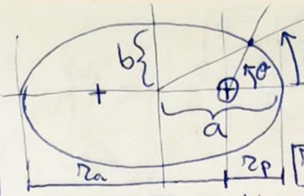
Formulas

Orbital Mechanics

All

Orbital period $\boxed{T^2 = \frac{4\pi^2 a^3}{\mu}}$ $\boxed{T = \frac{2\pi}{\omega}}$



Energy eq. $\boxed{\frac{1}{2}V^2 - \frac{\mu}{r} = -\frac{\mu}{2a}}$

$\boxed{r_a = a(1+e)}$ Apsapsis / Periapsis $\boxed{r_p = a(1-e)}$ $\boxed{\gamma_{par} = \frac{\theta}{2}}$

$\boxed{\frac{h^2}{\mu} = p = a(1-e^2)}$ semi-latus rectum

$\boxed{h = rV_\theta}$ $\boxed{\frac{\mu}{h} = \sqrt{\mu/p}}$ $\boxed{e = \frac{r_a - r_p}{2a}}$

Flight Path Angle $\boxed{\tan\gamma = \frac{V_r}{V_\theta}}$ $\boxed{\cos\theta|_{\gamma ext} = -e \rightarrow \gamma_{\substack{min\\max}}}$ $\boxed{b = a\sqrt{1-e^2}}$

$\boxed{r = \frac{p}{1 + e\cos\theta}}$

Hiperbolic: $\boxed{C_3 = V_\infty^2}$ Impact param. / Aiming radius $\boxed{\Delta = |b| = -ae\cos\frac{\delta}{2} = \frac{-a}{\tan\delta/2}}$

Deflection / Turn angle $\boxed{\sin\frac{\delta}{2} = \frac{1}{e}}$ $\boxed{\Delta V = 2V_\infty \sin\frac{\delta}{2}}$ $\boxed{a = -\mu/V_\infty^2}$ $\boxed{e = 1 + \frac{r_p V_\infty^2}{\mu}}$

Mean motion $\boxed{n^2 = \mu/a^3} = \frac{4\pi^2}{T^2}$

Local horizon $\rightarrow +\hat{\theta}$ Sidereal day: 23h56'4'' Sidereal year 365.25d Day 24·3600

Kepler's eq. $n(t_1 - t_0) = |\underbrace{E - e\sin E}_{\text{Mean anomaly } M}|_{t_0}^{t_1}$ $\quad tg\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}}\, tg\frac{E}{2}$

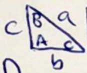
Hiperbolas $M = e\sinh H - H$ $\quad tg\frac{\theta}{2} = \sqrt{\frac{e+1}{e-1}}\tanh\frac{H}{2}$

Parabolas: Barker's eq. $D = tg\frac{\theta}{2}$ $\quad t - t_P = \sqrt{\frac{2r_p^3}{\mu}}\left(D + D^3/3\right)$

KIND OF ORBIT

$\boxed{V_{esc} = \sqrt{\frac{2\mu}{r}}}$ $\boxed{V = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}}$ $\boxed{\gamma_{parabolic} = \frac{\theta}{2}}$

Cos theorem $\boxed{\substack{\cos 360-\theta = \cos\theta \\ \sin 360 - \theta = -\sin\theta}}$

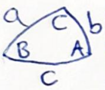


$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$

Sine theorem $\boxed{\frac{a}{\sin A} = \frac{b}{\cos B} = \frac{c}{\cos C}}$

$\cos -x = \cos x$
$\sin -x = -\sin x$
$\cos 90 - x = \sin x$
$\sin 90 - x = \cos x$

Cosine 3D



$\boxed{\begin{aligned} \cos A &= -\cos B\cos C + \sin B\sin C\cos a \\ \cos a &= \cos b\cos c + \sin b\sin c\cos A \end{aligned}}$

Sine 3D

$\boxed{\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}}$

	Spring	Summer	Autumn	Winter
date	21 March equinox	21 June solstice	21 Sept. equinox	21 Dec solstice
δ	0	ε	0	$-\varepsilon$
$\hat{\Sigma\gamma} \alpha$	0h	6h	12h	18h

$\boxed{\mu_\oplus = 398600 \frac{Km^3}{s^2}}$ $\boxed{R_\oplus = 6378.1\ Km}$ $\boxed{\varepsilon_{sun} = 23.4°}$

Heliocentric (γ, Y, Z) Geocentric (γ, Y, CNP) TNH $(\hat{t}, \hat{n}, \hat{h})$ Orb $(\hat{r}, \hat{\theta}, \hat{h})$ PF (Perifocal) $(\hat{e}, \hat{p}, \hat{h})$

