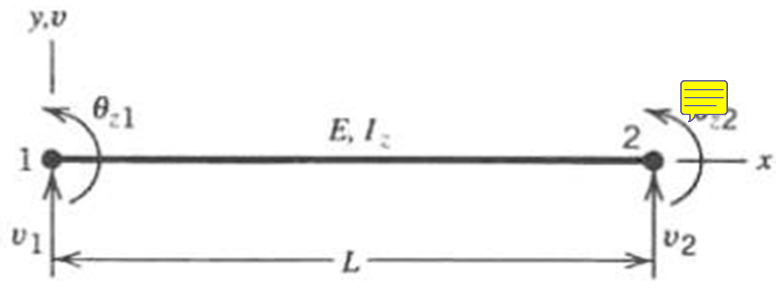
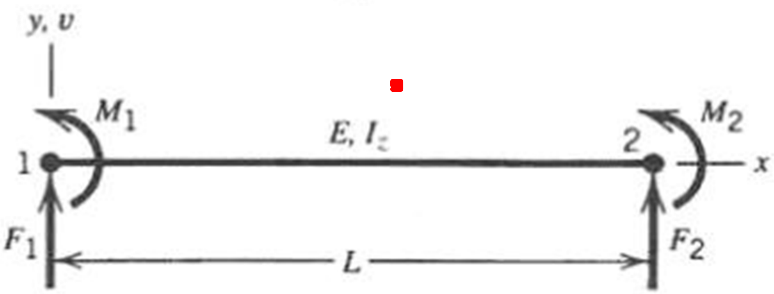
Elemento Viga
Flexión Plano Z

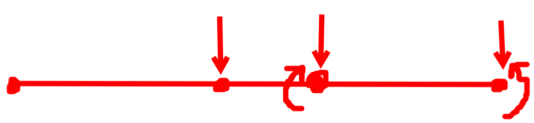


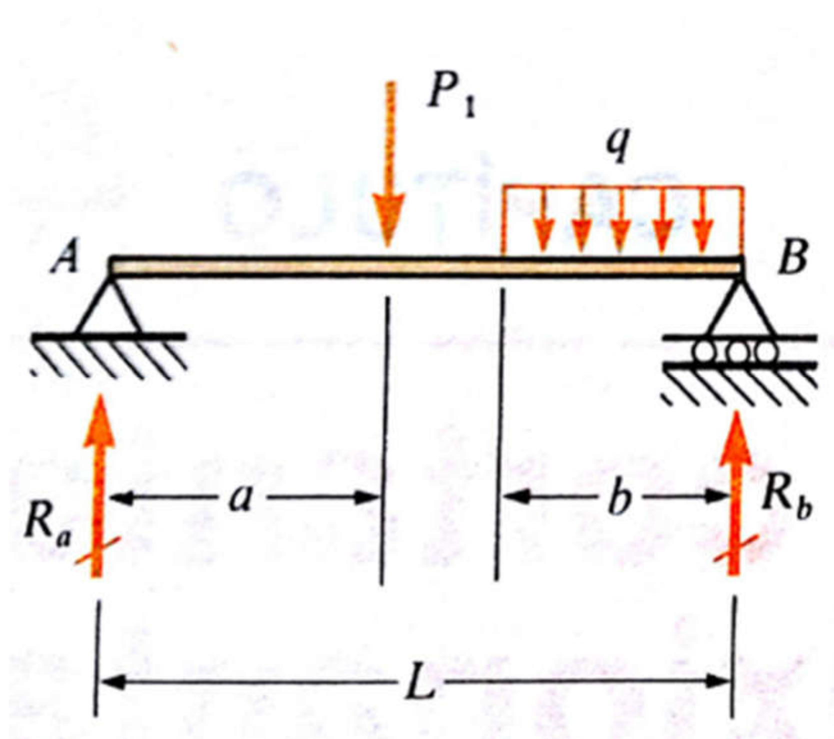
(a)



(b)

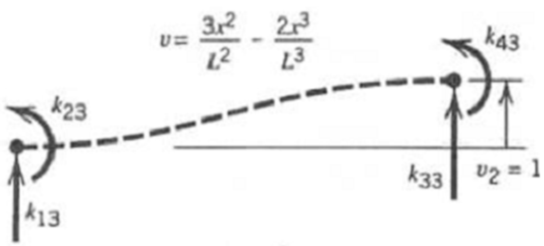
$$
\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{21} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} v_1 \\ \theta_{z1} \\ v_2 \\ \theta_{z2} \end{Bmatrix} = \begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{Bmatrix}
$$

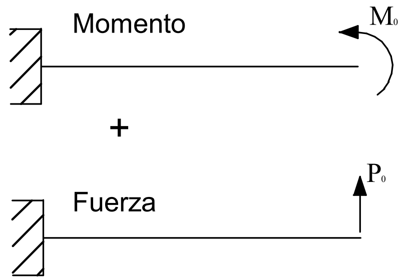




Elemento Viga Formulación

Desplazamiento Unitario





$$\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{21} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} v_1 \\ \theta_{z1} \\ v_2 \\ \theta_{z2} \end{Bmatrix} = \begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{Bmatrix}$$

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{21} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{Bmatrix} = \begin{Bmatrix} k_{13} \\ k_{23} \\ k_{33} \\ k_{43} \end{Bmatrix}$$

Desplazamiento y rotación por el momento en el extremo

$$\theta_{z2}^{1} = \frac{ML}{EI} \qquad v_2^1 = \frac{ML^2}{2EI}$$

Desplazamiento y rotación por la fuerza en el extremo

$$\theta_{z2}^{2} = \frac{PL^2}{2EI} \qquad v_2^2 = \frac{PL^3}{3EI}$$

Superponiendo ambos efectos se obtiene:

$$\theta_{z2} = \frac{PL^2}{2EI} + \frac{ML}{EI} = 0 \qquad v_2 = \frac{PL^3}{3EI} + \frac{ML^2}{2EI} = 1$$

