CONTEXT DEPENDENCE IN PERCEPTUAL AND PREFERENTIAL CHOICE

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ABSTRACT

CONTEXT DEPENDENCE IN PERCEPTUAL AND PREFERENTIAL CHOICE

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A large body of research in psychology has shown that context can systematically affect choice. These *context effects* are in violation of classical choice models and are both practically and theoretically interesting. Recent work has demonstrated context effects in simple perceptual choice, findings that suggest that these results are not restricted to high-level (e.g., consumer) choice. This dissertation explores the attraction and repulsion effects, two related context effects, with a particular emphasis on perceptual choice. Chapter 1 introduces the attraction and repulsion effects and reviews the empirical and theoretical literature surrounding them. Chapter 2 tests the ability of perceptual and decisional processes to account for these effects via a Thurstonian choice model. Chapter 3 uses this model to make predictions for best-worst choice and tests them empirically. Chapter 4 generalizes the Thurstonian model and experimental paradigm from Chapter 2 to consumer choice. Chapter 5 tests the

idea that an effect similar to that of Chapter 2 can be generated through ease of inter-option comparability in perceptual choice. Chapter 6 summarizes and discusses these results.

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CHAPTER 1

INTRODUCTION

1.1 Overview

A large body of decision-making research has shown that choice can systematically depend on context. In decision-making experiments, researchers present participants with a finite set of options on each trial and ask them to select a single option based on either an internal (e.g., most preferable) or external (e.g., the largest shape) criterion. Decision-making research spans multiple fields, including psychology, neuroscience, economics, marketing, and political science. In economics, for example, researchers have developed models based on the idea that people make choices to maximize their utility but are subject to resource constraints and noisy preferences. In psychology and marketing, however, decision-making researchers have identified a set of phenomena that violate assumptions of certain models of choice, by showing that choices can vary with the *choice set*, or the menu of available options. This class of phenomena is known as *context effects*.

One notable context effect, the attraction effect, occurs when the choice share of a *target* option increases upon the inclusion of a similar but inferior *decoy* option (Huber et al., 1982). Another finding, the repulsion effect, occurs when a decoy boosts the choice share of the dissimilar *competitor* option rather than the target (Simonson, 2014).

Context effects, originally studied in preferential choice, have been recently demonstrated in perceptual choice (Evans et al., 2021; Liao et al., 2021; Spektor et al., 2018, 2022; Trueblood & Pettibone, 2017; Trueblood et al., 2013; Turner et al., 2018;

Yearsley et al., 2022). The fact that context effects can appear in perceptual choice is theoretically interesting because it suggests that context effects are a theoretical primitive rather than simply a feature of high-level consumer choice (Trueblood et al., 2013).

This dissertation explores various forms of context dependence in both perceptual and preferential choice. Recent work has demonstrated inconsistency in context effects, particularly in perceptual choice. This dissertation uses behavioral experiments and statistical modeling in an attempt to reconcile these inconsistencies. The goal of this dissertation is to further understand why these effects - specifically the attraction and repulsion effects - occur in perceptual and preferential choice by employing well-studied statistical models from the psychology literature.

This dissertation is structured as follows. Chapter 2 develops and tests a statistical (Thurstonian) model of choice and applies it to perceptual choice. In Experiment 1, it is shown that the types of stimuli used in perceptual choice context effects experiments are easily confusable; this confusability varies systematically with theoretically relevant properties of the stimuli. In Experiment 2, the results of a high-powered psychophysics experiment show that the repulsion effect, but not the attraction effect, is naturally predicted by the Thurstonian choice model. Chapter 3 further tests the Thurstonian choice model by applying it to best-worst choice. Chapter 4 generalizes the Experiment 2 paradigm as well as the Thurstonian model to preferential choice. Finally, Chapter 5 uses a perceptual choice experiment to show that stimulus comparability affects choice, even when the decoy is equally similar to both focal options. Chapter 6 summarizes the findings of the dissertation, their implications, and discusses future directions for research in this domain.

Below, the attraction and repulsion effects are introduced, and the empirical and theoretical literature is reviewed.

1.2 The Attraction Effect

This section formally defines the attraction effect. Let A, B, D_A, and D_B be discrete choice options, [] denote the options in a choice set, and $P(A|[A, B])$ denote the probability of choosing option A from a set consisting of A and B, for example. See Figure 2.1 (left panel), which shows a graphical configuration of the options. These options vary on two dimensions (or attributes), where higher values of an attribute are always preferred. The dimensions are named generically to emphasize the generality of the attraction effect.

In Figure 1.1, options A and B trade off on attributes. A is high on dimension 2 but low on dimension 1, while B is high on dimension 1 but low on dimension 2. A decision-maker who assigns equal importance to both dimensions should be indifferent between both options when presented with choice set $[A, B]$[1]. Now, however, consider option D_A, which is inferior to A and B, but more similar to A than to B. In decision-making terminology, D_A is *asymmetrically dominated* by A. D_A, the *decoy* option, is dominated by A, so no rational agent who notices the dominance relationship should intentionally select D_A over A. Similarly, D_B is inferior to both A and B but more similar to B. The attraction effect is the finding that the relative choice for A over B is greater given set A, B, D_A then given set A, B, D_B.

In the context effects literature, it is common to refer to the similar, dominated option as the *decoy*, the similar dominating option as the *target*, and the dissimilar dominating option as the *competitor*. For example, in the choice set $[A, B, D_A]$, A is the target, B is the competitor, and D_A is the decoy. This terminology will be adopted throughout this dissertation.

The attraction effect was first demonstrated by Huber et al. (1982), who tested participants with sets of choice options, using products such as cars, beers, and TV

[1] The attraction effect does not require the assumption of equal dimension weighting, but it is assumed here to simplify the example.

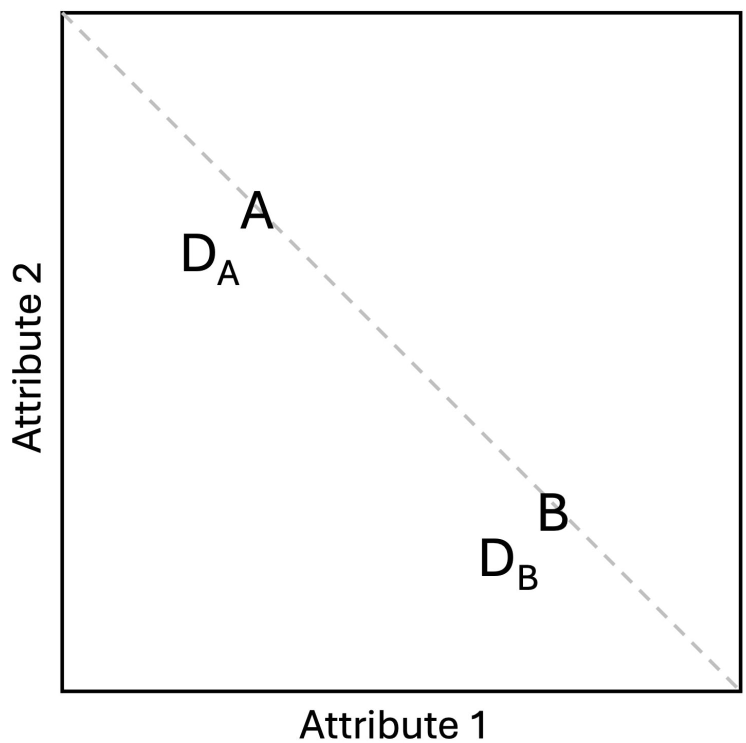

Figure 1.1. A graphical depiction of choice options in the attraction/repulsion effect. Attributes are named generically. The diagonal shows the line of indifference, where all options that fall on the line are assumed to be equally valuable, assuming additive utility and equally weighted dimensions.

sets. Participants first completed hypothetical choice tasks where they chose their most preferred option from ternary choice sets, each containing a target, competitor, and decoy option. Participants then returned two weeks later to choose from the same choice sets but with the decoy removed from all trials. Participants chose the target at a higher proportion when the decoy was present than when it was absent. The results of Huber et al. (1982) violate the regularity principle, which states that the choice proportion for a particular option cannot increase when more options are added to the choice set (MacKay & Zinnes, 1995; Marley, 1989).

According to regularity, the following inequality should hold:

$$P(A|[A,B]) \geq P(A|[A,B,D_A]) \tag{1.1}$$

Thus, the results demonstrated by Huber et al. (1982) violate regularity, because $P(A|[A,B,D_A]) \geq P(A|[A,B])$. Huber et al. (1982) referred to this finding as the asymmetric dominance effect because D_A (for example), is necessarily dominated by A but not B. B only dominates D_A assuming Attribute 1 is at least as important as attribute 2. This dissertation will generally use the term attraction effect, but the term asymmetric dominance is used where appropriate when reviewing the literature.

The attraction effect also violates the *Independence of Irrelevant Alternatives*(IIA) principle. IIA states that the relative likelihood of choosing a particular option over another is invariant of the choice set (Ray, 1973).

Written in the context of the above example, IIA requires the follow equality to hold[2]:

$$\frac{P(A|[A,B])}{P(B|[A,B])} = \frac{P(A|[A,B,D_A])}{P(B|[A,B,D_A])} \tag{1.2}$$

However, in the attraction effect, this equality is violated because of the following inequality:

---

[2]This is also referred to as the constant ratio rule.

$$\frac{P(A|[A,B,D_A])}{P(B|[A,B,D_A])} > \frac{P(A|[A,B])}{P(B|[A,B])} \tag{1.3}$$

The attraction effect is also demonstrated via a ternary-ternary comparison, where participants choose from two choice sets each containing a target, competitor, and decoy option (e.g., $[A, B, D_A]$ and $[A, B, D_B]$ in Figure 1.1). In the ternary-ternary form of the attraction effect, participants are more likely to choose A given $[A, B, D_A]$ than given $[A, B, D_B]$, and they are also more likely to choose B given $[A, B, D_B]$ than given $[A, B, D_A]$. This effect can be written using the following inequality, which also violates IIA:

$$\frac{P(A|[A,B,D_A])}{P(B|[A,B,D_A])} > \frac{P(A|[A,B,D_B])}{P(B|[A,B,D_B])} \tag{1.4}$$

Thus, IIA is violated by the attraction effect in this ternary-ternary example. Throughout this dissertation, results will often be presented by collapsing over choice sets and presenting the proportion of target, competitor, and decoy choices.

Since the initial work of Huber et al. (1982), a large body of basic and applied research has developed around the attraction effect. Doyle et al. (1999) demonstrated both the attraction effect and a related effect, the phantom decoy effect[3] in real-world supermarket purchases.van den Enden and Geyskens (2021) showed that the attraction effect can be used to induce people to choose healthier food items. Slaughter et al. (1999) showed that the attraction effect can be found even without the explicit attribute descriptions commonly used in laboratory experiments, that is when participants must infer option attributes.

O'Curry and Pitts (1995) demonstrated the attraction effect in political choice. They presented participants with actual Illinois state senate candidates described by numerical $0 - 100$ ratings on various policy questions (e.g., 55 on tax policy, 75 on

[3] A phantom decoy is a decoy which is both similar to and superior to the target but which made unavailable at the time of choice, but nonetheless increases target choice (Pratkanis & Farquhar, 1992).

education), and showed that the inclusion of an asymmetrically dominated decoy candidate increased participants' stated willingness to vote for the dominating candidate. In a second study, they had participants rate candidates for the 1992 presidential election (Clinton, Bush, Perot) on a seven-point scale for two policy issues (defense and health). O'Curry and Pitts (1995) showed that participants' ratings for a candidate, when plotted in a two-dimensional space, created an asymmetric dominance structure, the participants were more willing to vote for the dominating candidate than voters whose space did not form such a structure. Schwartz and Chapman (1999) demonstrated the attraction effect in physicians' decisions for medications treating various ailments (e.g., depression, sinusitis), where physicians were more likely to choose a medication if a similar, but inferior decoy was present. For example, when a decoy with identical efficacy but greater likelihood of side effects (when compared to a target option) was included, physicians chose the target more often.

Cataldo and Cohen (2019) showed that, in preferential choice, context effects can be reversed or eliminated simply by altering stimulus presentation format. For example, they showed that if participants can easily compare pairs of options (e.g., target and decoy) on each dimension, the attraction effect occurs quite strongly, but without this ease of comparison the attraction effect becomes negligible. They argued that these results suggest the importance of the comparison process in generating context effects. Hasan et al. (2025) failed to replicate this effect, albeit with slightly different decoys than those of Cataldo and Cohen (2019).

Hayes et al. (2024) manipulated attribute commensurability in a context effects experiment. When two dimensions are commensurable, they vary on a common unit (e.g., user ratings from 0-10), while incommensurable units exist on incomparable units (e.g., RAM and CPU speed in laptops). Hayes et al. (2024) found that when dimensions are commensurable, the attraction effect vanishes, while it still exists strongly when dimensions are incommensurable. This result suggests that the attraction effect occurs more strongly when the representation of options encourages between-option comparisons on a single attribute. Hayes et al. (2024)'s result is consistent with Cataldo and Cohen (2019), in that when the stimulus display facilitates between-option comparisons on individual attributes, the attraction effect emerges.

Other researchers have demonstrated related context effects. Tversky (1972) demonstrated that a similar, but not necessarily inferior, option can decrease choice for a focal option. For example, an option closer to A than to B on the diagonal indifference line in Figure 1.1 can cause a decrease in the choice proportion for A. This is known as the *similarity effect*. Simonson (1989) demonstrated the compromise effect, where the placement of an intermediate option between two extremes on a diagonal decreases choice for the extremes.

Theoretically, the attraction effect is interesting to decision-making researchers because, in violating IIA and regularity, the attraction effect is not predicted by many classical models of choice. Luce's Choice Model (Luce, 1959), for example, is a well-known choice model which states that the probability of choosing option x from set K can be written as:

$$P(x|K) = \frac{v_x}{\sum_{i \in K} v_i} \tag{1.5}$$

where v_i is a ratio scaled variable indicating the value, or utility, of option i. Luce's model has been highly influential to psychology and has been used to account for data in areas such as categorization (Nosofsky, 1986) and identification (Townsend, 1971). However, given that the model satisfies both IIA and regularity, the model cannot predict the attraction effect without incorporating additional mechanisms.

Traditional models of choice, as used in economics and marketing research (McFadden, 2001), treat the *utility*, or value, of each option as a random variable whose parameters are estimated from choice data. According to these models, on each trial of a decision-making experiment, the participant samples option values from these distributions and deterministically chooses the option with the highest sampled value. Typically, these models are used to estimate the latent utility values for each option and determine the participants' underlying preferences. These models are known as *Random Utility Models* (RUMs). When utilities are assumed to follow a Type 1 Generalized Extreme Value distribution, the logit or softmax model is used (Gensch & Recker, 1979). The probit model, another popular model, assumes Gaussian distributed utilities (McFadden, 1980). Typically, though not necessarily, these models assume independently distributed errors, which typically implies IIA. RUMs were considerably influenced by Thurstone (1927), who proposed the idea of representing psychological value as a random Gaussian variable.

Often RUMs assume IIA, though this assumption can be relaxed, however. Paetz and Steiner (2018) showed, via simulations, that the probit model can violate IIA by allowing correlations between alternatives. IIA can also be relaxed by estimating choice set or alternative-specific coefficients (Rooderkerk et al., 2011). Without such modifications, RUMs are unable to account for context effects.

Given the large body of published empirical data and the inability of existing models to explain context effects, researchers then developed mathematical process models to explain the cognitive processes that lead to context effects. One particularly notable model of context effects is Roe et al. (2001)'s Multialternative Decision Field Theory (MDFT) model. MDFT is noteworthy for being the first model to simultaneously account for the attraction, similarity, and compromise effects using a single set of parameters. MDFT builds on evidence accumulation models, which cast decision-making as a stochastic evidence accumulation process indexed by time (Ratcliff, 1978). In consumer choice, evidence is represented by preference for each option. According to MDFT, the valence $\boldsymbol{V}$ of each option at time t is computed as:

$$\boldsymbol{V}(t) = \boldsymbol{CMW}(t) + \boldsymbol{\epsilon}(t) \tag{1.6}$$

where $\boldsymbol{C}$ is a symmetric contrast matrix defining the comparison process, $\boldsymbol{M}$ is a matrix of attribute values, $\boldsymbol{W}$ is a vector of attention weights (defining which attribute is being attended to), and ϵ is a random error term.

The valences are then combined into a preference state vector $\boldsymbol{P}(t)$ and a new state is formed at time $t+1$ by:

$$\boldsymbol{P}(t+1) = \boldsymbol{SP}(t) + \boldsymbol{V}(t+1) \tag{1.7}$$

$\boldsymbol{S}$ is a feedback matrix where the diagonal elements allow memory decay for previous preference states, but the off-diagonal elements allow options in a choice set to influence one another through lateral inhibition. Roe et al. (2001) stated that this influence should be negative, and its strength should be a decreasing function of distance in attribute space. They did not specify a form to this function, but Hotaling et al. (2010) amended MDFT to specify a Gaussian distance function, where the value of $\boldsymbol{S}_{ij}$ is computed as:

$$\boldsymbol{S}_{ij} = \delta_{ij} - \varphi_2 \cdot e^{-\varphi_1 \cdot D_{ij}^2} \tag{1.8}$$

where $\delta_{ij} = 0$ if $i \neq j$ (indicating lateral inhibition), D_{ij}^2 is distance in attribute space between options i and j[4], and the φ are free parameters. In MDFT, as in other evidence accumulation models, preferences continue to accumulate until one option reaches an upper threshold, at which point a choice is made and the decision process stops.

MDFT can produce the attraction effect because of the strong connection between target and decoy (e.g., A and B in Figure 1.1), where the mutual negative inhibition cancels via multiplication, causing the decoy to boost the preference state for the target relative to the competitor. Mohr et al. (2017) demonstrated the attraction

[4]Hotaling et al. (2010) also specify that distance should be weighted differently depending on whether one option dominates another, i.e., the dominance dimension vs. the indifference dimension.

effect in risky choice and showed that the MDFT can account for decreases in the size of the attraction effect by allowing for stronger weighting of the subjective distance between target and decoy. In other words, the model predicts that as the decoy is further away, psychologically speaking, from the target, choice for the target will decrease.

Roe et al. (2001) noted that if the off-diagonal elements of $\boldsymbol{S}$ are set to 0, the MDFT reduces to a classical Thurstonian choice model indexed over time. Thurstone (1927) showed that given binary choice proportions (for example, participants' choices discriminating between two perceptual stimuli), the *psychological* distance between stimuli can be estimated by assuming Gaussian distributions along the underlying psychological scale. Thurstone's model has various forms (referred to as Cases I-V in the original article), each with various assumptions about the parameters of the model, in particular whether the distributions are independent or correlations are allowed. When the Gaussian distributions are independent, Thurstone's model is unable to predict the attraction effect or similar violations of IIA.

Berkowitsch et al. (2014) tested the independent logit and independent probit models against the MDFT. In their experiments, participants chose between digital cameras from choice sets designed to elicit context effects. Berkowitsch et al. (2014) found that MDFT, but not the logit or probit models, could account for the attraction effect. Independent RUMs, such as the independent logit and independent probit model, assume each option's utility is independently sampled, so these models cannot account for context effects. Furthermore, according to model comparison measures, the majority of participants were best described by MDFT. Participants whose data were best described by MDFT were also more likely to exhibit context effects.

The MDFT is not the only cognitive process model developed to explain context effects. Usher and McClelland (2004)'s Leaky Competing Accumulators (LCA) model offered an alternative explanation of the attraction effect. LCA uses loss aversion, the idea that losses on a given dimension are weighted more heavily than gains on the same dimension (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992), to account for the attraction effect via an asymmetric loss function. In LCA, each option in a choice set is a reference point to all other options. LCA explains the attraction effect through loss aversion because preference for the competitor is hurt by its relatively large subjective distance from both the target and decoy. Trueblood (2012) demonstrated context effects in an inference tasks, where participants chose the most likely suspect to have committed a crime from a set of three possible suspects, each described by two numerical ratings describing the strength of eyewitness accounts. Trueblood (2012) used this result to argue that the loss aversion account, which may be plausible in preferential choice, is an implausible mechanism for non-preferential tasks. Trueblood et al. (2013) demonstrated the attraction, similarity, and compromise effects in perceptual choice tasks, where participants were told to select the largest rectangle against a set of rectangles varying in height and width. Trueblood et al. (2013) argued similarly that loss aversion is an implausible mechanism for the attraction effect in a perceptual task. Other researchers have demonstrated context effects in perceptual choice tasks (Choplin & Hummel, 2005; Liao et al., 2021; Spektor et al., 2018, 2022; Yearsley et al., 2022), results which will be explored further in Chapter 2.

Numerous other cognitive models have emerged to explain context effects (Bergner et al., 2019; Bhatia, 2013; Noguchi & Stewart, 2018; Spektor et al., 2019a; Trueblood et al., 2014; Wollschläger & Diederich, 2012). Such models often assume an attribute-wise comparison process (Bhatia, 2013; Roe et al., 2001; Trueblood et al., 2013; Usher & McClelland, 2004). According to this framework, people arrive at a decision by comparing pairs of options on a single attribute, where the modeller assumes attribute values are veridical. This assumption is quite reasonable when modeling choices where each attribute is presented separately and discriminability issues are minimal or nonexistent. In perceptual choice experiments, however, like those presented by Trueblood et al. (2013), these assumptions are likely incorrect. However, the general modeling framework, where inter-stimulus comparison leads to preference, which then leads to choice, is still plausible.

There are, however, important limitations to the attraction effect. Frederick et al. (2014) argued that consumer choice researchers who study the attraction effect tend to use simple "stylized" stimuli varying on few attributes which are described by numeric values, but real-world consumer choice relies heavily on perception, often via images and/or verbal descriptions of products. They conducted 33 experiments attempting to demonstrate the attraction effect with both abstract numeric stimuli or with perceptual consumer choice stimuli (e.g., a TV with a lower price but an accompanying photo demonstrating slightly fuzzy picture quality as a decoy for a similar priced TV with superior picture quality). Frederick et al. (2014) found that very few studies (2/33) using perceptual consumer stimuli demonstrated the attraction effect. They argued that the attraction effect is overrepresented in the marketing literature relative to its real-world applications. Yang and Lynn (2014) independently conducted a large scale series of studies attempting to demonstrate the attraction effect with similar stimuli and found that only 2/54 studies using perceptual or verbally described consumer stimuli showed the attraction effect. Simonson (2014) acknowledged the results of Frederick et al. (2014) but argued that researchers should consider the factors that do generate the repulsion effect, such as participants' attention, and crucially, whether the asymmetric dominance relationship between target and decoy is perceived by the participant.

Trendl et al. (2021) also attempted to demonstrate the attraction with non-numerical, naturalistic stimuli. They had participants rate a number of popular movies which had pre-determined similarities between each other. The similarities were based on human similarity ratings gathered from pilot data. The researchers then used these ratings to create sets of target, competitor, and decoy options, tailored to individual participants, which were then presented to the participants at a later date. Participants were asked to select their favorite movie from each set. The results showed a clear null attraction effect, and Trendl et al. (2021) argued similarly to Frederick et al. (2014) that the attraction effect may not generalize to naturalistic choice stimuli.

Fang et al. (2024) argued that to demonstrate context effects, researchers need an adequate understanding of participants' subjective, psychological representation of the stimuli. They asked college basketball fans to plot NCAA men's basketball teams on a 2-dimensional plot, where the dimensions were offensive and defensive ability. From these representations, the researchers created triplets, tailor-made to each participant, with a target, competitor, and decoy option. In a second phase of the task, participants were presented with these triplets and, for each team in the triplet, they estimated the probability that the given team would out-rank the other two at the end of the season[5]. Fang et al. (2024) found the attraction effect, but only when the subjective target-decoy distance in psychological space was high; when it was low, they found the *repulsion effect*, where the competitor was chosen more than the target.

The repulsion effect is, empirically speaking, a reversal of the attraction effect. The repulsion effect was first demonstrated in a consumer choice experiment reported by Frederick and Lee (2008) who used TVs as choice stimuli and showed that the attraction effect could be reversed to a repulsion effect by representing picture quality perceptually rather than with numerical rating. Frederick et al. (2014) also reported a repulsion effect in one of their studies[6].

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[5]In NCAA basketball, teams are ranked weekly by coaches and sports journalists via polls, with the last ranking poll taking place after the NCAA tournament championship game.

[6]The TV study from Frederick and Lee (2008) may have been the same study reported in Frederick et al. (2014).

Though the repulsion effect has been studied far less than the attraction effect, a recent series of papers has revived interest in the effect. Spektor et al. (2018) demonstrated the repulsion effect using the same perceptual stimuli as Trueblood et al. (2013) but represented differently on screen[7]. Spektor et al. (2022) demonstrated the repulsion effect with both perceptual and risky choice stimuli in a format similar to Spektor et al. (2018). Liao et al. (2021) conducted a perceptual (rectangle) choice experiment, varying the target-decoy distance (TDD) in attribute space. Liao et al. (2021) found a non-monotonic relationship between TDD and the attraction/repulsion effect. In perceptual choice, low and high TDD values produced a repulsion effect while relatively moderate TDD values produced an attraction effect. In a separate inference task similar to that of Trueblood (2012), Liao et al. (2021) found a different non-monotonic relationship between TDD and the attraction effect; low and high TDD values produced an attraction effect while moderate TDD values produced the repulsion effect. Brendl et al. (2023) demonstrated the repulsion effect with consumer choice stimuli, but only when attributes were represented qualitatively; when attributes were represented quantitatively, they found the attraction effect.

There has been relatively little theoretical progress towards explaining the repulsion effect. Frederick and Lee (2008) proposed the *tainting hypothesis*, the idea that the decoy sometimes decreases the subjective value of the nearby target, though evidence for the tainting hypothesis is mixed. Spektor et al. (2018) found that the repulsion effect weakened with TDD, which will be explored more in Chapter 2. The tainting hypothesis is unable to explain the non-monotonic relationship between TDD and attraction/repulsion found by Liao et al. (2021). Spektor et al. (2021) argued that understanding how people subjectively represent stimuli in decision-making is crucial

[7] The details of this study are discussed at length in Chapter 2.

to understanding the repulsion effect; in particular they argue that attributes are represented less concretely in perceptual decision-making.

There is much progress to be made, empirically and theoretically, in understanding the conditions that produce the attraction and repulsion effects. The existing literature has demonstrated that the attraction effect can be robust, but perhaps only under certain conditions; the results are far less clear with the repulsion effect, where researchers are just beginning to hypothesize mechanisms to generate the effect.

The goal of this dissertation is to further understand the attraction and repulsion effects and the mechanisms that generate (some forms of) context-dependence in decision-making. This work builds on the literature discussed here but also in Chapter 2, where the repulsion effect is explored in greater depth.

CHAPTER 2

CORRELATED VALUATIONS AS A MECHANISM FOR THE REPULSION EFFECT

2.1 Introduction

The attraction effect is a choice phenomenon where an asymmetrically dominated decoy option increases the choice share of a similar, dominating, target option at the expense of a dissimilar competitor option (Huber et al., 1982). The attraction effect is a context effect, where the relative choice share of an option varies systematically with the set of available options[1].

The attraction effect was first demonstrated in a consumer choice study by Huber et al. (1982), though the effect is not solely limited to consumer choice. Researchers have also demonstrated the attraction effect in risky choice (Mohr et al., 2017), policy choice (Herne, 1997), voting (O'Curry & Pitts, 1995), intertemporal choice (Marini et al., 2020), probability judgment (Cai & Pleskac, 2023; Fang et al., 2024), medical decision-making (Schwartz & Chapman, 1999), episodic memory judgment (Maylor & Roberts, 2007), charitable donation (Pittarello et al., 2020), inference (Trueblood, 2012), job candidate selection (Highhouse, 1996), political choice (O'Curry & Pitts, 1995), sports prediction (Fang et al., 2024), and, as will be the focus of the current chapter, perceptual choice (Evans et al., 2021; Liao et al., 2021; Spektor et al., 2018, 2022; Trueblood et al., 2013, 2015; Turner et al., 2018; Yearsley et al., 2022).

[1]See Chapter 1 for a review of relevant literature on the attraction effect.

2.1.1 From Preferential to Perceptual Choice

Choplin and Hummel (2005) first demonstrated the attraction effect in perceptual choice. In their experiments, participants saw various perceptual stimuli (e.g., ovals, vertical lines) and chose the option most similar to a reference option. The researchers showed that participants were more likely to choose the option paired with an asymmetrically dominated decoy option. Yearsley et al. (2022) later replicated them with similar stimuli.

Trueblood et al. (2013) also demonstrated the attraction effect in perceptual choice. In their experiments, participants saw three rectangles on each trial, arranged in a horizontal array, and selected the option they perceived to have the largest area. For an example stimulus set, see Figure 2.1 (right panel). Note that options A and B have equal area but trade off in height and width. The decoy options (D_A and D_B, respectively) are smaller in area but are more similar to their respective targets than to their respective competitor. In Trueblood et al. (2013)'s experiments, participants chose the target option more than the competitor option, on average.

Notably, the title of Trueblood et al. (2013)'s paper was "Not Just for Consumers: Context Effects Are Fundamental to Decision-Making", and in their General Discussion, Trueblood et al. (2013) argued "our experiments suggest that these context effects are a general feature of human choice behavior because they are a fundamental part of decision-making processes. As such, our results challenge explanations of these effects exclusively in terms that are unique to high-level decision-making and thus call for a common theoretical explanation that applies across paradigms." (p. 907). According to Trueblood et al. (2013), context effects are not idiosyncratic to high-level decision-making but are a fundamental feature of choice. Trueblood et al. (2013) also used these perceptual results to argue against the context-dependent advantage (CDA) model (Tversky & Simonson, 1993) as well as the Leaky Competing Accumulators (LCA) model (Usher & McClelland, 2004), as these models use loss

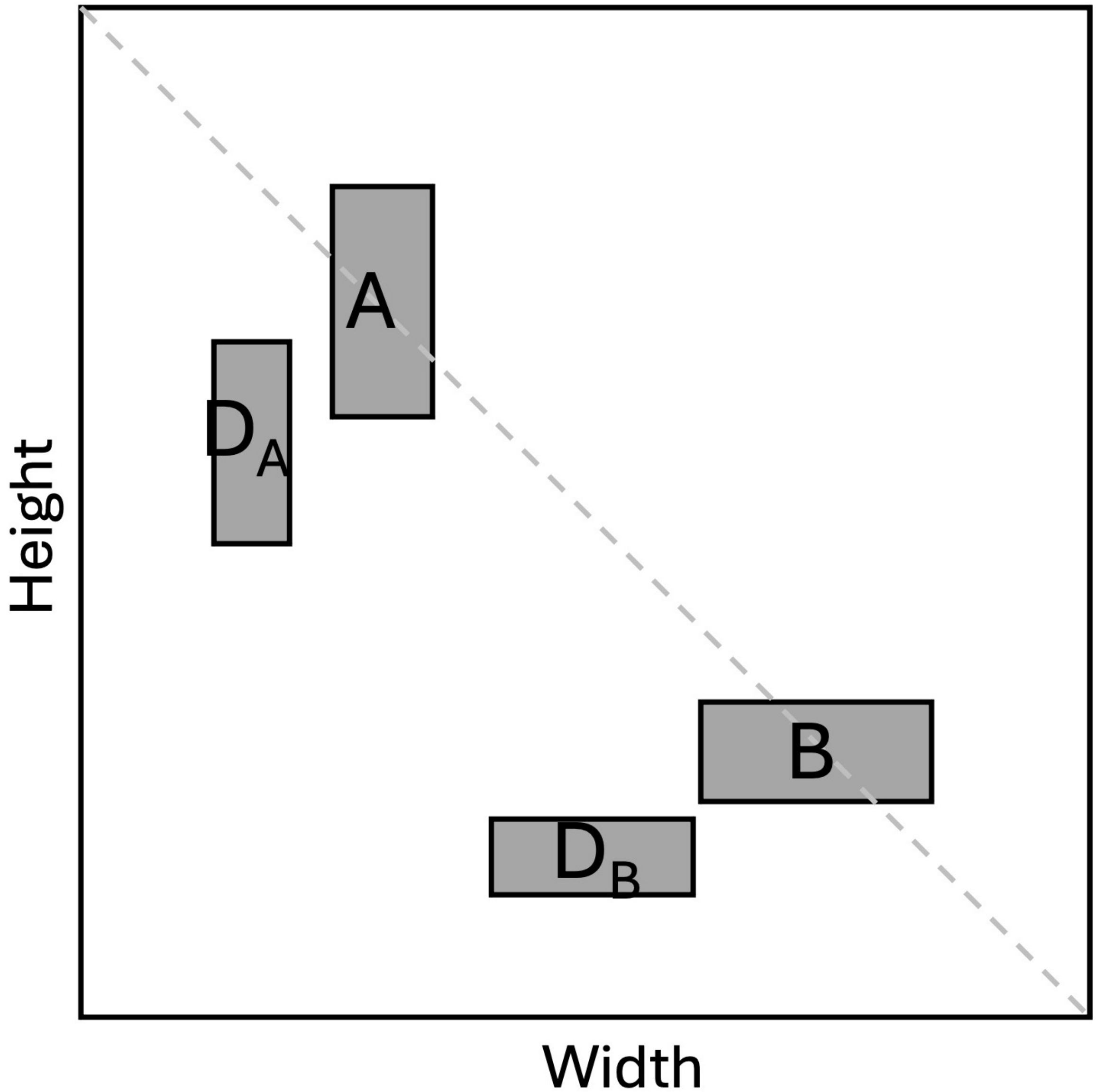

Figure 2.1. A graphical depiction of choice options in the attraction/repulsion effect.

aversion - the idea that an option's disadvantages on an attribute are weighted more strongly than its advantages on other attributes - to account for context effects.

Frederick et al. (2014) failed to replicate Trueblood et al. (2013)'s finding of the attraction effect in perceptual choice. Frederick et al. (2014) collected data online, which may have resulted in less precision than a laboratory task, potentially leading to the null finding.

Turner et al. (2018) replicated Trueblood et al. (2013)'s results and performed a large scale modeling analysis, comparing the ability of various model mechanisms to account for context effects. For example, Turner et al. (2018) concluded that pairwise comparisons on individual attributes greatly improves models' ability to account for context effects. This may not be appropriate, however, given a perceptual domain where dimensions may not be separable. Specifically, in a consumer choice experiment, participants may very well compare options on a single dimension at a time, e.g. comparing cars on their fuel efficiency. However, in a perceptual choice task, it is unlikely that participants are able to separately compare rectangles on the height dimension and on the width dimension and then arrive at an overall judgment. Rather, it is more plausible that participants compare these stimuli holistically.

Spektor et al. (2018) followed up on Trueblood et al. (2013) but found the *repulsion effect* in a similar perceptual choice experiment. In the repulsion effect, the competitor's choice share is higher than the target's choice share. In Spektor et al. (2018)'s experiments, the target and competitor options varied in area, such that one option was always larger, though on average they were the same size. The researchers also varied the *target-decoy attribute distance* (TDD), the percentage difference between the target and decoy areas. For example, if TDD is 2%, the decoy's area is 98% the area of the target. Also note that if the target and competitor have the same area, then the TDD is also the difference between the competitor and decoy areas.

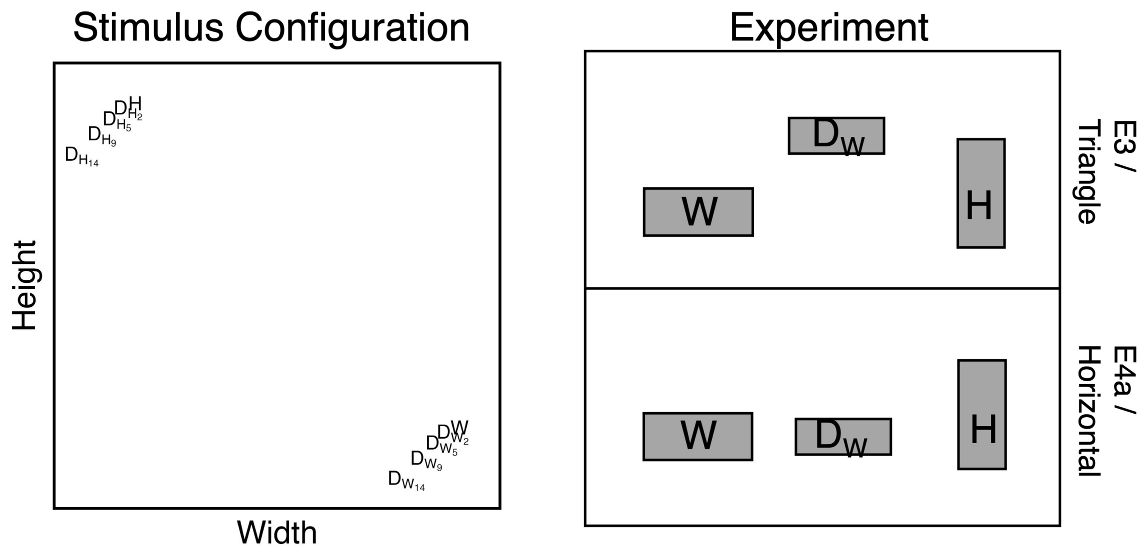

Figure 2.2. Stimulus configuration and example trials from Spektor et al. (2018), Experiments 3 and 4a. Decoy subscripts indicate TDD. Note that the labels in the right-hand plot were not shown in the experiment; these are shown here for the benefit of the reader.

Spektor et al. (2018) conducted a total of five experiments testing the repulsion and attraction effects in perceptual choice. All experiments showed similar results, so the current focus is on their experiments 3 and 4a, which are the most representative of their article. In Experiment 3, the authors varied TDD at four levels: 2%, 5%, 9%, and 14%. The rectangles were arranged in a triangular display on the screen (see Figure 2.2, Experiment 3), in contrast to Trueblood et al. (2013)'s horizontal display. Spektor et al. (2018) found a repulsion effect, such that the competitor was selected more than the target at all levels of TDD (see Figure 2.3).

