H_0 Uncertainties in the Era of LIGO

Shivani Shah

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Co-advisors: Prof. Steve Eikenberry & Prof. Anthony Gonzalez

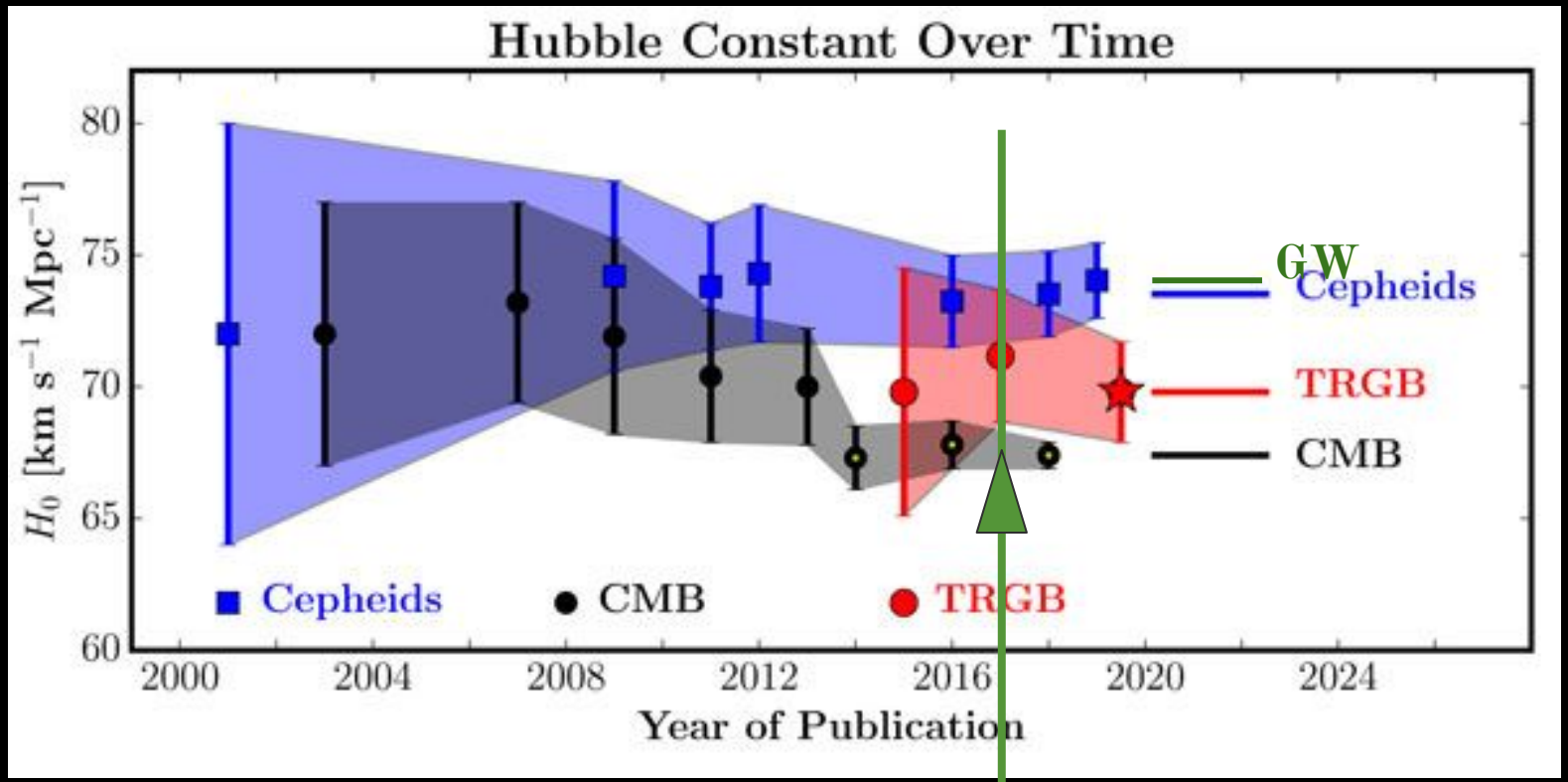


Image Credit: Freedman et al. 2019
2019

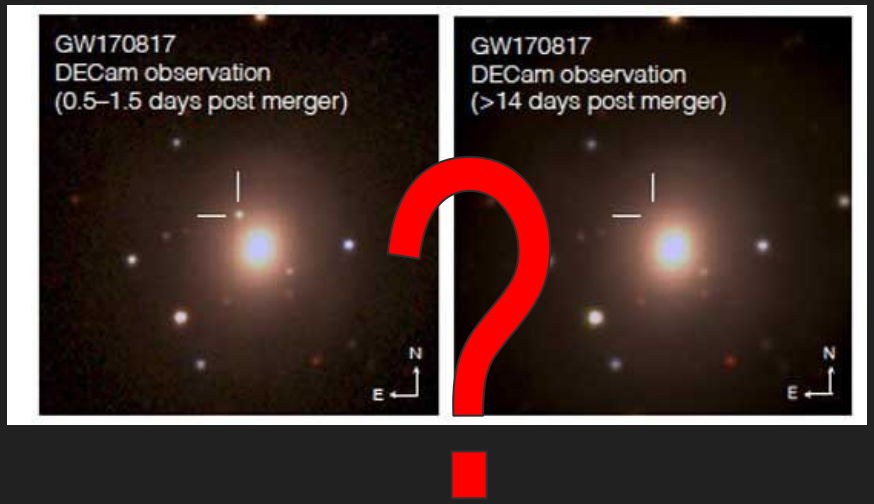


Image Credit: Soares-Santos et al. 2017

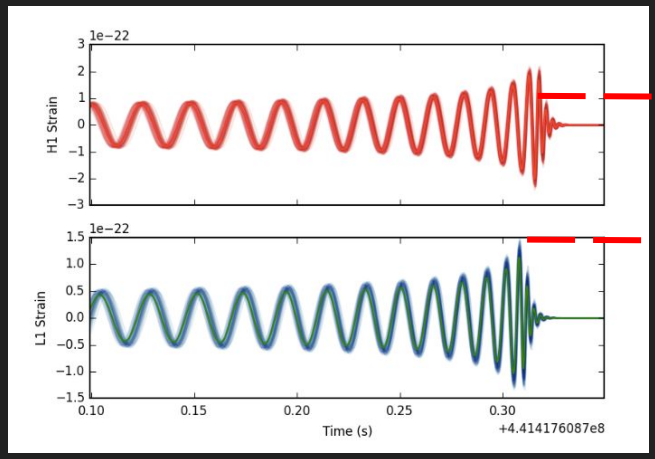


Image Credit: V. Raymond/AEI

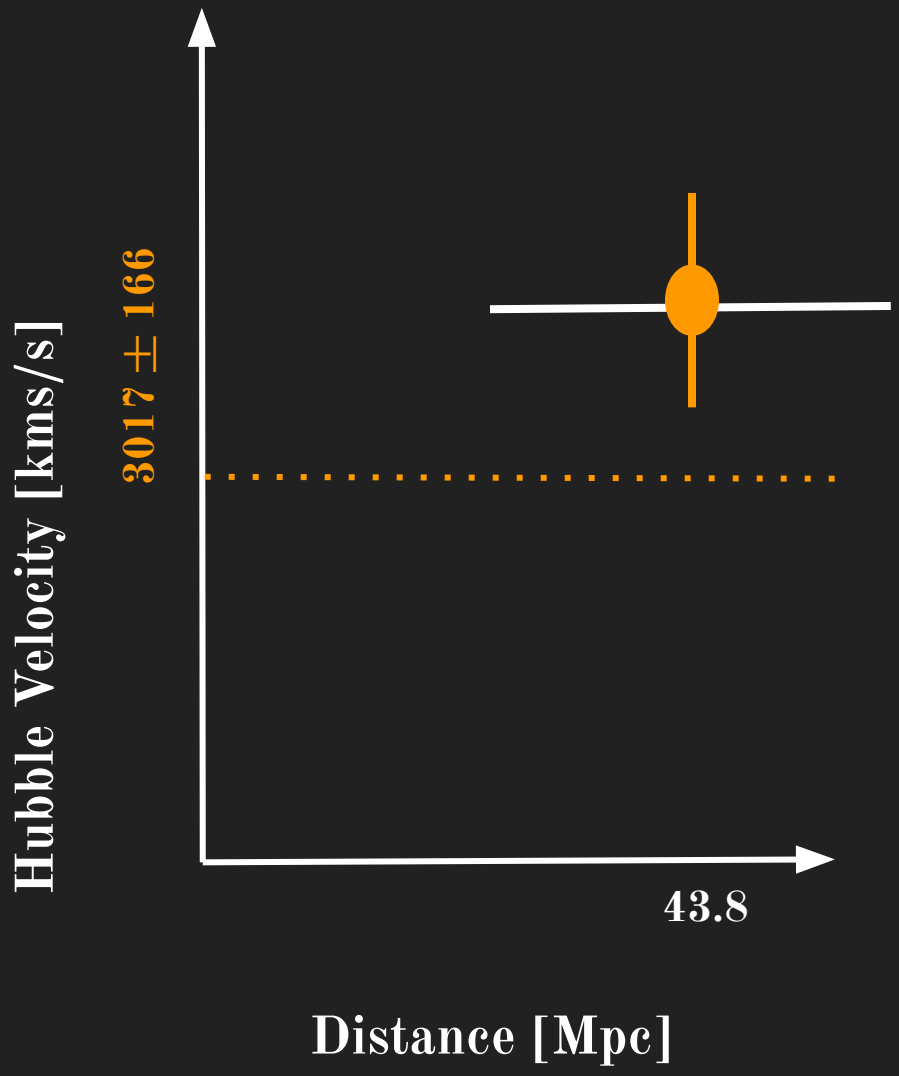
Hubble Velocity [km/s]