Despejando:

$$M = k_{43} = \frac{-6EI_z}{L^2} \qquad P = k_{33} = \frac{12EI_z}{L^3}$$

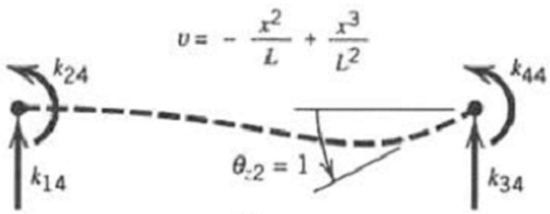
Reacciones:

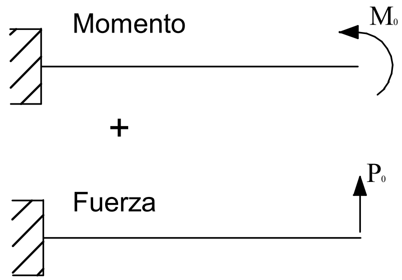
$$k_{13} = -P = -\frac{12EI_z}{L^3}$$

$$k_{23} = -PL - \frac{-6EI_z}{L^2} = -\frac{6EI_z}{L^2}$$

Elemento Viga Formulación

Giro Unitario





$$\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{21} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} v_1 \\ \theta_{z1} \\ v_2 \\ \theta_{z2} \end{Bmatrix} = \begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{Bmatrix}$$

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{21} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{Bmatrix} = \begin{Bmatrix} k_{14} \\ k_{24} \\ k_{34} \\ k_{44} \end{Bmatrix}$$

Desplazamiento y rotación por el momento en el extremo

$$\theta_{z2}^{1} = \frac{ML}{EI} \qquad v_2^1 = \frac{ML^2}{2EI}$$

Desplazamiento y rotación por la fuerza en el extremo

$$\theta_{z2}^{2} = \frac{PL^2}{2EI} \qquad v_2^2 = \frac{PL^3}{3EI}$$

Superponiendo ambos efectos se obtiene:

$$\theta_{z2} = \frac{PL^2}{2EI} + \frac{ML}{EI} = 1 \qquad v_2 = \frac{PL^3}{3EI} + \frac{ML^2}{2EI} = 0$$

Despejando:

$$M = k_{44} = \frac{4EI_z}{L} \qquad P = k_{34} = -\frac{6EI_z}{L^2}$$

Reacciones:

$$k_{14} = -P = \frac{6EI_z}{L^2}$$

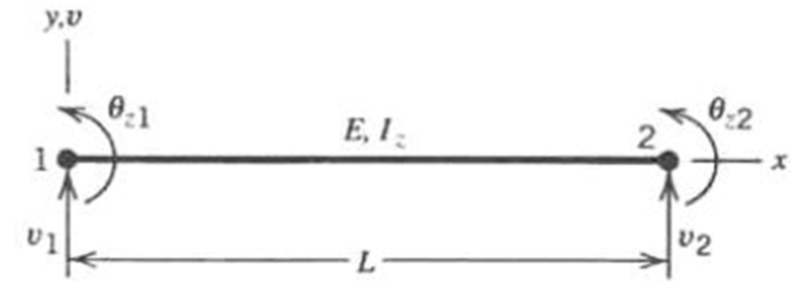
$$k_{24} = -PL - \frac{-6EI_z}{L^2} = \frac{2EI_z}{L}$$

Elemento Viga

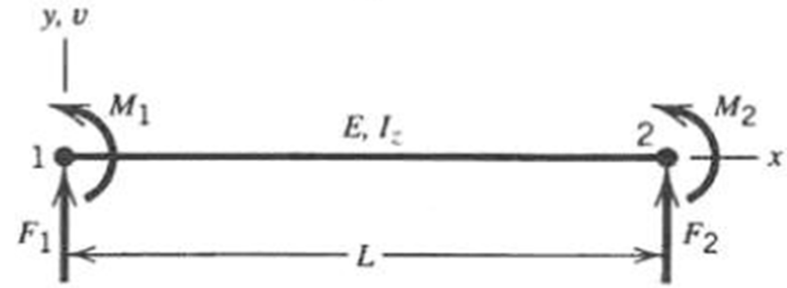
Flexión Plano Z

$$[K]=\begin{bmatrix} Y_1 & Y_2 & -Y_1 & Y_2 \\ Y_2 & Y_3 & -Y_2 & Y_4 \\ -Y_1 & -Y_2 & Y_1 & -Y_2 \\ Y_2 & Y_4 & -Y_2 & Y_3 \end{bmatrix} \quad \begin{matrix} v_1 \\ \theta_{z1} \\ v_2 \\ \theta_{z2} \end{matrix} \quad \begin{matrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{matrix}$$

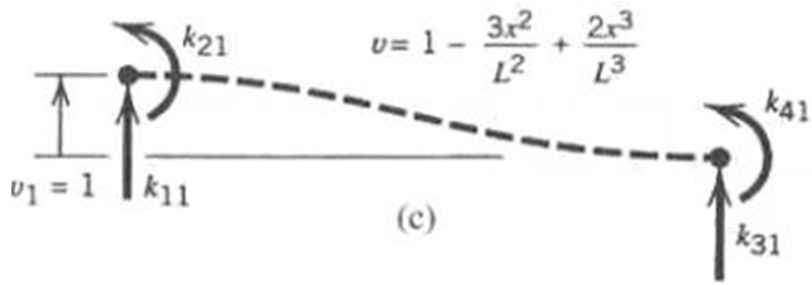
$$Y_1 = 12\frac{EI_z}{L^3};\quad Y_2 = 6\frac{EI_z}{L^2};\quad Y_3 = 4\frac{EI_z}{L};\quad Y_4 = 2\frac{EI_z}{L}$$



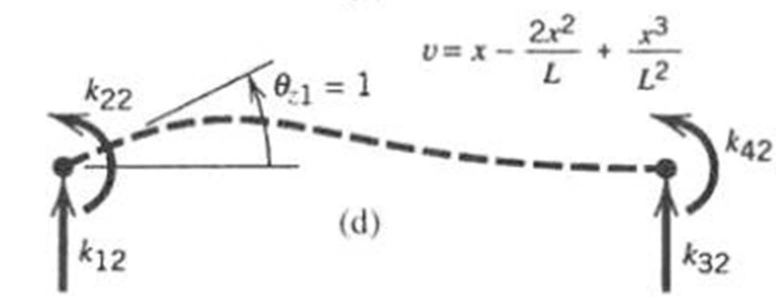
(a)



