EMSP - Part 2 Signal Processing - exercises in preparation of the final examination

Ex 1

A complex signal $g(t)$ with (two-sided) bandwidth equal to $B = 1$GHz is transmitted using a carrier frequency $f_0 = 50$ GHz and then received after $t_0 = 1$ microsecond.

1. Write the expression of the complex transmitted signal, of the complex received signal, and of the complex received signal after demodulation (the complex envelope).
2. Describe a procedure to measure the delay and assess the temporal resolution of such measurement.
3. Assume now that $g(t)$ is a chirp signal, $g(t) = rect\left(\frac{t}{T}\right) exp\left(j\pi K t^2\right)$. Determine the value of the chirp rate K assuming a total duration $T = 100$ microseconds.
4. Comment on why you can achieve a temporal resolution much better than pulse length
5. Draw a graph of the absolute value of the Fourier Transform of $g(t)$.
6. Write a pseudo-code to form the complex envelope (assuming complex tranmission and reception) and to measure the delay

tips for the solution

1. $s_{Tx}(t) = g(t) exp(j2\pi f_0 t)$, $s_{Rx}(t) = g(t - t_0) exp(j2\pi f_0 (t - t_0))$, $z(t) = s_{Rx}(t) exp(-j2\pi f_0 t) = g(t - t_0) exp(-j2\pi f_0 t_0)$
2. find the peak in the cross-correlation
3. chirp bandwidth = KT (look back on the Matlab codes)
4. What matters is the bandwidth
5. just look back on the Matlab codes - it is OK to approximate it to a rectangular pulse in the frequency domain!
6. just look back on the Matlab codes

Ex 2

Reconsider exercise 1, and assume now that the signal arriving at the receiver is contributed by the direct path, delayed by t_0, plus a reflection, delayed by $t_1 = t_0 + \Delta t$.

1. Write the expression of the complex received signal, and of the complex received signal after demodulation (the complex envelope).
2. Draw a graph of the absolute value of the complex envelope
3. Describe a procedure to compensate for the reflection and discuss the role of Δt
4. Write a pseud-code to implement point 3

tips for the solution

1. By linearity $z(t) = g(t - t_0)\, exp(-j2\pi f_0 t_0) + g(t - t_1)\, exp(-j2\pi f_0 t_1)$
2. $Z(f) = G(f) \cdot [exp(-j2\pi(f + f_0)t_0) + exp(-j2\pi(f + f_0)t_1)] = G(f) \cdot [1 + exp(-j2\pi(f + f_0)\Delta t)] \cdot exp(-j2\pi(f + f_0)t_0) =>$

$$|Z(f)| = rect\left(\frac{f}{B}\right) \cdot |1 + exp(-j2\pi(f + f_0)\Delta t)|$$

$$|Z(f)| = \sqrt{2} rect\left(\frac{f}{B}\right) \cdot \sqrt{1 + cos(2\pi(f + f_0)\Delta t)}$$

   hence: if $2\pi f_0 \Delta t = \pi$ we have nearly perfect destructive interferenc . In addition to that, any value of $\Delta t > \frac{1}{B}$ will determine amplitude oscillations. *Tip 1: don't focus on the square root - just try to understand when it reaches its minimim and maximum values. Tip 2: you can implement this on Matlab and take a look with your eyes.*

3. All it takes is an inverse filter $H_i(f) = ([1 + exp(-j2\pi(f + f_0)\Delta t)])^{-1}$. Of course, no complete inversion is possible at frequencies where $[1 + exp(-j2\pi(f + f_0)\Delta t)] = 0$. Hence you should require $\Delta t < \frac{1}{B}$ and $2\pi f_0 \Delta t \neq \pi$
4. just look back on the Matlab codes

Ex 3

Consider the 2D signal given as:

$$s(x, y) = rect(x^2 + y^2 - 1)$$

1. Draw the graph of $s(x, y)$. This can be done by considering s either as any other function of two variables, or simply as picture, by colouring all pixels for which $s(x, y) > 0$.

2. Draw a graph of $s(x, y)$ after low pass filtering in both directions
3. Draw a graph of $s(x, y)$ after high pass filtering in the x-direction
4. Draw a graph of $s(x, y)$ after high pass filtering in the y-direction
5. Write a simple algorithm to extract the contours of any picture

tips for the solution

1. Tip: just go back to the original definition of the rectangular pulse: $rect(t) = 1$ whenever $|t| < \frac{1}{2}$, otherwise it is 0.
2. you're just required to do it qualitatively, do not focus on mathematical aspects
3. same as 2 and 3
4. same as 2 and 3
5. a possibility is to generate a new signal $d(x, y) = \left(\frac{\partial s}{\partial x}\right)^2 + \left(\frac{\partial s}{\partial y}\right)^2$, where the partial derivatives $\frac{\partial s}{\partial x}$ and $\frac{\partial s}{\partial y}$ can be interpreted as high-pass filters (to see why, go back to the properties of the Fourier Transform. What is the Fourier Transform of a derivative?). Summing the squares of the partial derivatives is instrumental to ensuring that no cancellation occurs (i.e.: pixels at which one derivative is positive and the other is negative). Note that using the partial derivatives is just one possible solution. In general, any high-pass filter would do.