43.8

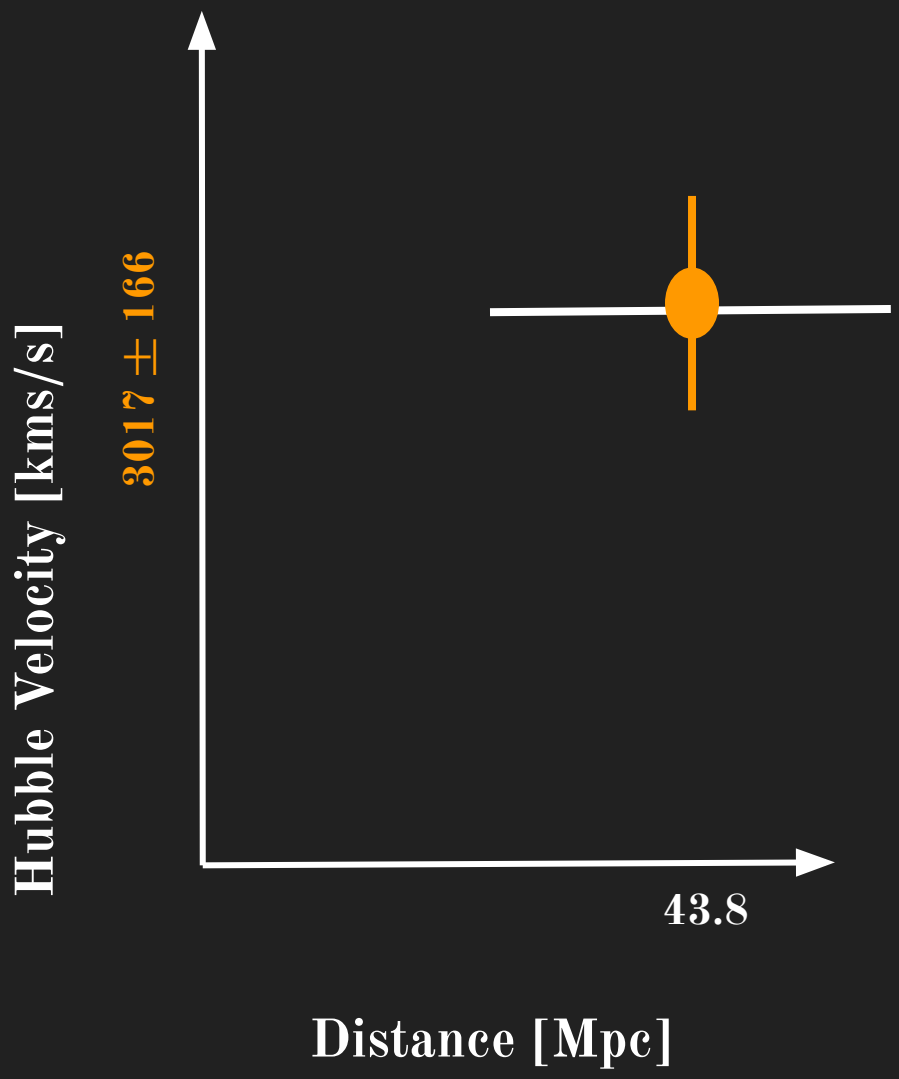
Distance [Mpc]

LIGO's correction for Peculiar Velocity of host galaxy

- ➔ Host galaxy identified with a group of galaxies
- ➔ Adopted the center-of-mass recessional velocity of the group
- ➔ Used peculiar velocity maps built using Fundamental Plane relation to correct for the bulk velocity of the group



- Is this a reliable measurement?



- Is this a reliable measurement?
- Is this an efficient technique?

Investigating reliability of the measurement & efficiency of the technique

1. What is the contamination and incompletion ratio of the grouping algorithm used
2. How accurate is the peculiar velocity map used ?
3. Given this treatment of recessional velocity, will the measurements be systematically biased?
4. Is this the most efficient treatment of recessional velocity if we want to obtain a precise measurement ASAP i.e., what is its rate of convergence compared to other techniques?

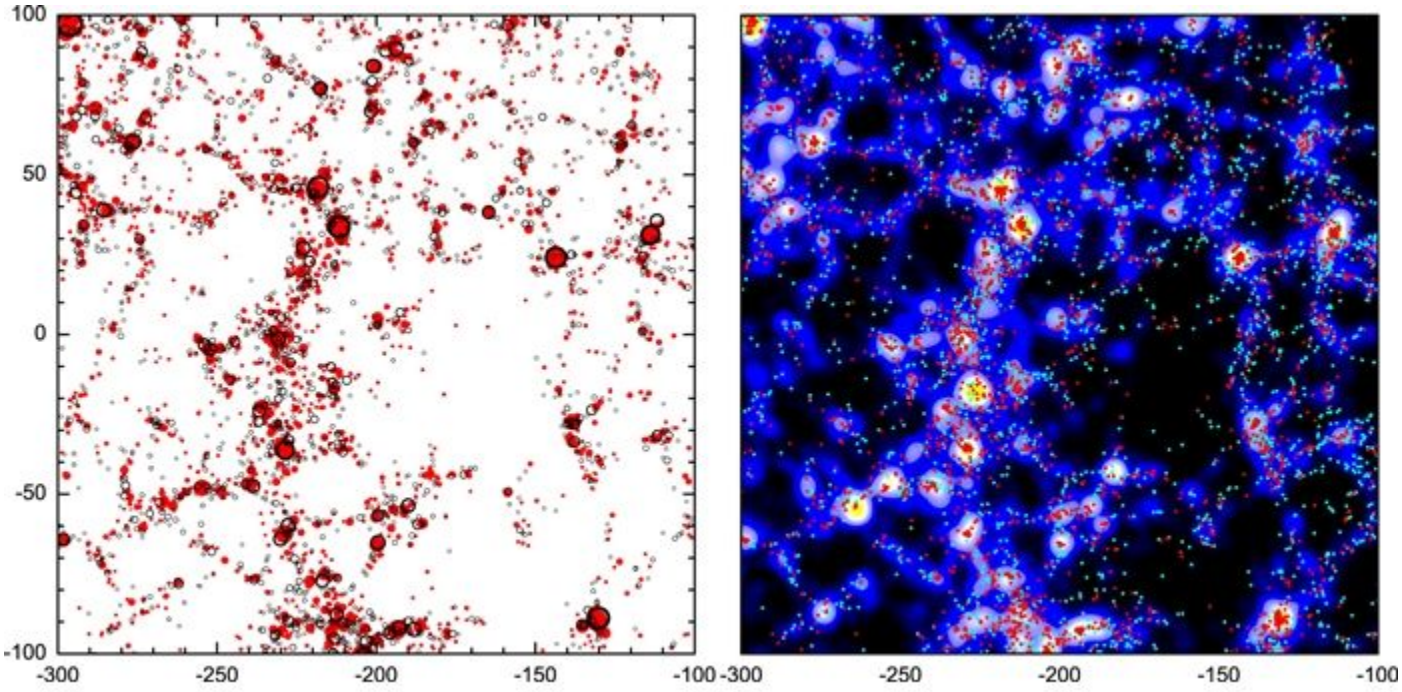


Figure 8 from ELUCID - Exploring the Local Universe with ReConstructed Initial Density Field III: Constrained Simulation in the SDSS Volume
Huiyuan Wang et al. 2016 ApJ 831 164 doi:10.3847/0004-637X/831/2/164

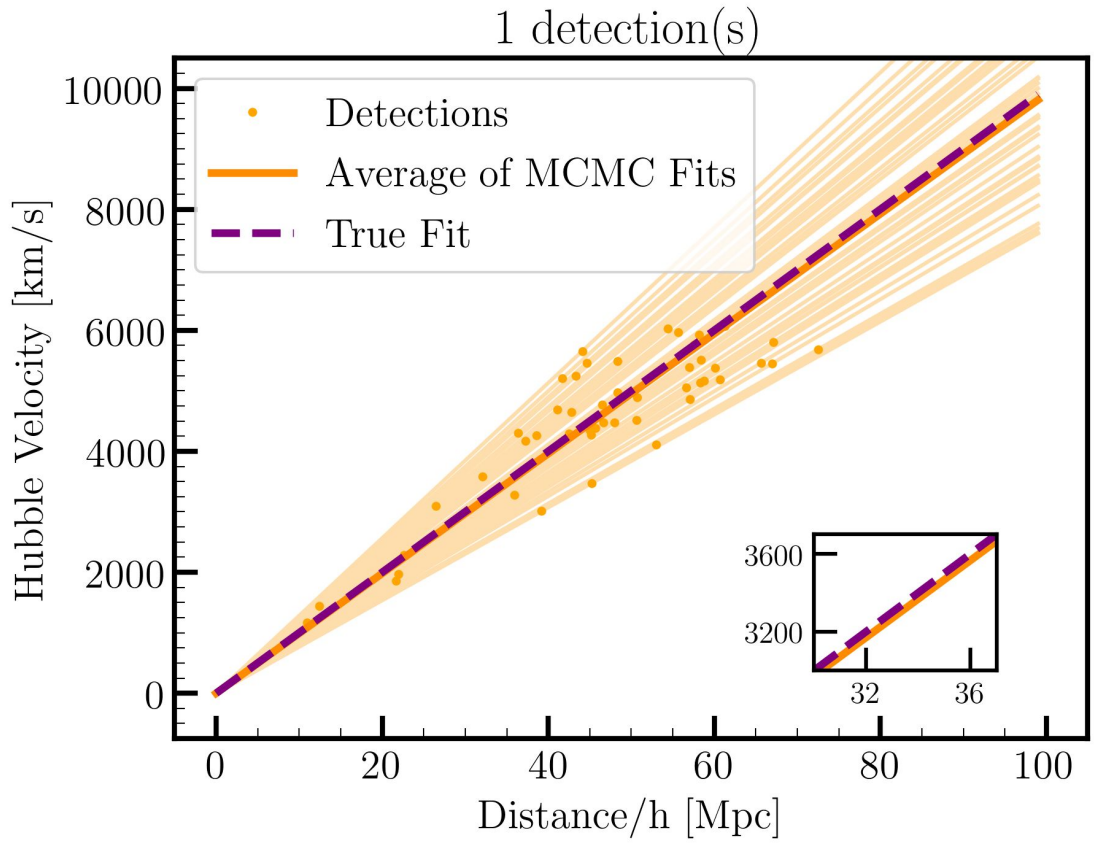
Tangible Questions

1. Contamination and incompletion ratio
2. Accuracy of peculiar velocity map?
3. Systematic bias?
4. Convergence rate?

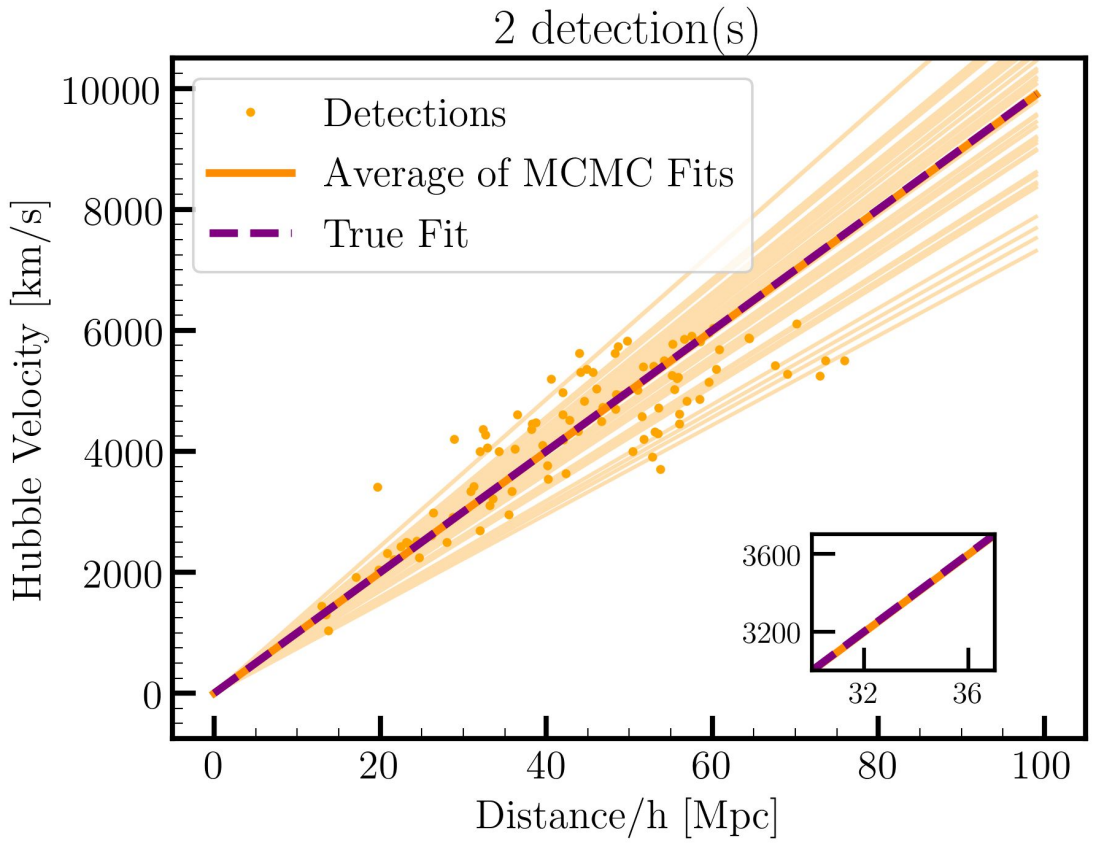
Methods

1. Add galaxies to the simulation - HODs
2. Build observational groups
   a. Build grouping algorithm
   b. Calibrate the algorithm
3. Compare observational groups to true groups
4. ?
5. Obtain distribution of H_0 at different number of detections

