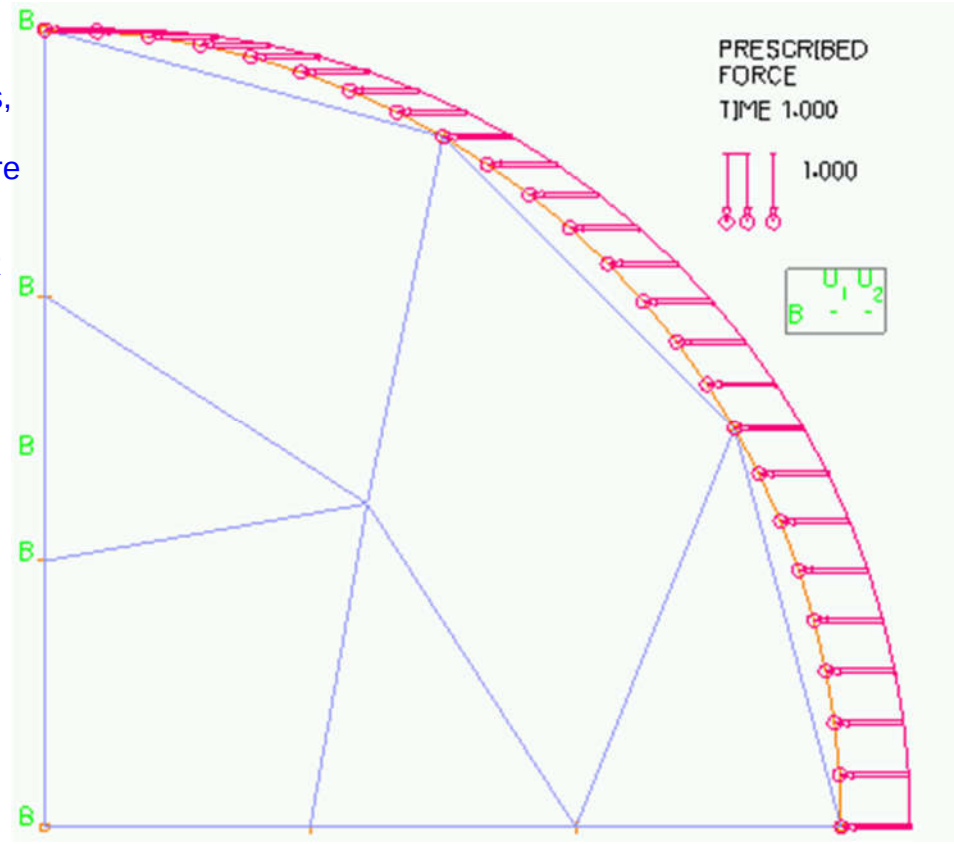
Análisis de Errores

Tipos de Error

Materiales: Siempre hay mucha incertidumbre ya que no se conoce el verdadero módulo de elasticidad. Nunca son homogeneos.
Fijaciones: Las fijaciones no son reales, en la realidad, no hay empotramientos.
Cargas: Es muy dificil conocerlas. Sobre todo cuando son dinámicas.

SI ESTAS BASES NO LAS CONOZCO CON EXACTITUD, REFINAR NO VA A CAMBIAR NADA. MODELO INEXACTO



- Modelo
  - Físico
    - Materiales
    - Fijaciones
    - Cargas
  - Geometría → Mallados
- Usuario
  - Análisis Previo
  - Control
    - Cargas externas
    - Desplazamientos
- Numérico → Condicionamiento
  - Truncado
  - Almacenamiento

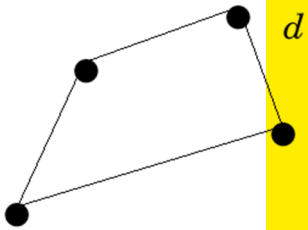
Error numérico, puntos flotantes

- Discretización
  - Estimación a priori (Calidad de elementos)
  - Estimación a posterior (Calidad de resultados)

Las deformaciones sólo pueden seguir las funciones de forma

Distorsiones de Mallado (2D)

Distorted Element

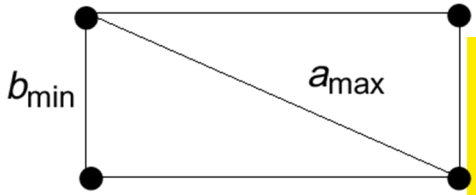


$$d = \frac{|J|_{min}}{A(x,y)} \times A(\xi,\eta)$$

$$Range \quad -1 \le d \le 1.0$$

óptimo

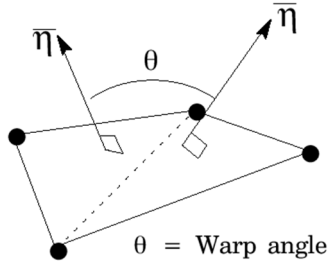
Stretched Element



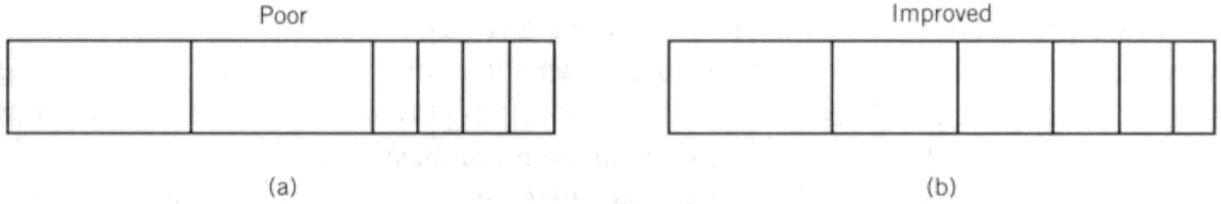
$$Stretch = \frac{b_{min}}{a_{max}}\sqrt{2}$$

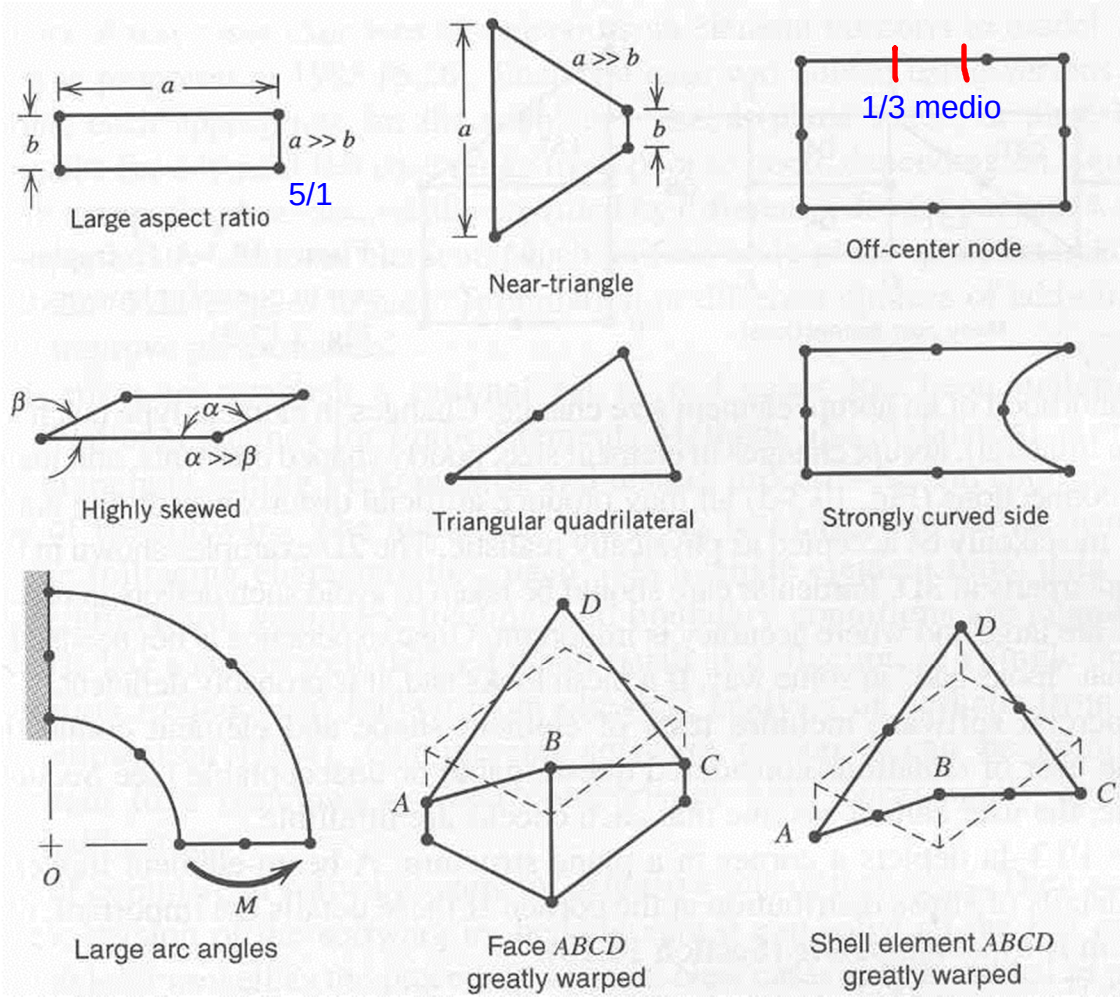
$$Range \quad 0 \le s \le 1.0$$

Warped Element



Biasing



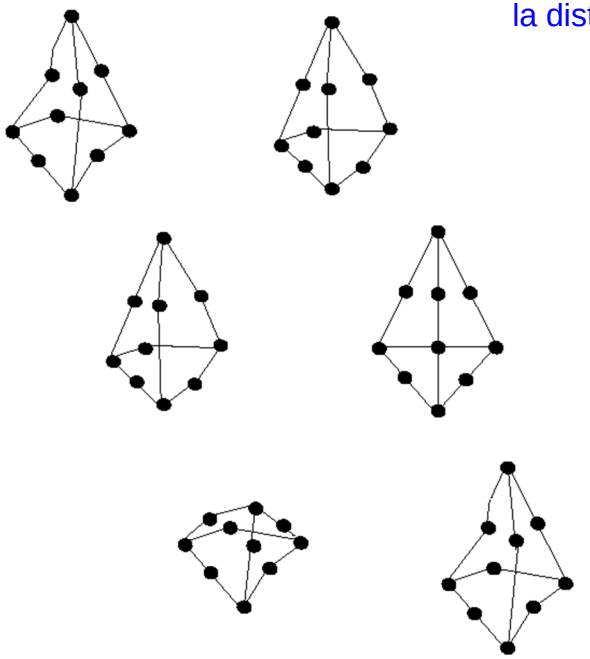


— critical:
distortion ≥ 0.7 stretch ≥ 0.7

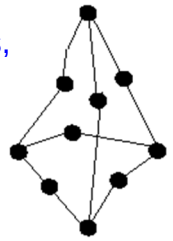
— general:
distortion ≥ 0.3 stretch ≥ 0.3

