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Solid rocket motors internal ballistics analysis

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Abstract

The following report will analyze the provided SRMs' experimental pressure traces with the aim of fully determining their Vieille's law coefficients, c^* and performing a Monte-Carlo simulation on the MEOP. Due to the nature of SRMs their performance can vary substantially and so experimental tests on downscaled versions are often needed. Using such motors allows the validation of their performance and the extrapolation to larger systems.

BATES motor and data

BATES (Ballistic Test and Evaluation System) is a standardized solid propellant performance characterization platform. In this exercise a BATES motor with pressure transducer at the back end was used (at a sampling rate of 1kHz) to generate the aforementioned pressure traces: 9 batches with three different throat configurations, as showed in Table 1, and a propellant mixture of Ammonium Perchlorate (AP) 68%, Al (aluminum) 18%, HTPB Binder 14%.

Table 1: Configurations.

P_c $[bar]$:	30	45	70
D_t $[mm]$	28,80	25,25	21,81

Small-scale SRM ballistic analysis

The propellant burn time t_b depends on the burning rate r_b, which in turn is function of the chamber pressure P_c as described by Saint Robert's (or Vielle's law) in Equation 1, assuming steady state behavior.

$$r_b = aP_c^n \tag{1}$$

P_c depends on several factors, Equation 2: the propellant density ρ_p (calculated with the propellant composition), Vieille's coefficients, throat area A_p, characteristic velocity c^* and burning area A_b. All but the burning area are assumed to be constant.

$$P_c = \left(\rho_p a \frac{A_b}{A_t} c^*\right)^{\frac{1}{1-n}} \tag{2}$$

The characteristic velocity can be calculated based on the experimental as described in Equation 3.

$$c^* = \frac{\int_0^{t_b} P_c A_t dt}{M_{tot}} \tag{3}$$

And finally the expression for A_b, Equation 4, which comes from the geometry of the SRM: central perforated and cylindrical-shaped. All known values are shown in Table 2.

$$A_b = 2\pi \left[(\frac{D_{ext}}{2})^2 - (\frac{D_{int}}{2} + \Delta x_b)^2 \right] + 2\pi(\frac{D_{int}}{2} + \Delta x_b)(L - 2\Delta x_b) \tag{4}$$

Table 2: Constants.

ρ_p $[kgm^{-3}]$	D_{ext} $[mm]$	D_{int} $[m]$	L $[m]$	Δx $[m]$	M_{total} $[kg]$
1761.9	0.16	0.1	0.29	0.03	6.2604

Having the pressure traces and the previously mentioned constants, the procedure follows with the calculation of the experimental effective $\hat{P}_c$ and $\hat{r}_b$ using the Bayern-Chemie method.[1] After, the results are linearly fitted for a mean and uncertainty estimation, Figure 1, Table 3. c_* is calculated as indicated before in Equation 3 but for its uncertainty the standard deviation of the sample is used.

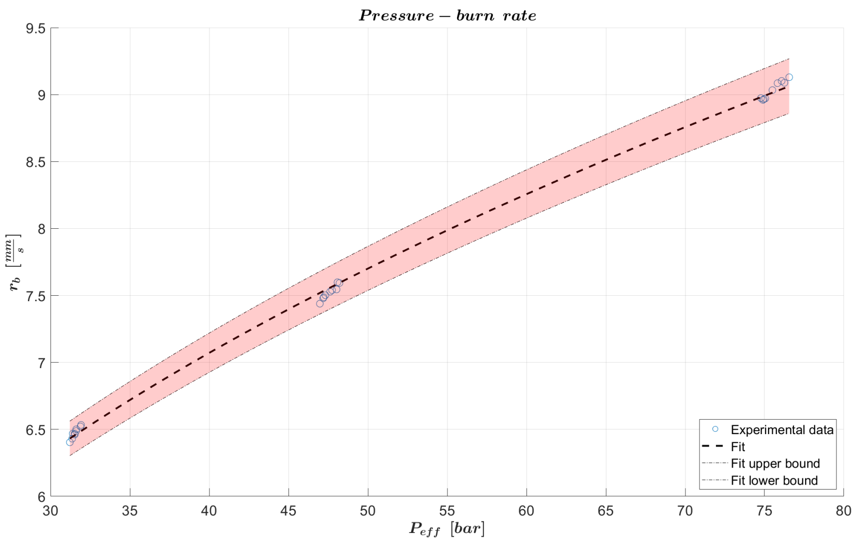

Figure 1: Vieille's fit

Table 3: Fitted coefficients and c_*.

	a $[mm/(s*bar)]$	n $[-]$	C* $[m/s]$
Mean value	1,7268	0,38217	1512,2
Uncertainty	$\pm 0,01841$	$\pm 0,002736$	$\pm 11,734$

SRM firing model

With the mean values for a, n and c_* the pressure of the SRM is now modeled by numerically integrating until the burnt web Δx_b exceeds its thickness Δx:

1. $t^i = t^{i-1} + \Delta t$ (At 1kHz, $\Delta t = 1ms$)
2. $A_b^i = f(\Delta x_b^{i-1})$ (Equation 4)
3. $P^i = f(A_b^i)$ (Equation 2)
4. $r_b^i = f(P^i)$ (Equation 1)
5. $\Delta x_b^i = \Delta x_b^{i-1} + r_b^i \Delta t$
6. Repeat if $\Delta x_b^i < \Delta x$ else stop

Substituting for the constants corresponding to each pressure level and following the scheme described above outputs the modeled pressure for each case, allowing for visualization against the experimental traces:

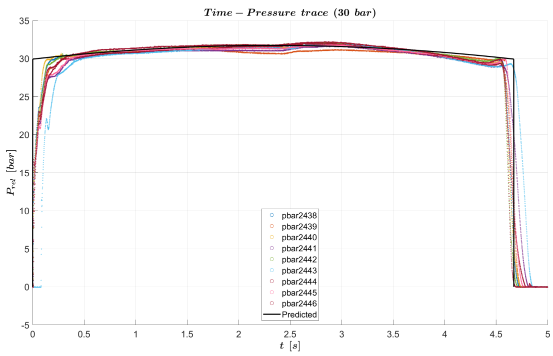

Figure 2: Low pressure traces.