Spektor et al. (2018)'s Experiment 4a, however, used the horizontal display of Trueblood et al. (2013) (see Figure 2.2, Experiment 4a). Here, they varied TDD at 5%, 9%, and 14%. In Experiment 4a, the data show a slight repulsion effect at low TDD levels that eventually becomes an attraction effect at high TDD levels.

Trueblood and Pettibone (2017) demonstrated a *phantom decoy* effect in perceptual choice. Phantom decoys are options that are similar to the target, but also

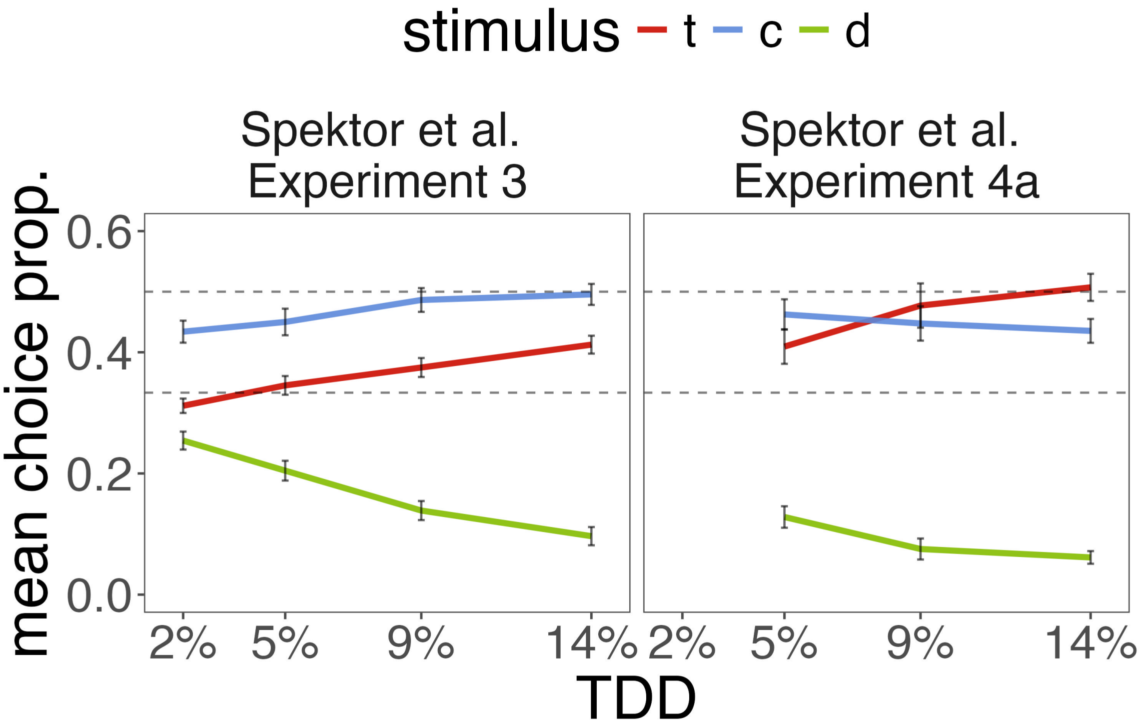

Figure 2.3. Data from Spektor et al. (2018), Experiments 3 (triangle display) and 4a (horizontal display), collapsed across choice set. Error bars are 95% CIs of the mean, computed using the within-participants error correction from Morey et al. (2008). Dashed lines are drawn at .5 and .33. Note that Spektor et al. (2018) did not test the 2% TDD level in Experiment 4a.

superior in value, and are initially presented but made unavailable at the time of choice. They showed that participants chose the target less than the competitor, a result at odds with phantom decoy effects in preferential choice (Pettibone & Wedell, 2000; Pratkanis & Farquhar, 1992).

Liao et al. (2021) replicated Spektor et al. (2018) while extending the TDD variable to 94%. When TDD was at 9% or 14%, they found a repulsion effect. However, when examining all levels of TDD, they found an inverse U-shaped relationship between TDD and the attraction/effects. Low and high TDD values created a repulsion effect, while intermediate TDD values created an attraction effect.

Spektor et al. (2022) demonstrated the repulsion effect in both preferential and perceptual choice, using similar stimuli and a similar display configuration to Spektor et al. (2018) and Liao et al. (2021). Here, the stimuli were squares containing two bars filled to varying degrees with color. In perceptual choice, participants were to select the stimulus with the largest cumulative filled area. In preferential choice, participants were told that each filled bar represented one possible outcome of a 50-50 gamble, and they were to select the gamble they wished to take. In both conditions participants were rewarded based on performance. Spektor et al. (2022)'s findings were similar to those of Spektor et al. (2018), where they found a general repulsion effect that weakened with TDD. Target choices also increased with TDD.

In a review of the recent context effects literature, Spektor et al. (2021) identified *attribute concreteness* as a crucial moderating factor on context effects. They defined attribute concreteness as the degree to which different observers can agree on the true value of options' attributes. For example, in a preferential choice experiment where attributes are presented numerically, e.g. RAM and Screen Resolution for laptops, attribute concreteness is high. In a perceptual rectangle choice experiment, where observers must rely on their own perception of area, attribute concreteness is low. Spektor et al. (2021) noted that high attribute concreteness tends to lead to robust context effects while low attribute concreteness has led to mixed results. According to their explanation, the lack of concreteness in perceptual choice is a crucial reason for the varied context effects data.

Researchers are clearly using perceptual experiments to demonstrate context effects and test theories of choice. These results are clearly informative and theoretically interesting. However, there are distinct differences between these perceptual choice experiments and the preferential choice experiments in which the attraction effect was originally demonstrated.

In preferential choice tasks, participants are typically shown a set of consumer choice options (e.g., laptops, apartments, washing machines), along with the attribute values associated with each option (e.g., 10 GB RAM, 1500 square feet, 2.7 cubic feet capacity). These attributes are typically represented numerically (Banerjee et al., 2024; Hayes et al., 2024) or with easily discriminable graphical representations (Cataldo & Cohen, 2019). The decoy option, therefore, is rarely selected (e.g., $<$ 5% of all trials in many studies), and these selections are often assumed to be the result of inattention or accidental responses. Researchers can generally assume that participants are able to detect the dominance relationship between target and decoy. Perceptual choice tasks, on the other hand, complicate participants' ability to detect this dominance relationship.

In Spektor et al. (2018)'s experiments, the decoy is selected as often as 25% of all trials in some conditions (see Figure 2.3), far more often than is common in preferential context effect experiments. The decoy is also selected less often in experiment 4a (triangle display) than in experiment 3 (horizontal display). Decoy selections also decrease as TDD increases. Finally, though both target and competitor increase in choice share as TDD increases, the target choice share increases at a higher rate than does the competitor, suggesting a strong trade-off between target and decoy choices; stronger, indeed, than the competitor-decoy trade-off. That is, the mean *Relative Choice Share of the Target*, or RST (Berkowitsch et al., 2014) increases with TDD in both experiments. Spektor et al. (2018) found that a relatively small change in stimulus display (arranging stimuli in a triangle rather than a horizontal line) reverses the attraction effect. A relatively subtle change in stimulus display led to a strong qualitative shift in the data.

Spektor et al. (2018) argued that their work provides partial evidence in favor of the tainting hypothesis, the idea that decoys sometimes "taint" the value of nearby targets, leading to a repulsion effect. They argued that this is supported by the decrease in competitor choice share as TDD increases. They did note that this high level explanation seems implausible for a perceptual rectangle task.

Another plausible account of Spektor et al. (2018)'s data is that participants occasionally perceive the decoy to be at least as large as, or larger than, the target. Indeed, the target and decoy are perceptually similar, more so than the decoy and competitor. Their results may be another form of the similarity effect (Tversky, 1972), where a similar decoy causes a decrease in the choice share of a target option. Simply put, participants may occasionally select the decoy over target, and they may do so more often than they select the decoy over the competitor. This account is consistent with the data shown in Figure 2.3. As TDD increases, decoy choices decrease while target choices increase. Though competitor choices also increase with TDD, the target choice shares increase at a higher rate. There appears to be a strong trade-off between target choices and decoy choices. Spektor et al. (2018) dismissed such an account because the target was always chosen more often the decoy. This does not, however, rule out that possibility that, on the trials where the decoy was chosen, it was more likely to be chosen over the target than the competitor. In other words, participants do not always select the decoy; however, when they do, they more often select it over the similar target than the dissimilar competitor.

Clearly, perceptual discriminability plays a role in Spektor et al. (2018)'s results. Participants clearly are better able to discriminate the target and competitor from the decoy as TDD increases. Any reasonable account of these data should parse discriminability, as well as target-decoy similarity, from genuine context effects.

In Experiment 1, a two-alternative forced-choice (2afc) task was used to show that participants regularly fail to discriminate the target from the decoy, given Spektor et al. (2018)'s stimuli. Moreover, overall discriminability is worse in Spektor et al. (2018)'s triangle display than in Trueblood et al. (2013)'s horizontal display.

2.2 Experiment 1

The goal of Experiment 1 was to test participants' ability to discriminate between rectangles in the perceptual choice tasks of Trueblood et al. (2013) and Spektor et al. (2018).

On each trial, participants saw three options (target, competitor, and decoy). After a short delay, two of the rectangles were highlighted, and participants were asked to select the larger rectangle from the highlighted pair. This 2afc task allows for testing discriminability between all pairs of options while also accounting for the possibility that overall discriminability is worse when three stimuli are presented instead of two.

This experiment also included a within-participants manipulation to compare discriminability across the triangle display of Spektor et al. (2018), Experiment 3, and the horizontal display of Spektor et al. (2018) Experiment 4a and Trueblood et al. (2013). Otherwise, with a few exceptions discussed below, the experiment followed the stimulus construction and experimental design of Spektor et al. (2018), Experiments 3 and 4a.

2.2.1 Methods

2.2.1.1 Participants.

Data collection took place at the University of Massachusetts Amherst. 86 undergraduate students participated in exchange for course credit. 1 participant who achieved less than 80% accuracy on catch trials (see below) was excluded from all analyses. $1,280$ and $1,296$ trials from the triangle and horizontal conditions, respectively, with response times (RTs) < 100ms or > 10000ms were also excluded from all analyses, leaving $18,069$ and $18,054$ trials remaining in the triangle and horizontal conditions, respectively.

2.2.1.2 Stimuli.

Stimuli were gray scale rectangles based on Trueblood et al. (2013) and Spektor et al. (2018).

Stimuli varied based on trial type. The experiment had two types of trials: critical trials and catch trials.

On each critical trial, the target and competitor had the same area[2] but differed on orientation, with one stimulus being wide and the other tall. The decoy always had the same orientation as the target. TDD was varied so that the decoy was always 0%, 2%, 5%, 9%, or 14% of the target area. Because the target and competitor always had the same area, this means that the decoy was also 0%, 2%, 5%, 9%, or 14% of the competitor area. These are the TDD values from Spektor et al. (2018), plus a 0% level which acted as a baseline[3].

On each catch trial, there was one large rectangle and two much smaller rectangles. The large rectangle was $260 \pm U(-40, 40) \times 200 \pm U(-40, 40)\text{px}^2$, with a random orientation. The smaller rectangles were $180 \pm U(-40, 40) \times 120 \pm U(-40, 40)\text{px}^2$, one wide and one tall.

2.2.1.3 Design.

There were 5 blocks of trials. In each block there were 60 critical trials, 12 at each TDD level, and 30 catch trials. Of the 12 critical trials at each TDD level, 6 trials were presented in the triangle display and 6 trials were presented in the horizontal display. Finally, 3 of the 6 targets in each display condition at each TDD level were wide and 3 were tall. Of each of these 3 trials, one trial was a target-decoy comparison, one trial was a target-competitor comparison, and one trial was a

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[2] Here Spektor et al. (2018)'s design was simplified as both focal stimuli had the same area.

[3] When TDD=0%, the target and decoy are identical, so labeling is arbitrary. Competitor labeling is not arbitrary because it is the rectangle that does not share an orientation with the other two.

competitor-decoy comparison. Trial order and rectangle order within each trial were randomized. Thus, this was a 5 (TDD: 0%, 2%, 5%, 9%, 14%) x 2 (display: triangle, horizontal) x 2 (target-decoy orientation: wide, tall) x 3 (comparison: target-decoy, target-competitor, competitor-decoy) within-participants design.

2.2.1.4 Procedure.

On each trial, participants saw three rectangles, labeled 1, 2, and 3 (from left to right), either in a horizontal or triangle display. The horizontal distance between all rectangles was constant, but 25 pixels of jitter was added to each rectangle's vertical location. The rectangles appeared for 1825ms total, but after 500ms, two of the rectangles changed to a darker shade. Next, all three rectangles disappeared from the screen, and participants were asked to select which of the two darker rectangles had the larger area. The procedure was identical for critical trials and catch trials.

Stimuli were presented on computer monitors with a resolution of 1920 x 1080 pixels. The experiment was programmed in jsPsych (De Leeuw, 2015).

2.2.2 Results

2.2.2.1 Catch Trials.

Participants performed well on the catch trials. The mean percentage correct in the triangle display was 92.6%($SD = 3.77$), and the mean percentage correct in the horizontal display was 93.2%($SD = 3.52$).

2.2.2.2 Critical Trials.

First the baseline TDD level data (TDD=0%) are presented to demonstrate that participants were indifferent between pairs of options when they had identical area. The mean percentage of target choices in target-competitor trials was 48.99% (SD=10.18, 95% CI [46.80, 51.20]). The mean percentage of competitor choices in competitor-decoy trials was 49.80% (SD=11.30, 95% CI [47.40, 52.20]). The mean percentage of target choices in target-decoy trials was 49.47% (SD=12.06, 95% CI [46.90, 52.10]). Participants were indifferent between all pairs of options in the TDD = 0% trials, so these trials are not considered in further analyses.

Next, the target-competitor trials are presented. Note that in these trials, because both stimuli had the same area, there is no correct response. The results are plotted in Figure 2.4, with the y-axis (arbitrarily) showing the mean proportion of target selections. Though participants were occasionally biased to select either the target or competitor in certain TDD levels, there was no systematic pattern to their choices across TDD. These data are not discussed further.

The primary analysis was performed on the target-decoy and competitor-decoy trials, excluding all TDD=0% trials and the target-competitor trials. Note that participants were to select the option with the largest area. Because both the target and competitor were always larger than the decoy, a correct response was one in which the decoy was not chosen.

Mean choice proportions across conditions are shown in Figure 2.5. Participants' performance indeed improves with TDD. Furthermore, their performance is better when stimuli are displayed in the horizontal configuration than in the triangle configuration, and it is also better in target-decoy trials compared to competitor-decoy trials. Finally, there is an interaction, such that as TDD increases, the target-decoy performance is even better than the competitor-decoy performance. See the Appendix for inferential statistics which support these conclusions.

2.2.3 Discussion

The goal of Experiment 1 was to test participants' ability discriminate between target-decoy and competitor-decoy stimuli in binary perceptual choice. Though participants choose the decoy at non-trivial rates in ternary choice (see Figure 2.3), previous researchers have not tested participants' binary discriminability.

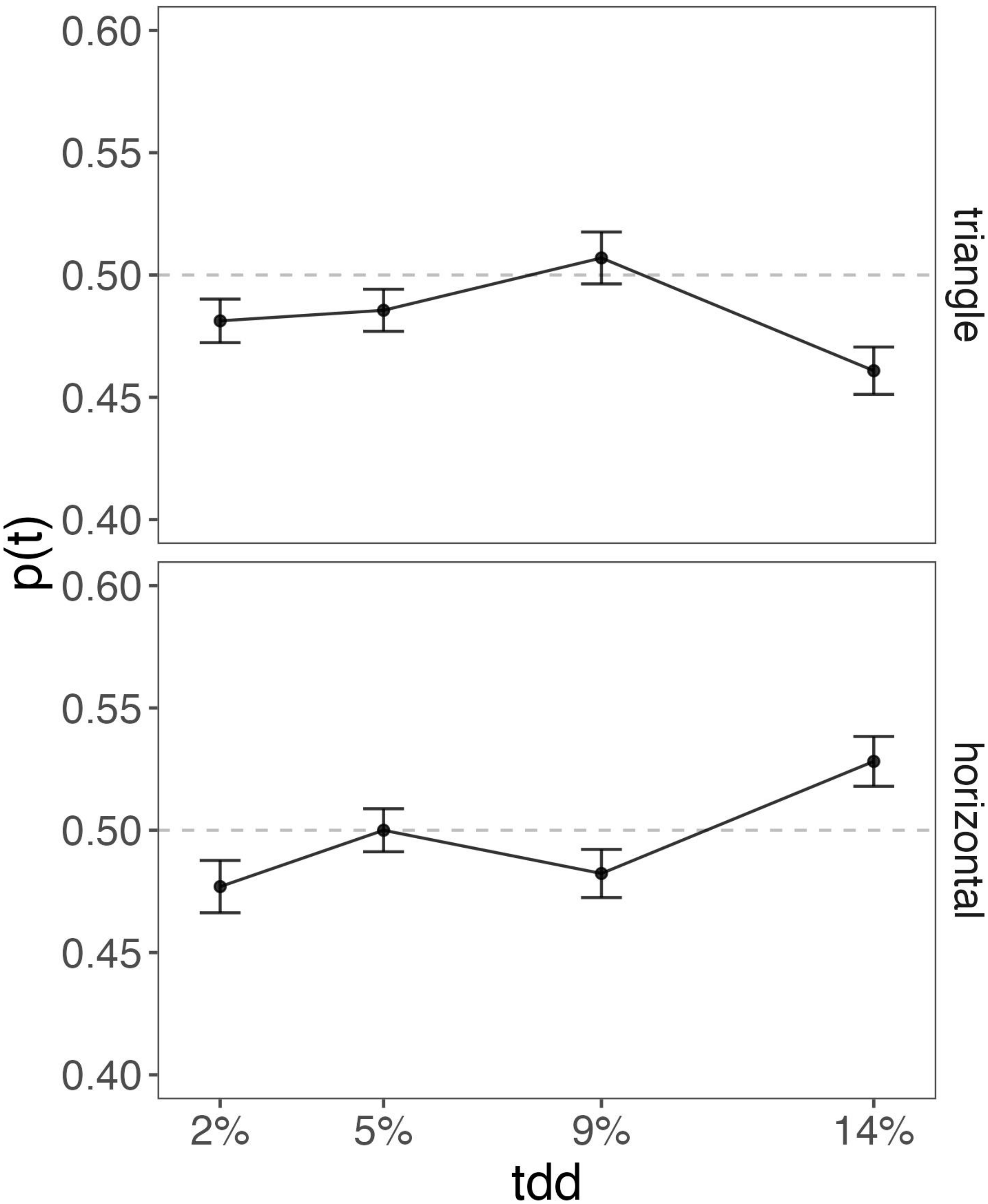

Figure 2.4. Experiment 1, mean target choice proportions by stimulus display and TDD for all target-competitor trials. Error bars are 95% CIs. Dashed line is shown at .5.

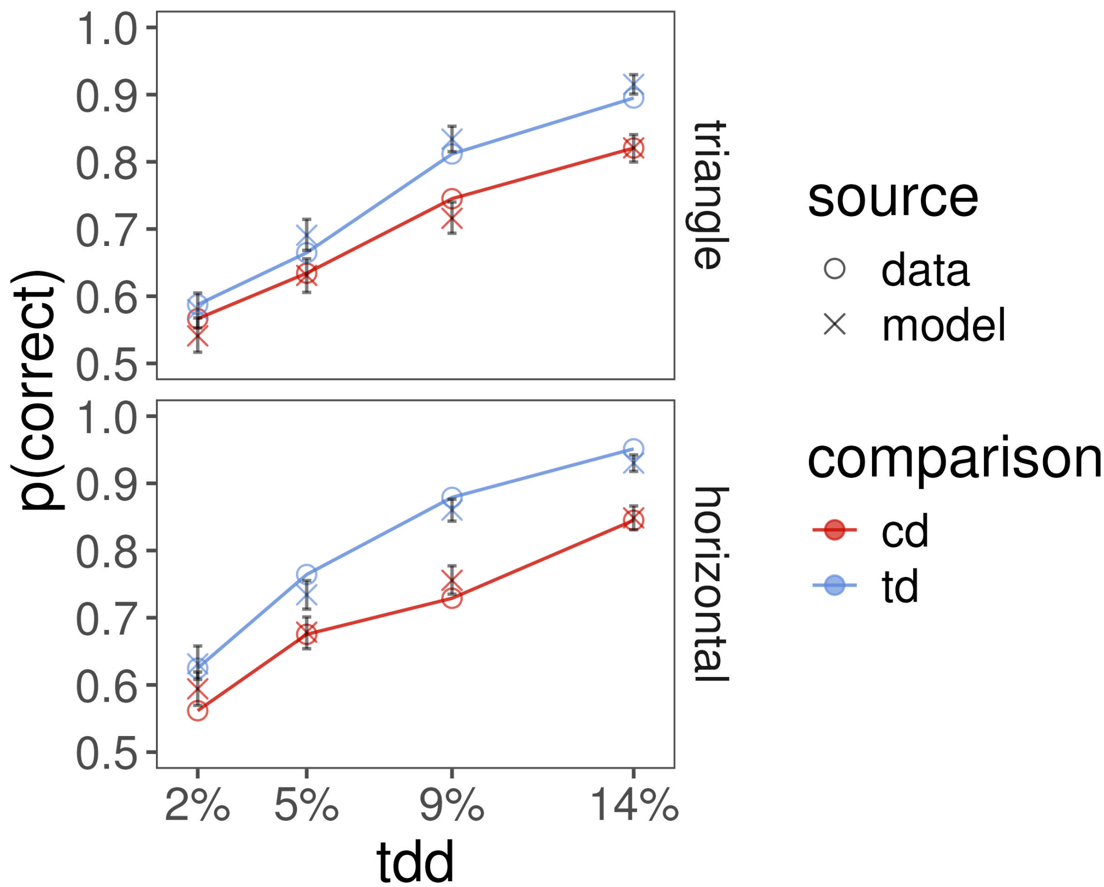

Figure 2.5. Experiment 1, mean choice proportions by stimulus display, TDD, and comparison. td=target-decoy trials, cd=competitor-decoy trials. Model predictions come from the Bayesian hierarchical logistic regression presented in the Appendix and are presented to demonstrate that the model used for inference adequately fits the data. Error bars are 95% HDIs on the means.

Experiment 1 showed that participants are not always able to discriminate between target-decoy and competitor-decoy stimuli. Additionally, this discriminability increases with TDD and that overall discriminability is better in the horizontal compared to the triangle display. The increased discriminability in the horizontal condition can be attributed to the fact that the stimuli exist in a horizontal array and are thus easier to compare to one another. Finally, through the interaction of comparison and TDD, it was shown that target-decoy discriminability increases with TDD at a higher rate than competitor-decoy discriminability.

These results are important because they show that target-decoy (and indeed, competitor-decoy) discriminability cannot be taken as a given in perceptual context effect experiments. Researchers must carefully consider how perception of the decoy affects choice. The role of perception is theoretically important because any conclusions about context effects being fundamental to choice (Trueblood et al., 2013) rely on the assumption that context effects are qualitatively similar across choice domains.

In the next section, a Thurstonian choice model is introduced, with the goal of using it to understand the attraction and repulsion effect.

2.3 Modeling Context-Dependent Perceptual Choice

One goal of this work is to separate the role of perceptual and decisional processes in the attraction and repulsion effects. To do so, we begin with an extreme stance - that such experiments are solely perceptual experiments rather than decision-making experiments and that no high level decision processes are occurring. This extreme assumption is quite likely incorrect, but it may be helpful in understanding existing data. For example, if a perceptual model can account for the repulsion effect without incorporating higher-level decision processes, (such as loss aversion or distance-dependent inhibition, as invoked by some cognitive models of context effects), this provides evidence that the repulsion effect is not a decision-making phenomenon, but a perceptual one. On the other hand, if perceptual processes explain part, but not all, of the repulsion effect, this provides evidence that decision processes must be invoked in a complete theoretical account of these data.

There is a large body of psychological research, beginning with the work of Thurstone (1927), that treats stimulus perception as a random variable. In his famous "Law of Comparative Judgment" paper, Thurstone (1927) showed that researchers can use binary choice proportions to estimate the psychological distance between stimuli, by treating psychological value as a Gaussian random variable. According to Thurstone's models, given a proportion of the trials on which participants were able to discriminate between two stimuli, researchers can estimate the average psychological distance between these perceptions, scaled by their standard deviations. Thurstone considered multiple forms of this model, but the most relevant model here is his Case II model, which allows for correlations between perceptions. Models of this class are often called *Thurstonian*, a term used throughout this chapter and in subsequent chapters.

Thurstone's work has been particularly influential on the choice modeling literature. In marketing and economics, researchers treat the utilities of choice options as random variables, which are often assumed to be Gaussian or Extreme Value Distributed and estimate choice models known as Random Utility Models (RUMs) (McFadden, 2001; Train, 2009). Often, though not necessarily, these models share the common property that value - whether the utility of a consumer product, the perception of magnitude in a perceptual task, or memory signal in a recognition task - is stochastic while choice is deterministic.

Based on this research, we introduce a Thurstonian choice model which will be applied to perceptual choice. This model treats the experiments of Trueblood et al. (2013) and Spektor et al. (2018) as perceptual tasks, rather than decision tasks. This assumption may be incorrect, but the extent to which it can explain existing data can still be theoretically informative. According to the Thurstonian model, the perceived areas of each rectangle on each trial are sampled from a multivariate normal probability distribution. Participants then deterministically choose the option with the largest perceived area. Thus, while perception is stochastic, choice is deterministic.

Though a large class of cognitive models has been developed to account for context effects, these models are quite complex. When these models are fit to data, the goal is typically to test the ability of various high-level decision mechanisms to account for context effects (Turner et al., 2018). The goal of the Thurstonian analysis, however, is to examine whether a simple choice model with well-known measurement properties can explain the attraction/repulsion effects, when correlations between options are included in the model.

2.3.1 A Thurstonian Choice Model

In a perceptual choice task such as that of Spektor et al. (2018), the value/utility of each option is its perceived area. According to the model, on each trial i with choice set K, $\boldsymbol{X_i}$ is a vector of length 3, where each element is the perceived area of a rectangle. $\boldsymbol{X_i}$ is sampled from a multivariate Gaussian distribution:

$$\boldsymbol{X}_i \sim \boldsymbol{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \tag{2.1}$$

where $\boldsymbol{\mu}$ is the column vector:

$$\begin{pmatrix} \mu_T \\ \mu_C \\ \mu_D \end{pmatrix} \tag{2.2}$$

where the subscripts T, C, and D indicate target, competitor, and decoy, respectively.

$\boldsymbol{\Sigma}$ is a positive semi-definite 3×3 covariance matrix computed by:

$$\boldsymbol{\Sigma} = \boldsymbol{S}\boldsymbol{\Omega}\boldsymbol{S} \tag{2.3}$$

where $\boldsymbol{S}$ is a diagonal matrix consisting of:

$$\begin{pmatrix} \sigma_T & 0 & 0 \\ 0 & \sigma_C & 0 \\ 0 & 0 & \sigma_D \end{pmatrix} \tag{2.4}$$

with σ_T, σ_C, σ_D denoting the standard deviations for target, competitor, and decoy, respectively. These parameters reflect the extent to which area perception varies from trial to trial for each stimulus type. $\boldsymbol{\Omega}$ is a 3×3 correlation matrix:

$$\begin{pmatrix} 1 & \rho_{TC} & \rho_{TD} \\ \rho_{TC} & 1 & \rho_{CD} \\ \rho_{TD} & \rho_{CD} & 1 \end{pmatrix} \tag{2.5}$$

with ρ_{TD}, for example, denoting correlation between target and decoy perception. These parameters measure the extent to which area perceptions move together for certain pairs of stimuli. For example, as will be shown below, the target and decoy perceptions are strongly positively correlated, meaning that on trials where the target is perceived to be relatively large, the decoy will also tend to be perceived relatively large.

As mentioned above, the model assumes that value is stochastic while choice is deterministic[4]. The model always chooses the option perceived as largest, regardless of the magnitude of the difference between the "winner" and "runners-up". That is, given a vector $\mathbf{X}_i$ of perceived areas on trial i with set K, the probability a participant selects stimulus j as largest is:

$$P(j|i,K) = P(\mathbf{X}_{ij} > \mathbf{X}_{ik}), \forall k \in K, j \neq k \tag{2.6}$$

If all correlations = 0 and $\sigma_T = \sigma_C = \sigma_D$, the model collapses to a trivariate version of the standard Thurstonian Case V model (Thurstone, 1927) often used by psychology researchers. Models of this form have closed form solutions and their predictions are easy to compute.

On the other hand, if any correlations are non-zero, the closed form solution of this model does not exist, and to compute predictions and estimate model parameters, researchers must use simulation or numerical integration methods (Train, 2009).

This model is easily capable of generating a(n) attraction (repulsion) effect by assuming $\mu_T > \mu_C$ ($\mu_C > \mu_T$), i.e., that on average target and competitor stimuli differ in their perceived areas. This is an ad hoc assumption that may describe the data well but will generate limited theoretical insight.

On the other hand, the correlation parameters ρ allow the model to predict context effects. Other researchers have considered correlations in RUMs as a mechanism for context effects. Kamakura and Srivastava (1984) argued that the correlation between a pair of options in the multinomial probit model reflect their similarity, or substitutability. They implemented a model where the correlation between options is parameterized as a decreasing function of their distance in attribute space. They

[4]This also assumes ties are not possible, which is true if and only if perceived area is continuous.

showed, via simulations, that this model is able to successfully predict the similarity effect. Kamakura and Srivastava (1984) also fitted the data to risky choice data, which were collected using stimuli designed to elicit the similarity effect, and found that the fitted model could successfully account for the data.

Natenzon (2019) implemented a Bayesian probit model, in which participants are assumed to sample from a multivariate normal distribution on each trial, with the correlation between options being related to their similarity in attribute space. Natenzon (2019) also suggested that similarity is related to ease of comparability. They fitted the model to frog mating choice data and showed that not only can the model explain choice reversals (i.e., context effects), but the estimated correlations between pairs of options were greater for options closer in multiattribute space.

It is worthwhile considering the role of correlations (i.e., ρ_{TC}, ρ_{TD}, and ρ_{CD}) in predicting the attraction and repulsion effects in perceptual choice. A set of simulations in which the ρ parameters varied systematically is reported below.

ρ_{TD} and ρ_{TC} were independently varied from -1 to 1 in increments of .005; it was assumed that $\rho_{TC} = \rho_{CD}$. In other words, all rectangles oriented the same way (i.e., target and decoy) shared one correlation and those oriented differently (i.e., target and competitor, decoy and competitor) shared another correlation. It was also assumed that $\mu_T = \mu_C > \mu_D$ and that $\sigma_T = \sigma_C = \sigma_D$[5].

The model was simulated for $10,000$ trials at each parameter set. On each trial, a set of target, competitor, and decoy areas were sampled from the model, and the stimulus with the largest sampled area was selected. Choice proportions were then computed for each parameter set.

[5]Experiment 2 presents evidence supporting these assumptions.

Figure 2.6 shows simulated model predictions in the form of RST[6], where RST values above .5 indicate an attraction effect and values below .5 indicate a repulsion effect. The model can, depending on the relationship between ρ_{TD} and ρ_{TC}, predict a repulsion, attraction, or a null context effect.

First, if $\rho_{TD} = \rho_{TC} = \rho_{CD}$ (diagonal line in Figure 2.6), the model predicts a null effect. In this case, no pair of stimuli are more correlated than any other pair, so the predictions are identical to a model where $\rho_{TD} = \rho_{TC} = \rho_{CD} = 0$. The decoy will still be chosen occasionally (depending on the relative difference between μ_D and μ_T / μ_C), it does not differentially take choice shares away from the target or competitor.

If $\rho_{TD} > \rho_{TC}$ (region above the diagonal line in Figure 2.6), the model predicts a repulsion effect. The correlation between target and decoy causes them to "move" together. If, on a particular trial, the perceived target area is relatively high, the perceived decoy area will also be relatively high. It is then more likely that the decoy will occasionally exceed the target than it is that the decoy occasionally exceeds the competitor. Though the decoy is chosen at a lower rate than target or competitor, the ρ_{TD} parameter causes it to be chosen over the target more often than it is chosen over the competitor.

Lastly If $\rho_{TC} > \rho_{TD}$ (region below the diagonal line in Figure 2.6), the model predicts an attraction effect. The logic here is the similar to the above logic; the decoy and competitor tend to move together, and the decoy is more likely to exceed the competitor than the target.

The black regions are the regions in which a positive semi-definite variance-covariance matrix could not be performed, so predictions are unavailable.

[6]Recall that RST is *Relative Share of the Target*, $\frac{P(T)}{P(T)+P(C)}$, reflecting the proportion of trials in which the target is chosen over the target, conditional on not having selected the decoy.

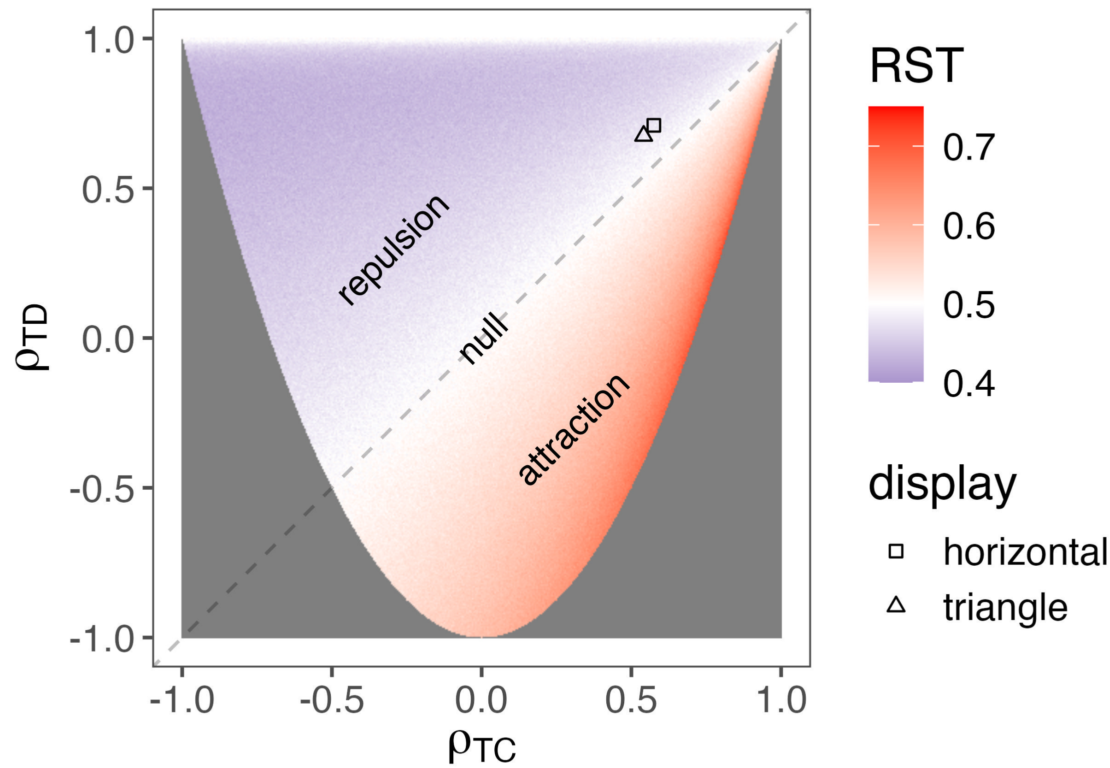

Figure 2.6. Model simulations for the attraction and repulsion effects based on the variation of ρ_{TD} and ρ_{TC}. Regions of the plot are labeled based on their qualitative predictions for attraction, null, and repulsion effects. The gray region is the area where, due to extreme correlations, a positive semi-definite variance-covariance matrix could not be formed and predictions are unavailable. The triangle and circle mark the estimated mean correlations from the Experiment 2 triangle and horizontal conditions, respectively (see Experiment 2 results for further details).

2.3.2 Perceptual Correlations as Mechanism for the Repulsion Effect

These perceptual correlations, which cause the decoy to occasionally exceed the target in perceived area, may be, at least partially, generating the repulsion effect in Spektor et al. (2018)'s data.

Experiment 1 demonstrated that participants do not always discriminate between the target and decoy (or indeed, competitor and decoy). Experiment 1 also demonstrated the triangle display decreases discriminability relative to the horizontal display.

The target and decoy, however, are quite perceptually similar and easier to compare. Simultaneously, however, the fact that target and decoy share an orientation (i.e., both wide or both tall) makes the comparison of these two options easier. At lower levels of TDD, when discriminability is quite difficult (i.e., the difference between μ_D and μ_T/μ_C is small), the similarity of target and decoy makes it more likely the decoy is occasionally perceived as larger than the target, than the competitor.

In statistical terms, the perception of the decoy is more strongly correlated with the perception of the target than with perception of the competitor. These correlations are measured with the ρ_{TD} and ρ_{CD} parameters. According to this account, if $\rho_{TD} > \rho_{CD}$, the perceived areas of target and decoy "move" together, allowing the competitor to exceed the target at a higher rate than the target exceeds the competitor, particularly if perceptual discriminability is low. This account explains the repulsion effect as a perceptual phenomenon rather than a decision-making phenomenon.

Experiment 2, shown below, combined a psychophysics task with a choice task to estimate the parameters of the Thurstonian choice model. In the first phase of the experiment, participants estimated the sizes of target, decoy, and competitor rectangles on each trial. The second phase of the experiment was a standard choice experiment, replicating Spektor et al. (2018)'s procedure. The data from the first phase of Experiment 2 were used to obtain stable estimates of $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ for the Thurstonian choice model. The Thurstonian model, conditioned on these parameter estimates, was then used to make predictions for the choice phase. The Thurstonian model, conditioned on these parameter estimates, naturally predicts a repulsion effect but not an attraction effect.