— error:
distortion ≤ 0 N/A

Distorsiones de Mallado (3D)

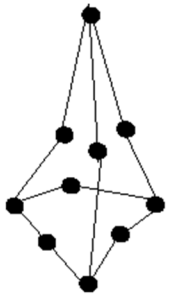


En 3D viaja más información entre elementos, la distorición y el strech son más aceptables.



$$d = \frac{|J|\min}{V(x,y,z)} \times V(\xi,\eta,\zeta)$$

$$Range \quad -1.0 \leq d \leq 1.0$$



Stretched Element

$$\text{Stretch} = \frac{R}{L_{max}}\sqrt{24}$$

$$Range: \ 0 \leq s \leq 1.0$$

Valores Recomendados

	Distortion	Stretch	Warp Angle	Interior Angle
Quadrilateral	0.6	0.3	5 deg.	135−45
Triangle	0.35	0.3	na	90−30
Brick	0.5	0.3	5 deg.	135−45
Wedge	0.35	0.3	na	90−30
Tetrahedron	0.1	0.1	na	90−30

- critical:
  distortion ≥ 0.3 stretch ≥ 0.3
- general:
  distortion ≥ 0.1 stretch ≥ 0.05
- error:
  distortion ≤ 0 N/A

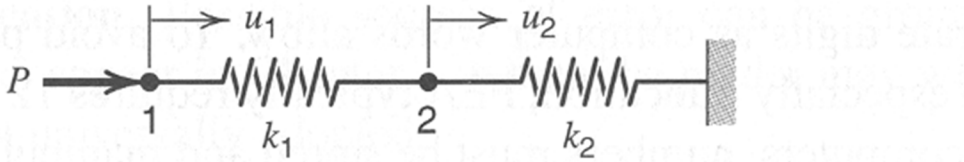
Condicionamiento Numérico

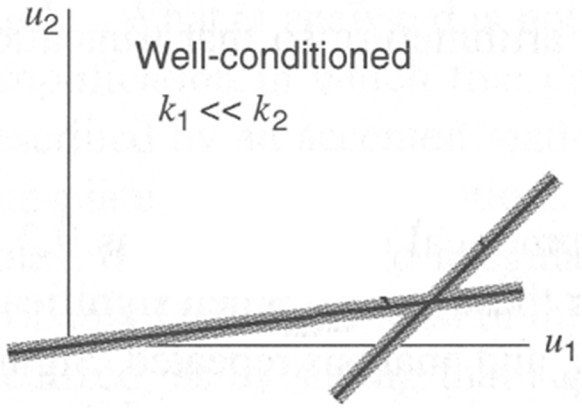
$$\Pi_p = \tfrac{1}{2} k_1 (u_1 - u_2)^2 + \tfrac{1}{2} k_2 (u_2)^2 - P u_1$$

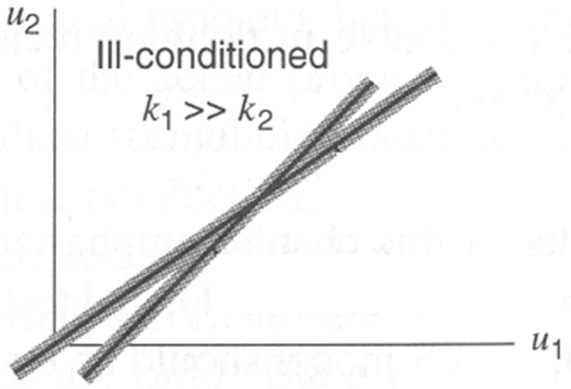
$$\begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} P \\ 0 \end{Bmatrix}$$

$$\left((k_1 + k_2) - k_1\right) u_2 = P$$

Si K1 >> K2, la fuerza necesaria es 0.
No se pueden comparar valores tan lejanos.







Numero de Condición:

$$[K]\{D\} = \{R\} \rightarrow [K_s] = [S][K][S] \text{ con } S_{ii} = \frac{1}{\sqrt{K_{ii}}} \Rightarrow K_{s_{ii}} = 1 \rightarrow C(K_s) = \frac{\lambda_{máx}(K_s)}{\lambda_{mín}(K_s)}$$

Matlab testea con rcond(K) los digistos perdidos. El log de ese número son los digitos perdidos

Precisión: $d_{precisos} = d - d_{perdidos}$ $\qquad d_{perdidos} \leq \log(C(K_s))$

Condicionamiento Numérico

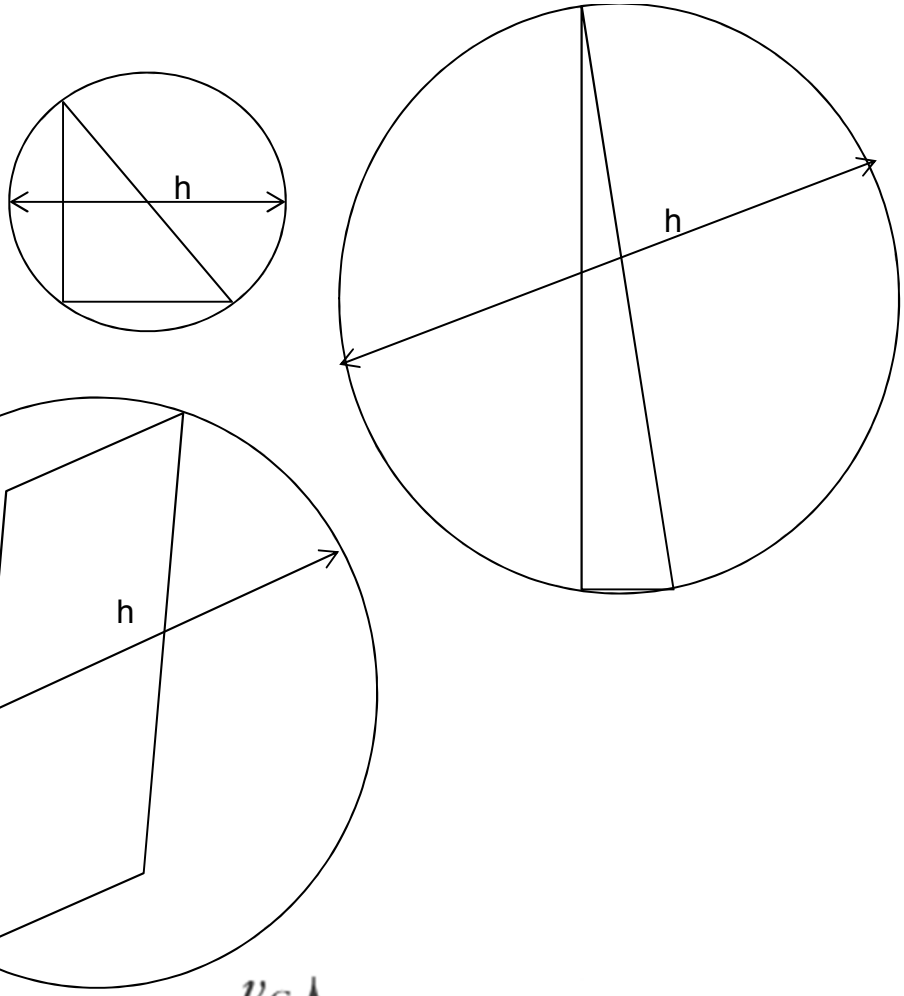
Influencia de parámetros del modelo

$$C[K] = b\left(\frac{h_{máx}}{h_{mín}}\right)^{2m-1} N_{els}^{\frac{2m}{n}}$$

b: constante

m:orden PDE

n: dimensión



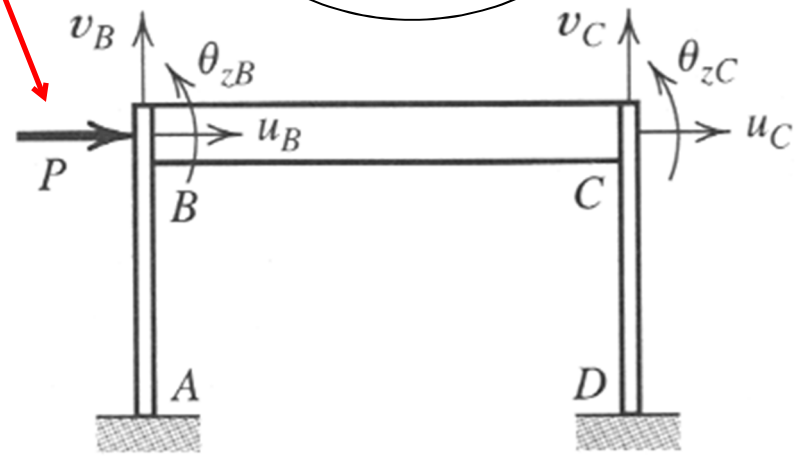
Como los desplazamientos que tiene la viga son muy grandes pero no así su deformació, intentar calcular las tensiones cuando se pierden digitos daría malos resultados. Ej u1 = 1.000000000000001m el ultimo digito ni importa.

