EMSP - Part 2 Signal Processing - exercises to test your understanding

Ex 1

Consider two signal $s_1(t)$ and $s_2(t)$, emitted from two ideal point-like sources in two distinct locations in space (like music played by two loudspeakers). Then, assume those signals are recorded by two receivers (imagine two microphones), so that the resuling data is modeled as:

$$d_1(t) = s_1(t - t_{11}) + s_2(t - t_{12})$$
$$d_2(t) = s_1(t - t_{21}) + s_2(t - t_{22})$$

where t_{nm} represents the delay from the $m - th$ source to the $n - th$ receiver.

1. Assume that you know the signals $s_1(t)$ and $s_2(t)$. Describe a procedure to measure all the delays t_{nm}. *Tip: you can assume that the cross-correlation between $s_1(t)$ and $s_2(t)$ is neglibile.*
2. Now assume that you do not know $s_1(t)$ and $s_2(t)$, so all you can work with is $d_1(t)$ and $d_2(t)$. Can you still measure the delays? If not, can you at least measure some *differences* of the delays? How?
3. Assume that both $s_1(t)$ and $s_2(t)$ can be described as having an effective (two-sided) bandwidth $B = 20$ KHz. What is the temporal resolution of your measurement?
4. Imagine now that two microphones are spaced by $L = 10$ cm, and assume wave propagation velocity to be 340 m/s. Describe a procedure to measure the angular positions of the two sources based on the measurement of delays carried out in previous points.
5. Can you tell the angular resolution associated with this measurement?

Ex 2

Spaceborne Radars operate by transmitting radio frequency pulses to the Earth surface and receiving the echoes back-scattered by the targets. Imagine the presence of a single target, so that the received signal is modeled as:

$$s_{rx}(t) = s_{tx}\left(t - 2\frac{R}{c}\right)$$

where R is the distance between the satellite and the target and c is the speed of light (the factor 2 accounts for the fact that the wave has travelled the distance R twice when returning back to the satellite). In complex notation, the transmitted waveform is given as:

$$g(t) = rect\left(\frac{t}{T}\right) \cdot exp\left(j\pi\alpha t^2\right)$$

where $T = 30$ microseconds is pulse duration, and $\alpha = 2 \cdot 10^{11}$ is the chirp rate. The transmitted signal is

$$s_{tx}(t) = g(t) \cdot exp(j2\pi f_0 t)$$

where: $f_0 = 500$ MHz is the carrier frequency.

1. Write the expression of the real-valued transmitted signal.
2. Write the expression of the complex signal at the receiver after demodulation (i.e.: the complex envelope).
3. Describe a procedure to measure the distance R given the received signal and assuming knowledge of the waveform $g(t)$.
4. What is the resolution in distance (a.k.a: range resolution)?

Ex 3

Based on the material in the slides relative to the third part of the course, design an array of receiving antennas to measure the angular position of impinging RF sources, trying to minimize the number of antennas while optimizing angular resolution for a given Field of View (FoV), i..e: the angular region surveyed by the system. Discuss in particular:

- The radiation pattern and approximate size of aindividual ntenna elements.
- The spacing between antenna elements.

Ex 4

Write a Matlab script to simulate and solve one (or even all) the problems above.