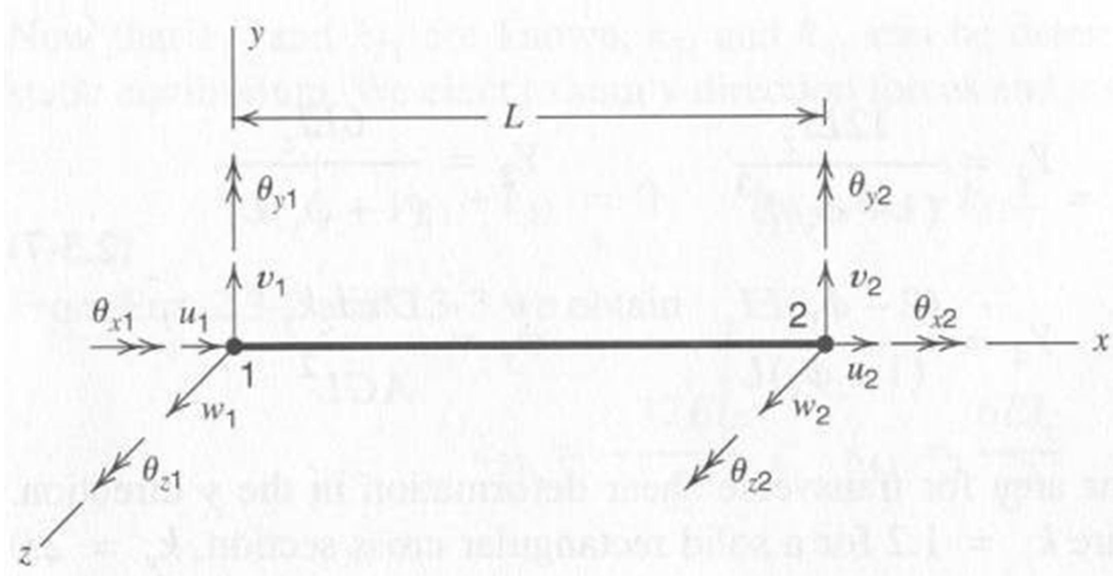
Elemento Viga – Generalizado



Esfuerzo Axial

$$[K]_{axial} = \begin{array}{c} \begin{bmatrix} X & -X \\ -X & X \end{bmatrix} \end{array} \begin{array}{c} u_1 \\ u_2 \end{array} \quad ; X = \frac{AE}{L}$$

Flexión Plano Z

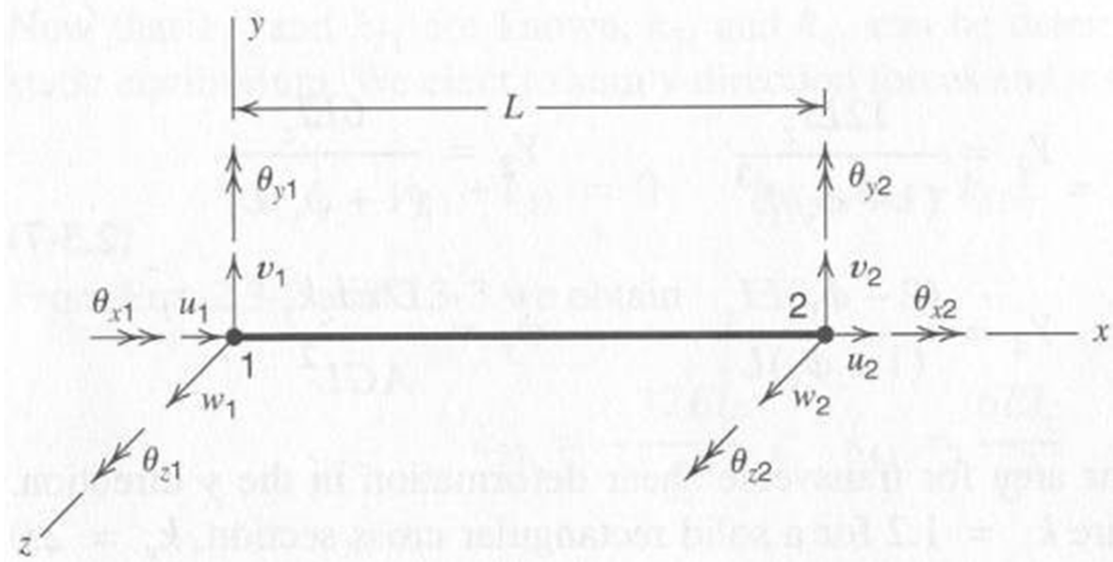
$$[K] = \begin{bmatrix} Y_1 & Y_2 & -Y_1 & Y_2 \\ Y_2 & Y_3 & -Y_2 & Y_4 \\ -Y_1 & -Y_2 & Y_1 & -Y_2 \\ Y_2 & Y_4 & -Y_2 & Y_3 \end{bmatrix} \begin{array}{c} v_1 \\ \theta_{z1} \\ v_2 \\ \theta_{z2} \end{array}$$

