Formulario

Electromagnetics & Signal Processing (EMSP) 1

$\bar{n} = \cos(\omega t - x) \rightarrow$ Phasor $\tilde{n} = e^{-j(x)}$

*Casio for complex numbers
- Complex mode (Write j after numbers, not before)
- "ENG" for $j = \sqrt{-1}$
- Polar/Rect. conversion $\rightarrow$ OPTN
- Shift + ENG for polar angle $\angle$

Free space: $\mu_r = 1$, $\varepsilon_r = 1$

$\cos\alpha = \sin(\alpha + \frac{\pi}{2})$

Intrinsic impedance: $\eta = \frac{E_0}{H_0} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$ $\quad \mathrm{Re}(\eta) > 0$

Speed of light (vacuum): $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$V = \frac{\omega}{\beta} = \frac{c}{\sqrt{\varepsilon_r \mu_r}}$ (non lossy case)

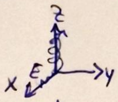
$\hookrightarrow$ Phase velocity $[m/s]$

$\varepsilon = \varepsilon_r \varepsilon_0$ — $8.84 \cdot 10^{-12}$ Electric permittivity

$\mu = \mu_r \mu_0$ — $4\pi \cdot 10^{-7}$ Magnetic permeability

$\eta_0 = \sqrt{\mu_0/\varepsilon_0} = 377\,\Omega$

Wavelength $\lambda = \frac{2\pi}{\beta}$



$\bar{E}_x = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\bar{\mu}_x$ $\rightarrow +x$ E

Attenuation const. $[Np/m] > 0$

Phase const. $[1/m]$

$-\beta z$: $+z$ prop. direction

$\hat{E}$ over $\hat{H}$ must // to prop dir. $+z \Rightarrow H \,//\, +\bar{\mu}_y$

$f = \frac{\omega}{2\pi}$

Loss tangent $\delta = \frac{\sigma}{\omega\varepsilon}$ $\;(+\varepsilon_r''/\varepsilon_r')$ $\quad > 10 \rightarrow$ Good conductor $\quad < 0.01 \rightarrow$ Good dielectric

Propagation constant $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \sqrt{-\omega^2\mu\varepsilon + j\omega\mu\sigma} = \alpha + j\beta$ (Has to be positive)

Free space wavelength $\lambda_0 = \frac{c}{f}$ $\quad (c = 3 \cdot 10^8)$

Careful with complex ε $\rightarrow$ Can't use lossless equations even if $\sigma = 0$

Lossless ($\sigma = 0$) or small loss ($\delta \ll 1$)-dielectric shortcuts $\rightarrow$ $\beta \approx \omega\sqrt{\mu\varepsilon}$ $\quad v = \frac{\omega}{\beta} \approx \frac{1}{\sqrt{\varepsilon\mu}}$ $\quad \lambda \approx \frac{\lambda_0}{\sqrt{\mu_r \varepsilon_r}}$ $\quad \eta \approx \sqrt{\mu/\varepsilon}$ $\quad \alpha = \frac{\sigma}{2}\sqrt{\mu/\varepsilon}$

High loss ($\delta \gg 1$)-conductor shortcuts $\rightarrow$ $\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$ $\quad \eta = (1+j)\sqrt{\frac{\mu\omega}{2\sigma}}$ $\quad V = \sqrt{\frac{2\omega}{\mu\sigma}}$

Skin depth $\delta_{SD} = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\sigma}}$ $\quad \hookrightarrow E(\delta_{SD}) = E/e$

Power density $S = \frac{1}{2}\mathrm{Re}\{\bar{E} \wedge \bar{H}^*\}$

Transmission coeff. $T = 1 + \Gamma = \frac{E_{20}^+}{E_{10}^+} = \frac{2\eta_2}{\eta_2 + \eta_1}$

Reflection coeff. $\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{E_{10}^-}{E_{10}^+}$

$\alpha = \frac{\alpha_{dB/km}}{8.636 \cdot 10^3}$ $\frac{Np}{m}$

$\bar{H} = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \measuredangle\eta)\bar{\mu}_y$

Antennas

$P_R = S A_e$ (Power) $\quad A_e = \frac{\lambda^2}{4\pi} D \eta_A$ (Gain G)

$\mathrm{Gain(dB)} = 10\log_{10} \mathrm{Gain}$

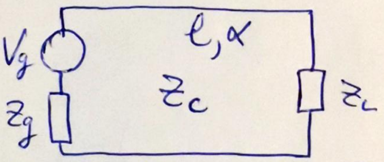
$P_{RX} = \frac{1}{2}\frac{|E|^2}{\eta_0}\frac{\lambda_0^2}{4\pi} G$