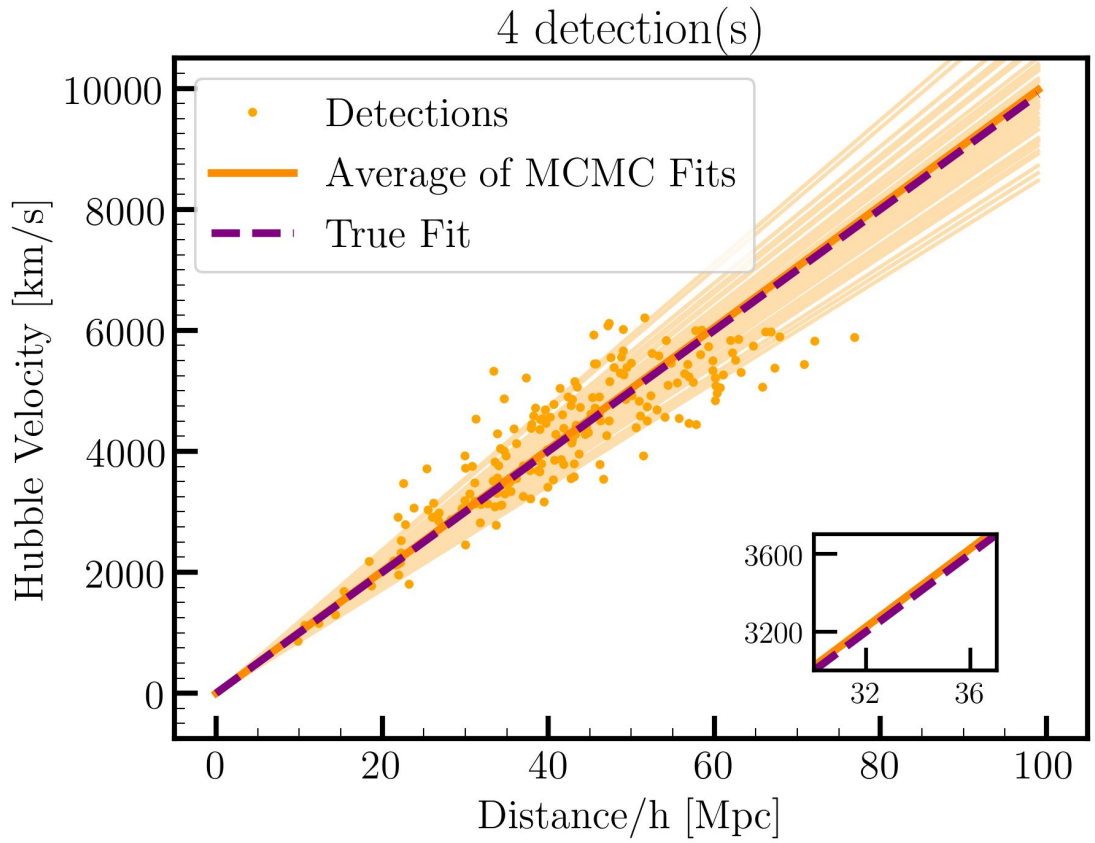
Obtaining H_0 distribution



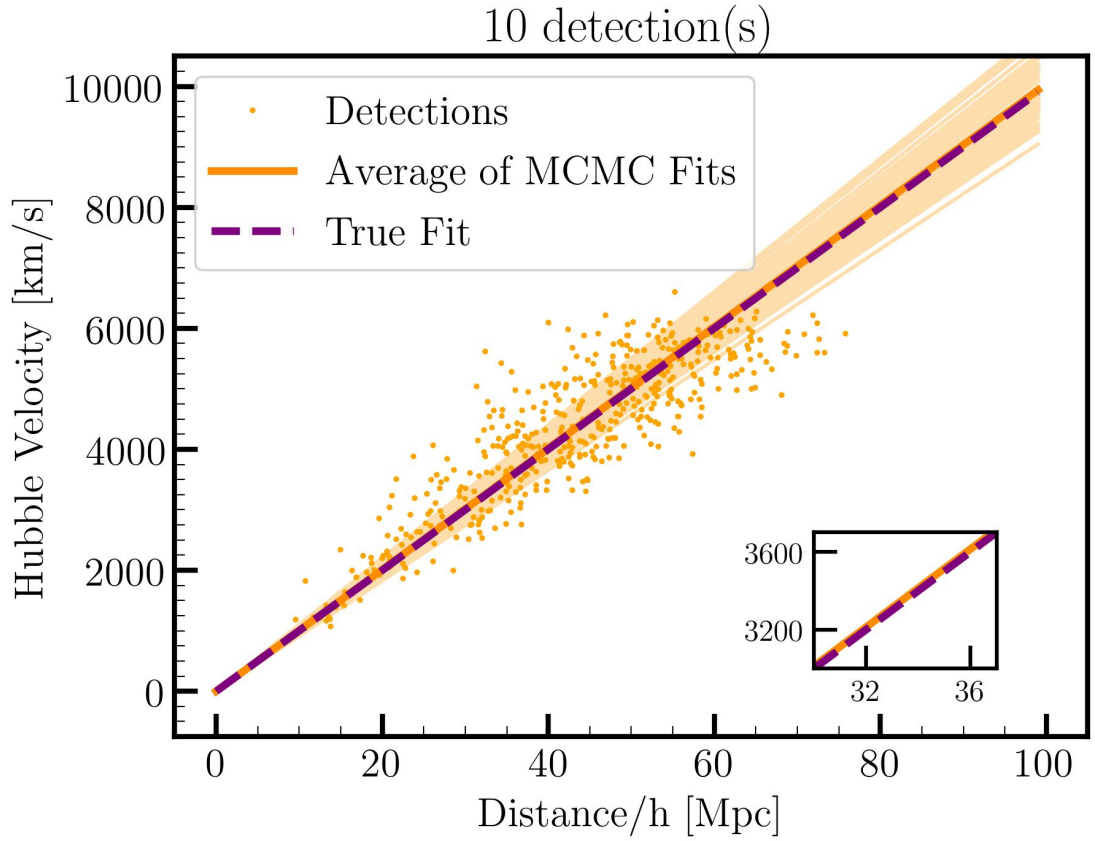
Obtaining H_0 distribution



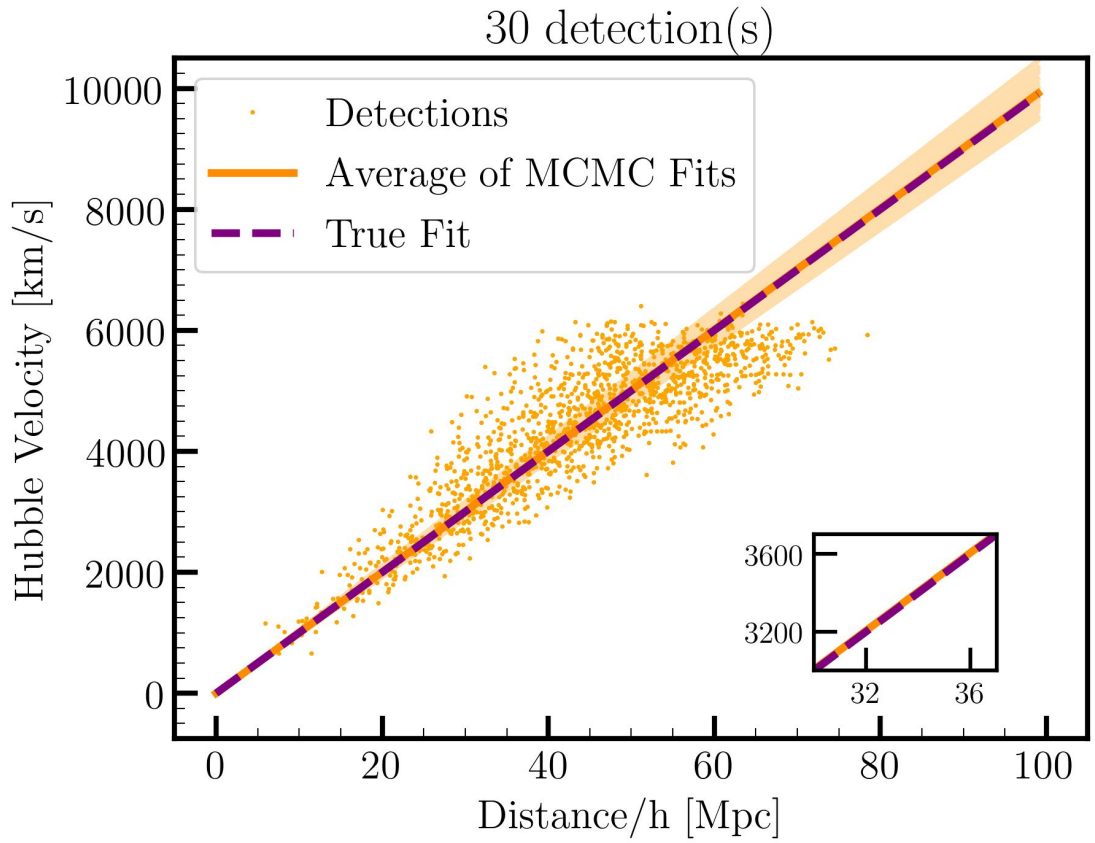
Obtaining H_0 distribution



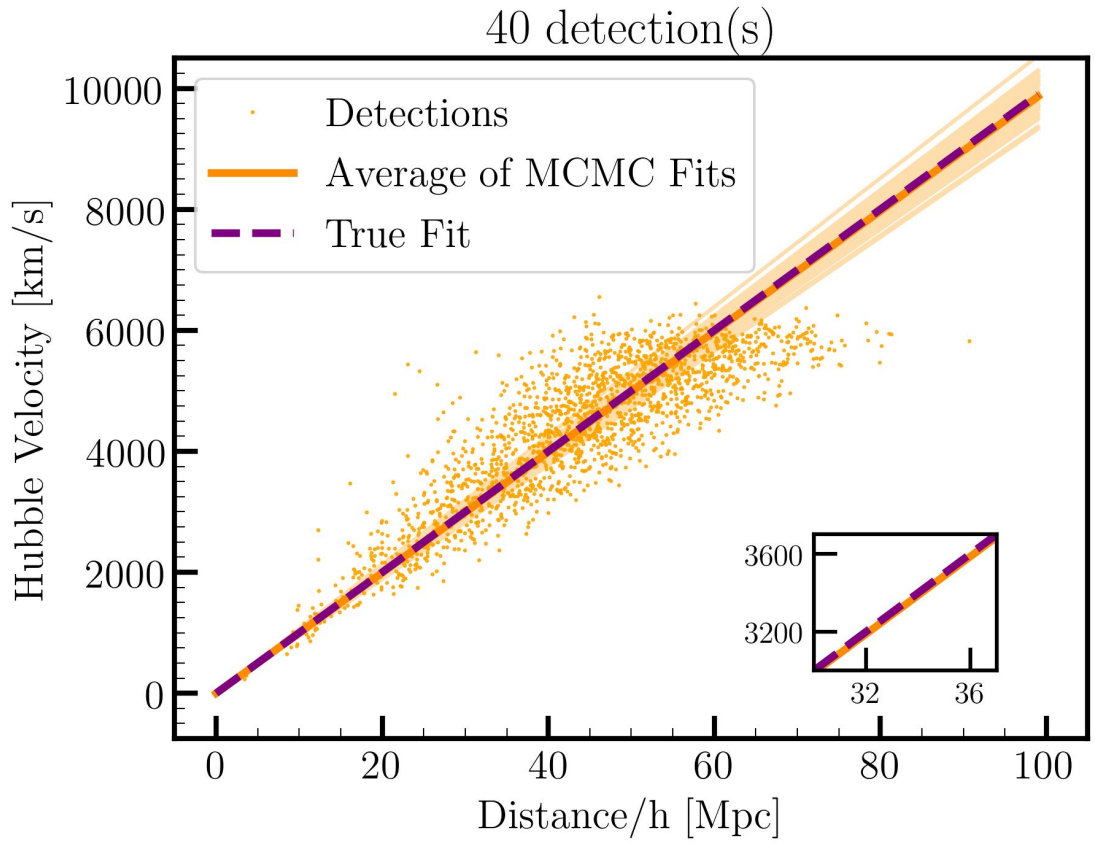
Obtaining H_0 distribution



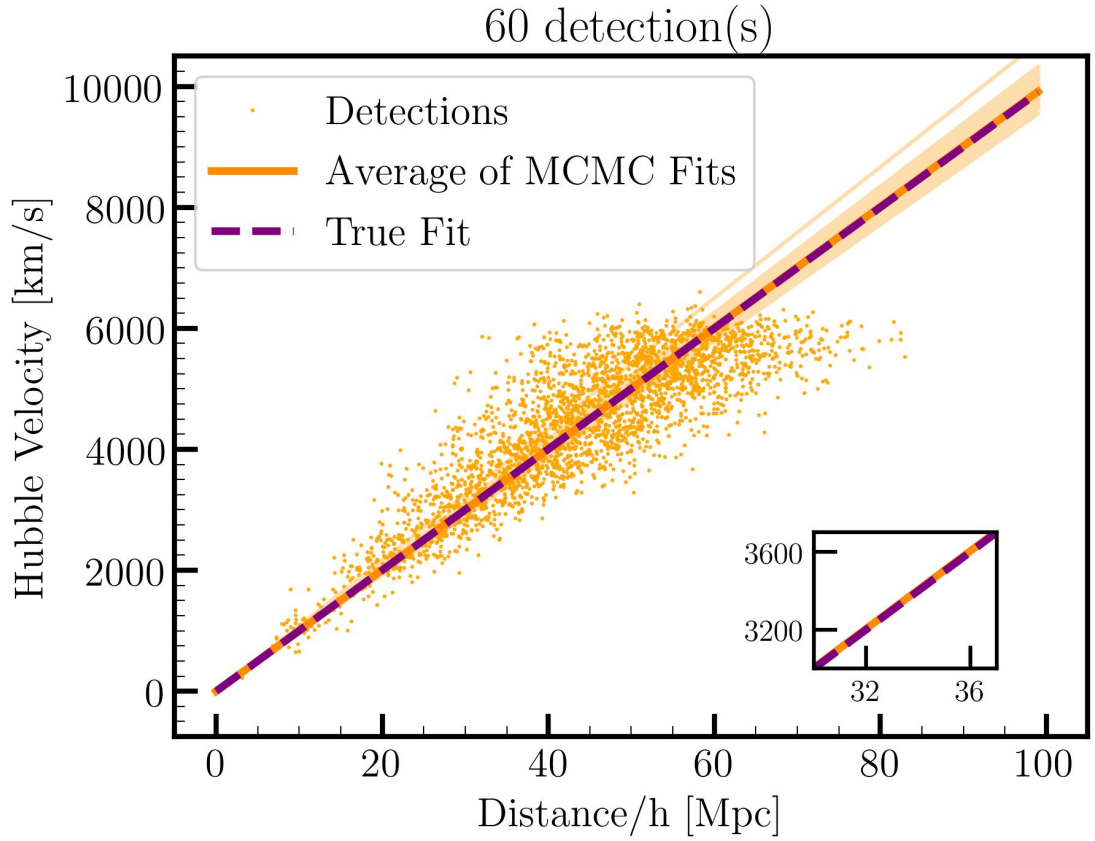
Obtaining H_0 distribution



