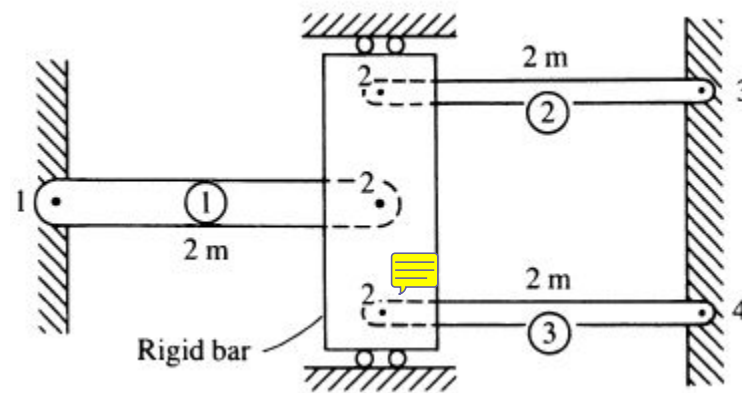


Carga térmica

Ejemplo



① $E = 70 \text{ GPa}$
 $A = 12 \times 10^{-4} \text{ m}^2$
 $\alpha = 23 \times 10^{-6} \text{ (mm/mm)/}^\circ\text{C}$

② $E = 100 \text{ GPa}$
 $A = 6 \times 10^{-4} \text{ m}^2$
 $\alpha = 20 \times 10^{-6} \text{ (mm/mm)/}^\circ\text{C}$

$$\underline{k}^{(1)} = \frac{(12 \times 10^{-4})(70 \times 10^6)}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 42,000 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \frac{\text{kN}}{\text{m}}$$

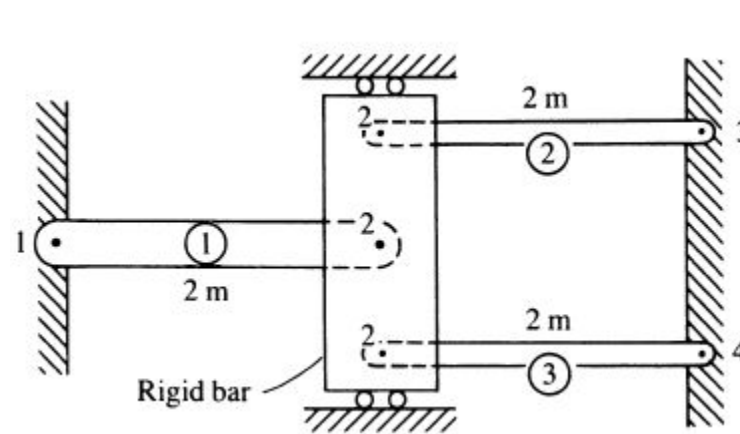
$$\underline{k}^{(2)} = \underline{k}^{(3)} = \frac{(6 \times 10^{-4})(100 \times 10^6)}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 30,000 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \frac{\text{kN}}{\text{m}}$$

$$-E\alpha TA = 19.32 \text{ kN} \quad \underline{f}^{(1)} = \begin{Bmatrix} f_{1x} \\ f_{2x} \end{Bmatrix} = \begin{Bmatrix} 19.32 \\ -19.32 \end{Bmatrix} \text{ kN} \quad \underline{f}^{(2)} = \underline{f}^{(3)} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$1000 \begin{bmatrix} 42 & -42 & 0 & 0 \\ -42 & 42 + 30 + 30 & -30 & -30 \\ 0 & -30 & 30 & 0 \\ 0 & -30 & 0 & 30 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{2x} \\ d_{3x} \\ d_{4x} \end{Bmatrix} = \begin{Bmatrix} F_{1x} + 19.32 \\ -19.32 \\ F_{3x} \\ F_{4x} \end{Bmatrix}$$

Carga térmica

Ejemplo



① $E = 70 \text{ GPa}$
 $A = 12 \times 10^{-4} \text{ m}^2$
 $\alpha = 23 \times 10^{-6} (\text{mm/mm})/^{\circ}\text{C}$