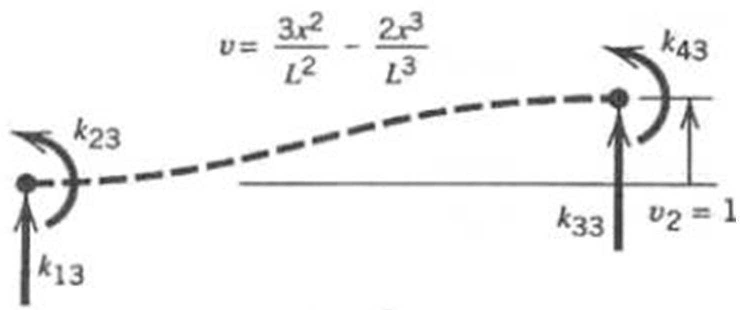
(b)



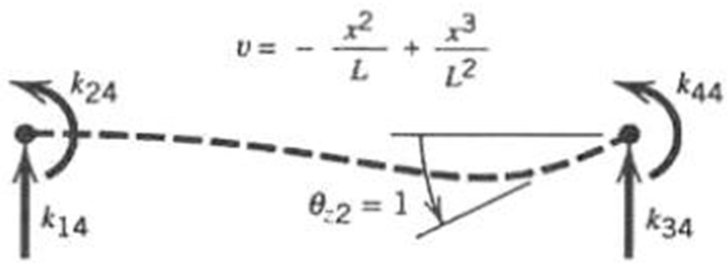
(c)



(d)



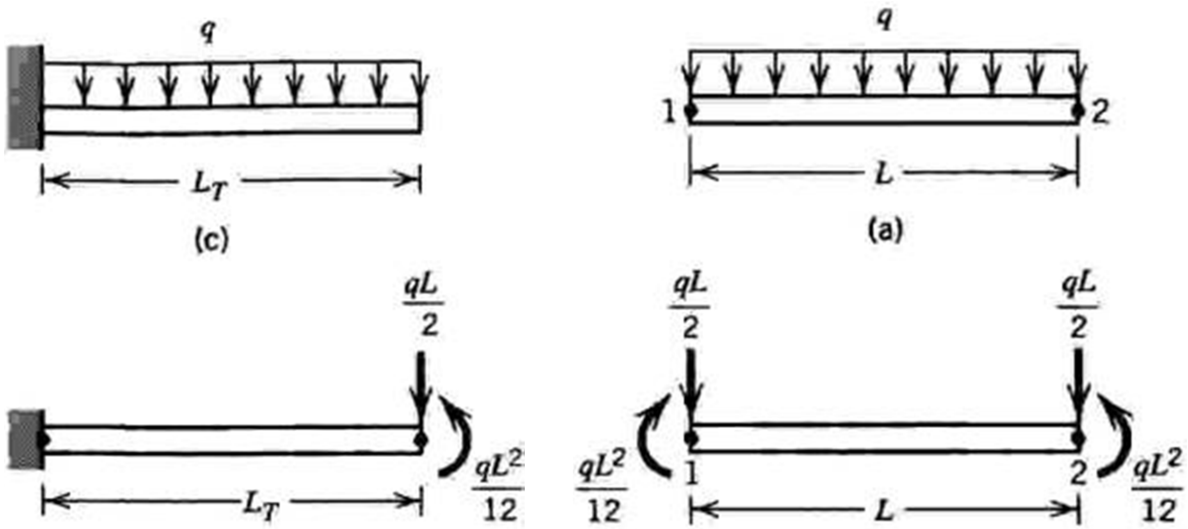
(e)



(f)

Elemento Viga

Cargas Nodales $F_2 = \dfrac{qL}{2}$; $M_2 = \dfrac{qL^2}{12}$



Condiciones de borde $v_1 = 0; \quad \theta_{z1} = 0$

$$\begin{bmatrix} Y_1 & Y_2 & -Y_1 & Y_2 \\ Y_2 & Y_3 & -Y_2 & Y_4 \\ -Y_1 & -Y_2 & Y_1 & -Y_2 \\ Y_2 & Y_4 & -Y_2 & Y_3 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ v_2 \\ \theta_{z2} \end{Bmatrix} = \begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{Bmatrix}$$

Resolución

$$\begin{bmatrix} -Y_1 & Y_2 \\ -Y_2 & Y_4 \\ Y_1 & -Y_2 \\ -Y_2 & Y_3 \end{bmatrix} \begin{Bmatrix} v_2 \\ \theta_{z2} \end{Bmatrix} = \begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \end{Bmatrix}$$

$$\begin{bmatrix} -Y_1 & Y_2 \\ -Y_2 & Y_4 \end{bmatrix} \begin{Bmatrix} v_2 \\ \theta_{z2} \end{Bmatrix} = \begin{Bmatrix} F_1 \\ M_1 \end{Bmatrix}$$

Reacciones de Vínculo

Indeterminado

$$\begin{bmatrix} Y_1 & -Y_2 \\ -Y_2 & Y_3 \end{bmatrix} \begin{Bmatrix} v_2 \\ \theta_{z2} \end{Bmatrix} = \begin{Bmatrix} F_2 \\ M_2 \end{Bmatrix}$$

Cargas

Determinado

$$\begin{Bmatrix} v_2 \\ \theta_{z2} \end{Bmatrix} = \left(\frac{EI}{L^3} \begin{bmatrix} 12 & -6L \\ \cdot & 4L^2 \end{bmatrix} \right)^{-1} \begin{Bmatrix} -\dfrac{qL}{2} \\ \dfrac{qL^2}{12} \end{Bmatrix}; \qquad \begin{Bmatrix} v_2 \\ \theta_{z2} \end{Bmatrix} = \begin{Bmatrix} -\dfrac{qL^4}{8EI} \\ -\dfrac{qL^3}{6EI} \end{Bmatrix}$$

