Properties and image compensation for the F20's CCD camera

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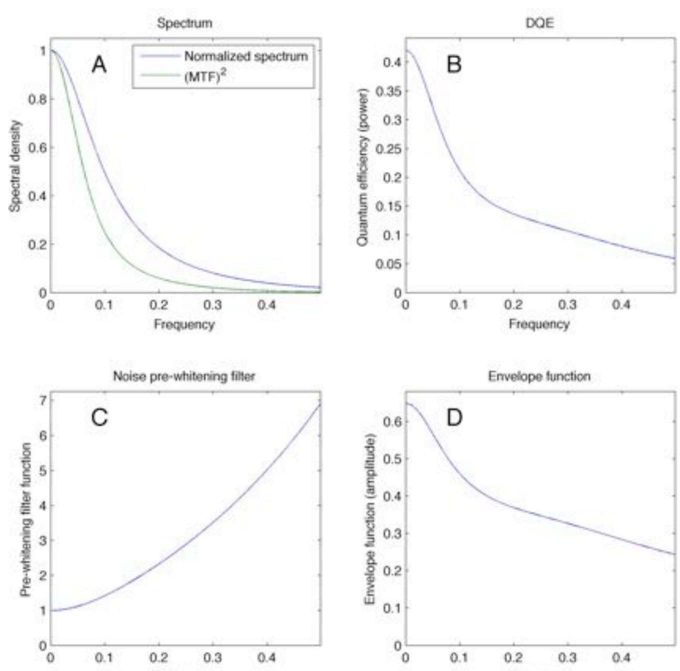
The recorded image I of a specimen X can be described (in Fourier terms) as

$$I = M_S ECX + M_N N \qquad (1)$$

where M_S is the modulation-transfer function (MTF) of the camera for a signal, C is the "pure" CTF equal to $\sin(\chi)$, E is the envelope function, M_N is the MTF for noise, and N is the shot noise random field. We can measure M_N by taking the power spectrum of an image of pure noise (just the beam alone), obtaining the power spectrum

$$\begin{aligned} S_I &= M_N^2 \langle N^2 \rangle \\ &= M_N^2 n_e \end{aligned} \qquad (2)$$

where n_e is the number of electrons accumulated per pixel. M_N^2 is larger than unity at some frequencies, because of the variability of response to individual electron events.



Meanwhile M_S can be estimated by taking the image of a step function (e.g. the shadow of the pointer), differentiating it (to give the effective image of a delta function) and taking the Fourier transform. Fig. 1A compares M_N^2 with M_S^2. If these functions were the same, then the signal and noise would be filtered identically and the SNR of the signal and noise would remain the same over spatial frequency. As it turns out, the normalized spectrum (proportional to M_N^2) remains larger at high frequencies than the squared MTF (M_S^2). Thus the signal-to-noise ratio decreases with frequency.

Note that what is plotted here are polynomial fits, rather than the experimentally-determined functions. The fitted functions are nice because they make it easy to compute the function values for any frequency.

A first step in compensating images is to filter them with a “pre-whitening” filter, which will render the noise spectral density constant. The resulting image is then

$$\begin{aligned} I_{pw} &= M_N^{-1} M_S ECX + M_N^{-1} M_N N \\ &= (M_S / M_N) ECX + N \end{aligned} \qquad (3)$$