DOF Relativos o Jerárquicos

$$u_r = u_1 - u_2$$

$$\Pi_p = \tfrac{1}{2} k_1 (u_r)^2 + \tfrac{1}{2} k_2 (u_2)^2 - P(u_r + u_2)$$

$$\begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \begin{Bmatrix} u_r \\ u_2 \end{Bmatrix} = \begin{Bmatrix} P \\ P \end{Bmatrix}$$



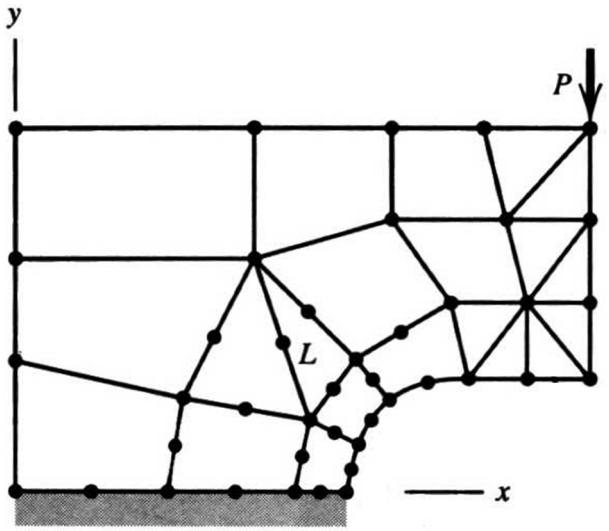
Error de Discretización

Estimación

Flagherty - libro donde está esto:

$$\|e\|_m \le C\, h^{p+1-r} \|u\|_{p+1} = C\, N_{els}^{\frac{p+1-r}{-n}} \|u\|_{p+1}$$

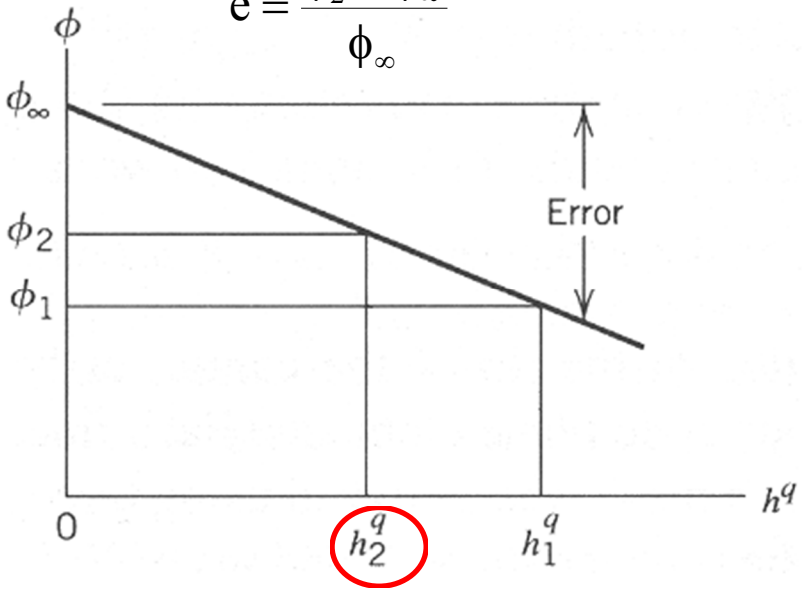
$$\|e\| = O\left(h^{p+1-r}\right) = O\left(N_{els}^{-\frac{p+1-r}{n}}\right)$$



Extrapolación Multimallado: q=p+1-r

$$\phi_1 = \frac{\phi_2 - \phi_1}{h_2^q - h_1^q} h_1^q + \phi_\infty \; ; \; \phi_\infty = \frac{\phi_1 h_2^q - \phi_2 h_1^q}{h_2^q - h_1^q}$$

$$e = \frac{\phi_2 - \phi_\infty}{\phi_\infty}$$

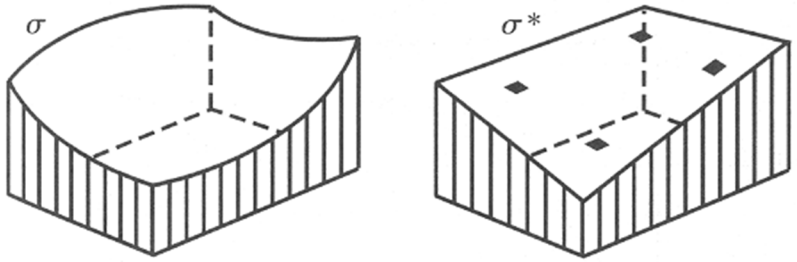


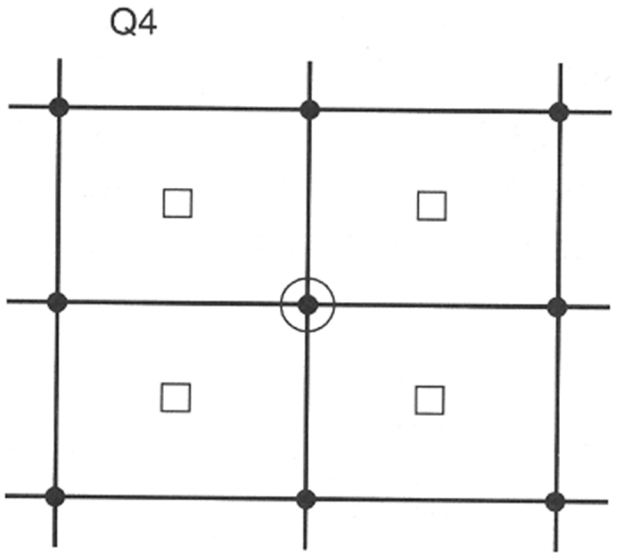
Ojo, h^q

Estimación de Error

Mejorado de Tensiones

- Elemental
- Patch
- Global





Estimación a Posteriori

$$\left.\begin{aligned}
\|U\|^2 &= \sum_{i=1}^{m} \int_{v_e} \{\varepsilon\}_i^T [E] \{\varepsilon\}_i \, dv \\
\|e\|^2 &= \sum_{i=1}^{m} \int_{v_e} \left(\{\varepsilon^*\}_i - \{\varepsilon\}_i\right)^T [E] \left(\{\varepsilon^*\}_i - \{\varepsilon\}_i\right) dv \\
\|e\|^2 &= \sum_{i=1}^{m} \int_{v_e} \left(\{\sigma^*\}_i - \{\sigma\}_i\right)^T [E]^{-1} \left(\{\sigma^*\}_i - \{\sigma\}_i\right) dv
\end{aligned}\right\}$$

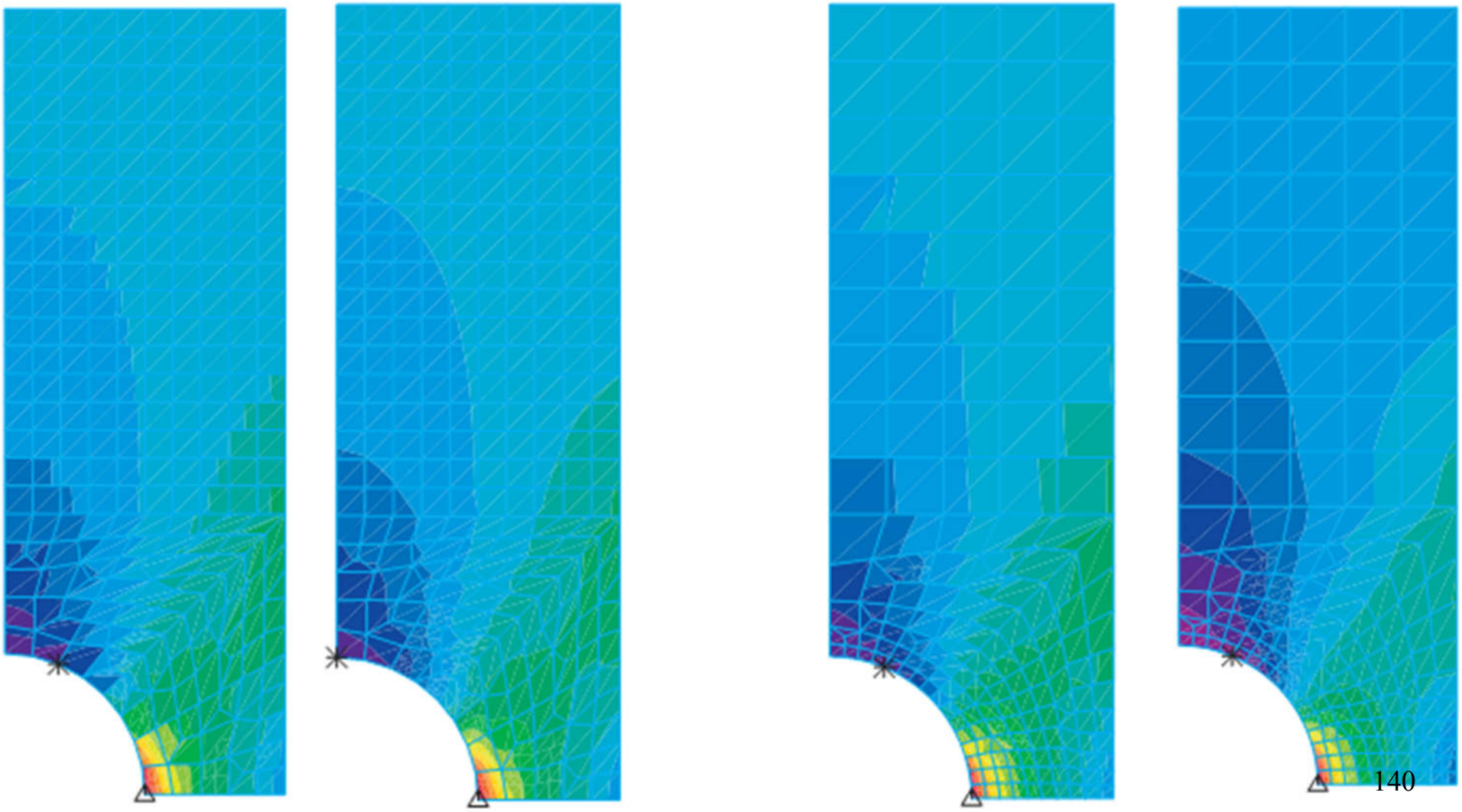
$$\eta = \sqrt{\frac{\|e\|^2}{\|e\|^2 + \|U\|^2}} \quad ; \eta \le 0.05$$

Sólamente cuando el error es menor a 5% se puede aceptar que el error estimado es efectivamente menor al 5% ya que convirgió.