Obtaining H_0 distribution



Obtaining H_0 distribution



Evolution of H_0 distribution parameters

Fiducial Model: $V_H = V_{recessional} \pm V_{pec,SD}$

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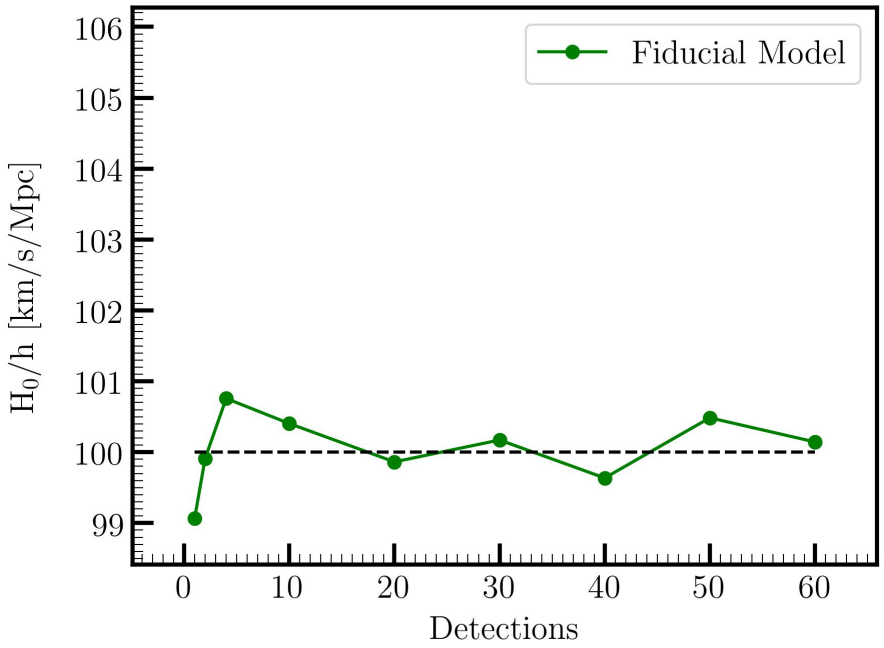
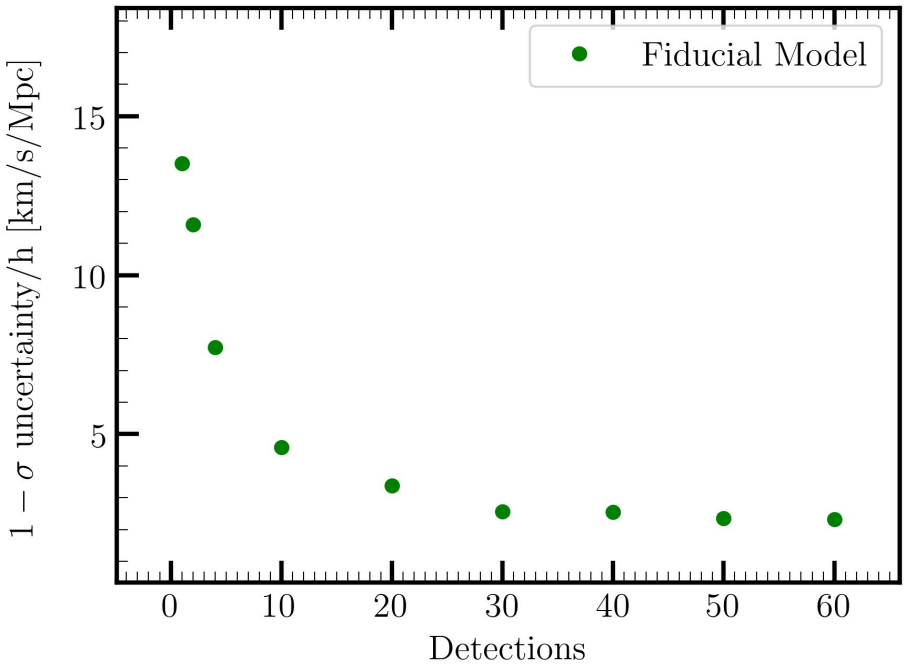
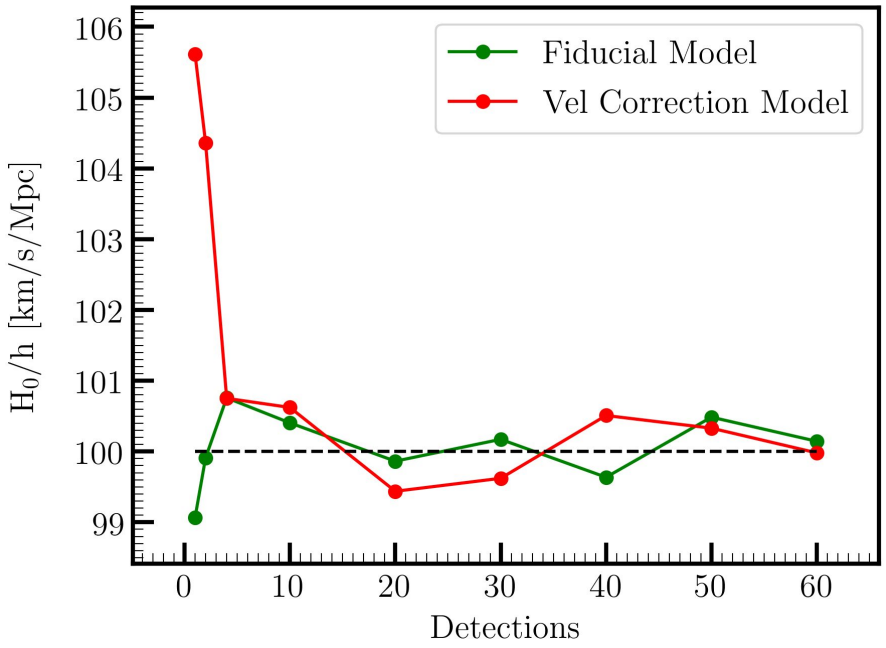


