LABS 1-

LAB 1 – Tutorial simulink

LAB 2 – Pendulum dynamics

$$\boxed{\ddot{\theta} = -\sin\theta}$$

LAB 2 – Spring-mass-damper dynamics

$$\boxed{\ddot{x} = \frac{1}{m}\left(\sin\omega t - c\dot{x} - Kx\right)}$$

LAB 3 – Rotation of a massive object (Euler equations)

$$\underline{M} = \dot{\underline{h}} + \underline{\omega} \wedge \underline{h} \; ; \; I\underline{\omega} = \underline{h} \; ; \Rightarrow I^{-1}\underline{M} = \dot{\underline{\omega}} + I^{-1} I \underline{\omega} \wedge \underline{\omega}$$

$$I\dot{\underline{\omega}} = \dot{\underline{h}} \qquad \hookrightarrow \boxed{\dot{\underline{\omega}} = I^{-1}M + \underline{\omega} \wedge \underline{\omega}}$$

LAB 3 – Comparison with analytic solution of the symmetric case

$I_x = 0.0504$

$I_y = 0.0504 \longrightarrow$

$I_z = 0.0109$

$$\boxed{\begin{aligned} \omega_x &= \omega_{x0}\cos\lambda t - \omega_{y0}\cos\lambda t \\ \omega_y &= \omega_{x0}\sin\lambda t + \omega_{y0}\cos\lambda t \\ \omega_z &= \omega_{z0} \end{aligned}} \qquad \boxed{\lambda = \frac{I_z - I_x}{I_x}\,\omega_{z0}}$$

LAB 3 – Assess stability of equilibrium points

1.- Test what happens when it's spinning around major, minor or medium axis.

major → stable; minor → Can be stable; medium → Unstable

2.- Plot the ellipsoids of angular momentum and kinetic energy, plus their intersection if they have one.

$$T = \frac{1}{2}\left(\underline{\omega}^T I \underline{\omega}\right) \xrightarrow{\text{Principal axis}} \boxed{\frac{\omega_x^2}{2T/I_x} + \frac{\omega_y^2}{2T/I_y} + \frac{\omega_z^2}{2T/I_z} = 1}$$

$$\underline{h} = I\underline{\omega} \xrightarrow{\text{Principal axis}} \boxed{\frac{\omega_x^2}{h^2/I_x^2} + \frac{\omega_y^2}{h^2/I_y^2} + \frac{\omega_z^2}{h^2/I_z^2} = 1}$$

LAB 3 – Dual-spin s/c

Inertia wheel → ω_r – Rot; I_r – Inertia; K_r – Axis

$$\rightarrow \underline{M} + \cancel{\underline{M_r}} = I\dot{\underline{\omega}} + \cancel{I_r\dot{\omega}_r\underline{K_r}} - \left(I\underline{\omega} + I_r\omega_r\underline{K_r}\right)\wedge\underline{\omega}$$

Inertia of the static wheel is included in I → Decoupling in $\dot{\omega}_r$

$$\hookrightarrow \boxed{\dot{\underline{\omega}} = I^{-1}\underline{M} + I^{-1}\left(I\underline{\omega} + I_r\omega_r\underline{K_r}\right)\wedge\underline{\omega}} \quad \hookrightarrow \boxed{\underline{M_r} = I_r\dot{\omega}_r\underline{K_r}}$$

LAB 4 – Rotation dynamics + Kinematics (DCM) – Direct cosine matrix

Define an inertial frame → $\boxed{\underline{a}_{inertial_i} = A_0\, \underline{a}_{body_i}}$

Now the body moves, so we must change A accordingly: $\boxed{\dot{A} = -[\omega\wedge]\cdot A}$

$$[\omega\wedge] \equiv \text{Skew-symmetric matrix of } \omega_{3x1} = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}$$

LAB 4 – Orthonormalize A in every step

$$\boxed{A_{t+1} = 1.5A_t - 0.5A_t \cdot A_t^T \cdot A_t} \leftarrow \text{(The integration induces noise)}$$