Estimación de Error Global

Mejorado de Tensiones

- Global - Promediado

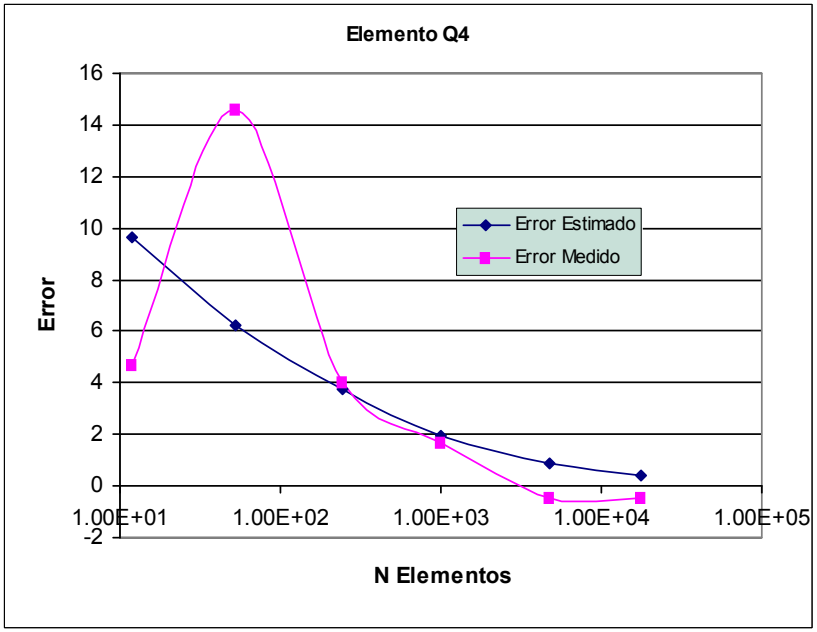


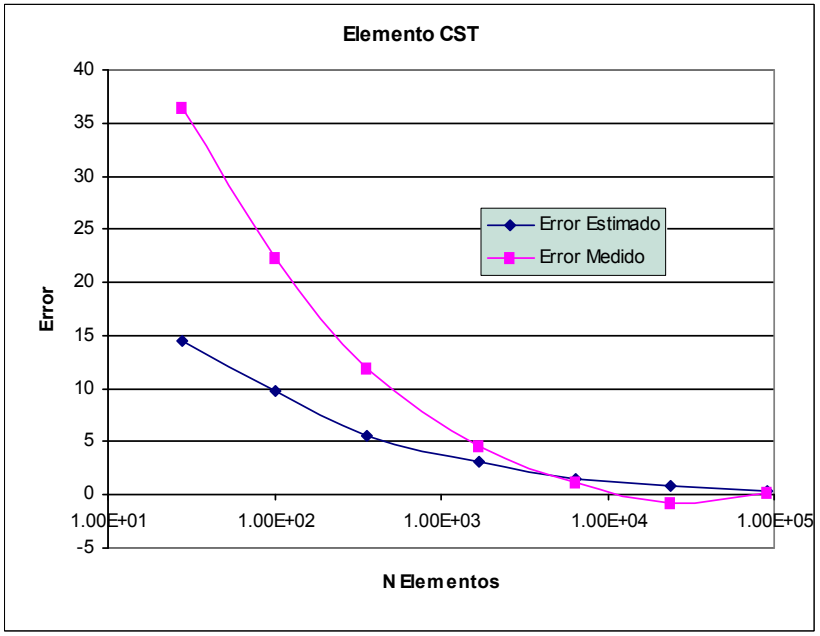
Estimación de Error

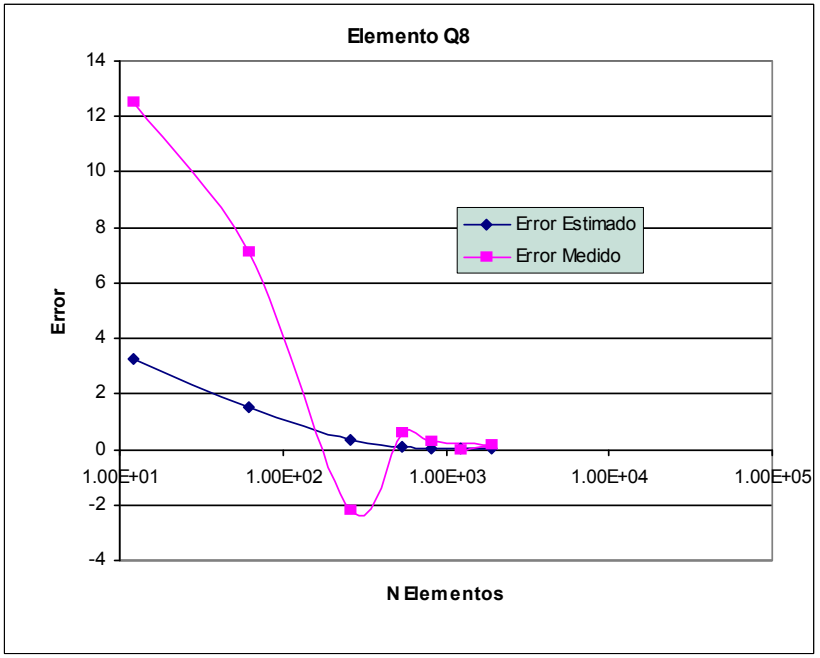
Estimación de error

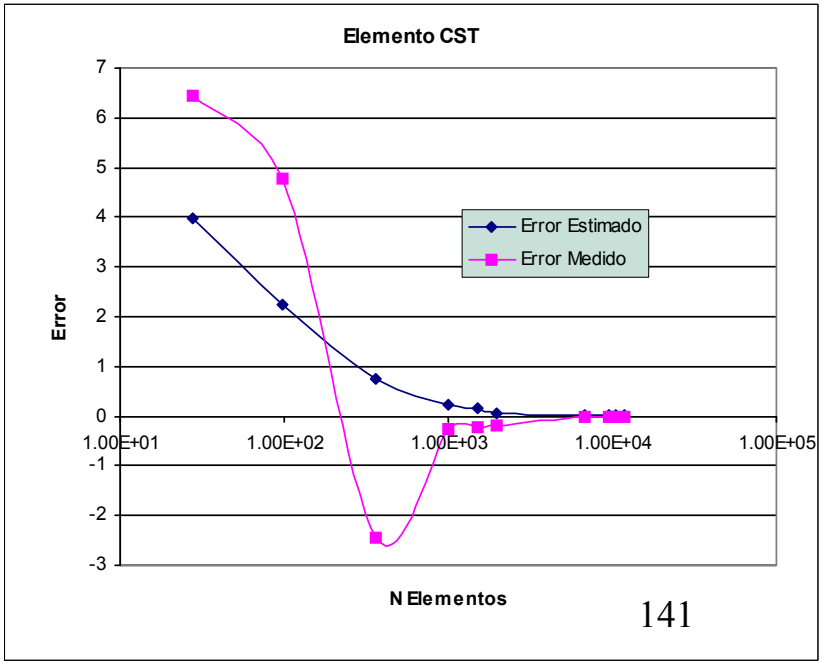
vs

Error medido









Estimación de Error

Adaptividad – Malla Optima

donde eta_total = eta_elemental

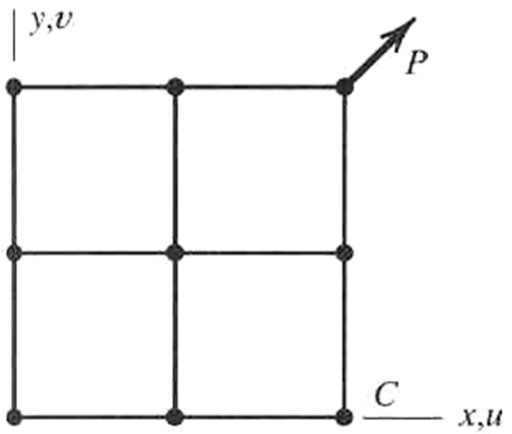
h^(p+1-r)

$$\|e\|_{med} = \eta_{med}\sqrt{\frac{\|e\|^2 + \|U\|^2}{m}}$$

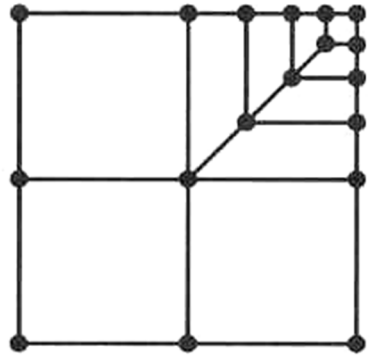
$$\|e\|_i = \int_{v_e} \left(\{\varepsilon^*\}_i - \{\varepsilon\}_i\right)^T [E] \left(\{\varepsilon^*\}_i - \{\varepsilon\}_i\right) dv$$

$$\xi_i = \frac{\|e\|_i}{\|e\|_{med}}$$

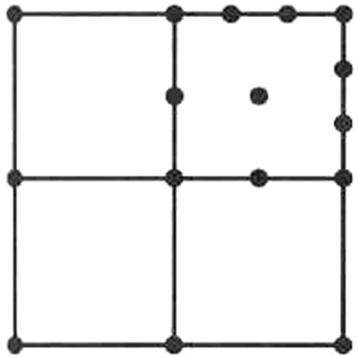
$$(h_i)_{remallado} = \frac{(h_i)_{original}}{\xi_i^{\alpha}} \quad ; \begin{cases} \alpha = \dfrac{1}{p} \\ \alpha = \lambda = \dfrac{1}{2} : \quad Singularidades \end{cases}$$