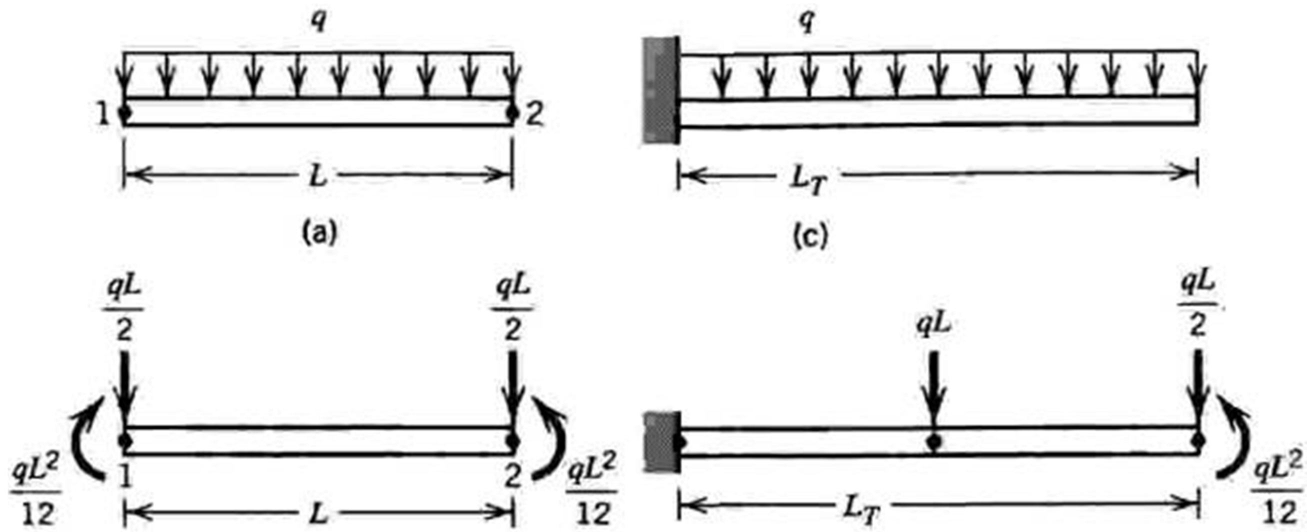
Remallado

Condiciones de borde

$$v_1 = 0; \quad \theta_{z1} = 0$$

Cargas Nodales

$$F_2 = qL \quad ; F_3 = \frac{qL}{2} \quad ; M_3 = \frac{qL^2}{12}$$



$$[K_1] = \begin{bmatrix} Y_1 & Y_2 & -Y_1 & Y_2 & 0 & 0 \\ Y_2 & Y_3 & -Y_2 & Y_4 & 0 & 0 \\ -Y_1 & -Y_2 & Y_1 & -Y_2 & 0 & 0 \\ Y_2 & Y_4 & -Y_2 & Y_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} v_1 \\ \theta_{z1} \\ v_2 \\ \theta_{z2} \\ v_3 \\ \theta_{z3} \end{matrix} \quad \begin{matrix} F_1 \\ M_1 \\ F_2 \\ M_2 \\ F_3 \\ M_3 \end{matrix}$$

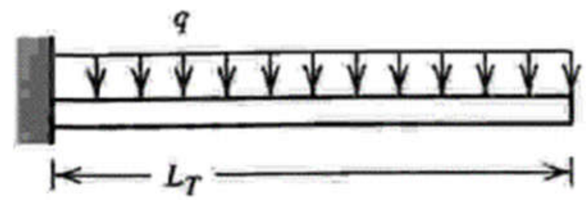
$$[K_2] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_1 & Y_2 & -Y_1 & Y_2 \\ 0 & 0 & Y_2 & Y_3 & -Y_2 & Y_4 \\ 0 & 0 & -Y_1 & -Y_2 & Y_1 & -Y_2 \\ 0 & 0 & Y_2 & Y_4 & -Y_2 & Y_3 \end{bmatrix} \quad \begin{matrix} v_1 \\ \theta_{z1} \\ v_2 \\ \theta_{z2} \\ v_3 \\ \theta_{z3} \end{matrix} \quad \begin{matrix} F_1 \\ M_1 \\ F_2 \\ M_2 \\ F_3 \\ M_3 \end{matrix}$$

$$[K] = \begin{bmatrix} Y_1 & Y_2 & -Y_1 & Y_2 & 0 & 0 \\ Y_2 & Y_3 & -Y_2 & Y_4 & 0 & 0 \\ -Y_1 & -Y_2 & 2Y_1 & 0 & -Y_1 & Y_2 \\ Y_2 & Y_4 & 0 & 2Y_3 & -Y_2 & Y_4 \\ 0 & 0 & -Y_1 & -Y_2 & Y_1 & -Y_2 \\ 0 & 0 & Y_2 & Y_4 & -Y_2 & Y_3 \end{bmatrix}$$

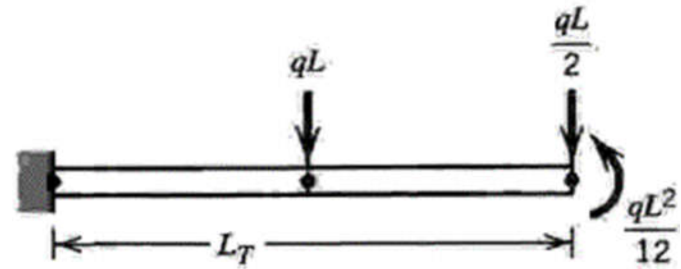
- Diagonal No Negativa
- Singular - Desplazamiento Cuerpo Rígido

Remallado

$$\begin{bmatrix} Y_1 & Y_2 & -Y_1 & Y_2 & 0 & 0\\ Y_2 & Y_3 & -Y_2 & Y_4 & 0 & 0\\ -Y_1 & -Y_2 & 2Y_1 & 0 & -Y_1 & Y_2\\ Y_2 & Y_4 & 0 & 2Y_3 & -Y_2 & Y_4\\ 0 & 0 & -Y_1 & -Y_2 & Y_1 & -Y_2\\ 0 & 0 & Y_2 & Y_4 & -Y_2 & Y_3 \end{bmatrix}\begin{Bmatrix} v_1\\ \theta_{z1}\\ v_2\\ \theta_{z2}\\ v_3\\ \theta_{z3} \end{Bmatrix}=\begin{Bmatrix} F_1\\ M_1\\ F_2\\ M_2\\ F_3\\ M_3 \end{Bmatrix}$$



