

Cálculo de Fuerzas

Vector de cargas
nodales:

$$\{r_e\} = \int_{V_e} [N]^T [F] dV_e + \int_{S_e} [N]^T [\Phi] dS_e + \int_{V_e} [B]^T [E] [\varepsilon_0] dV_e - \int_{V_e} [B]^T [\sigma_0] dV_e$$

Cálculo de Fuerzas de Volumen

$$\{r_e\}_F = \int_{\Omega} [N]^T [N] dV \{f\}$$

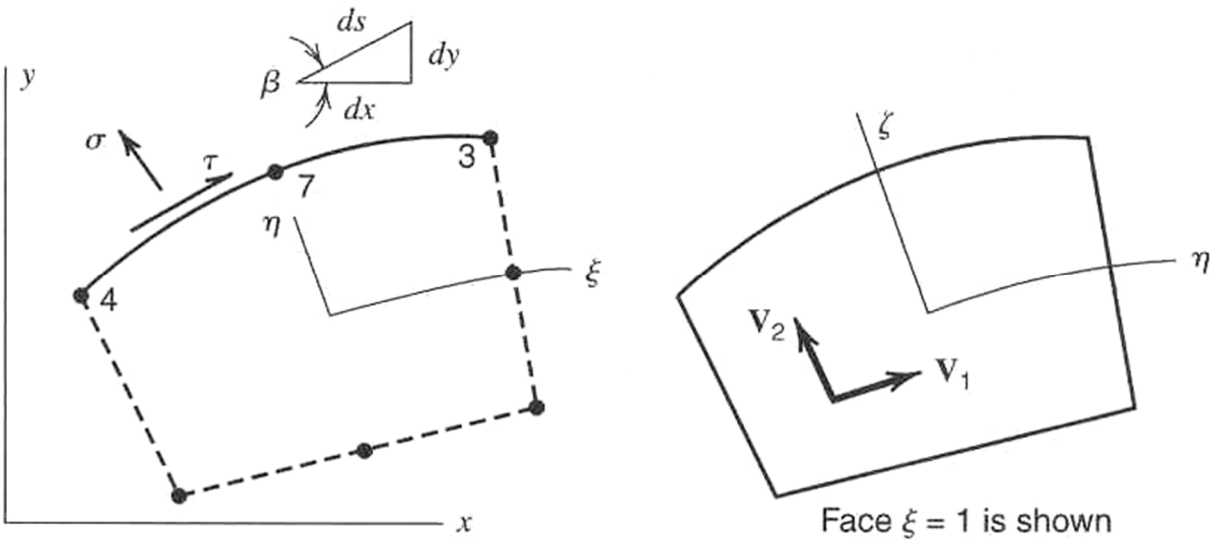
$$\{r_e\}_F = \int_{-1}^1 \int_{-1}^1 \begin{bmatrix} N_1 & 0 & \dots & 0 \\ 0 & N_1 & \dots & N_n \end{bmatrix}^T \begin{bmatrix} N_1 & 0 & \dots & 0 \\ 0 & N_1 & \dots & N_n \end{bmatrix} |J| t d\eta d\xi \begin{Bmatrix} f_{1_x} \\ f_{1_y} \\ \vdots \\ f_{n_y} \end{Bmatrix}$$

Cálculo de Fuerzas de Superficie

Las direcciones de los versores isoparamétricos depende de la numeración de nodos.

$$\{r_e\} = \int_{4-7-3} [N(\xi,\eta=1)]^T \begin{Bmatrix} \Phi_x \\ \Phi_y \end{Bmatrix} t ds$$

$$\begin{Bmatrix} \Phi_x \\ \Phi_y \end{Bmatrix} t ds = \begin{Bmatrix} \tau dx - \sigma dy \\ \sigma dx + \tau dy \end{Bmatrix} t$$



Los puntos de gauss en J11 y J12 estan evaluados sobre el arco!!!!!!

$$\left. \begin{aligned} dx &= \frac{\partial x}{\partial \xi} d\xi = J_{11}(\xi,\eta=1) d\xi \\ dy &= \frac{\partial y}{\partial \xi} d\xi = J_{12}(\xi,\eta=1) d\xi \end{aligned} \right\} \rightarrow \begin{Bmatrix} r_{xi} \\ r_{yi} \end{Bmatrix} = \begin{Bmatrix} \int_{-1}^1 N_i (\tau J_{11} - \sigma J_{12}) t d\xi \\ \int_{-1}^1 N_i (\sigma J_{11} + \tau J_{12}) t d\xi \end{Bmatrix}$$

$$\left. \begin{aligned} \tau &= N_k \tau_k \\ \sigma &= N_k \sigma_k \end{aligned} \right\} \rightarrow \begin{Bmatrix} r_{xi} \\ r_{yi} \end{Bmatrix} = \begin{Bmatrix} \int_{-1}^1 N_i (\tau J_{11} - \sigma J_{12}) t d\xi \\ \int_{-1}^1 N_i (\sigma J_{11} + \tau J_{12}) t d\xi \end{Bmatrix} = \begin{Bmatrix} \int_{-1}^1 N_i \left(\sum_k N_k \tau_k J_{11} - \sum_k N_k \sigma_k J_{12} \right) t d\xi \\ \int_{-1}^1 N_i \left(\sum_k N_k \sigma_k J_{11} + \sum_k N_k \tau_k J_{12} \right) t d\xi \end{Bmatrix}_{1,2}$$