$$Y_1 = 12\frac{EI_z}{L^3}; \quad Y_2 = 6\frac{EI_z}{L^2};$$

$$Y_3 = 4\frac{EI_z}{L}; \quad Y_4 = 2\frac{EI_z}{L}$$

$$[\mathbf{k}] = \left[\begin{array}{cccccc:cccccc} X & 0 & 0 & 0 & 0 & 0 & -X & 0 & 0 & 0 & 0 & 0 \\ & Y_1 & 0 & 0 & 0 & Y_2 & 0 & -Y_1 & 0 & 0 & 0 & Y_2 \\ & & Z_1 & 0 & -Z_2 & 0 & 0 & 0 & -Z_1 & 0 & -Z_2 & 0 \\ & & & S & 0 & 0 & 0 & 0 & 0 & -S & 0 & 0 \\ & & & & Z_3 & 0 & 0 & 0 & Z_2 & 0 & Z_4 & 0 \\ & & & & & Y_3 & 0 & -Y_2 & 0 & 0 & 0 & Y_4 \\ \hdashline & & & & & & X & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & Y_1 & 0 & 0 & 0 & -Y_2 \\ & & \text{symmetric} & & & & & & Z_1 & 0 & Z_2 & 0 \\ & & & & & & & & & S & 0 & 0 \\ & & & & & & & & & & Z_3 & 0 \\ & & & & & & & & & & & Y_3 \end{array}\right] \begin{array}{c} u_1 \\ v_1 \\ w_1 \\ \theta_{x1} \\ \theta_{y1} \\ \theta_{z1} \\ u_2 \\ v_2 \\ w_2 \\ \theta_{x2} \\ \theta_{y2} \\ \theta_{z2} \end{array}$$

Elemento Viga – Generalizado



Esfuerzo Torsor

$$[K]_{\text{Torsor}} = \begin{array}{c} \begin{bmatrix} S & -S \\ -S & S \end{bmatrix} \end{array} \begin{array}{c} \theta_{x1} \\ \theta_{x2} \end{array} \quad ; S = \frac{GK}{L}$$

$$[\mathbf{k}] = \left[\begin{array}{cccccc|cccccc} X & 0 & 0 & 0 & 0 & 0 & -X & 0 & 0 & 0 & 0 & 0 \\ & Y_1 & 0 & 0 & 0 & Y_2 & 0 & -Y_1 & 0 & 0 & 0 & Y_2 \\ & & Z_1 & 0 & -Z_2 & 0 & 0 & 0 & -Z_1 & 0 & -Z_2 & 0 \\ & & & S & 0 & 0 & 0 & 0 & 0 & -S & 0 & 0 \\ & & & & Z_3 & 0 & 0 & 0 & Z_2 & 0 & Z_4 & 0 \\ & & & & & Y_3 & 0 & -Y_2 & 0 & 0 & 0 & Y_4 \\ \hline & & & & & & X & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & Y_1 & 0 & 0 & 0 & -Y_2 \\ & & \text{symmetric} & & & & & & Z_1 & 0 & Z_2 & 0 \\ & & & & & & & & & S & 0 & 0 \\ & & & & & & & & & & Z_3 & 0 \\ & & & & & & & & & & & Y_3 \end{array}\right] \begin{array}{c} u_1 \\ v_1 \\ w_1 \\ \theta_{x1} \\ \theta_{y1} \\ \theta_{z1} \\ u_2 \\ v_2 \\ w_2 \\ \theta_{x2} \\ \theta_{y2} \\ \theta_{z2} \end{array}$$

Flexión Plano Y

$$[K] = \begin{bmatrix} Z_1 & Z_2 & -Z_1 & Z_2 \\ Z_2 & Z_3 & -Z_2 & Z_4 \\ -Z_1 & -Z_2 & Z_1 & -Z_2 \\ Z_2 & Z_4 & -Z_2 & Z_3 \end{bmatrix} \begin{array}{c} w_1 \\ \theta_{y1} \\ w_2 \\ \theta_{y2} \end{array}$$

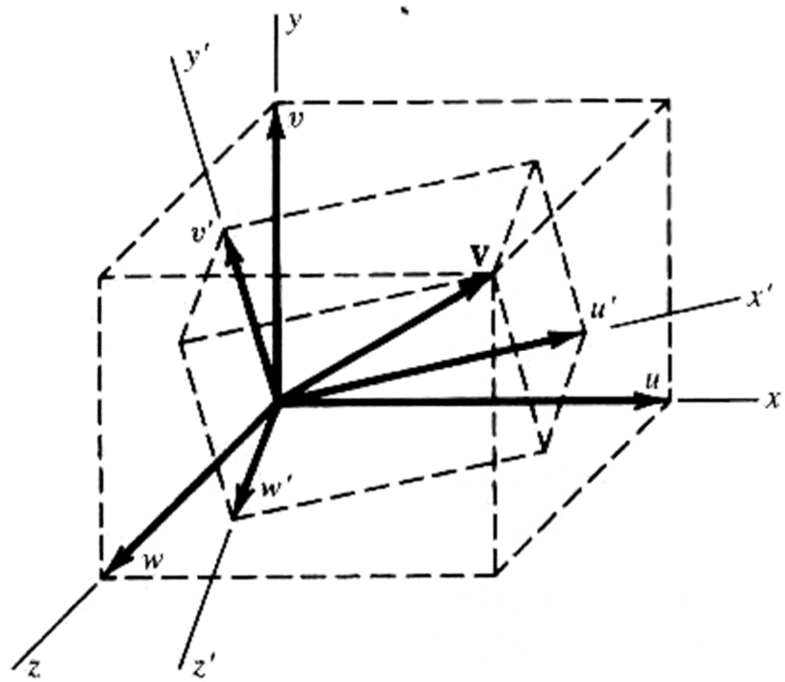
$$Z_1 = 12\frac{EI_y}{L^3}; \quad Z_2 = 6\frac{EI_y}{L^2};$$

$$Z_3 = 4\frac{EI_y}{L}; \quad Z_4 = 2\frac{EI_y}{L}$$

Elemento Viga – Rotación

$$\{d'\} = [T]\{d\} \rightarrow \begin{cases} u_1' = l_1 u_1 + m_1 v_1 + n_1 w_1 \\ v_1' = l_2 u_1 + m_2 v_1 + n_2 w_1 \\ w_1' = l_3 u_1 + m_3 v_1 + n_3 w_1 \end{cases}$$

$$\begin{Bmatrix} u_1' \\ v_1' \\ w_1' \end{Bmatrix} = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ w_1 \end{Bmatrix} \; ; \; [\Lambda] = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$$



