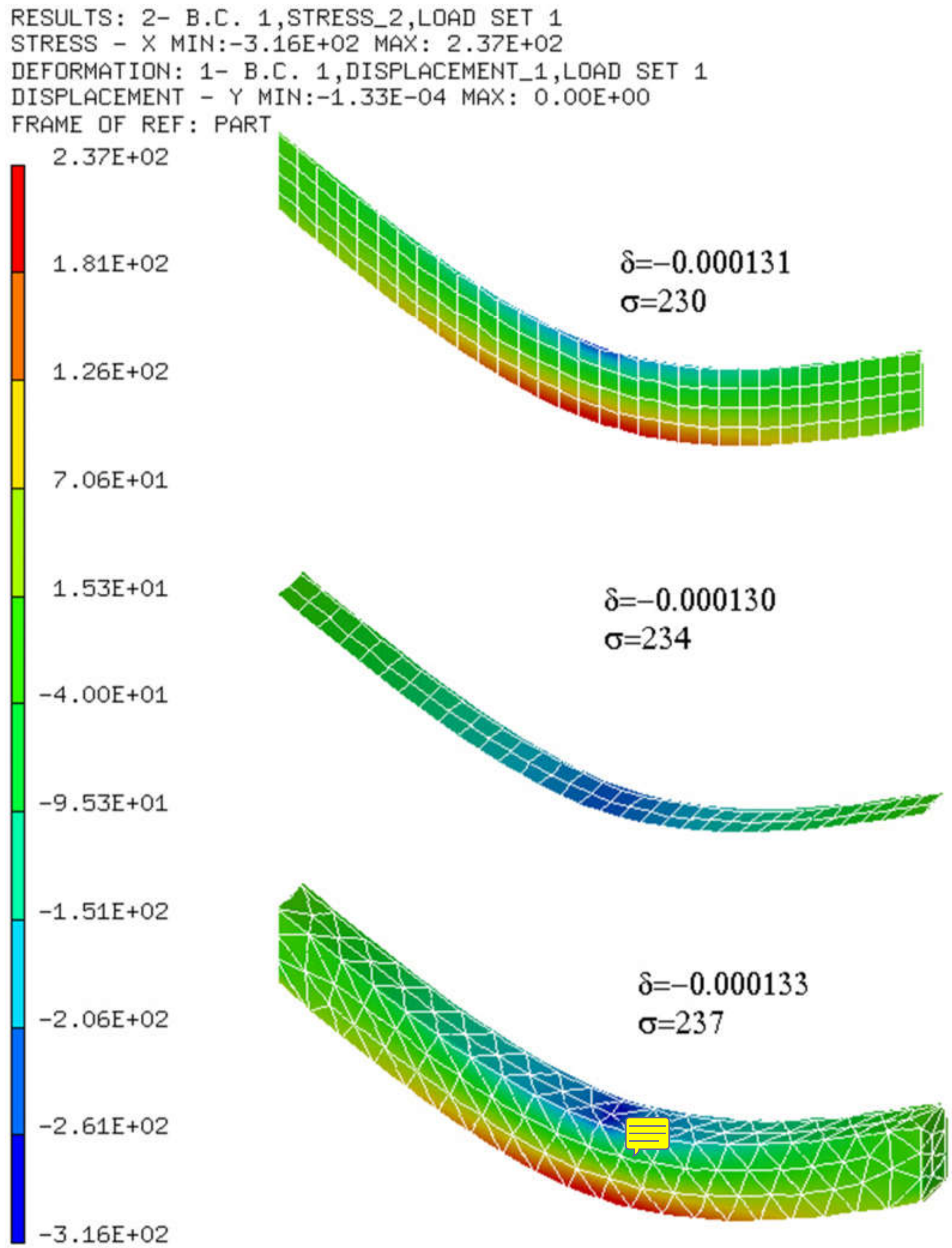
$$\{r_e\}_F = \left(\int_\Omega [N]^T [N] dV\right)\{f\}$$



Comparación Elementos

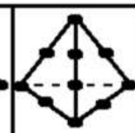
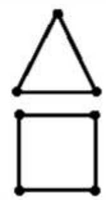
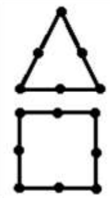
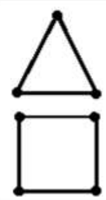
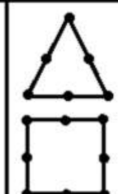
Resultados δ=-0.000124mm / σ= 240mN/mm^2