2.4 Experiment 2

The goal of this experiment was to estimate the parameters of the model presented above and to clarify the role of perception and decision in producing the repulsion and attraction effects in the design of Spektor et al. (2018). The Thurstonian model introduced above can predict the repulsion effect and the attraction effect through the perceptual correlation parameters. However, this model can produce many data patterns (as shown in Figure 2.6), some of which may be consistent with existing choice data but many others which are not. This experiment allows us to estimate the model parameters using a non-decision task and examine how the model predictions compare with empirical choice data.

Experiment 2 utilized the *method of cross-modal matching* (Stevens & Marks, 1965), a psychophysics technique where participants estimate the intensity of a stimulus presented in one modality using a different response modality, for example by estimating the loudness of a tone by adjusting the brightness of a light. Here, participants were told to adjust the size of three circles to match the areas of three corresponding rectangles.

The experiment also incorporated Spektor et al. (2018)'s stimulus display. In both phases, there was between-participants manipulation to display the rectangle stimuli in either the horizontal or triangle displays of Spektor et al. (2018).

2.4.1 Methods

2.4.1.1 Participants.

Data collection took place at the University of Massachusetts Amherst. 521 undergraduate students participated in exchange for course credit. 68 participants did not complete the full experiment within the 1-hour time limit and were removed from all analyses. 10 additional participants were failing at least 2/8 catch trials from the circle adjustment phase (see below).

After removing participants, this left 218 participants in the horizontal display condition and 225 participants in the triangle display condition.

$4,414$ choice trials with RTs < 100ms or $> 10,000$ms were removed from analysis, leaving a total of $56,552$ total choice trials.

2.4.1.2 Stimuli.

Stimuli were gray-scale rectangles, based on the experiments of Spektor et al. (2018) and Trueblood et al. (2013).

Both the circle adjustment phase and the choice phase (see below) had critical trials, the main focus of the experiment. On each critical trial, there was a target, competitor, and decoy rectangle. Target and competitor always had identical areas. The target and competitor rectangles were always oriented differently (i.e., one wider than tall and the other taller than wide), while the decoy was oriented the same as the target. Target-decoy distance (TDD) also varied, with the decoy being either 2%, 5%, 9%, and 14% smaller than the target and competitor. For example, if TDD=2%, then the decoy's area is $\approx$ 98% the area of the target and competitor. Decoys were created by reducing both the height and width equally. For example, on the lowest diagonal (see below), the target and competitor had dimension values of 60 and 135 px, and the TDD=2% decoy had dimension values of 59 and 134 px. Decoys where both dimension values are lower than that of the target are known as range-frequency decoys in the context effects literature (Wedell, 1991).

To introduce variation in the stimuli, the absolute area of the target and competitor also varied. The target and competitor were either 8100, 14850, or 23400 px^2 (with the decoy area changing proportionally based on TDD), falling along one of three diagonals when plotted. See Figure 2.7 for a plot of all critical stimuli. In pixels, the smaller/larger dimension values on the lower, middle, and upper diagonals were 60/135, 90/165, and 120/195, respectively.

Participants were randomly assigned to either the triangle or horizontal display condition, where, on each trial both rectangles and circles were either arranged in a triangle or in a horizontal array.

The experiment also contained filler trials. On filler trials, stimuli were randomly generated by independently sampling three rectangles' dimension values from the distribution $U(56, 195)$px, the full range of dimension values from the critical trials, where $56px$ was the smallest possible decoy dimension value and 195 was the largest possible target/competitor dimension value.

On the catch trials, one larger stimulus was generated by sampling uniformly along the upper diagonal line from the critical trials (see Figure 2.7). Two additional smaller stimuli were generated by sampling uniformly from the lower diagonal line from the critical trials (see Figure 2.7). This ensured that one rectangle was always larger than the other two rectangles and allowed for the removal of participants who could not complete the task. Specifically, on a given trial, a participant who did not judge the larger rectangle to be greater in area than the other two rectangles was considered to have failed this trial. Only participants who passed 6/8 judgment catch trials were retained in the analysis (see Participants).

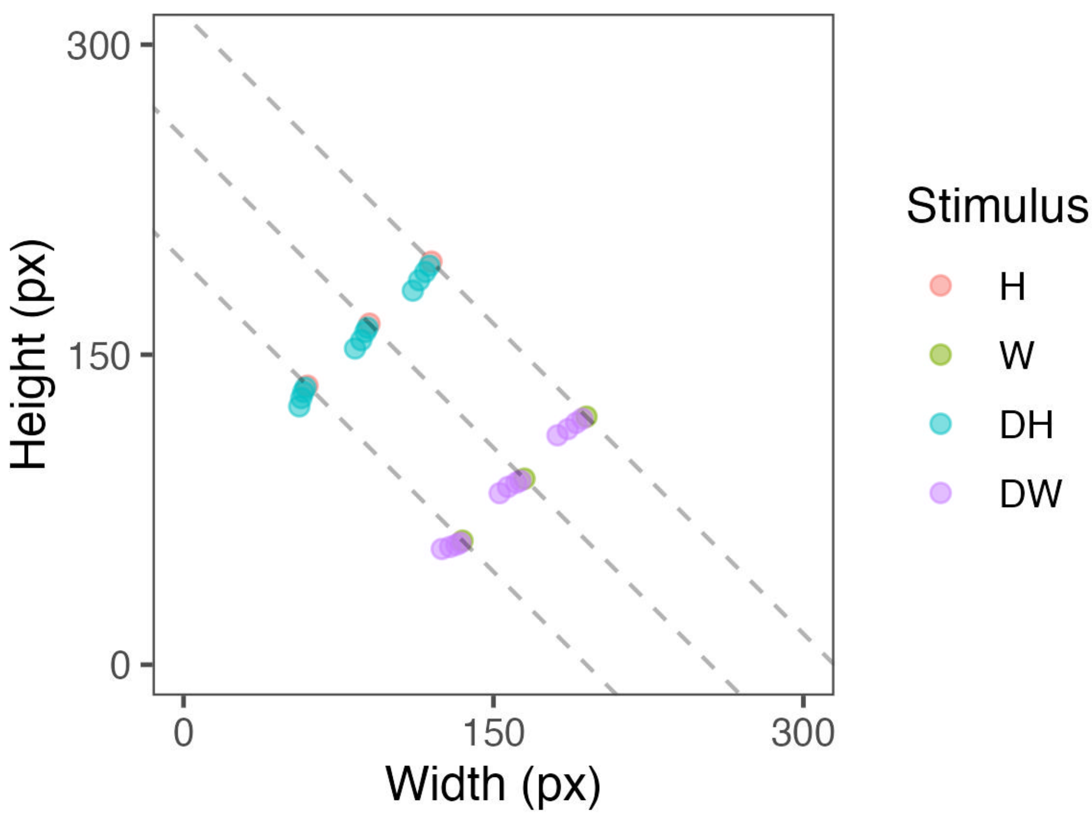

Figure 2.7. Stimulus configuration for Experiment 2. DH and DW indicate the decoys for H and W, respectively.

The choice phase had identical stimuli with the exception that there were no catch trials, only critical and filler trials. Additionally, there were no circles in this phase (see Procedure).

2.4.1.3 Design.

Across both phases, display condition (triangle, horizontal) varied between-participants and TDD (2%, 5%, 9%, 14%), diagonal (lower, middle, upper), and target-decoy orientation (wide, tall) varied within-participants.

In the circle adjustment phase, there were 4 blocks, each with 40 trials. Each block consisted of 24 critical trials, 14 filler trials, and 2 catch trials. Within the critical trials, there were 6 trials at each level of TDD. In 3 of these 6 trials the target and decoy were oriented wide (choice set $[W, H, D_W]$), and in the other 3 trials the target and decoy were oriented tall (choice set $[W, H, D_H]$).

In the choice phase, there were 4 blocks, each with 34 trials. 24 of these trials were critical trials and 10 were filler trials. Of these 24 critical trials, there were 6 trials at each level of TDD. Within each 6, there were 3 trials where target and decoy were oriented wide and 3 trials where target and decoy were oriented tall.

2.4.1.4 Procedure.

The experiment was presented on computer monitors with a resolution of 1920 x 1080 pixels and programmed with GNU Octave (Eaton et al., 2021) and PsychoPhysics Toolbox (Brainard, 1997).

The experiment took place in two phases: a circle adjustment phase, followed by a choice phase:

On each circle adjustment trial, participants saw three rectangles and three circles, labeled 1, 2, and 3. The rectangles appeared in the lower left corner of the screen, either in a triangle or horizontal display, depending on the condition to which the participant was assigned. The circles appeared in the upper right of the screen, in the same display as the rectangles (see Figure 2.8 for a sample trial). Stimuli were positioned this way to ensure that participants could not easily compare the circles to the rectangles. A small amount of jitter ($U(-15, 15)\text{px}^2$) was also added to the vertical position of each rectangle and the corresponding circle. Each circle started with an area of 78px^2, the minimum size allowed in the experiment.

The circle adjustment phase began with three calibration trials. Calibration trials were identical to filler trials, except that participants received feedback after their responses. After participants submitted their responses on a calibration trial, a red circle appeared around each adjusted circle, showing the true area of the corresponding rectangle.

After the calibration trials, the experimental circle adjustment trials began. There were 4 blocks of experimental trials. At the beginning of each block, participants completed 3 calibration trials.

Participants were told to adjust the size of each circle until it had the same area as the rectangle with the corresponding label. They used the mouse to adjust the circles. They were not given instructions on the order they should adjust the circles in or the time they were to spend on the adjustments. The maximum circle area allowed was 65144px^2, and the minimum area was 78px^2[7]. When a participant finished adjusting the circles on a trial, they clicked a "Submit" button located on the lower right-hand corner of the screen and advanced to the next trial.

At the end of each experimental block, participants were told that they were either over or under-adjusting, on average, based on the current mean deviation of their

[7] 65144px^2 was the maximum area the circles could be while only appearing on the right half of the screen and maintaining the same horizontal distance from each other as the corresponding rectangles. The minimum possible rectangle area in the experiment was $3136px^2$, while the maximum possible area was $38025px^2$. Thus, the range of possible responses was well within the true area of all rectangles participants saw.

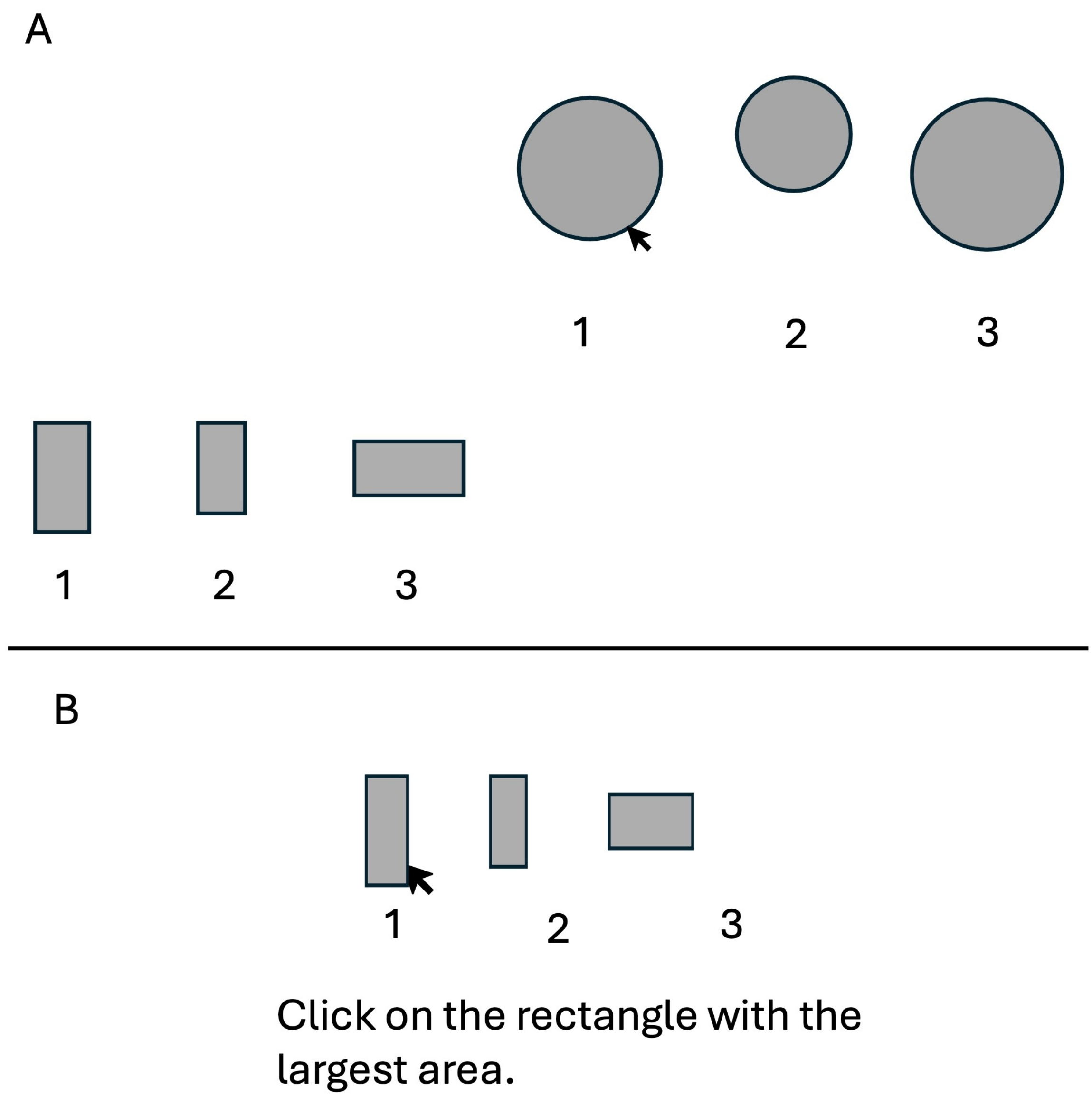

Figure 2.8. Example trials from Experiment 2. A: Circle adjustment phase. B: Choice phase. This is an example from the horizontal display condition.

responses from the true areas. They were not told the magnitude of their estimation error.

After the circle adjustment phase, the choice phase began.

The choice phase began with 3 practice trials, which were identical to the filler trials but separate from the experimental trials. Participants did not receive feedback during choice practice trials.

During the choice phase, participants were shown three rectangles on each trial and asked to select the rectangle with the largest area. They were told that they would know their percentage of correct choices at the conclusion of the experiment. Note that in a critical trial, a correct response is one where the participant did not select the decoy, given that the target and competitor rectangles always had the same area.

At the end of the choice phase, participants were told their percentage of correct choices.

In both the circle phase and the choice phase, stimulus order was randomized on each trial.

2.4.2 Results

2.4.2.1 Data Processing

The most substantial data processing occurred with the circle phase data, so these are discussed first. These data required processing to ensure that no outliers influenced estimates of $\boldsymbol{\mu}$ or $\boldsymbol{\Omega}$, as it is widely known that outliers can detrimentally influence correlation estimates.

First, all trials where at least one circle was not adjusted (i.e., at least one circle was left at the starting size) were removed. Given the discrepancy between the starting circle area (78px^2) and the smallest rectangle area shown in the experiment

$(3, 136\text{px}^2)$, these are unlikely to be representative of any participant's true perception of rectangle area.

Next, outlier participants were removed, followed by outlier trials from the remaining participants. An outlier participant is a participant whose responses are substantially different from most other participants, while an outlier trial is a trial, from a valid participant, which differs substantially from the rest of a given participant's trials. Outlier participants were first removed, and then outlier trials were removed from the remaining participants.

Outlier participants were identified and removed using the following procedure:

Each individual participant's data were log-transformed[8]. Each participant's data were then fit to a linear regression, regressing each log circle area on each corresponding log rectangle area. Participants whose corresponding R^2 value fell below the 5% quantile for all R^2s (.3975 in this case) were removed from all analysis. The 5% quantile was chosen arbitrarily as a conservative threshold to ensure maximum participant retention while removing outliers.

This procedure removed 23 participants, leaving $N = 420$ participants, 213 in the triangle display condition and 207 in the horizontal display condition. Of the remaining participants, R^2 values were high ($M = .67, SD = .12$), indicating that participants could generally perform the task.

Next, outlier trials were identified and removed using the following procedure:

From the 420 participants whose data were analyzed, outlier trials were removed from the critical trial data. All data points were normalized within each participant and diagonal[9]. All critical trials with at least one z-score with an absolute value above 3.5 were dropped, removing a total of 102 trials. This removed 0, 1, 2, and 4 critical trials from 339, 62, 18, and 1 participants, respectively. This left $20, 371$

[8]This was done to reduce the skewness observed in the raw data.

[9]This was done solely for the removal of outliers, not for the substantive analysis that follows.

data points in the triangle display condition and $19,809$ data points in the horizontal display condition, where a data point is a vector of the participant's estimated target, competitor, and decoy areas on a given trial.

Only those participants whose data were retained in the circle adjustment phase were retained in the choice phase.

2.4.2.2 Judgment Phase Results

First, it is important to show that participants could adequately perform the adjustment task. The mean difference between actual log area and estimated log area was computed for each participant, stimulus pair (i.e., target-competitor, target-decoy, competitor-decoy), and actual difference. These results are plotted in Figure 2.9. The x-axis shows the actual difference between areas, while the y-axis shows the estimated difference in areas according to participants' judgments. Participants' judgments varied considerably along the y-axis, but on average, the estimated difference increased with the actual difference.

Next scatterplots of all pairwise circle area estimations are shown in Figure 2.10.

Correlation coefficients in each display condition were also computed. For the triangle condition, $r_{td} = .61$, $r_{cd} = .54$, and $r_{tc} = .55$. For the horizontal condition, $r_{td} = .66$, $r_{cd} = .56$, and $r_{tc} = .57$. In both conditions, the target-decoy correlation is stronger than both the competitor-decoy and the target-decoy correlations.

Bayesian hierarchical modeling was used to estimate the parameters of the Thurstonian perceptual model introduced earlier, as well as perform inference on the means and correlations. As an alternative to computing descriptive statistics for the relevant components of a multivariate Gaussian (i.e., computing raw means, variances, and correlations), the Bayesian analysis allows us to model the obtained trial-level area judgments as coming from a single joint Gaussian distribution of estimated areas. Additionally, this is also a novel application of the multinomial probit (MNP) model,

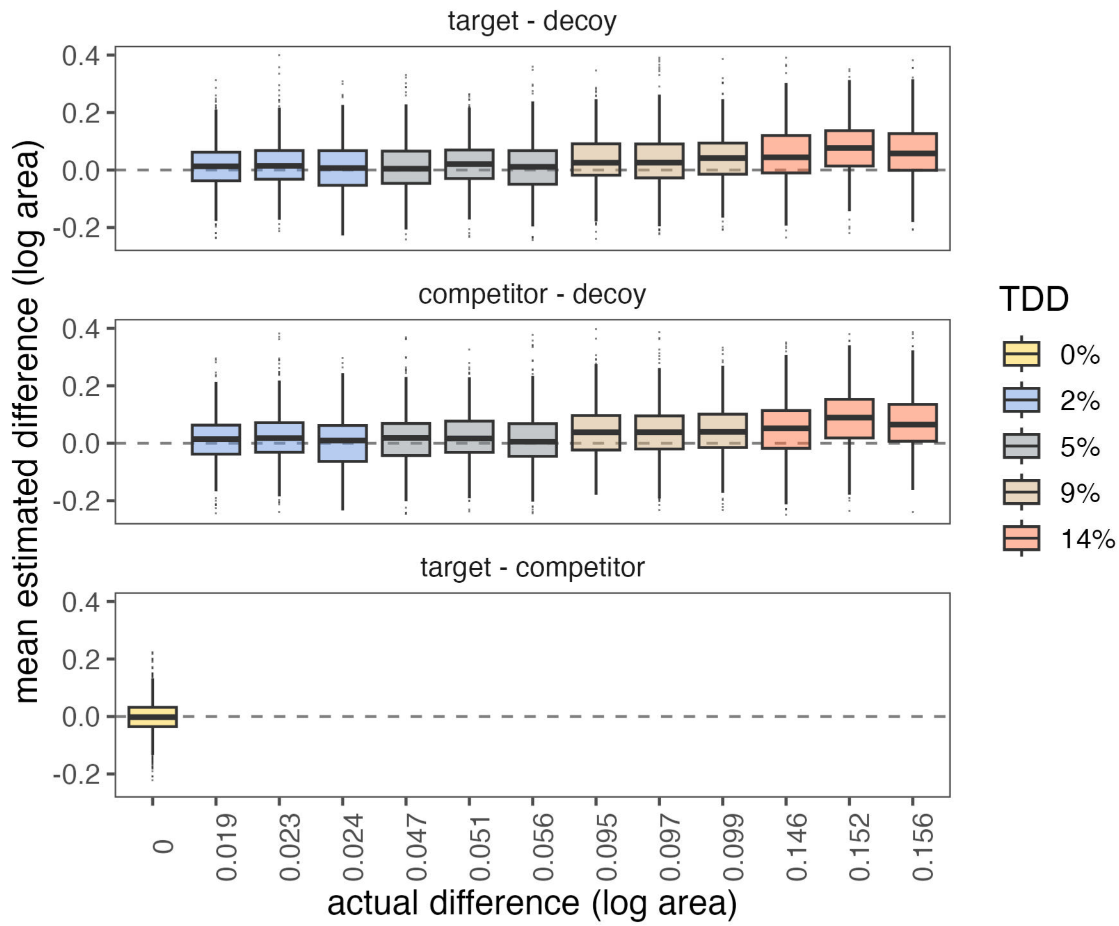

Figure 2.9. Experiment 2 plots of subject-level mean error in log-transformed area estimations, split by stimulus pair (e.g., target-decoy), TDD, and actual difference in rectangle area. Because the difference in physical area at a given TDD differed across the three diagonals, there are three plots per each TDD level. Note that because the target and competitor rectangles always had equal areas, the true difference is always 0.

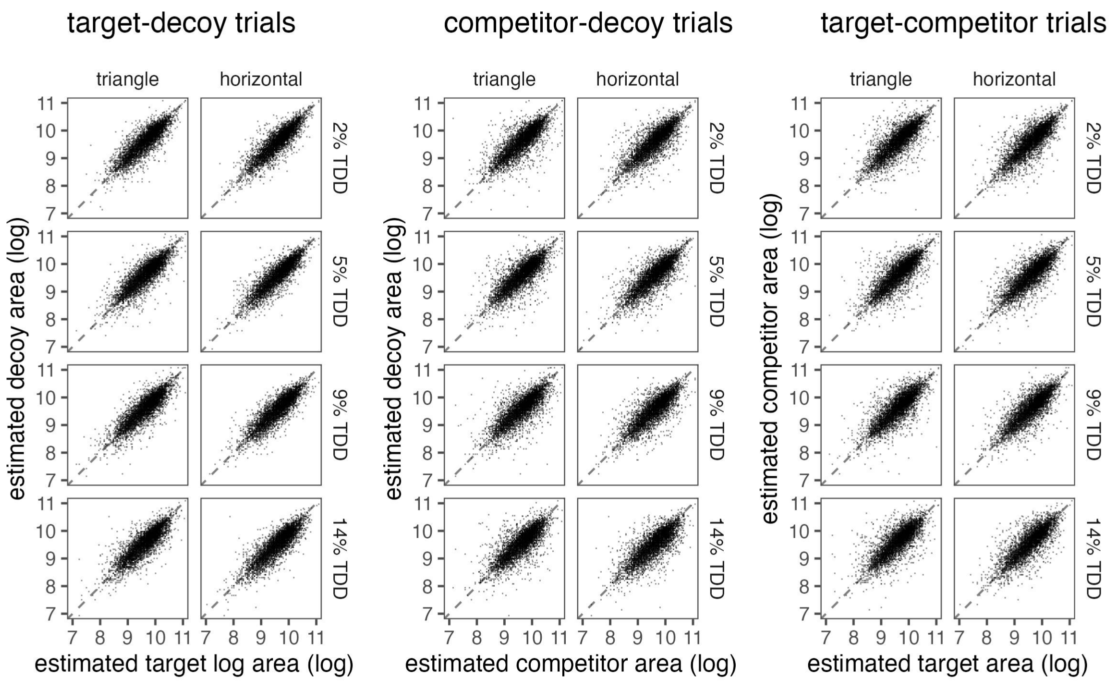

Figure 2.10. Scatterplots of target-decoy (left), competitor-decoy (middle), and target-competitor (right) log estimated areas, split by display condition (columns) and TDD (rows).

whose parameters are usually estimated by inferring utilities from choice data (Train, 2009). In the current application, the parameters were estimated from the judgment data using Bayesian modeling and were then used to predict choice data collected from the same participants.

The main text contains estimates of $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ (the vector of means and the variance-covariance, respectively), while the details of the estimation procedure are found in the Appendix. As discussed above $\boldsymbol{\Sigma}$ is a positive semi-definite variance-covariance matrix, computed as $\boldsymbol{\Sigma} = S\boldsymbol{\Omega}S$, where S is a vector of standard deviations and $\boldsymbol{\Omega}$ is a 3×3 correlation matrix.

The estimates of S suggest no systematic differences in the variance of area judgments variance across target, decoy, and competitor rectangles. In the triangle condition, the parameters were as follows: σ_T:M= 0.34,SD= 0.002,95%HDI $[0.332, 0.338]$, σ_C:M= 0.34,SD= 0.002,95%HDI $[0.335, 0.341]$, σ_D:M= 0.34,SD= 0.002,95%HDI $[0.331, 0.338]$. In the horizontal condition, the parameters were as follows: σ_T:M= 0.34,SD= 0.002,95%HDI $[0.334, 0.340]$, σ_C:M= 0.34,SD= 0.002,95%HDI $[0.338, 0.345]$, σ_D:M= 0.34,SD= 0.002,95%HDI $[0.333, 0.340]$.

Estimates of the $\boldsymbol{\mu}$ components at each TDD level (averaged across participants) are plotted in Figure 2.11. First, note that the decoy mean is always lower than that of the target and competitor. Furthermore, as TDD increases, the decoy mean decreases, indicating that participants were sensitive to decreases in absolute area, judging smaller rectangles as having smaller areas. Both target and competitor mean areas also decrease with TDD, suggesting that participants somewhat judged target and competitor relative to the decoy.

Next, consider only the target and competitor means. The competitor mean is always larger than the target mean, albeit only slightly (see Appendix for details). This pattern is somewhat consistent with the tainting hypothesis, the idea that nearby decoys can taint the value of the target option (Frederick & Lee, 2008; Simonson, 2014). However, the tainting hypothesis predicts that tainting is a decreasing function of distance in attribute space, and the current observed difference between target and competitor means appears to be constant. There is also limited statistical evidence for this difference, as the 95% HDIs overlap considerably at each TDD level in both display conditions (see Figure 2.11).

There is, however, evidence from the Bayesian Hierarchical Regression (see Appendix), via the competitor-specific regression coefficient β_{comp}, that the competitor is rated higher than the target, at least in the triangle condition. The β_{comp} parameter captures the tendency for participants to rate the competitor higher than the target, regardless of TDD or rectangle orientation. In the triangle condition, this parameter was reliably above 0, $M = 0.005$, $SD = 0.0023$, 95%HDI $[0.0001, 0.009]$. Note that this effect is quite small. Given that the data were transformed via the natural logarithm, this estimate corresponds to a $e^{0.005} = 1.005\text{px}^2$ difference in mean judged area, a considerably small effect size. In the horizontal condition, this parameter was not reliably different from zero, $M = 0.003$, $SD = 0.002$, 95%HDI $[-0.002, 0.007]$.

The estimates of $\mathbf{\Omega}$ (the correlations) are plotted in Figure 2.12. First, it is worth noting that all correlations are positive. This means that in general, there are strong trial-level dependencies between area judgments. However, the crucial pattern emerges when comparing the correlations to one another within each display condition. As predicted, in both display conditions, ρ_{TD} is larger than both ρ_{CD} and ρ_{TC}, while ρ_{CD} and ρ_{TC} do not differ from each other. This shows that there is a stronger dependency between judgments of the target and decoy rectangles - which are perceptually similar to one another - than of the target and competitor or the competitor and decoy rectangles. Note that these correlation estimates are lower than might be expected after inspecting the scatterplots in Figure 2.10. However, the estimates reported here account for individual participant effects, reducing the correlation estimates.

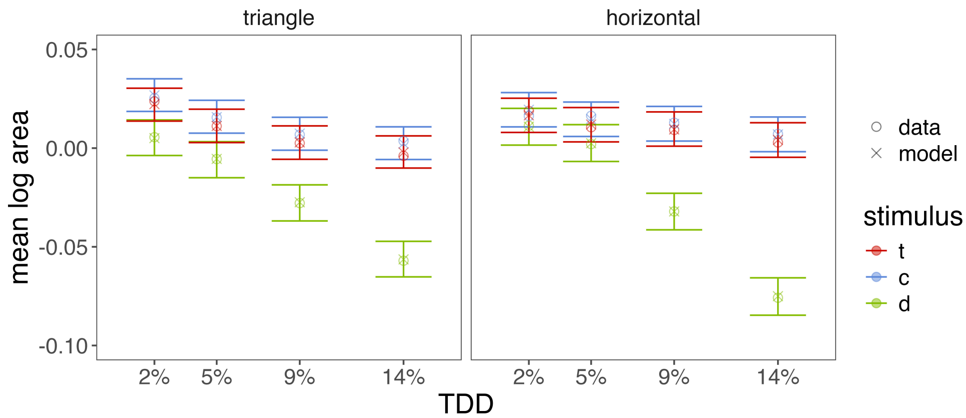

Figure 2.11. Experiment 2 $\boldsymbol{\mu}$ estimates. Model values are means, and the error bars are 95% HDIs.

The relatively high ρ_{TD} estimate predicts that the target and decoy tend to move together. On trials where the target is judged relatively large, the decoy is also judged to be relatively large. Furthermore, because $\rho_{TD} > \rho_{CD}$, this means that the decoy is more likely to be judged larger than the target than it is to be judged larger than the competitor. This provides a plausible mechanism for the repulsion effect; the decoy is more likely to take choice shares away from the target than from the competitor. This can generate a data pattern where $P(C) > P(T)$ without requiring $\mu_C > \mu_T$.

Furthermore, all ρ parameters from the horizontal condition are larger than the corresponding parameters from the triangle condition. This suggests that the difference between area judgments for target, competitor, and decoy rectangles in the horizontal condition can be attributed more to the true differences between their means than to general noise, when compared to the triangle condition. However, given that the model was fit separately to each display condition, these inferences should be taken cautiously. These results do align with those of Experiment 1, where participants' overall discriminability was higher in the horizontal condition than in the triangle condition, presumably due to the ease of comparison when all stimuli are arranged in a single horizontal array.

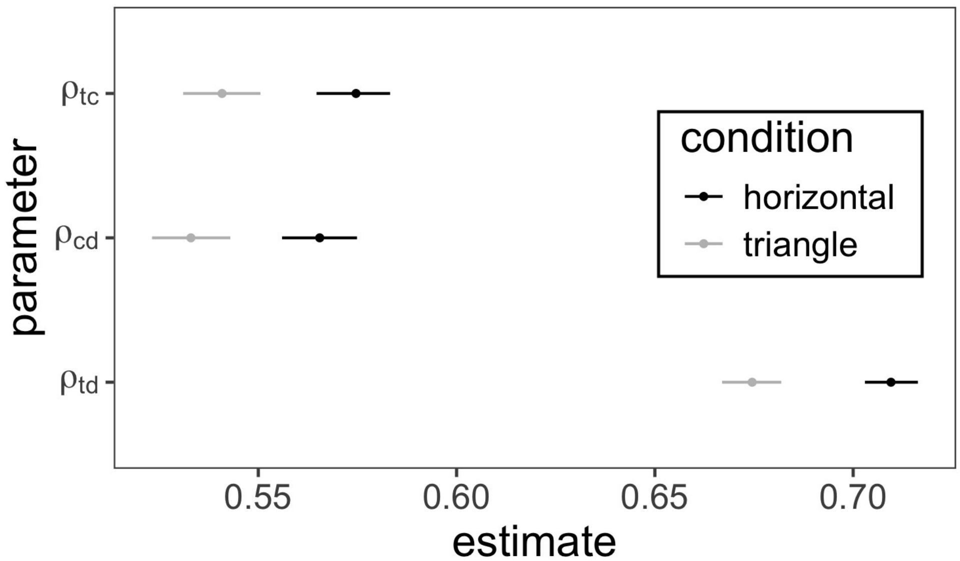

Figure 2.12. Experiment 2 posterior estimates of $\mathbf{\Omega}$ off-diagonal parameters across display conditions. Lines show 95% HDIs. Dots indicate means.

2.4.2.3 Choice Phase Results

In each trial of the choice phase, participants saw three rectangles and selected the rectangle with the largest area. This was a replication of Spektor et al. (2018).

Mean choice proportions across display condition and TDD are plotted in Figure 2.13, collapsed across choice set (i.e., ignoring whether the target was wide/tall).

First, in both display conditions, note that decoy choice decreases with TDD. Participants were sensitive to the absolute size of the decoy and chose it less often as the decoy decreased in area.

In the triangle condition (Figure 2.13, left panel), at the first three levels of TDD (i.e., 2%, 5%, and 9%), the data show a fairly strong repulsion effect, as the competitor was clearly chosen more often than the target. Though this figure suggests a null effect at $TDD = 14\%$, inferential statistics reported in the Appendix show that there is also a small repulsion effect at $TDD = 14\%$.

In the horizontal condition (Figure 2.13, right panel), at the first two levels of TDD (i.e., 2% and 5%), the data show a fairly strong repulsion effect, as the competitor was

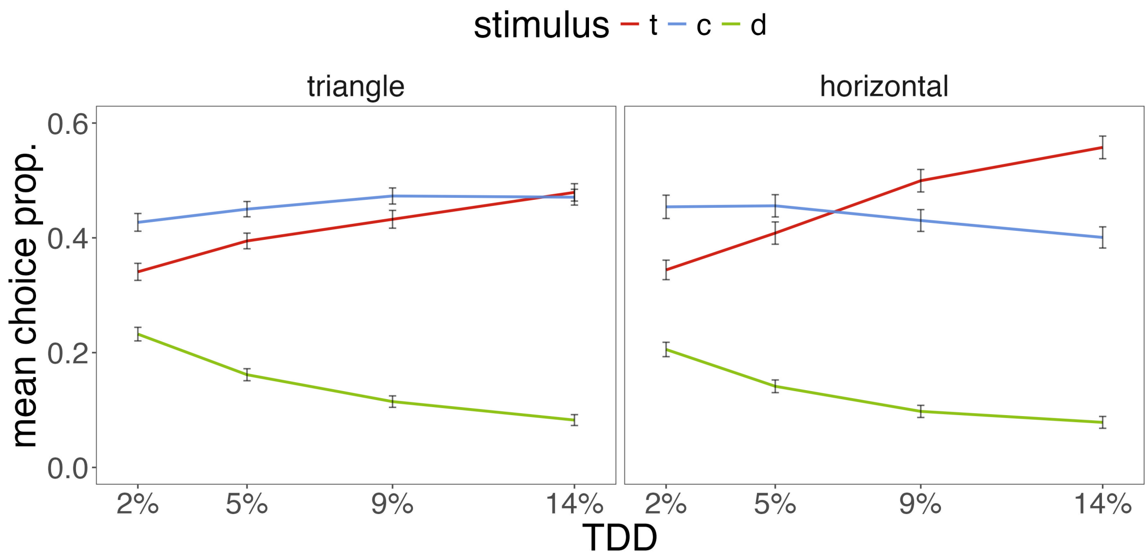

Figure 2.13. Experiment 2 mean choice proportions for target, competitor, and decoy options, by TDD and display condition. Error bars are 95% CIs on the means.

clearly chosen more often than the target. Though this figure suggests an attraction effect at $TDD = 9\%$, inferential statistics reported in the appendix show that there is a null effect at $TDD = 9\%$. The data clearly show, however, a strong attraction effect at $TDD = 14\%$. See the Appendix for inferential statistics which support these conclusions.

These data show that the repulsion effect, at least in the current paradigm, is strongly related to the similarity between target and decoy along with participants' ability to discriminate the target/competitor from the decoy. As TDD increases, the repulsion effect either decreases in magnitude (triangle condition) or reverses entirely to an attraction effect (horizontal condition). The results are also consistent with the overall discriminability difference between display conditions. The results of Experiment 1 showed that participants are better at discriminating the target and competitor in the horizontal condition than in the triangle condition.

To demonstrate the strong relationship between discriminability and the repulsion/attraction effects, the choice data from Experiment 2 are plotted against the two-alternative forced choice data from Experiment 1 (see Figure 2.14). The y-axis is identical to that of Figure 2.13. However, rather than the TDD - the physical difference between target and decoy - the x-axis is now the mean target-decoy discriminability. In other words, the x-axis is now the psychological target-decoy difference rather than the physical difference.

When the data are plotted this way, a critical pattern emerges. The crossover point - the point at which the repulsion effect becomes null or weak - occurs at lower discriminability levels in the horizontal condition compared to the triangle condition. This strongly suggests that the target-decoy relationship is crucial for generating the repulsion effect.

2.4.3 Model Simulations

Earlier, it was demonstrated that the Thurstonian Choice Model could generate a repulsion effect, an attraction effect, or a null effect, depending on the correlation parameters. However, it was open question whether, conditioned on actual estimated parameters, the model could generate data that match empirical choice data. The goal of the following analysis is to test the model's ability to predict the choice data.

Predictions were generated at each level of TDD using the mean estimates of $\boldsymbol{\mu}$[10] and $\boldsymbol{\Sigma}$, in each display condition. For simplicity, the mean estimates for each parameter were used, rather than the full distribution of parameter values.

To generate predictions, at each TDD level in each display condition $1,000,000$ samples were drawn from a multivariate Gaussian distribution with parameters $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$, where a sample is a vector of perceived target, competitor, and decoy areas. The choice proportion for each rectangle is simply the proportion of samples in which a particular rectangle was perceived to be the largest in that sample. The results are

[10] Given the considerable overlap in HDIs (see Figure 2.11), the μ_T and μ_C parameters were constrained to be equal at each level of TDD.