② $E = 100 \text{ GPa}$
 $A = 6 \times 10^{-4} \text{ m}^2$
 $\alpha = 20 \times 10^{-6} (\text{mm/mm})/^{\circ}\text{C}$

$$d_{1x} = 0 \quad d_{3x} = 0 \quad d_{4x} = 0$$

$$1000(102)d_{2x} = -19.32$$

$$d_{2x} = -1.89 \times 10^{-4} \text{ m}$$

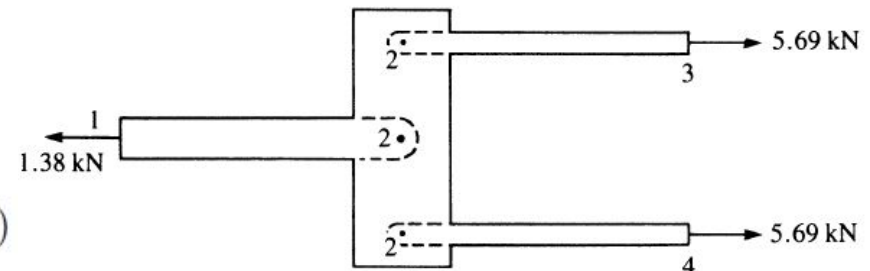
Fuerzas nodales

$$\begin{Bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \end{Bmatrix} = 1000 \begin{bmatrix} 42 & -42 & 0 & 0 \\ -42 & 102 & -30 & -30 \\ 0 & -30 & 30 & 0 \\ 0 & -30 & 0 & 30 \end{bmatrix} \begin{Bmatrix} 0 \\ -1.89 \times 10^{-4} \\ 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 19.32 \\ -19.32 \\ 0 \\ 0 \end{Bmatrix}$$

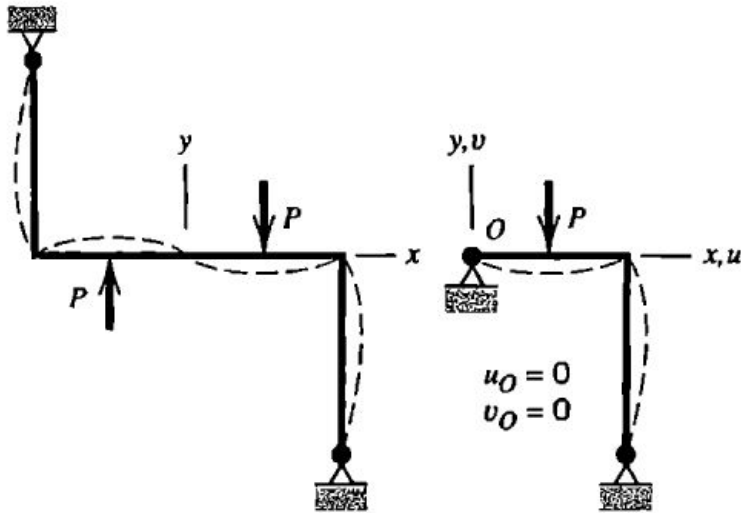
$F_{1x} = -11.38 \text{ kN}$
 $F_{2x} = 0.0 \text{ kN}$
 $F_{3x} = 5.69 \text{ kN}$
 $F_{4x} = 5.69 \text{ kN}$

$$\sigma^{(1)} = \frac{11.38}{12 \times 10^{-4}} = 9.48 \times 10^3 \text{ kN/m}^2 \quad (9.48 \text{ MPa})$$