Cálculo de Fuerzas

Ejemplo Q8:

$$\tau = 0; \sigma = -10; t = 1$$

$$N_2(1, \eta) = \frac{1}{4}(1 + \xi)(1 - \eta) - \frac{1}{2}(N_5 + N_6) = \frac{\eta^2 - \eta}{2}$$

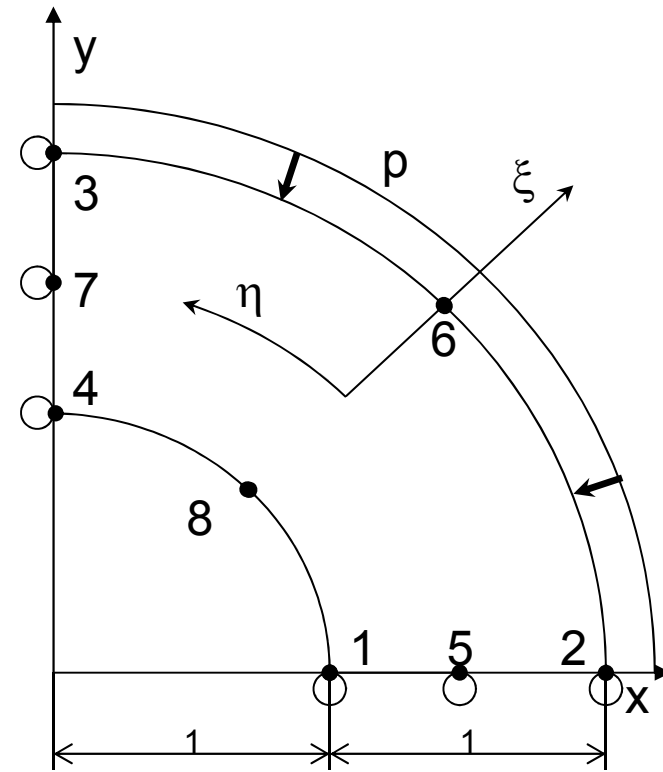
$$N_3(1, \eta) = \frac{1}{4}(1 + \xi)(1 + \eta) - \frac{1}{2}(N_7 + N_6) = \frac{\eta^2 + \eta}{2}$$

$$N_6(1, \eta) = \frac{1}{2}(1 + \xi)(1 - \eta^2) = 1 - \eta^2$$

$$dx = \frac{\partial x}{\partial \eta} d\eta = J_{21} d\eta \quad ; \quad dy = \frac{\partial y}{\partial \eta} d\eta = J_{22} d\eta$$

$$J_{21} = \frac{\partial \sum_{\text{nnode}} N_i(1, \eta) x_i}{\partial \eta} = \frac{\partial N_2}{\partial \eta} x_2 + \frac{\partial N_3}{\partial \eta} x_3 + \frac{\partial N_6}{\partial \eta} x_6 = 2\eta - 1 - \eta\sqrt{8}$$

$$J_{22} = \frac{\partial \sum_{\text{nnode}} N_i(1, \eta) y_i}{\partial \eta} = \frac{\partial N_2}{\partial \eta} y_2 + \frac{\partial N_3}{\partial \eta} y_3 + \frac{\partial N_6}{\partial \eta} y_6 = 2\eta + 1 - \eta\sqrt{8}$$



Ejemplo Q8:

$$r_{2x} = \int_{-1}^1 -N_2(1, \eta) \sigma J_{22} d\eta = \int_{-1}^1 \left(\frac{\eta^2 - \eta}{2} \right) \sigma (2\eta + 1 - \eta\sqrt{8}) d\eta = -6.095$$

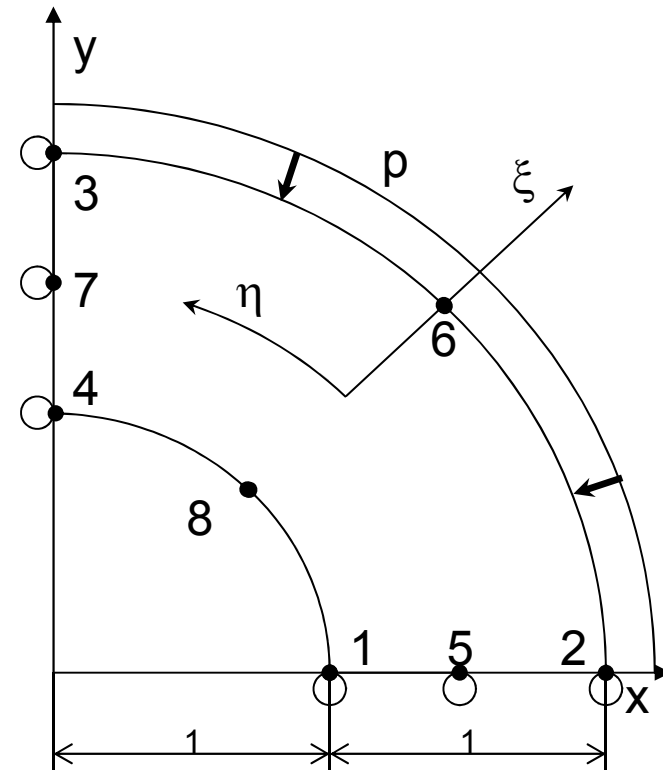
$$r_{2_y} = \int_{-1}^1 N_2(1, \eta) \sigma J_{21} d\eta = \int_{-1}^1 \left(\frac{\eta^2 - \eta}{2} \right) \sigma (2\eta - 1 - \eta\sqrt{8}) d\eta = -0.572$$