$r_{orb} = (r, 0, 0)$ $\quad V_{orb} = (\underbrace{e\sin\theta}_{V_r = V\sin\gamma}, \underbrace{1 + e\cos\theta}_{V\cos\gamma = V_\theta}, 0)\,\mu/h$ $\quad r_{PF} = (r\cos\theta, r\sin\theta, 0)$ $\quad V_{PF} = (-\sin\theta, e + \cos\theta, 0)\,\mu/h$

Hour angle of star t_*: Clockwise as $\hat{\Sigma\Sigma^*}$ (Observer/Star meridian)

Declination $\boxed{\delta = 90 - a\cos\frac{z}{r}}$

Right Ascension $\boxed{\alpha = a\cos\frac{x}{|x, y|}}$ $\rightarrow y > 0$; $\rightarrow 2\pi - \text{angle}$, $y < 0$

Local Sidereal Time $\boxed{LST = \hat{\Sigma\gamma} = t + \alpha = t^* + \alpha^*}$

Local Mean Time $\boxed{LMT = t_M + 12h = GMT + \lambda}$

Local Apparent Time $\boxed{LAT = t_{sun} + 12h = GAT + \lambda}$ $GAT = GMT + E$

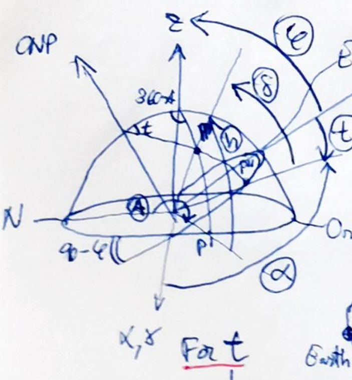
Correction $\boxed{E = LAT - LMT}$, tabulated wrt. GMT

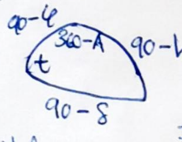
West is negative

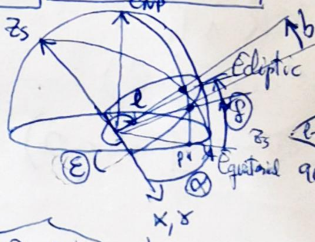
$\boxed{h_{max} = 90 - |\varphi - \delta|}$

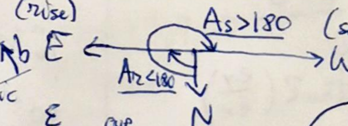
$\boxed{h_{sunrise} = h_{sunset} = 0}$

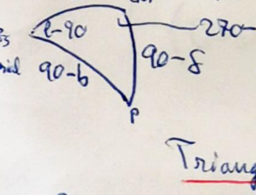
Local Civil Time $\boxed{LCT = GMT + 1h \cdot N}$ $\boxed{N = ceil\left(\frac{\lambda}{15°}\right)}$

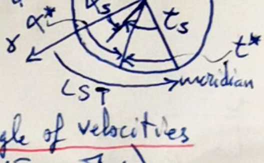








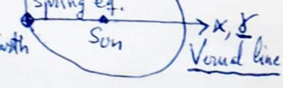


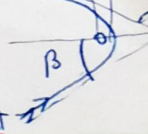


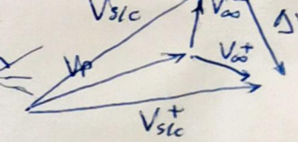
$\frac{Z\ P\ \text{ecliptic}}{} \rightarrow b$

$\frac{CNP\ P\ \text{equat.}}{} \rightarrow \delta$

Triangle of velocities



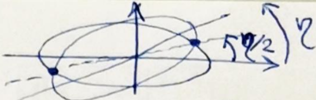




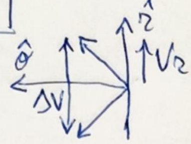
For t ↓

$\delta \in [-90, 90]$ Need to use sine and cosine theorem to determine the quadrant

Phasing $\boxed{KT_x = KT + \Delta t_\lambda}$ $\boxed{\Delta t_\lambda = \frac{\Delta\lambda}{360° \cdot K} T}$



Apse line rotation $r_1 = r_2 \quad \theta_2 = \theta_1 - \eta$ with constant shape



$$\theta = \eta/2 \begin{cases} \theta \\ 180 + \theta \end{cases}$$

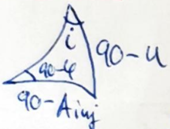
Grav. $\ddot{\underline{r}} = -\frac{\mu}{r^3}\underline{r} + \underline{a}_{pert}$ 3rd body $\underline{a} = \mu_{3rd}\left(\frac{\underline{r}_{s/c-3rd}}{r_{s/c-3rd}^3} - \frac{\underline{r}_{main-3rd}}{r_{main-3rd}^3}\right)$