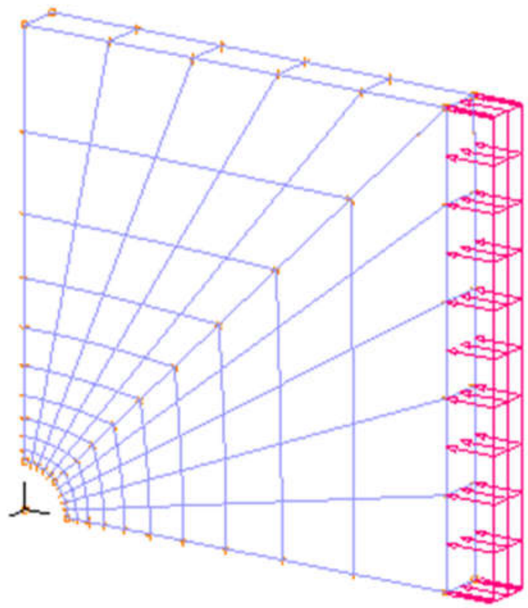


No vale la pena mallar en 3D cuando 1D resuelve mas facil y mejor

Tipos de Elementos

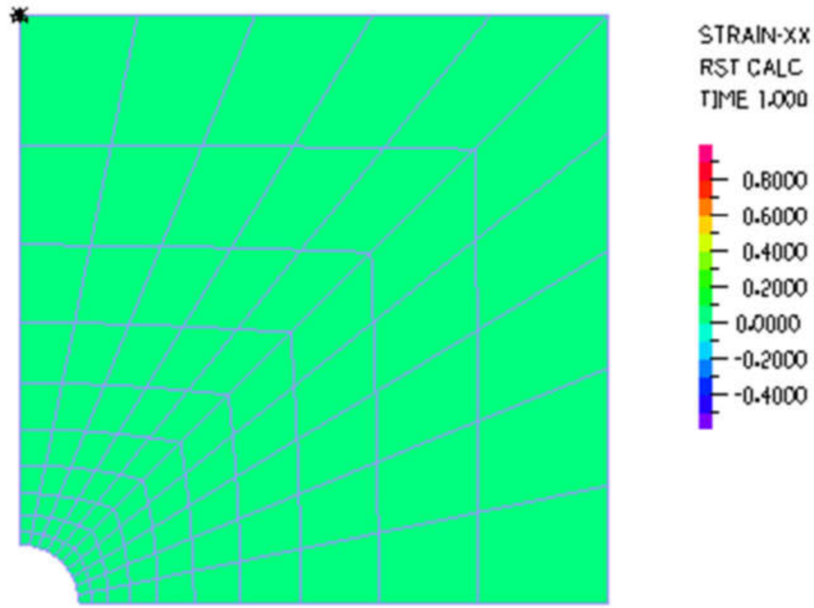
	Topology linear	parabolic	Modeled Stiffness Trans.	Rot.	Required Properties	Best Application
3D solid			3		material	general 3D structures (those that appear solid)
axis symmet-ric solids			2		material	general 3D structures that are symmetric about an axis
2D shell			3	3	material thickness non–structural mass formulation option	thin structures (>10:1 length/thickness) where bending is important
plane stress/ strain			2		material thickness	structures where only in–plane behavior is impor-tant
axis symmet-ric shells			2	1	material thickness	thin structures that are symmetric about an axis
1D rod			3		material non–structural mass cross section	axial deformation and rigid body motion of space frames
1D beam			3	3	material non–structural mass cross section orientation end offset end shortening release warping restraint factor	general space frame components and stiffen-ers for shell structures