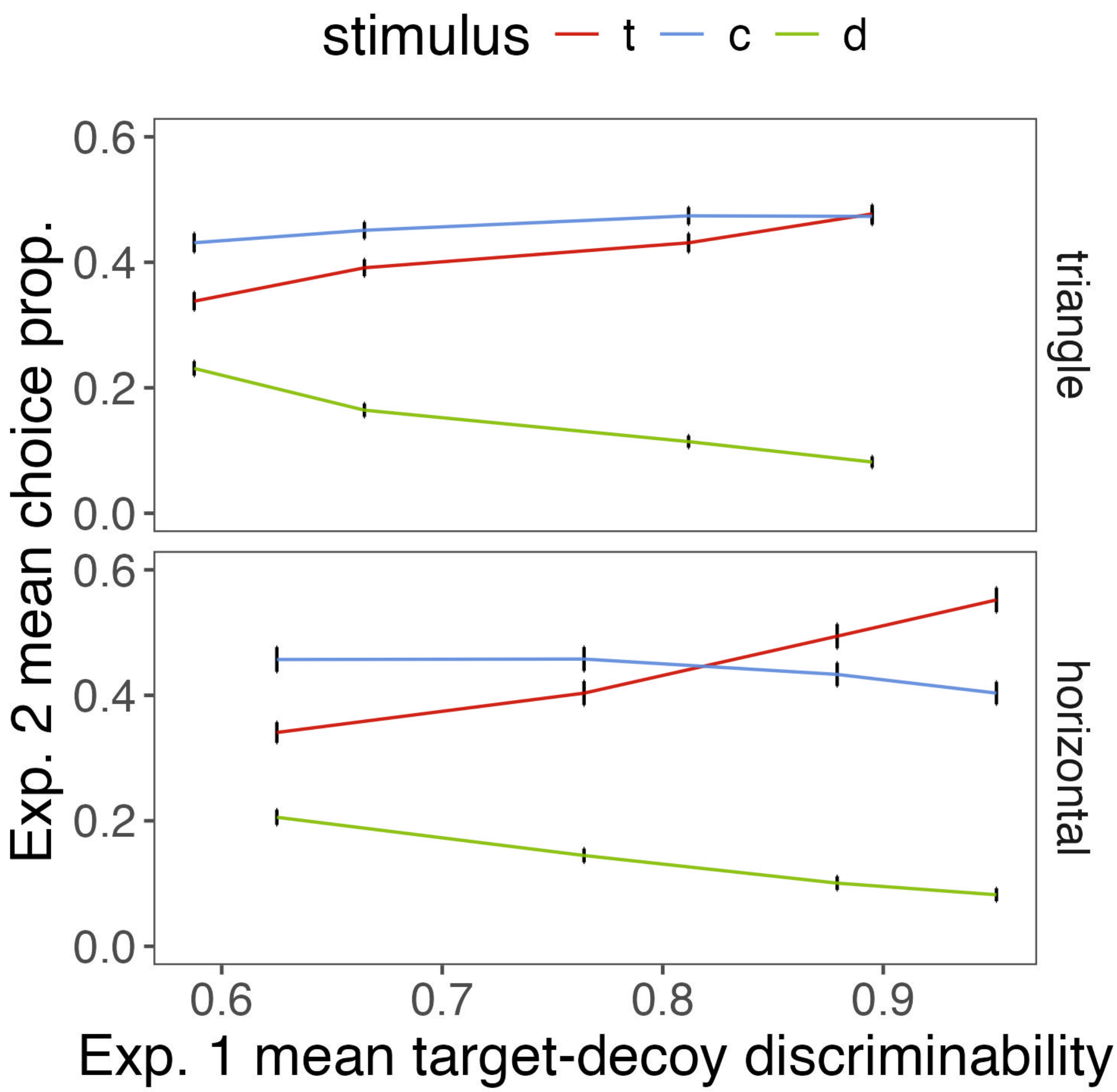

Figure 2.14. Mean target, competitor, and decoy choice proportions (y-axis) at each TDD level in Experiment 2, plotted against mean target-decoy discriminability for each TDD level from Experiment 1 (x-axis). The y-axis error bars are 95% CIs on the mean.

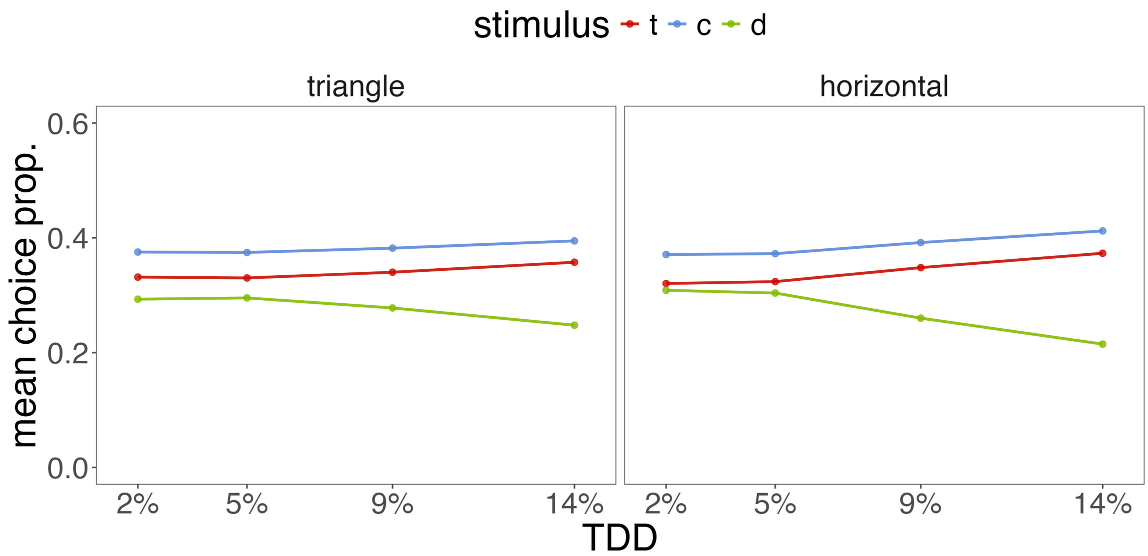

Figure 2.15. Model predictions for the choice data, conditioned on the mean estimated parameters from Experiment 2.

plotted in Figure 2.15 at each TDD level in each display condition. The reader can also see Figure 2.6 to examine where these predictions fall, though Figure 2.6 was generated by making assumptions about the mean and variance parameters.

Conditioned on the estimated parameters, the model is able to produce a repulsion effect. This result aligns with our predictions; the repulsion effect, at least in some forms, can be generated by the Thurstonian choice model if the correlation between target and decoy perceptions is greater than the target-competitor and competitor-decoy correlations. In other words, the dependence between target and decoy perceptions is sufficient to produce the repulsion effect.

The model fails in predicting the weakened repulsion effect found at high TDD levels in the triangle condition, and it also fails completely in predicting the attraction effect. This is unsurprising, as to produce the attraction effect the model requires either $\mu_T > \mu_C$ or $\rho_{CD} > \rho_{TD}$, patterns not found in the current parameter estimates.

The model can produce the repulsion effect solely based on perceptual factors. That is, the model predicts the repulsion effect because the decoy is more likely to exceed the target in perceived area, than it is to exceed the competitor in perceived area, decreasing the RST.

Conditioned on the observed parameter estimates, the model cannot predict the attraction effect. Assuming the validity of the Thurstonian model, additional processes are needed to explain the attraction effect.

2.5 Discussion

The goal of Experiments 1 and 2 was to test the mechanisms generating the repulsion effect in perceptual choice.

In Experiment 1, participants performed a two-alternative forced choice task, where on each trial they first saw a ternary set of target, competitor, and decoy rectangles before selecting the largest option from a particular pair (i.e., target/decoy, competitor/decoy, or target/competitor). The results showed that participants were not always able to discriminate between stimuli of differing areas. Indeed, they selected the decoy a non-trivial proportion of all trials. Furthermore, participants performed worse when the rectangles were arranged in the triangle display of Spektor et al. (2018) than in the horizontal display of Trueblood et al. (2013). Finally, participants were better at discriminating the target from the decoy than they were at discriminating the competitor from the decoy, a result predicted by a Thurstonian model where $\rho_{TD} > \rho_{TC}$.

Experiment 2 was designed to estimate the parameters of a Thurstonian choice model and test the model's ability to predict the empirical results of a choice experiment. The experiment took place in two phases: a judgment phase, followed by a choice phase. On each trial in the judgment phase, participants saw a set of target, competitor, and decoy rectangles and estimated the areas of each rectangle by adjusting the size of corresponding circles. In each trial of the choice phase, participants saw three rectangles and were told to choose the rectangle with the largest area. The choice phase was a replication of Trueblood et al. (2013) and Spektor et al. (2018).

The results of Experiments 1 and 2 show that participants are not always able to discriminate the decoy from the target and the competitor, and that target-decoy perceptions are highly (positively) correlated. The observed correlations can, when embedded in a Thurstonian choice model, naturally produce the repulsion effect but not the attraction effect. This result is highly informative for decision-making research, as it shows to what extent the effects reported by Trueblood et al. (2013) and Spektor et al. (2018) can be explained by valuation noise alone, and to what extent researchers must invoke higher-level decision processes to explain them. This finding does not rule out the possibility that the Thurstonian model is incorrect, but the model does provide a parsimonious and empirically validated account of the repulsion effect.

To understand how these correlations produces the repulsion effect, consider the following simulation.

First, two sets of parameters were generated. In each set, the $\boldsymbol{\mu}$ and S values were similar to those estimated in Experiment 2, $\mu_T = \mu_C = 0$, $\mu_D = -0.1$, $\sigma_T = \sigma_C = \sigma_D = \frac{1}{3}$. However, for the first set of parameters, all correlations were set equal, $\rho_{TC} = \rho_{CD} = \rho_{TD} = .75$, a pattern not observed in Experiment 2. For the second set of parameters, the correlations were set to comparable values from Experiment 2, $\rho_{TC} = \rho_{CD} = .65$, and $\rho_{TD} = .75$.

Next, $1,000,000$ samples were drawn from the model. For each sample, rankings were computed; for example, a ranking of TCD indicates that the target was perceived as largest, the competitor as second-largest, and the decoy as smallest. The simulation results are plotted in Figure 2.16.

First, consider the parameter set where all correlations are equal (Figure 2.16, top panel). The results show that $P(CTD) = P(TCD) > P(TDC) = P(CDT) >$

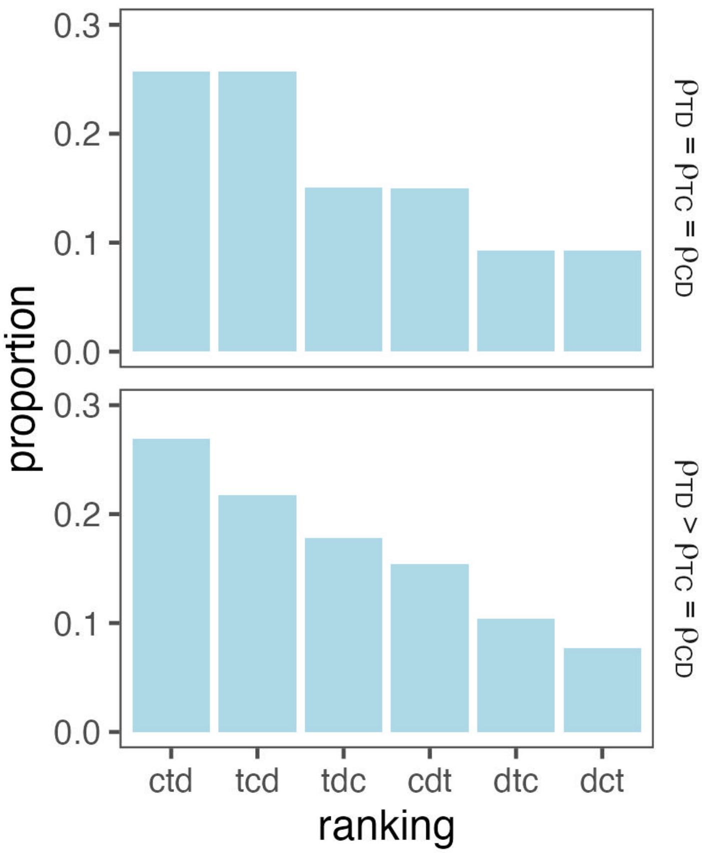

Figure 2.16. Model-simulated ranking proportions, in the order of largest to smallest.

$P(DTC) = P(DCT)$. The decoy is most likely be perceived smallest, least likely to be perceived largest, and the perceived size of the decoy does not influence the relative perception of target and competitor; The target and competitor are equally likely to be perceived as largest (e.g., $P(CTD) = P(TCD)$).

Now consider the parameter set where $\rho_{TD} > \rho_{TC} = \rho_{CD}$ (Figure 2.16, bottom panel). Here, $P(CTD) > P(TCD)$, because the target and decoy tend to move together, making it easier for the competitor to exceed both options. Furthermore, $P(DTC) > P(DCT)$, because the relatively large ρ_{TD} also allows the decoy to be pulled up with the target. These correlations also cause $P(TDC) > P(CDT)$. In simple terms, if the decoy is in the middle, it is more likely that the target is the largest than it is that the competitor is largest. Note that if we sum up these orderings we can obtain the marginal choice proportions for T, C, and D which will show a repulsion effect, i.e., $P(C) > P(T) > P(D)$.

Thus far the research provides a statistical account of these data, but a process account is (one) ultimate goal of cognitive psychology research. It is argued that the target and decoy, which are inherently more similar to one another than either is to the competitor, are more likely to be compared to one another. This ease of comparison leads to correlated valuations which in turn affects choice. This account is plausible, based both on this research and on prior decision-making research.

2.5.1 Generating Correlations via Comparisons

One possible mechanism for generating the correlations is that the similarity between target and decoy cause the target and decoy to be compared more than competitor and decoy or competitor and target. The process by which comparisons create correlations is shown via a simple simulation, presented here.

In the simulation, on each trial, the perceived area of competitor and target are sampled from independent Gaussian distributions, with equal means and variances (parameters μ_{TC} and σ, respectively). As above, $\boldsymbol{X}$ is a vector of perceived target, competitor, and decoy areas.

$$\boldsymbol{X_T} \sim N(\mu_{TC}, \sigma) \tag{2.7}$$

$$\boldsymbol{X_C} \sim N(\mu_{TC}, \sigma) \tag{2.8}$$

From here, the decoy area is sampled according to the following process: First, a binary variable x is sampled from a Bernoulli distribution according to parameter p_{dom}, which defines the probability that the dominance relationship (i.e., that target is greater than decoy) is correctly noticed. If $x = 1$, then the decoy is correctly perceived to be smaller than the target, and the decoy area is sampled from a truncated normal distribution TN with mean $\mu = \mu_D$[11], lower bound $a = 0$, and upper bound $b = \boldsymbol{X_T}$, ensuring the decoy is perceived with noise but always smaller than the target:

$$\boldsymbol{X_D} \sim TN(\mu_D, \sigma, a = 0, b = \boldsymbol{X_T}) \tag{2.9}$$

If $x = 0$, then a binary variable y is sampled from another Bernoulli distribution according to parameter p_{err}, the probability that the dominance relationship is perceived incorrectly (i.e., the decoy is greater than target). If $y = 1$, then the decoy is incorrectly perceived to be smaller than the target, and the decoy area is sampled from a truncated normal distribution TN with mean $\mu = \mu_D$, lower bound $a = \boldsymbol{X_T}$, and upper bound $b = \infty$, ensuring the decoy is perceived with noise but always greater than the target:

$$\boldsymbol{X_D} \sim TN(\mu_D, \sigma, a = \boldsymbol{X_T}, b = \infty) \tag{2.10}$$

[11]Note that $\mu_{TC} > \mu_D$

If $y = 0$, then the decoy is sampled from an independent Gaussian distribution and is not compared to the target:

$$\boldsymbol{X_D} \sim N(mu_D, \sigma) \tag{2.11}$$

Finally, Gaussian noise ϵ with mean $\mu = 0$ and standard deviation $\sigma = 1$ is added to $\boldsymbol{X}$, creating a variable notated as $\boldsymbol{Xg}$. The single source of Gaussian noise accounts for "global" noise, that is the general trial-level dependencies in area estimation that created positive correlations between all pairs of judgments in Experiments 2.

$$\boldsymbol{Xg} = \boldsymbol{X} + \epsilon \tag{2.12}$$

From here, all three pairwise correlations were computed from $\boldsymbol{Xg}$.

The simulation ran for $10,000$ trials, and the following parameters were used:

- $\mu_T = \mu_C = 0.11$
- $\mu_D = 0.08$
- $\sigma = 0.1$
- $p_{dom} = .75$
- $p_{err} = .25$

The results of the simulation are plotted via correlations and scatterplots in Figure 2.17 below.

The simulation correctly generates the critical pattern observed in Experiment 2: that the correlation between target and decoy is stronger than both the correlation between the correlation between target and competitor and the correlation between competitor and decoy. It does so because the decoy is systematically compared to the target more than it is compared to the competitor. The "global" noise, which allows

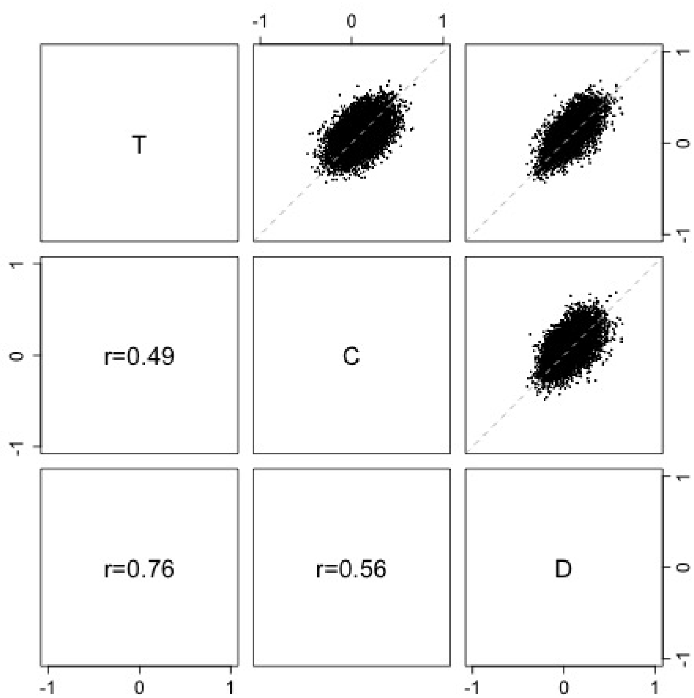

Figure 2.17. Results of a simulation demonstrating how target-decoy comparability can generate positive correlations.

for trial-level correlations, then generates the positive pairwise correlations found in Experiment 2. The simulation does predict that target-competitor are more strongly correlated than target-decoy, a prediction not found in the Experiment 2 results. This simulation provides a simple demonstration of how comparability can produce the observed correlations.

2.5.2 Correlations and Similarity

Other researchers have suggested that correlated valuations may be a measure of similarity between choice options. Kamakura and Srivastava (1984) and Natenzon (2019) both estimated correlations indirectly from choice data, by applying constraints to variants of the multinomial probit model. The current research is, to the author's knowledge, the first to directly estimate these correlations through trial-level valuations.

Multialternative Decision Field Theory (MDFT) (Roe et al., 2001) relies on within-trial correlations between similar options to produce the similarity effect, though this mechanism is distinct from the current model, which relies on *across-trial* correlated valuations. It may be possible that the across-trial correlated valuations are generated from a similar process within trials, as the modeling from Experiment 2 did not provide a process account by which area judgments are made.

Bhui and Xiang (2021) showed that target and decoy valuations may be correlated, and they further argued that the repulsion effect is rational given a consumer's belief that the target and decoy come from the same distribution (e.g., two products from the same brand). Bhui and Dubey (2024) similarly argued that context effects can be rational given limited information about options in a choice set.

The results of Experiments 1 and 2 have important methodological implications. Trueblood et al. (2013) and Spektor et al. (2018) designed their studies to test the hypothesis that context effects occur even in perceptual decision-making. They designed experiments similar to other attraction/repulsion effect experiments, with two equally viable focal options and an inferior decoy option which is similar to a focal target option. Spektor et al. (2018) acknowledged the possibility that the repulsion effect may be a form of the *similarity effect*, where two similar options split choice shares, but they dismissed this hypothesis because participants chose the target more often than they chose the decoy. They did not fully consider the possibility, however, that the target-decoy comparability could cause perceptual correlations which manifest themselves in the choice data. Experiment 2 showed that, even though $P(T) > P(D)$, the decoy is systematically chosen over the target more than it is chosen over the competitor. Thus, the correlation between target and decoy can generate a repulsion effect which is qualitatively different from a reversal of the attraction effect. Spektor et al. (2022) considered a multinomial processing tree (MPT) account of the repulsion effect, where the similarity of target and decoy causes the decoy to take choice shares away from the target, but they dismissed the model because they deemed the best-fitting parameters implausible.

Researchers have also argued in favor of the "tainting hypothesis" (Simonson, 2014; Spektor et al., 2018), where the inferior decoy "taints" similar options (i.e., the target), as a mechanism for the repulsion effect. On average, participants did generally rate the competitor as larger than the target in Experiment 2. These results were, however, quite small and not always statistically apparent. Moreover, the model does not require this tainting to produce the repulsion effect, as demonstrated in the simulations reported above.

Researchers should be careful in assuming that an empirical context effect is generated solely by decision processes whenever the choice environment precludes full stimulus discriminability. Though data from Spektor et al. (2018) showed an empirical repulsion effect (i.e., target chosen less than the competitor), this chapter has presented evidence that the data are generated from a fundamentally different process than the attraction effect.

There are some limitations to the current experiment. Due to the large amount of data required to estimate a variance-covariance matrix in the Bayesian framework (Martin, 2021; Merkle et al., 2023), the correlation parameters were estimated by collapsing across participants. The aggregation of data may obscure individual differences, an area of increasing concern in context effect research (Cataldo & Cohen, 2019; Davis-Stober et al., 2023; Liew et al., 2016; Trueblood et al., 2015). Future studies should collect enough participant-level data to allow participant-level variation in correlation estimates, perhaps by asking participants to return for multiple experimental sessions. It would be theoretically informative to find that individual differences in the correlation parameters could explain individual variation in the attraction and repulsion effects. If the repulsion effect in perceptual choice is truly generated by target-decoy similarity, then at an individual participant level, the strength of the repulsion effect should be positively related to the difference between ρ_{TD} and ρ_{CD}.

Participants also appeared to use both relative and absolute judgment processes when judging the areas of the rectangles. Mean judgments of decoy size decreased with TDD, but mean target and competitor judgments decreased as well. Thus, the measurements of area perception are not ideal, though they were sufficient for the current purposes.

As discussed in the Appendix, there was a bias for wide rectangles during the choice phase but a bias for tall rectangles during the judgment phase (at least for the triangle condition). In the choice phase, participants chose wide rectangles more than tall rectangles. In the judgment phase, however, they judged tall rectangles to be larger than wide, at least in the triangle condition. The latter effect is quite small, but it is nonetheless present in the data. There is currently no clear explanation for this result. Gronau et al. (2023) compared participants' ability to perform a perceptual discrimination task based on whether they responded via an ordinal rating scale or a choice. The authors concluded that participants relied on the same representation in both cases, but the participants were more sensitive to stimulus-level differences when choice responses were collected. It may be that, in Experiment 2, the relative lack of perceptual sensitivity caused participants to slightly underestimate their reported responses to stimuli they actually *perceived* as larger. This inconsistency may also be idiosyncratic statistical variation which may not appear in future samples.

This chapter shows that the repulsion effect in perceptual choice can be explained by target-decoy similarity alone, but the attraction effect may require invoking higher-level decision processes. To explain the repulsion effect, a Thurstonian choice model was developed and tested. In particular, the means, variances, and correlations of multivariate Gaussian distribution of area perception were jointly estimated from participants' judgments of perceptual stimuli. This model was then tested on its ability to explain choice data collected from the same participants. This research furthers the study of context effects, provides a new experimental paradigm for testing them, and helps explain inconsistencies in prior results.

CHAPTER 3

VIOLATIONS OF INDEPENDENCE IN BEST-WORST CHOICE

3.1 Introduction

Chapter 2 introduced a Thurstonian choice model and showed that, conditioned on estimated parameters, the model can systematically predict the repulsion effect, but not the attraction effect. This chapter tests another prediction of the model while demonstrating a novel empirical result in another domain: best-worst choice.

3.1.1 Introducing Best-Worst Choice

Best-worst choice is an experimental paradigm where participants select their most and least preferred options from a choice set. Originally proposed by Finn and Louviere (1992), best-worst choice is widely used in a number of applied fields, such as transportation (Beck & Rose, 2016) and healthcare economics (Cheung et al., 2016; Flynn et al., 2007a; Mühlbacher et al., 2016). One key advantage, when compared to standard choice research, is that researchers can use best-worst choices to gain information about participants' ranking of options while never requiring them to complete a full ranking task (Flynn & Marley, 2014).

Researchers have developed theoretical models to account for best-worst choice data. These models relate best-worst choices to underlying utility functions. Marley and Louviere (2005) developed a class of models known as the *maxdiff* (maximum difference) model of best-worst choice[1]. According to the maxdiff model, the joint probability of selecting both option j as best and also option k as worst (denoted $BW(j,k|K)$) from set K, where $j \neq k$, is computed as:

$$BW(j,k|K) = \frac{e^{u_j - u_k}}{\sum_{\substack{p,q \in K \\ p \neq q}} e^{u_p - u_q}} \tag{3.1}$$

where u_i is the utility of option i. This model proposes that a single utility scale determines the selection of both the best option and the worst option in a choice set. It assumes that best choice probabilities are an increasing function of u, while worst choice probabilities are a decreasing function of u. Furthermore, the maxdiff model relies on the differences of utilities; From a given choice set, the most likely best-worst pair is the pair with the largest difference in utility values. Utilities are also assumed to be independent of each other. Note that the use of the exponential function means that the maxdiff model is another form of the widely used multinomial logit (MNL) choice model (Hausman & McFadden, 1984).

There are several variants of the maxdiff model along with other best-worst choice models (Flynn & Marley, 2014; Flynn et al., 2007b; Marley & Louviere, 2005; Marley & Pihlens, 2012; Marley et al., 2008), though the maxdiff model from Equation 3.1 remains the dominant model for analyzing best-worst choice data (Hawkins et al., 2014a).

The marginal probability $B(j|K)$ of selecting option j as best from set K can be written as:

$$B(j|K) = \frac{e^{u_j}}{\sum_{p \in K} e^{u_p}} \tag{3.2}$$

---

[1]Note that the term maxdiff is sometimes erroneously used to refer to best-worst experiments in the generic sense. Following Marley and Louviere (2005), maxdiff refers to a specific class and parameterization of a best-worst choice model.

while the marginal probability $W(k|K)$ of selecting option k as worst from set K can be written as:

$$W(j|K) = \frac{e^{-u_j}}{\sum_{p \in K} e^{-u_p}} \tag{3.3}$$

Best-choice probabilities are an increasing function of u, while worst choice probabilities are a decreasing function of u.

The maxdiff model predicts a decreasing monotonic relationship between best choice probabilities and worst choice probabilities (Hawkins et al., 2014a). Options most likely to be chosen as best are necessarily least likely to be chosen as worst, and vice versa. That is, if we assume all options' utilities are independent, the model predicts that best choice probabilities are negatively, monotonically related to worst choice probabilities.

To demonstrate this, consider the following simulation. 500 ternary (three-option) best-worst choice trials were simulated, by sampling 3 utilities from the standard normal distribution. For each trial, the marginal best and worst choice probabilities were computed using the maxdiff model. These probabilities were then plotted against each other in a state-trace plot (see Figure 3.1) (Bamber, 1979). In a state-trace plot, the analyst plots two dependent variables against one another in each experimental condition.

Researchers have explored whether this monotonicity holds empirically. Hawkins et al. (2014b) tested both preferential and perceptual best-worst choice data using response time modeling. They used the Linear Ballistic Accumulator model (LBA) (Brown & Heathcote, 2008), which casts the decision process as a race between evidence accumulators towards a threshold, where the average accumulation across trials is captured by the drift rate parameter. Modeling both preferential and perceptual best-worst choice data, they were able to successfully account for choice data by assuming a parallel race between best and worst accumulators for each option. Fur-

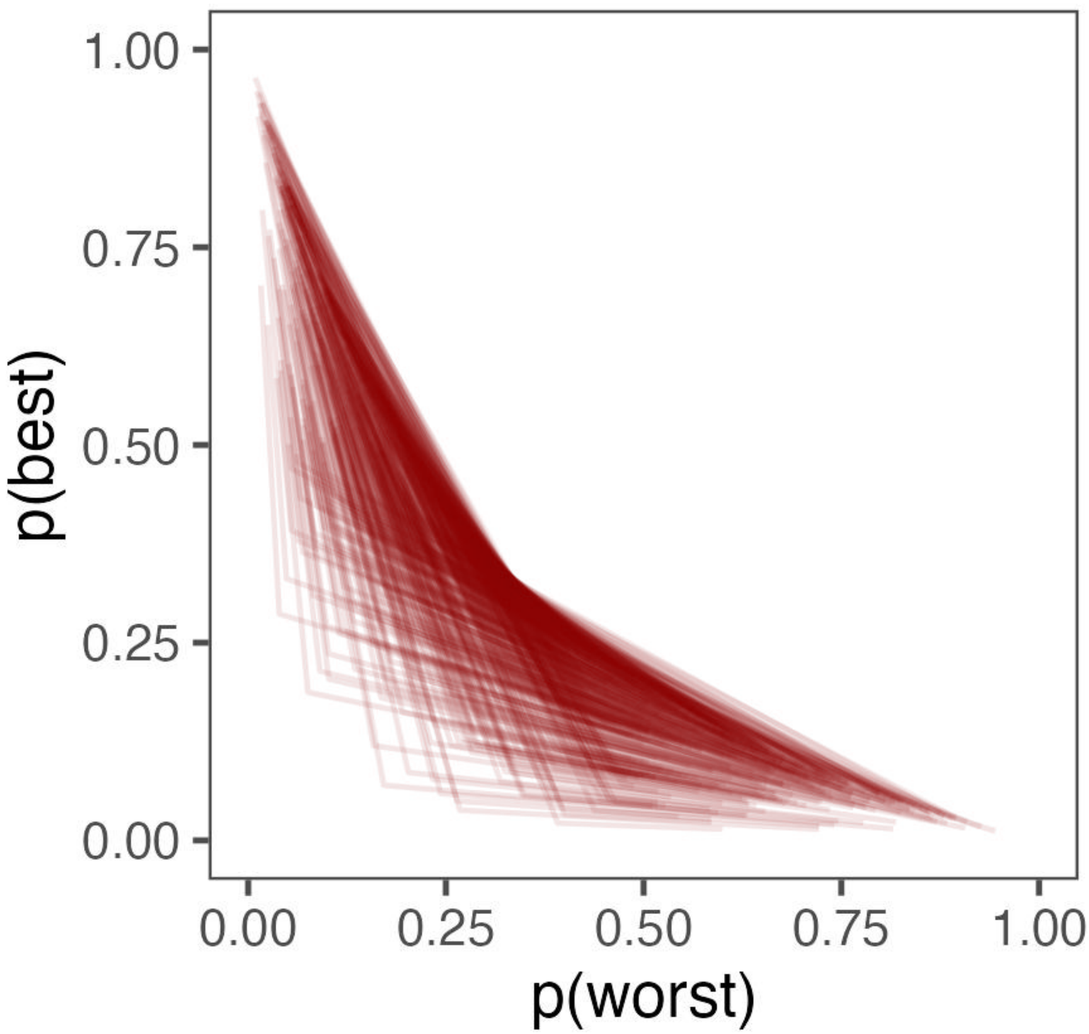

Figure 3.1. Best-worst choice simulations using the maxdiff model. Each curve is a separate simulation. The model predicts a negative, monotonic relationship between the marginal best choice and worst choice probabilities.

thermore, they showed that the utility values estimated for each option using an MNL model were positively linearly related to the log drift rate values from the LBA, suggesting an underlying utility representation that explains both best and worst choices.

In a follow-up article, Hawkins et al. (2014a) found that, collapsing across choice sets, best choice probabilities are monotonically related to worst choice probabilities. Options that were most likely to be selected as best were least likely to be selected as worst, and vice versa. This finding held for perceptual choice and consumer choice. They also showed that, using the parallel best-worst LBA, the drift rate parameter for worst choice can be parameterized as the reciprocal of the best choice drift rate. Formally, if $d_b(i)$ is the drift rate for selecting option i as best, then $d_w(i) = 1/d_b(i)$, where $d_w(i)$ is option i's drift rate for worst choices.

Implementations of both the parallel best-worst LBA and the maxdiff model assume that the utilities of all options presented are independent. However, as shown below, the Thurstonian choice model from Chapters 1 and 2 predicts, under certain conditions, a dissociation between best and worst choices inconsistent with this assumption.

3.1.2 Model-Based Dissociations in Best-Worst Choice

Below, the Thurstonian choice model from Chapter 2 is used to make a prediction regarding a dissociation between best choices and worst choices.

Let K be a choice set consisting of options T, C, and D (i.e., target, competitor, and decoy). As in Experiments 1 and 2, the options are rectangles in a perceptual choice experiment. As in Chapter 2, it is assumed that on each trial i with choice set K, the vector $\mathbf{X_i}$ of perceived areas is sampled from a multivariate Gaussian distribution with a mean vector $\boldsymbol{\mu}$ and variance-covariance matrix $\boldsymbol{\Sigma}$:

$$\boldsymbol{X}_{ij} \sim \boldsymbol{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \tag{3.4}$$

Given a vector $\mathbf{X_i}$ of perceived areas on trial i with set K, according to model, the probability a participant selects stimulus j as best and option k as worst is:

$$BW_K(j,k) = P(\mathbf{X}_{ij} - \mathbf{X}_{ik} > \mathbf{X}_{ip} - \mathbf{X}_{iq}), \forall p, q \in K, p \neq q \tag{3.5}$$

Simply put, on any given trial, the option with the largest perceived area is always selected as best, while the option with the smallest perceived area is always selected as worst.

Applied to best-worst choice the Thurstonian choice model is somewhat similar to the maxdiff model. There are two main differences between these models, however. First, the Thurstonian model is a version of the Multinomial Probit (MNP) model, while the maxdiff model is a variant of the MNL model. Crucially, however, the Thurstonian model provides a distribution for option utilities and allows for correlations between these utilities, while the maxdiff model generally assumes that these utilities are independent of one another. Note that the latter is a convenience assumption and can of course be relaxed.

However, given the correlations estimated from Experiment 2 , the Thurstonian predicts that, in the best-worst choice paradigm, there is a non-monotonic relationship between marginal best choices and worst choices. This dissociation is explained below.

The Thurstonian choice model, conditioned on the parameters $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ estimated from Experiment 2 was simulated for $1,000,000$ best-worst choice trials[2]. As in Chapter 2, results are collapsed over choice set (i.e., whether the target is wide or tall). Marginal choice proportions for the target, competitor, and decoy options were computed at each level of TDD. These results are presented in Figure 3.2, in a series of state-trace plots.

[2]Only the parameters estimated from the triangle condition (See Experiment 2) were used here, as the parameters estimated from the horizontal condition made identical predictions.

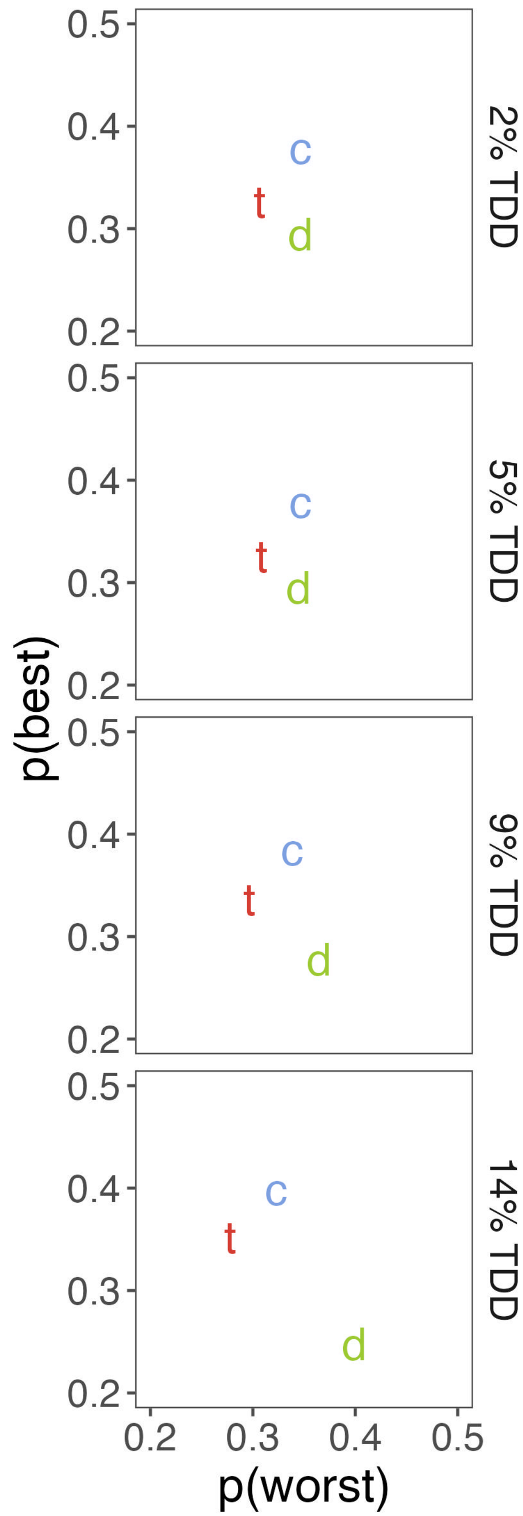

Figure 3.2. Simulated best-worst predictions per the Thurstonian choice model. Each row is a different level of TDD.

