Formuleo Turbomáquinas

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Ecuaciones 1 (Termodinámica) *Ecuaciones Cardinales:* $T\,ds = dh - v\,dp \parallel T\,ds = du + p\,dv$

Ecuaciones 2 (Modelo gas ideal) *Gas Ideales:* $\tilde{R} = 8,314$*[J/mol/K]*$\parallel \tilde{R} = \bar{M}R$, $\bar{M}$*[kg/mol]*$\parallel pV = mRT \parallel pv = RT \parallel pV = N\tilde{R}T \parallel a = \sqrt{Z_a kRT} \parallel T_0 = T\left(1 + \frac{k-1}{2}M^2\right) \parallel p_0 = p\left(1 + \frac{k-1}{2}M^2\right)^{\frac{k}{k-1}} \parallel \rho_0 = \rho\left(1 + \frac{k-1}{2}M^2\right)^{\frac{1}{k-1}}$

Procesos incompresibles: $s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_v(T)}{T} dT$

Procesos compresibles isoentropicos: $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = \left(\frac{v_1}{v_2}\right)^{k-1} \parallel \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{k} \parallel \left(\frac{T_1}{T_2}\right)^{\frac{1}{k-1}} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{k}} = \frac{v_2}{v_1}$

Procesos compresibles isoentropicos: $p_0 = cte = p + \frac{1}{2}\rho c^2 \parallel h_0 = h + \frac{1}{2}c^2$

Gas perfecto: $h = c_pT$; $c_p = kR/(k-1) \rightarrow T_0 = T + \frac{1}{2}c^2/c_p \parallel s_x^0 = s_x - s_0 = \int_{T_0}^{T_x} \frac{c_p}{T}\mathrm{d}T \rightarrow s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$

Ecuaciones 3 (Fundamentales de turbomáquinas) $\frac{\dot{W}}{\dot{m}} = U_1c_{\theta 1} - U_2c_{\theta 2} = \left(\frac{w_2^2 - w_1^2}{2}\right) + \left(\frac{c_1^2 - c_2^2}{2}\right) + \left(\frac{U_1^2 - U_2^2}{2}\right) \parallel$

$\eta_{tob} = \frac{\text{Energía cinética actual de salida}}{\text{Energía cinética si el proceso fuera reversible}} = \frac{1 - T_2/T_{01}}{1 - T_{2s}/T_{01}} = \frac{\frac{1}{2}c_2^2}{\frac{1}{2}c_{2s}^2} = K_{tob}^2 \parallel \eta_{dif} = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{c_1^2 - c_{2S}^2}{c_1^2 - c_2^2} = \frac{(p_2/p_1)^{(k-1)/k} - 1}{[(p_{01}/p_{02})(p_2/p_1)]^{(k-1)/k} - 1}$

$\parallel$ *Grado de reacción:* $\boldsymbol{R} = \frac{\text{Salto entálpico en rotor}}{\text{Salto entálpico total en etapa (rotor + estator)}} = \frac{\Delta h_{\text{rotor}}}{\Delta h_{\text{etapa}}} \parallel$ *Maq. Hidraulicas:* $\boldsymbol{R} = \frac{p_3 - p_2}{p_3 - p_1} \parallel$ *Pelton:* $F_x = \rho A(c-u)^2(1 - \cos\theta) \parallel \eta_{\text{generadora}} = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}} = \frac{\text{Trabajo adiabatico mínimo por seg.}}{\text{Trabajo adiabatico hecho por seg.}} \parallel$ *Turbinas:* $\eta_{\text{motora}} = \frac{h_{02} - h_{01}}{h_{02s} - h_{01}}$

Ecuaciones 4 (Centrifugos) *Adimensionalizacion dinámica:* $\pi_1 = \frac{p_{02}}{p_{01}}$, $\pi_2 = \dot{m}\frac{\sqrt{RT_{01}}}{D^2P_{01}}$, $\pi_3 = \frac{\Omega}{\sqrt{RT_{01}}}$, $\pi_4 = \eta_{\text{iso}} \parallel \tan\beta_1 = \frac{c_{a1}}{U_1}$ $\parallel \xi = \frac{c_{\theta d}}{c_\theta} = \frac{c_\theta - u_d}{c_\theta} \parallel D_{\text{eddy}} \simeq \frac{\pi D_2 \cos\beta_2}{Z} \parallel \frac{\dot{W}_d}{\dot{m}} = e_d = h_{02} - h_{01} = c_{\theta 2d}U_2 - c_{\theta 1}U_1 = (\xi c_{\theta 2})U_2 - c_{\theta 1}U_1 \parallel \frac{p_{03}}{p_{01}} = \left(\frac{T_{03s}}{T_{01}}\right)^{\frac{k}{k-1}} = \left[1 + \frac{\eta_{\text{iso}}(\xi c_{\theta 2}U_2 - c_{\theta 1}U_1)}{c_pT_{01}}\right]^{\frac{k}{k-1}} \parallel T_{03} - T_{01} = \frac{\xi c_{\theta 2}U_2 - c_{\theta 1}U_1}{c_p} \parallel \mathrm{M}_1 = \frac{c_{a1}}{a} = \frac{c_{a1}}{\sqrt{kRT_1}} \parallel \mathrm{M}_2 = \frac{c_{2d}}{\sqrt{Z_2kRT_2}} \parallel$ *Coef. Lift:* $\mathrm{C_L} = \frac{L}{\frac{1}{2}\rho c^2}bt$; *$t$ es cuerda, b es long. de perfil*