(c)



$$\begin{Bmatrix} v_2\\ \theta_{z2}\\ v_3\\ \theta_{z3} \end{Bmatrix}=\left(\frac{EI}{L^3}\begin{bmatrix} 24 & 0 & -12 & 6L\\ \cdot & 8L^2 & -6L & 2L^2\\ \cdot & \cdot & 12 & -6L\\ \cdot & \cdot & \cdot & 4L^2 \end{bmatrix}\right)^{-1}\begin{Bmatrix} -qL\\ 0\\ -\dfrac{qL}{2}\\ \dfrac{qL}{12} \end{Bmatrix}$$

$$\begin{Bmatrix} v_2\\ \theta_{z2}\\ v_3\\ \theta_{z3} \end{Bmatrix}=\begin{Bmatrix} \dfrac{-17L^4q}{24EI}\\ \dfrac{-7L^3q}{6EI}\\ \dfrac{-2L^4q}{EI}\\ \dfrac{-4L^3q}{3EI} \end{Bmatrix}\quad L=\frac{L_T}{2}\rightarrow\begin{Bmatrix} v_2\\ \theta_{z2}\\ v_3\\ \theta_{z3} \end{Bmatrix}=\begin{Bmatrix} \dfrac{-17{L_T}^4q}{384EI}\\ \dfrac{-7{L_T}^3q}{48EI}\\ \dfrac{-{L_T}^4q}{8EI}\\ \dfrac{-{L_T}^3q}{6EI} \end{Bmatrix}$$

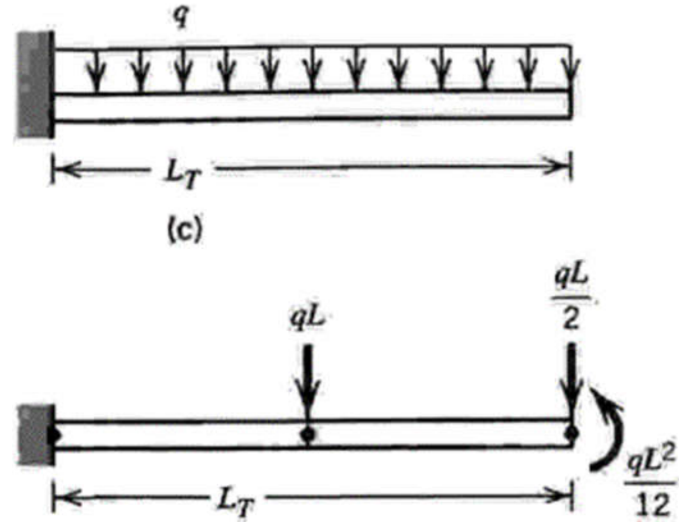
- Idéntica a la solución con 1 elemento
- Solución exacta en nodos

Desplazamiento lateral

Real

$$v(x)=\frac{qx^2}{24EI}\left(6L^2-4Lx+x^2\right)\quad\leftrightarrow\quad \underbrace{\begin{Bmatrix}1-\dfrac{3x^2}{L^2}+\dfrac{2x^3}{L^3}\\ x-\dfrac{2x^2}{L}+\dfrac{x^3}{L^2}\\ \dfrac{3x^2}{L^2}-\dfrac{2x^3}{L^3}\\ -\dfrac{x^2}{L}+\dfrac{x^3}{L^2}\end{Bmatrix}^T}_{[N]}\begin{Bmatrix}v_2\\ \theta_{z2}\\ v_3\\ \theta_{z3}\end{Bmatrix}=\tilde{v}(x)$$