$S_r = S_i |\Gamma|^2$ $\quad S = \frac{1}{2}\frac{|E|^2}{|\eta|}\cos(\measuredangle\eta_1) e^{-2\alpha z} \rightarrow S = S_0 e^{-2\alpha z}$

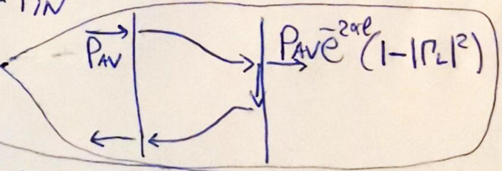
η no es η_{TM} ni η_{TE}!

No info on ε, μ or σ $\rightarrow \varepsilon_0, \mu_0, 0$.

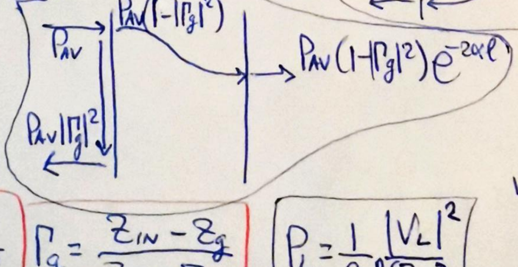
$\frac{1}{|\eta|\angle \mathrm{ang}} = \frac{1}{|\eta|}\angle(-)\mathrm{ang}$



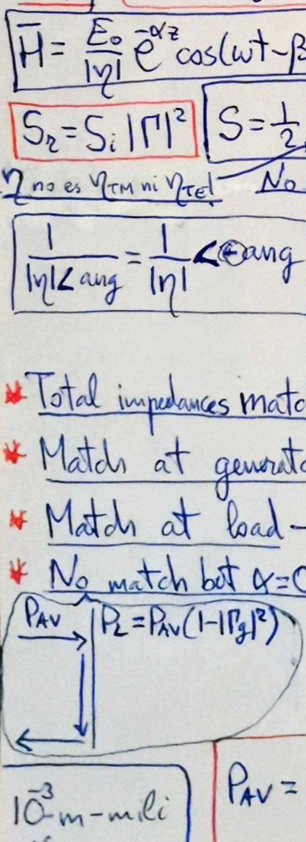
- Total impedances match $\rightarrow \Gamma_L = \Gamma_g = \Gamma_{IN} = 0$, $P_{AV} = P_{IN}$
- Match at generator side $\rightarrow Z_g = Z_C \rightarrow \Gamma_g = \Gamma_{IN}$



- Match at load $\rightarrow Z_C = Z_L$



- No match but $\alpha = 0$



$P_{AV} = \frac{V_g^2}{8\,\mathrm{Re}\{Z_g\}}$ $\quad \Gamma_g = \frac{Z_{IN} - Z_g}{Z_{IN} + Z_g}$ $\quad P_L = \frac{1}{2}\frac{|V_L|^2}{\mathrm{Re}\{Z_L\}}$

"Temporal trend" $\equiv V(t) = V\cos(\omega t)$

$V_{IN} = V_g \frac{Z_{IN}}{Z_{IN} + Z_g}$ $\quad \Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C}$

Z_{IN}: $\Gamma_{IN} = \Gamma_L e^{-2\alpha\ell} e^{-2j\beta\ell} \rightarrow Z_{IN} = Z_C \frac{1 + \Gamma_{IN}}{1 - \Gamma_{IN}}$

$Z_{IN} = \frac{Z_C^2}{Z_L}$ $\leftarrow$ Only if ℓ is multiple of $\lambda/4$

10^{-3} - m - mili	Kilo - K - 10^3
10^{-6} - μ - micro	mega - M - 10^6
10^{-9} - n - nano	giga - G - 10^9
10^{-12} - p - pico	

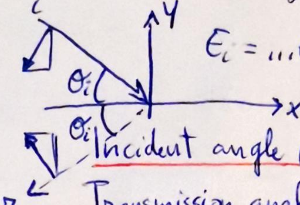
Additional concepts

EM

$\bar{E} = (a\bar{\mu}_x + b\bar{\mu}_y + c_j\bar{\mu}_z)(e^{-jx} e^{-jy}) \rightarrow$ Polarization $\begin{cases} \text{Transverse electric} - TE \text{ in } \hat{z} \\ \text{Transverse magnetic} - TM \text{ in } \hat{x} \text{ and } \hat{y} \end{cases}$