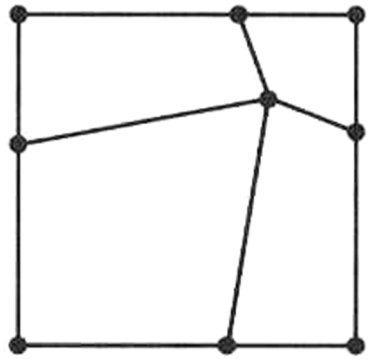
An *h* refinement



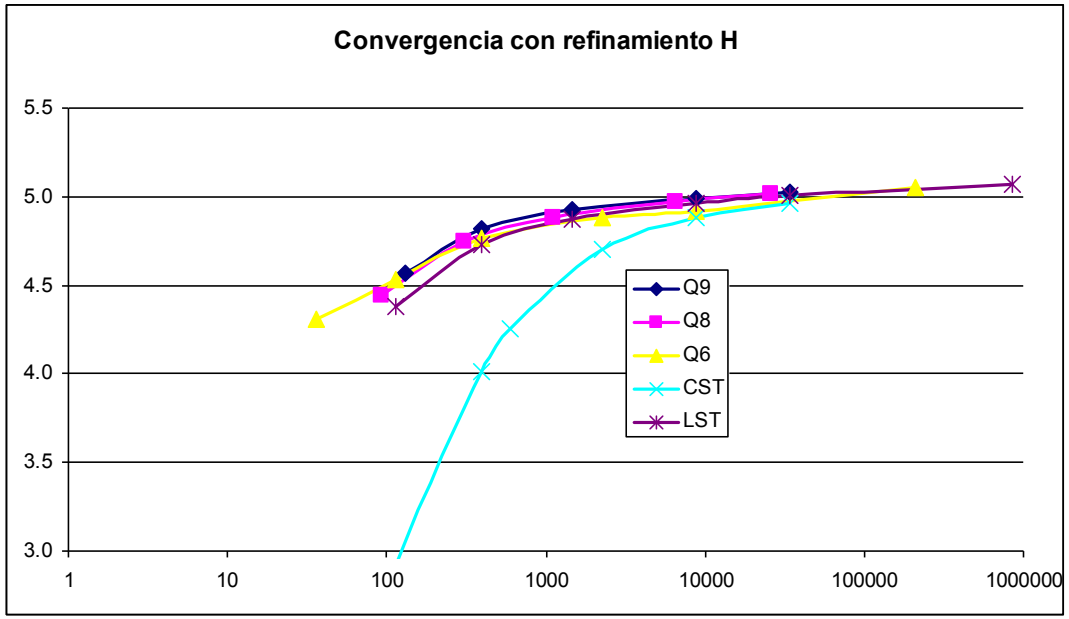
A *p* refinement

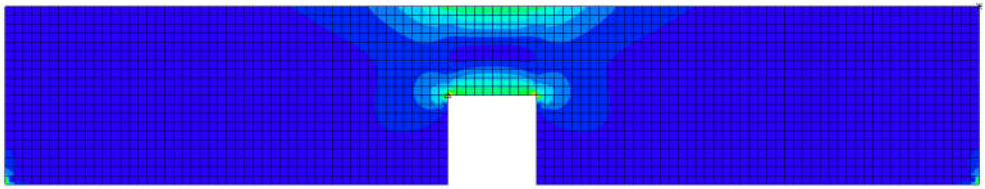


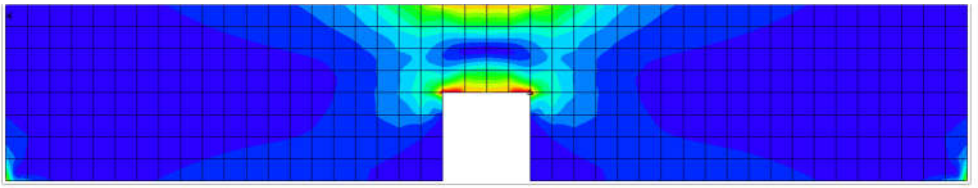
An *r* refinement

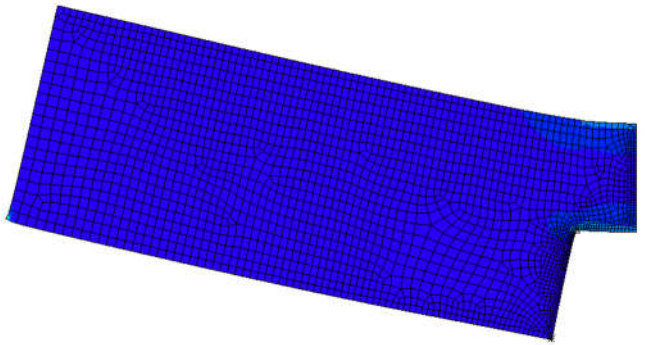


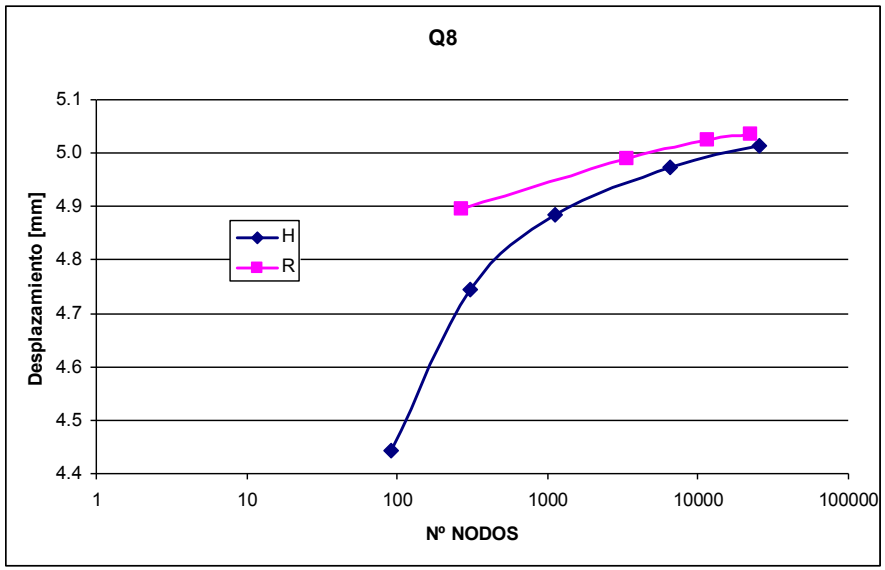
Refinamiento H - R

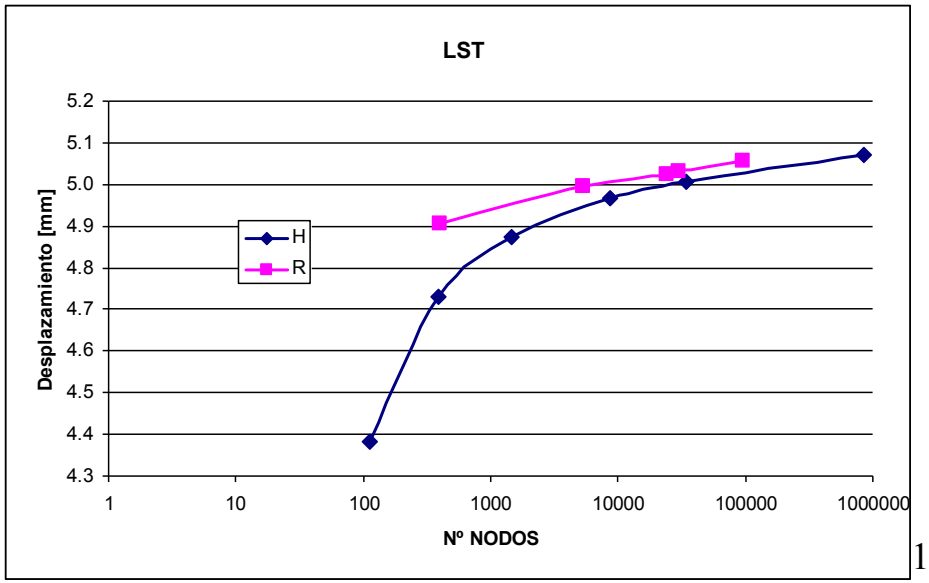




