Elemento isoparamétrico: cálculo de fuerzas

Formulación isoparamétrica (Referencias: Teóricas / Logan 10.2)

Fuerzas de volumen: La integración se realiza numéricamente con los puntos y pesos de Gauss análogamente al cómputo de la matriz de rigidez.

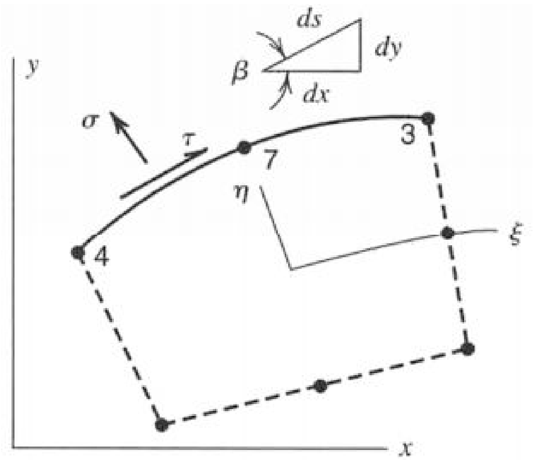
$$\{r_e\}_F = \int_{-1}^{1}\int_{-1}^{1} \begin{bmatrix} N_1 & 0 & \dots & 0 \\ 0 & N_1 & \dots & N_n \end{bmatrix}^T \begin{bmatrix} N_1 & 0 & \dots & 0 \\ 0 & N_1 & \dots & N_n \end{bmatrix} |J| t\, d\eta\, d\xi \begin{Bmatrix} f_{1_x} \\ f_{1_y} \\ \vdots \\ f_{n_y} \end{Bmatrix}$$

Interpolación de la carga

Fuerzas de superficie

$$\{r_e\} = \int_{4-7-3} [N(\xi, \eta = 1)]^T \begin{Bmatrix} \Phi_x \\ \Phi_y \end{Bmatrix} t\, ds$$

$$\begin{Bmatrix} \Phi_x \\ \Phi_y \end{Bmatrix} t\, ds = \begin{Bmatrix} \tau dx - \sigma dy \\ \sigma dx + \tau dy \end{Bmatrix} t$$



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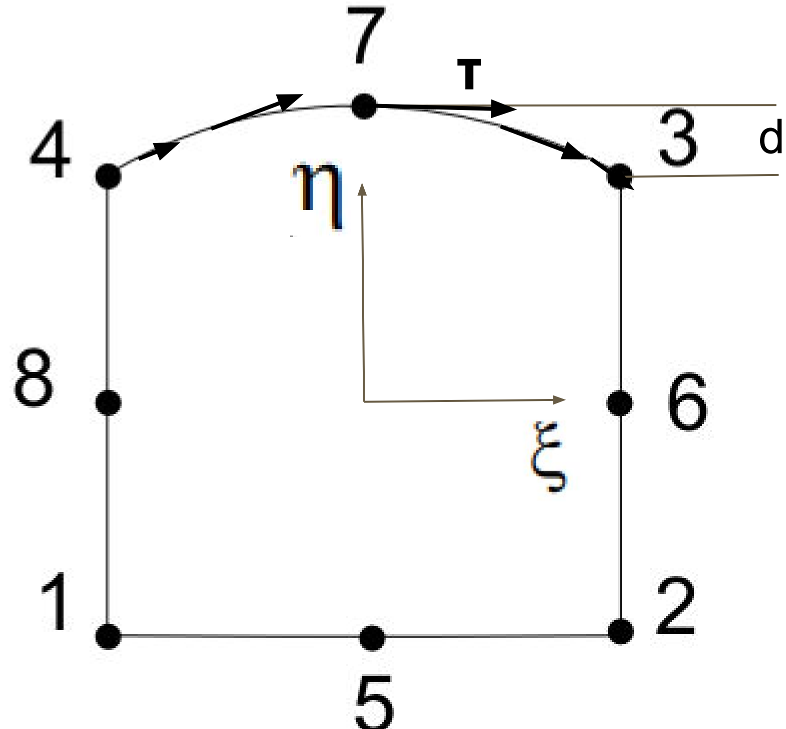
Ejemplo Q8



Fuerza tangencial con variación lineal de 0 a T.