Case 2D: $E_x\bar{\mu}_x e^{-jz}_{\pm} E_y\bar{\mu}_y e^{-jz}$

Types:
- Linear $\rightarrow \phi = 0,\ z = 0,\ \forall E_x, \forall E_y$
- Circular $\rightarrow |E_x| = |E_y|,\ \phi = \pm\pi/2$
- Elliptical $\rightarrow \forall E_x, \forall E_y, \forall\phi$



$E_i = \dots e^{-j\beta\sin\theta y} e^{-j\beta\cos\theta x}$, with θ under each exponent

Incident angle θ_i

Transmission angle $\theta_t \leftarrow \boxed{\sin\theta_t = \sin\theta_i\sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r2}}}}$

Another example (Different abs. values for TE & TM components $\rightarrow$ Elliptical polarization)

$\bar{E} = a\bar{\mu}_x(e^{-jy} e^{-jz}) \rightarrow$ Polarization $\rightarrow$ Linear along $-\hat{x}$ (TE)

If $\theta_t \notin \mathbb{R} \rightarrow$ Total reflection of the wave, $|\Gamma| = 1$. Evanescent wave as the electric field penetrates.

$\boxed{\eta_{TE_x} = \frac{\eta_x}{\cos\theta_x}}$ $\boxed{\eta_{TM} = \eta_x\cos\theta_x}$

Brewster angle $\boxed{\tan\theta_B = \sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}}$ If $\theta_B = \theta_i$, total TM transmission $\hookrightarrow \Gamma_{TM} = 0$

$|z_1 + z_2| = \sqrt{z_1^2 + z_2^2 + |z_1\bar{z}_2 + \bar{z}_1 z_2|}$

SP

Disclaimer $\rightarrow X(f) = F(x(t))$; $Y(f) = F(y(t))$; $H(f) = F(h(t))$

$X^* \equiv$ Complex conjugate of X

$\boxed{\operatorname{sinc} x = \frac{\sin\pi x}{\pi x}}$

$z_{TX} \rightarrow \otimes \rightarrow S_{TX}$ (with $e^{j2\pi f_0 t}$ into the mixer: Modulation) $\quad h(t) \quad S_{RX} \rightarrow \otimes \rightarrow z_{RX}$ (with $e^{-j2\pi f_0 t}$: Demodulation)

Complex signal: S_{RX}; Complex envelope: z_{RX}

Properties of the F.T.

Linearity	$F(ax(t)+by(t)) = aF(x(t)) + bF(y(t))$
Delay	$F(x(t-t_0)) = F(x(t))\, e^{-j2\pi f t_0}$
Scaling	$F(x(at)) = \frac{1}{\lvert a\rvert} X(f/a)$
Convolution	$F(x(t) * y(t)) = F(x(t)) \cdot F(y(t))$
Duality	$F(X(-t)) = x(f)$ $\rightarrow$ $F(\operatorname{sinc}) = \operatorname{rect}$, $F(\operatorname{rect} t) = \operatorname{sinc}$
Complex	$F(x^*(t)) = X^*(-f)$

$\boxed{F(x(t)\, e^{+j2\pi f_0 t}) = X(f - f_0)}$

$\sin\alpha = \frac{1}{2j}(e^{j\alpha} - e^{-j\alpha})$

$\cos\alpha = \frac{1}{2}(e^{j\alpha} + e^{-j\alpha})$

$F(e^{j2\pi f_0 t}) = \delta(f - f_0)$ | $x(t) * \delta(t - \tau) = x(t - \tau)$

$F(\operatorname{rect}(\frac{t}{T})) = \frac{\sin(\pi f T)}{\pi f} = T\operatorname{sinc}(fT)$