$$r_{6x} = \int_{-1}^1 -N_6(1, \eta) \sigma J_{22} d\eta = \int_{-1}^1 (1 - \eta^2) \sigma (2\eta + 1 - \eta\sqrt{8}) d\eta = -13.333$$

$$r_{6y} = \int_{-1}^1 N_6(1, \eta) \sigma J_{21} d\eta = \int_{-1}^1 (1 - \eta^2) \sigma (2\eta - 1 - \eta\sqrt{8}) d\eta = -13.333$$

$$r_{3x} = \int_{-1}^1 -N_3(1, \eta) \sigma J_{22} d\eta = \int_{-1}^1 \left(\frac{\eta^2 + \eta}{2} \right) \sigma (2\eta + 1 - \eta\sqrt{8}) d\eta = -0.572$$

$$r_{3_y} = \int_{-1}^1 N_3(1, \eta) \sigma J_{21} d\eta = \int_{-1}^1 \left(\frac{\eta^2 + \eta}{2} \right) \sigma (2\eta - 1 - \eta\sqrt{8}) d\eta = -6.095$$



Integración por Gauss:

$$r_{6x} = (1 - \eta_1^2) \sigma(2\eta_1 + 1 - \eta_1 \sqrt{8}) + (1 - \eta_2^2) \sigma(2\eta_2 + 1 - \eta_2 \sqrt{8}) = -13.333$$

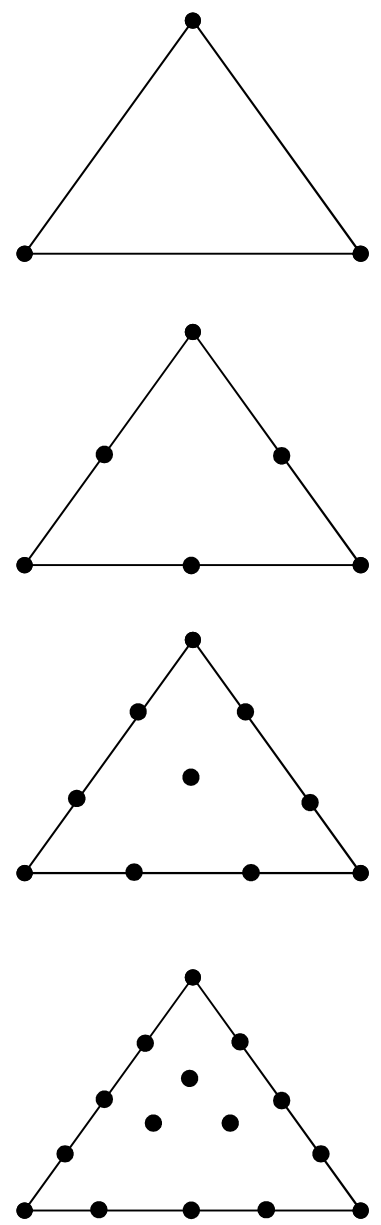
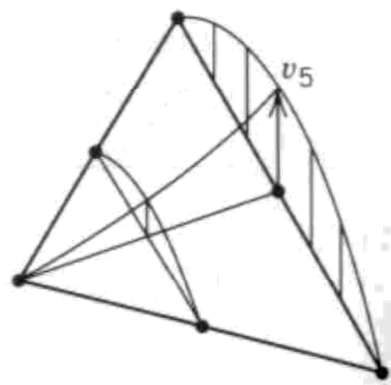
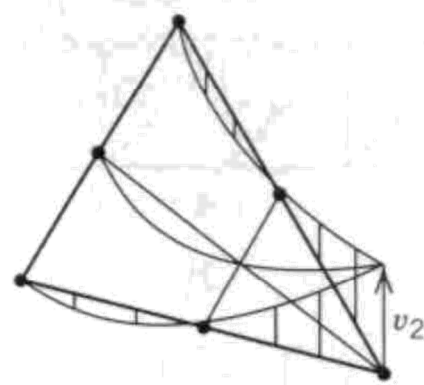
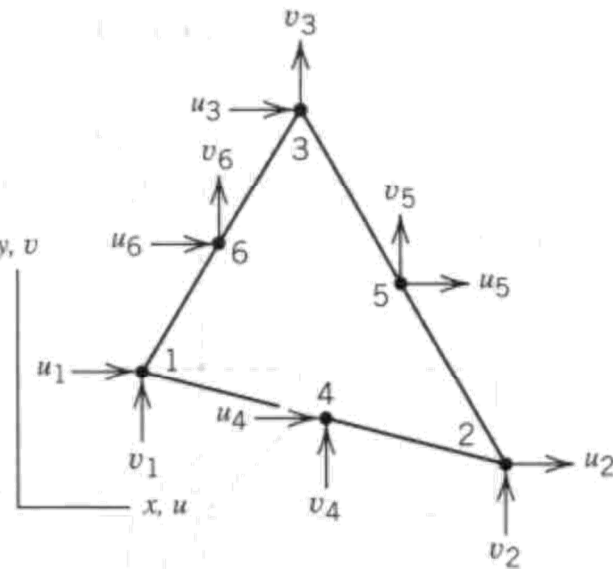
$$r_{6_y} = (1 - \eta_1^2) \sigma(2\eta_1 - 1 - \eta_1 \sqrt{8}) + (1 - \eta_2^2) \sigma(2\eta_2 - 1 - \eta_2 \sqrt{8}) = -13.333$$

