Problemas axisimétricos

Repaso

$$\begin{Bmatrix}\sigma_r\\ \sigma_\theta\\ \sigma_z\\ \tau_{zr}\end{Bmatrix} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1 & f & f & 0\\ & 1 & f & 0\\ & & 1 & 0\\ \text{symm.} & & & g\end{bmatrix}\left(\begin{Bmatrix}\varepsilon_r\\ \varepsilon_\theta\\ \varepsilon_z\\ \gamma_{zr}\end{Bmatrix} - \begin{Bmatrix}\alpha T\\ \alpha T\\ \alpha T\\ 0\end{Bmatrix}\right)$$

$$f = \frac{\nu}{1-\nu} \quad \text{and} \quad g = \frac{1-2\nu}{2(1-\nu)}$$

$$\begin{Bmatrix}u\\ w\end{Bmatrix} = \underbrace{\begin{bmatrix}N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0\\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4\end{bmatrix}}_{[\mathbf{N}]}\begin{Bmatrix}u_1\\ w_1\\ u_2\\ \vdots\\ w_4\end{Bmatrix}$$

(Q4)

$$\begin{Bmatrix}\varepsilon_r\\ \varepsilon_\theta\\ \varepsilon_z\\ \gamma_{zr}\end{Bmatrix} = [\partial]\begin{Bmatrix}u\\ w\end{Bmatrix}$$

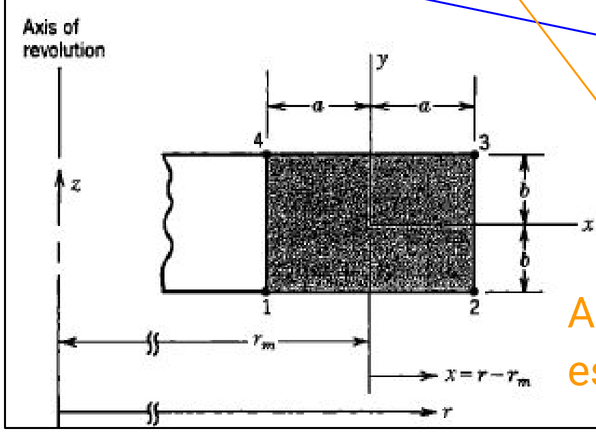
$$\varepsilon_\theta = \frac{2\pi(r+u) - 2\pi r}{2\pi r} = \frac{u}{r}$$

$$[\partial] = \begin{bmatrix}\partial/\partial r & 0\\ 1/r & 0\\ 0 & \partial/\partial z\\ \partial/\partial z & \partial/\partial r\end{bmatrix}$$

$$[\mathbf{B}] = [\partial][\mathbf{N}]$$

$$\underset{8\times 8}{[\mathbf{k}]} = \int_{-b}^{b}\int_{-a}^{a}\int_{-\pi}^{\pi} \underset{8\times 4}{[\mathbf{B}]^T}\,\underset{4\times 4}{[\mathbf{E}]}\,\underset{4\times 8}{[\mathbf{B}]}\, r\, d\theta\, dx\, dy \qquad \text{where} \qquad r = r_m + x$$

=2 π



Análogo a “t” de estados planos

Cargas de revolución en dirección radial

$$\{\mathbf{r}_e\} = \iint_{-\pi}^{\pi} [\mathbf{N}]^T \begin{Bmatrix}\rho r\omega^2\\ 0\end{Bmatrix} r\, d\theta\, dA$$