LAB4 – Idem but with quaternions

$q = [\underline{u}; s] = [?, ?, ?, ?]$

$\boxed{q0 \text{ from } A0} \longrightarrow \begin{pmatrix} \frac{0.25}{q_4}(A_{23}-A_{32}) \\ \frac{0.25}{q_4}(A_{31}-A_{13}) \\ \frac{0.25}{q_4}(A_{12}-A_{21}) \\ 0.5\sqrt{1+\mathrm{trace}(A)} \end{pmatrix} = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix}$

(so we transform to the same inertial system)

$$\boxed{\dot{q} = 0.5[\Omega]\cdot q}$$

$$\boxed{[\Omega] = \begin{pmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{pmatrix}}$$

$\boxed{\text{Also normalize}}$ with $q_{t+1} = \frac{q}{|q|}$

LAB5 – Idem but using Euler angles 312.

$312: \; \omega \longrightarrow \boxed{} \longrightarrow \begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix}_{312} \longrightarrow A_{312}$

Formulas too long, they are in the material

$\boxed{\text{To compute a valid } A_{312_0}\text{, use vrrotvec, vrrotvecmat}}$

Problem → Singularity in $90°, 270° = \theta_{312} \Rightarrow \boxed{\text{When } \|\cos\theta_{312}\| \ll 1}$

Happens to every system with 3 different rotations.

$\boxed{\text{Before the singularity, we switch to } A_{313}}$, with singularity in $0°, 180° \Rightarrow \sin\theta_{313}$

Same story, different formulas.

$\boxed{\text{For easier code}} \longrightarrow A_{312} = A_2(\psi)\, A_1(\theta)\, A_3(\phi)$

$$A_2(\psi) = \begin{pmatrix} \sim & 0 & \sim \\ 0 & 1 & 0 \\ \sim & 0 & \sim \end{pmatrix}, \quad A_1(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sim & \sim \\ 0 & \sim & \sim \end{pmatrix}, \quad A_3(\phi) = \begin{pmatrix} \sim & \sim & 0 \\ \sim & \sim & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

LAB5 – Pointing error of spin stabilized S/C

$\omega_x = \uparrow\uparrow \longleftarrow$ Spin axis

$\omega_y = 0.1$

$\omega_z = 0.1$

$I_x = 0.07$

Major axis → $I_y = 0.0504$

$I_z = 0.0109$

⇒ The spacecraft is trying to keep the x axis stable because it's pointing there

↳ $\boxed{\Gamma_0 = [1 \; 0 \; 0]}$

$\boxed{\dot{\underline{\Gamma}} = \underline{\Gamma} \wedge \underline{\omega}} \longleftarrow$ Change over time ($\boxed{\text{Also normalize}}$)

$\Gamma \cdot \Gamma_0 = |\Gamma| \cdot |\Gamma_0| \cos\theta = \cos\theta$

$\boxed{\text{Pointing error} \equiv \theta = \mathrm{acos}(\Gamma \cdot \Gamma_0)}$

Efficient way of rotating vectors with quaternions

$q = [\underline{u}; s]$; $\boxed{\underline{V_{new}} = 2(\underline{u}\cdot\underline{v})\underline{u} + (s^2 - \underline{u}\cdot\underline{u})\underline{v} + 2s(\underline{u} \wedge \underline{v})}$

Hard way (expensive) → $p = [\underline{V_{old}}; 0]$; $\boxed{P_{new} = q \wedge p \wedge q^*}$; where $q^* = [-\underline{u}, s]$

Reference frames nomenclature

B → Body

N → Inertial

L → Moving reference

LAB 6 - Attitude error

LVLH → Local Vertical Local Horizontal

Define a new moving reference frame L such that $\underline{\omega}_{L/N} = n\underline{K}$, with $n = \sqrt{\mu/r^3}$. Eg. for r, r = 6371+200 → n = 0.0012

If $\boxed{\underline{\omega}_{L/N} = n\underline{K}} \rightarrow \boxed{A_{L/N} = Rot_3(nt)} = \begin{pmatrix} \cos nt & \sin nt & 0 \\ -\sin nt & \cos nt & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\boxed{\text{Attitude error} \equiv A_{B/L} = A_{B/N} A_{L/N}^T} \rightarrow \text{Scalar measure} = \boxed{\cancel{a\cos}(\text{trace}(A_{B/L}))}$