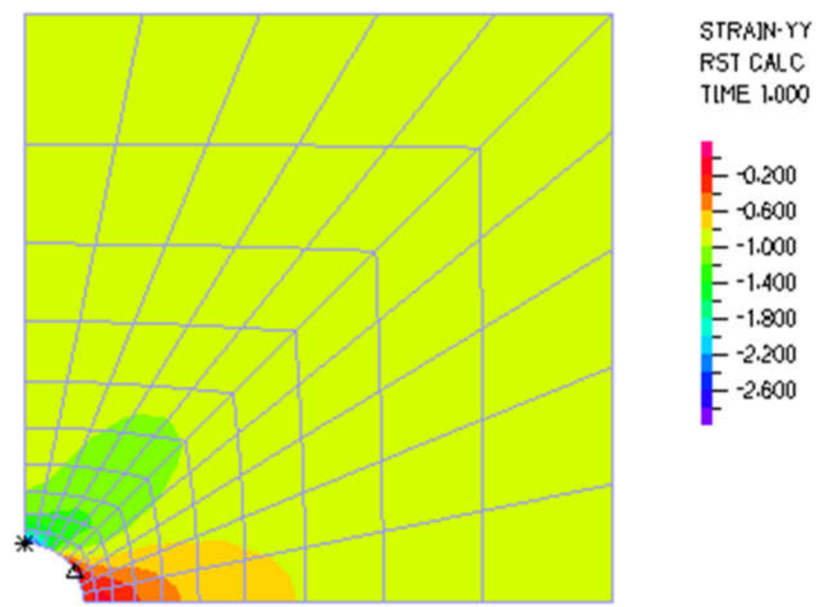
Comparación de casos 2D



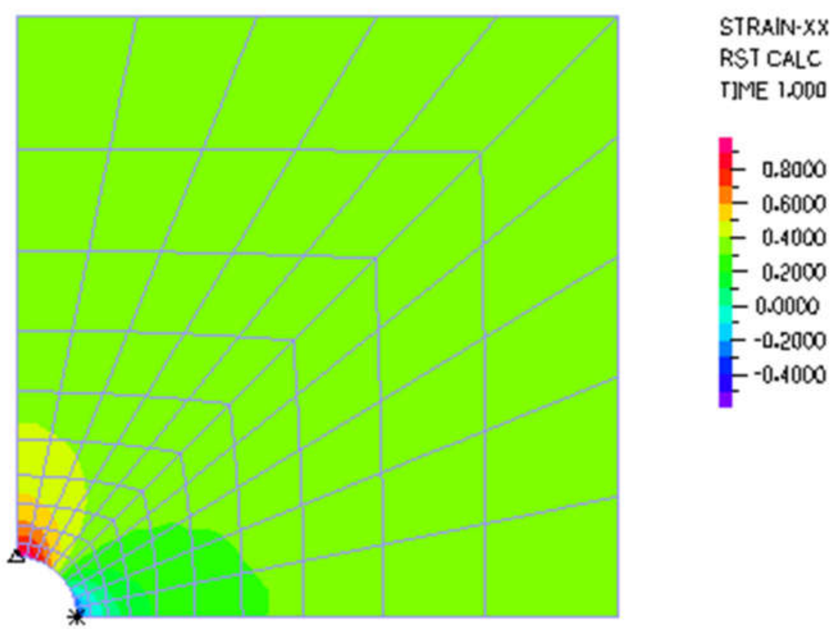
Comparación de casos 2D

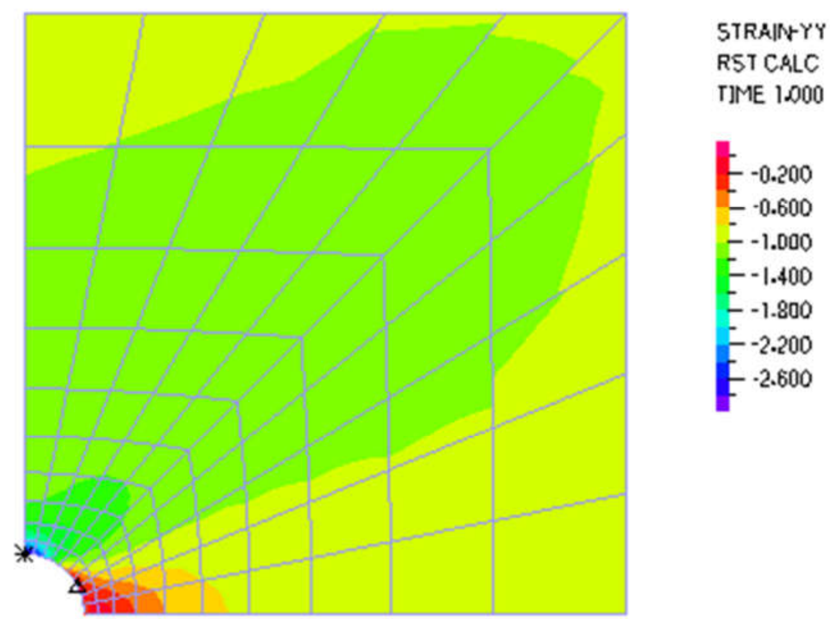
