Space Propulsion

Formulario

$$\Gamma(\gamma)=\sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{T_0}{T}=\left(\frac{P_0}{P}\right)^{\frac{\gamma-1}{\gamma}}=\left(\frac{\rho_0}{\rho}\right)^{\gamma-1}=1+\frac{\gamma-1}{2}M^2$$

$1\,in = 2.54\,cm$

$1\,cal = 4.184\,J$

$A = 6.022\cdot 10^{23}$

Relación de areas

$$\frac{A}{A_t}=\frac{\Gamma(\gamma)}{\sqrt{\gamma}\,M}\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}=\frac{\Gamma(\gamma)}{(P/P_c)^{\frac{1}{\gamma}}\sqrt{\frac{2\gamma}{\gamma-1}\left[1-(P/P_c)^{\frac{\gamma-1}{\gamma}}\right]}}=\varepsilon_{(A=A_e)}$$

$M = V/a$

$a=\sqrt{\gamma R_g T}$

Thrust coeff.

$$C_T=\frac{T_E}{P_c A_t}=\Gamma(\gamma)\sqrt{\frac{2\gamma}{\gamma-1}\left[1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}+\varepsilon\left(\frac{P_e}{P_c}-\frac{P_a}{P_c}\right)$$

$$V=c^*\Gamma(\gamma)\sqrt{\frac{2\gamma}{\gamma-1}\left[1-\left(\frac{P}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

Pressure model

$$P_a(h)=101325\,e^{-\frac{h}{7338}}$$ ISA

$$R_g=\frac{R}{M_m}\qquad R=8.314\,\frac{J}{mol\,K}$$

$$\gamma=\frac{C_p}{C_v}$$

$C_p=C_v+R_g$ Mayer's relation

Monoatomic $C_p=\frac{5R}{2}$

Diatomic $C_p=\frac{7R}{2}$

$$C^*=\frac{\sqrt{R_g T_c}}{\Gamma(\gamma)}$$

$$I_s=C^*C_T=\frac{T_E}{\dot m}$$ (gravimetric is /9.81)

$$T_E=\dot m V_e+(P_e-P_a)A_e$$

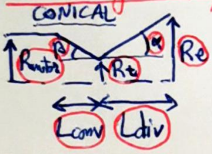
$$\dot m=\frac{P_c A_t}{C^*}$$

$I_v=\langle\rho\rangle I_s$ (volumetric)

$I_{total}=\int T\,dt$

Nozzle geometry

CONICAL



Typical
$\alpha\sim 18^\circ$
$\beta\sim 45^\circ$

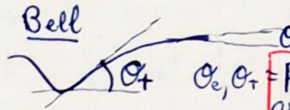
$$L_{conv}=\frac{R_m-R_t}{\tan\beta}$$

$$L_{div}=\frac{R_e-R_t}{\tan\alpha}$$

$$\lambda_{2D}=\frac{1+\cos\alpha}{2}$$

$T_{2D}=$ dyn.$\cdot\lambda_{2D}$ + static

Bell



θ_e,θ_t = Rao curves $(\varepsilon, x\%$ length of $\alpha=15^\circ$ cone$)$

Maximizing the x% length leads to less losses.

$$\lambda_{2D\,bell}=\frac{1}{2}\left(1+\cos\frac{\alpha+\theta_e}{2}\right)$$

Solid propulsion

$r_b=b+aP^n$ VIEILLE

(b~0 for P<200bar)

$$P_c=\left(c^*\rho_p a\frac{A_b}{A_t}\right)^{\frac{1}{1-n}}$$

$\dot m=r_b\rho_p A_b$

Sensitivity to temperature $\rightarrow$ $a=a_{T_{ref}}\,e^{\sigma_P(T_0-T_{ref})}$

$V_p=\frac{m_p}{\rho_p}$

$m_p=\dot m\,t_b$

Imperfect combustion

We know T_c, we want the composition.

$A\rightarrow\alpha B+\beta C+\gamma D$

$\hookrightarrow xC\rightleftarrows yD\rightarrow \Delta\nu=y-x \rightarrow K_p=\frac{P_D{}^y}{P_C{}^x}$; $\gamma^y=\beta^x K_n$

$h_0=\Delta h_f^0+h+\frac{v^2}{2}$

Penner table (Given T_c)

$$K_n=K_p\left(\frac{n_{tot}}{P_c}\right)^{\Delta\nu}$$

Use atomic equilibrium too

Arrhenius $K=A\,e^{-E_a/RT}$

Iterative process depending on n_{TOT}.