The model, conditioned on the estimated parameters, predicts a theoretically informative data pattern. Although the competitor is most frequently chosen as best (as in the empirical results from Experiment 2), it is not least frequently chosen as worst. Specifically, the Thurstonian model predicts that $B(C) > B(T)$ and $W(C) > W(C)$. At lower levels of TDD, the model even predicts that competitor and decoy are chosen as best at similar rates, a prediction likely due to the fact that participants are generally less sensitive to perceptual differences when providing ratings than when making choices (Gronau et al., 2023).

The Thurstonian choice model predicts this pattern because $\rho_{TD} > \rho_{CD} \approx \rho_{TC}$. On the relatively few trials where X_{iD} is largest, it is more likely that $X_{iD} > X_{iT} > X_{iC}$ than $X_{iD} > X_{iC} > X_{iT}$. In other words, the high ρ_{TD} value pulls up the target more than the competitor. Though the similarity, and comparability, of target and decoy thus entail that the competitor is chosen more as best compared to the target (as in Experiment 2), this does not necessarily entail that the competitor is chosen less than the target as worst.

This effect is subtle, and the predicted effect size is small. Indeed, all differences in predicted $W(C) - W(T)$ values were $< .05$. Experiment 3 contains the empirical and modeling results from a best-worst choice experiment designed to test this prediction. The dissociation between best and worst choices does indeed occur empirically. Here the monotonicity assumption required by the independent utilities assumption, typical in applications of the maxdiff model, also fails empirically.

3.2 Experiment 3

The goal of Experiment 3 was to test the predictions of the Thurstonian choice model. Specifically, the perceptual model predicts that both $B(C) > B(T)$ and $W(C) > W(T)$. To test this prediction, the stimuli were identical to those of Experiment 2, including the triangle display. The aforementioned prediction holds empirically and the maxdiff model (with independent utilities) cannot account for these results.

3.2.1 Methods

3.2.1.1 Participants.

Data collection took place at the University of Massachusetts Amherst. 392 undergraduate students participated in exchange for course credit. 23 participants who achieved less than 80% accuracy on catch trials (see below) were excluded from all analyses.

From the remaining 369 participants, $2,604$ trials with first-choice response times (RTs) < 100ms or > 10000ms were also excluded from all analyses, leaving $107,727$ trials across all participants.

Participants were randomly assigned into one of two conditions: best-worst or worst-best. On each trial, participants in the best-worst condition first chose the largest rectangle and then chose the smallest rectangle. Participants in the worst-best condition chose in the opposite order. The order condition was included to account for the possibility that best-worst choice order impacts choice, though results were qualitatively identical across conditions, so all results are collapsed across order.

After removing participants, there were 185 participants in the best-worst condition and 184 participants in the worst-best condition.

3.2.1.2 Stimuli.

The experiment had three types of trials: critical trials, filler trials, and catch trials.

Stimuli on critical trials were identical to those of Experiment 2. On each critical trial, the target and competitor had the same area but differed on orientation, with one stimulus being wide and the other tall. The decoy always had the same orientation as the target. TDD varied at 2%, 5%, 9%, and 14%. Target, competitor, and decoy rectangles along three diagonals, as in Experiment 2 (see Figure 2.7).

On each filler trial, three stimuli were uniformly sampled from the space between the largest and smallest diagonals (see Figure 2.7).

On each catch trial, one stimulus was sampled from the largest diagonal, while two stimuli were sampled from the smallest diagonal.

3.2.1.3 Design.

There were 8 blocks of trials. In each block there were 24 critical trials, 6 at each TDD level. There were 8 trials per diagonal. In addition to the critical trials, there were 10 filler trials and 3 catch trials per block.

Participants were randomly assigned into one of two conditions: best-worst or worst-best, as described above.

Stimuli were presented on computer monitors with a resolution of 1920 x 1080 pixels. The experiment was programmed with GNU Octave and Psychtoolbox (Brainard, 1997; Eaton et al., 2021).

3.2.1.4 Procedure.

The experiment began with three practice trials, which were identical to the filler trials.

On each trial, participants saw three rectangles, labeled 1, 2, and 3 (from left to right), arranged in a triangle display. Participants in the best-worst (worst-best) condition saw a prompt asking them to select the largest (smallest) rectangle on screen. Participants used the mouse to select their choice. After they made their choice, their chosen rectangle changed color to indicate that it was no longer available as an option. Next, participants in the best-worst (worst-best) condition selected the smallest (largest) rectangle, at which point the trial ended. See Figure 3.3 for an example trial.

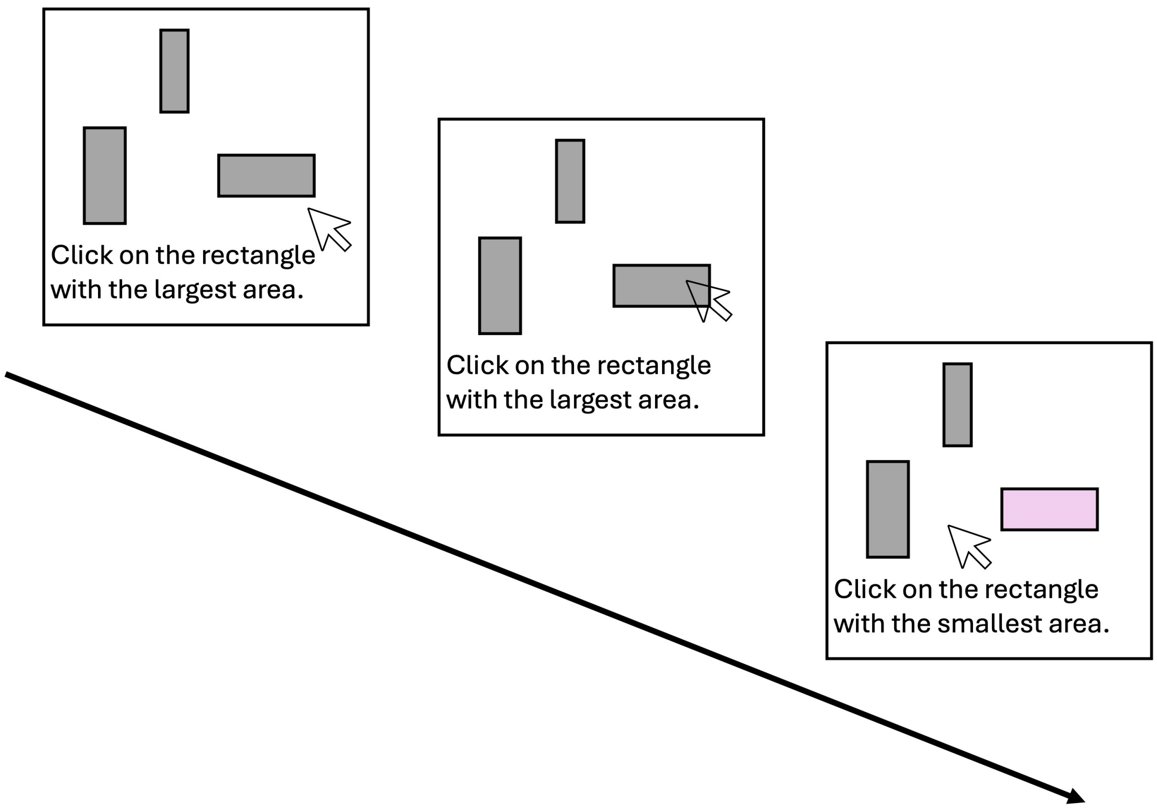

Figure 3.3. A sample experimental trial from Experiment 3. This is a trial in the best-worst condition.

Stimulus order was randomized on each trial.

Participants were told their percentage correct for best choices, worst choices, and overall choices at the end of the experiment.

3.2.2 Results

3.2.2.1 Catch Trials.

Participants performed well on the catch trials. The mean percentage correct for best choices was 97.97%(SD = 14.09), and the mean percentage correct for worst choices was 98.26%(SD = 13.09). The mean percentage correct for both best and worst choices (i.e., the mean percentage of the trials on which participants were able to correctly identify the largest and smallest rectangles) was 96.98%(SD = 17.12).

3.2.2.2 Filler Trials.

Participants performed worse on the filler trials compared to the catch trials, but still well above chance. The mean percentage correct for best choices was 89.83%(SD = 30.23), and the mean percentage correct for worst choices was 88.95%(SD = 41.35). The mean percentage correct for both best and worst choices was 80.95%(SD = 39.37).

3.2.2.3 Critical Trials.

First, data were analyzed by computing choice proportions for each option conditioned on TDD and choice set. Mean choice proportions for these data are plotted in Figure 3.4 (left and middle columns).

Participants showed a consistent bias to choose W (the wider rectangle) as largest, also found in Experiments 1, 2 and 5. Participants also (on average) regularly choose the decoy rectangle as smallest, except for the choice set H, W, D_W and $TDD = 2\%$, where they selected the H rectangle as smallest, on average. This can be attributed to the difficulty of the $TDD = 2\%$ condition and the overall wide rectangle bias. How-

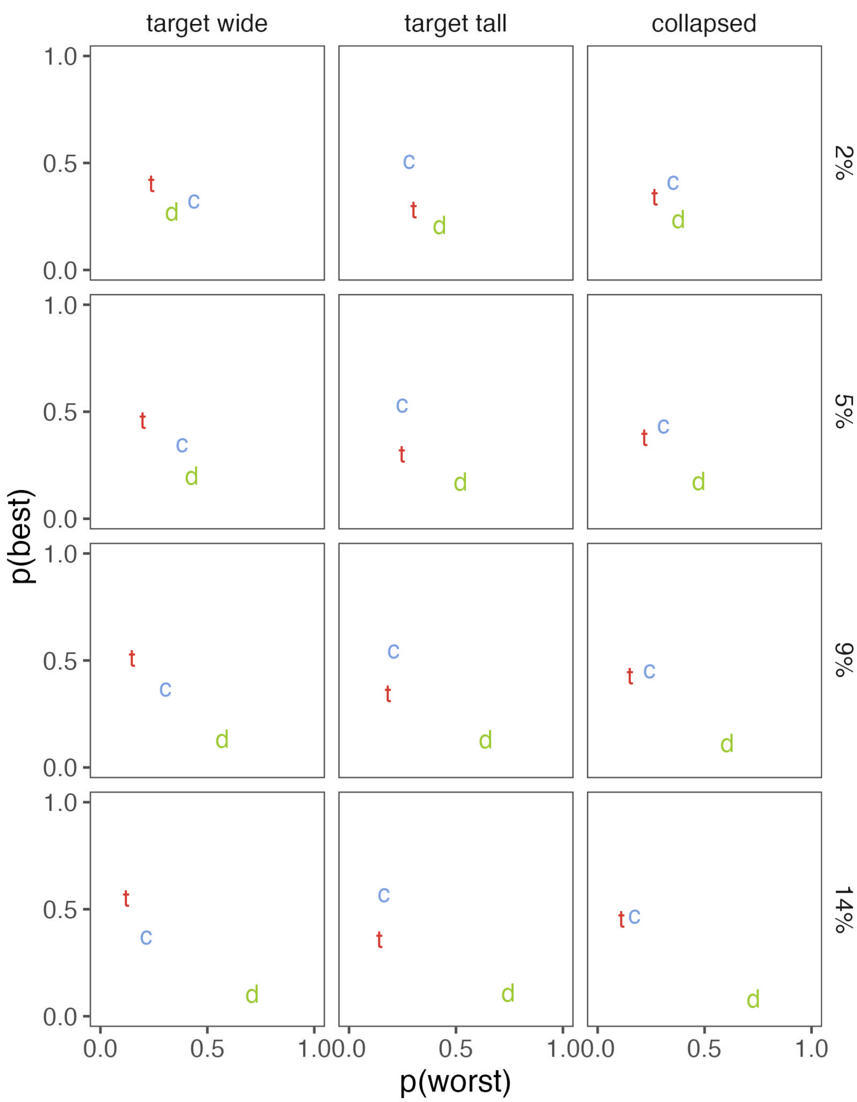

Figure 3.4. Experiment 3 mean best and worst-choice proportions, conditioned on TDD (rows) and choice set (columns). The left column shows data when the target was wide, the middle when the target was tall, while the right column collapses over orientation.

ever, consistent with the predictions of the model, the target was still less likely to be chosen as worst than the competitor, $W(H|[H, W, D_H]) < W(H|[H, W, D_W])$ and $W(W|[H, W, D_W]) < W(W|[H, W, D_H])$, while the competitor option was also more likely to be chosen as best, $B(H|[H, W, D_W]) > B(H|[H, W, D_H])$ and $B(W|[H, W, D_H]) > B(W|H, W, D_W)$.

These results are more easily understood by plotting mean target, competitor, and decoy choice proportions across TDD levels, collapsed over choice set. See Figure 3.4 (right column) for these data.

The best choice proportions replicated the results of Spektor et al. (2018) and the current Experiment 2, where the competitor was more likely to be chosen as best at low TDD levels, while competitor best-choice proportions also decreased with TDD. Decoy best-choice proportions also decreased systematically with TDD. Furthermore, the target was always less likely to be chosen as worst, compared to the competitor, at all TDD levels, $W(C) > W(T)$, as predicted by the Thurstonian choice model outlined in Chapter 2.

3.2.2.4 Maxdiff Modeling

Now consider the maxdiff model (Marley & Louviere, 2005), which was outlined in the introduction to this chapter. This model predicts that the probability of choosing options x as best and y as worst, $x \neq y$, increases monotonically with the difference in their estimated utilities (see Equation D.1). This model is the most commonly used analysis technique for best-worst choice data (Flynn & Marley, 2014). The maxdiff model was applied to the current experiment, where it is unable to predict the observed dissociations in best-worst choices, even with the best fitting parameter set.

Bayesian hierarchical model was used to fit the model. See the Appendix for the fitting procedure, including parameterization, priors, and parameter estimates. The model predictions for the mean best and worst choices are shown in Figure 3.5.

The model clearly mispredicts the data. The model predicts that the target and competitor are chosen at roughly equal amounts for both best and worst choice. Furthermore, note that the model predictions fall on a straight line rather than the curve shown by the data. This misprediction stems from the fact that the model choices are a function of the utility of each option, computed independently through a linear combination of experimental factors multiplied by model coefficients, including target/competitor/decoy status. The model could, if the data suggest it, predict that the target has greater utility than the competitor or vice versa. However, because best choice proportions are positively related to utility and worst choice proportions are negatively related to utility, the model cannot simultaneously predict $B(C) > B(T)$ and $W(T) < W(C)$. Instead, the mostly likely parameter set is that which predicts $B(C) \approx B(T)$ and $W(T) \approx W(C)$

Participant-level data and model predictions are shown in Figure 3.6. The model generally does a poor job at accounting for participant worst choice proportions.

3.3 Discussion

Experiment 3 demonstrated that correlated valuations can induce dissociations in best-worst choice. Specifically, given a target, competitor, and decoy option (borrowing the terminology of the attraction effect), the competitor is more likely than the target to be selected as best ($B(C) > B(T)$), but the competitor is also more likely than the target to be selected as worst ($W(C) > W(T)$). This prediction was made using the Thurstonian choice model of Chapter 2, conditioned on the parameters estimated from Experiment 3. Furthermore, the prediction was made with a different set of participants and a somewhat different experimental task. The maxdiff model, the most common model for best-worst choice analysis, cannot simultaneously predict $B(C) > B(T)$ and $W(C) > W(T)$.

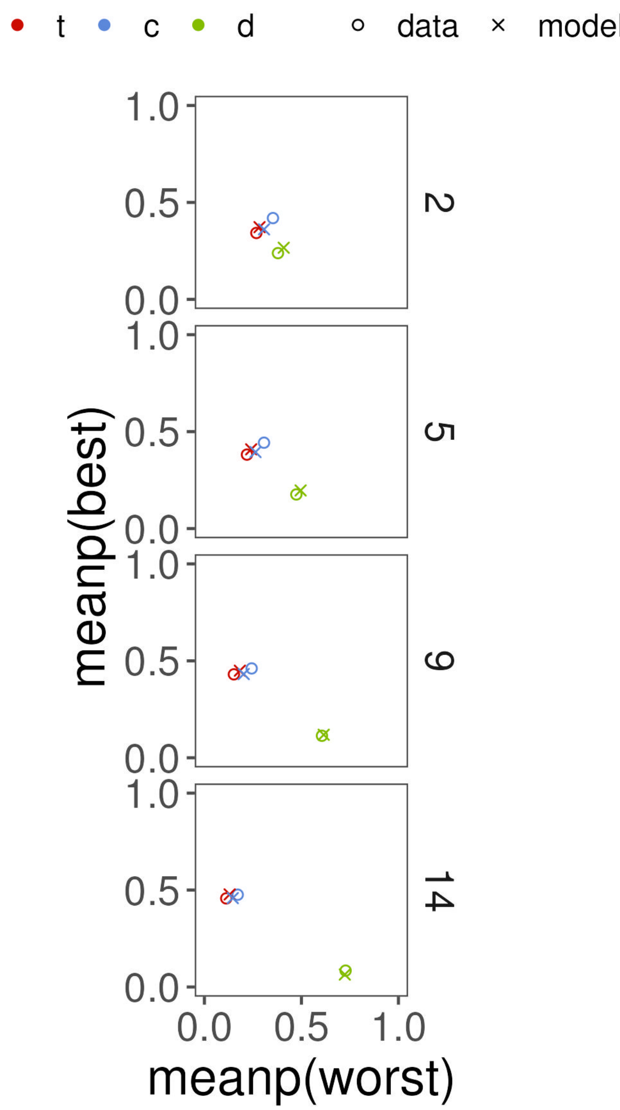

Figure 3.5. Experiment 3 maxdiff model predictions for the mean target, competitor, and decoy best-worst choice proportions. The model predictions fall on a single diagonal line, consistent with the monotonic best-worst choice predictions outlined earlier. Note that 95% HDIs on the mean were too small to represent on the plot, but the data means are not included in any of these HDIs.

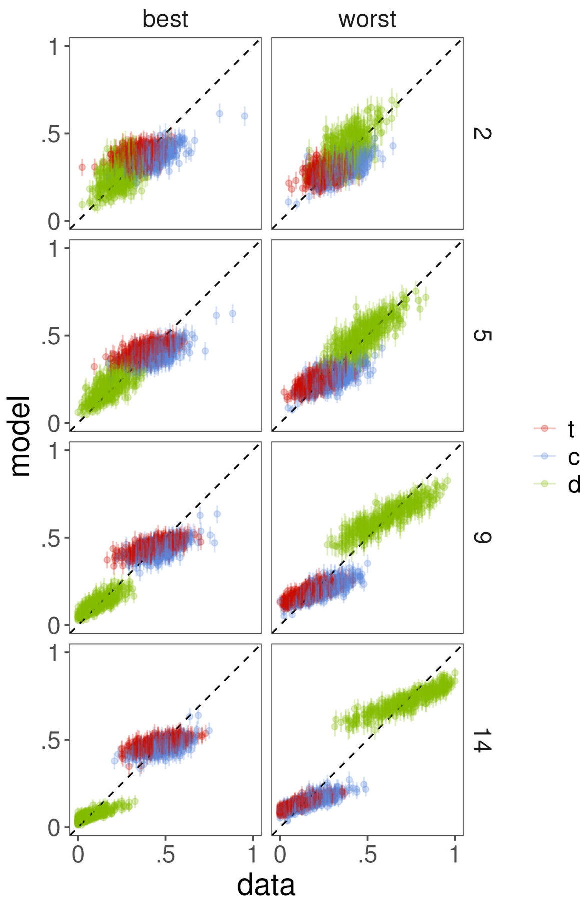

Figure 3.6. Experiment 3 maxdiff model predictions for the mean target, competitor, and decoy best-worst participant-level choice proportions, conditioned on TDD (rows) and choice type, i.e. best v. worst (columns). Vertical error bars are 95% HDIs.

The maxdiff model assumes that, when selecting the best and worst option, the decision-maker picks the option with the highest and lowest utility, respectively. Most applications of the model assumes that these utilities are independently distributed (Flynn & Marley, 2014), and that the decision-maker uses the same utility for both best and worst choices. The use of a single utility scale is to some extent a convenience assumption. Marley and Louviere (2005) explored several theoretical models, including the case where best and worst utilities exist on independent ratio scales, though such a model does not seem to have been adopted by substantive researchers. Marley et al. (2008) introduced set-dependent best-worst choice models, which allow for context dependence based on choice set, albeit with best and worst choices still a function on a common underlying utility scale.

Hawkins et al. (2019) also argued that best and worst choices rely on a common utility representation, i.e., that the best option is chosen because it is highest in utility and the worst option is chosen because it is lowest in utility. They fit the maxdiff model to 5 best-worst datasets and showed, via Bayesian mixture modeling, that the overwhelming majority of participants were best fit by a model with a single utility representation.

Geržinič et al. (2021) argued (and provided evidence for) the claim that while best and worst choices rely on a common utility scale, people use two distinct decision rules for best choices and worst choices. They argued that the former is compensatory (i.e., allowing trade-offs between attributes), while the latter is non-compensatory (i.e., disallowing trade-offs to minimize future regret).

The Thurstonian choice model, used for the current predictions, also employs a common utility scale for both best and worst choices. However, the model does not assume independently distributed utilities, as assumed by most applications of the maxdiff model. Thus, the claims of Hawkins et al. (2014a) and Hawkins et al. (2019)

are not necessarily falsified; rather, they are amended to account for correlations between option utilities.

Due to the small effect size, this experiment required a large amount of data to estimate these dissociations. Most best-worst choice research is applied (for example in transportation and healthcare economics), where researchers do not typically have access a large amount of data at the individual participant level. Thus, researchers are unlikely to observe the dissociations in best-worst choice and will analyze the data using the maxdiff model. They may then arrive at incorrect conclusions regarding participants' preferences.

The Thurstonian choice model was not fit to the Experiment 3 data. In Experiment 2, participants judged the size of the rectangles, and these judgments were used to estimate the model parameters. It also seems unreasonable to expect researchers to fit a multivariate Thurstonian model to most best-worst choice studies, given limitations in data and potential issues with parameter identifiability.

The central purpose for conducting best-worst choice studies is to identify participants' preference distributions on a set of options. Best-worst choice is less cognitively demanding on participants than asking them to rank all options and far more efficient than pairwise forced choices on all combinations of options (Louviere et al., 2008). In many cases, analyzing best-worst data with the maxdiff model may be the best approach, especially if researchers have no reason to believe that options are strongly correlated, or if all pairs of options have equally strong correlations. It is an open question, left for future research, whether correlations between options in applied choice research can create similar dissociations in best-worst choice.

The maxdiff model's utility comes from its simplicity; it is essentially an extension of the MNL model to best-worst choice data. When choice sets are not designed to systematically various choice axioms (e.g., as in context effects), the MNL model is fairly effective in accounting for choice data; similarly, so is the maxdiff model effective in accounting for choice data where independence is not violated.

The current study only considered Case 3 best-worst choice (Marley & Pihlens, 2012), where the attributes of options (in this case, height/width, TDD, diagonal) are systematically manipulated. This study did not consider Case 1 best-worst choice, where researchers are interested in preference for each option as a whole (e.g., a consumer's preference for cars over bicycles) or Case 2 best-worst choice, where researchers ask participants to select their preferred attribute from a set (e.g., a consumer's preference for short waiting times over physician experience in an urgent care clinic). Future research should consider ways to generalize the current paradigm and results to the other best-worst choice types.

CHAPTER 4

CORRELATED VALUATIONS IN PREFERENTIAL CHOICE

4.1 Introduction

Thus far, the dissertation has focused on perceptual choice. This has allowed for the reconciliation of conflicting findings from other researchers (Spektor et al., 2018; Trueblood et al., 2013), using a Thurstonian choice model which was developed from the ground up in a simplified choice environment.

However, many decision theorists, in particular those who study context effects, are interested in a wide variety of choice environments. For example, the original demonstration of the attraction effect came from the marketing literature (Huber et al., 1982), where participants selected amongst hypothetical consumer products. This chapter generalizes the paradigm and model from Chapter 2 to preferential choice. Participants were presented with sets of consumer stimuli, designed by previous researchers to elicit the repulsion effect. In one phase of the experiment, participants provided the selling prices for consumer goods, a measure analogous to the area judgments from Experiment 2. In a second phase of the experiment, they chose their preferred alternative from the same choice sets. The parameters of the Thurstonian choice model (e.g., means, variances, correlations) were estimated from the pricing data. Crucially, the target-decoy correlation ρ_{TD} was quite strong, a conceptual replication of the results of Experiment 2. The Thurstonian model, conditional on the estimated parameters, was then used to make predictions for the choice data. The model was generally successful in making predictions, albeit with some limitations which are discussed.

4.1.1 Expanding the Paradigm to Preferential Choice

In Experiment 2, participants provided psychophysical judgments, which were used to estimate the parameters of a Thurstonian choice model. This model, conditional on estimated parameters, was then used to make predictions for choices collected from the same set of participants.

The next step, in developing and testing both the experimental paradigm and the Thurstonian choice model, was to generalize the paradigm to a preferential choice environment. Rather than use the same set of parameters as found in Experiment 2 (a perceptual choice task), it was crucial to estimate new parameters in a different choice environment. The goal of this study was to further the understanding of context effects, particularly the attraction and repulsion effect, by estimating means and correlations in preferential choice. Context effects have their empirical and theoretical origins in preferential (consumer) choice, so preferential choice provides a natural next step in this research program.

To test this approach in preferential choice, we collected preference ratings from participants in the form of selling prices, an analogous measure to the continuous area judgments collected in Experiment 2.

In most studies of consumer preference, researchers collect choice data rather than ratings. There are good reasons for doing so. The literature on willingness to pay (WTP; the largest amount a consumer is willing to pay for a given product) has shown that, when responding to hypothetical survey questions, participants tend to over-estimate their WTP by a sizeable amount (Breidert et al., 2006; Schmidt & Bijmolt, 2020), c.f. Miller et al. (2011). Furthermore, WTP estimates may not simply be off by a constant; Jedidi and Zhang (2002) asked participants to state the price they would be willing to pay for a series of Compaq and Dell laptops, and, in a separate phase of the study, to indicate whether they would be willing to pay various actual listing prices for each laptop. Comparing the former stated WTP estimates to those estimated from those estimated from the latter choice data, the results showed that the stated WTP values were sizeably higher, on average, than those estimated from choice data. The stated WTP and estimated WTP values were positively correlated, albeit fairly weakly (maximum $r = .43$). Furthermore, the authors fit two demand curves (one for stated WTP and the other for WTP estimated from choice data) to estimate the percentage of respondents willing to pay various prices for a laptop. The curve based on the stated WTP estimates tended to simultaneously overestimate demand at low prices and underestimate demand at high prices.

Bridges et al. (2012) conducted a study comparing WTP methods by asking hearing-loss patients to provide both Likert-scale ratings of the importance of each hearing aid attribute as well as to complete a series of pairwise choices between hearing aids with varying attributes (a research technique known as conjoint analysis). The researchers then estimated WTP values for each attribute (e.g., how much would the average consumer be willing to pay for good battery life) using logistic regression. The mean estimated WTP values were ordinally related across methods, though the authors found that the Likert scale method tended to lead to floor and ceiling effects in WTP.

In one review of the WTP literature, Breidert et al. (2006) strongly recommended that researchers do not attempt to estimate WTP by directly asking consumers to state their price estimates. Other authors have made similar recommendations Jedidi and Jagpal (2009). Miller et al. (2011) compared the empirical value of both stated WTP and choice-based conjoint (CBC) analysis to two other approaches: the Becker, DeGroot, and Marschak auction (BDM, Becker et al. (1964)), where participants bid a price for a product and are required to purchase the product if a price drawn from a lottery is not larger than their bid; and the incentive-aligned choice-based conjoint (ICBC) analysis, a more complex method where the participant completes a hypothetical choice, the researcher infers their WTP based on these data, and this WTP value is put through a BDM mechanism.

Using a cleaning product as the stimulus, Miller et al. (2011) found that all methods overestimated consumers' true WTP (using actual purchase data as the benchmark). BDM showed the smallest bias, followed by ICBC, stated WTP, and CBC. All methods perform reasonably well, however, in predicting actual purchase data, even given these biases. The authors argued that when possible, researchers should use methods based on incentive alignment (i.e., BDM and ICBC) to estimate WTP. They do note that for many applications, however, stated WTP may work reasonably well, particularly when resources are limited and incentive-based methods would be difficult to implement.

In a meta-analysis, Schmidt and Bijmolt (2020) found that stated WTP methods actually showed less overestimation bias than indirect methods (e.g., BDM, ICBC, CBC). However, they also found that bias increased with actual prices of products, and that the bias was stronger in within compared to between-participant studies.

There is not a strong consensus on the best way to estimate pricing data from research participants, nor do the measurement properties of WTP data appear to be well understood. These concerns, while crucial to applied researchers, are less pertinent to the current study, as the goal is to estimate prices on a relative, rather than absolute, scale. In other words, if participants over (or under) estimate their preferences by a constant, but generally rate higher valued options more highly than lower valued options, it may still be possible to obtain reasonable estimates on the ρ parameters required by the Thurstonian model. The lack of firm understanding on the measurement properties, however, may still limit the current approach.

Other researchers have studied context effects with ratings measures. Wedell and Pettibone (1996) collected Likert scale attractiveness ratings for attraction effect stimuli, generally finding that the presence of a decoy increased mean ratings for a target option. Windschitl and Chambers (2004) asked participants to judge the likelihood of various events (also on a Likert scale), finding that the presence of a *dud* (highly unlikely) alternative increased participants' ratings of focal options. Cai and Pleskac (2023) and Fang et al. (2024) demonstrated similar effects with continuous probability judgments.

To the author's knowledge, however, there has been no research systematically connecting valuations and choices in a single context effects experiment through a Thurstonian choice model. This chapter seeks to fill this gap in the literature while also generalizing Chapter 2 to consumer choice. Pricing data were collected to estimate the multivariate normal parameters $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ for the choice model from Chapter 2 which was then used these to predict consumer choice data collected from the same group of participants.

4.1.2 Correlations in Preferential Choice

Chapter 2 demonstrated that the model can capture the repulsion effect in perceptual choice (Spektor et al., 2018), through the estimated target-decoy correlation ρ_{TD}. The current goal is to test whether 1) preferential stimuli also elicit these correlations and 2) the model can capture the repulsion effect in preferential choice, conditional on these estimated parameters.

The current experiment focuses on the attraction and repulsion effects. In the attraction effect, an asymmetrically dominated *decoy* option increases the choice share of a similar, but dominating *target* option at the expense of a dissimilar *competitor* option (Huber et al., 1982). The repulsion effect occurs when the competitor is chosen more often than the target (Frederick & Lee, 2008).

The literature on the repulsion effect in preferential choice is relatively sparse. Liao et al. (2021) varied TDD (target-decoy distance) in an inference task and found a U-shaped relationship between TDD and the Relative Share of the Target (RST), with the attraction effect occurring at low and high TDD levels but the repulsion effect occurring at more intermediate TDD levels. Spektor et al. (2022) demonstrated the repulsion effect in (risky) preferential choice, displaying stimuli graphically but in the same triangle display as Spektor et al. (2018). Participants chose the competitor more than the target at all levels of TDD, but the effect was strongest when TDD was low (similar to the results of Spektor et al. (2018), see Chapter 2). Brendl et al. (2023) showed the repulsion effect when consumer choice stimuli were represented by qualitative attributes but found the attraction effect with quantitative attributes, a finding also shown by Frederick et al. (2014).

Banerjee et al. (2024) demonstrated a binary-ternary form of the repulsion effect using the stimuli depicted in Figure 4.1, across multiple experiments. Participants saw either two or three options on each trial, each option varying on two dimensions. The options were consumer choice products from a number of categories (e.g., cameras, coffee makers, laptops), and the dimension names varied by product category (e.g., coffee makers' dimensions were brew-speed and features). Attribute values were always displayed numerically using ratings of 1-100.

In each choice set of Banerjee et al. (2024)'s experiments, the target (t) was always the most extreme option - particularly high on one dimension and particularly low on the other dimension. The competitor (c) was a more intermediate option. For example, consider the blue-colored stimuli in Figure 4.1. t is very high on X and very low on Y. Compared to t, c is slightly worse on X but slightly better on Y. d, however, is as high as t on X but even worse on Y.

Using these stimuli, Banerjee et al. (2024) showed that the competitor's choice share increased from binary to ternary choice sets, $P(C|[T,C,D]) > P(C|[T,C])$, in violation of the regularity principle (MacKay & Zinnes, 1995; Marley, 1989). They also showed that the repulsion effect decreased with TDD, a conceptual replication of Spektor et al. (2018).

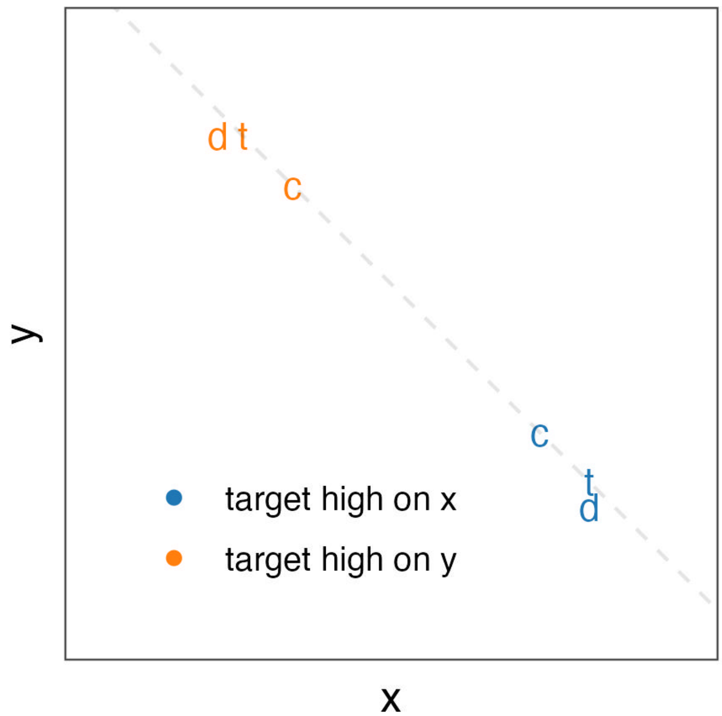

Figure 4.1. Graphical depiction of a subset of the stimuli used in Banerjee et al. (2024), Experiment 5. Target, competitor, and decoy are labeled t, c, and d, respectively. Dimensions are (generically) labeled X and Y. The choice sets vary based on whether the target is higher on the X or Y dimension. The dashed line is the diagonal line of indifference, assuming equal weighting of both the X and Y dimensions.

Banerjee et al. (2024)'s experiments compared binary to ternary choice rather than ternary to ternary choice, as in Spektor et al. (2018). To do a ternary-ternary comparison, one would switch the target and competitor labels, such that the target is the intermediate option, the competitor is the extreme option, and the decoy is nearer to the new, intermediate target. Though Banerjee et al. (2024) were able to generate violations of regularity in their binary-ternary comparisons, these results are somewhat limited by the fact that the target was always more extreme than the competitor. In other words, though Banerjee et al. (2024) demonstrated that the choice share of the competitor option increased with the introduction of the decoy, this result may be due to extremeness aversion - decision-makers' tendency to not prefer options positioned more extremely i n attribute space (Simonson & Tversky, 1992). The introduction of the decoy may highlight the fact that the target is positioned particularly extremely compared to the target. For example, see their stimuli plotted in Figure 4.1. Consider the blue choice set, where the target is high on dimension X. Both T and C are relatively high on the X dimension and low on the Y dimension. C is, however, higher on Y while T is higher on X. In choosing T over C, consumers are forced to trade a small improvement in the X dimension for an option with a particularly dismal Y value. The introduction of the decoy may draw further attention to the Y dimension, leading to greater choice for C over T. This result is qualitatively different from the results of Spektor et al. (2018), because the decoy was never positioned near the less extreme option. Banerjee et al. (2024) argued this as well, suggesting that the decoy may have caused participants to focus more attention on the competitor's superior dimension. For example, in the blue choice set of Figure 4.1, the decoy is quite poor on Y while being equally good as the target on X, so participants may have focused more attention on Y, leading to a preference for the competitor. Indeed, in Experiment 5, the repulsion was strongest when all options were particularly high on the target's superior dimension - i.e., located at more extreme locations in attribute space.

Banerjee et al. (2024)'s results are worth exploring further. The current study uses their stimuli to measure participants' valuations of consumer products as well as collect choices from the same sets of products.

4.2 Experiment 4

The goal of Experiment 4 was to collect ratings and choice data in a preferential choice setting using (a subset of) Banerjee et al. (2024)'s Experiment 5 stimuli. These data were used to estimate the parameters of the choice model from Chapter 2.

For a ratings measure, participants were told to assign selling prices to each option. Though people often overestimate prices (Breidert et al., 2006), pricing measures are approximately continuous[1] and monotonic with value (Miller et al., 2011) and are thus comparable to estimated area, the value measure used in Experiment 2.

4.2.1 Methods

4.2.1.1 Participants

137 U.S. adults participated in the experiment. Participants were recruited from Prolific, an online research platform and were paid \$5 for their participation. 24 participants were removed from all analyses for failing catch trials (see below), leaving a final sample size of $N = 113$.

4.2.1.2 Stimuli

The stimuli were adapted from Banerjee et al. (2024)'s Experiment 5. The stimuli were hypothetical consumer choice products. All stimuli varied on two attributes,

[1] In the current experiment participants could give price responses down to a single dollar. Thus, prices are approximately, but not perfectly, continuous.

each of which ranged from 0-100. The products came from four categories: televisions, washing machines, laptops, and microwave ovens.

In each phase, there were two types of trials: critical trials and catch trials. The critical trial stimuli are shown in Figure 4.2.

There were two types of critical trials: those designed to elicit the repulsion effect (a replication of Banerjee et al.), and those designed to elicit the attraction effect (because the stimuli are located in the center of the attribute space with a decoy near one of the two focal options). These trials will be referred to as repulsion and attraction trials, respectively.