Direction cosines between axes:

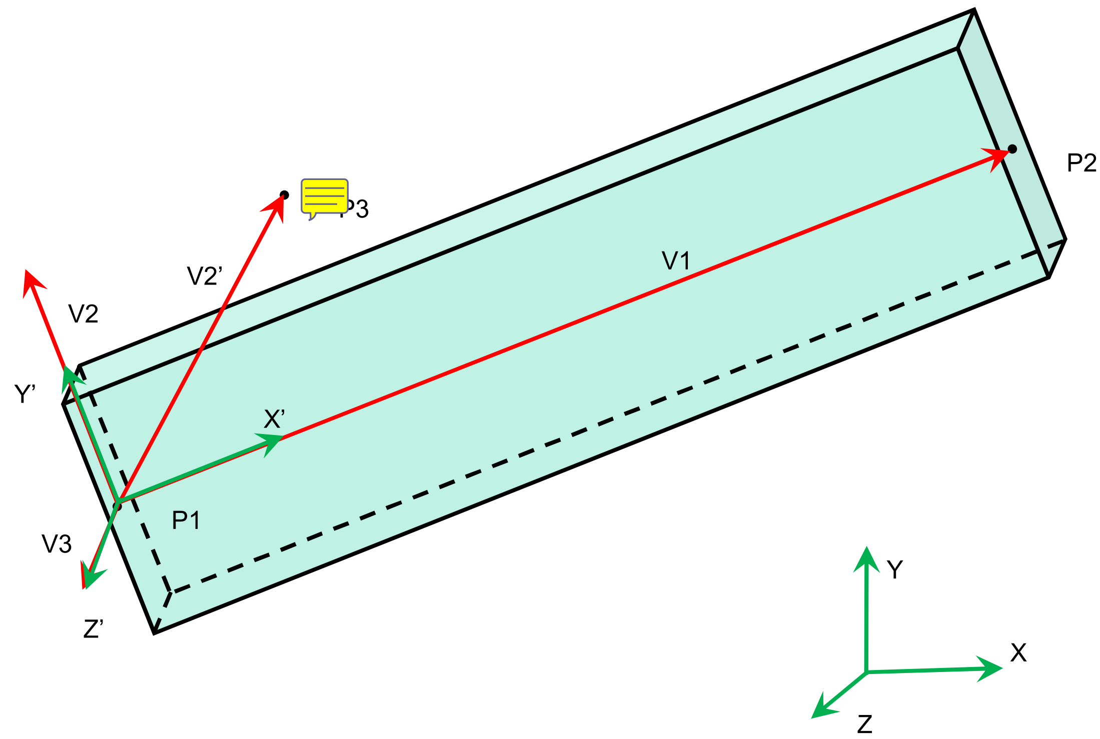
	x	y	z
x'	l_1	m_1	n_1
y'	l_2	m_2	n_2
z'	l_3	m_3	n_3

$$\theta_{x1}' = l_1\theta_{x1} + m_1\theta_{y1} + n_1\theta_{z1}$$
$$\theta_{y1}' = l_2\theta_{x1} + m_2\theta_{y1} + n_2\theta_{z1}$$
$$\theta_{z1}' = l_3\theta_{x1} + m_3\theta_{y1} + n_3\theta_{z1}$$

$$\begin{Bmatrix} \theta_{x1}' \\ \theta_{y1}' \\ \theta_{z1}' \end{Bmatrix} = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{Bmatrix} \theta_{x1} \\ \theta_{y1} \\ \theta_{z1} \end{Bmatrix} \; ; \; [\Lambda] = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \quad \Rightarrow \quad [T] = \begin{bmatrix} \Lambda & 0 & 0 & 0 \\ 0 & \Lambda & 0 & 0 \\ 0 & 0 & \Lambda & 0 \\ 0 & 0 & 0 & \Lambda \end{bmatrix}$$