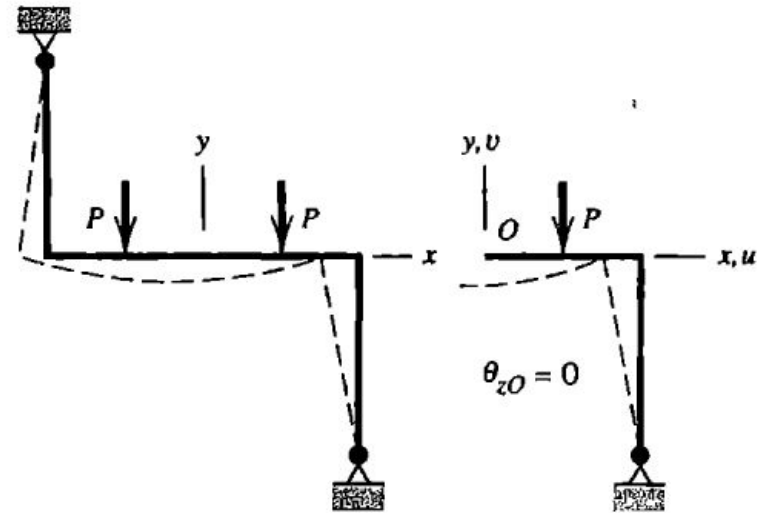
$$\sigma^{(2)} = \sigma^{(3)} = \frac{5.69}{6 \times 10^{-4}} = 9.48 \times 10^3 \text{ kN/m}^2 \quad (9.48 \text{ MPa})$$



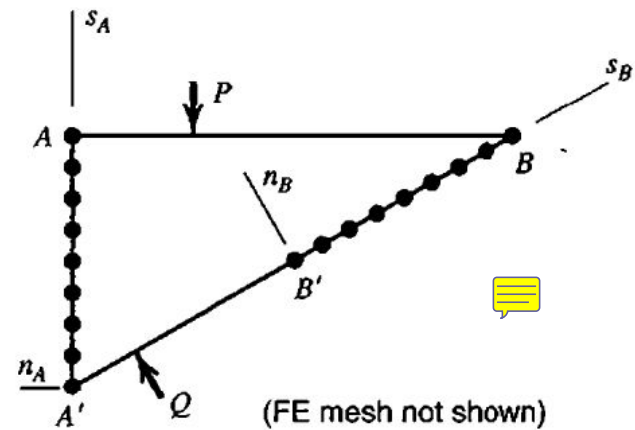
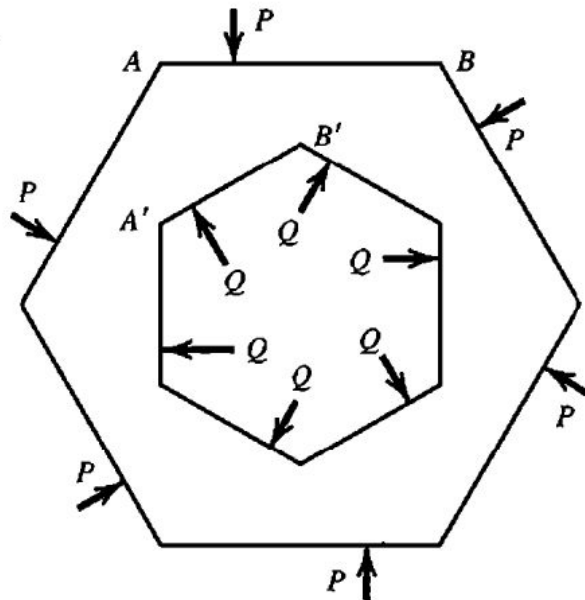
Simetría "oblicua" y circular