Ecuaciones 5 (Axiales) $\nu = \frac{r_{\text{ext}}}{r_{\text{base}}} \parallel \frac{\dot{W}}{\dot{m}} = Uc_a(\cos\beta_2 - \cos\beta_1) \parallel$ *Isoentropico:* $h_3 - h_1 = \frac{p_3 - p_1}{\rho} = U(c_{\theta 3} - c_{\theta 1}) \parallel$

$\mu = \frac{\Delta T_{\text{real}}}{\Delta T_{\text{euler}}}$ *tal que* $\dot{W}_{\text{real}} = \mu\dot{W} \parallel \frac{p_{03}}{p_{01}} = \left[1 + \frac{\eta_{\text{iso}}\mu}{c_pT_{01}}Uc_a(\tan\beta_1 - \tan\beta_2)\right]^{\frac{k}{k-1}} \parallel \boldsymbol{R} = \frac{h_2 - h_1}{h_3 - h_1} \rightarrow \rho = cte \Rightarrow \boldsymbol{R} = \frac{p_3}{p_1}$

Ecuaciones 6 (Alternativos) $\varepsilon_{vol} = 1 - C(r_c^{\frac{1}{n}} - 1) = \frac{V_1 - V_4}{V_1 - V_3} \parallel r_{\text{cmáx}} = (1 + C^{-1})^n \parallel \varepsilon_{term} = T_{asp}/T_{int} \parallel \varepsilon_{pca} = \frac{P_{asp}}{P_{int}}$ $\parallel \varepsilon_{fugas} = \frac{1}{1+f} \parallel \eta_{vol} = \prod_i^4 \varepsilon_i \parallel \dot{V} = \pi r^2 L_{carr} f_{hz} X \eta_{\text{vol}} \parallel \dot{m} = \frac{p\dot{V}}{Z_eR} \parallel p_{int} = p_e\frac{A_p}{A_v} - \frac{kx}{A_v} - \Delta p$

Ecuaciones 7 (Refrigeración) *Sin refrigeración:* $\dot{W}_{\text{ad/rev}} = \dot{m}\left[\frac{k}{k-1}R(T_{\text{out}} - T_{\text{in}})\right] \parallel$

Refrig. intermedia:$\dot{W}_{\text{pol/rev}} = \dot{m}\left\{\frac{n}{n-1}T_{\text{in}}R\left[\left(\frac{p_{\text{out}}}{p_{\text{in}}}\right)^{\frac{n-1}{n}} - 1\right]\right\} \parallel$ *Refrig. máxima:* $\dot{W}_{\text{isot/rev}} = \dot{m}RT_{\text{in}}\ln\left(\frac{p_{\text{out}}}{p_{\text{in}}}\right)$

Ecuaciones 8 (TdC) *Ecuación de calor:* $\nabla^2 T + \dot{q_G} = \rho c\frac{\partial T}{\partial t} \parallel q_k = -k \cdot A\frac{\Delta T}{\Delta x} \parallel$ *cilindricas:* $q_k = 2\pi Lk\frac{\Delta T}{\ln\frac{r_o}{r_i}} \parallel$ *esféricas:* $q_k = \frac{\Delta T}{\left(\frac{r_o - r_i}{4\pi k r_o r_i}\right)} \parallel$ *Resistencias:* $R_k = \frac{L}{kA}$, *cilindro:* $R_k = \frac{\ln\frac{r_o}{r_i}}{2\pi Lk}$, *esfera:* $R_k = \left(\frac{r_o - r_i}{4\pi k r_o r_i}\right) \parallel$ *Aletas:* $\eta = \frac{q}{q_{max}} = \frac{q}{hPL\Delta T_{b\infty}} \parallel$