Elemento Viga – Ejemplo

$$\frac{P2-P1}{|P2-P1|}=V1=\begin{Bmatrix}-0.3536\\0.7071\\0.6124\end{Bmatrix}$$



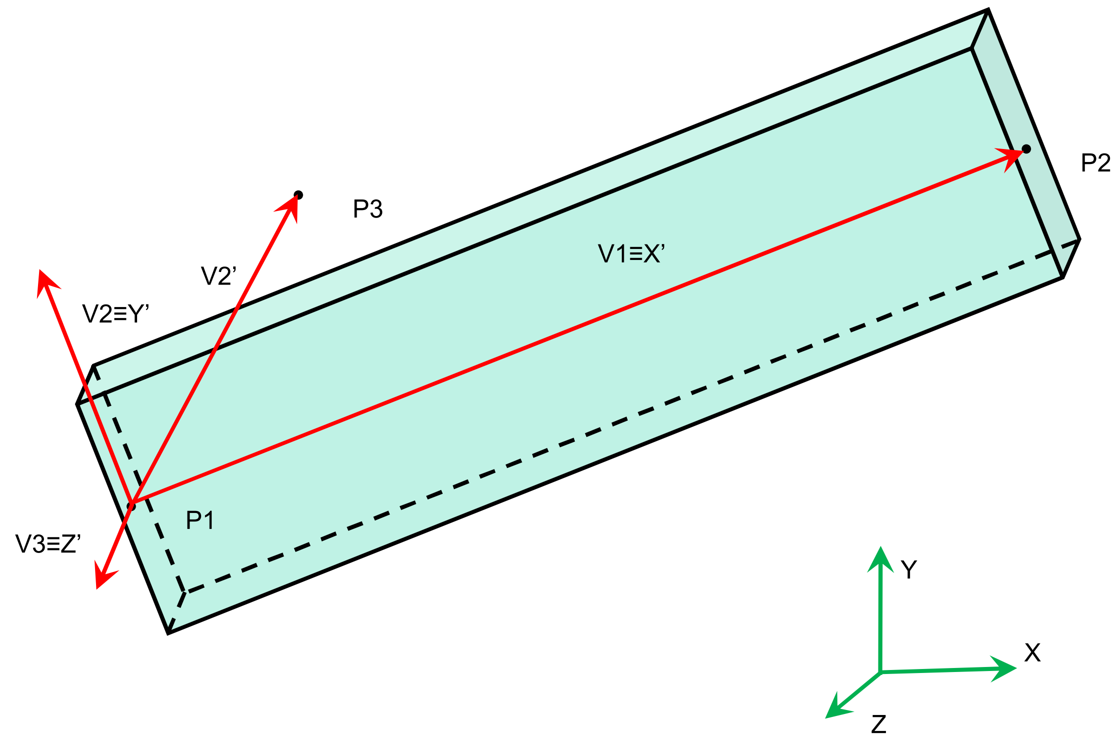
$$P3-P1=V2'$$

$$\frac{\mathrm{V1xV2'}}{|\mathrm{V1xV2'}|}=\mathrm{V3}=\begin{Bmatrix}0.8660\\0.0\\0.5\end{Bmatrix}$$

$$\begin{Bmatrix}x'\\y'\\z'\end{Bmatrix}=\begin{bmatrix}-0.3536 & 0.7071 & 0.6124\\-0.3536 & -0.7071 & 0.6124\\0.8660 & 0.0 & 0.5\end{bmatrix}\begin{Bmatrix}x\\y\\z\end{Bmatrix}$$

$$\mathrm{V3xV1}=\mathrm{V2}=\begin{Bmatrix}-0.3536\\-0.7071\\0.6124\end{Bmatrix}$$

Elemento Viga – Ejemplo

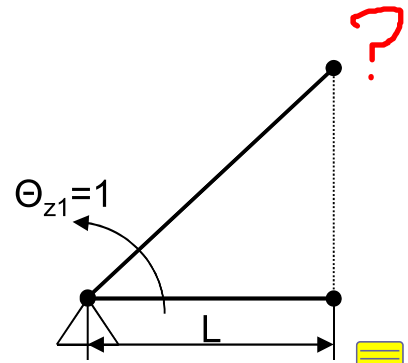


$$Sección\begin{cases} a = 20 \\ b = 10 \end{cases} \Rightarrow \quad \begin{array}{ll} K = \beta ab^3 = 0.229 \;\; 20000 = 4580 & I_{Z'} = \dfrac{ba^3}{12} = 6666 \\ A = ab = 200 & I_{Y'} = \dfrac{ab^3}{12} = 1666 \end{array}$$

Desplazamientos Rígidos

Valores Nodales

$$[K]\{D\}=\{R\}$$



$$\begin{bmatrix} 12\dfrac{EI_z}{L^3} & 6\dfrac{EI_z}{L^2} & -12\dfrac{EI_z}{L^3} & 6\dfrac{EI_z}{L^2} \\ 6\dfrac{EI_z}{L^2} & 4\dfrac{EI_z}{L} & -6\dfrac{EI_z}{L^2} & 2\dfrac{EI_z}{L} \\ -12\dfrac{EI_z}{L^3} & -6\dfrac{EI_z}{L^2} & 12\dfrac{EI_z}{L^3} & -6\dfrac{EI_z}{L^2} \\ 6\dfrac{EI_z}{L^2} & 2\dfrac{EI_z}{L} & -6\dfrac{EI_z}{L^2} & 4\dfrac{EI_z}{L} \end{bmatrix} \begin{Bmatrix} 0 \\ 1 \\ v_2 \\ \theta_{z2} \end{Bmatrix} = \begin{Bmatrix} F_1 \\ M_1 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} -6\dfrac{EI_z}{L^2} \\ 2\dfrac{EI_z}{L} \end{bmatrix} + \begin{bmatrix} 12\dfrac{EI_z}{L^3} & -6\dfrac{EI_z}{L^2} \\ -6\dfrac{EI_z}{L^2} & 4\dfrac{EI_z}{L} \end{bmatrix} \begin{Bmatrix} v_2 \\ \theta_{z2} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} v_2 \\ \theta_{z2} \end{Bmatrix} = \frac{L^4}{12} \begin{bmatrix} \dfrac{4}{L} & \dfrac{6}{L^2} \\ \dfrac{6}{L^2} & \dfrac{12}{L^3} \end{bmatrix} \begin{Bmatrix} \dfrac{6}{L^2} \\ \dfrac{-2}{L} \end{Bmatrix} = \frac{L^4}{12} \begin{Bmatrix} \dfrac{4}{L}\dfrac{6}{L^2} + \dfrac{6}{L^2}\dfrac{-2}{L} \\ \dfrac{6}{L^2}\dfrac{6}{L^2} + \dfrac{12}{L^3}\dfrac{-2}{L} \end{Bmatrix} = \begin{Bmatrix} L \\ 1 \end{Bmatrix}$$