Within both the attraction and repulsion trials, the stimuli varied on which dimension the target was higher on (1 or 2), TDD (designated near or far from the target), and product category (microwaves, washing machines, laptops, and televisions). Note that to create the attraction trials, the target and competitor were shifted towards the center of the attribute space. That is, target and competitor were equidistant in both repulsion and attraction trials, but they were both more extreme in the repulsion trials.

The attribute names varied by category. Televisions varied on screen size and average lifespan. Washing machines varied on average lifespan and energy savings. Laptops varied on processing speed and memory (RAM). Microwave ovens varied on warranty and cooking power.

Within each category, one attribute was arbitrarily designated as dimension 1 and another as dimension 2. However, the order of attributes was randomized on each trial.

The catch trials were designed such that one option was clearly superior to the other two. On each catch trial, the superior option's dimension values were each independently sampled from the vector $[50, 55, 60, 65, 70, 75, 80, 85, 90, 95]$, while the

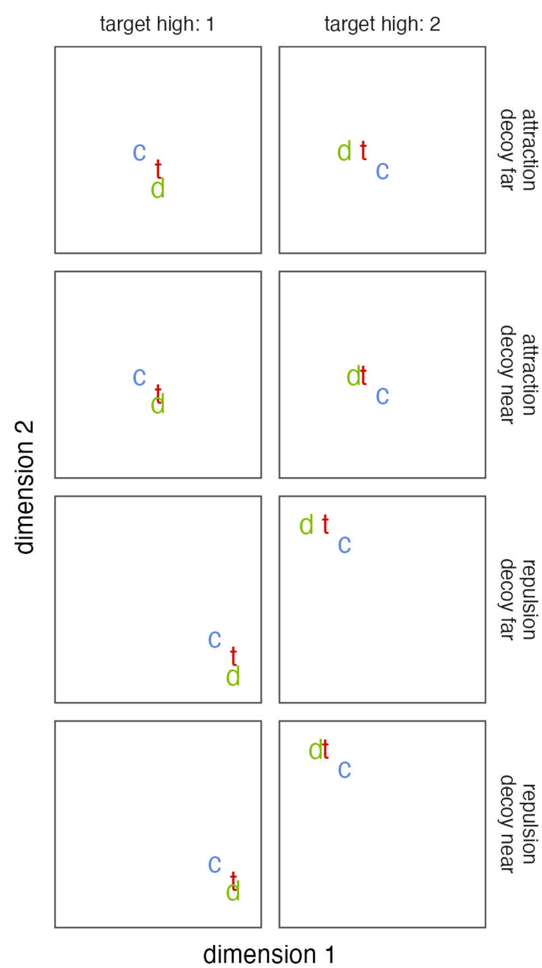

Figure 4.2. Graphical depiction of the critical stimuli from Experiment 4. Rows show the different choice sets designed to elicit the attraction/repulsion effect, with the label also specifying whether the decoy is near or far from the target in attribute space. The columns indicate which dimension the target is high on (1 or 2). Dimensions are labelled generically because their names vary with product category.

two inferior options' dimension values were independently sampled from the vector $[5, 10, 15, 20, 25, 30, 35, 40, 45, 50]$.

4.2.1.3 Design

The experiment took place in two phases: pricing and choice. The stimuli were identical for both phases.

As discussed above, there were both critical trials and catch trials in each phase.

In both the rating phase and the choice phase, there were 40 trials: 32 critical trials - 16 repulsion trials and 16 attraction trials - and 8 catch trials.

The experiment did not contain binary repulsion effect trials, as in Banerjee et al. (2024)'s experiments, so this chapter cannot assess the repulsion effect to the same degree as those authors. The goal of this experiment was to measure valuation correlations in consumer preference and relate them to choice.

4.2.1.4 Procedure

The experiment took place in two phases: a pricing phase and a choice phase.

Prior to the pricing phase, participants were provided with a cover story. They were told to imagine that they run an online consumer goods resale business. On each trial, they would see three products, and they needed to determine which price to sell each product for. Participants were told that the ratings varied from 0 to 100, where 0 was the lowest possible rating and 100 was the highest possible rating. Participants were also told that they should determine a price that maximizes both profit and the likelihood the product is purchased.

During the pricing trials, the three options were presented in a table, with the options in rows and the attributes in columns. All attributes were represented numerically. The options were labeled A, B, and C. The last column of the table contained three boxes, which participants used to type in their selling price for each option. Participants typed in their selling price and then clicked a button on the screen to advance to the next trial. Both option order and dimension order was randomized on each trial. See Figure 4.3 (left panel) for an example trial. Participants were only allowed to enter in whole numbers (i.e., US dollars but not cents).

After completing all pricing trials, participants moved onto the choice phase. Prior to the choice phase, participants were told to imagine that they were purchasing consumer goods in bulk. On each trial, they were to select the option they wanted to purchase. Participants were provided no price information during the choice phase. Though participants may have remembered some of their prices from the initial phase and used these prices as reference points when making choices, prices were omitted from the choice phase to best ensure that participants only made choices by using the attributes provided.

As in the pricing phase, options were presented in a table. See Figure 4.3 (bottom) for an example trial.

In both phases, trial order and option order within each trial were randomized.

After the choice phase, participants completed a short demographics form before the experiment ended.

4.2.2 Results

4.2.2.1 Data Processing

First, 24 participants were removed from the data because they did not pass at least 5/8 catch trials in both the pricing phase and the choice phase. To pass a pricing catch trial, the participant needed to price the superior option at least as high as the other two inferior options. To pass a choice catch trial, the participant needed to select the superior option.

4.2.2.2 Pricing Trials

First, mean prices were computed for the target, competitor, and decoy options within each trial type and product category. These means are shown in Figure 4.4.

Microwave ovens

Please enter the price you want to sell each product for.

Product	Cooking power (1=worst, 100=best)	Warranty (1=worst, 100=best)	Your Selling Price
Product A	60	75	$ 0
Product B	5	30	$ 0
Product C	45	25	$ 0

Televisions

Please select one of the following products::

Product	Average lifespan (1=worst, 100=best)	Screen size (1=worst, 100=best)	
Product A	25	30	○
Product B	35	10	○
Product C	70	85	○

Figure 4.3. Sample trials from the pricing phase (top) and choice phase (bottom) in Experiment 4.

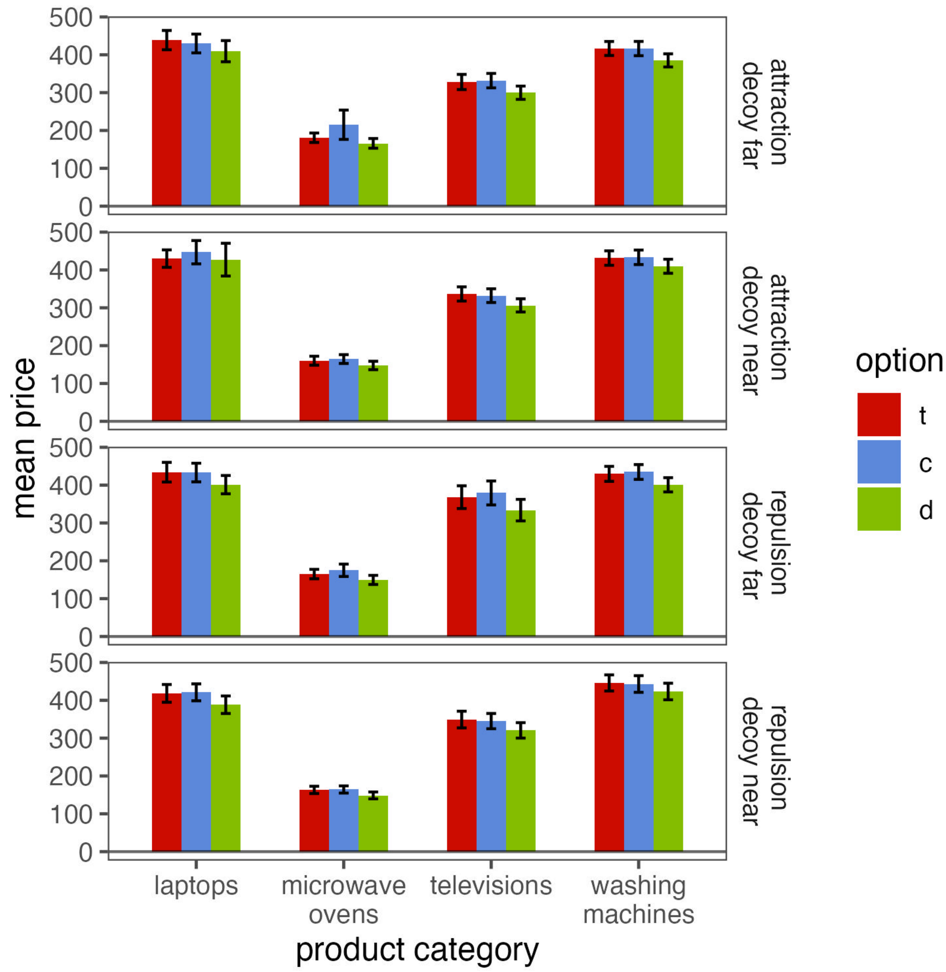

Figure 4.4. Experiment 4 mean prices by product category, option, and trial type. Error bars are ±1 SEM.

On average, participants priced the target and competitor higher than the decoy. They also assigned higher prices to products that are typically more expensive (e.g., washing machines are more expensive than microwave ovens). This suggests that participants were engaged with the task and, in a relative sense, performed the task well.

A Bayesian modeling analysis was performed to estimate the parameters of the Thurstonian choice model, comparable to that of Experiment 2. This analysis was used to estimate the means, variances, and correlations for the pricing data and perform inference on these data. The details of the estimation and the results are presented in the Apppendix, with the exceptions of the posterior distributions for several crucial parameters (i.e., means and correlations). The main text discusses descriptive statistics for these parameters, but whenever it is claimed that one parameter value is greater than another, the reader can see the Appendix for statistical inference which supports these conclusions.

To account for participant-level differences in pricing, all prices were normalized within-participants. To avoid the influence of outliers on correlation estimates, 33 trials in which at least one normalized price had an absolute value > 3 were also removed, leaving $3,583$ trials remaining.

The mean prices were computed for target, competitor, and decoy options within each trial type, collapsing over product category and TDD, shown in Figure 4.5. In both repulsion and attraction effect trials, the target and competitor did not differ in mean price, while both were priced higher than the decoy option.

Correlations between the prices assigned to options were computed within each trial type. The normalized prices are plotted in a series of scatterplots, with the Pearson correlations included. See Figure 4.6 for attraction effect scatterplots and Figure 4.7 for repulsion effect scatterplots.

Note that in both Figure 4.6 and Figure 4.7, the decoy was occasionally priced higher than the target (i.e., the points above the diagonals in the target-decoy scatterplots). This may be due to participant inattention or response error, as the decoy should never be priced higher than the target given adequate participant attention.

Posterior distributions of the correlation parameters are plotted in Figure 4.8. The repulsion trials replicated the results of Experiment 2, in that $\rho_{TD} > \rho_{TC}$ and $\rho_{TD} > \rho_{CD}$. The results also showed that $\rho_{TC} > \rho_{CD}$, while in Experiment 2 $\rho_{TC} \approx \rho_{CD}$. In the attraction trials, the correlations showed a slightly different pattern, where $\rho_{TC} \approx \rho_{TD} > \rho_{CD}$. In other words, the target and competitor are approximately equally

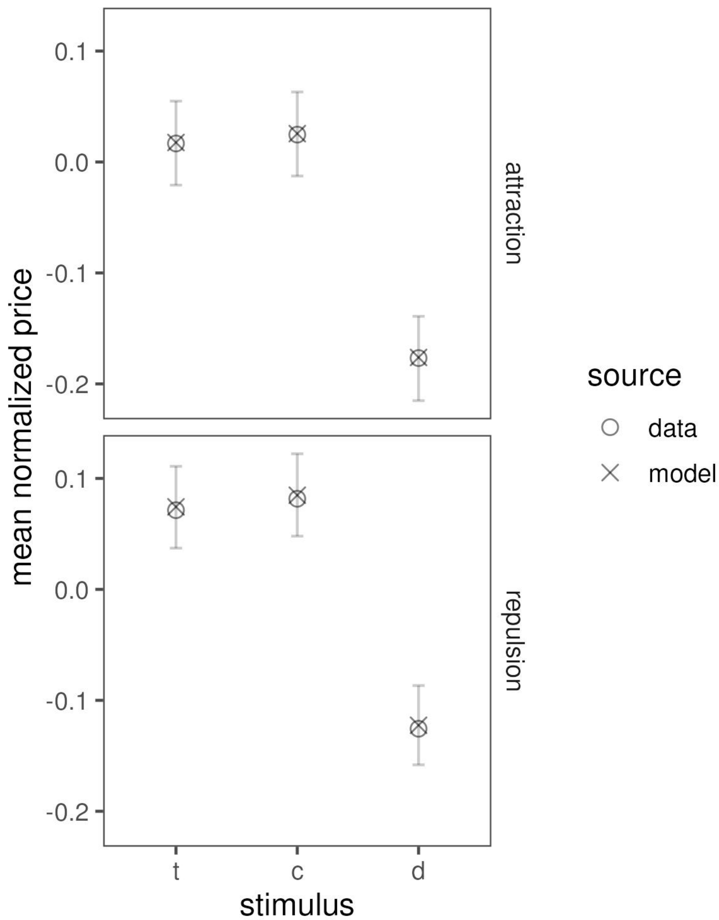

Figure 4.5. Experiment 4 mean prices for target, competitor, and decoy options in both repulsion and attraction trials. Model values are the means from the posterior distribution, and error bars are 95% HDIs.

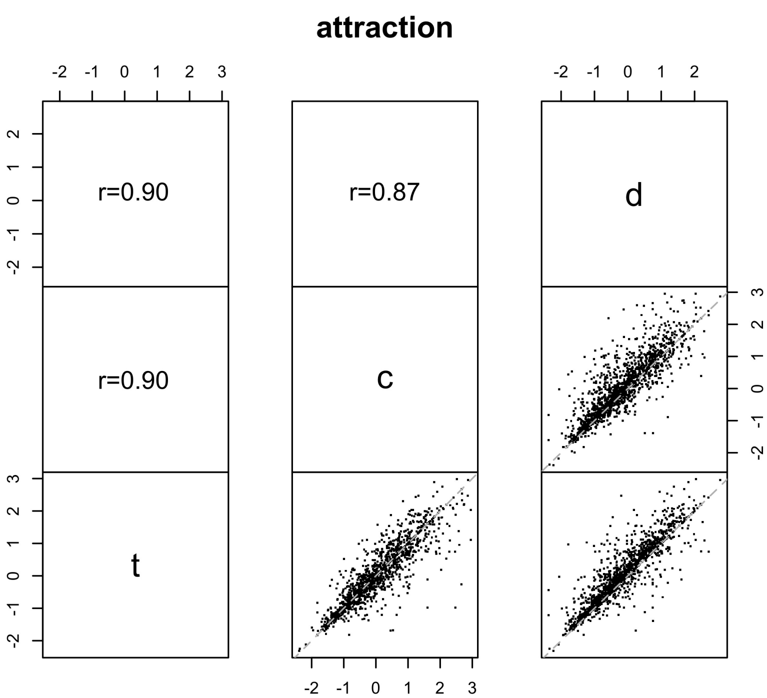

Figure 4.6. Experiment 4 correlation plots for all pairs of stimuli, in trials designed to elicit the attraction effect. t=target, c=competitor, and d=decoy.

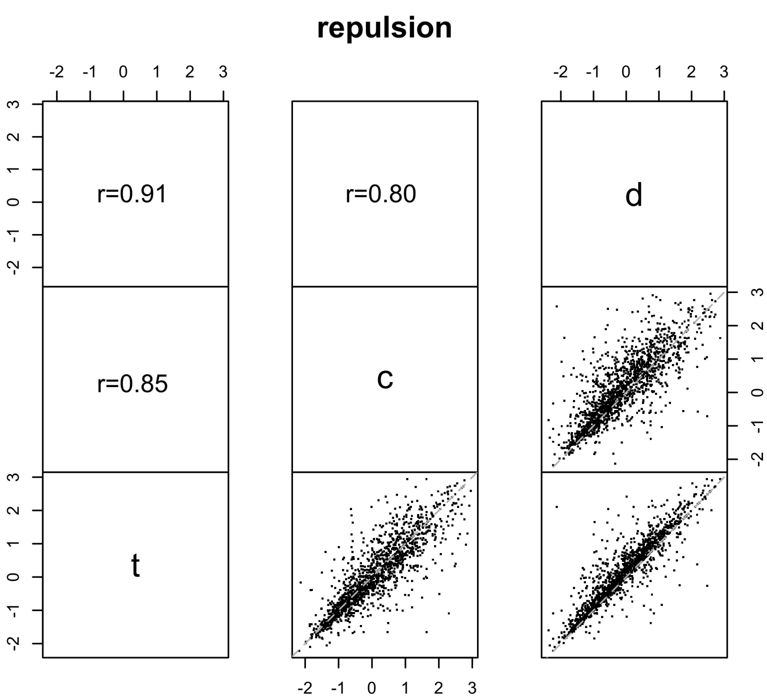

Figure 4.7. Experiment 4 correlation plots for all pairs of stimuli, in trials designed to elicit the repulsion effect. t=target, c=competitor, and d=decoy.

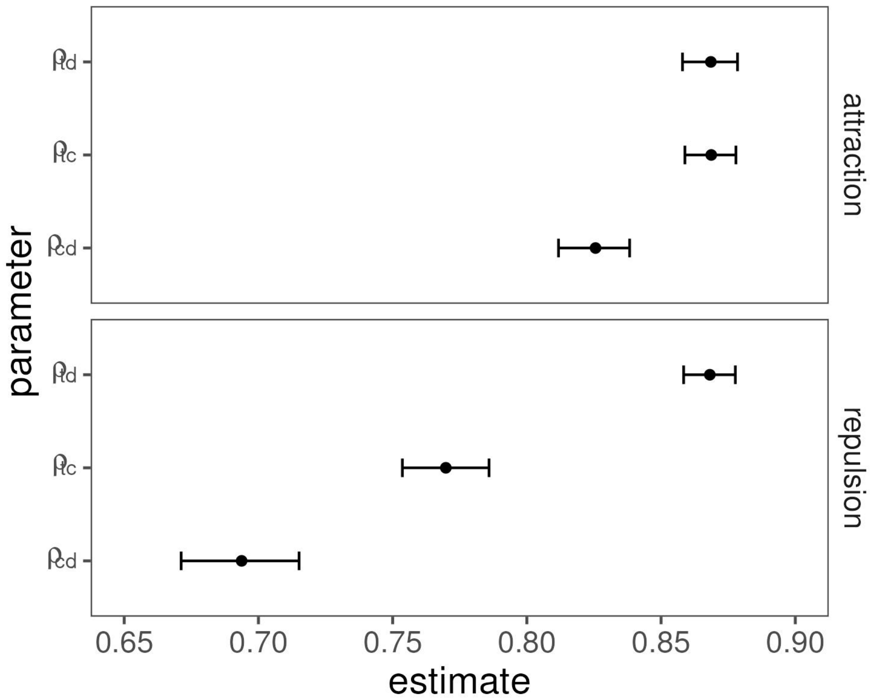

Figure 4.8. Experiment 4 posterior distributions on correlation parameters. Dots are means, and error bars are 95% HDIs.

similar as are target and decoy, and both pairs are more similar than competitor and decoy.

4.2.2.3 Choice Trials

Next, the choice proportions were computed for the critical choice trials. These results are collapsed across participant (due to the small n per subject), product category, and the target's superior dimension. These aggregate choice proportions are plotted in Figure 4.9.

Participants seldom chose the decoy, an indication that they were attentive to the task. This also provides evidence that decoy selection is far more common in perceptual choice than preferential choice, as noted in Chapter 2. The results clearly show a null attraction effect, regardless of TDD. Participants chose the target and competitor options at equal rates. This is likely due to the strong similarity of target

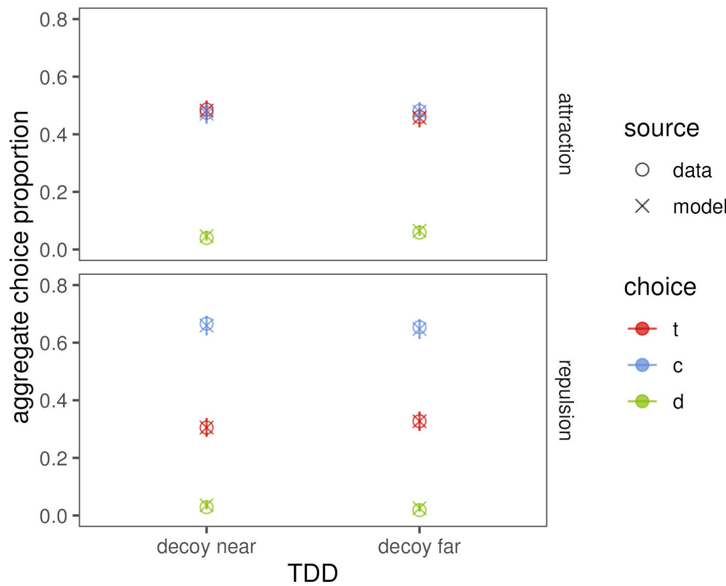

Figure 4.9. Experiment 4 aggregate choice proportions for each trial type, TDD, and option. Data points are aggregate choice proportions, while model points are posterior means computed from the Bayesian Dirichlet-multinomial model presented in the Appendix. Error bars are 95% HDIs.

and competitor. The target and competitor were particularly close to one another in attribute space, a design which is atypical of attraction effect studies.

The results also show a strong repulsion effect, where the participants generally preferred the competitor option to the target option. These data replicate the results of Banerjee et al. (2024), albeit without the binary-ternary comparison included in their experiments. These results may be due to participants simply preferring the less extreme option, as discussed in the introduction to this chapter.

See the Appendix for statistical inference which supports these conclusions.

4.2.2.4 Model Simulations

As in Chapter 2, the Thurstonian choice model, conditional on the current parameter estimates (i.e., $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$), was used to simulate choice. Though the model was originally applied to perceptual choice, it is a general purpose choice model where the primary dimension is value, and the model is agnostic to the meaning of value.

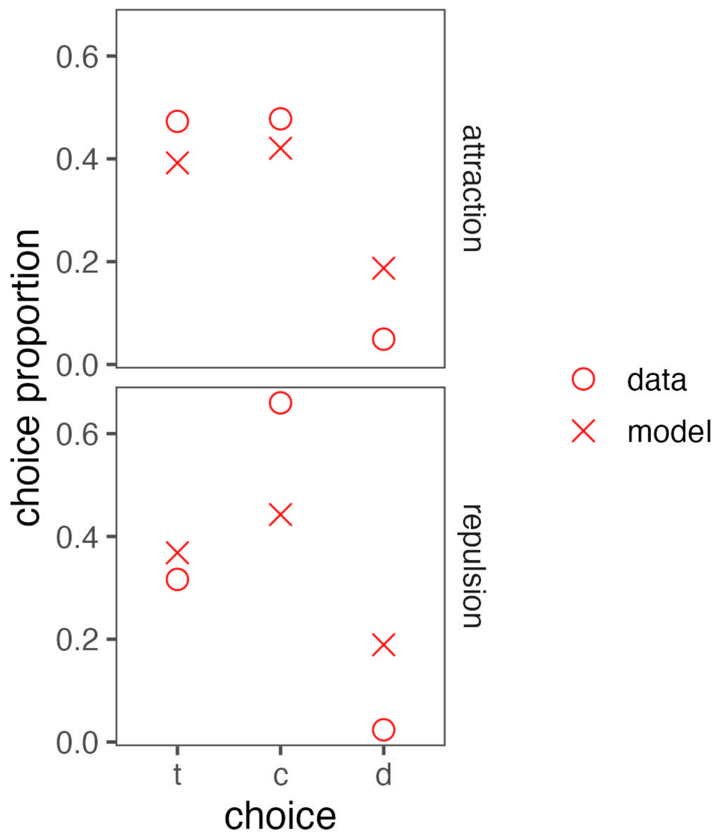

Figure 4.10. Experiment 4 data vs. Thurstonian model predictions.

As in Chapter 2, the model assumes that value is stochastic while choice is deterministic. The model always chooses the option perceived as most valuable, regardless of the magnitude of the difference between the winner and runners-up. That is, given a vector $\mathbf{X}_i$ of perceived values on trial i with set K, the probability a participant selects stimulus j is:

$$P(j|i,K) = P(\mathbf{X}_{ij} > \mathbf{X}_{ik}), \forall k \in K, j \neq k \tag{4.1}$$

$1,000,000$ simulations of the model were run, with the results plotted against the data in Figure 4.10.

The model mispredicts the attraction trials. It predicts a slight repulsion effect when in fact the data suggest a null effect. This is similar to the results of Experiment 2 (horizontal condition), where the model predicted a repulsion effect even when the empirical data showed an attraction effect because of the influence of the target-decoy correlation.

The model does, however, successfully predict a qualitative repulsion effect, i.e., $P(C) > P(T)$. However, it strongly overpredicts the decoy choice proportions. It also underpredicts competitor choice proportions. Furthermore, the model relies on the correlation between target and decoy choice proportions to predict the repulsion effect. According to the Thurstonian model, because the target and decoy are strongly correlated, it is more likely that the utility of the decoy exceeds the target than the competitor. The decoy then takes away choice shares from the target. This prediction makes sense in perceptual choice, where participants pick the decoy roughly 20-30% of all trials (see Experiment 2). In preferential choice, participants almost never pick the decoy, and researchers can generally assume that any participant with sufficient attention can discriminate the target from the decoy. Thus, though the model can qualitatively predict a repulsion effect, the mechanism by for doing so is implausible and inconsistent with the empirical decoy choice data.

The Thurstonian model does not account for the extremeness aversion discussed in the introduction to this chapter. That is, it is plausible that the competitor and target are indeed equally valuable on average, but that the competitor is chosen more often due to participants' unwillingness to choose the target because it is more extreme than the competitor. The Thurstonian model does not account for this bias, and thus the conclusion that the repulsion effect cannot be predicted by the Thurstonian choice model may be premature.

4.3 Discussion

Experiment 4 generalized the paradigm from Chapter 2 to preferential choice. Participants completed two phases; in the first phase, they assigned a selling price to each of three options on each trial. In the second phase, they selected the option they would most prefer. Choice sets were designed to either elicit the attraction effect or the repulsion effect. These prices were used to estimate the mean value of each option as well as the correlations between all pairs of options. In doing so, this work extends not only the experimental paradigm of Chapter 2, but also the Thurstonian modeling framework, to a preferential choice setting.

Crucially, when estimating the correlations, the data replicated the result of Experiment 2, that, in the repulsion effect $\rho_{TD} > \rho_{TC}$ and $\rho_{TD} > \rho_{CD}$. The inferential statistics also showed that $\rho_{TC} > \rho_{CD}$. This result was unexpected. This finding may be due to the fact that, because both target and competitor are high on one dimension and low on another, the competitor draws attention to the dimension on which it is best but that the decoy is slightly worse on. It may be easier, or at least more likely, for participants to compare target and competitor to one another than it is to compare the competitor to the decoy. Participants also typically (but not always) priced the decoy as less than the target and competitor. There thus may have been an upper limit to the decoy pricing which could affect the correlation estimates.

In the attraction trials, it was found that $\rho_{TC} \approx \rho_{TD} > \rho_{CD}$. This pattern is likely due to the similarity of target and competitor on both attributes (i.e., one option's dimension values were $[50, 60]$ while the other's was $[60, 50]$). The target and competitor are easily comparable to one another, and the trade-off on attributes is negligible. Furthermore, the target and competitor are both located in an intermediate region of attribute space rather than in an extreme region as in the repulsion effect trials.

The choice results replicated those of Banerjee et al. (2024), in that participants chose the competitor more than the target in each of the repulsion effect choice sets. However, given that the decoy location did not vary (i.e., the competitor was always less extreme than the target), nor did the experiment include a binary-ternary choice comparison, these results may be due to a bias for the less extreme option, which happens to be the competitor in this case. Future research should include binary-ternary and ternary-ternary trials to generalize these results.

In Banerjee et al. (2024)'s article, they conducted five total experiments. Though the current experiment focused on their Experiment 5, their experiments 1-4 tested the effect of TDD on the repulsion effect. They found mixed evidence that varying TDD affects the repulsion effect. The tainting hypothesis Frederick and Lee (2008), which predicts that the decoy can taint the nearby target and cause the repulsion effect, predicts that the repulsion effect should weaken with increasing TDD. Only two of Banerjee et al. (2024)'s four experiments found evidence for tainting; moreover, the current experiment found no evidence supporting the tainting hypothesis, whether in choice or in rating. Indeed, the target and competitor were rated equally valuable on average.

The strong correlation between target and decoy valuations, compared to competitor-decoy valuations, appears to be a robust finding, holding across perceptual and preferential choice. It is worth exploring, in greater detail, what causes these correlations. One strong hypothesis, considered by other researchers, is that these correlations are measures of similarity.

In the attraction and repulsion effect, the target and decoy are designed such to be more similar to each other than either option is to the competitor. It may be that, by measuring the correlation in valuations, we are actually measuring the *similarity* between options. Indeed, the similarity of target and decoy was a primary motivation for the original demonstration of the attraction effect (Huber et al., 1982).

Other researchers have argued that, in models of choice, the correlation between options is a measure of similarity. Kamakura and Srivastava (1984) parameterized correlations in a choice model as an exponentially decreasing function of distance in attribute space, such that options located more closely in attribute space are more strongly correlated. They also showed that, when embedded in the multinomial probit model, a model based on this parameterization can successfully predict the similarity effect. The similarity effect is the finding that the introduction of a similar option can sometimes decrease the choice share of a focal option Tversky (1972). Kamakura and Srivastava (1984)'s model is nearly identical to the model of similarity developed by Shepard (1987), which also models similarity as an exponentially decreasing function of distance in psychological space and has successfully accounted for empirical data across multiple domains (Hotaling et al., 2010; Nosofsky, 1986; Roads & Love, 2024; Townsend, 1971).

Natenzon (2019) implemented a model using the multinomial probit (MNP), also allowing the correlations between options to be decreasing function of distance in attribute space. They used cosine similarity, the cosine of the angle formed by options in bivariate space, to measure correlations and showed that the model explains choice reversals (i.e., context effects) in frog mating data.

Spektor et al. (2019a) developed a model, known as the accentuation of differences (AOD) model, to account for context effects in choice. According to the AOD, the subjective value of each option can be either increased or decreased by the presence of similar options, where similarity is also an exponentially decreasing function of distance in attribute space. This similarity-based mechanism is governed by a free parameter; the model can account for either the similarity effect (if option value is decreased by similar options) or the attraction effect (if option value is increased by similar options), but not both effects simultaneously.

The current correlations may also be measuring the ease of comparability between pairs of options. Comparability has previously been shown to drive the attraction effect (Cataldo & Cohen, 2019; Hayes et al., 2024; Noguchi & Stewart, 2014), and several models of context effects rely on comparisons of options on single attributes to generate context effects (Roe et al., 2001; Trueblood et al., 2014). Supporting this hypothesis is the finding that $\rho_{TC} > \rho_{TD}$ in the attraction trials, when the target and competitor were quite similar on each attribute and thus easier to compare.

Future work could test these hypotheses by systematically manipulating option comparability and assessing whether the correlations vary with comparability. Chapter 5 works towards this, by testing the effect of comparability on choice, though future work should measure inter-option correlations in choice environments of varying comparability.

CHAPTER 5

COMPARABILITY IN PERCEPTUAL CHOICE

5.1 Introduction

Thus far, the dissertation has explored perceptual and decisional processes in both perceptual and preferential choice. Experiment 2 showed that stimuli which are more similar to one another, and are thus more easily comparable, generate valuations with stronger correlations. This result holds across both perceptual choice (Chapter 2) and preferential choice (Chapter 4). The comparison process is defined as a cognitive operation where a participant attends to the relative difference between two options on a choice set, typically (though not necessarily) on a single dimension.

A Thurstonian choice model, conditional on parameters estimated from actual data, can produce the repulsion effect (Simonson, 2014; Spektor et al., 2018) because the decoy option, whose value is tightly correlated with the target, occasionally exceeds the target in perceived value and thus takes choice shares away from the target. The model can also predict the attraction effect, but the model with parameters which were estimated from actual empirical data do not predict the attraction effect.

In this chapter, inter-stimulus comparability is manipulated directly in a perceptual choice task. The goal of this manipulation was to empirically test the relationship between comparability and choice. First, the previous literature on comparability is reviewed, and then the results of a perceptual choice experiment are presented.

5.1.1 Previous Literature on Comparability

Other researchers have studied the comparison process in decision-making, particularly in high-level (e.g., preferential) choice. Below, relevant preferential choice research is introduced before transitioning to relevant research on perceptual choice.

Chang and Liu (2008) tested the compromise effect by varying the presentation of options. In the compromise effect, a middle-ground option decreases the choice share of two dissimilar, extreme options. Chang and Liu (2008) displayed consumers options in either a by-alternative format, where option names are listed as columns while attribute values are listed as rows, or a by-attribute format, where option attributes are columns while option names are rows. The former display makes it more difficult to compare options on a single attribute, while the latter makes it easier. Chang and Liu (2008) found that listing options by-attribute increased the choice share of the compromise option, relative to a by-alternative display.

Cataldo and Cohen (2019) replicated this result, also finding that a by-alternative format nullified the attraction effect while a by-attribute format strengthens the attraction effect. Cataldo and Cohen (2019) attributed these results to a flexible comparison process, where the comparison strategy is influenced by display format. According to this account, the by-attribute format increases the ease of target-decoy comparisons relative to the by-alternative format.

Noguchi and Stewart (2014) studied context effects using eye-tracking, showing that people tend to compare pairs of options on a single attribute, and that these comparisons appear to drive the attraction, similarity, and compromise effect. In Noguchi and Stewart (2014)'s study, participants' eye movements were more likely to transition between options on a single dimension than they were to transition between dimensions within a single option. Transitions between two options were also negatively related to the choice share of a third option.

Hayes et al. (2024) manipulated attribute comparability, such that the dimensions of each option were either represented in a common unit (high comparability, e.g., 0-10 ratings on all attributes) or in different units (low comparability, e.g., CPU speed vs. RAM for laptops). The attraction effect only occurred in the low comparability condition, with the high comparability condition creating a null effect. The rationale behind this result is that by reducing across-attribute comparability, the researchers are encouraging *within-attribute* comparability, generating the attraction effect in a form similar to Cataldo and Cohen (2019).

Hasan et al. (2025) conducted a large scale replication study on factors that impact the attraction effect, systematically varying option order, presentation mode (numerical or graphical), and presentation format (by-attribute or by-alternative). Hasan et al. (2025) found that the attraction effect was stronger when the target and decoy options were adjacent to one another, presumably because this allows for easier target-decoy comparison. The attraction effect was stronger when attributes were presented numerically compared to graphically, a result found by other researchers (Frederick et al., 2014; Yang & Lynn, 2014). They did, however, fail to replicate Cataldo and Cohen (2019)'s finding that the attraction effect varies with by-alternative vs. by-attribute format. One possible explanation for this failure to replicate is that Cataldo and Cohen (2020) held absolute TDD constant, while Hasan et al. (2025) allowed it to vary with product category. Given that TDD can be a crucial and factor in generating the repulsion effect Liao et al. (2021) and Spektor et al. (2018), the varied TDDs (and perhaps, limited target-decoy comparability) may have nullified the by-attribute vs. by-alternative difference demonstrated by Cataldo and Cohen (2019).

Hsee and colleagues (Hsee et al., 1999; Hsee, 1996, 1998; Hsee & Leclerc, 1998) have also shown that the comparison of options affects consumer behavior. For example, they repeatedly showed that participants' evaluation of a given option can change with the addition of a reference point. For example, lower valued options improve with a high reference point and vice versa. Participants' judgments can reverse when options are evaluated jointly, compared to separately (Hsee et al., 1999).

Several computational models of decision-making also rely on the comparison process. Roe et al. (2001)'s Multialternative Decision Field Theory (MDFT) model assumes that options accumulate evidence through comparisons on individual attributes, nearby options exhibit greater influence on a given option's preference state than do farther options. According to Trueblood et al. (2014)'s Multiattribute Linear Ballistic Accumulator Model (MLBA), each option accumulates evidence through pairwise comparisons to all other available options. These comparisons are modulated by several processes, such as distance in attribute space and extremeness aversion. Other decision models incorporate similar mechanisms (Landry & Webb, 2021; Noguchi & Stewart, 2018; Spektor et al., 2019a; Usher & McClelland, 2004; Wollschläger & Diederich, 2012) (c.f. Bergner et al. (2019) and Bhatia (2013)).

Trueblood et al. (2022) argued that pairs of similar options garner more attention in the comparison process. They presented a simple Markov model where pairwise comparisons on a single attribute determine the accumulation of preference, and the time spent on a comparison is an increasing function of the similarity of options on the attribute. Their model can successfully, and parsimoniously, account for the attraction, compromise, and similarity effects.

5.1.2 Comparability Effects in Perceptual Choice

There has been other research, albeit relatively limited, studying the comparison process in perceptual choice. Much of this work has focused on the spatial layout of the options and its effect on perceptual context effects.

Trueblood et al. (2022) re-analyzed previous perceptual choice context effect data (Trueblood et al., 2015) by examining the order of the options presented to participants. They found that the attraction effect was strongest when the target and decoy were next to each other, while the effect was weak (or even absent) when the options were separated spatially. Their conclusion, supported by a modeling analysis, was that people tend to compare pairs of options which are spatially closer to one another more often than pairs further away from one another.

Evans et al. (2021) found a similar result in perceptual choice, though in their experiment the options were separated both spatially and temporally. In their experiment, participants saw three rectangles, presented sequentially, and selected the largest rectangle after all stimuli were presented. They found that orders in which the target and decoy were presented in the latter two positions elicited an attraction effect, whereas orders in which the competitor and decoy were presented in the latter two positions tended to elicit a repulsion effect. They interpreted their results as evidence that the comparison process can be altered through spatial and temporal properties of the stimuli.