Aproximado



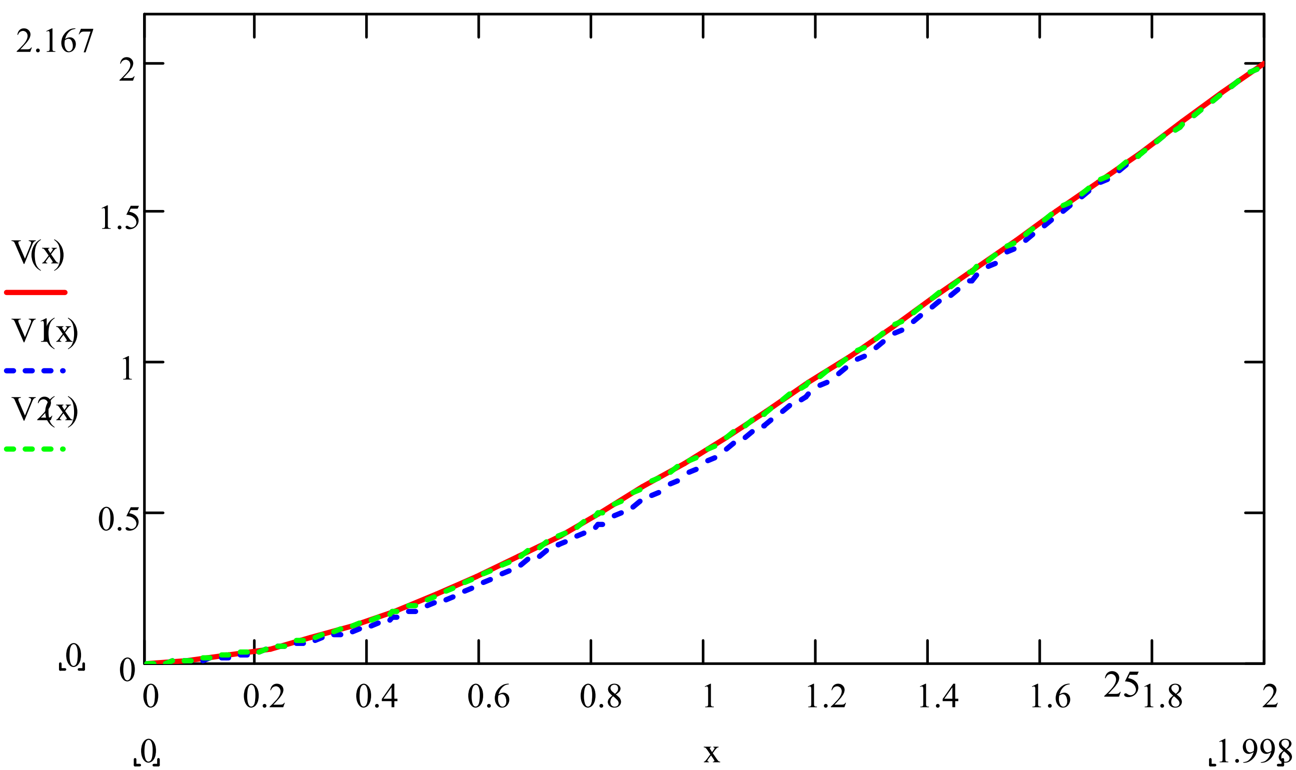
Comparación de Resultados

$$v(x)=\frac{qx^2}{24EI}\left(6L^2-4Lx+x^2\right)$$

$$\tilde{v}(x)=\left(\frac{3x^2}{L^2}-\frac{2x^3}{L^3}\right)v_3+\left(\frac{x^3}{L^2}-\frac{x^2}{L}\right)\theta_{z3}+$$

$$+\left(1-\frac{3x^2}{L^2}+\frac{2x^3}{L^3}\right)v_2+\left(x-\frac{2x^2}{L}+\frac{x^3}{L^2}\right)\theta_{z2}$$

$$\begin{Bmatrix}v_e\\ \theta_{ze}\end{Bmatrix}=\begin{Bmatrix}2\\ 1.333\end{Bmatrix}$$



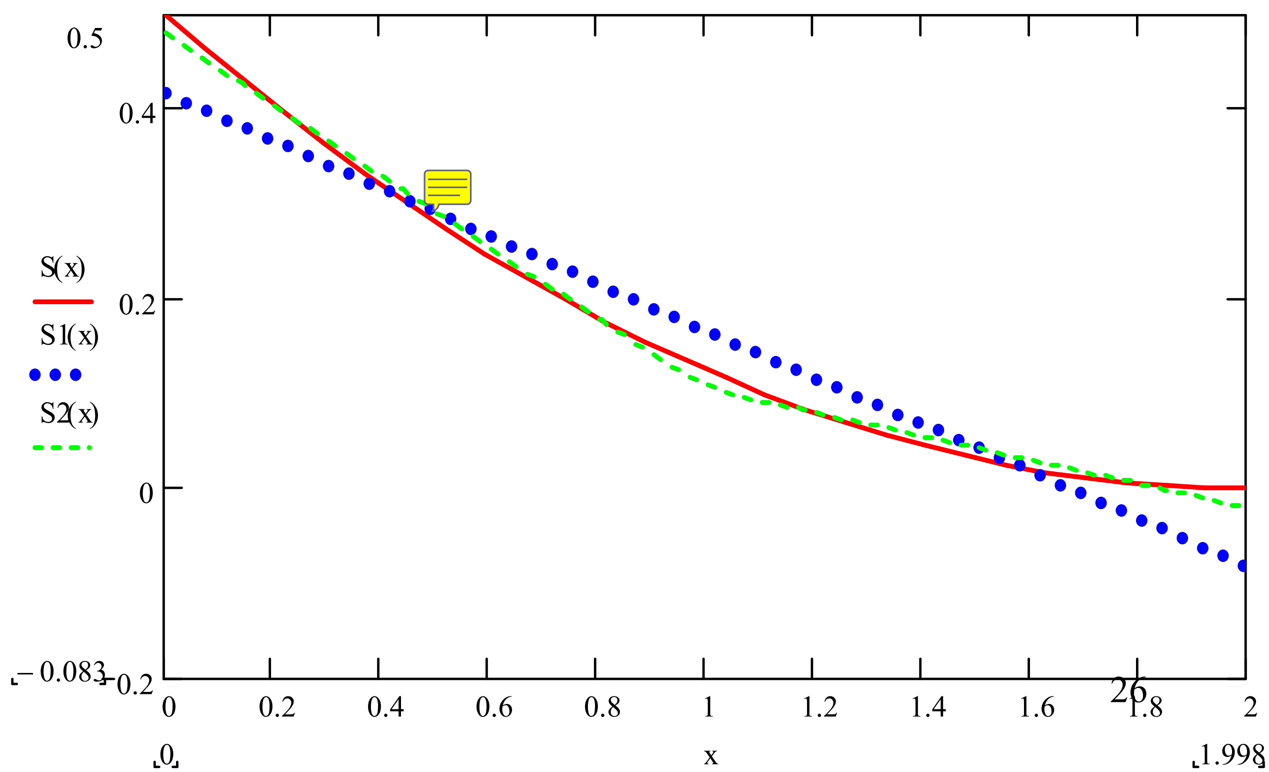
Tensiones

$$\tilde{\sigma}(x) = E\varepsilon = E\frac{\partial u}{\partial x} = E\frac{\partial\left(y\frac{\partial\tilde{v}(x)}{\partial x}\right)}{\partial x} = Ey\frac{\partial^2\tilde{v}(x)}{\partial x^2}$$