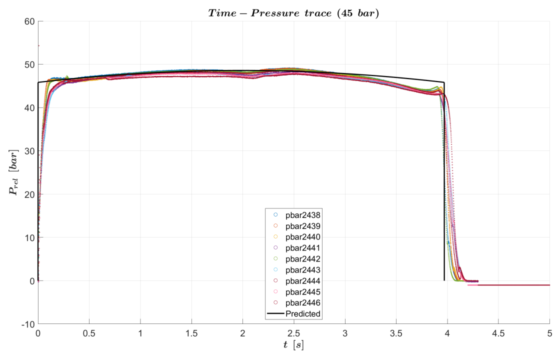

Figure 3: Medium pressure traces.

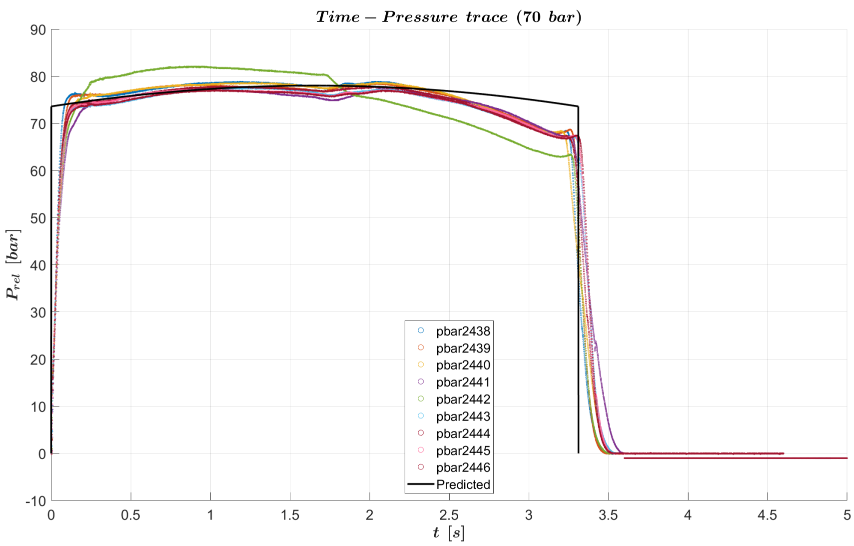

Figure 4: High pressure traces.

70 bar SRM MEOP and Monte-Carlo simulation

The final point consists in obtaining the expected value and uncertainty of the 70 bar SRM MEOP, through Monte-Carlo analysis. For the setup, 30-sized samples of a, n and c^* are drawn from normal distributions with the same mean and variance as the parameters that were calculated previously, for a total of 27,000 combinations. This assuming those SRM parameters come in fact from normally distributed populations. In each one, the process employed was the same as in the last section, with the caveat of selecting at the end the MEOP as $max(P)$.

After computing, Monte-Carlo offers a sample of MEOPs from which the mean and standard deviation can be calculated:
$MEOP_{70bar} = 78.3057 \pm 7.3189$ bar

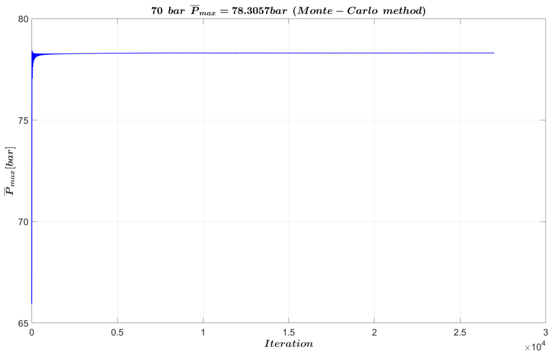

Figure 5: MEOP mean, Monte-Carlo.

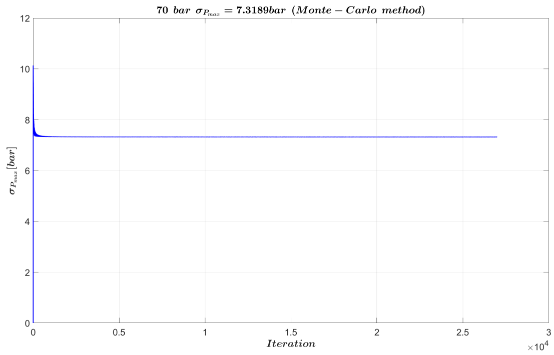

Figure 6: MEOP uncertainty, Monte-Carlo.

Conclusions

From a set of experimental traces, the steady state ballistics of the studied SRM, at different pressure levels, are estimated. This also includes an additional estimation for its MEOP in the 70 bar case. When stacked against real pressure traces, the model based only on Vieille's law has good behavior, specially in lower pressure levels and the steady state part of the firing, but is incapable of reproducing other phenomena such as the hump effect, which is specially evident in higher pressure levels, or the initial and final transient processes.[2]

Bibliography

[1] *Fereoli, G. et al. (2022) Ballistic Analysis Through Bayern-Chemie Method and MCM Sensitivity Analysis On a Solid Propellant Grain*

[2] *Utah University, Mechanical and Aerospace Engineering (2018) Modeling Transient Rocket Operation* http://mae-nas.eng.usu.edu/MAE_5540_Web/propulsion_systems/section6/section.6.2_2018.pdf