Skew symmetric



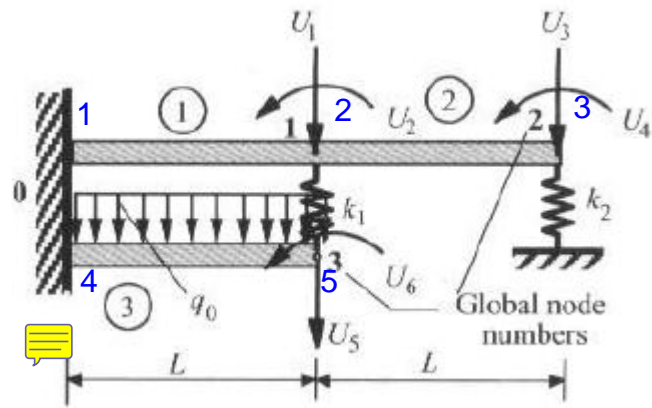
Skew antisymmetric



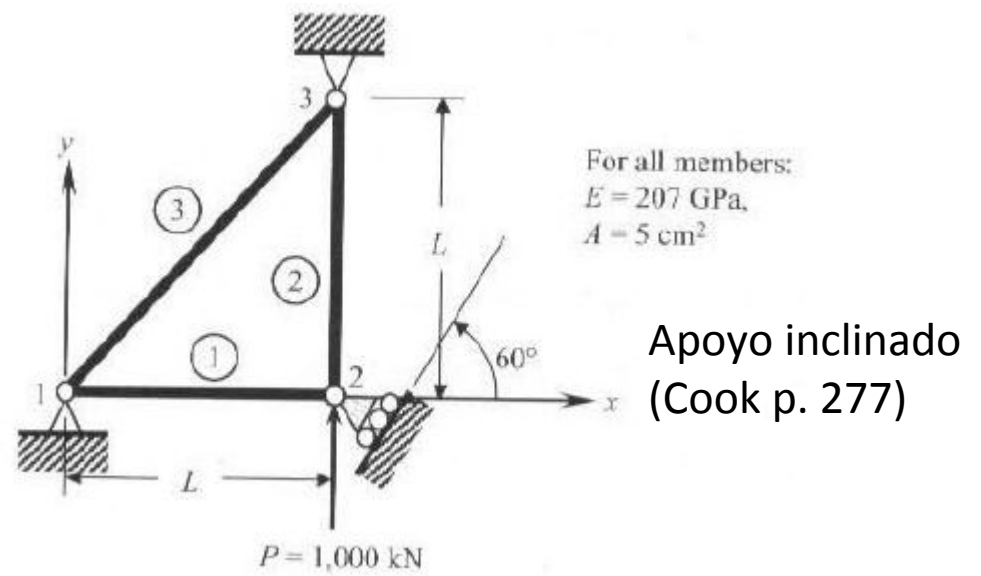
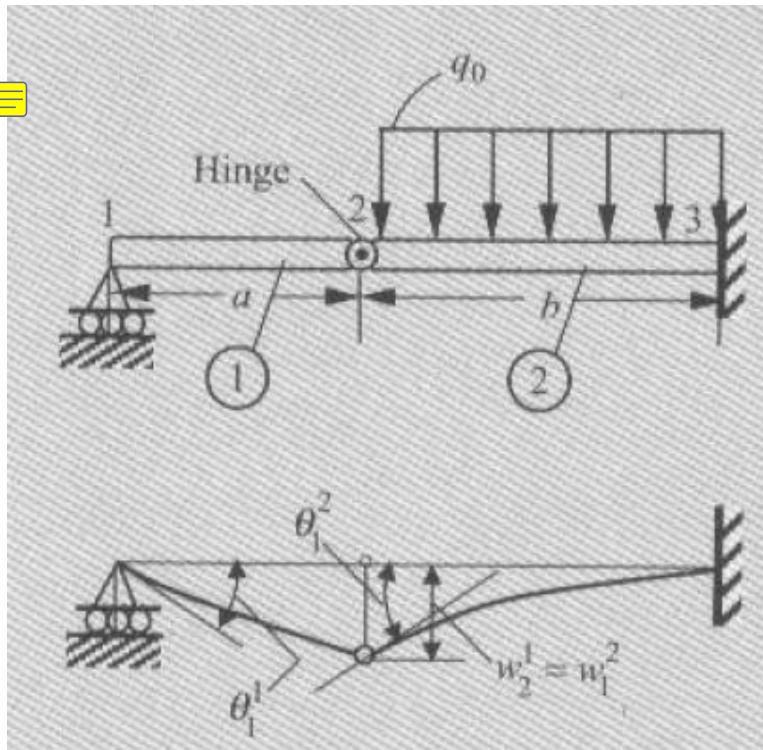
(FE mesh not shown)