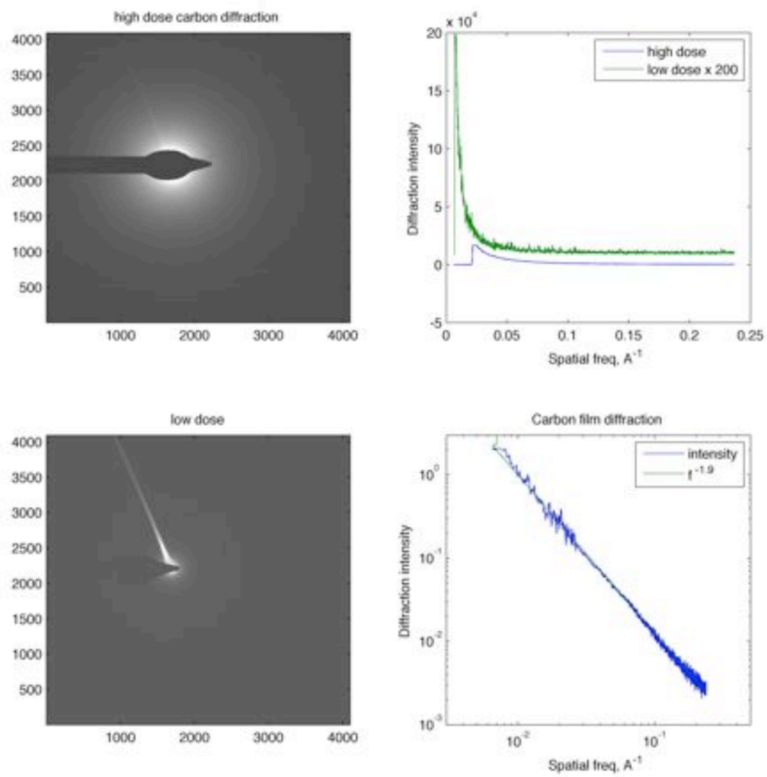
and it is clear that the signal to noise (power) ratio contribution from the camera will be $(M_S / M_N)^2$. This quantity is called the detector quantum efficiency (DQE) and is plotted in Fig. 1B. Although M_S is equal to one at low frequencies, M_N is greater than one, so the DQE starts out at less than unity and decreases with frequency. The DQE is the fundamental measure of the quality of a camera. In our case the DQE is about 10% at 0.3 times the sampling frequency (0.6 of Nyquist). This means that to obtain the same SNR at this frequency as would be gotten from a perfect camera, 10 times as many images need to be averaged.

The frequency response of the prewhitening filter M_N^{-1} is given in Fig. 1C. It starts out at unity and increases to a magnification of about 7 times at the Nyquist frequency. The contribution to the overall envelope function, equal to M_S / M_N, is given in Fig. 1D. In describing the imaging process the composite contrast transfer function becomes

$$(M_S / M_N)EC.$$

With these functions I set out to evaluate an experimental image of the carbon film in one of Liguo's homemade holey carbon grids, using the room-temperature holder and at about 1 μm defocus and 200 kV. Carbon film is not a "white noise" object, and to estimate the spectrum of the true object, Liguo obtained a diffraction pattern of the film. In Fig. 2 (lower right) is shown the diffraction intensity as a function of the spatial frequency *f*. Over the range from $(200\text{Å})^{-1}$ to $(5\text{Å})^{-1}$ the intensity shows a decay that is well fitted by the function

$$\langle X^2 \rangle = af^{-1.9} \qquad (4)$$



Knowing the spectrum of the actual object, we can then compare the measured power spectrum, of an image of the same carbon film, with the power spectrum predicted from eqns. (3) and (4).