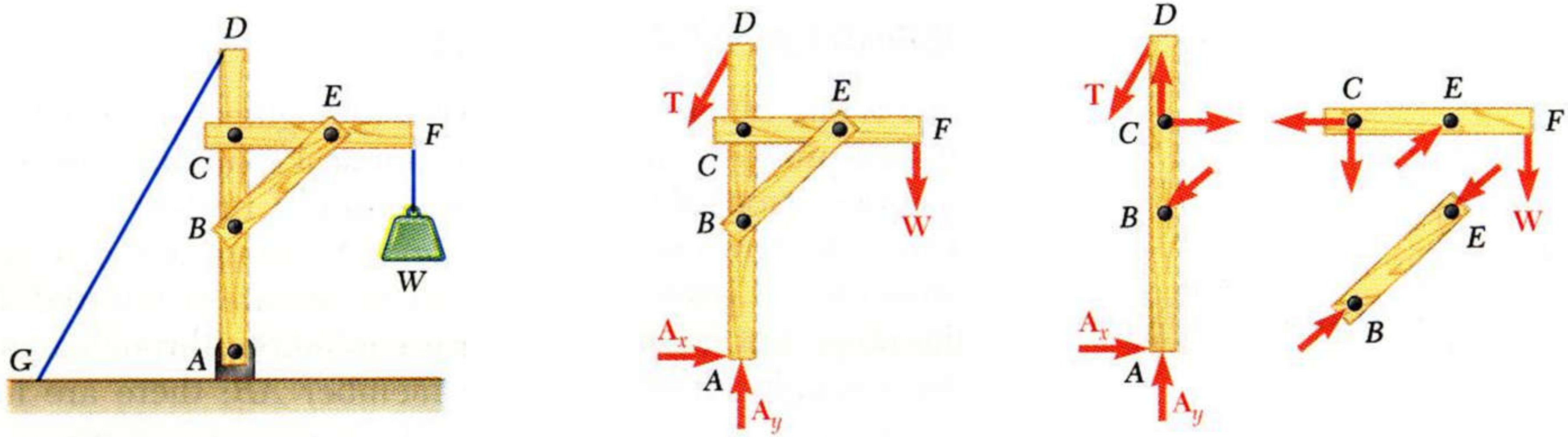
$$\tilde{M}(x) = \int_{-b/2}^{b/2}\int_{-h/2}^{h/2}\tilde{\sigma}(x)\,y\,dy\,dz = \int_{-b/2}^{b/2}\int_{-h/2}^{h/2}E\frac{\partial^2\tilde{v}(x)}{\partial x^2}y^2\,dy\,dz = E\underbrace{\int_{-b/2}^{b/2}\int_{-h/2}^{h/2}y^2\,dy\,dz}_{I_z}\frac{\partial^2\tilde{v}(x)}{\partial x^2} = EI_z\frac{\partial^2\tilde{v}(x)}{\partial x^2}$$

$$\frac{\partial^2\tilde{v}(x)}{\partial x^2} = \frac{\tilde{M}(x)}{EI_z} = \frac{\tilde{\sigma}(x)}{Ey} \Rightarrow \tilde{\sigma}(x) \propto \tilde{M}(x) \propto \frac{\partial^2\tilde{v}(x)}{\partial x^2}$$

$$\frac{\tilde{M}(x)}{EI_z} = \frac{\tilde{\sigma}(x)}{Ey} = \underbrace{\begin{Bmatrix} -\frac{6}{L^2}+\frac{12x}{L^3} \\ -\frac{4}{L}+\frac{6x}{L^2} \\ \frac{6}{L^2}-\frac{12x}{L^3} \\ -\frac{2}{L}+\frac{6x}{L^2} \end{Bmatrix}^T}_{[\partial][N]=[B]} \begin{Bmatrix} v_2 \\ \theta_{z2} \\ v_3 \\ \theta_{z3} \end{Bmatrix}$$



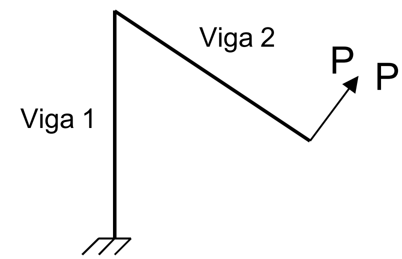
Elemento Barra y Viga - Usos



Elemento Viga

Ejemplo

$$X=\frac{AE}{L};\ Y_1=12\frac{EI_z}{L^3};\quad Y_2=6\frac{EI_z}{L^2};\quad Y_3=4\frac{EI_z}{L};\quad Y_4=2\frac{EI_z}{L}$$



$$[K']=\begin{bmatrix} X & 0 & 0 & -X & 0 & 0 \\ 0 & Y_1 & Y_2 & 0 & -Y_1 & Y_2 \\ 0 & Y_2 & Y_3 & 0 & -Y_2 & Y_4 \\ -X & 0 & 0 & X & 0 & 0 \\ 0 & -Y_1 & -Y_2 & 0 & Y_1 & -Y_2 \\ 0 & Y_2 & Y_4 & 0 & -Y_2 & Y_3 \end{bmatrix}$$

$$[K]=[T]^T[K'][T]$$

$$[T]=\begin{bmatrix} c\beta & s\beta & 0 & & 0 & 0 \\ -s\beta & c\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c\beta & s\beta & 0 \\ 0 & 0 & 0 & -s\beta & c\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Para la Viga 1

$$[K_1]=\begin{bmatrix} Y_1 & 0 & -Y_2 & -Y_1 & 0 & -Y_2 \\ 0 & X & 0 & 0 & -X & 0 \\ -Y_2 & 0 & Y_3 & Y_2 & 0 & Y_4 \\ -Y_1 & 0 & Y_2 & Y_1 & 0 & Y_2 \\ 0 & -X & 0 & 0 & X & 0 \\ -Y_2 & 0 & Y_4 & Y_2 & 0 & Y_3 \end{bmatrix}$$

Para la Viga 2

$$[K_2]=\begin{bmatrix} c^2X+s^2Y_1 & csX-csY_1 & -sY_2 & -c^2X-s^2Y_1 & -csX+csY_1 & -sY_2 \\ csX-csY_1 & s^2X+c^2Y_1 & cY_2 & -csX+csY_1 & -s^2X-c^2Y_1 & cY_2 \\ -sY_2 & cY_2 & Y_3 & sY_2 & -cY_2 & Y_4 \\ -c^2X-s^2Y & -csX+csY_1 & sY_2 & c^2X+s^2Y & csX-csY_1 & sY_2 \\ -csX+csY_1 & -s^2X-c^2Y_1 & -cY_2 & csX-csY_1 & s^2X+c^2Y_1 & -cY_2 \\ -sY_2 & cY_2 & Y_4 & sY_2 & -cY_2 & Y_3 \end{bmatrix}$$