Elementos Triangulares

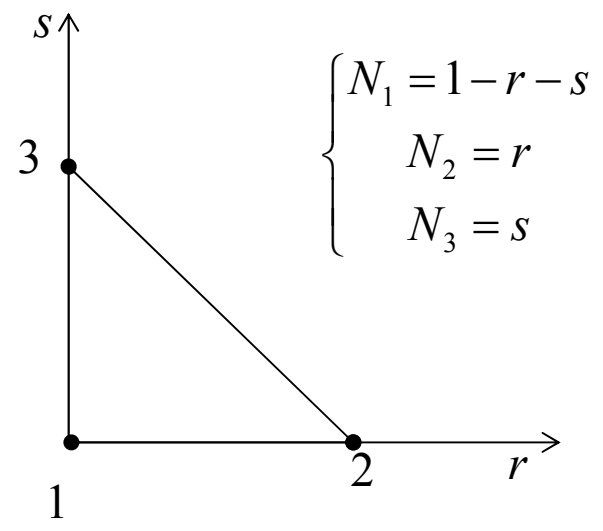
$p = 0$
 $p = 1$
 $p = 2$
 $p = 3$
 $p = 4$

$\rightarrow \left\{ \begin{array}{ccccccccc} & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{array} \right.$

1
 x
 y
 x^2
 xy
 y^2
 x^3
 x^2y
 xy^2
 y^3
 x^4
 x^3y
 x^2y^2
 xy^3
 y^4



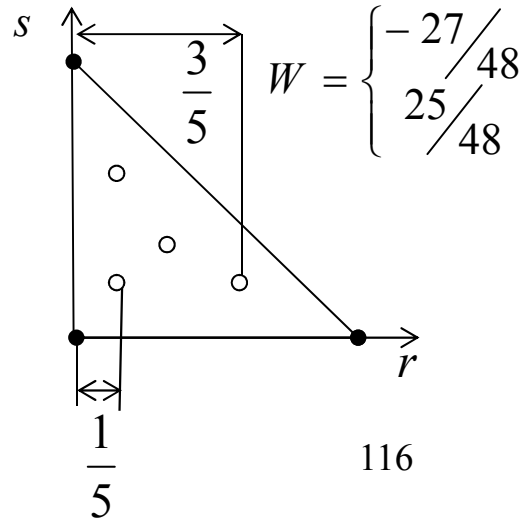
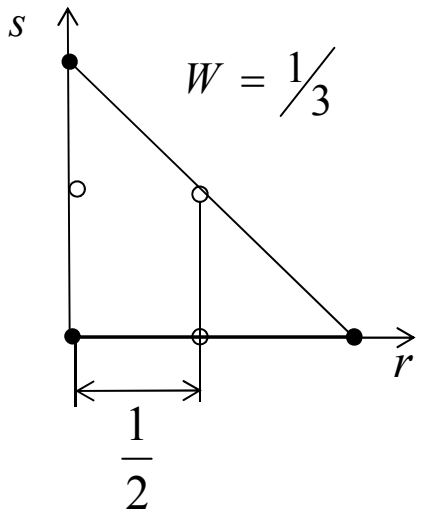
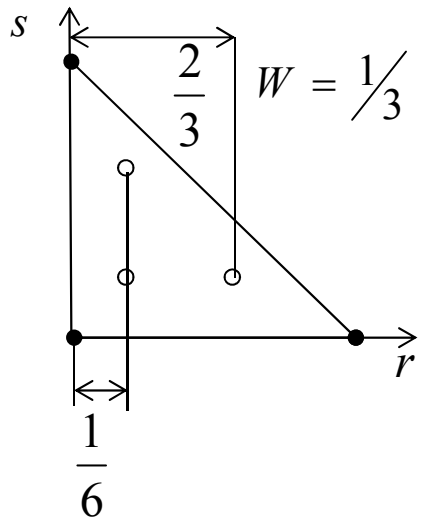
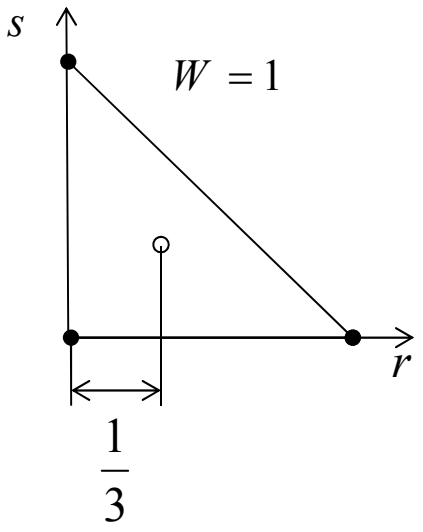
Formulación Isoparamétrica