Usually $n_{TOT}\approx 1\sim1.1\,n_{stochiometric}$

$$M_m=\frac{\sum M_{mi}n_i}{n_{TOT}}\qquad C_p=\frac{\sum C_{pi}n_i}{n_{TOT}}$$

Other times we know the composition, not the T_c.

$$\Delta h_R=\sum^{prod}n_i\Delta h_{fi}-\sum^{react}n_i\Delta h_{fi}$$

Iterative process on T_c $\longrightarrow$ $\Delta h_{sensible}=\sum^{prod}n_i\int_0^{T_c}C_p dT\,(\text{table})-\sum^{prod}n_i\int_0^{T_{ref}}C_p dT\,(\text{table})=-\Delta h_R$

Nonreacting flows - heat 1→2

Rayleigh table $\xrightarrow{\gamma,M_1} T_{01}/T_0^* \rightarrow T_0^*\rightarrow q_{max}=\dot m_p C_p(T_0^*-T_{01})\xrightarrow{q} T_{02}\rightarrow \frac{T_{01}}{T_{02}}=\frac{T_{01}}{T_0^*}\Big|_{M_1}\frac{T_0^*}{T_{02}}\Big|_{M_2}\xrightarrow{Table} M_2$

Knowing M_1 and M_2 $\xrightarrow{table}$ P_{01}/P_0^*, $P_{02}/P_0^*\rightarrow P_{02}$

Nonreacting flows - friction 2→3

Rel. (ε) roughness $=\frac{\text{Roughness}}{D}$

$$Re=\frac{\rho v D}{\mu}$$

$(\varepsilon, Re)\xrightarrow{\text{Moody Diagram}}$ Darcy factor $f\rightarrow C_f=f/4$; $f\frac{4}{D}\Big|_3=\frac{L^*}{D}\Big|_2-\frac{L}{D}\rightarrow$ Fanno tables

Injectors

$$C_D\sim0.7=\frac{\dot m_{real}}{\dot m_{ideal}}$$

$$\dot m_{inj}=A_{inj}C_D\sqrt{2\Delta P\rho_l}$$

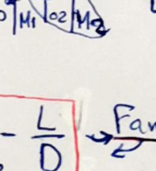
$$V_{liq}=C_D\sqrt{\frac{2\Delta P}{\rho}}$$

Impinging mom. balance $\rightarrow$ $\dot m_1 V_1\sin\theta_1=\dot m_2 V_2\sin\theta_2$

Heat transfer

$$P_r=\frac{C_p\mu}{K}\qquad N_u=\frac{hL}{K}$$

$$D_H=\frac{4A_{cross}}{Perim.}\qquad q=h\Delta T\qquad q=-K\frac{\Delta T}{\Delta x}$$



$T_{aw}=RT_0$

$$R=\frac{1+P_r^{1/3}\frac{\gamma-1}{2}M^2}{1+\frac{\gamma-1}{2}M^2}$$

Group	Category	Flavour	Isp[s]
Solid	Homogeneous	(single) (double base) NC or NC-NG or triple	~
	Composite	AP+HTPB+Al	
Liquid	Monoprop	H_2O_2 or N_2H_4 or N_2O	150-200
	Biprop.	Storable (MMH, N_2O_4)	~310
		Cryogenic (LOX, H_2)	~450
Hybrid	—		
Gas	Cold		50-150
Nuclear	Thermal		~900
Electro-	Thermal	Resistojets	~300
		Arcjets	~1000
	Static	Ion thrusters	1500-10000
		Hall effect " "	~2000
	Magnetic	Magneto Plasma Dynamic	1500-8000
		Pulsed Plasma Thruster	~5000

Theory reminders

Semi-empirical Granular Diffuse Flame

$$\boxed{1/r_b = \frac{a_{KIN}}{p} + \frac{b_{DIF}}{\sqrt[3]{p}}}$$

$$\boxed{P_e/T_E = \frac{I_s g_0}{2\eta_t}} \quad \boxed{\eta_T = \frac{\frac{1}{2}\dot{m}V_e^2}{VI}}$$

$$\boxed{\alpha = \frac{P_e}{M_{PP}+M_{str}}} \quad \boxed{\beta = \frac{1}{\alpha}} \quad \boxed{V_c = \sqrt{\frac{2\eta_t}{\alpha} t_p}}$$

$$\boxed{\frac{M_{pay}}{M_0} = e^{-\Delta V/V_e}\left(1+\frac{V_e^2}{V_c^2}\right) - \frac{V_e^2}{V_c^2}}$$

$$\boxed{I_{sp} = V_{acc} = \sqrt{2\Delta V\, q/\mu}} \quad \boxed{q = \kappa C} \quad \boxed{\mu = \frac{M_m}{A}}$$

$$\boxed{\gamma = \rho_i V_{acc}} \quad \boxed{T_E/A_{eu} = \gamma \frac{\mu}{q} V_{acc}}$$

$$\boxed{r_g = a\, G^n p^m}$$

$$\dot{m}_{IN} = \dot{m}_{out} + \frac{d}{dt}(\rho_g V) \xrightarrow[\boxed{\frac{dV}{dt} = A_b r_b} \quad \frac{d\rho}{dt} = \frac{dP}{dt}\frac{1}{R_g T} \quad C^* = \frac{\sqrt{R_g T}}{\Gamma(\gamma)}]{\dot{m}_{IN} = \rho_p r_b A_b \quad \dot{m}_{out} = \frac{P_c A_t}{C^*}} \frac{dP_c}{dt} = \frac{(C^*\Gamma(\gamma))^2}{V}\left((\rho_p - \rho_g)A_b r_b - \frac{P_c A_t}{C^*}\right)$$