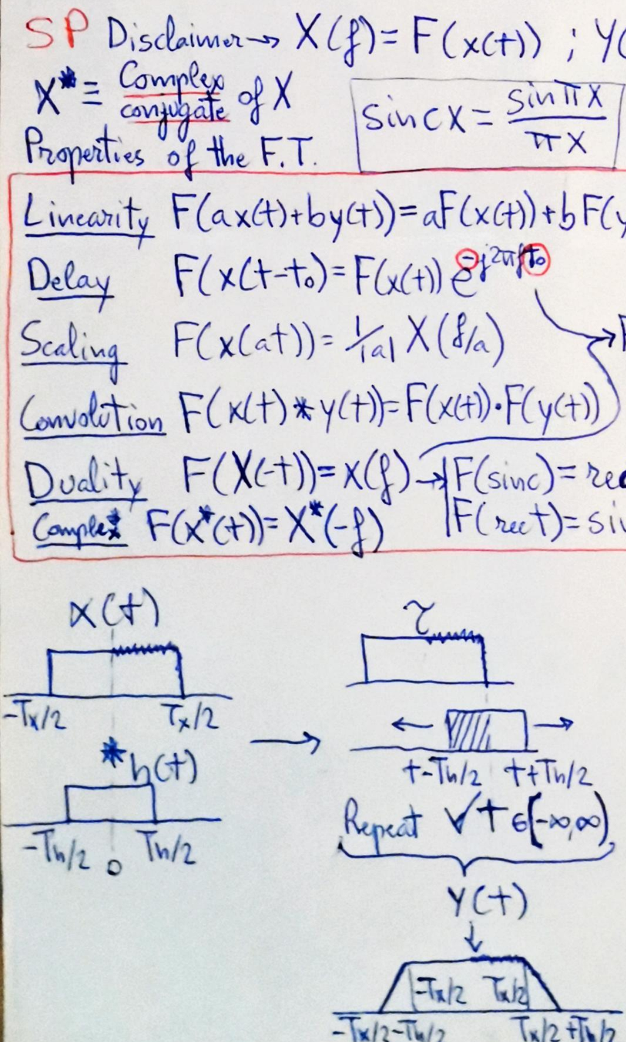


Table of Common Functions and their Fourier Transforms

Function name	Function in the time domain	Fourier Transform (in the frequency domain)
	$w(t)$	$\hat{W}(f)$
Dirac delta	$\delta(t)$	1
Constant	1	$\delta(f)$
Cosine	$\cos(2\pi f_0 t)$	$\frac{\delta(f-f_0)+\delta(f+f_0)}{2}$
Sine	$\sin(2\pi f_0 t)$	$\frac{\delta(f-f_0)-\delta(f+f_0)}{2j}$
Unit step function	$u(t) = \begin{cases} 0, \text{ if } t < 0 \\ 1, \text{ if } t \geq 0 \end{cases}$	$\frac{1}{j\omega}$ (for $\omega = 2\pi f$)
Decaying exponential (for t > 0)	$e^{-at}u(t)$.	$\frac{1}{a+j2\pi f}$, $a > 0$
Box or rectangle function	$\text{rect}(at) = \begin{cases} 0, \text{ if } \lvert at\rvert > \frac{1}{2} \\ 1, \text{ if } \lvert at\rvert \leq \frac{1}{2} \end{cases}$	$\frac{1}{\lvert a\rvert}\text{sinc}\left(\frac{f}{a}\right) = \frac{\sin(\pi f/a)}{\pi f/a}$
Sinc function	$\text{sinc}(at) = \frac{\sin(\pi at)}{\pi at}$	$\frac{1}{\lvert a\rvert}\text{rect}\left(\frac{f}{a}\right)$
Comb function	$\sum_{n=-\infty}^{\infty} \delta(t-nT)$	$\frac{1}{T}\sum_{k=-\infty}^{\infty} \delta(f-\frac{k}{T})$
Gaussian	e^{-at^2}	$\sqrt{\frac{\pi}{\alpha}}e^{-\frac{(\pi f)^2}{\alpha}}$

Table of Fourier Transform Pairs and Properties

	Expression in the time domain	Expression's Fourier Transform (in the frequency domain)
Definition	$w(t) = \int_{-\infty}^{\infty} \hat{W}(f) e^{j2\pi f t} df$	$\hat{W}(f) = \int_{-\infty}^{\infty} w(t) e^{-j2\pi f t} dt$
Linearity	$a\, w_1(t) + b\, w_2(t)$	$a\, \hat{W}_1(f) + b\, \hat{W}_2(f)$
Shift	$w(t-a)$	$\hat{W}(f) e^{-2\pi j a f}$
Convolution	$\int_{-\infty}^{\infty} w(\tau) v(t-\tau) d\tau$	$\hat{W}(f)\hat{V}(f)$
Product	$w(t)v(t)$	$\int_{-\infty}^{\infty} \hat{W}(v)\hat{V}(f-v) dv$
Scaling	$w(at)$	$\frac{1}{\lvert a \rvert} \hat{W}\left(\frac{f}{a}\right)$
Differentiation	$\frac{d}{dt} w(t)$	$j\omega \hat{W}(\omega)$, where $\omega = 2\pi f$
Integration	$\int_{-\infty}^{\infty} w(t)\, dt$	$\frac{\hat{W}(\omega)}{j\omega}$
Two Dimensional	$w(x,y) = \int\int_{-\infty}^{\infty} \hat{W}(u,v) e^{j2\pi(ux+vy)} du\, dv$	$\hat{W}(u,v) = \int\int_{-\infty}^{\infty} w(x,y) e^{-j2\pi(ux+vy)} dx\, dy$