$$I = \int_A \phi(r,s) dr ds \cong \sum_{i=1}^n W_i \phi(r_i, s_i) \frac{1}{2} |J_i|$$

$$J(r,s) = \begin{bmatrix} x_{,r} & y_{,r} \\ x_{,s} & y_{,s} \end{bmatrix} = \begin{bmatrix} \sum_k N_k(r,s)_{,r} x_k & \sum_k N_k(r,s)_{,r} y_k \\ \sum_k N_k(r,s)_{,s} x_k & \sum_k N_k(r,s)_{,s} y_k \end{bmatrix}$$

Precisión	$pg = 1$	$O = 1$
	$pg = 3$	$O = 2$
	$pg = 3$	$O = 2$
	$pg = 4$	$O = 3$



Mallados

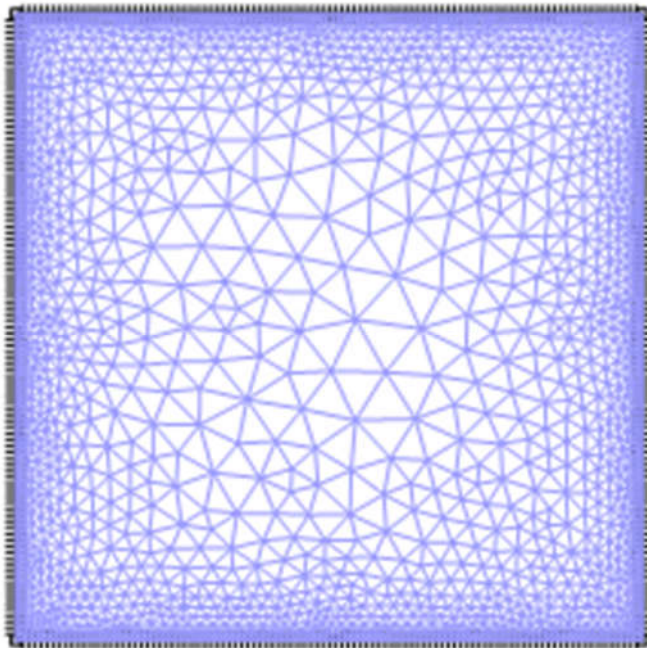
Subdivisión

- Tamaño de elemento
- Biasing

Cualquier espacio se puede rellenar con tetrahedros y cualquier área con trigulos, no así con cubos y rectángulos respectivamente.

Control de tamaño de elementos

- De menor dimensión a mayor

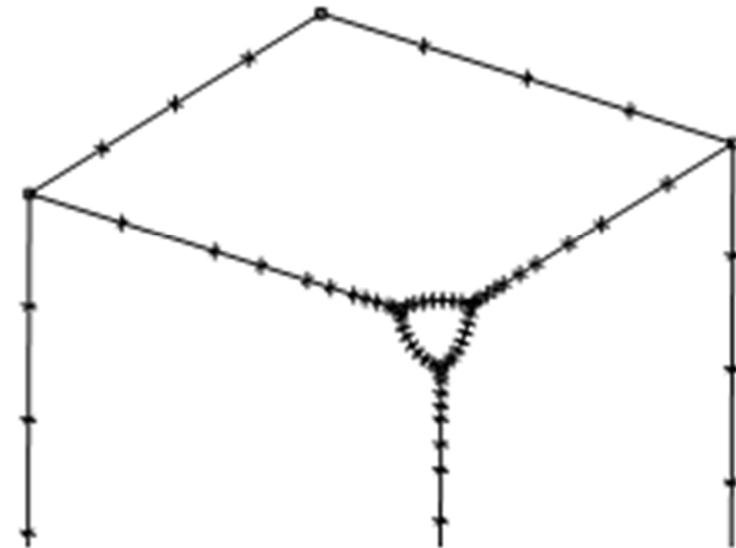


INVESTIGAR GRIFFITHS



Primero se malla todo parejo, se calcula y se remalla (biasing) donde se encuentren las mayores tensiones.

No conviene hacer un salto muy grande en el tamaño de los elementos ya que rigidiza la solución, no da un buen resultado. La energía es una función suave, así tendrá que ser el resultado.