$\tau(\xi)$ = T (ξ+1), -1≤ ξ ≤ 0
$\tau(\xi)$ = T (1-ξ), 0≤ ξ ≤ 1

El lado 4-7-3 corresponde a (**η**=1, ξ) en coordenadas locales

$$J_{11} = \frac{\partial \sum_{\text{nnod}} N_i(\xi, \eta = 1)\, x_i}{\partial \xi} = \frac{\partial N_3}{\partial \xi} x_3 + \frac{\partial N_4}{\partial \xi} x_4 + \frac{\partial N_7}{\partial \xi} x_7 = 1\left(\xi + \frac{1}{2}\right) - \left(\xi - \frac{1}{2}\right) + 0(-2)\xi = 1$$

$$J_{12} = \frac{\partial \sum_{\text{nnod}} N_i(\xi, \eta = 1)\, y_i}{\partial \xi} = \frac{\partial N_3}{\partial \xi} y_3 + \frac{\partial N_4}{\partial \xi} y_4 + \frac{\partial N_7}{\partial \xi} y_7 = (d+1)(-2\xi) + 1\left(\xi + \frac{1}{2}\right) + 1\left(\xi - \frac{1}{2}\right) = -2d\xi$$

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$$\left.\begin{aligned} dx &= \frac{\partial x}{\partial \xi} d\xi = J_{11}(\xi, \eta = 1) d\xi \\ dy &= \frac{\partial y}{\partial \xi} d\xi = J_{12}(\xi, \eta = 1) d\xi \end{aligned}\right\} \rightarrow \quad \begin{Bmatrix} r_{xi} \\ r_{yi} \end{Bmatrix} = \begin{Bmatrix} \int_{-1}^{1} N_i \left(\tau J_{11} - \sigma J_{12}\right) t\, d\xi \\ \int_{-1}^{1} N_i \left(\sigma J_{11} + \tau J_{12}\right) t\, d\xi \end{Bmatrix}$$

$$\left.\begin{aligned} \tau &= N_k \tau_k \\ \sigma &= N_k \sigma_k \end{aligned}\right\} \rightarrow \quad \begin{Bmatrix} r_{xi} \\ r_{yi} \end{Bmatrix} = \begin{Bmatrix} \int_{-1}^{1} N_i \left(\tau J_{11} - \sigma J_{12}\right) t\, d\xi \\ \int_{-1}^{1} N_i \left(\sigma J_{11} + \tau J_{12}\right) t\, d\xi \end{Bmatrix} = \begin{Bmatrix} \int_{-1}^{1} N_i \left(\sum_k N_k \tau_k J_{11} - \sum_k N_k \sigma_k J_{12} \right) t\, d\xi \\ \int_{-1}^{1} N_i \left(\sum_k N_k \sigma_k J_{11} + \sum_k N_k \tau_k J_{12} \right) t\, d\xi \end{Bmatrix}$$

Interpolación de la carga
(en nuestro caso usamos
directamente la expresión analítica)

$$r_{3x} = t \int_{-1}^{1} N_3(1,\xi)\ \tau \xi J_{11} d\xi = t \int_{-1}^{1} \frac{1}{2}\left(\xi^2 + \xi\right) \tau(\xi)\, d\xi = \frac{tT}{12}$$

$$r_{4x} = t \int_{-1}^{1} N_3(1,\xi)\ \tau \xi J_{11} d\xi = t \int_{-1}^{1} \frac{1}{2}\left(\xi^2 + \xi\right) \tau(\xi)\, d\xi = \frac{tT}{12}$$

$$r_{7x} = t \int_{-1}^{1} N_3(1,\xi)\ \tau \xi J_{12} d\xi = t \int_{-1}^{1} \frac{1}{2}\left(\xi^2 + \xi\right) \tau(\xi)\, d\xi = \frac{5tT}{6}$$

$$r_{3y} = t \int_{-1}^{1} N_3(1,\xi)\ \tau \xi J_{12} d\xi = t \int_{-1}^{1} \frac{1}{2}\left(\xi^2 + \xi\right) (-2d\xi) \tau(\xi)\, d\xi = -\frac{1}{6} dtT$$

$$r_{4y} = t \int_{-1}^{1} N_3(1,\xi)\ \tau \xi J_{12} d\xi = t \int_{-1}^{1} \frac{1}{2}\left(\xi^2 + \xi\right) (-2d\xi) \tau(\xi)\, d\xi = \frac{dtT}{6}$$

$$r_{7y} = t \int_{-1}^{1} N_3(1,\xi)\ \tau \xi J_{12} d\xi = t \int_{-1}^{1} \frac{1}{2}\left(\xi^2 + \xi\right) (-2d\xi) \tau(\xi)\, d\xi = 0$$

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<u>Otros ejercicios</u> (para elementos Q4, Q8 o Q9)

-Repetir el anterior con σ≠0

-Calcular con el método general y comparar con un cálculo directo.