Deformada

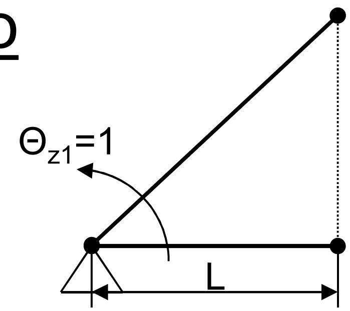
$$v(x) = [N]\begin{Bmatrix} v_2 \\ \theta_{z2} \\ v_3 \\ \theta_{z3} \end{Bmatrix} = \begin{Bmatrix} 1-\dfrac{3x^2}{L^2}+\dfrac{2x^3}{L^3} \\ x-\dfrac{2x^2}{L}+\dfrac{x^3}{L^2} \\ \dfrac{3x^2}{L^2}-\dfrac{2x^3}{L^3} \\ -\dfrac{x^2}{L}+\dfrac{x^3}{L^2} \end{Bmatrix}^T \begin{Bmatrix} 0 \\ 1 \\ L \\ 1 \end{Bmatrix}$$

$$v(x) = x-\frac{2x^2}{L}+\frac{x^3}{L^2}+L\left(\frac{3x^2}{L^2}-\frac{2x^3}{L^3}\right)-\frac{x^2}{L}+\frac{x^3}{L^2}=x$$

Reacciones

$$\{R\}=\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Condiciones de Borde – Desplazamiento



$$[K]\{D\} = \{R\}$$

$$\begin{bmatrix} 12\frac{EI_z}{L^3} & 6\frac{EI_z}{L^2} & -12\frac{EI_z}{L^3} & 6\frac{EI_z}{L^2} \\ 6\frac{EI_z}{L^2} & 4\frac{EI_z}{L} & -6\frac{EI_z}{L^2} & 2\frac{EI_z}{L} \\ -12\frac{EI_z}{L^3} & -6\frac{EI_z}{L^2} & 12\frac{EI_z}{L^3} & -6\frac{EI_z}{L^2} \\ 6\frac{EI_z}{L^2} & 2\frac{EI_z}{L} & -6\frac{EI_z}{L^2} & 4\frac{EI_z}{L} \end{bmatrix} \begin{Bmatrix} 0 \\ 1 \\ v_2 \\ \theta_{z2} \end{Bmatrix} = \begin{Bmatrix} F_1 \\ M_1 \\ 0 \\ 0 \end{Bmatrix}$$

$$\{D_x\} = [K_{xx}]^{-1}\left(\{R_c\} - [K_{xc}]\{D_c\}\right)$$

$$\begin{Bmatrix} v_2 \\ \theta_{z2} \end{Bmatrix} = \begin{bmatrix} 12\frac{EI_z}{L^3} & -6\frac{EI_z}{L^2} \\ -6\frac{EI_z}{L^2} & 4\frac{EI_z}{L} \end{bmatrix}^{-1} \left(\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \begin{bmatrix} -12\frac{EI_z}{L^3} & -6\frac{EI_z}{L^2} \\ 6\frac{EI_z}{L^2} & 2\frac{EI_z}{L} \end{bmatrix} \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \right) = \begin{Bmatrix} L \\ 1 \end{Bmatrix}$$

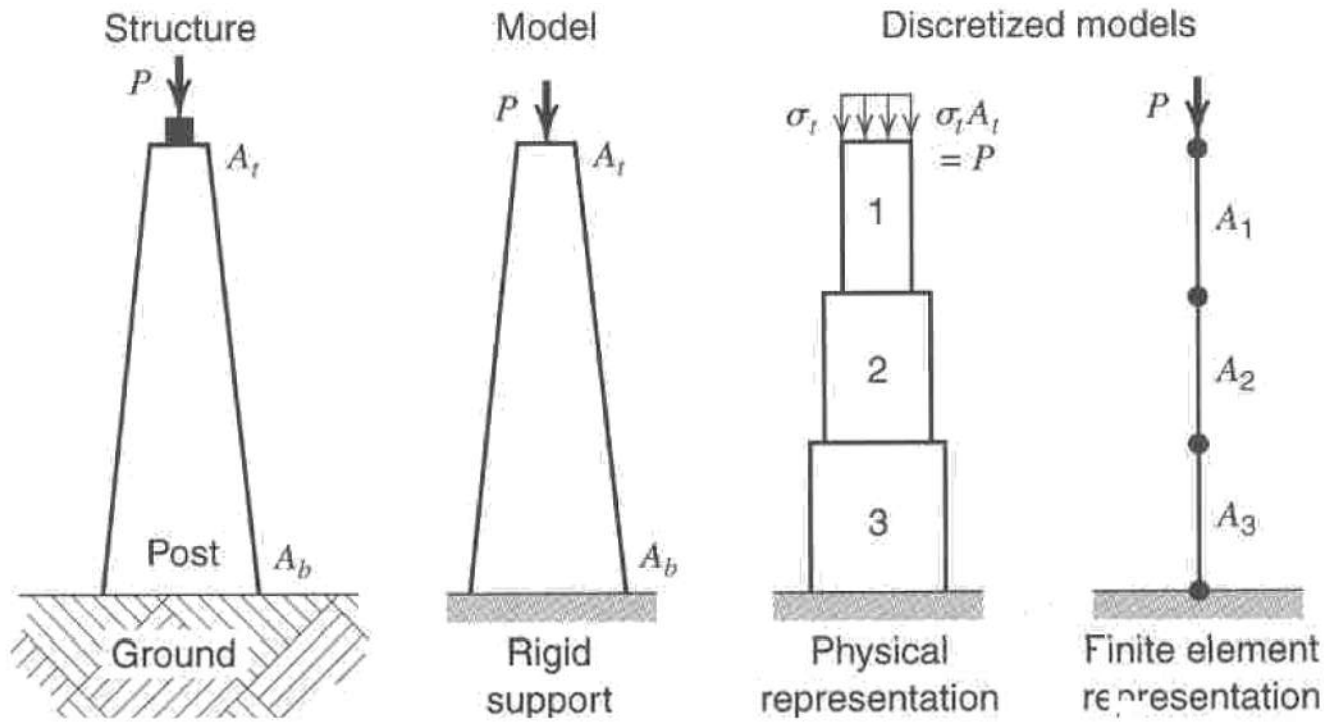
$$\{R_x\} = [K_{cc}]\{D_c\} + [K_{cx}]\{D_x\}$$

$$\begin{Bmatrix} F_1 \\ M_1 \end{Bmatrix} = \begin{bmatrix} 12\frac{EI_z}{L^3} & 6\frac{EI_z}{L^2} \\ 6\frac{EI_z}{L^2} & 4\frac{EI_z}{L} \end{bmatrix} \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} + \begin{bmatrix} -12\frac{EI_z}{L^3} & 6\frac{EI_z}{L^2} \\ -6\frac{EI_z}{L^2} & 2\frac{EI_z}{L} \end{bmatrix} \begin{Bmatrix} L \\ 1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Modelado Matemático