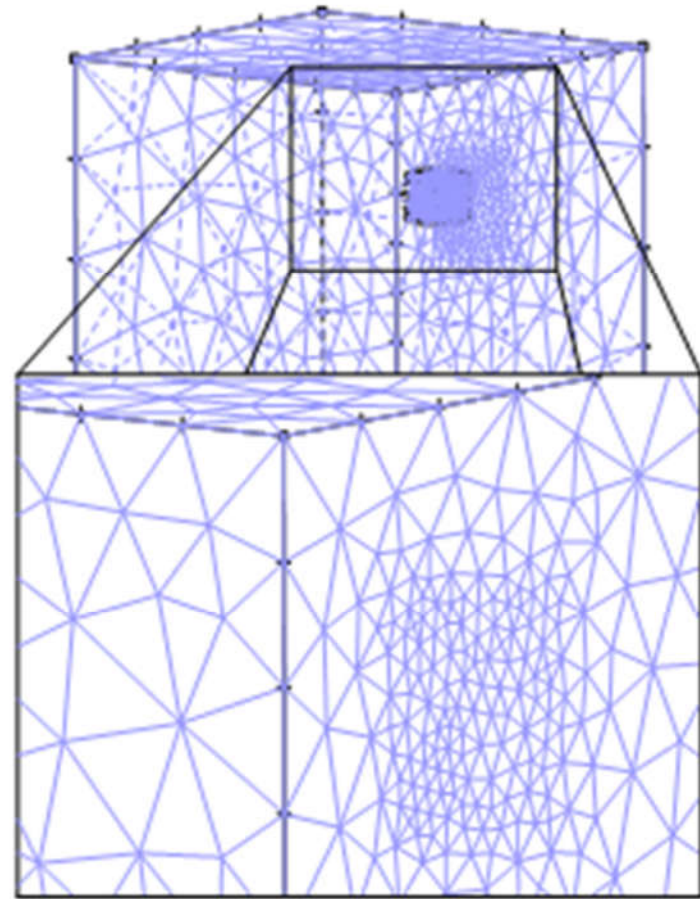
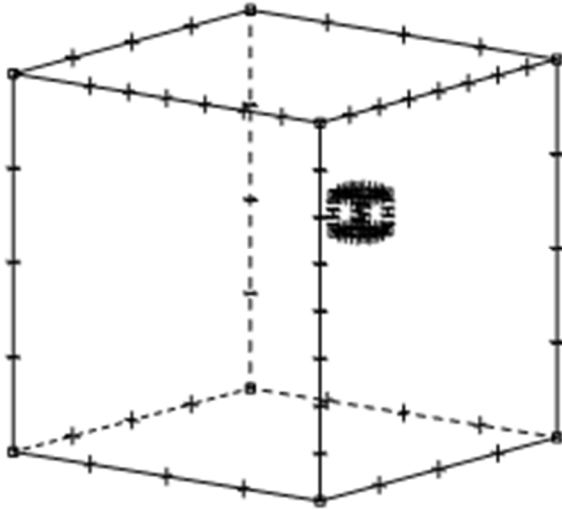


Conviene llenar las areas comprometidas con mallas ordenadas¹¹⁷ ya que es más facil evaluar tensiones. El resto puede estar desordenado con tetrahedros. Siempre mallar los más chico primero y dejar que crezca hacia lo grande.

Mallados

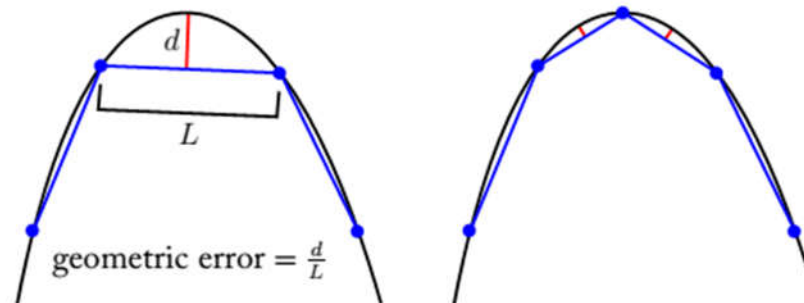
Control de tamaño de elementos – Subdominios