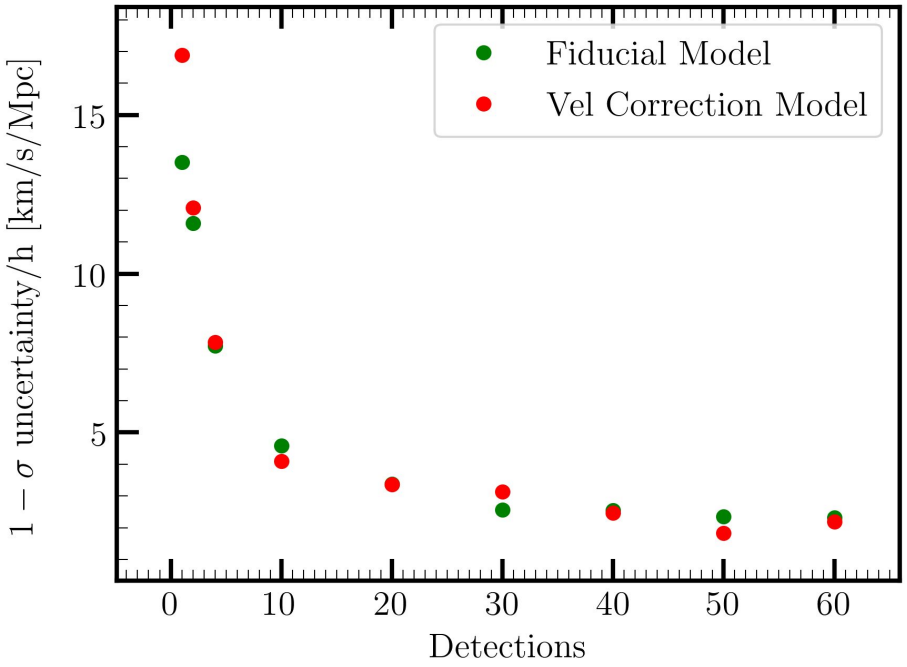


Evolution of H_0 distribution parameters

Fiducial Model: $V_H = V_{recessional} \pm V_{peculiar,SD}$

Velocity Correction Model: $V_H = (V_{recessional} - \langle V_{pec} \rangle) \pm V_{pec,SD}$



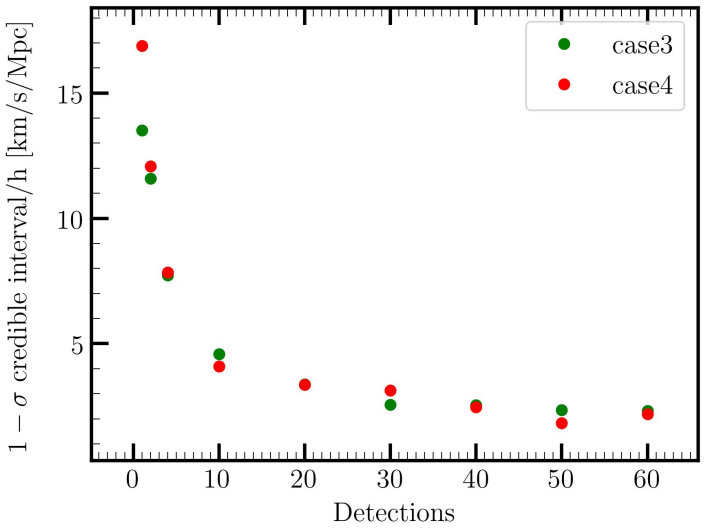


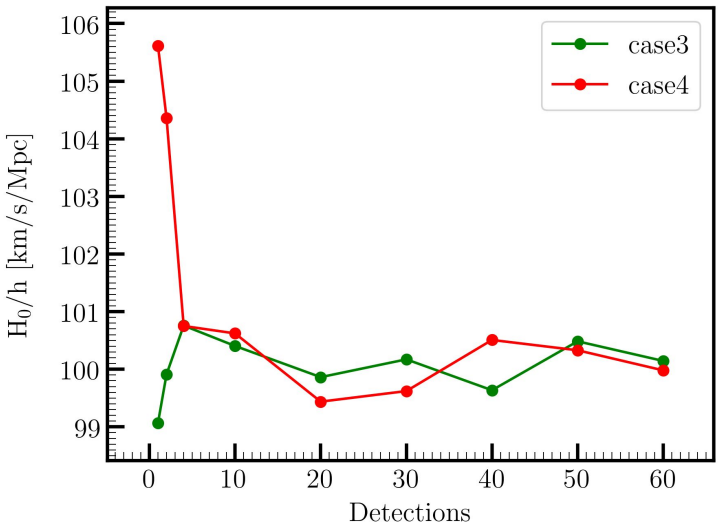
Summary

I am studying the **reliability** of the Hubble constant measurement made using GW170817 event and the **efficiency** of the technique used by asking the following questions:

1. What is the contamination and incompletion ratio of the grouping algorithm used
2. How accurate is the peculiar velocity map used?
3. Given this treatment of recessional velocity, will the measurements be systematically biased?
4. Is this the most efficient treatment of recessional velocity if we want to obtain a precise measurement ASAP i.e., what is the convergence rate of the measurement?

II. A Stepping Stone





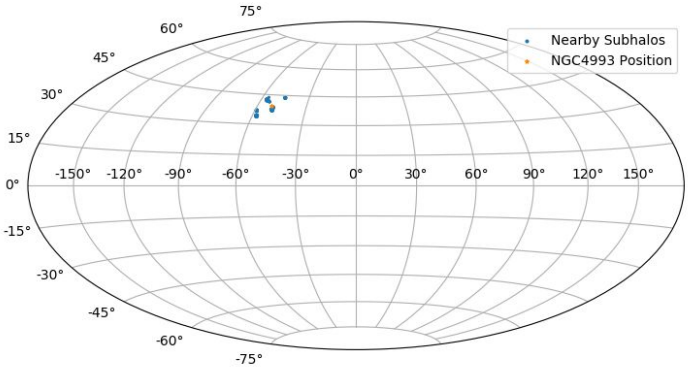
Studying the reliability of Hubble velocity measurement

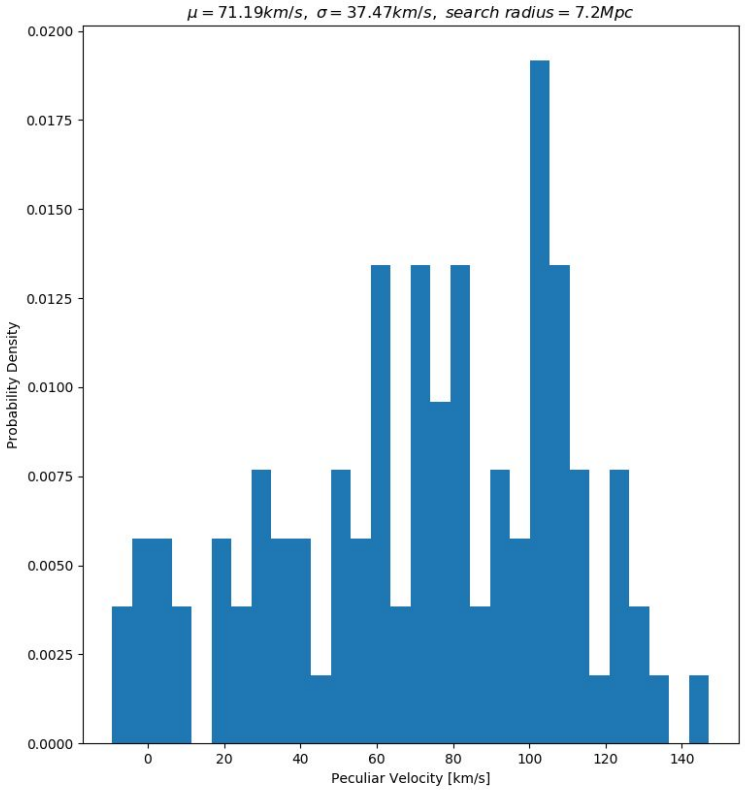
➔ Could it affect the convergence rate of the Hubble constant's measurement to a more precise value?

➔ Could this type of correction for peculiar velocity introduce a systematic bias?