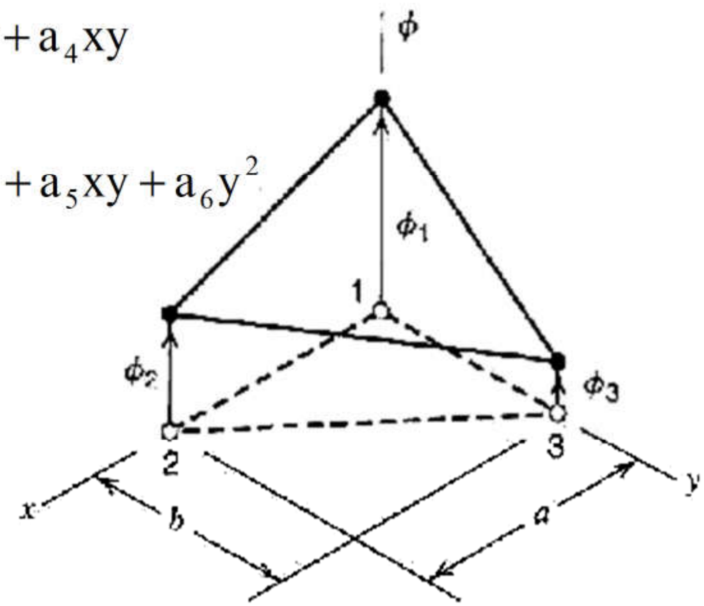
Pasos

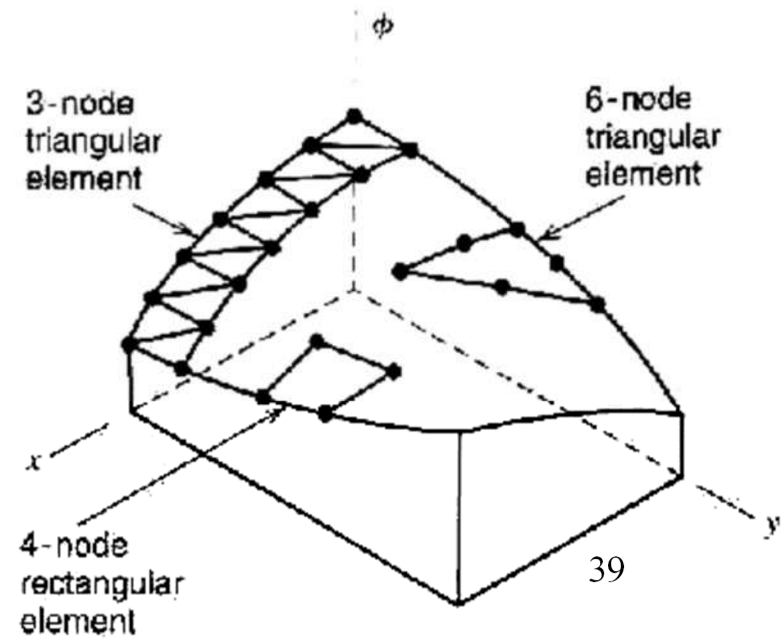
- Problema Físico
- Modelo Matemático
- Modelo Numérico