- Malla de más refinado a menos refinado



Geometría

- Control por Curvatura



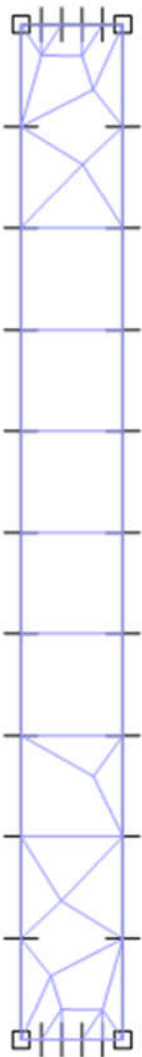
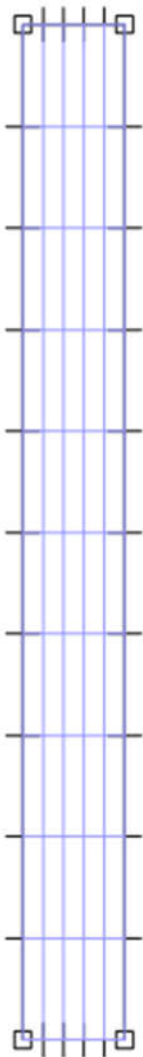
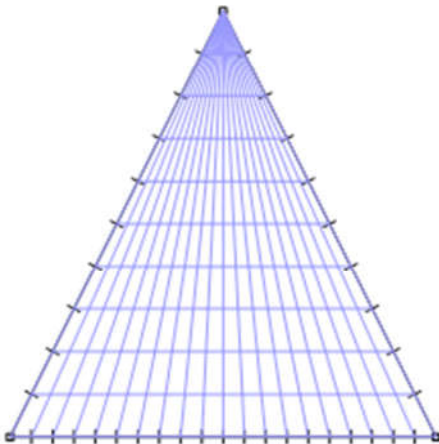
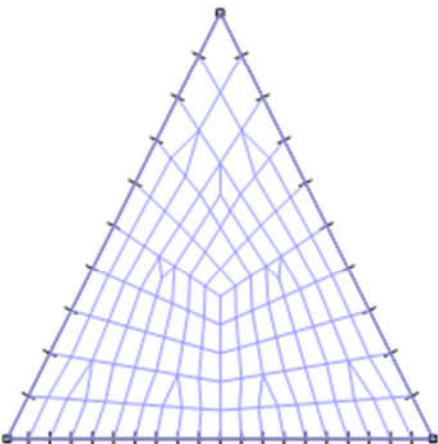
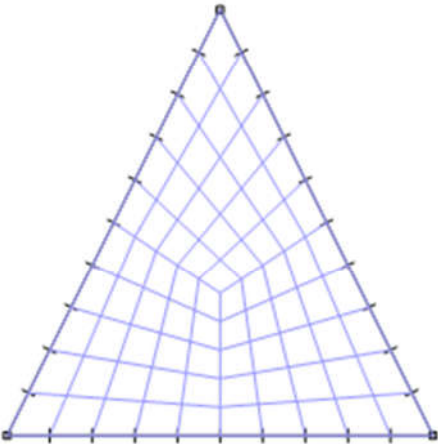
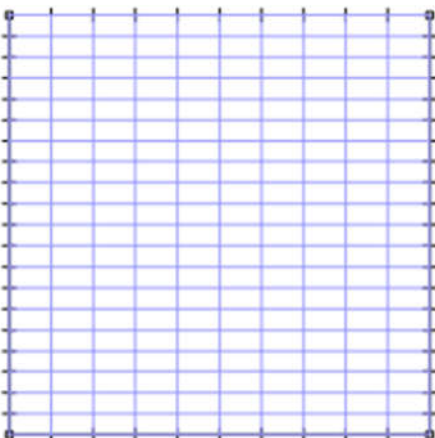
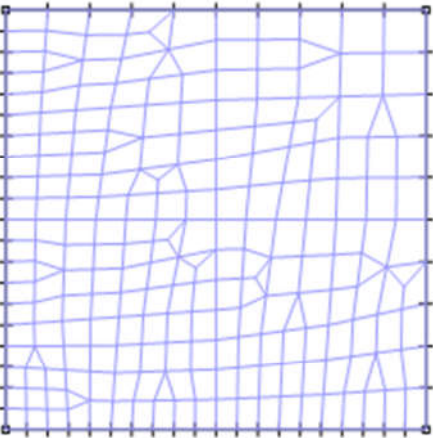
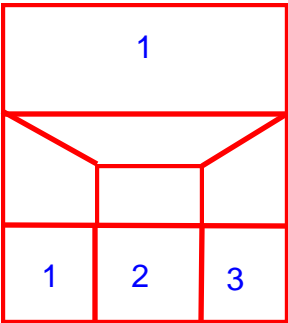
Mallados

Arranco ordenado con elementos Q donde claramente hay concentración de tensiones, que el programa rellene el resto.

Mallado de superficies y volúmenes

- Mapeado o estructurado
- Libre
- Tipos de elementos - ¿Combinar?