Drag $\underline{a} = -\frac{1}{2}\frac{A_{cross}}{m} C_D \rho V_{rel}^2 \hat{V}_{rel}$ $\quad \underline{V_{rel}} = \dot{\underline{r}} - \underline{\omega}_\oplus \wedge \underline{r}$ $\quad \rho = \rho_0 e^{-\frac{h-h_0}{H}}$ $\quad \underline{a} = -\rho_{SR \text{ at } 1AU} \frac{(AU)^2}{r_{s/c-sun}^3} \frac{A_{sor}}{m} C_R \underline{r}_{s/c-sun}$

J2 secular effects $\dot{\Omega} = -\frac{3}{2}\frac{nR_\oplus^2 J_2}{p^2}\cos i$ $\quad \dot{\omega} = \frac{3}{4}\frac{nR_\oplus^2 J_2}{p^2}(4 - 5\sin^2 i)$ $\quad \dot{M} = -\frac{3}{4}\frac{nR_\oplus^2 J_2}{p^2}\sqrt{1-e^2}(3\sin^2 i - 2)$

Sphere of influence $r_{SOI} = \left(\frac{m_E}{m_S}\right)^{2/5} r_E = r_E \left(\frac{\mu_E}{\mu_S}\right)^{2/5}$

Launchers



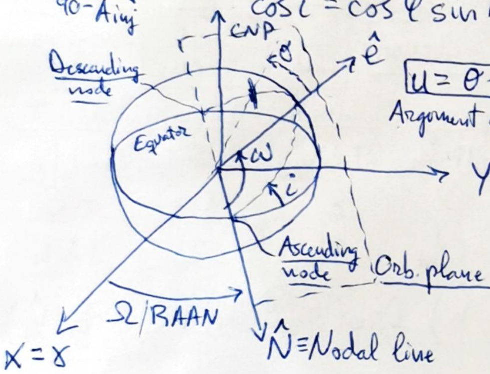
$A_{\Delta V \text{ or launch}} \neq A_{injection}$ because $V_{Earth} > 0$

$\underline{V_{Earth}} = R\omega_\oplus \cos\varphi \, \hat{E}$

$\cos i = \cos\varphi \sin A_{inj}$

$\underline{\Delta V} = \Delta V \sin A_{\Delta V} \hat{E} + \Delta V \cos A_{\Delta V} \hat{N}$

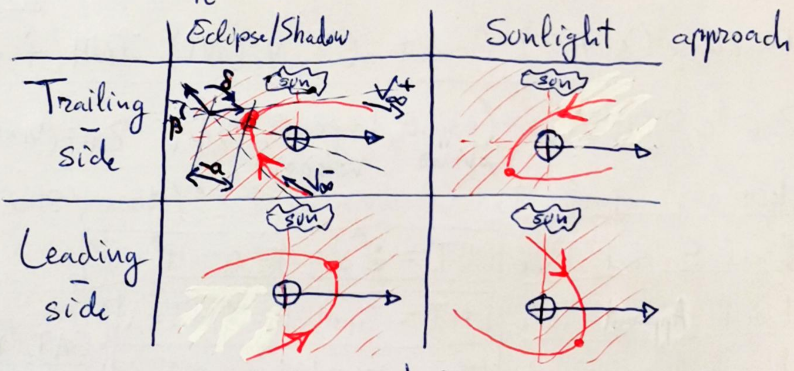
$\underline{\Delta V} + \underline{V_{Earth}} \Rightarrow A_{inj}$



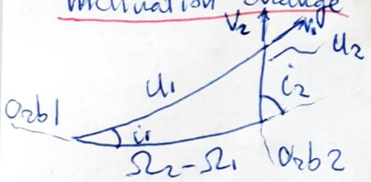
$\boxed{u = \theta + \omega}$ Argument of latitude

Repeating GT

$\hookrightarrow \frac{T}{T_E} = \frac{m}{K}$ ← Earth rotations / ← Orbit revolutions



Inclination change



$\underline{h} = \underline{r} \wedge \underline{V}$ $\quad i = \operatorname{acos}\frac{h_z}{h}$ $\quad \Omega = \operatorname{acos} N_x$ $\quad \underline{e} = \frac{1}{\mu}(\underline{V} \wedge \underline{h}) - \hat{r}$

$$a = \frac{1}{2/r - V^2/\mu}$$

$i = 0$	$i \neq 0$
$\hat{N} = \hat{i}$	$\hat{N} = \hat{K} \wedge \hat{h}$
$\omega = \operatorname{atan2}\left(\frac{e_y}{e_x}\right)$	$\omega = \operatorname{acos}(\hat{N} \cdot \hat{e})$
$(2\pi - \omega$ if $h_z < 0)$	$(2\pi - \omega$ if $e_z < 0)$

$e = 0$

$\hookrightarrow \underline{e} = \hat{N}$

$\omega = 0$