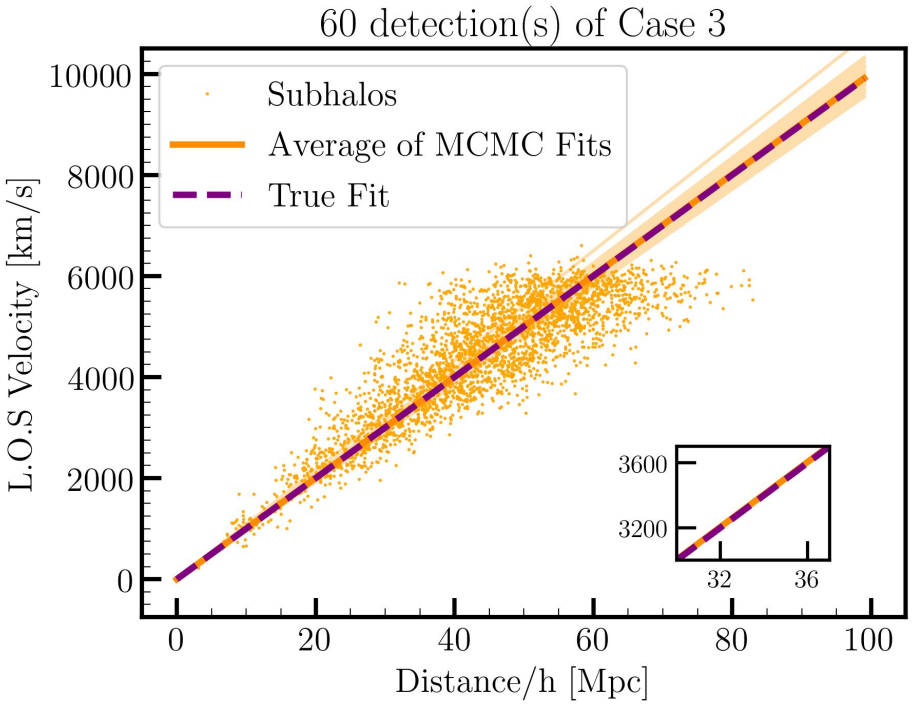
Comparing

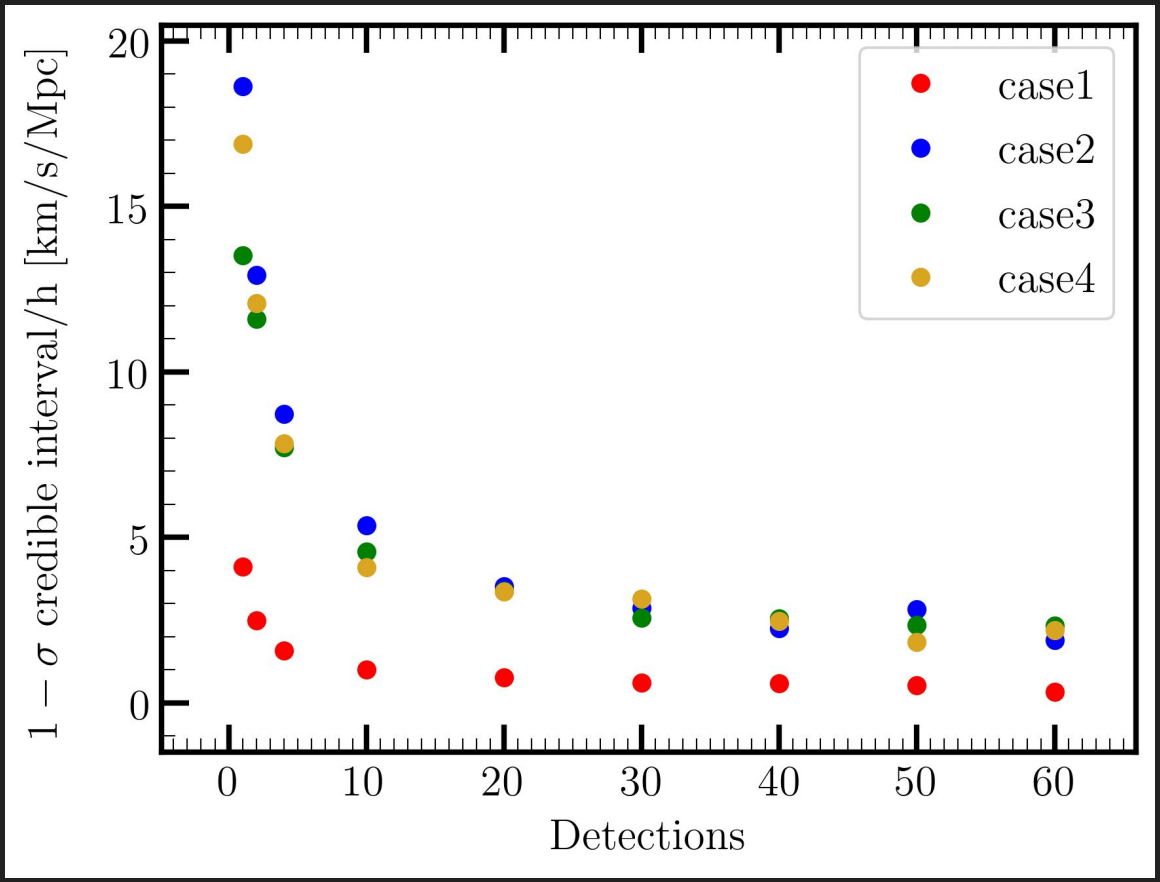
1. We modify the source recessional velocity as per LIGO analysis
2. We add appropriate error bars to the recessional velocity of the source





Peculiar Velocity Distribution at the position of NGC 4993
(EM counterpart of the first NS-NS merger)


60 detection(s) of Case 3
Subhalos
Average of MCMC Fits
True Fit
L.O.S Velocity [km/s]
Distance/h [Mpc]
0
2000
4000
6000
8000
10000
0
20
40
60
80
100
3600
3200
32
36


20
15
10
5
0
0
10
20
30
40
50
60
Detections
$1-\sigma$ credible interval/h [km/s/Mpc]
case1
case2
case3
case4

Scenario	Distance Measurements	Velocity Measurements
I.	$D_{LIGO} = (D_{true} + D_{random}) \pm 0.15*D_{true}$	$V_H = V_{Tot,true} \pm V_{pec,SD}$
II.	$D_{LIGO} = (D_{true} + D_{random}) \pm 0.15*D_{true}$	$V_H = (V_{Tot,true} - \langle V_{pec} \rangle) \pm V_{pec,SD}$ LIGO Way

LIGO's correction for Peculiar Velocity of NGC 4993

- ➔ NGC 4993 identified with HDC 763 group

- ➔ Adopted the mass-weighted average group recessional velocity as the recessional velocity of the source

- ➔ Used peculiar velocity maps built using Fundamental Plane relation to correct for the bulk velocity of the group

Comparing

1. We modify the source recessional velocity as per LIGO analysis
2. We add appropriate error bars to the recessional velocity of the source

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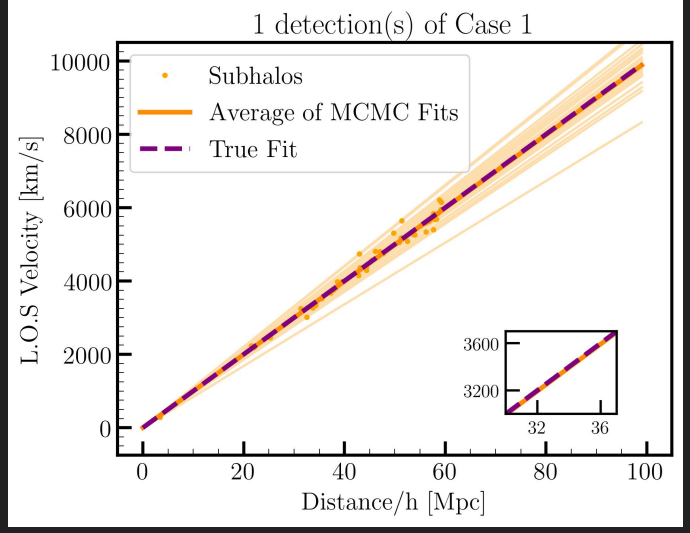
- ➔ NGC 4993 identified with HDC 763 group
- ➔ Assigned the mass-weighted average group velocity to the recessional velocity of NGC 4993
- ➔ Used peculiar velocity maps built using Fundamental Plane to correct for the bulk velocity of the group

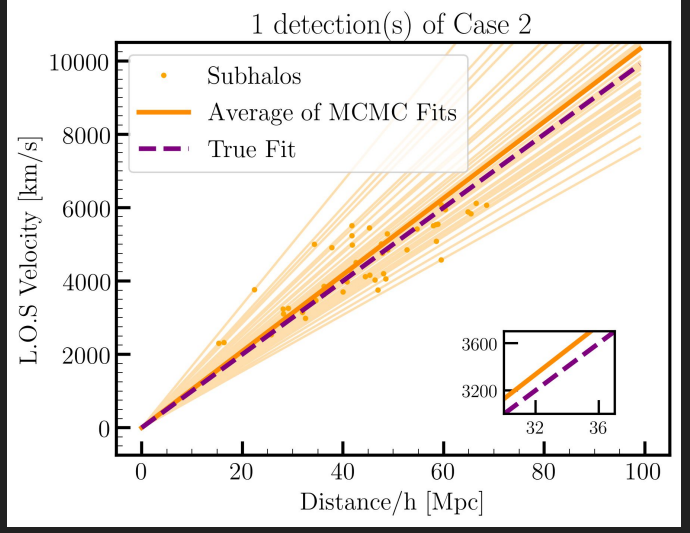
Setup

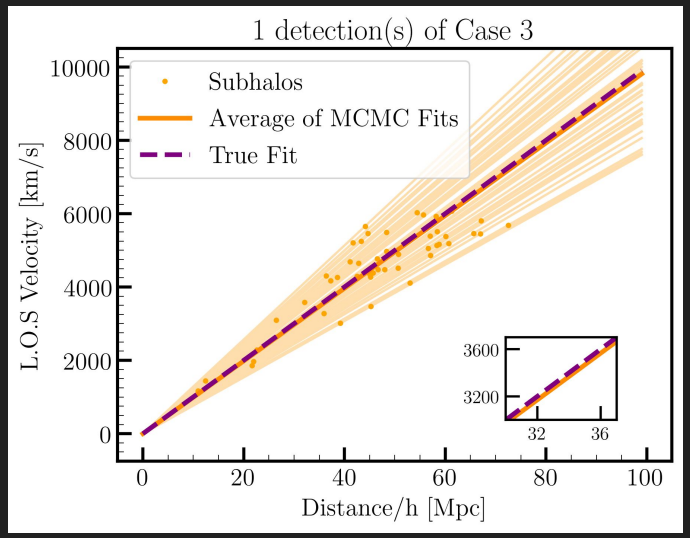
1. We use 5 800^3 simulations.
2. We consider 1,2,4,10,20,30,40,50 and 60 detections in 48 universes.
3. To represent a detection we draw a random subhalo weighted by stellar mass.
4. For each detection set, we consider four scenarios:

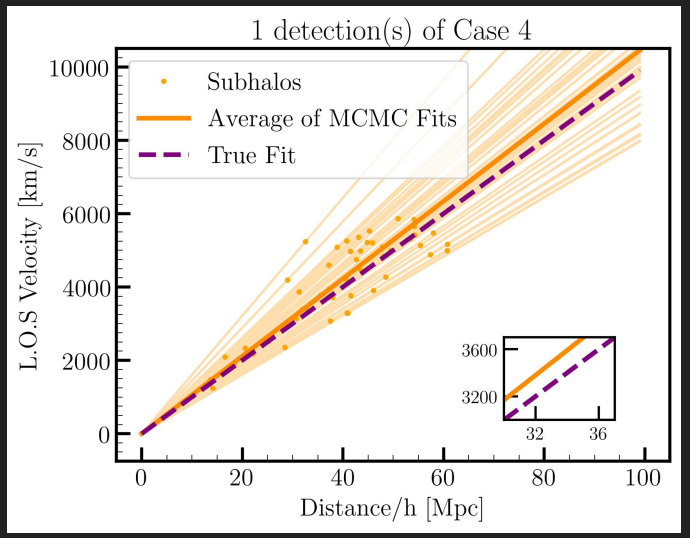
Case	Distance Measurements	Velocity Measurements
I.	D_{true}	$V_{Tot,true}$
II.	$D_{LIGO} = (D_{true} + D_{random}) +/- 0.15*D_{true}$	$V_{Tot,true}$
III.	$D_{LIGO} = (D_{true} + D_{random}) +/- 0.15*D_{true}$	$V_{Tot,true} +/- V_{pec,SD}$
IV.	$D_{LIGO} = (D_{true} + D_{random}) +/- 0.15*D_{true}$	$V_H = (V_{Tot,true} - \langle V_{pec} \rangle) +/- V_{pec,SD}$

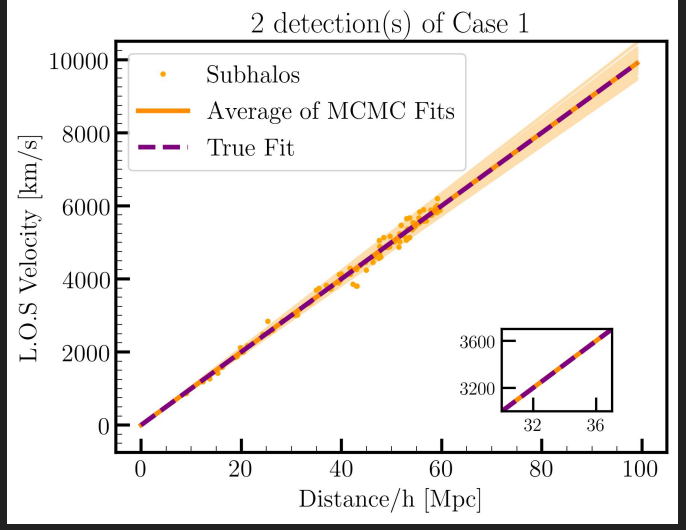
5. We use MCMC to obtain the best H_0 fit to each set of detections in each universe.