Another interpretation of Evans et al. (2021)'s results, as well as those of Trueblood et al. (2022), is that by altering the location and timing of the stimuli, the researchers are also altering the comparability. In the Thurstonian model of Chapter 2, increased comparability is represented by an increase in perceptual correlation. As shown previously, these perceptual correlations can create a repulsion effect by allowing the decoy to more easily take choice shares from the target.

This chapter extends the experimental work of Chapter 2. The Thurstonian model from Chapter 2 predicts that when two options (target and competitor) are on average equally viable, but if the target and decoy perceptions are more strongly correlated than competitor and decoy perceptions, participants will choose the target less than the competitor. In Experiment 2, these strong correlations occurred because the target and decoy are perceptually similar (i.e., oriented identically) and particularly comparable. In this Chapter, however, the decoy is equally similar to both focal options, but the correlation between decoy and target is now created through ease of comparability on screen.

The goal of this experiment was to isolate the effect of comparability from standard context effects. Results of a perceptual choice experiment are presented, where participants saw three rectangles at a time and were told to select the largest rectangle. On critical trials, two of these rectangles were equally large but oriented differently (i.e., *focal* rectangles, as in Experiments 1, 2, and 3). A third option was a decoy equally similar to both focal options. One of the two focal options was designated as the *target* based on its proximity and comparability to the decoy.

To manipulate comparability, the positioning of the options on screen was systematically manipulated. Options were displayed in one of four ways: all aligned in a horizontal line, as in Trueblood et al. (2013); two adjacent options aligned vertically, with a third option positioned in a different vertical location; none aligned, with all options taking up different vertical and horizontal positions; and the triangle alignment from Spektor et al. (2018). See Figure 5.2 for example stimuli.

Comparability is defined as the ease of comparison between two stimuli and is determined based on proximity in space. The critical condition was the two-aligned condition, where the target and decoy were positioned such that they were easily compared, while the competitor was located where comparisons were more difficult. Based on the results of Chapter 2, ease of comparability should increase the correlation between the target and decoy and result in a *decrease* in the target's choice share in the two-aligned condition, relative to the none aligned and all aligned conditions. In the model and experiment presented in Chapter 2, the similarity of the target and decoy caused the decoy to take choice shares away from the target. Here, both focal options were equally similar to the target, but the target was located closer to the decoy and is more comparable. These predictions were generally borne out in the data, albeit with limitations which will be discussed below.

5.2 Experiment 5

Experiment 5 addressed the effect of comparability in perceptual choice using *symmetrically dominated* decoys. A symmetrically dominated decoy is dominated by both focal options but is also equally similar to both options. Thus, the terms target and competitor, which have been used throughout this dissertation, take on a different meaning here. Here the target is defined as the option that is both adjacent to and easily comparable to the decoy. Based on the experimental and modeling results of Chapter 2, it was predicted that the choice share of the target will be reduced when it is particularly comparable to the decoy.

5.2.1 Methods

5.2.1.1 Participants

231 undergraduate students at the University of Massachusetts Amherst participated in the lab, in exchange for course credit. 17 participants' data were removed from all analyses because they failed to achieve at least 80% correct on catch trials (see below), leaving a final sample size of $N = 214$.

5.2.1.2 Stimuli

The stimuli were gray-scale rectangles and squares, varying systematically on height and width.

The critical stimuli are depicted in Figure 5.1. The focal stimuli (H and W) were equal in area and fell on two diagonals, the upper diagonal area being 25000 square pixels and the lower diagonal being 7581px^2. On the lower diagonal, the focal stimuli had dimension values of 57 and 133 pixels, while on the upper diagonal, the focal stimuli had dimension values of 127 and 141 pixels. The decoy options were either 20% or 35% smaller than the focal options. This was determined based on the results of pilot data and with the goal of making the decoy somewhat difficult to discriminate

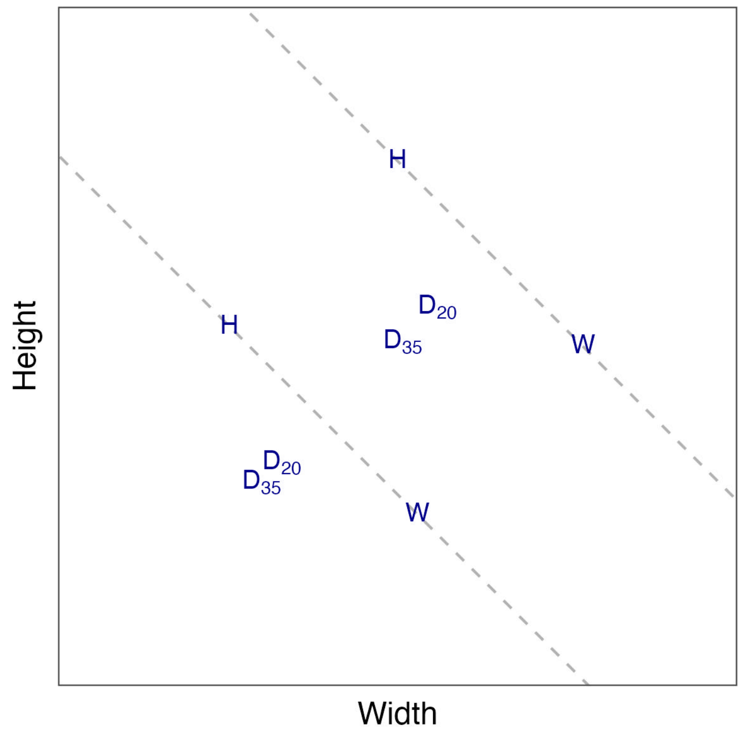

Figure 5.1. Graphical depiction of critical stimuli from Experiment 5. The stimuli fall on two diagonals, referred to as upper and lower. The H rectangles are taller than wide, while the W rectangles are wider than tall. The decoy (D) rectangles are equally wide and tall (i.e., squares). The decoy subscripts indicate the TDD %.

from the focal options. For the manipulation to work, participants needed incentive to consider the decoy and thus to compare it to the two focal options.

The stimuli were arranged in one of four displays: all-aligned, two-aligned, none-aligned, or triangle. See Figure 5.2 for sample trials.

In the all-aligned display, all stimuli were arranged in a horizontal array, as in the experiments of Trueblood et al. (2013) and Spektor et al. (2018), Experiment 4.

In the two-aligned display, two options were aligned horizontally in the top-left and top-middle positions, while the third option was placed in the bottom-right position of the screen. This is a crucial condition and an instantiation of the comparability hypothesis, in which the comparison of the two aligned options was far easier than all other pairwise comparisons.

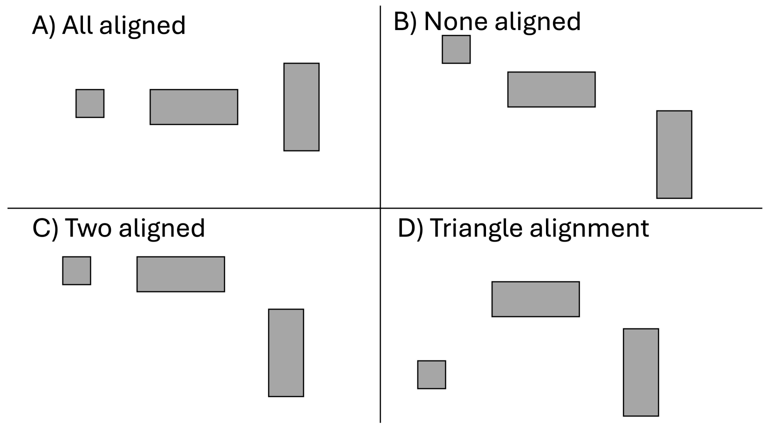

Figure 5.2. Sample critical trials from Experiment 5. A) all aligned. B) Two aligned. C) None aligned. D) Triangle alignment.

In the none-aligned display, all options are located in different vertical and horizontal positions, such that all comparisons should be more difficult, as determining the relative areas between rectangles is less straightforward.

The triangle display was identical to the triangle display of Experiments 1 and 2, with the exception that on half of all triangle display trials, the triangle was inverted. Note that this condition is distinct from the two aligned condition, as the options were located in distinct vertical and horizontal positions, and it was difficult to determine the relative sizes between rectangles.

In all displays, the horizontal distances between all options was constant.

In addition to varying diagonal, TDD, and display, option order also varied, such that all six orders (DHW, DWH, HDW, HWD, WDH, and WHD) were equally common.

There were other types of stimuli on non-critical trials: filler-random, filler-square, and catch.

On filler-random trials, three options were randomly generated by independently sampling both a height and width from the $U(57, 200)px$ distribution. Filler-random trials were included to avoid participants noticing the manipulation on the critical trials and to include enough challenging trials to keep participants engaged in the task.

On filler-square trials, a square was randomly generated by sampling a side length from the $U(57, 200)px$ distribution. Then, two non-square rectangles were generated by sampling a height and width from the $U(57, S)px$ distribution, where S is the side length of the square. This procedure ensured that the square was always the largest option on filler-square trials. Filler-square trials were included to ensure that participants did not learn to ignore the squares, as the critical trials rely on participants comparing the squares to the focal options.

On catch trials, one large option was randomly generated by randomly sampling a stimulus from the upper diagonal (see Figure 5.1). Two smaller options were randomly sampled from the lower diagonal.

5.2.1.3 Design

As discussed above, there were four types of trials: critical trials, filler-square trials, filler-random trials, and catch trials. The study took place in four blocks.

This was a 2 (diagonal: lower, upper) x 2 (TDD: 20%, 35%) x 4 (display: all-aligned, two-aligned, none-aligned, triangle) x 6 (order: DHW, DWH, HDW, HWD, WDH, and WHD) within-participants study. Each participant completed 4 trials for all combinations of these factors (1 per each of the four blocks), except for the two-aligned trials, for which they completed 8 trials (2 per each of the four blocks). Thus, there were a total of 480 critical trials ($(2 * 2 * 3 * 6 * 4) + (2 * 2 * 1 * 6 * 8) = 480$).

On each of the non-critical trials, stimulus order was randomized. Additionally, one of the four displays (all-aligned, two-aligned, none-aligned, triangle) was selected at random.

There were 40 filler-random trials per block for each of four blocks, a total of 160 filler-random trials. There were 40 filler-square trials per block for each of four blocks, a total of 160 filler-square trials. There were 10 catch trials per block for each of four blocks, a total of 40 catch trials.

There were 840 total trials in the experiment (480 critical + 160 filler-random + 160 filler-square + 40 catch=840).

5.2.1.4 Procedure

On each trial, participants saw three rectangles, labeled 1, 2, and 3. Participants were told to select the largest rectangle. They made their choice by pressing the corresponding key on the keyboard.

The experiment was split into four blocks. There was a 15-second break between blocks.

5.2.2 Results

5.2.2.1 Data Processing

In addition to removing participants who failed to meet the 80% correct criterion for catch trials, $2,319$ trials with RTs $< 100ms$ or $> 10,000ms$ were removed from all analyses.

5.2.2.2 Catch and Filler Trials

On average, participants performed well above chance on the catch, filler-random, and filler-square trials, including on all display types. See Figure 5.3 for these data.

As will be relevant below, participants showed position biases. The order of stimuli on each trial was random, so on average, participants should be equally likely to select the left, middle, rightmost rectangle. However, participants tended to select the middle rectangle the most, followed by the rightmost rectangle, followed by the leftmost rectangle. This bias occurred in each display condition. Mean choice proportions for each position for all non-critical trials (collapsed across trial type) are plotted in Figure 5.4.

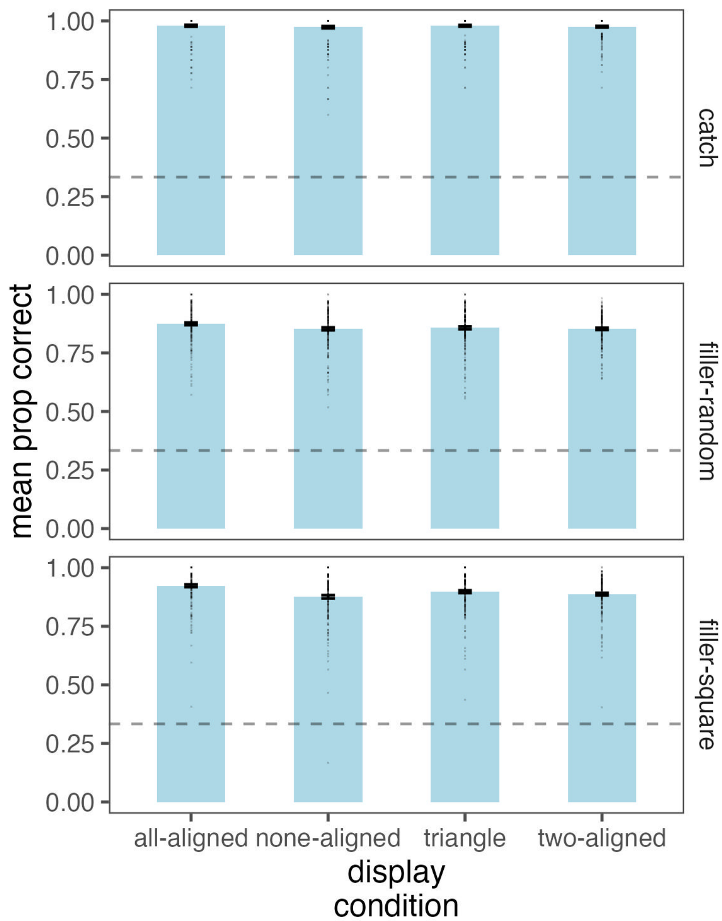

Figure 5.3. Results from non-critical trials in Experiment 5. Rows show trial types. Bars show mean proportion correct in a given condition, with the error bars showing ±1 SEM. Dots show individual participant data. Dashed line is at 1/3 (chance performance).

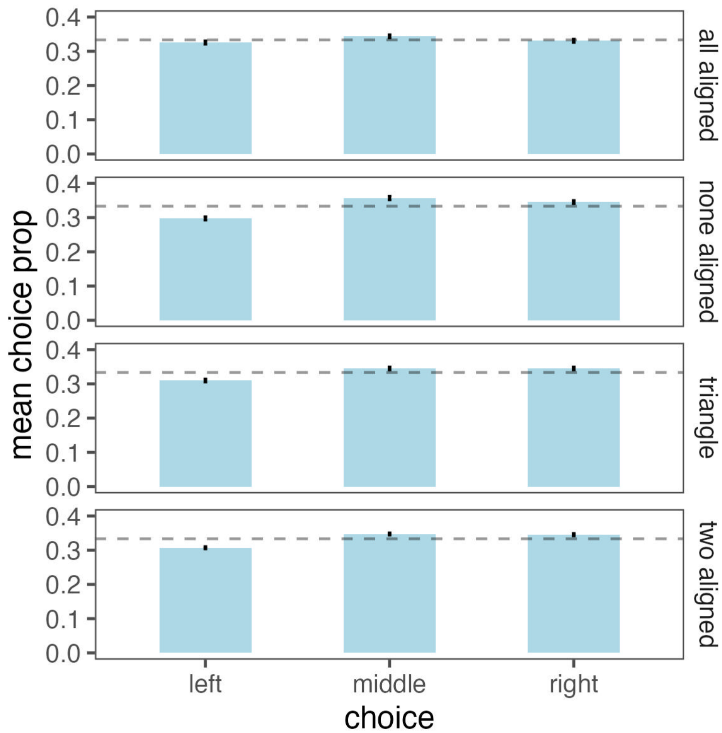

Figure 5.4. Position biases from non-critical trials in Experiment 5. Rows show trial types. Bars show mean choice proportions for a given position, with the error bars showing ± 1 SEM. Dashed line is at chance (1/3).

5.2.2.3 Critical Trials

The goal of the critical trial analysis was to determine how alignment affected choice for the target option, defined as the rectangle located next to the decoy.

Consider the order of options on screen. There were six possible orderings: DHW, DWH, HDW, HWD, WDH, and WHD. Given that the crucial display is the two-aligned display - where the first two options were aligned vertically while a third option was unaligned both vertically and horizontally - the crucial trials were those trials where the decoy was among the first two options.

The ordering was re-classified into a variable referred to as alignment. Because the two-aligned condition is the critical one, the alignment variable is labeled based on the order in the two-aligned condition. If the ordering is DHW or HDW, the alignment is "H aligned with D". If the ordering is WDH or DWH, the alignment is "D aligned with W". The WHD and HWD trials were removed from further analysis.

The mean choice proportion for the H, W, and D rectangles based on alignment were computed and are shown in Figure 5.5. The data show that, in the two-aligned condition, participants were less likely to choose the W option when it was aligned with D than when H was aligned with D. This effect also appears to occur in the none-aligned and all-aligned conditions, which suggests a position bias because choosing H more when W is aligned with D but choosing W more when H is aligned with D amounts to having a bias for the third (rightmost) rectangle.

Next, the analyses were collapsed over H/W and instead, choices were re-classified into target, competitor, and decoy. The target was the option aligned with the decoy, while the competitor was the option not aligned with the decoy. The decoy label did not change. Mean choice proportions were computed and plotted in Figure 5.6. Note that this classification is most appropriate for the two-aligned condition; the other conditions are included as a point of comparison.

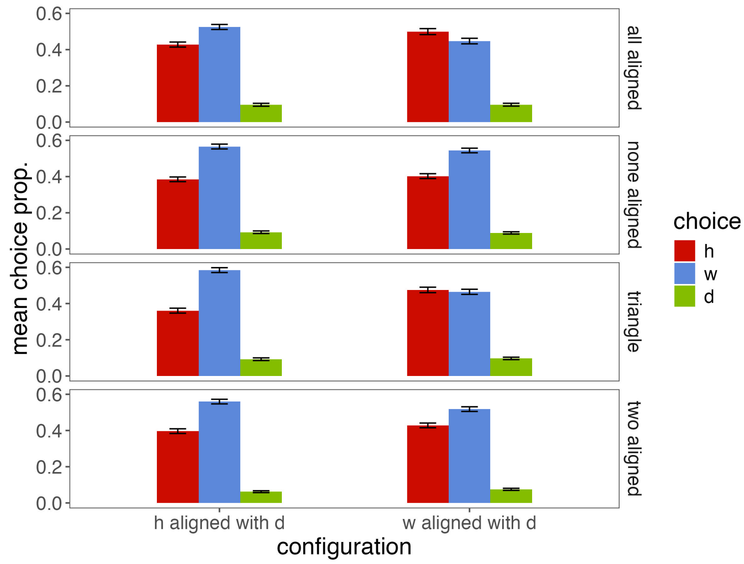

Figure 5.5. Results from critical trials in Experiment 5. Mean choice proportions for the H, W, and D rectangles conditioned on alignment and display. Error bars are ±1SEM.

Participants reliably chose the competitor more than the target; however, this effect appears to be stronger in the triangle condition, followed by the all aligned condition, the two aligned condition, and then the none aligned condition. Given that the triangle condition does not readily facilitate comparison between the decoy and the target, choosing the competitor more often in the triangle condition amounts to a bias for the third (rightmost) rectangle. A similar interpretation can be made for the none aligned condition.

Next, all trials in which participants chose the decoy were removed and Relative Share of the Target (RST) was computed. The mean RST is plotted in Figure 5.7.

On average, participants selected the target option on less than 1/2 of trials (conditional on not having selected the decoy). This again may suggest a position bias; that is, participants appeared to have a bias for the third (rightmost rectangle) in the choice set. However, crucially, they selected the target less when it is aligned with the decoy in the two-aligned condition than when it is adjacent to the decoy in

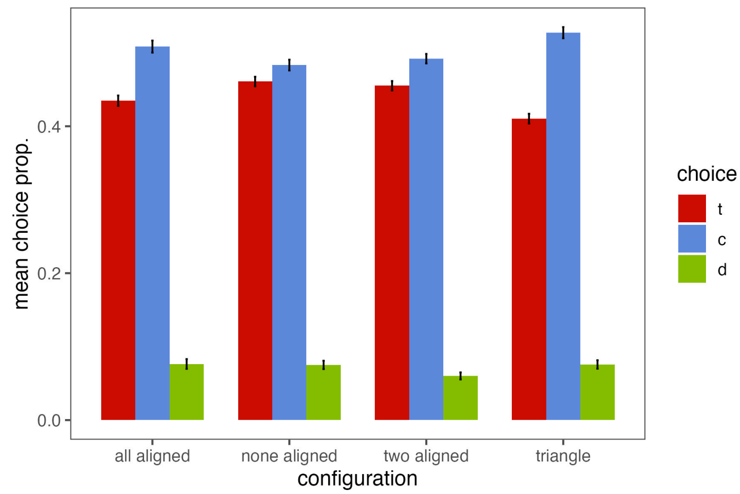

Figure 5.6. Experiment 5 mean target, competitor, and decoy choice proportions by display. Error bars are ±1SEM.

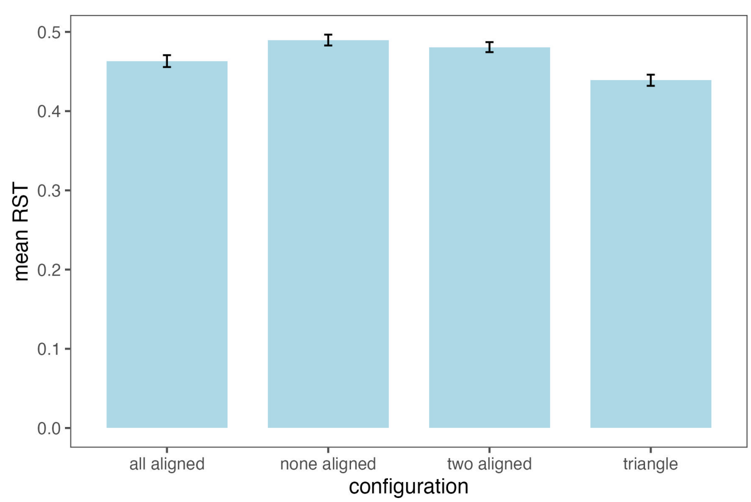

Figure 5.7. Experiment 5 mean RST by display. Error bars are ±1SEM.

the none-aligned condition. See the Appendix for inferential statistics which support these conclusions.

5.3 Discussion

Experiment 5 showed that comparability can affect choice. Given two equally viable dissimilar options, when one of these options (i.e., the target) was more easily comparable to a symmetrically dominated decoy option, participants chose the target less than the competitor. This effect is small but nonetheless present.

This result aligns with the predictions from the Thurstonian perceptual choice model from Chapter 2. Increasing the comparability of a focal option and a decoy option increases the perceptual correlations between the two, which in turn decreases the choice share of the comparable *target*, for the benefit of the non-comparable competitor option. This result is also similar to previous results presented by Trueblood et al. (2022) and Evans et al. (2021), who also manipulated the arrangement of options on screen in perceptual choice.

These results are, however, somewhat limited by the positioning of the target and decoy options. In the two-aligned condition, the target and decoy were always presented in the first and second position. The statistical model accounted for this through a position bias effect, though a more thorough design would include a variety of presentation formats to more effectively test this effect.

Predictions were made assuming that increasing comparability decreased target choice share through perceptual correlation; however, given that these correlations were not measured directly, this may not be the case. It would be interesting to use the paradigm from Experiment 2 to directly measure the correlations between options when the decoy is symmetrically dominated and the comparability is systematically manipulated.

Future research should generalize this paradigm to high-level preferential choice. For example, given three consumer products, does the comparability of two of them affect the choice for a third option? For example, given three cars available for purchase, two of which are hybrids and the other is a traditional combustion engine vehicle, is the correlation between the hybrids stronger than the hybrid-combustion correlation? This question is left for future research.

When perceptual decisions are difficult, and a decoy option is easily comparable to one of two viable options, this viable option is chosen less than it otherwise would have been chosen. The effect was predicted using the Thurstonian model developed earlier in this dissertation, and the empirical results support it, albeit with limitations. The empirical results of Experiment 5 demonstrate another form of context dependence, the main focus of this dissertation. Here, the context is not the choice set (which remains constant), but rather the presentation of the options. In this sense, the repulsion effect in this experiment is qualitatively similar to the repulsion effect of Spektor et al. (2018) and Experiment 2 of this dissertation.

CHAPTER 6

GENERAL DISCUSSION AND CONCLUSIONS

6.1 General Discussion

Decisions, even simple ones, may depend on context. Context effects occur when the relative choice for a given option varies with situational properties. In particular, the set of other available options affects choice. The attraction effect occurs when a decoy option boosts the choice share of a similar, superior target option. The repulsion effect, a reversal of the attraction effect, occurs when participants select the dissimilar competitor more than the target.

This dissertation has explored context dependence in choice, largely by studying the attraction and repulsion effects (Chapters 2-4) but also through studying context dependence induced by the presentation format of options in perceptual choice (Chapter 5).

In Chapter 2, a Thurstonian choice model was developed to measure the correlations between valuations in the attraction and repulsion effects. Experiment 1 demonstrated systematic discriminability issues in a 2AFC task in accordance with the Thurstonian model. In Experiment 2, participants provided both valuations (area judgments for rectangles) and choices (participants selected the largest rectangle from each ternary choice set). The Thurstonian model, conditional on parameters estimated from the judgment data, was used to make predictions for the choice data. When the ρ_{TD} parameter is greater than both the ρ_{TC} and ρ_{CD} parameters (as observed in the judgment data), the Thurstonian model can parsimoniously account for the repulsion effect without invoking higher-level decision processes (Experiment 2). Conditional on the observed parameters, the model cannot, however, account for the attraction effect, suggesting that higher-level decision processes may be required to explain this effect.

In Chapter 3, the Thurstonian model was generalized to best-worst choice. In best-worst choice, participants select their most and least preferred options from a choice set. Given the stimuli from Experiment 2 (a set of target, competitor, and decoy rectangles in a perceptual choice experiment), the model predicts a non-monotonic relationship between best and worst choice probabilities. Specifically, the model predicts that the target is both less likely to be chosen as worst, compared to the competitor, and also less likely to be chosen as best. This effect is not predicted by the maxdiff model, a commonly used model for best-worst choice, when independently distributed utilities are assumed. These results were verified with empirical data in Experiment 3.

Chapter 4 generalized the paradigm and model from Chapter 2 to preferential choice. In Experiment 4, on each experimental trial, participants saw three consumer products (e.g., microwave ovens, laptops) and assigned each product a selling price. In later experimental trials, they saw the same three options and selected the option they preferred the most. The results of Experiment 4 found similar correlational patterns as found in Experiment 2. The choice results also replicated previous researchers' choice results (Banerjee et al., 2024), albeit with limitations described in the text.

The model from Experiment 4 is able to qualitatively account for the repulsion effect, with one crucial limitation; the model accounts for the repulsion effect through the correlation between target and decoy evaluations, which causes the decoy to take choice shares away from the target. In preferential choice, unlike in perceptual choice, participants seldom if ever select the decoy. In the absence of these correlations, or if all correlations are equal (i.e., $\rho_{TD} = \rho_{TC} = \rho_{CD}$), the model will be unable to predict the effect[1]. Thus, the form of the repulsion effect in Experiment 4 may occur through higher-level decision processes, or a hitherto unidentified low-level process. Nonetheless, the demonstration of target-decoy correlations across choice environments is a novel result. Though other researchers have proposed correlations as a measure of the similarity between options in a choice set (Kamakura & Srivastava, 1984; Natenzon, 2019), these studies were the first (to the author's knowledge) to systematically measure these correlations using valuations, incorporate them into a Thurstonian choice model, and connect this model to choices obtained from the same experimental participants.

Chapter 5 demonstrated a different form of context dependence, where choice systematically varied based on option comparability. In Chapter 5, Given a *symmetrically dominated decoy* option, placing a focal target option in a nearby position, such that participants can more easily compare it to the decoy, caused a decrease in the target's choice share.

The results of this dissertation have important methodological considerations. In particular researchers should carefully consider the assumptions made when designing and analyzing experiments. For example, the experiments of Spektor et al. (2018) contain a crucial assumption: that because participants chose the target more often the decoy, the repulsion effect cannot be explained by the decoy taking choice shares from the competitor. The results of Experiment 2 provide strong evidence that the repulsion effect can, on the contrary, be entirely explained by target-decoy correlations. Chapter 3 showed that the independence assumption of the maxdiff choice model is incorrect in certain cases and that a failure to consider whether the stimuli of a given experiment can cause these violations may lead to incorrect conclusions about participants' preferences. The results of Chapter 5 show that the comparability, and

[1]See the diagonal line from Figure 2.6 for evidence of this theoretical result.

even order on screen, can systematically affect choice. Previous researchers have also argued in favor of this point (Evans et al., 2021; Hasan et al., 2025; Trueblood et al., 2022).

The approach outlined in Chapter 2 can be used for measurement purposes, specifically for measuring context effects in perceptual choice, after taking into account systematic covariance in perceptual noise. For example, the model showed that the repulsion effect, at least in the form demonstrated by Spektor et al. (2018), can be generated entirely from a process by which the decoy takes choice shares away from the target, rather than one in which the competitor is systematically chosen over the target. Spektor et al. (2022) considered the possibility that the decoy was taking choice shares from the target, and they tested this hypothesis by estimating the parameters of a multinomial processing tree model. They considered the parameters implausible and dismissed the decoy similarity account on these grounds. The current approach minimizes these concerns because the parameters of the Thurstonian choice model were estimated independently of the choice data.

Other researchers have introduced statistical measures of context effects. Berkowitsch et al. (2014) introduced the *Relative Share of the Target* (RST), defined as the proportion of times the target is chosen over the competitor. Katsimpokis et al. (2022) refined this measure by introducing *Absolute Share of the Target* (AST), which corrects RST by equally weighting each choice set (e.g., $[A, B, D_A]$), reducing bias. These approaches, while valuable in statistically testing for context effects, do not test process accounts of context effects. RST and AST only test for whether the target is systematically chosen more or less often than the competitor. Neither RST nor AST can, for example, test the hypothesis that the decoy systematically takes away choice shares from the target. RST and AST are still quite useful; indeed, these measures were used to test for context effects in Experiments 2 and 5. Their utility stems from their ability to provide reliable statistical tests of context effects.

The current work identifies a crucial measurement factor - namely, correlations between valuations in perceptual choice - which has hitherto been ignored by decision researchers, with a few exceptions (Kamakura & Srivastava, 1984; Natenzon, 2019). Future researchers should be able to use the current approach to identify data generating processes in various context effect data (e.g., compromise, similarity, phantom decoy effects).

Other researchers have developed numerous cognitive process models to account for context effects (Bergner et al., 2019; Bhatia, 2013; Noguchi & Stewart, 2018; Roe et al., 2001; Spektor et al., 2019b; Trueblood et al., 2014; Tversky, 1972; Tversky & Simonson, 1993; Usher & McClelland, 2004; Wollschläger & Diederich, 2012); these models are generally quite complex. Generally, researchers fit them to choice data by estimating free parameters from choice data. These parameters have strong psychological interpretations, such as loss aversion (Usher & McClelland, 2004), extremeness aversion (Trueblood et al., 2014), or lateral inhibition (Roe et al., 2001). However, success at identifying which model or mechanisms can best explain context effects has been mixed (Turner et al., 2018). Furthermore, there have been limited attempts to test the psychological reality of these models' parameters or make a priori predictions from model mechanisms[2]. The Thurstonian model is simpler and far more parsimonious than any of these cognitive process models. While it is almost certainly the case that additional mechanisms operate in the decision-maker's mind, we can also be fairly confident in the psychological reality of the mechanisms assumed by the Thurstonian model. For example, we have independent datasets across multiple paradigms supporting the case that $\rho_{TD} > \rho_{TC}$ and $\mu_T = \mu_C$.

The correlations observed here could perhaps be incorporated into other models. For example, Roe et al. (2001)'s MDFT model includes within-trial correlations as a

[2]See Cataldo and Cohen (2020), Tsetsos et al. (2010), Hotaling et al. (2010), and Trueblood et al. (2013) for a few notable exceptions.

mechanism for accounting for the similarity effect; the current experimental results could provide a benchmark for across-trial correlations, which the MDFT and similar models need to satisfy.

Furthermore, though other researchers have hypothesized the aforementioned pattern of correlations (Kamakura & Srivastava, 1984; Natenzon, 2019), the current work is the first (to the author's knowledge) to independently measure them using valuations and predict choice data, rather than simply estimating them freely from choice data.

This work also has implications for real-world decision-making. For example, the maxdiff model is frequently used in applied choice research (Beck & Rose, 2016; Cheung et al., 2016; Flynn & Marley, 2014; Flynn et al., 2007b; Mühlbacher et al., 2016), as a method of identifying consumers' preferences. Though the model may work well when there are no systematic patterns of correlations, as observed in the current stimuli, when these patterns do exist, the researcher may be led astray by use of the maxdiff model with independent utilities.

Applied researchers are also interested in using the multinomial probit (MNP) model to account for choice data (Train, 2009). Identifying the model parameters can be complex, and to some extent, the model is a "black box". However, the current work provides a method for estimating the multinomial probit model parameters separately from choice data, such that the researcher can be more confident in the parameter estimates. Furthermore, to the extent that the model predictions agree with the choice data (as in Experiment 2, triangle condition), the researcher can be confident that no additional data generation processes are required to explain empirical results. Such a test can be more powerful than model-fitting or traditional cross-validation approaches.

There are numerous directions that this research could take, beyond this dissertation. Future work could generalize the experimental and modeling paradigm of

Experiment 2 to other context effects (e.g., compromise, similarity, phantom decoy). Researchers should also measure correlations between option valuations at the individual participant level, which is currently limited by the quantity of data available.

Regarding best-worst choice, future work should explore models of best-worst choice that can be used when the independence assumption is violated. Exploration or development of such models is necessary but is beyond the scope of this dissertation. However, given the numerous applied uses for best-worst choice, this avenue of research may improve researchers' ability to identify participants' preferences.

Future work should also continue the line of research begun in Experiment 4. For example, research could collect both choice and pricing data from various binary and ternary sets.

In addition to the experimental modifications discussed earlier, future work in comparability should generalize the paradigm to various choice types. Additionally, the effect observed in Experiment 5 was quite small; practically speaking, this may have limited impact on actual choices. Future work should address the limitations of comparability in affecting choice.

6.2 Conclusions

This dissertation has identified various forms of context dependence, in both perceptual and preferential choice. The research has provided theoretical explanations for these results in the form of a mathematical model of choice. To ensure falsifiability, The model's predictions were tested on out-of-sample data, to varying degrees of success. This work will further the study of context effects and decision-making in general.

APPENDIX A

BAYESIAN LOGISTIC REGRESSION MODEL OF 2AFC DISCRIMINABILITY FROM EXPERIMENT 1

The 2AFC data from Experiment 1 was analyzed with Bayesian Hierarchical Logistic Regression.

Recall that in this experiment, participants were presented with three stimuli (target, competitor, and decoy rectangles). They were then asked to select the largest rectangle from a subset of two options. There were three trial types: target/competitor (TC), target/decoy (TD), and competitor/decoy (CD). As discussed in the main text, all TC trials and all $TDD = 0\%$ trials were removed from the main analyses.

A.1 Model Details

The model predicts the probability of discriminating the target/competitor from the decoy option. According to the model, discrimination D for participant i on trial j is:

$$D_{ij} \sim Bernoulli(\theta_{ij}) \tag{A.1}$$

To compute θ_{ij}, a parameter η_{ij} is first predicted from a linear combination of the relevant variables and model coefficients (slopes).

$$
\begin{aligned}
\eta_{ij} = (\beta_0 + S_{0_i}) + (\beta_{\text{or}} + S_{\text{or}_i}) \times \boldsymbol{or}_{ij} \\
(\beta_{\text{horiz}} + S_{\text{horiz}_i}) \times \boldsymbol{horiz}_{ij} + \\
(\beta_{\text{TD}} + S_{\text{TD}_i}) \times \boldsymbol{TD}_{ij} + \\
(\beta_{\text{TDD5}} + S_{\text{TDD5}_i}) \times \boldsymbol{TDD5}_{ij} + \\
(\beta_{\text{TDD9}} + S_{\text{TDD9}_i}) \times \boldsymbol{TDD9}_{ij} + \\
(\beta_{\text{TDD14}} + S_{\text{TDD14}_i}) \times \boldsymbol{TDD14}_{ij} + \\
(\beta_{\text{TDD5xTD}} + S_{\text{TDD5xTD}_i}) \times \boldsymbol{TDD5}_{ij} \times \boldsymbol{TD}_{ij} + \\
(\beta_{\text{TDD9xTD}} + S_{\text{TDD9xTD}_i}) \times \boldsymbol{TDD9}_{ij} \times \boldsymbol{TD}_{ij} + \\
(\beta_{\text{TDD14xTD}} + S_{\text{TDD14xTD}_i}) \times \boldsymbol{TDD14}_{ij} \times \boldsymbol{TD}_{ij}
\end{aligned}
\tag{A.2}
$$

All β terms are fixed effects, and all S terms are random (participant) effects. β_{or} is the fixed effect of orientation, where or_{ij} is a dummy variable which $= 0$ if the target and decoy are taller than wide and $= 1$ if the target and decoy are wider than tall. β_{horiz} is the fixed effect of the horizontal display, where $\boldsymbol{horiz}_{ij}$ is a dummy variable which $= 0$ for triangle display trials and $= 1$ for horizontal display trials. β_{TD} is the fixed effect of comparison, where $\boldsymbol{TD}_{ij}$=0 for CD trials and $TD_{ij} = 1$ for TD trials. The $\boldsymbol{TDD}$ variable has 4 levels (2%, 5%, 9%, and 14%), so there are three dummy variables ($\boldsymbol{TDD5}$, $\boldsymbol{TDD9}$, and $\boldsymbol{TDD14}$) with 2% being the reference level for $\boldsymbol{TDD}$. The interaction captures the additional boost / decrement to TD over CD trials at each level of $\boldsymbol{TDD}$.