Plain Strain





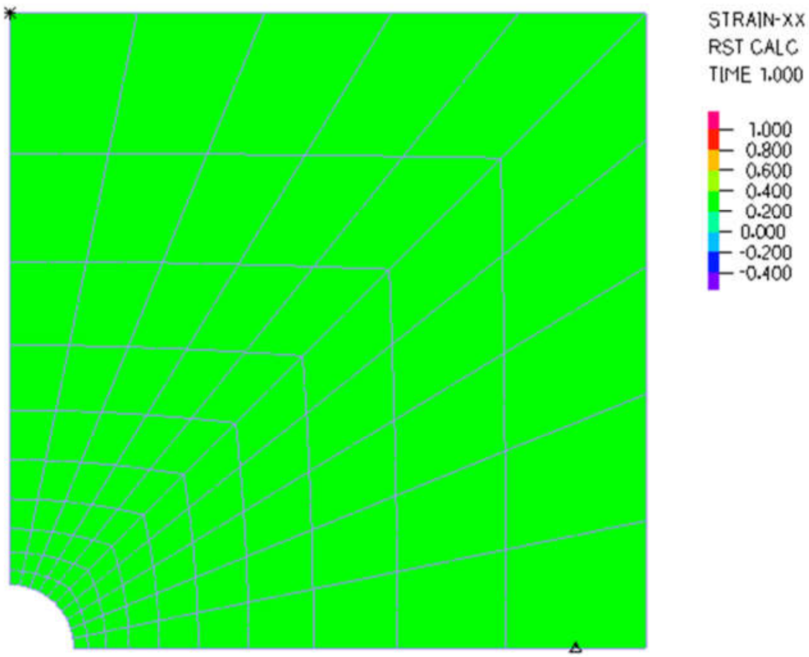
Plain Stress

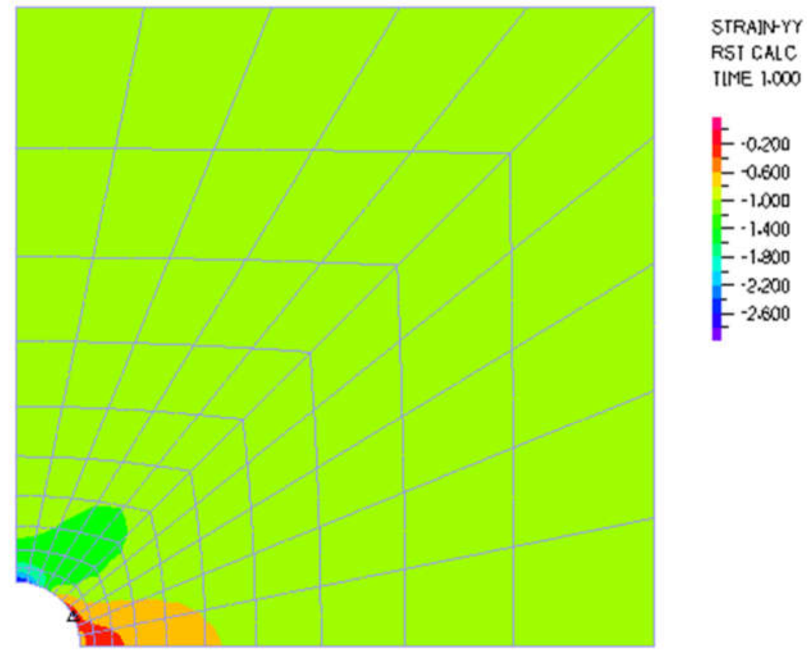




Comparación de casos 2D

Gran espesor





Bajo espesor