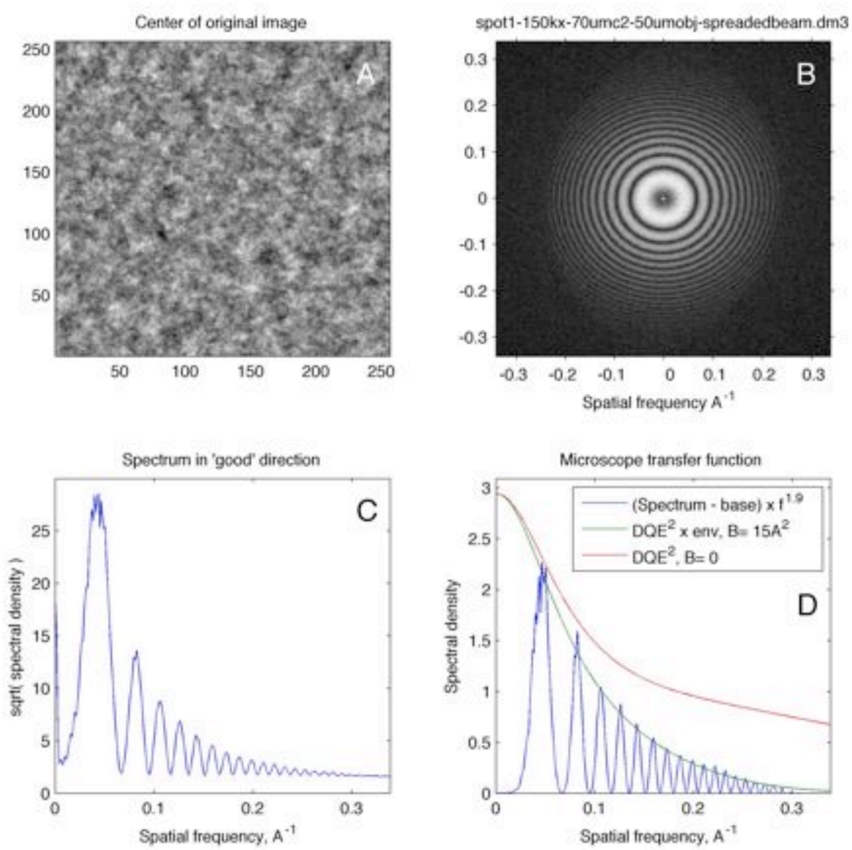
Figure 3 shows the experiment. The original image (Fig. 3A) is subjected to the pre-whitening filter so that it is given by eqn. (3). The resulting power spectrum is shown in Fig. 3B. As there was apparently some drift so that Thon rings were cut off on the sides, I take a vertical section through the power spectrum to be the true spectral response. This is plotted in Fig. 3C. The spectrum in this case is expected to be

$$\begin{aligned}\left\langle I_{pw}^2 \right\rangle &= (M_S / M_N)^2 E^2 C^2 \left\langle X^2 \right\rangle + \left\langle N^2 \right\rangle \\ &= (M_S / M_N)^2 E^2 C^2 (af^{-1.9}) + n_e\end{aligned} \qquad (5)$$

where a is a constant.

The pre-whitened spectrum in Fig. 3C shows the important property that the background noise is constant, as is represented by the n_e term in eqn. (5). We can further manipulate the spectrum by subtracting the constant term and then multiplying by $f^{1.9}$. This modified spectrum should, according to eqn. (5), correspond to

$$\left(\left\langle I_{pw}^2 \right\rangle - n_e\right) f^{1.9} = (M_S / M_N)^2 E^2 C^2 a \qquad (6)$$



and is shown in Fig. 3D. The envelope of this modified power spectrum should be proportional to $(M_S / M_N)^2 E^2$; that is, the product of the DQE and the microscope's envelope function. Fig. 3D shows two fitted curves, one a scaled DQE alone, and the other the product of the DQE with the square of an envelope function

$$E(f) = e^{-Bf^2}$$

with the quite reasonable factor B=15 Å^2. The beautiful fit to the envelope suggests that the modeling of this carbon image is quite good out to a resolution of 3 Å.

In my new Matlab library EMCCD, there are two functions CCDModelSpectrum(f) and CCDModelMTF(f). These functions accept a frequency argument (in units of the sampling frequency, in the range of 0 to 0.5) and return the quantities $M_N^2(f)$ and $M_S(f)$. (At the moment these functions have hard-wired parameters for the camera on our F20.) From these are derived a function CCDPreFilter(n) that produces the pre-whitening filter function $M_N^{-1}(f)$ as an n x n array, and also the function CCDSqrtDQE(n), which returns the ratio $M_S(f) / M_N(f)$ as an n x n array which multiplies the CTF in eqn. (3). The parameter n will normally be the size of the CCD, n=4096.

Finally, the procedure for processing an image can be summarized in this way.

1. Pre-whiten the image, by multiplying its Fourier transform by the filter function obtained from CCDPreFilter. See the comments in the file for an example.

2. In evaluating the image, use a composite contrast-transfer function that consists of the product of CCDSqrtDQE() and the result from either of the usual functions CTF() or ContrastTransfer() which represent the product EC. The result is a model of the pre-whitened image like that in eqn. (3) where the noise is white and the overall contrast transfer function $(M_S / M_N)EC$ is known. See the comments in CCDSqrtDQE.m for an example.