η_{ij} is then transformed to the probability scale using the logit function:

$$
\theta_{ij} = \frac{1}{1 + e^{-\eta_{ij}}} \tag{A.3}
$$

A.2 Prior Distributions on Parameters

- $\beta_0 \sim \mathcal{N}(0, 5)$

- $\beta_{or} \sim \mathcal{N}(0, 5)$
- $\beta_{horiz} \sim \mathcal{N}(0, 5)$
- $\beta_{TD} \sim \mathcal{N}(0, 5)$
- $\beta_{TDD5} \sim \mathcal{N}(0, 5)$
- $\beta_{TDD9} \sim \mathcal{N}(0, 5)$
- $\beta_{TDD14} \sim \mathcal{N}(0, 5)$
- $\beta_{TDD5xTD} \sim \mathcal{N}(0, 2.5)$
- $\beta_{TDD9xTD} \sim \mathcal{N}(0, 2.5)$
- $\beta_{TDD14xTD} \sim \mathcal{N}(0, 2.5)$
- $S_{0_i} \sim \mathcal{N}(0, \sigma_{S_0})$
- $S_{or_i} \sim \mathcal{N}(0, \sigma_S)$
- $S_{horiz_i} \sim \mathcal{N}(0, \sigma_S)$
- $S_{TD_i} \sim \mathcal{N}(0, \sigma_S)$
- $S_{TDD5_i} \sim \mathcal{N}(0, \sigma_S)$
- $S_{TDD9_i} \sim \mathcal{N}(0, \sigma_S)$
- $S_{TDD14_i} \sim \mathcal{N}(0, \sigma_S)$
- $S_{TDD5xTD_i} \sim \mathcal{N}(0, \sigma_S)$
- $S_{TDD9xTD_i} \sim \mathcal{N}(0, \sigma_S)$
- $S_{TDD14xTD_i} \sim \mathcal{N}(0, \sigma_S)$
- $\sigma_{S_0} \sim \mathrm{LogNormal}(0, 2.5)$

- $\sigma_S \sim \text{LogNormal}(0, 2.5)$

Note that the model assumes equal variance for all random effect distributions aside from the random intercepts.

A.3 Modeling Results

The model was coded using the Stan programming language (Carpenter et al., 2017) and implemented with the RStan package (Stan Development Team, n.d.). The sampler ran 5 chains, each for 2500 iterations. Posterior diagnostics indicated that the sampler converged.

A.3.1 Parameter estimates.

Table A.1 shows parameter estimates, including means and 95% credible intervals.

Parameter	M	SD	CI low	CI high
β_0	0.44	0.06	0.32	0.57
β_{or}	-0.54	0.06	-0.66	-0.43
β_{horiz}	0.23	0.06	0.12	0.34
β_{TD}	0.17	0.08	0.01	0.33
β_{TDD5}	0.40	0.08	0.23	0.56
β_{TDD9}	0.81	0.09	0.64	0.98
β_{TDD14}	1.45	0.10	1.25	1.64
$\beta_{TDD5xTD}$	0.14	0.12	-0.09	0.36
$\beta_{TDD9xTD}$	0.61	0.13	0.36	0.85
$\beta_{TDD14xTD}$	0.79	0.15	0.50	1.10
σ_{S_0}	0.20	0.04	0.12	0.29
σ_S	0.34	0.03	0.29	0.39

Table A.1. Parameter estimates for Bayesian Hierarchical Logistic Regression from Experiment 1 Data, including means, standard deviations, and 95% Credible Intervals.

Inference is made by examining the posterior distributions of the fixed effect parameters (i.e., all β values). First, consider the main effects. The intercept β_0 was above 0, $M = 0.44, 95\% CI[0.32, 0.57]$, indicating that participants could perform the discrimination task, even at the lowest TDD level. There was also a main effect of orientation, $M = -0.54, 95\% CI[-0.66, -0.43]$, indicating that participants performed worse when the target and decoy were oriented wide than when they were oriented tall. The main effect of horizontal, $M = 0.23, 95\% CI[0.12, 0.34]$, indicates that participants performed better in the horizontal display condition than in the triangle display condition. The main effect of TD comparison was above 0, $M = 0.17, 95\% CI[0.01, 0.33]$, indicating participants performed better in target-decoy trials than in competitor decoy trials.

Crucially, there was an interaction between comparison (TC vs TD) and TDD. The effect of TD at $TDD = 5\%$ was not reliably above 0, $M = 0.14, 95\% CI[-0.09, 0.36]$. However, the effect of TD at $TDD = 9\%$ was above 0, $M = 0.61, 95\% CI[0.36, 0.85]$, even after accounting for the main effect of TD comparison. The effect of TD at $TDD = 14\%$ was also above 0, $M = 0.79, 95\% CI[0.50, 1.10]$.

APPENDIX B

BAYESIAN HIERARCHICAL MODELING OF CIRCLE JUDGMENT DATA FROM EXPERIMENT 2

The circle judgment data were analyzed with the multivariate Thurstonian choice model first presented in Chapter 2.

B.1 Model Details

The model assumes that, for participant i on trial j, the vector of perceived areas $\boldsymbol{X}_{ij}$ is sampled from a multivariate normal distribution with parameters $\boldsymbol{\mu}_{ij}$ and $\boldsymbol{\Sigma}$. That is,

$$\boldsymbol{X}_{ij} \sim \boldsymbol{N}(\boldsymbol{\mu}_{ij}, \boldsymbol{\Sigma}) \tag{B.1}$$

Using Bayesian statistical modeling, the parameters $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ were estimated. Note that $\boldsymbol{\mu}$ is allowed to vary systematically over trials and participants, but $\boldsymbol{\Sigma}$ remains constant. $\boldsymbol{\mu}$ is estimated through hierarchical regression while the components of $\boldsymbol{\Sigma}$ (i.e., σ_T, σ_C, σ_D, ρ_{TD}, ρ_{TC}, ρ_{CD}) were estimated freely.

The model was fit separately for the triangle and horizontal condition. First, the computation of $\boldsymbol{\mu}$ is described, followed by the computation of $\boldsymbol{\Sigma}$. The prior distributions on each parameter separately for each of these components are shown, followed by an explanation of the modeling procedure and results. Note that the model predicts mean-centered log-transformed estimated area.

B.1.1 $\boldsymbol{\mu}$ Parameterization

The model predicts the mean area $\boldsymbol{\mu}_{ijk}$ for the ith participant on the jth trial for the kth stimulus. There were $k = 3$ stimuli on each trial. If $k = 1$, the stimulus is the target; If $k = 2$, the stimulus is the competitor; If $k = 3$, the stimulus is the decoy. Thus, $\boldsymbol{\mu}$ can be broken down into μ_{ij1} (target), μ_{ij2} (competitor), and μ_{ij3} (decoy). Note that some parameters are common to all stimuli (e.g., the effects of diagonal or orientation), while others are common only to a particular option (e.g., the effect of competitor vs. decoy vs. target).

μ_{ij1} is computed as:

$$\begin{aligned} \mu_{ij1} = (S_{0_i} + \beta_0) + \beta_{\text{or}} \times \boldsymbol{or}_{ij1} + \beta_{\text{diag2}} \times \boldsymbol{diag2}_{ij} + \\ \beta_{\text{diag3}} \times \boldsymbol{diag3}_{ij} + \beta_{\text{TDD5}} \times \boldsymbol{TDD5}_{ij} + \\ \beta_{\text{TDD9}} \times \boldsymbol{TDD9}_{ij} + \beta_{\text{TDD14}} \times \boldsymbol{TDD14}_{ij} \end{aligned} \tag{B.2}$$

S_{0_i} is a random intercept for participant i. β_0 is the fixed intercept. β_{or} is the fixed effect of orientation, where $\boldsymbol{or}_{ij1}$ is a dummy variable which $= 0$ if the target is taller than wide and $= 1$ if the target is wider than tall. β_{diag2} is the fixed effect of the middle diagonal, where $\boldsymbol{diag2}_{ij1} = 1$ if the all stimuli on the trial fall on the middle diagonal and $= 0$ otherwise. β_{diag3} is the fixed effect of the upper diagonal, where $\boldsymbol{diag3}_{ij1} = 1$ if all stimuli on the trial fall on the upper diagonal and $= 0$ otherwise. β_{TDD5} is the fixed effect of TDD 5, and $TDD5_{ij1}$ is a dummy variable which $= 1$ if $TDD = 5\%$ and $= 0$ otherwise. β_{TDD9} is the fixed effect of TDD 9, and $TDD9_{ij1}$ is a dummy variable which $= 1$ if $TDD = 9\%$ and $= 0$ otherwise. β_{TDD14} is the fixed effect of TDD 14, and $TDD14_{ij1}$ is a dummy variable which $= 1$ if $TDD = 14\%$ and $= 0$ otherwise.

μ_{ij2} is computed as:

$$\begin{aligned}\mu_{ij2} = (S_{0_i} + \beta_0) + \beta_{\text{or}} \times \boldsymbol{or}_{ij2} + \beta_{\text{diag2}} \times \boldsymbol{diag}2_{ij} + \\ \beta_{\text{diag3}} \times \boldsymbol{diag}3_{ij} + \beta_{\text{TDD5}} \times \boldsymbol{TDD}5_{ij} + \\ \beta_{\text{TDD9}} \times \boldsymbol{TDD}9_{ij} + \beta_{\text{TDD14}} \times \boldsymbol{TDD}14_{ij} + \beta_{\text{comp}}\end{aligned} \tag{B.3}$$

S_{0_i} is a random intercept for participant i. β_0 is the fixed intercept. β_{or} is the fixed effect of orientation, where $\boldsymbol{or}_{ij2}$ is a dummy variable which = 0 if the competitor is taller than wide and = 1 if the competitor is wider than tall. β_{diag2} is the fixed effect of the middle diagonal, where $\boldsymbol{diag2}_{ij2} = 1$ if the all stimuli on the trial fall on the middle diagonal and = 0 otherwise. β_{diag3} is the fixed effect of the upper diagonal, where $\boldsymbol{diag3}_{ij2} = 1$ if all stimuli on the trial fall on the upper diagonal and = 0 otherwise. β_{TDD5} is the fixed effect of TDD 5, and $\boldsymbol{TDD5}_{ij2}$ is a dummy variable which = 1 if $TDD = 5\%$ and = 0 otherwise. β_{TDD9} is the fixed effect of TDD 9, and $TDD9_{ij2}$ is a dummy variable which = 1 if $TDD = 9\%$ and = 0 otherwise. β_{TDD14} is the fixed effect of TDD 14, and $TDD14_{ij2}$ is a dummy variable which = 1 if $TDD = 14\%$ and = 0 otherwise. β_{comp} is a parameter that reflects the possibility of bias for the competitor.

μ_{ij3} is computed as:

$$\begin{aligned}\mu_{ij3} = (S_{0_i} + \beta_0) + \beta_{\text{or}} \times \boldsymbol{or}_{ij3} + \beta_{\text{diag2}} \times \boldsymbol{diag}2_{ij} + \\ \beta_{\text{diag3}} \times \boldsymbol{diag}3_{ij} + \beta_{TDD2D} \times \boldsymbol{TDD2D}_{ij} + \\ (\beta_{\text{TDD5}} + \beta_{\text{TDD5}D}) \times \boldsymbol{TDD5D}_{ij} + \\ (\beta_{\text{TDD9}} + \beta_{\text{TDD9}D}) \times \boldsymbol{TDD9D}_{ij} + \\ (\beta_{\text{TDD14}} + \beta_{\text{TDD14}D}) \times \boldsymbol{TDD14D}_{ij}\end{aligned} \tag{B.4}$$

S_{0_i} is a random intercept for participant i. β_0 is the fixed intercept. β_{or} is the fixed effect of orientation, where $\boldsymbol{or}_{ij3}$ is a dummy variable which = 0 if the decoy is taller than wide and = 1 if the decoy is wider than tall. β_{diag2} is the fixed effect of the middle diagonal, where $\boldsymbol{diag2}_{ij3} = 1$ if the all stimuli on the trial fall on the middle diagonal and = 0 otherwise. β_{diag3} is the fixed effect of the upper diagonal, where

$\boldsymbol{diag3}_{ij3} = 1$ if all stimuli on the trial fall on the upper diagonal and $= 0$ otherwise. β_{TDD2D} is the fixed effect of TDD=2, and $TDD2D_{ij3}$ is a dummy variable which $= 1$ if $TDD = 2\%$ for the decoy and $= 0$ otherwise (including for target and competitor regardless). β_{TDD5D} is the fixed effect of TDD=5, and $TDD5D_{ij3}$ is a dummy variable which $= 1$ if $TDD = 5\%$ for the decoy and $= 0$ otherwise. β_{TDD9D} is the fixed effect of TDD 9 for the decoy, and $TDD9D_{ij3}$ is a dummy variable which $= 1$ if $TDD = 9\%$ and $= 0$ otherwise (including for target and competitor regardless). β_{TDD14D} is the fixed effect of TDD 14 for the decoy, and $TDD14D_{ij3}$ is a dummy variable which $= 1$ if $TDD = 14\%$ and $= 0$ otherwise (including for target and competitor regardless).

Note that there is a common set of parameters for each level of TDD and additional set of parameters for each level of TDD that only apply to the decoy. In the data, it was clear that participants often adjusted the target and competitor relative to the decoy. In other words, even though the physical size of both target and competitor remains constant across TDD, participants' *judgment* of their size varied with TDD. The inclusion of a separate set of parameters for TDD that only apply to the decoy allows for a "deflection" of the decoy size, relative to target and competitor size.

Note the following reference points for the variables:

- TDD: 2%
- Orientation: taller than wide
- Diagonal: lower
- Stimulus: target

The β_0 parameter captures the fixed effect of a tall target on the lower diagonal at 2% TDD, and all other parameters reflect deflections from this stimulus.

B.1.1.1 Prior Distributions on Parameters

Below are shown the following prior distributions on each parameter relevant to $\boldsymbol{\mu}$:

- $\beta_0 \sim \mathcal{N}(0, 5)$
- $\beta_{or} \sim \mathcal{N}(0, 5)$
- $\beta_{diag2} \sim \mathcal{N}(0, 5)$
- $\beta_{diag3} \sim \mathcal{N}(0, 5)$
- $\beta_{TDD5} \sim \mathcal{N}(0, 5)$
- $\beta_{TDD9} \sim \mathcal{N}(0, 5)$
- $\beta_{TDD14} \sim \mathcal{N}(0, 5)$
- $\beta_{TDD2D} \sim \mathcal{N}(0, 5)$
- $\beta_{TDD5D} \sim \mathcal{N}(0, 5)$
- $\beta_{TDD9D} \sim \mathcal{N}(0, 5)$
- $\beta_{TDD14D} \sim \mathcal{N}(0, 5)$
- $\beta_{comp} \sim \mathcal{N}(0, 5)$
- $S_{0_i} \sim \mathcal{N}(0, \sigma_{S_0})$
- $\sigma_{S_0} \sim$ Half-Cauchy$(0, 2.5)$

B.2 Σ Parameterization

Σ is a positive semi-definite 3×3 covariance matrix computed as:

$$\boldsymbol{\Sigma} = \boldsymbol{S}\boldsymbol{\Omega}\boldsymbol{S} \tag{B.5}$$

where $\boldsymbol{S}$ is a diagonal matrix consisting of:

$$\begin{pmatrix} \sigma_T & 0 & 0 \\ 0 & \sigma_C & 0 \\ 0 & 0 & \sigma_D \end{pmatrix} \tag{B.6}$$

and $\boldsymbol{\Omega}$ is a correlation matrix:

$$\begin{pmatrix} 1 & \rho_{TC} & \rho_{TD} \\ \rho_{TC} & 1 & \rho_{CD} \\ \rho_{TD} & \rho_{CD} & 1 \end{pmatrix} \tag{B.7}$$

The three standard deviation parameters σ_T, σ_C, and σ_D were freely estimated.

When estimating $\boldsymbol{\Omega}$, the LKJ distribution (Lewandowski et al., 2009) was used to set priors on the Cholesky factorization of the correlation matrix Ω. This was done to ensure that the resulting variance-covariance matrix $\boldsymbol{\Sigma}$ is positive semi-definite, a requirement of the multivariate Gaussian distribution. The critical inferences, however, are performed on the off-diagonal elements ρ_{TC}, ρ_{TD}, ρ_{CD} in each display condition. Priors were set on the σ parameters using the Half-Cauchy distribution (Gelman, 2006).

B.2.1 Prior Distributions on Parameters

Below are shown the following prior distributions on each parameter relevant to $\boldsymbol{\Sigma}$.

- $\sigma_T \sim \text{HalfCauchy}(0, 2.5)$

- $\sigma_C \sim \text{HalfCauchy}(0, 2.5)$

- $\sigma_D \sim \text{HalfCauchy}(0, 2.5)$

- $\mathbf{\Omega} \sim \text{LKJCorr}(\eta = 1)$

B.3 Modeling Results

The model was implemented using the Stan programming language (Carpenter et al., 2017) using the cmdstanr interface (Gabry et al., 2024) in R .

The model ran for 2500 iterations (not including warm-up) with 4 chains for each display condition. Posterior diagnostics indicated that the sampler converged in each condition.

Below are parameter estimates for each display condition and relevant parameter. The estimates of the participant effects S_{0_i} are excluded for brevity. Estimates are rounded to two or three decimal places, depending on the size of the parameter.

The posterior estimates indicate that $\rho_{TD} > \rho_{TC} \approx \rho_{CD}$ in each display condition, in accordance with the predictions. Furthermore, the absolute values of all ρ values are greater in the horizontal condition than in the triangle condition, suggesting that the horizontal condition better facilitates comparisons.

The β estimates are generally as expected. The β_{diag} estimates show that participants increased their area estimations with the absolute size of the stimuli. Participants also decreased the size of decoy estimations as TDD increased. They also, to some extent, decreased the size of target and competitor estimations as TDD increased (captured by the β_{TDD5}, β_{TDD9}, and β_{TDD14}) parameters, indicating that participants adjusted the target and competitor relative to the decoy.

The β_{or} estimates indicated that participants judged tall stimuli larger than wide stimuli in the triangle condition. This effect is quite small, but is nonetheless present in the parameter estimates. This is opposite to what was found in the choice data, where participants selected the wide rectangle more than the tall rectangle. Participants had a slight bias to judge wide stimuli larger than tall stimuli in the horizontal condition, though the 95% HDI included 0.

Participants also judged the competitor slightly larger than the target, particularly in the triangle condition, although this effect is quite small. This effect was indeed too small to show differences in any single TDD level (see Figure 2.11).

Display Condition	Parameter	M	SD	HDI lower	HDI upper
Horizontal	β_0	-0.41	0.02	-0.44	-0.38
	β_{or}	0.003	0.002	-0.001	0.007
	β_{diag2}	0.47	0.01	0.46	0.48
	β_{diag3}	0.80	0.01	0.79	0.81
	β_{TDD5}	-0.005	0.01	-0.017	0.007
	β_{TDD9}	-0.007	0.01	-0.019	0.005
	β_{TDD14}	-0.01	0.01	-0.02	-0.0005
	β_{TDD2D}	-0.006	0.004	-0.013	0.001
	β_{TDD5D}	-0.01	0.004	-0.016	-0.003
	β_{TDD9D}	-0.04	0.004	-0.05	-0.04
	β_{TDD14D}	-0.08	0.004	-0.09	-0.07
	β_{comp}	0.003	0.002	-0.002	0.007
	σ_{S_0}	0.19	0.01	0.17	0.21
	σ_T	0.337	0.002	0.334	0.340
	σ_C	0.341	0.002	0.338	0.345
	σ_D	0.337	0.002	0.333	0.340
	ρ_{TC}	0.575	0.005	0.565	0.584
	ρ_{TD}	0.710	0.004	0.703	0.716
	ρ_{CD}	0.575	0.005	0.565	0.584
Triangle	β_0	-0.40	0.01	-0.42	-0.38
	β_{or}	-0.006	0.002	-0.01	-0.002
	β_{diag2}	0.47	0.005	0.455	0.474
	β_{diag3}	0.81	0.005	0.80	0.82
	β_{TDD5}	-0.01	0.006	-0.03	0.0003
	β_{TDD9}	-0.02	0.006	-0.03	-0.008
	β_{TDD14}	-0.03	0.006	-0.04	-0.01
	β_{TDD2D}	-0.0172	0.004	-0.024	-0.01
	β_{TDD5D}	-0.0167	0.004	-0.0237	-0.01
	β_{TDD9D}	-0.03	0.004	-0.037	-0.02
	β_{TDD14D}	-0.05	0.004	-0.06	-0.05
	β_{comp}	0.005	0.002	0.0001	0.009
	σ_{S_0}	0.15	0.01	0.14	0.17
	σ_T	0.335	0.002	0.332	0.338
	σ_C	0.338	0.002	0.335	0.341
	σ_D	0.335	0.002	0.331	0.338
	ρ_{TC}	0.541	0.005	0.531	0.551
	ρ_{TD}	0.675	0.004	0.667	0.682
	ρ_{CD}	0.533	0.005	0.523	0.543

Table B.1. Parameter estimates for Bayesian Hierarchical Thurstonian Model from Experiment 2 Circle Phase Data, including means, standard deviations, and 95% Credible Intervals.

APPENDIX C

INFERENTIAL STATISTICS FOR EXPERIMENT 2 CHOICE DATA

Following Katsimpokis et al. (2022), I performed inference on *Absolute Share of the Target*, a variant of RST that corrects for a bias in RST. AST is an unweighted average of the target choice proportion from each choice set. Here, AST is computed as:

$$AST = 0.5 \times \left(\frac{P(H|H,W,D_H)}{P(H|H,W,D_H) + P(W|H,W,D_H)} + \frac{P(W|H,W,D_W)}{P(W|H,W,D_W) + P(H|H,W,D_W)}\right) \tag{C.1}$$

AST was computed for each participant in each display condition at each level of TDD. First are the analyses from the triangle condition followed by those from the horizontal condition.

C.1 Triangle Condition Analysis

A one-way within-groups ANOVA testing the effect of TDD on AST in the triangle condition was significant, $F(3, 636) = 79.97$,$p < .001$.

I then performed a follow-up one-sample t-test on AST at each level of TDD, using the within-participants error correction from Cousineau and O'Brien (2014) and comparing the mean AST value to the null value .5. I compared each p-value to a Bonferroni-corrected α level of $\alpha = \frac{.05}{4} = .0125$.

The AST value was significantly different from .5 at TDD=2%, $t(212) = -18.4, p < .001$, $M = .34$, 95% CI[.33, .36], indicating a repulsion effect.

The AST value was significantly different from .5 at TDD=5%, $t(212) = -13.2, p < .001$, $M = .39$, 95% CI[.38, .41], indicating a repulsion effect.

The AST value was significantly different from .5 at TDD=9%, $t(212) = -7.45, p < .001$, $M = .43$, 95% CI[.42, .45], indicating a repulsion effect.

The AST value was significantly different from .5 at TDD=14%, $t(212) = -2.34, p = .002$, $M = .48$, 95% CI[.46, .49], indicating a slight repulsion effect. Note that the mean $P(T) > P(C)$ in Figure 2.13; however, those values are not equally weighted.

C.2 Horizontal Condition Analysis

A one-way within-groups ANOVA testing the effect of TDD on AST in the horizontal condition was significant, $F(3, 618) = 176.10$,$p < .001$.

I then performed a follow-up one-sample t-test on AST at each level of TDD, using the within-participants error correction from Cousineau and O'Brien (2014) and comparing the mean AST value to the null value .5. Each p-value was compared to a Bonferroni-corrected α level of $\alpha = \frac{.05}{4} = .0125$.

The AST value was significantly different from .5 at TDD=2%, $t(206) = -15.6, p < .001$, $M = .34$, 95% CI[.33, .36], indicating a repulsion effect.

The AST value was significantly different from .5 at TDD=5%, $t(206) = -8.12, p < .001$, $M = .41$, 95% CI[.39, .43], indicating a repulsion effect.

The AST value was not significantly different from .5 at TDD=9%, $t(206) = -0.04, p = .10$, $M = .50$, 95% CI[.48, .52], indicating a null effect.

The AST value was significantly different from .5 at TDD=14%, $t(206) = 5.00, p < .001$, $M = .56$, 95% CI[.54, 58], indicating an attraction effect.

Mean AST values for each TDD level in each display condition are plotted in Figure C.1.

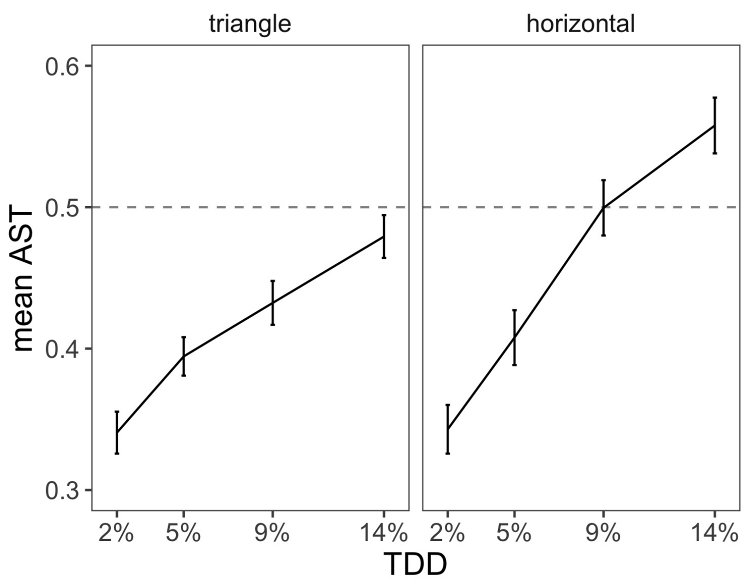

Figure C.1. Mean AST values for each display condition and TDD level from Experiment 2. Error bars are 95% CIs with the within-participants error correction from Cousineau and O'Brien (2014).

APPENDIX D

MAXDIFF MODELING FROM EXPERIMENT 3

According to the maxdiff model of best-worst choice (Marley & Louviere, 2005), the probability $BW_K(x, y)$ of selecting option x as best and y as worst from choice set K is computed as:

$$BW_K(x, y) = \frac{e^{u_x - u_y}}{\sum_{\substack{p,q \in K \\ p \neq q}} e^{u_p - u_q}} \tag{D.1}$$

where u_i is the utility of option i.

Below are the details of the maxdiff modeling, as applied to the data from Experiment 3.

D.1 Model Details

Following typical approaches in the choice modeling literature, the utility of each option in a choice set was estimated using linear regression. There was no intercept.

According to the implementation of the model, the utility U_{ijk} for participant i on trial j and option k is computed as:

$$\begin{aligned} U_{ijk} = {} & \beta_{or} \times \boldsymbol{or}_{ijk} + \beta_{\text{TDD5}} \times \text{TDD5}_{ij} + \beta_{\text{TDD9}} \times \boldsymbol{TDD}9_{ij} + \beta_{\text{TDD14}} \times \boldsymbol{TDD}14_{ij} + \\ & (\beta_{comp} + S_{comp_i}) \times \boldsymbol{comp}_{ijk} + (\beta_{decoy} + S_{decoy_i}) \times \boldsymbol{decoy}_{ijk} \end{aligned} \tag{D.2}$$

As in the modeling of Experiment 2, there was an effect of option (i.e., target, competitor, decoy) where β_{comp} and S_{comp_i} are the fixed and random effects for the competitor stimulus, respectively, and $comp_{ijk}$ is a dummy variable which equals 1 if the stimulus is a competitor and 0 otherwise. Similarly, β_{decoy} and S_{decoy_i} are the fixed and random effects for the decoy stimulus, respectively, and $comp_{ijk}$ is a dummy variable which equals 1 if the stimulus is a decoy and 0 otherwise. Note that the reference level is the target. These analyses collapse over diagonal.

According to the model, the probability a participant i on trial j, given set K, selects option k as best and l as worst, $k \neq l$, is computed as:

$$BW_{ij_K}(k,l) = \frac{e^{U_{ijk}-U_{ijl}}}{\sum_{\substack{p,q\in K \\ p\neq q}} e^{u_p - u_q}} \tag{D.3}$$

The vector $\boldsymbol{\theta_{ij}}$ is a vector of all possible best-worst choices for participant i and trial j, where $\sum \boldsymbol{\theta_{ij}} = 1$.

The count vector of all possible BW choice pairs $\boldsymbol{C_{ij}}$ is distributed:

$$\boldsymbol{C}_{ij} \sim \text{Multinomial}(\boldsymbol{\theta}_{ij}) \tag{D.4}$$

Below are the prior distributions on all parameters.

D.2 Prior Distributions on Parameters

- $\beta_{or} \sim \mathcal{N}(0, 5)$
- $\beta_{TDD5} \sim \mathcal{N}(0, 5)$
- $\beta_{TDD9} \sim \mathcal{N}(0, 5)$
- $\beta_{TDD14} \sim \mathcal{N}(0, 5)$
- $\beta_{comp} \sim \mathcal{N}(0, 5)$

- $\beta_{decoy} \sim \mathcal{N}(0, 5)$
- $S_{comp_i} \sim \mathcal{N}(0, \sigma_{S_comp})$
- $S_{decoy_i} \sim \mathcal{N}(0, \sigma_{S_decoy})$
- $\sigma_{S_comp} \sim$ Half-Cauchy$(0, 2.5)$
- $\sigma_{S_decoy} \sim$ Half-Cauchy$(0, 2.5)$

D.3 Parameter Estimates

Table D.1 shows parameter estimates, including means and 95% credible intervals.

Parameter	M	SD	CI low	CI high
β_{or}	0.27	0.01	0.26	0.28
β_{TDD5}	0.27	0.01	0.24	0.29
β_{TDD9}	0.64	0.01	0.61	0.67
β_{TDD14}	1.05	0.02	1.02	1.08
β_{comp}	-0.03	0.01	-0.05	-0.01
β_{decoy}	-0.25	0.02	-0.28	-0.21
σ_{comp}	0.14	0.01	0.12	0.16
σ_{decoy}	0.29	0.01	0.27	0.32

Table D.1. Parameter estimates for maxdiff modeling from Experiment 3, including means, SDs, and 95% Credible Intervals.

APPENDIX E

BAYESIAN MODELING OF PRICE DATA FROM EXPERIMENT 4

The pricing data from Experiment 4 were modeled using the Thurstonian model from Experiment 2.

E.1 Thurstonian Price Model

The model assumed that on each trial i, a vector of prices $\boldsymbol{X}_i$ is drawn from a multivariate normal distribution:

$$\boldsymbol{X}_i \sim \boldsymbol{N}(\boldsymbol{\mu}_{jk}, \boldsymbol{\Sigma}_k) \tag{E.1}$$

where j is the product category (washing machines, laptops, televisions, microwave ovens) and k is the trial type (attraction, repulsion).

$\boldsymbol{\mu}_{jk}$ is the column vector:

$$\begin{pmatrix} \mu_{T_j k} \\ \mu_{C_j k} \\ \mu_{D_j k} \end{pmatrix} \tag{E.2}$$

and $\boldsymbol{\Sigma}_k$ is a 3×3 positive semi-definite variance-covariance matrix:

$$\boldsymbol{\Sigma}_k = \boldsymbol{S}\boldsymbol{\Omega}_k\boldsymbol{S} \tag{E.3}$$

where $\boldsymbol{S}$ is a diagonal matrix consisting of:

$$\begin{pmatrix} \sigma_T & 0 & 0 \\ 0 & \sigma_C & 0 \\ 0 & 0 & \sigma_D \end{pmatrix} \tag{E.4}$$

with σ_T, σ_C, σ_D being the standard deviations for target, competitor, and decoy, respectively. $\boldsymbol{\Omega}_k$ is a correlation matrix:

$$\begin{pmatrix} 1 & \rho_{TC_k} & \rho_{TD_k} \\ \rho_{TC_k} & 1 & \rho_{CD_k} \\ \rho_{TD_k} & \rho_{CD_k} & 1 \end{pmatrix} \tag{E.5}$$

with ρ_{TD_1}, for example, indicating the population-level correlation between target and decoy valuations in the attraction condition.

This model has 24 free parameters, relatively few compared to the several hundred from Experiment 2.

There were no a priori predictions about the size or even direction of the price differences here. All $\boldsymbol{\mu}$ parameters were freely estimated rather than estimated through linear regression as in Experiment 2.

Prior to model estimation, all prices were normalized within-participants.

E.2 Prior Distributions on all Parameters

- $\boldsymbol{\mu}_{jk} \sim \mathcal{N}(0, 1)$

- $\sigma_T \sim \text{Half-Cauchy}(0, 2.5)$

- $\sigma_C \sim \text{Half-Cauchy}(0, 2.5)$

- $\sigma_D \sim \text{Half-Cauchy}(0, 2.5)$

- $\boldsymbol{\Omega}_k \sim \text{LKJCorr}(\eta = 0.5)$

E.3 Parameter Estimates

For brevity, the $\boldsymbol{\mu}$ and σ estimates are omitted, as mean prices collapsed across product category are plotted in Figure 4.5. ρ estimates are shown below.

Trial Type	Parameter	*M*	*SD*	HDI lower	HDI upper
Attraction	ρ_{TC}	.87	0.005	.86	.88
	ρ_{TD}	.87	0.005	.86	.88
	ρ_{CD}	.83	0.007	.81	.84
Repulsion	ρ_{TC}	.77	0.008	.75	.79
	ρ_{TD}	.87	0.005	.86	.88
	ρ_{CD}	.69	0.011	.67	.72

Table E.1. ρ Parameter estimates for ρ parameters from the Bayesian Hierarchical Thurstonian Model from Experiment 4 Pricing Data, including means, standard deviations, and 95% Credible Intervals.

APPENDIX F

BAYESIAN MODELING OF CHOICE DATA FROM EXPERIMENT 4

The choice data from Experiment 4 were analyzed with a Dirichlet-Multinomial model. The Dirichlet distribution is a generalization of the Beta distribution to more than two dimensions. Here the Dirichlet distribution is used to model the variability in ternary choice proportions for the attraction and repulsion trials from Experiment 4.

This analysis collapsed over participants. This is a limitation, particularly given recent concerns about participant-level variability in context effects (Liew et al., 2016; Trueblood et al., 2015). However, given that the main goal of Experiment 4 was to measure correlations in pricing, and that each participant made relatively few choices, this analysis was performed on aggregate choice data.

F.1 Dirichlet-Multinomial Choice Model

According to the model, the vector $\boldsymbol{C_{ijkl}}$ of target, competitor, and decoy counts for trial type (attraction, repulsion) i, TDD (near, far) j, product category (laptops, microwave ovens, televisions, washing machines) k, target high dimension (1,2) l is distributed:

$$\boldsymbol{C}_{ijkl} \sim \text{Multinomial}(\boldsymbol{\theta}_{ijkl}) \tag{F.1}$$

$\boldsymbol{\theta_{ijkl}}$ is a vector of choice probabilities which is turn distributed:

$$\boldsymbol{\theta_{ijkl}} \sim \text{Dirichlet}(\boldsymbol{\alpha}_{ijkl}) \tag{F.2}$$

where all $\boldsymbol{\alpha}_{ijkl} > 0$.

For a prior distribution on $\boldsymbol{\alpha}$, it was assumed that:

$$\boldsymbol{\alpha}_{ijkl} \sim \text{LogNormal}(1, 1) \tag{F.3}$$

Inference was performed using the mean target, competitor, and decoy choice probabilities, collapsed across product category and the target's high dimension. See Table F.1.

Trial Type	**TDD**	**Option**	*M*	HDI lower	HDI upper
Attraction	Near	Target	.48	.45	.51
		Competitor	.47	.44	.50
		Decoy	.05	.03	.06
	Far	Target	.46	.43	.49
		Competitor	.48	.45	.51
		Decoy	.07	.05	.08
Repulsion	Near	Target	.31	.28	.34
		Competitor	.66	.63	.69
		Decoy	.04	.03	.05
	Far	Target	.33	.30	.36
		Competitor	.65	.62	.68
		Decoy	.03	.02	.04

Table F.1. Experiment 4 Mean Posterior Choice Proportions from the Bayesian Dirichlet-Multinomial Model.

On average, participants chose the competitor more than the target in both TDD levels, in the repulsion effect trials. In the attraction effect trials, participants chose the target and competitor at equal rates.

APPENDIX G

BAYESIAN HIERARCHICAL LOGISTIC REGRESSION FOR EXPERIMENT 5 CHOICE DATA

The comparability data from Experiment 5 were analyzed with a Bayesian hierarchical logistic regression model.

All trials where participants chose the decoy option were removed. Trials where the W and H rectangles were in the first two positions were also removed.

The target was classified as the option aligned with the decoy in the two-aligned option. The model predicted choice via a linear combination of an intercept, the display (none-aligned, two-aligned, all-aligned) and whether the target was in the middle position (1, 0). The reference level for display was none-aligned. There were no interactions included in the model. There was also a random effect of participant included in the model.

The model was fit as a Bayesian hierarchical logistic regression model using the rstanarm package (Goodrich et al., 2020), using default priors for all parameters. Posterior estimates for all fixed effects are shown in Table G.1.

Parameter	*M*	*SD*	HDI lower	HDI upper
β_0	-0.18	0.03	-0.24	-0.13
$\beta_{\text{two-aligned}}$	-0.03	0.02	-0.08	0.01
$\beta_{\text{all-aligned}}$	-0.11	0.03	-0.16	-0.06
$\beta_{\text{target middle}}$	0.27	0.02	0.24	0.31

Table G.1. Experiment 5 Posterior Estimates for all Fixed Effects from the Bayesian Hierarchical Logistic Regression Model.

The main parameter of interest is $\beta_{\text{two-aligned}}$, which captures the change in target choice when comparing the none-aligned condition to the two-aligned condition. The posterior mean was below 0 ($M = -0.03$), and while the 95% HDI included 0 ($[-0.08, 0.01]$), 93.23% of all samples were < 0, which is taken as moderate evidence for an effect.

Participants also chosen the target less in the all-aligned condition ($M = -0.11$) compared to the none aligned condition, suggesting a bias away from the aligned option even when all options were aligned.

There was also a position effect, such that participants chose the target more when it was in the middle of the screen ($M = .27$, 95% CI $[0.24, 0.41]$).

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