$\boxed{\text{Relative rotation} \equiv \underline{\omega}_{B/L} = \underline{\omega}_B - A_{B/L}\underline{\omega}_L}$

LAB 6 - Idem + Gravity gradient effect

$$M = -\int_B \underline{r} \wedge \frac{\mu}{(R+r)^3}(\underline{R}+\underline{r})\,dm \underset{\substack{R+r \simeq R \\ + \text{Maclaurin} \\ \text{series}}}{=} -\underbrace{\frac{\mu}{R^3}}_{n^2}\int_B \underline{r} \wedge \left(1 - 3\frac{\underline{R}\cdot\underline{r}}{R^2}\right)(\underline{R}+\underline{r})\,dm$$

If $\underline{R} = R\underline{i}$ (LVLH)

$\hookrightarrow M = 3n^2(-I_{xz}\hat{j} + I_{xy}\hat{k})$

In body frame

$$\underline{R} = R[c_1; c_2; c_3] \rightarrow \underline{M} = 3n^2 \begin{vmatrix} (I_z - I_y)c_2c_3 \\ (I_x - I_z)c_1c_3 \\ (I_y - I_x)c_1c_2 \end{vmatrix} \quad \text{and} \quad \boxed{\underline{c} = A_{B/L}\begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}}$$

$\hookrightarrow \boxed{\underline{M} = 3n^2 I\underline{c} \wedge \underline{c}}$

LAB7 - Orbit propagator

We just need theta, the other $\underbrace{a, e, i}_{\text{needed too}}$, ω, Ω will be assumed constant.

$$\text{orbit} = \begin{cases} a \\ e \\ i \end{cases} \quad \text{Elliptical} \rightarrow \begin{cases} h^2 = a\mu(1-e^2) \\ r = \dfrac{h^2/\mu}{1+e\cos\theta} \\ \dot{\theta} = h/r^2 \end{cases} \rightarrow \boxed{\dot{\theta} = n\frac{(1+e\cos\theta)^2}{(1-e^2)^{3/2}}}$$

(theta0 = 0)

mu = mu_Earth → $\boxed{n = \sqrt{\frac{mu}{a^3}}}$

LAB7 - Sun direction, Earth direction

$$\boxed{n_{sun} = \frac{mu_{sun}}{r_s}} \qquad r_s = R_{sat-sun} \approx R_{E-sun} \qquad \boxed{r = \frac{a(1-e^2)}{1+e\cos\theta}}$$

$$\boxed{\underline{R}_N = r\begin{pmatrix} \cos\theta \\ \sin\theta\cos i \\ \sin\theta\sin i \end{pmatrix}} \rightarrow \boxed{\begin{aligned} \underline{R}_B &= A_{B/N}\underline{R}_N \\ \underline{S}_B &= A_{B/N}\underline{S}_N \end{aligned}} \leftarrow \boxed{\underline{S}_N = R_{sun}\begin{pmatrix} \cos n_{sun}t \\ \sin n_{sun}t\cos\varepsilon \\ \sin n_{sun}t\sin\varepsilon \end{pmatrix}}$$

ε: $\boxed{23.45°}$

LAB7 - Magnetic disturbance

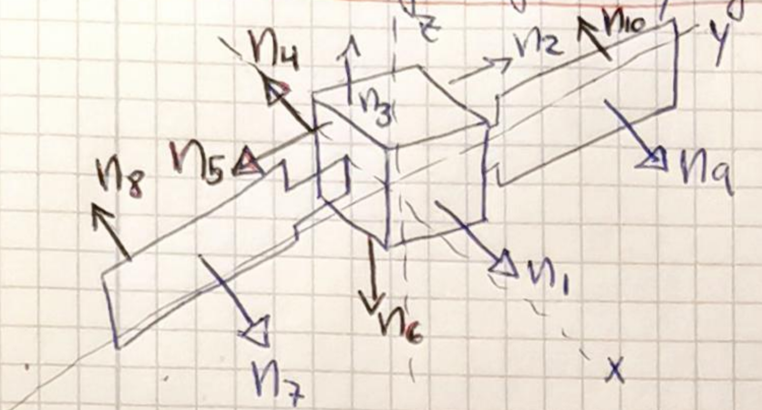
The earth magnetic field can interact with satellite's parasitic magnetic induction