$$[K]=\begin{bmatrix} Y_1 & 0 & -Y_2 & -Y_1 & 0 & -Y_2 & 0 & 0 & 0 \\ 0 & X & 0 & 0 & -X & 0 & 0 & 0 & 0 \\ -Y_2 & 0 & Y_3 & Y_2 & 0 & Y_4 & 0 & 0 & 0 \\ -Y_1 & 0 & Y_2 & c^2X+s^2Y_1+Y & csX-csY_1 & -sY_2+Y_2 & -c^2X-s^2Y_1 & -csX+csY & -sY_2 \\ 0 & -X & 0 & csX-csY_1 & s^2X+c^2Y_1+X & cY_2 & -csX+csY_1 & -s^2X-c^2Y_1 & cY_2 \\ -Y_2 & 0 & Y_4 & -sY_2+Y_2 & cY_2 & Y_3+Y_3 & sY_2 & -cY_2 & Y_4 \\ 0 & 0 & 0 & -c^2X-s^2Y & -csX+csY & sY_2 & c^2X+s^2Y_1 & csX-csY_1 & sY_2 \\ 0 & 0 & 0 & -csX+csY_1 & -s^2X-c^2Y_1 & -cY_2 & csX-csY_1 & s^2X+c^2Y_1 & -cY_2 \\ 0 & 0 & 0 & -sY_2 & cY_2 & Y_4 & sY_2 & -cY_2 & Y_3 \end{bmatrix}$$

$$[F]=\begin{bmatrix} R_x^1 \\ R_y^1 \\ M^1 \\ 0 \\ 0 \\ 0 \\ P_x \\ P_y \\ 0 \end{bmatrix}$$

Condiciones de Borde – Desplazamiento

R_c: Cargas impuestas

D_c: Desplazamientos impuesto

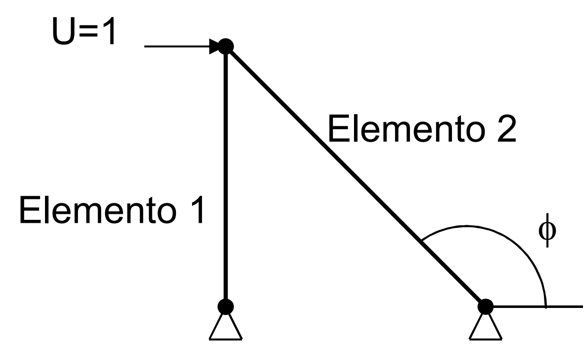
R_x: Reacciones Incógnitas

D_x: Desplazamientos Incógnitas

$$\begin{bmatrix} K_{xx} & K_{xc} \\ K_{cx} & K_{cc} \end{bmatrix} \begin{Bmatrix} D_x \\ D_c \end{Bmatrix} = \begin{Bmatrix} R_c \\ R_x \end{Bmatrix}$$

$$\begin{cases} [K_{xx}]\{D_x\} + [K_{xc}]\{D_c\} = \{R_c\} \\ \quad \rightarrow \quad \{D_x\} = [K_{xx}]^{-1}\left(\{R_c\} - [K_{xc}]\{D_c\}\right) \\ \qquad\qquad\qquad\qquad \downarrow \\ [K_{cx}]\{D_x\} + [K_{cc}]\{D_c\} = \{R_x\} \\ \quad \rightarrow \quad \{R_x\} = [K_{cc}]\{D_c\} + [K_{cx}]\{D_x\} \end{cases}$$

Condiciones de Borde – Desplazamiento



$$[K]=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & k_1 & 0 & -k_1 & 0 & 0 \\ 0 & 0 & k_2c^2\phi & k_2\cdot c\phi\cdot s\phi & -k_2c^2\phi & -k_2\cdot c\phi\cdot s\phi \\ 0 & -k_1 & k_2\cdot c\phi\cdot s\phi & k_1+k_2s^2\phi & -k_2\cdot c\phi\cdot s\phi & -k_2s^2\phi \\ 0 & 0 & -k_2c^2\phi & -k_2\cdot c\phi\cdot s\phi & k_2c^2\phi & k_2\cdot c\phi\cdot s\phi \\ 0 & 0 & -k_2\cdot c\phi\cdot s\phi & -k_2s^2\phi & k_2\cdot c\phi\cdot s\phi & k_2s^2\phi \end{bmatrix};\ \{U\}=\begin{bmatrix} 0 \\ 0 \\ 1 \\ v_2 \\ 0 \\ 0 \end{bmatrix}$$

Descomposición

$$\begin{bmatrix} K_{xx} & K_{xc} \\ & \boxed{K_{cc}} \end{bmatrix}=\begin{bmatrix} \begin{bmatrix} k_1+k_2s^2\phi \end{bmatrix} & \begin{bmatrix} 0 & -k_1 & k_2\cdot c\phi\cdot s\phi & -k_2\cdot c\phi\cdot s\phi & -k_2s^2\phi \end{bmatrix} \\ \begin{bmatrix} 0 \\ -k_1 \\ k_2\cdot c\phi\cdot s\phi \\ -k_2\cdot c\phi\cdot s\phi \\ -k_2s^2\phi \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & k_1 & 0 & 0 & 0 \\ 0 & 0 & k_2c^2\phi & -k_2c^2\phi & -k_2\cdot c\phi\cdot s\phi \\ 0 & 0 & -k_2c^2\phi & k_2c^2\phi & k_2\cdot c\phi\cdot s\phi \\ 0 & 0 & -k_2\cdot c\phi\cdot s\phi & k_2\cdot c\phi\cdot s\phi & k_2s^2\phi \end{bmatrix} \end{bmatrix};\ \begin{Bmatrix} D_c \\ D_x \end{Bmatrix}=\begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \\ [v_2] \end{bmatrix};\ \begin{Bmatrix} R_x \\ R_c \end{Bmatrix}=\begin{bmatrix} \begin{bmatrix} R_{x1} \\ R_{y1} \\ R_{x2} \\ R_{x3} \\ R_{y3} \end{bmatrix} \\ [0] \end{bmatrix}$$

$$\{D_x\}=\left[k_1+k_2s^2\phi\right]^{-1}\left([0]-\begin{bmatrix} 0 & -k_1 & k_2\cdot c\phi\cdot s\phi & -k_2\cdot c\phi\cdot s\phi & -k_2s^2\phi \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}\right)=\frac{k_2\cdot c\phi\cdot s\phi}{\left[k_1+k_2s^2\phi\right]}$$