Truco para pasar de 1 a 3

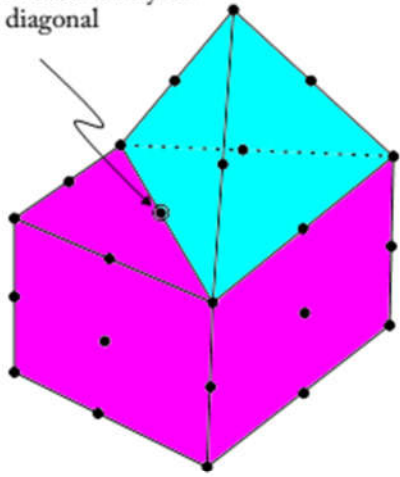


Mallados

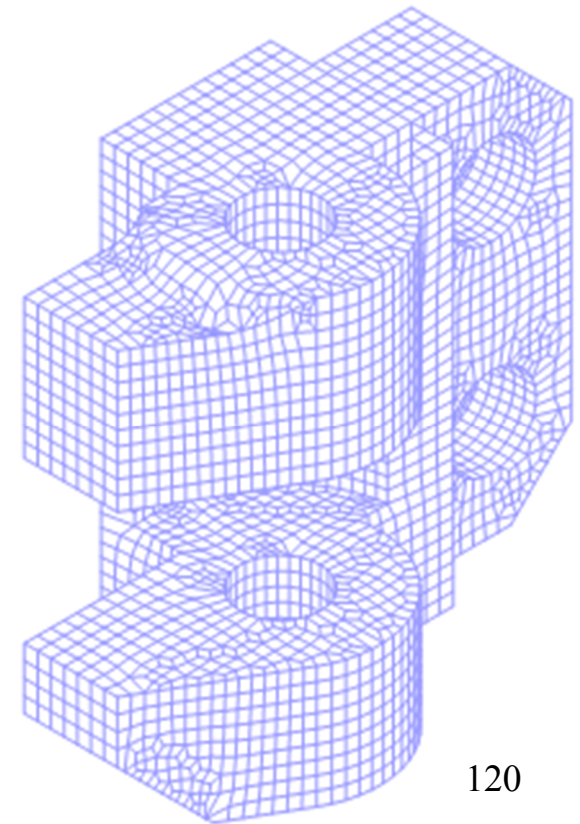
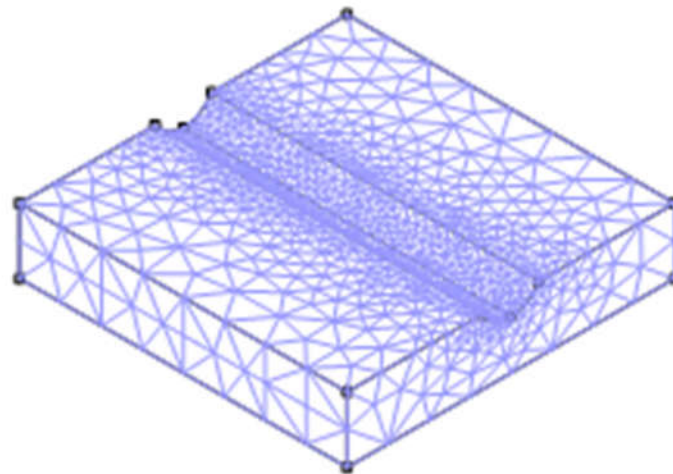
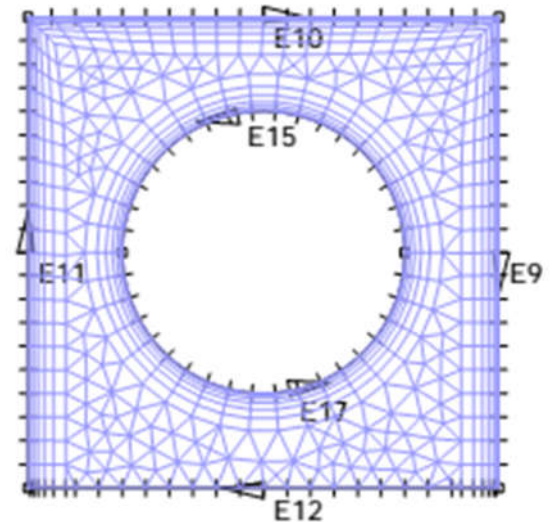
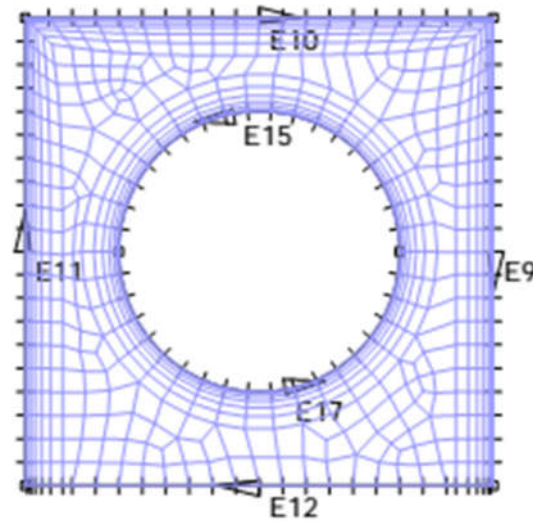
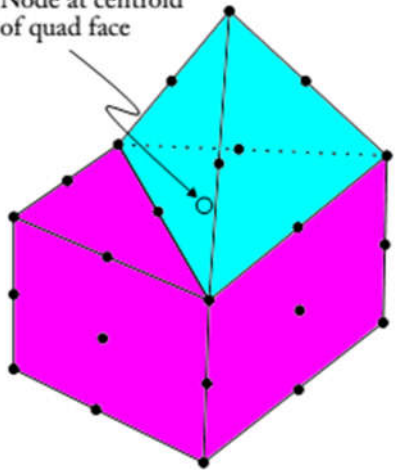
Transiciones

- Nodos libres
- Tetraedros
- Hexaedros

Node midway on diagonal



Node at centroid of quad face



Mallados 3D

- Regularidad
- Extrusión
- Revolución