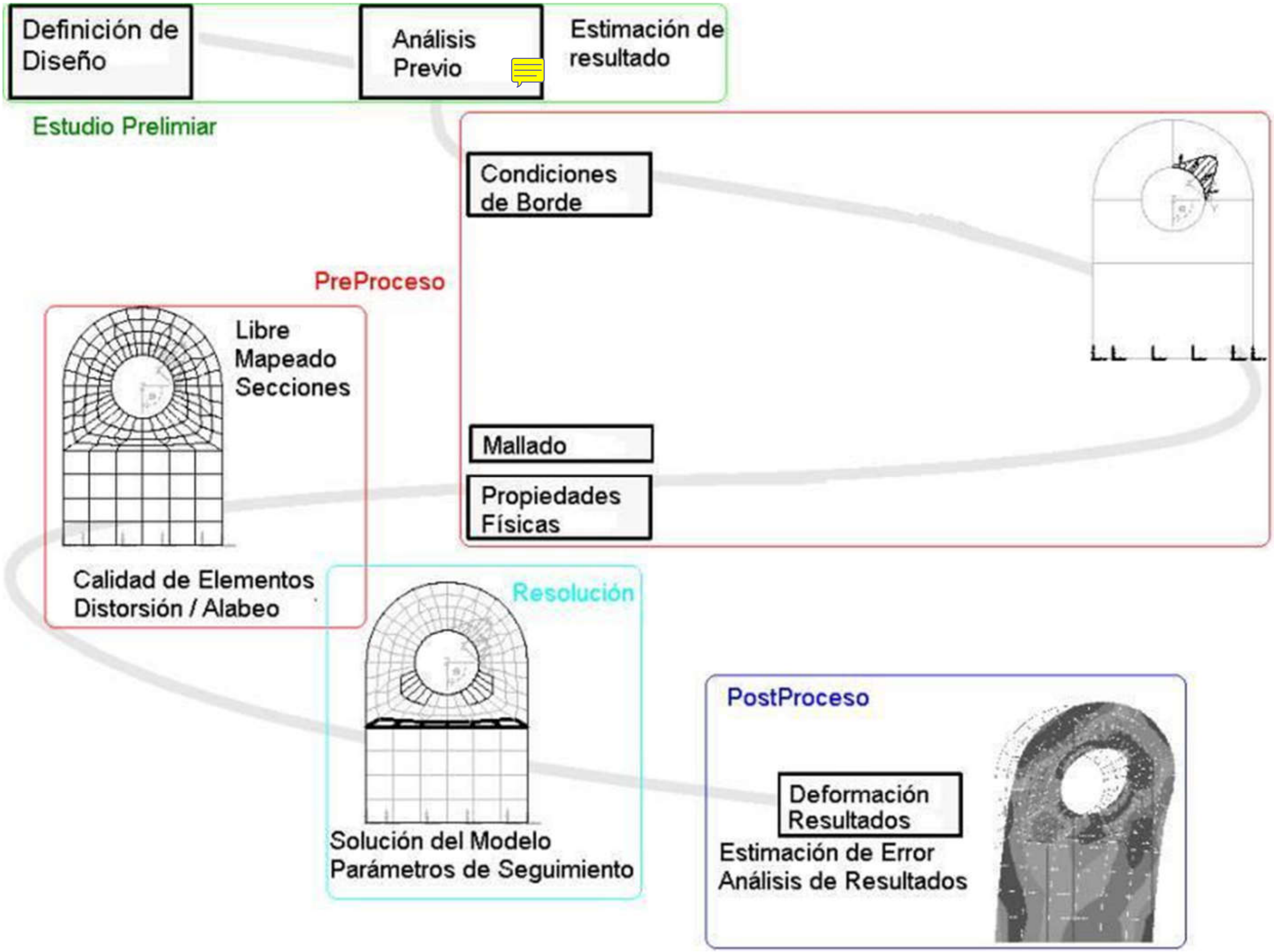
Funciones de Interpolación

$$\phi = \begin{cases} a_1 + a_2 x + a_3 y \\ a_1 + a_2 x + a_3 y + a_4 xy \\ \vdots \\ a_1 + a_2 x + a_3 y + a_4 x^2 + a_5 xy + a_6 y^2 \end{cases}$$





Modelado por Elementos Finitos



Historia del Método

•1943: Uso de elementos triangulares para problema de Torsión (St. Venant).
R. Courant, "Variational Methods for the Solutions of Problems of Equilibrium and Vibrations," *Bull. Am. Math.*, 1943.

•1954 – 60: Solución computacional de problemas lineales de analisis estructural
J. H. Argyris, "Energy Theorems and Structural Analysis," *Aircraft Eng*., 1954.
J. H. Argyris, "Energy Theorems and Structural Analysis,", *Aircraft Eng*., 1955.
J. H. Argyris, "The Matrix Theory of Statics" , *Ingenieur Archiv*, 1957.
J. H. Argyris, "The Analysis of Fuselages of Arbitrary Cross-Section and Taper,"*Aircraft Eng*., 1959.
J. H. Argyris and S. Kelsey, *Energy Theorems and Structural Analysis*, Butter-worth, 1960.

•1953: IBM Crea la primer computadora de propósitos generales.

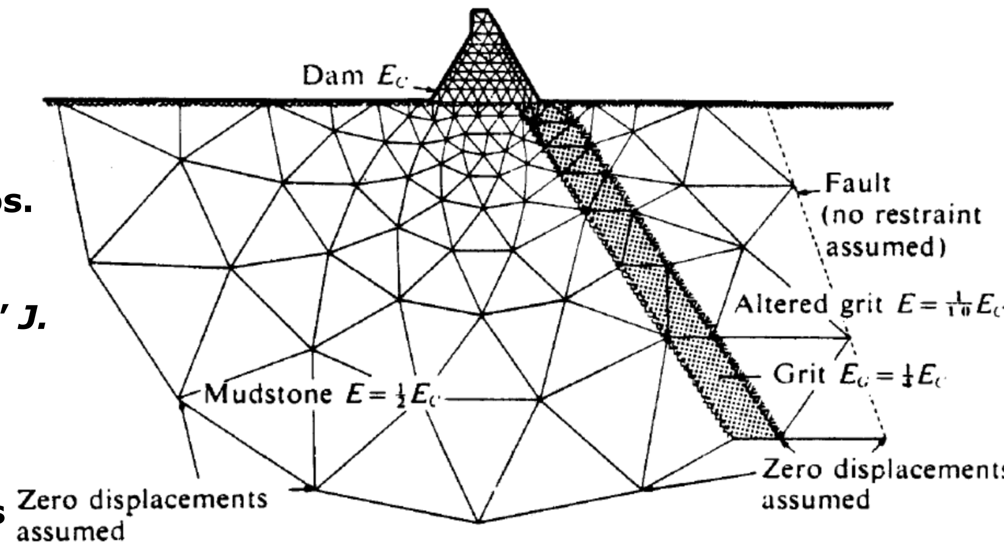
•1954: Nace Fortran.

•1956: Método directo para desarrollo de elementos finitos. Boeing.
•M. J. Turner, R. W. Clough, H. C. Martin, and L. C. Topp, "Stiffness and Deflection Analysis of Complex Structures," *J. Aeronaut. Sci*., 1956.

•1960: Estudios de elasticidad plana. Nombre "Elemento Finito".
•R. W. Clough, "The Finite Element Method in Plane Stress Analysis," *Proceedings of 2nd ASCE Conf. on Elect. Comp.*, 1960.