$\alpha = \frac{k}{\rho c_p} \parallel$ *Modelo resistencia despreciable:* $\frac{\Delta T_{t\infty}}{\Delta T_{0\infty}} = e^{\mathrm{BiFo}} \parallel$ *Radiación:* $q_r = \sigma T^4 \parallel q_{r1\leftarrow 2} = A_1\mathcal{F}_{1,2}\sigma T^4; \mathcal{F}_{1,2} = f(\epsilon_2, \alpha_1, forma)$ $\parallel \rho + \alpha + \epsilon = 1 \parallel$ *Wien:* $\lambda_{\text{máx}}T = 2,8976 \times 10^{-3} \parallel r_{\text{crit}} = \frac{k}{h}$

Ecuaciones 9 (Adimensionales) $\mathrm{Re} = \frac{UL}{\nu} = \frac{\rho UL}{\mu} \parallel \mathrm{Nu} = \frac{\bar{h}_cL}{k_{\text{fluido}}} = \frac{\text{conveccion}}{\text{conduccion en fluido}}$

$\mathrm{Pr} = \frac{\mu c_p}{k} = \frac{\nu}{\alpha} = \frac{\text{difusion cantidad cantidad movimiento}}{\text{difusion calor}} \parallel \mathrm{Bi} = \frac{h\cdot L_c}{k} = \frac{R_k}{R_c} \parallel \mathrm{Fo} = \frac{\alpha t}{L^2} \parallel \mathrm{Gr} = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{\text{conveccion natural}}{\text{conveccion viscosa}}$

Ecuaciones 10 (Correlaciones TdC) *Correlaciones conveccion interna: Turb:* $\mathrm{Nu}_D = 0.023Re_D^{4/5}Pr^n \rightarrow n = 0,4$ *heating*, $n = 0,3$ *cooling* ‖ *Lam:* $q'' = cte, \mathrm{Nu}_D = 4,36 \rightarrow T_s = cte, \mathrm{Nu}_D = 3,66$ ‖ $D_H = \frac{4\cdot \textit{Seccion}}{\textit{Perimetro mojado}}$
"Integrables" Capa Límite: $\mathrm{Re}_{\mathrm{crit}} \approx 5 \times 10^5$, *Laminar:*$\mathrm{Nu}_x = \frac{h_x x}{k} = 0,332\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3}$ ‖ *Turbulento:* $\mathrm{Nu}_x = 0,0288\mathrm{Pr}^{1/3}\mathrm{Re}_x^{0,8}$ ‖ *Velocidad Altas Mach*$\gtrsim 1$ $T^* = T_\infty + 0,5(T_s - T_\infty) + 0,22(T_{as} - T_\infty)$ *donde* $T_{as}^{\mathrm{lam}} = T_\infty + \mathrm{Pr}^{1/2}(T_0 - T_\infty)$
$T_{as}^{\mathrm{turb}} = T_\infty + \mathrm{Pr}^{1/3}(T_0 - T_\infty)$ *Hi-speed laminar:* $\mathrm{St}_x^* = \left(\frac{h_{cx}}{c_p \rho U_\infty}\right)^* = 0,332(\mathrm{Re}_x^*)^{-1/2}(\mathrm{Pr}^*)^{-2/3}$, *Hi-speed* $10^5 < \mathrm{Re}_x^* < 10^7$: $\mathrm{St}_x^* = 0,0288(\mathrm{Re}_x^*)^{-1/5}(\mathrm{Pr}^*)^{-2/3}$ ‖ *HiHi-speed* $10^7 < \mathrm{Re}_x^* < 10^9$: $\mathrm{St}_x^* = \frac{2,46}{(\ln \mathrm{Re}_x^*)^{2,584}}(\mathrm{Pr}^*)^{-2/3}$

Ecuaciones 11 (Intercambiadores) $U = \frac{1}{\sum_i^n R_i} = \frac{1}{\frac{1}{h_1} + \frac{t_1}{k} + \frac{1}{h_2} + \ldots}$ ‖ *balance:* $\dot{U}_{VC} = \dot{W} = 0 \Rightarrow \dot{Q} = \dot{m}\Delta h = \rho u_\infty A c_p (T_2 - T_1)$ ‖ $\dot{Q} = UA \cdot \mathrm{LMTD}$ ‖ $\mathrm{LMTD} = \frac{\Delta T_A - \Delta T_B}{\ln\left(\frac{\Delta T_A}{\Delta T_B}\right)}$ ‖ $\Delta T_B = T^{fluido1}\big|_{x=B} - T^{fluido2}\big|_{x=B}$ ‖ $\beta_i = \frac{\dot{m}_i c_{p_i}}{U\ell_{\mathrm{iperimetro}}}$
$(T_2 - T_1|_x = (T_2 - T_1|_{x=0}\, e^{-\left(\frac{1}{\beta_1} + \frac{1}{\beta_2}\right)x}$ ‖ $T_1 =$ *efectividad:* $\epsilon = \frac{T_{h,i} - T_{h,o}}{T_{h,i} - T_{c,i}}$