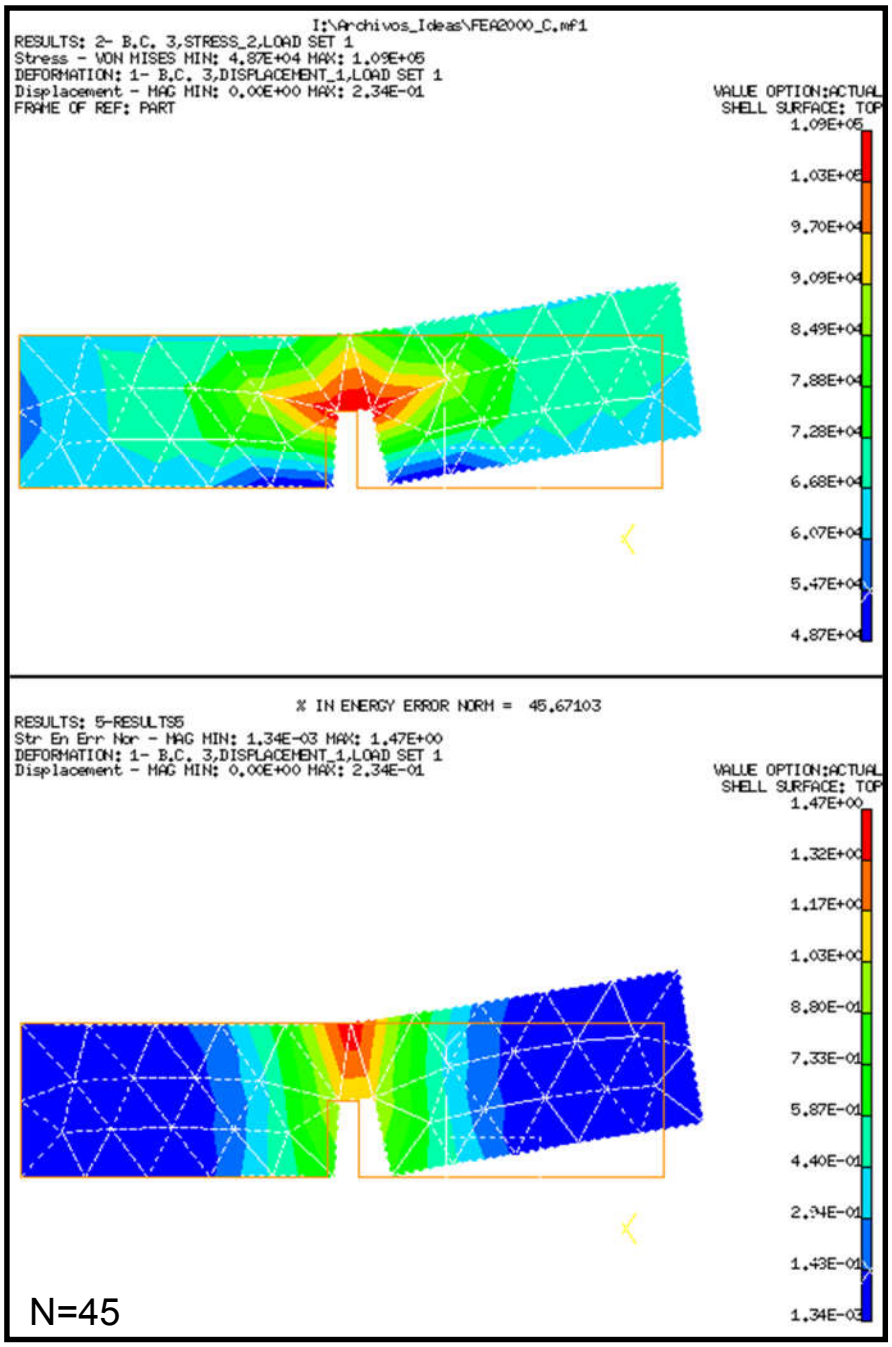


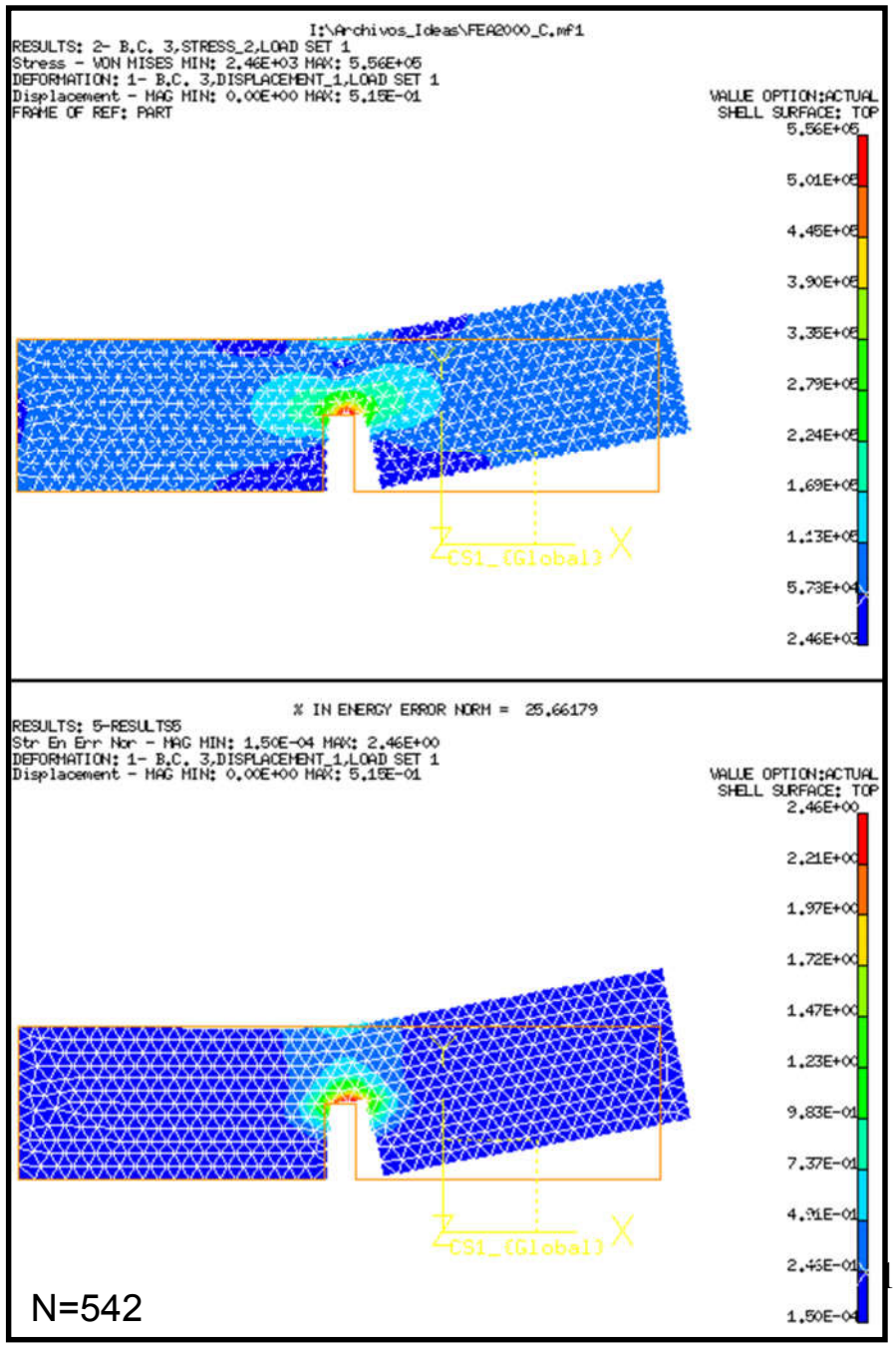






Refinamiento H

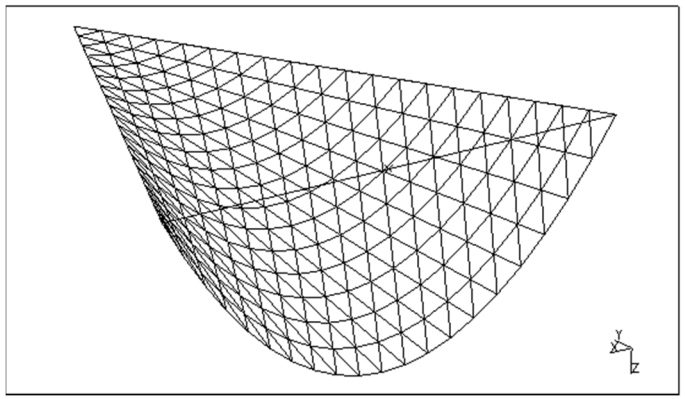




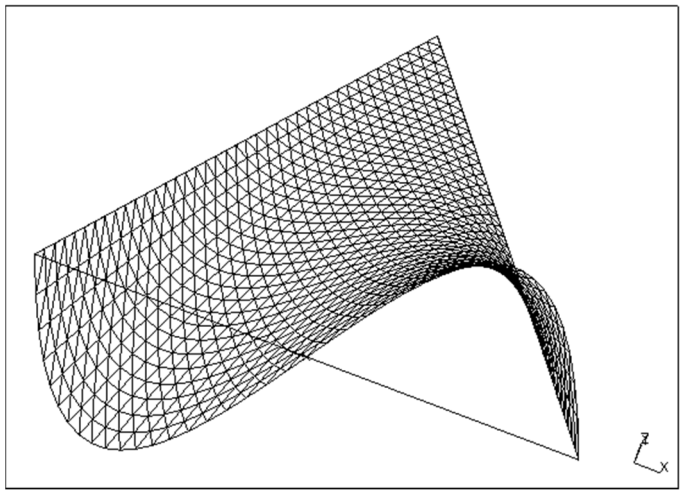
Refinamiento P

Funciones de Forma Jerárquicas

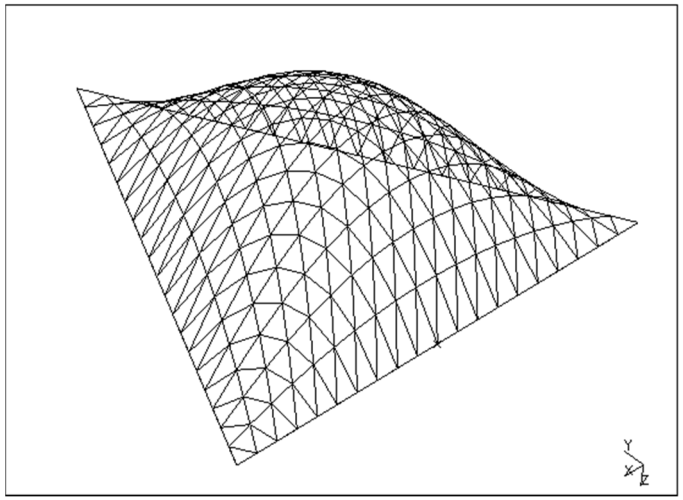
Orden 2 - Lado



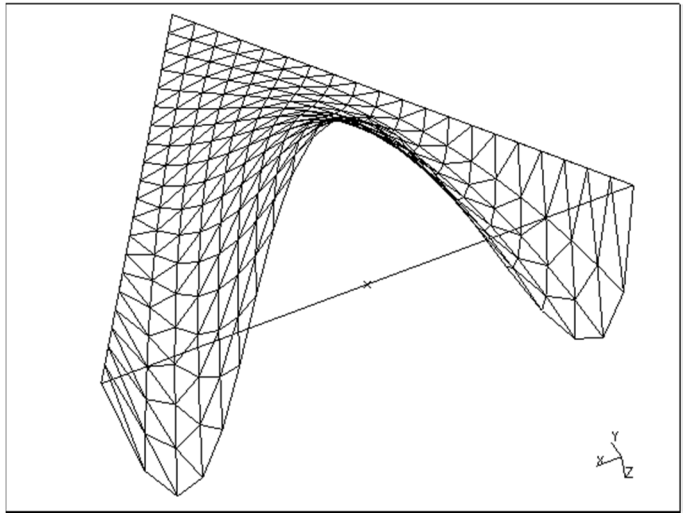