•1964: Control Data Corporation desarrolla el primer software

•1965: Formulación variacional. Aplicación a problemas de campo.
•O. C. Zienkiewicz and Y. K. Cheung, "Finite Elements in the Solution of Field Problems," *Engineer*, 1965.



Zienkiewicz, O. C. "Origins, milestones and directions of the finite element method— A personal view", Archives of Computational Methods in Engineering Volume 2, Issue 1 , pp 1-48

Historia del Método

•1964: NASA solicita desarrollo de software para análisis de estructuras. Nace NASA STRucture ANalysis (1968)

•1971: NASTRAN liberado al público. Se expanden las aplicaciones

•1972: Patch Test (Irons and Razzaque 1972)

•1978: Estimación del error. (Babuška and Rheinboldt 1978,79)

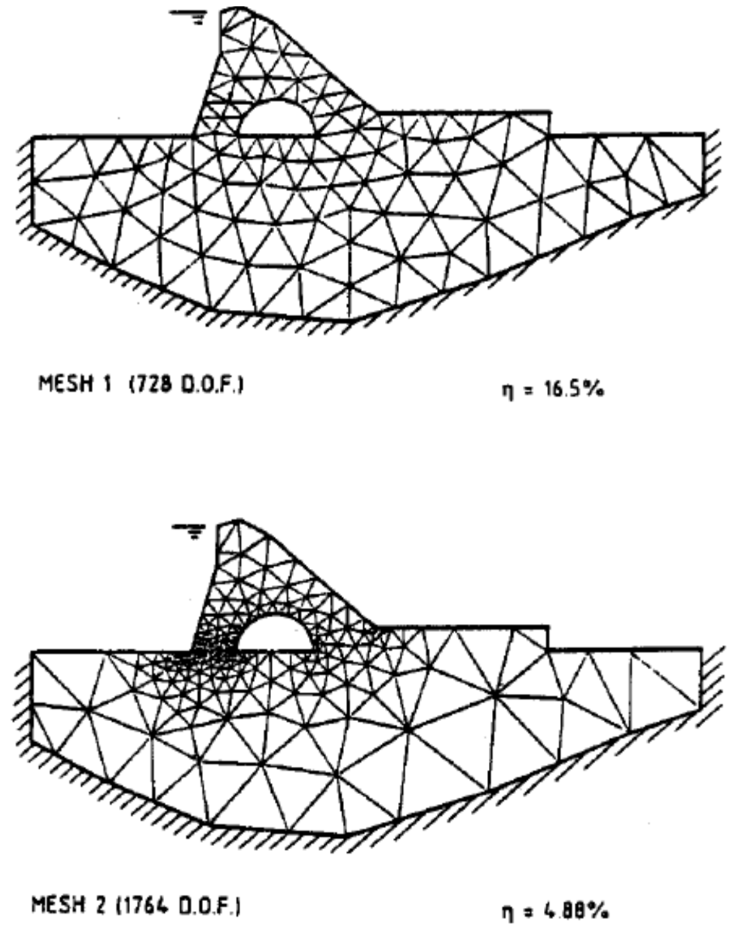
•1980's: Desarrollo de workstations y software comercial.

•1987: Adaptividad (Zienkiewicz and Zhu 1987, 89)

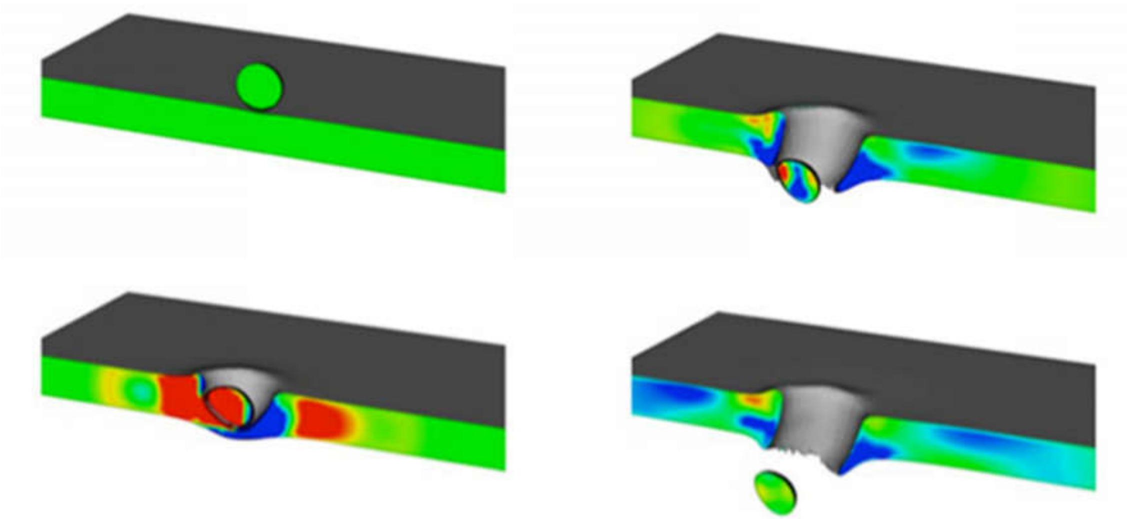
•1990's: Computación personal.

•2000's: Multiproceso, Multifísica, Multiescala, Multi…

•2010's: Límites experimentales. The further development of a general Optimal Uncertainty Quantification (OUQ) framework (cf. H. Owhadi, C. Scovel, T. J. Sullivan, M. McKerns, and M. Ortiz, "Optimal uncertainty quantification," arXiv:1009.0679, 2010).



Zienkiewicz, O. C. "Origins, milestones and directions of the finite element method— A personal view", Archives of Computational Methods in Engineering Volume 2, Issue 1 , pp 1-48



Simulated 3-D oblique impact of a 440C steel spherical projectile on an Al-6061 plate

http://www.psaap.caltech.edu/