$\hookrightarrow \boxed{M_{mag} = \underline{m}_{par_B} \wedge \underline{b}_B} \leftarrow \boxed{\underline{b}_B = A_{BN}\underline{b}_N}$ $\qquad \underline{m}_{par_B}$ (given)

Magnetic field → $\boxed{\underline{b}_N = \frac{R_{equator}^3}{r^3}H_0\left(3(\hat{m}\cdot\hat{r}_N)\hat{r}_N - \hat{m}\right)}$ where $\hat{r}_N = \frac{\underline{r}_N}{\|\underline{r}_N\|}$; Unit dipole vector → $\boxed{\hat{m} = \begin{pmatrix} \sin 11.5° \cos\omega_\oplus t \\ \sin 11.5° \sin\omega_\oplus t \\ \cos 11.5° \end{pmatrix}}$

$\boxed{H_0^2 = g_1^{0\,2} + g_1^{1\,2} + h_1^{1\,2}}$ ← Coefficients in tables, in nanoteslas $\qquad$ ↑ Magnetic tilt

LAB7 - Define the geometry of the s/c



Main body → $n_1 \not{n_4}$, $n_2 \not{n_5}$, $n_3 \not{n_6}$

Solar panels → $n_7 \not{n_8}$, $n_9 \not{n_{10}}$

We need to have for each surface: $\boxed{\text{Area, center position, normal, ~~cte~~ specular coef and diffuse coef.}}$

$\boxed{\text{+ Position of the overall CG}}$

LAB7 - Solar radiation pressure

Radiation $\boxed{F}$ (given) Radiation pressure $\boxed{P = \frac{F}{c}}$ ← Light speed 3e8

$$\boxed{\underline{Force_i} = -PA_i(\hat{S}_B \cdot \hat{N}_{Bi})\left[(1-\rho_{si})\hat{S}_B + \left(2\rho_{si}(\hat{S}_B \cdot \hat{N}_{Bi}) + \frac{2}{3}\rho_{di}\right)\hat{N}_{Bi}\right]}$$

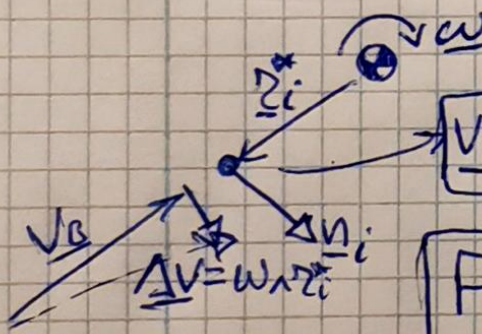
↳ $\boxed{\text{If } \hat{S}_B \cdot \hat{N}_{Bi} < 0,\ F_i = 0}$

Torque = $\boxed{M_{SRP} = \sum \underline{r}_i^* \wedge \underline{F}_i}$ where $\boxed{r_i^* = \underline{r}_i - \underline{r}_{CG}}$

LAB - Air drag

$\boxed{\underline{V_B} = A_{BN}\underline{V_N}}$; $\boxed{\underline{V_N} = \underline{V_{orb}} - \underline{\omega}_\oplus \wedge \underline{r_N}} = (\dot{x} + \omega_\oplus y,\ \dot{y} - \omega_\oplus x,\ \dot{z})$

$\boxed{(0,\ 0,\ \omega_\oplus)}$



$\boxed{\underline{V_i} = \underline{V_B} + \underline{\omega_{sat}} \wedge \underline{r_i^*}}$ $\boxed{\underline{M_{aero}} = \sum \underline{r_i^*} \wedge \underline{F_i}}$

$$\boxed{F_i = -\frac{1}{2}\rho C_D V_i^2 \hat{V}_i(\hat{N}_{Bi} \cdot \hat{V}_i)A_i}$$