Orden 3 - Lado



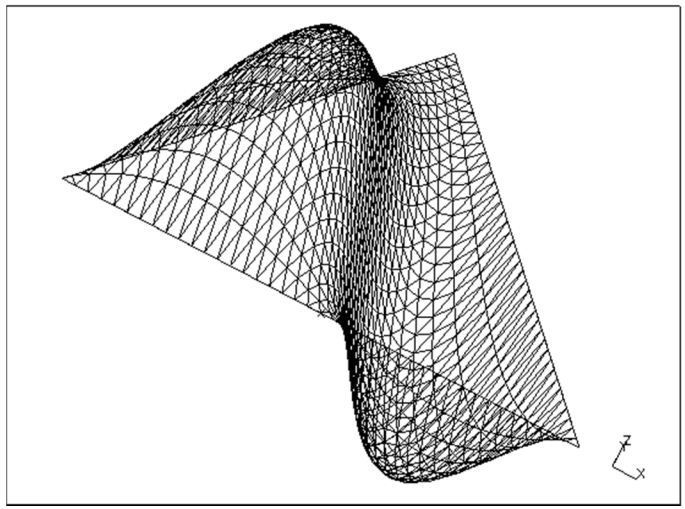
Orden 3 - Cara



Orden 4 - Lado

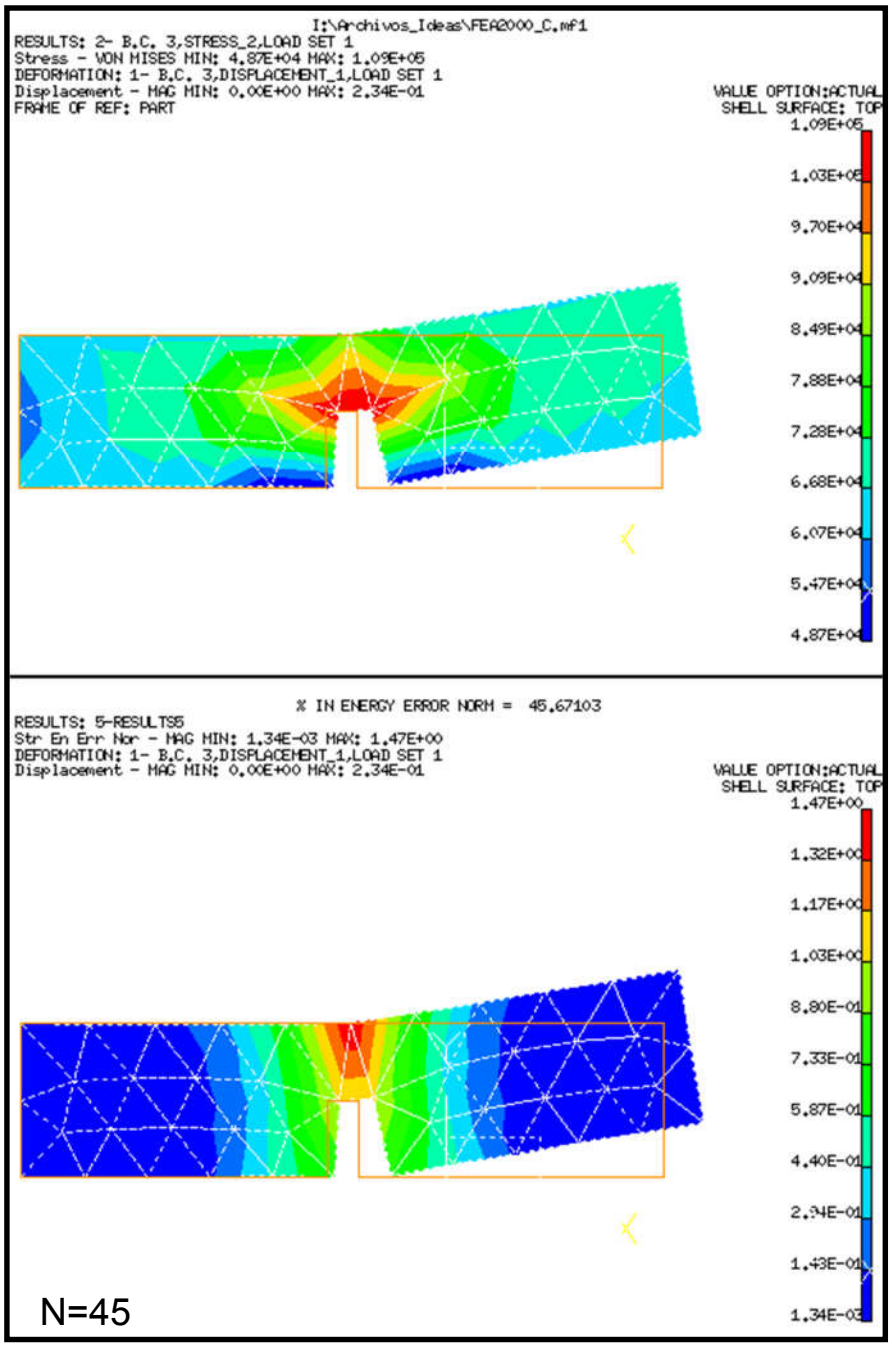


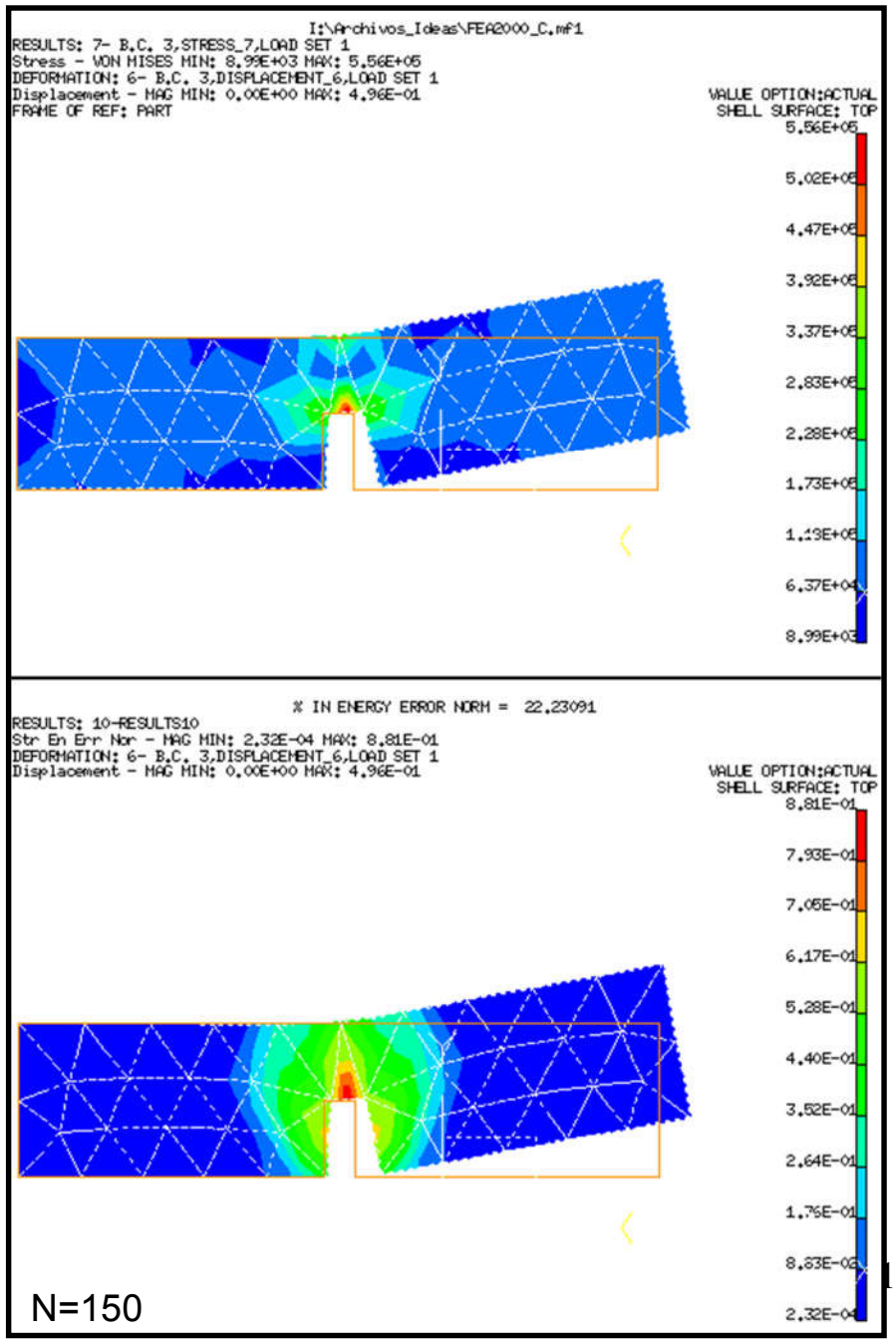
Orden 4 – Cara 1



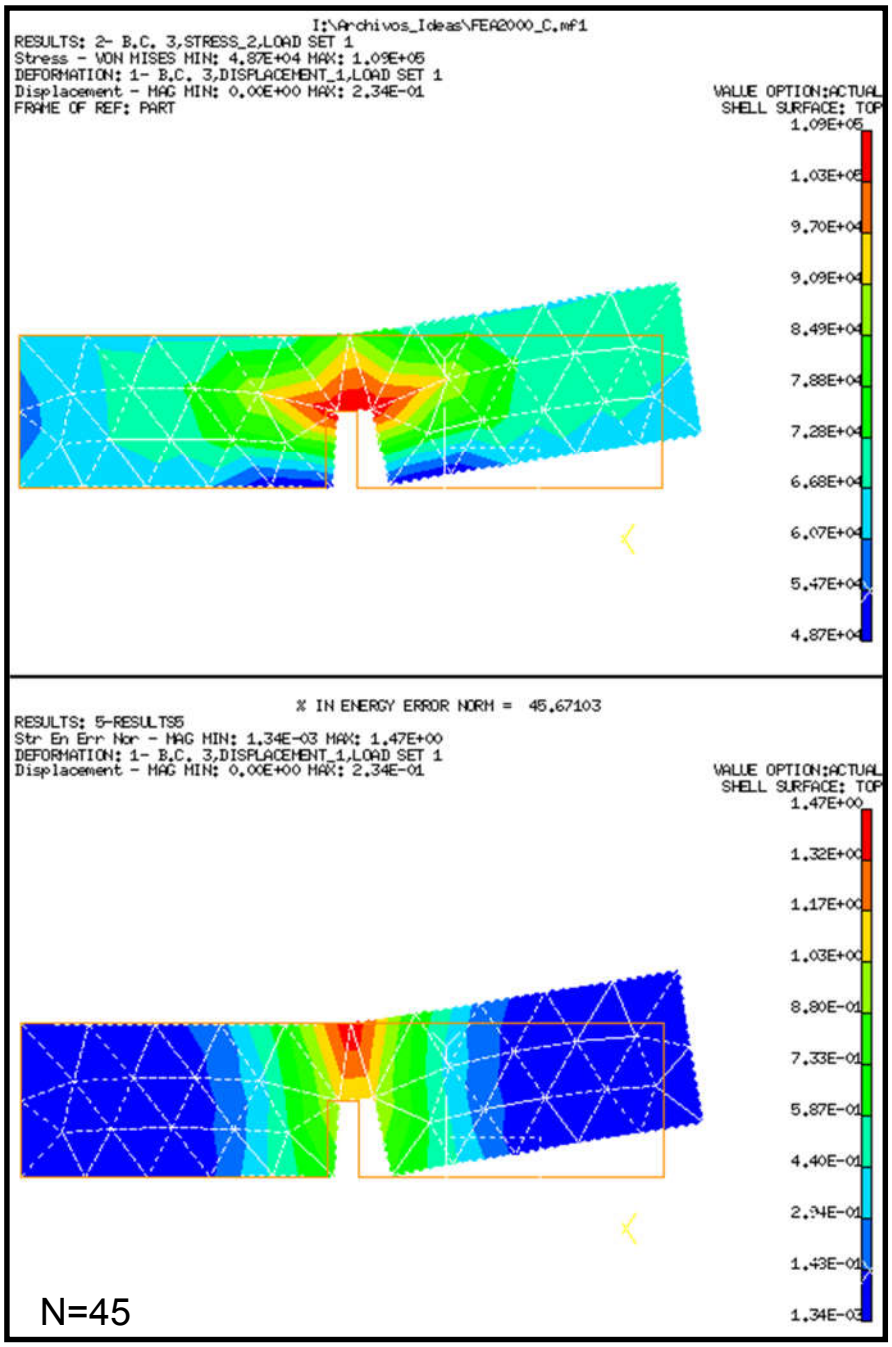
Orden 4 – Cara 2

Refinamiento P

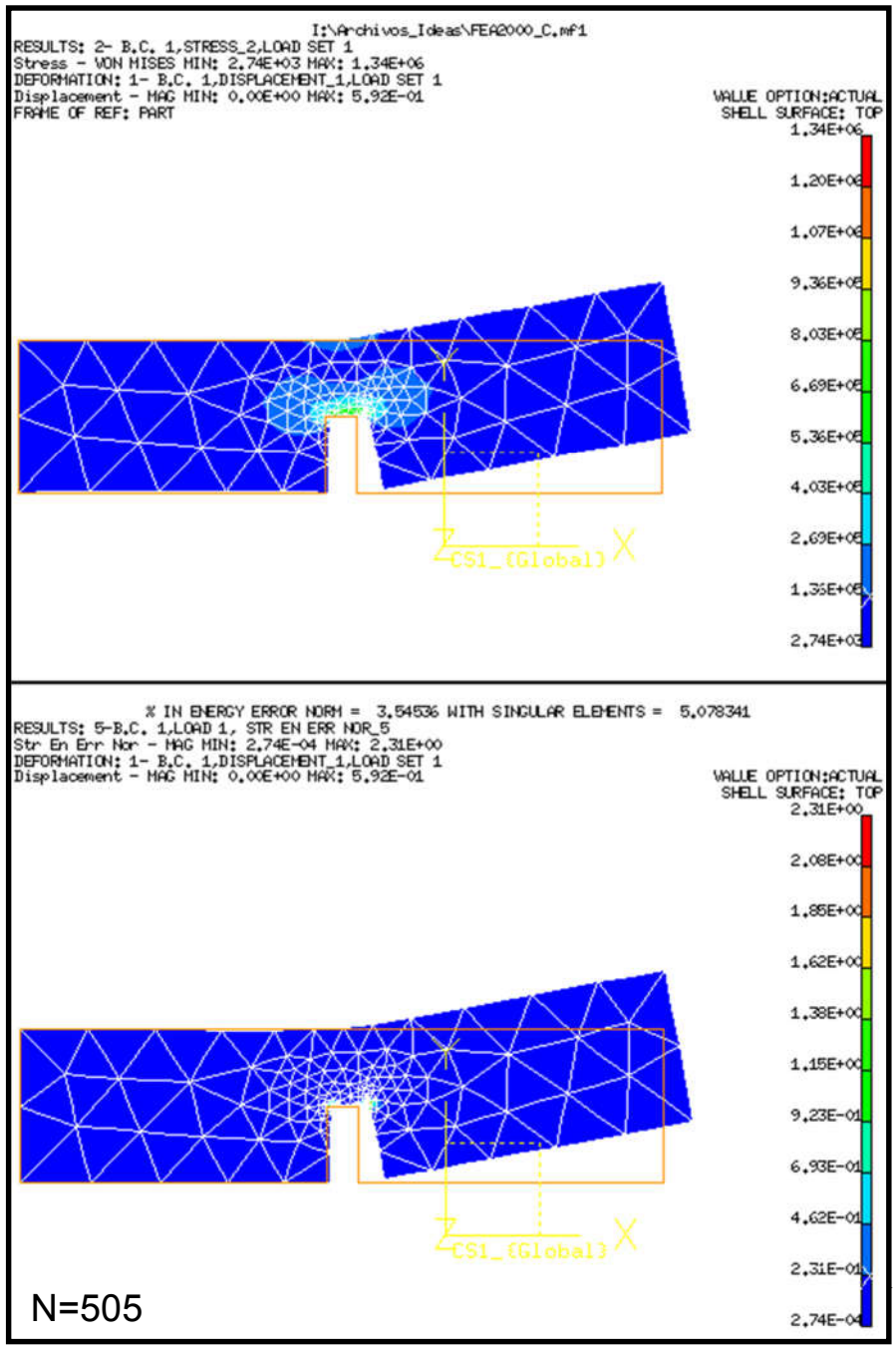




Refinamiento R - Adaptivo