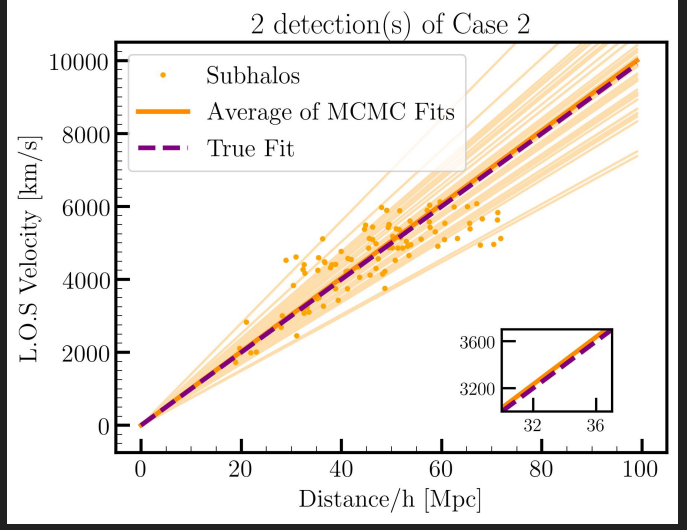

1 detection(s) of Case 1
Subhalos
Average of MCMC Fits
True Fit
L.O.S Velocity [km/s]
Distance/h [Mpc]
0
2000
4000
6000
8000
10000
0
20
40
60
80
100
3600
3200
32
36

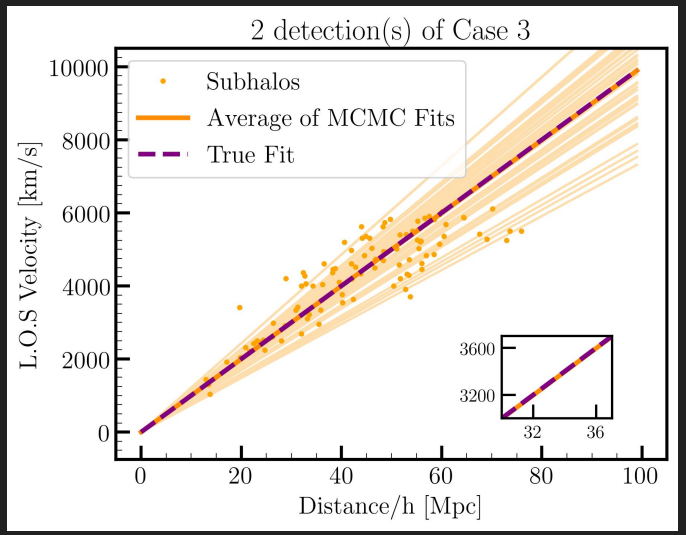

1 detection(s) of Case 2
Subhalos
Average of MCMC Fits
True Fit
L.O.S Velocity [km/s]
Distance/h [Mpc]
0
2000
4000
6000
8000
10000
0
20
40
60
80
100
3600
3200
32
36

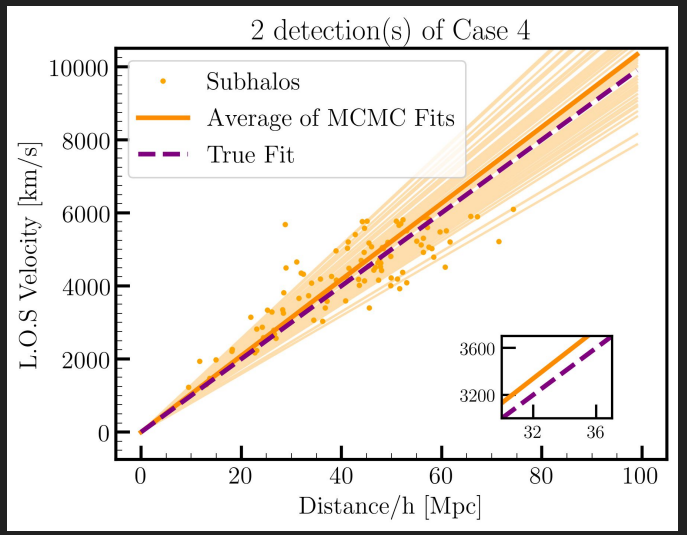

1 detection(s) of Case 3
Subhalos
Average of MCMC Fits
True Fit
L.O.S Velocity [km/s]
Distance/h [Mpc]
0
2000
4000
6000
8000
10000
0
20
40
60
80
100
3600
3200
32
36

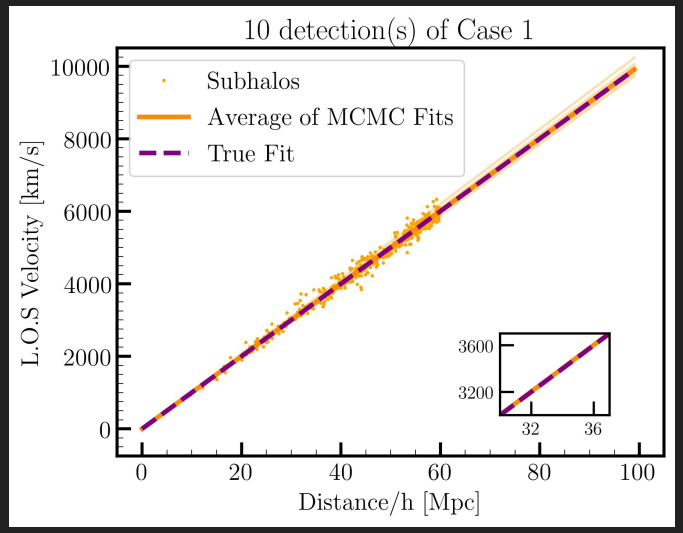

1 detection(s) of Case 4
Subhalos
Average of MCMC Fits
True Fit
L.O.S Velocity [km/s]
Distance/h [Mpc]
0
2000
4000
6000
8000
10000
0
20
40
60
80
100
3600
3200
32
36

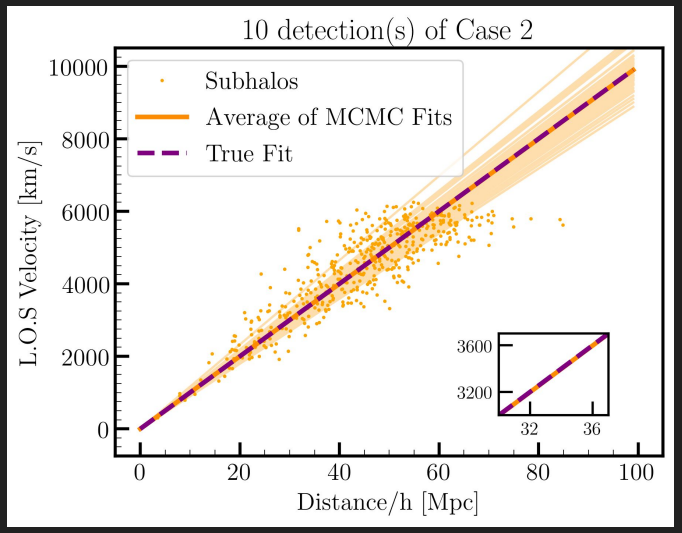

2 detection(s) of Case 1
Subhalos
Average of MCMC Fits
True Fit
L.O.S Velocity [km/s]
Distance/h [Mpc]
0
2000
4000
6000
8000
10000
0
20
40
60
80
100
3600
3200
32
36

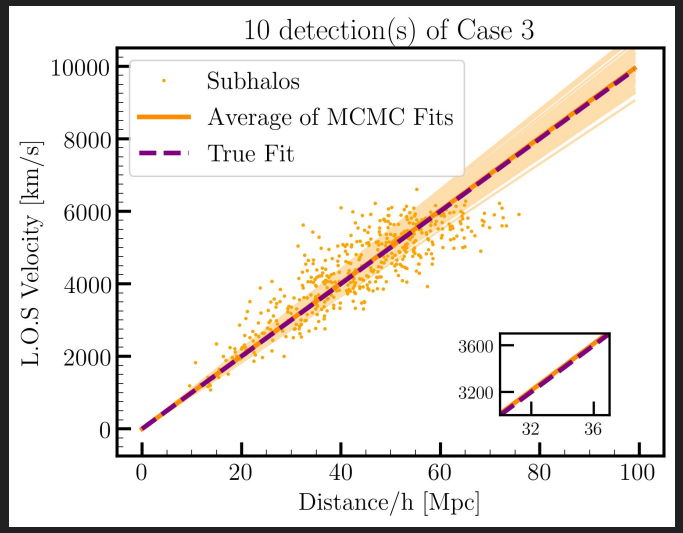

2 detection(s) of Case 2
Subhalos
Average of MCMC Fits
True Fit
L.O.S Velocity [km/s]
Distance/h [Mpc]
0
2000
4000
6000
8000
10000
0
20
40
60
80
100
3600
3200
32
36

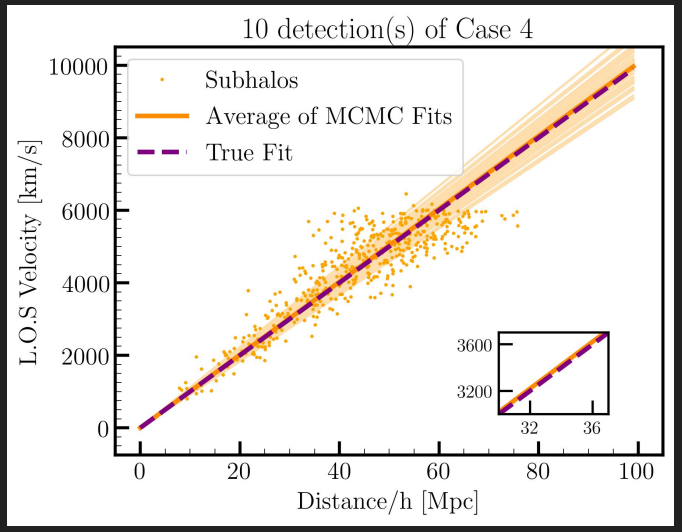

2 detection(s) of Case 3
Subhalos
Average of MCMC Fits
True Fit
L.O.S Velocity [km/s]
Distance/h [Mpc]
0
2000
4000
6000
8000
10000
0
20
40
60
80
100
3600
3200
32
36