Problemas varios

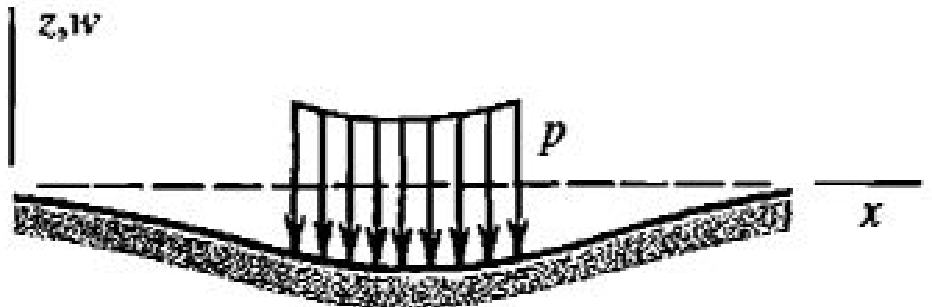
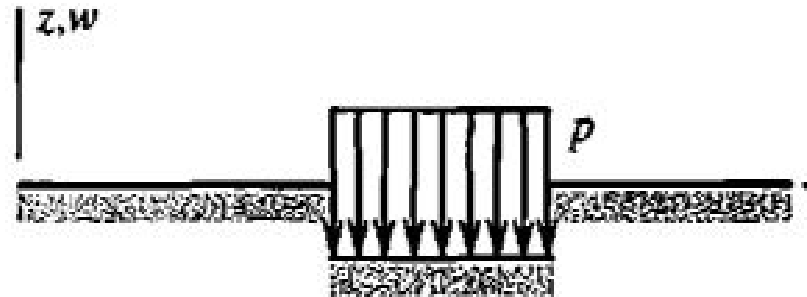
Vigas y resortes



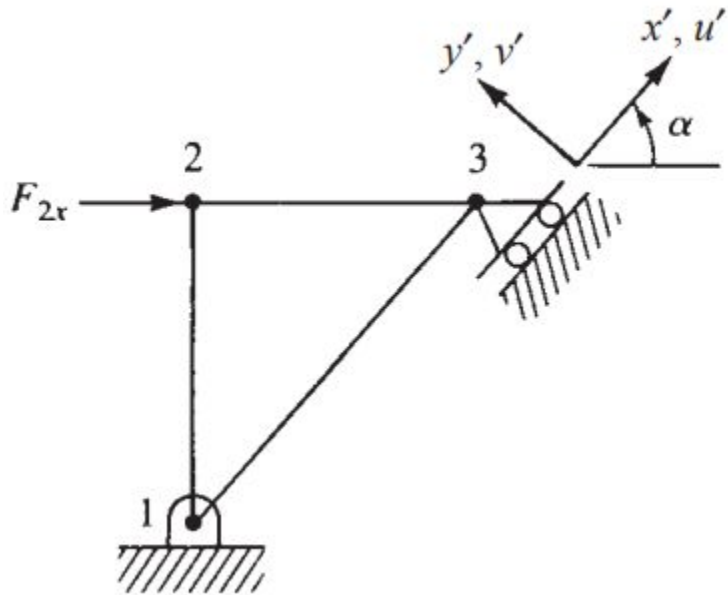
Viga con articulación interna (Logan 211)



Fundación elástica (Cook 287)



Ejemplo (apoyo inclinado)



$$\begin{Bmatrix} u'_3 \\ v'_3 \end{Bmatrix} = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{Bmatrix} u_3 \\ v_3 \end{Bmatrix}$$

$$[t_3] = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$\begin{aligned} \{d'\} &= [T_1]\{d\} \\ \{d\} &= [T_1]^T \{d'\} \end{aligned} \quad \leftarrow [T_1] = \begin{bmatrix} [I] & [0] & [0] \\ [0] & [I] & [0] \\ [0] & [0] & [t_3] \end{bmatrix}$$

$$\boxed{\{f\} = [K]\{d\}}$$

$$\boxed{[T_1]\{f\} = [T_1][K][T_1]^T \{d'\}}$$

Aplico BC y cargas

$$\begin{array}{ccc} u_1 = 0 & v_1 = 0 & v'_3 = 0 \\ F_{2x} & F_{2y} = 0 & F'_{3x} = 0 \end{array}$$

$$\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} = \begin{bmatrix} [I] & [0] & [0] \\ [0] & [I] & [0] \\ [0] & [0] & [t_3]^T \end{bmatrix} \begin{Bmatrix} u'_1 \\ v'_1 \\ u'_2 \\ v'_2 \\ u'_3 \\ v'_3 \end{Bmatrix}$$