Segundo Parcial ↓

Ecuaciones 12 (Fundamentalismos) *Euler:* $-\frac{\dot{W}}{\dot{m}} = \tau \cdot \Omega = -e = U_2 c_{\theta 2} - U_1 c_{\theta 1} = (h_{02} - h_{01}) = \left(\frac{c_2^2 - c_1^2}{2}\right) + \left(\frac{U_2^2 - U_1^2}{2}\right) + \left(\frac{w_1^2 - w_2^2}{2}\right)$
Primera Ley: $\dot{U} = \dot{Q} - \dot{W} - \dot{m}\left[(h_2 - h_1) + \frac{1}{2}\left(c_2^2 - c_1^2\right) + g\left(z_2 - z_1\right)\right]$ *Rend. de la instalación:* $\frac{\textit{Potencia efectiva}}{\textit{Potencia entregada a la instalación}}$
Ley de Coseno: $c^2 = a^2 + b^2 - 2ab\cos\theta_{\widehat{ab}}$

Ecuaciones 13 (T. Hidráulicas) *Pelton: aprox Dixon* $w_2 \approx U \rightarrow c_2 \approx 2U \sin\frac{\beta_2}{2}$ *donde* $\beta_2 = 180° - \vartheta_2$ $\eta_{\mathrm{h}}^{\mathrm{pelton}} = \frac{U(c_1 - c_{\theta 2})}{gH}$
Kaplan: $c_{a1} = c_{a2} = \frac{Q}{\pi(r_{\mathrm{ext}}^2 - r_{\mathrm{int}}^2)}$ ‖ $\eta_{\mathrm{h}}^{\mathrm{kaplan}} = \frac{\dot{W}/\dot{m}}{gH}$
Eficiencia Hidráulica $= \frac{\textit{Potencia entregada al rotor}}{\textit{Potencia que se puede entregar a la instalación (potencial)}}$

Ecuaciones 14 (T. de Vapor) *Trabajo:* $\mathrm{de} = \frac{1}{\rho}\mathrm{d}p + \mathrm{d}\frac{1}{2}c^2 + \mathrm{d}q$‖ *Tobera:* $\eta_{\mathrm{iso}}^{\mathrm{tobera}} = \frac{c_1^2/2}{c_{1s}^2/2}$; $K_f = \frac{c_1}{c_{1s}}$ ‖ *Rotor:* $dp = 0:$ $\eta_{\mathrm{iso}}^{\mathrm{rotor}} = \frac{\dot{W}/\dot{m}}{c_1^2/2}$ *con* $K_m = \frac{w_2}{w_1}$ ‖ *Eficiencia interna de etapa:* $\eta_i = \eta_{\mathrm{iso}}^{\mathrm{tobera}} \cdot \eta_{\mathrm{liso}}^{\mathrm{rotor}} \cdot \ldots = \frac{\dot{W}/\dot{m}}{\Delta h_s}$ *Eficiencia maxima:* $\frac{U}{c_1} = \frac{\cos\alpha_1}{2m\cdot(1 - \boldsymbol{R})}$
Escalonamiento de reacción. *Fijo:* $c_{2s} = \sqrt{2\Delta h_s(1 - \boldsymbol{R}) + c_1^2}$ *Movil:* $w_{2s} = \sqrt{2\Delta h_s \boldsymbol{R} + w_1^2}$ *donde* $K_m = \frac{w_2}{w_{2s}}$
Perdidas tobera: $Y_{\mathrm{tob.}} = \dot{m}(c_{1s}^2/2 - c_1^2/2)$ *Perdidas movil:* $Y_{\mathrm{m}} = \dot{m}(w_1^2/2 - w_2^2/2)$ *Perdidas roz. entre fijo/movil:* $k_{\mathrm{axial}} = 0,009$,$k_{\mathrm{radial}} = 0,027$: $Y_{\mathrm{roz.}} = k\rho n_{\mathrm{Hz}}^3 D_m^5$ *[W]*‖ *Perdidas ventil.* ε *es grado adm.* l *es largo alabes en* cm, D_m *es diametro medio y* k *depende del nro. de ruedas* $k_1 = 3,8$; $k_2 = 4,5$; $k_3 = 6 \rightarrow Y_{\mathrm{vent.}} = (1 - \varepsilon)k\rho n^3 D_m^4 l$ *[W]. donde* $\varepsilon = \frac{\textit{Long arco inyeccion}}{\textit{circunferencia media (U/omega)}}$
donde $\ell_a = \frac{\dot{m}}{\rho h c_0}$ *para una ÚNICA tobera cuadrada* $A = \ell_a \cdot h$
Etapa acción: $c_0 \approx 0$ *para tobera. Rend. Maximo de UNA etapa accion (*c_1 *salida tobera):* $\eta_{\mathrm{máx}} = \frac{\dot{W}/\dot{m}}{c_{1s}^2/2} = K_f^2 \cdot \frac{\cos^2(\alpha_1)}{2} \cdot (1 + K_m \frac{\cos\beta_1}{\cos\beta_2})$