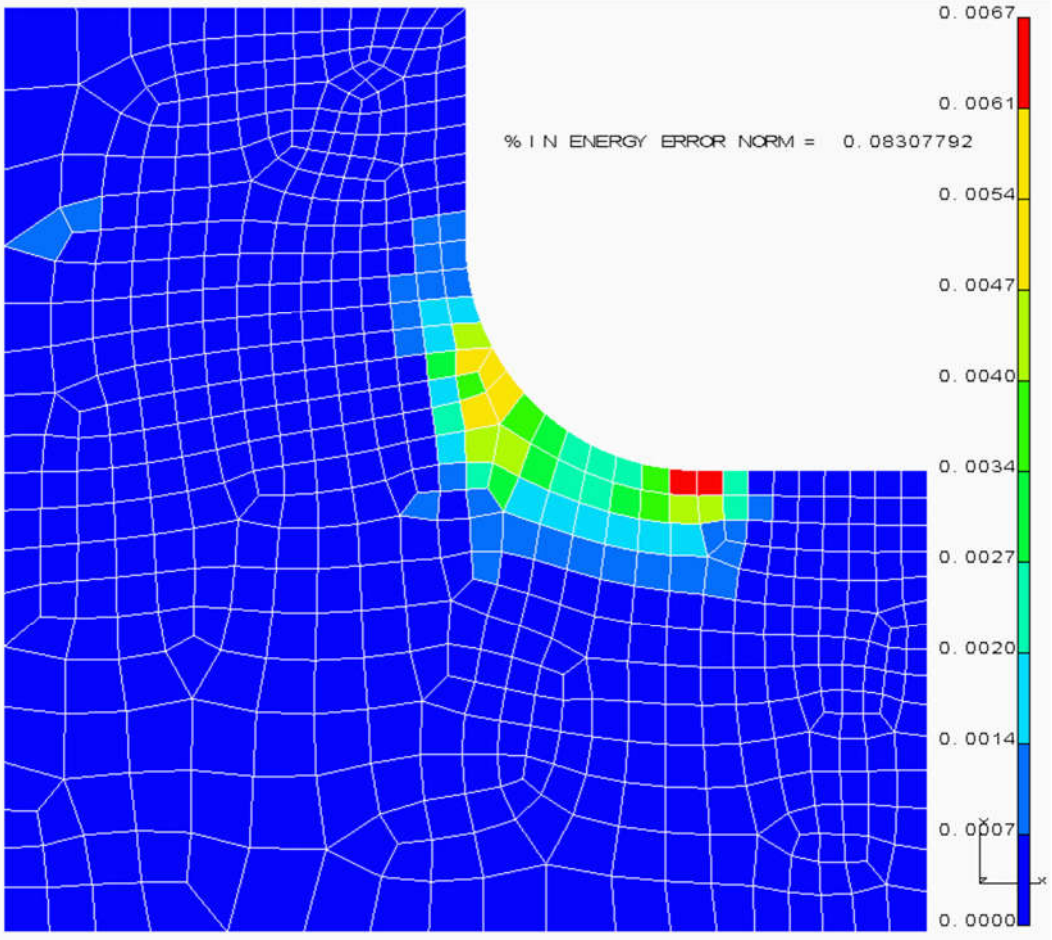


N=45

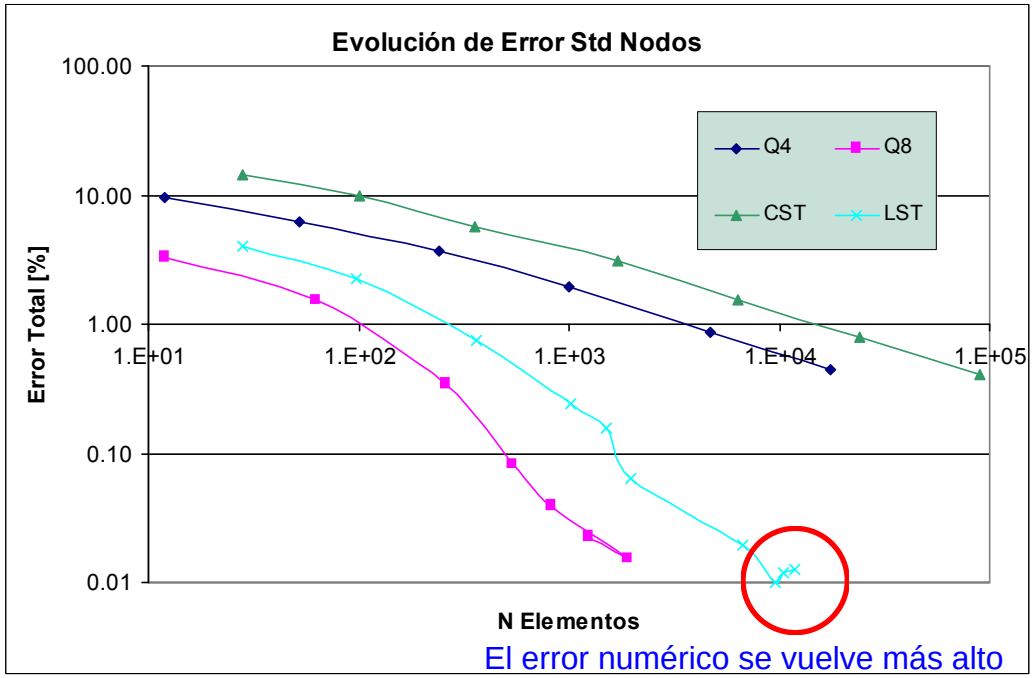


N=505

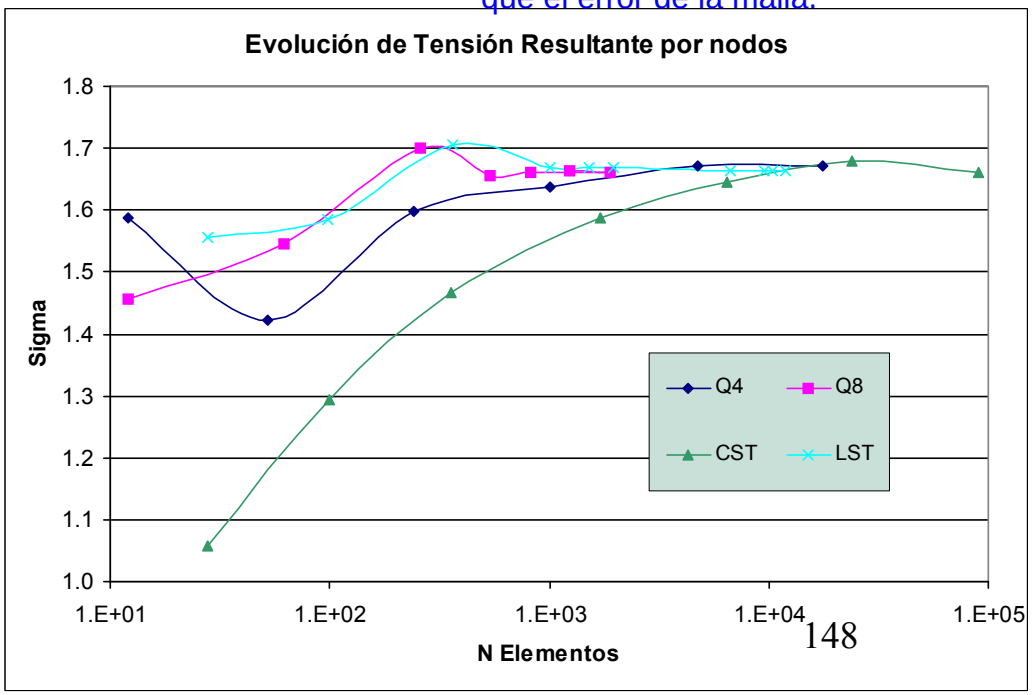
Refinamiento R - Adaptivo



Los cuadrilateros convergen a la misma velocidad que los triangulares y tiene menos error.



El error numérico se vuelve más alto que el error de la malla.



Estrategias de Remallado

Malla Optima - Tetrahedros