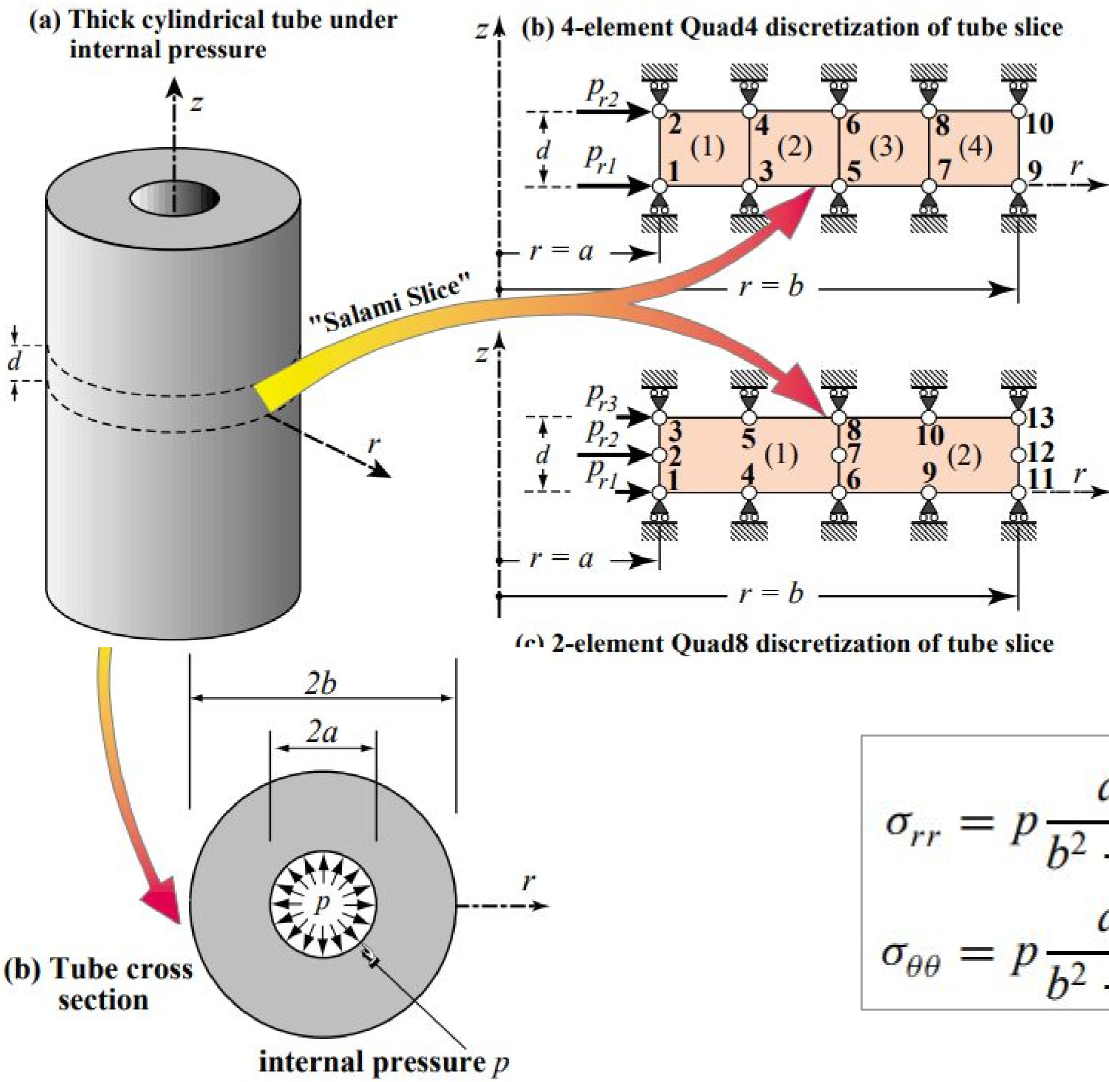
Carga axisimétrica q

$$P = 2\pi r q$$

Problemas axisimétricos

Problema 1: Cilindro hueco grueso con presión interna

Graficar tensiones y desplazamientos en función de r para la solución exacta y la discreta obtenida con elementos Q4 y Q8/9



Datos:

E=1000

nu=0.25

d=2

a=4

b=10

p=10

Solución exacta

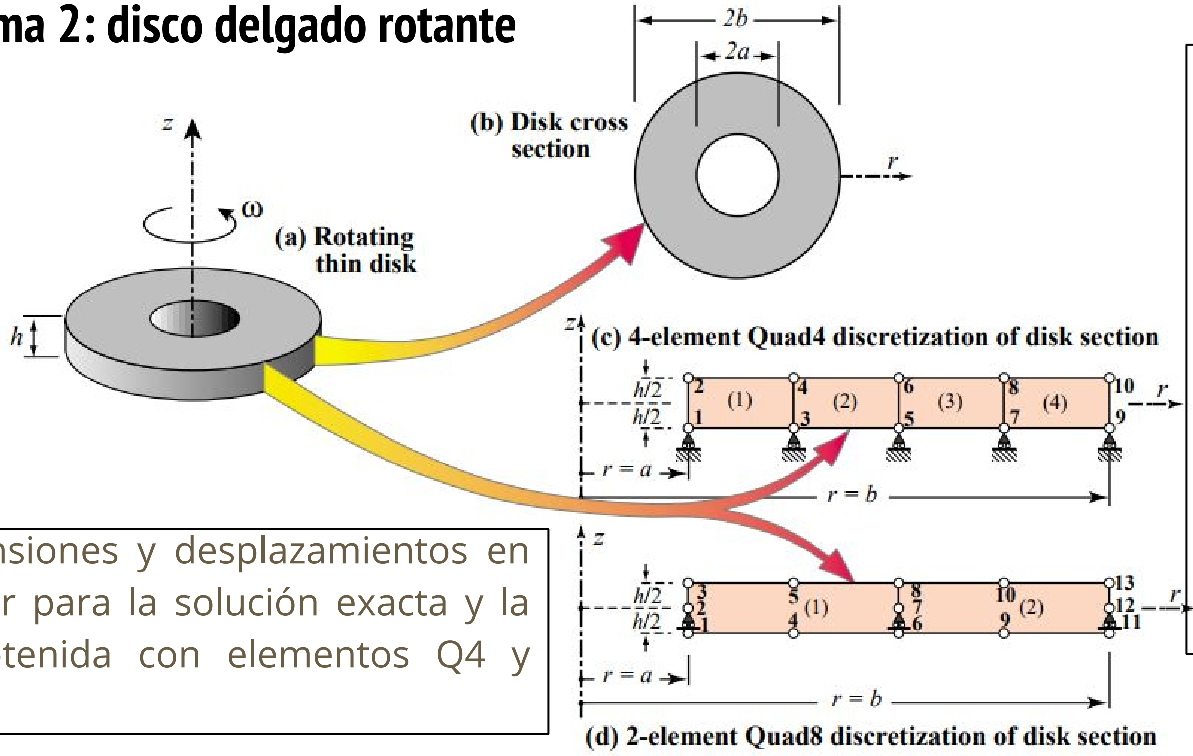
$$u_r = p\frac{a^2(1+\nu)(b^2+r^2(1-2\nu))}{E(b^2-a^2)r}$$

$$\sigma_{rr} = p\frac{a^2}{b^2-a^2}\left(1-\frac{b^2}{r^2}\right), \qquad \sigma_{zz} = p\frac{2a^2\nu}{b^2-a^2},$$

$$\sigma_{\theta\theta} = p\frac{a^2}{b^2-a^2}\left(1+\frac{b^2}{r^2}\right), \qquad \text{others zero.}$$

Problemas axisimétricos

Problema 2: disco delgado rotante



Datos:
- E=1000
- nu=1/3
- h=1
- a=4
- b=10
- p=10
- rho=3
- w=0.5

Graficar tensiones y desplazamientos en función de r para la solución exacta y la discreta obtenida con elementos Q4 y Q8/9

$$\sigma_{rr} = \rho\,\omega^2\, r\, \frac{3+\nu}{8}\left(b^2 + a^2 - \frac{a^2\, b^2}{r^2} - r^2\right),$$

$$\sigma_{\theta\theta} = \rho\,\omega^2\, r\, \frac{3+\nu}{8}\left(b^2 + a^2 + \frac{a^2\, b^2}{r^2} - \frac{(1+3\nu)r^2}{(3+\nu)}\right)$$

Solución exacta

$$u_r = \rho\,\omega^2\, \frac{a^2(3+\nu)\big(r^2(1-\nu) + b^2(1+\nu)\big) + r^2(1-\nu)\big(b^2(3+\nu) - r^2(1+\nu)\big)}{8\,E\,r},$$

$$u_z = \rho\,\omega^2\, z\, \nu\, \frac{(1-\nu-2\nu^2)\big(2r^2(1+\nu) - a^2(3+\nu) - b^2(3+\nu)\big)}{4E\,(1+3\nu)}.$$