Ecuaciones 15 (T.Gas) *Valores:* $c_{p_{comb}} \approx 1,15$ *[kJ/kg]*; $k_{comb.} \approx 1,33$; $\eta_{\mathrm{comb}} = \frac{\mathrm{FAC}_t}{\mathrm{FAC}_c}$
Relacion de compresion (Segun Hilal): $r_c = \frac{p_2}{p_1}$ *Potencia eff donde* $w = \Delta h$
$\dot{W}_e = (\dot{m}_{\mathrm{air}} + \dot{m}_{\mathrm{comb.}}) \cdot |w_{\mathrm{turb.}}| - \dot{m}_{\mathrm{air}} \cdot |w_{\mathrm{comp.}}|$ *donde* $w_t = h_{\mathrm{in}} - h_{\mathrm{out}} = c_{p_{\mathrm{cte}}}(T_{\mathrm{in}} - T_{\mathrm{out}})$
Eficiencias Internas: $\eta_{\mathrm{int}}^{\mathrm{turb.}} = \frac{\textit{Potencia ciclo indicado}}{\textit{Potencia ciclo ideal}} = \frac{\Delta h}{\Delta h_s}$ *Efic. mecánicas(al reves para bombas):* ${\eta_{\mathrm{m}}}^{\mathrm{turb.}} = \frac{\textit{Potencia entregada}}{\textit{Potencia indicada}} = \frac{w_t}{w_t^{\mathrm{ind}}}$
Optimo ς: ${r_c}^* = \left(\frac{T_3}{T_1}\eta_c\eta_t\frac{c_{34}}{c_{12}}\right)^{\left(\frac{k}{2(k-1)}\right)}$ *Regeneración:* $\sigma = \frac{T_A - T_2}{T_4 - T_2}$,$\varsigma = \frac{T_2}{T_1}$,$\vartheta = \frac{T_3}{T_1}$,$\eta_{\mathrm{regen}} = \frac{\varsigma - 1}{\varsigma}\frac{\vartheta - \varsigma}{\vartheta - \varsigma - \sigma\frac{\vartheta - \varsigma^2}{\varsigma}}$ ‖ *Poder Calorico del combustible (PCI) [J/kg] Consumo especifico* $\mathbf{C}_e = \frac{\dot{m}_{\mathrm{comb.}}}{P_e} \rightarrow \eta_{\mathrm{instalacion}} = \frac{\dot{W}_e}{\dot{m}_{\mathrm{comb.}}\cdot\mathrm{PCI}} = \frac{1}{\mathbf{C}_e\cdot\mathrm{PCI}}$

Ecuaciones 16 (Ciclo combinado) *Rankine:* $\eta = \frac{\dot{W}_{\mathrm{t}}/\dot{m} - \dot{W}_{\mathrm{b}}/\dot{m}}{\dot{Q}_{\mathrm{in}}/\dot{m}} = \frac{|\Delta h_{\mathrm{turb.}}| - \Delta h_{\mathrm{bomba}}}{\Delta h_{\mathrm{cald}}} = 1 - \frac{|\Delta h_{\mathrm{cond.}}|}{\Delta h_{\mathrm{cald}}}$‖ $\mathrm{BWR} = \frac{\dot{W}_{\mathrm{b}}/\dot{m}}{\dot{W}_{\mathrm{t}}/\dot{m}} = \frac{\Delta h_{\mathrm{bomba}}}{|\Delta h_{\mathrm{turb}}|}$
‖ *Turb/bomba (b al revés):* $\eta_{\mathrm{t}} = \frac{\Delta h_{\,\textit{turb}}}{(\Delta h_{\,\textit{turb.}})_s}$