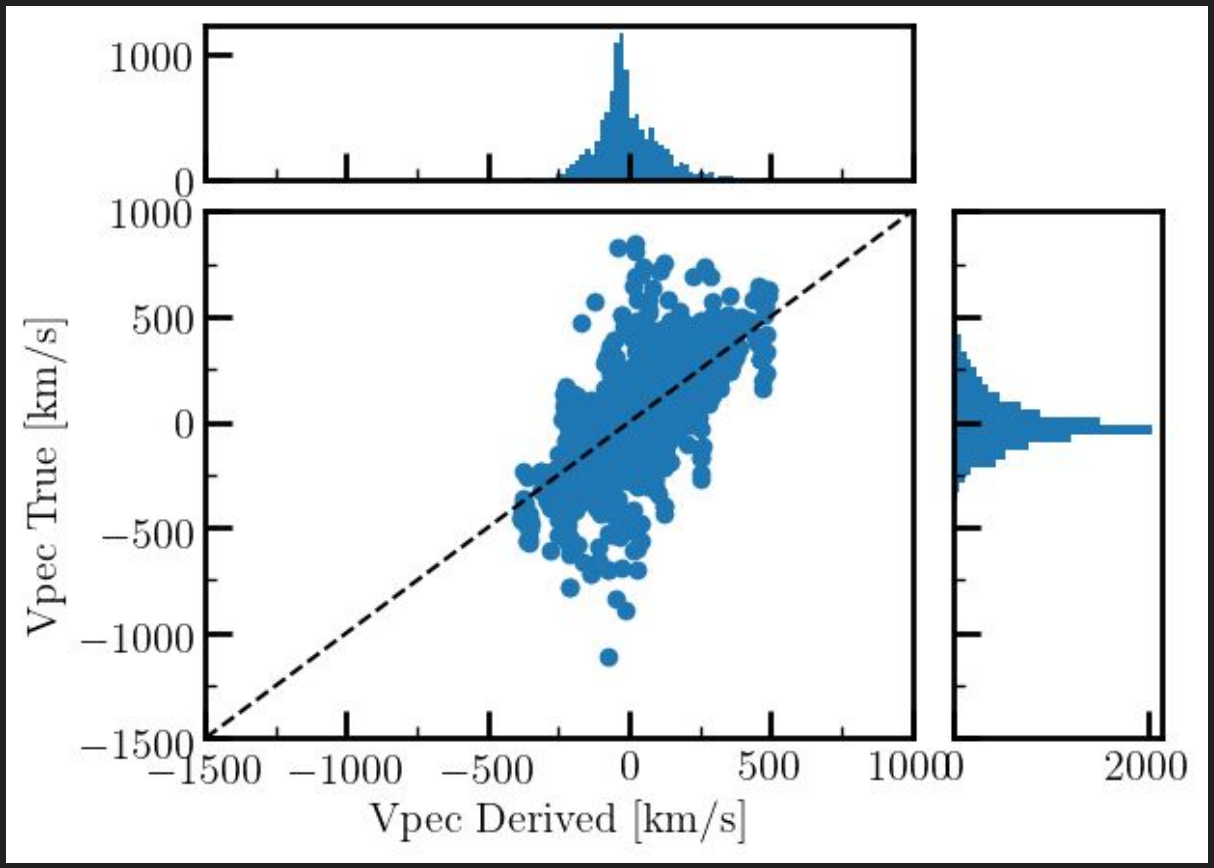

2 detection(s) of Case 4
Subhalos
Average of MCMC Fits
True Fit
L.O.S Velocity [km/s]
Distance/h [Mpc]
0
2000
4000
6000
8000
10000
0
20
40
60
80
100
3600
3200
32
36

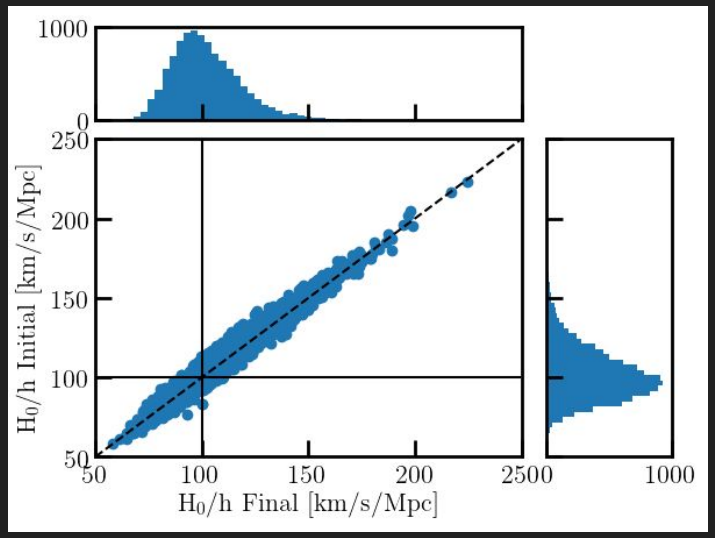

10 detection(s) of Case 1
Subhalos
Average of MCMC Fits
True Fit
L.O.S Velocity [km/s]
Distance/h [Mpc]
0
2000
4000
6000
8000
10000
0
20
40
60
80
100
3600
3200
32
36


10 detection(s) of Case 2
Subhalos
Average of MCMC Fits
True Fit
L.O.S Velocity [km/s]
Distance/h [Mpc]
0
2000
4000
6000
8000
10000
0
20
40
60
80
100
3600
3200
32
36

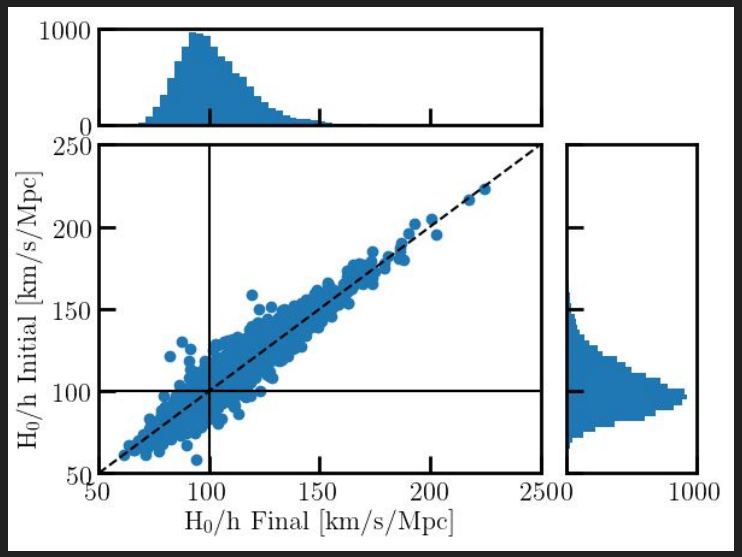

10 detection(s) of Case 3
Subhalos
Average of MCMC Fits
True Fit
L.O.S Velocity [km/s]
Distance/h [Mpc]
0
2000
4000
6000
8000
10000
0
20
40
60
80
100
3600
3200
32
36


10 detection(s) of Case 4
Subhalos
Average of MCMC Fits
True Fit
L.O.S Velocity [km/s]
Distance/h [Mpc]
0
2000
4000
6000
8000
10000
0
20
40
60
80
100
3600
3200
32
36


1000
0
1000
500
0
−500
−1000
−1500
Vpec True [km/s]
−1500
−1000
−500
0
500
1000
2000
Vpec Derived [km/s]



Total velocity corrected by the mean of peculiar velocity distribution.



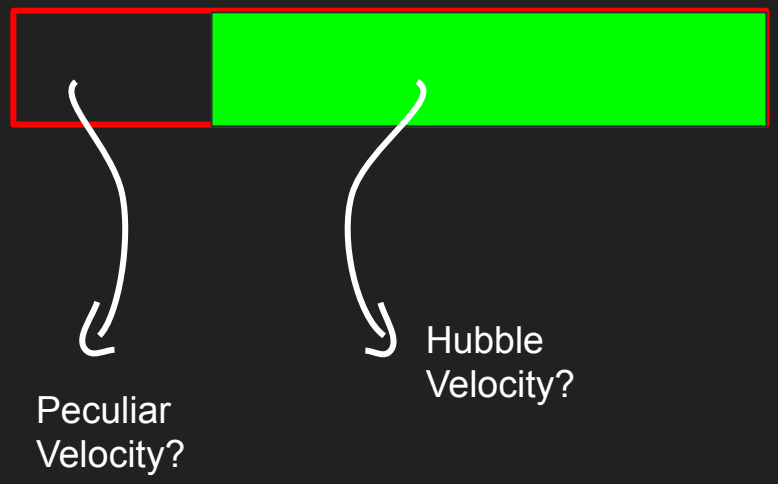
Total velocity corrected by the true peculiar velocity of the subhalo.

Questions

1. How will correcting for peculiar velocity using simulations with constrained initial condition impact the convergence rate of H0? Will there be a systematic bias?

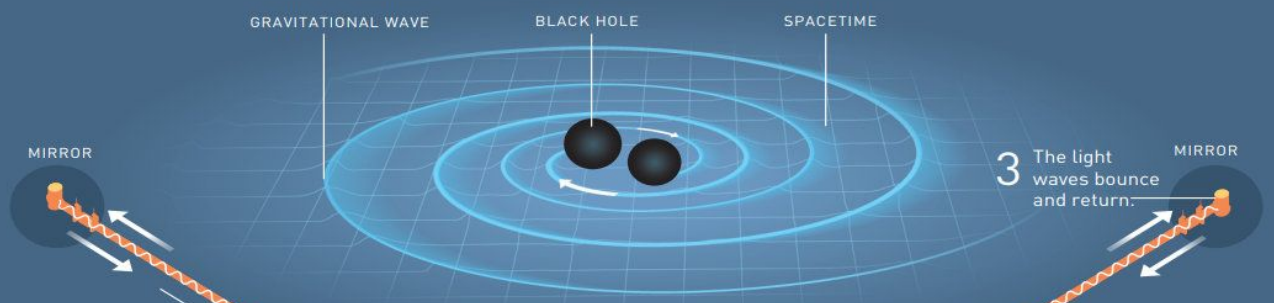
2. What if we continue to follow LIGO's analysis? Will this introduce a systematic bias? What will the convergence rate be?

3. What is the best way to correct for peculiar velocity?



END

LIGO – A GIGANTIC INTERFEROMETER



1 Laser light is sent into the instrument to measure changes in the length of the two arms.

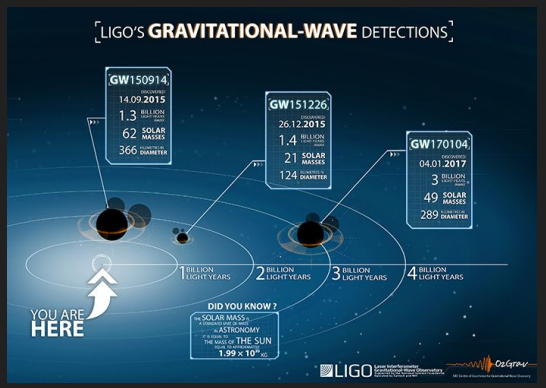
2 A "beam splitter" splits the light and sends out two identical beams along the 4 km long arms.

3 The light waves bounce and return.

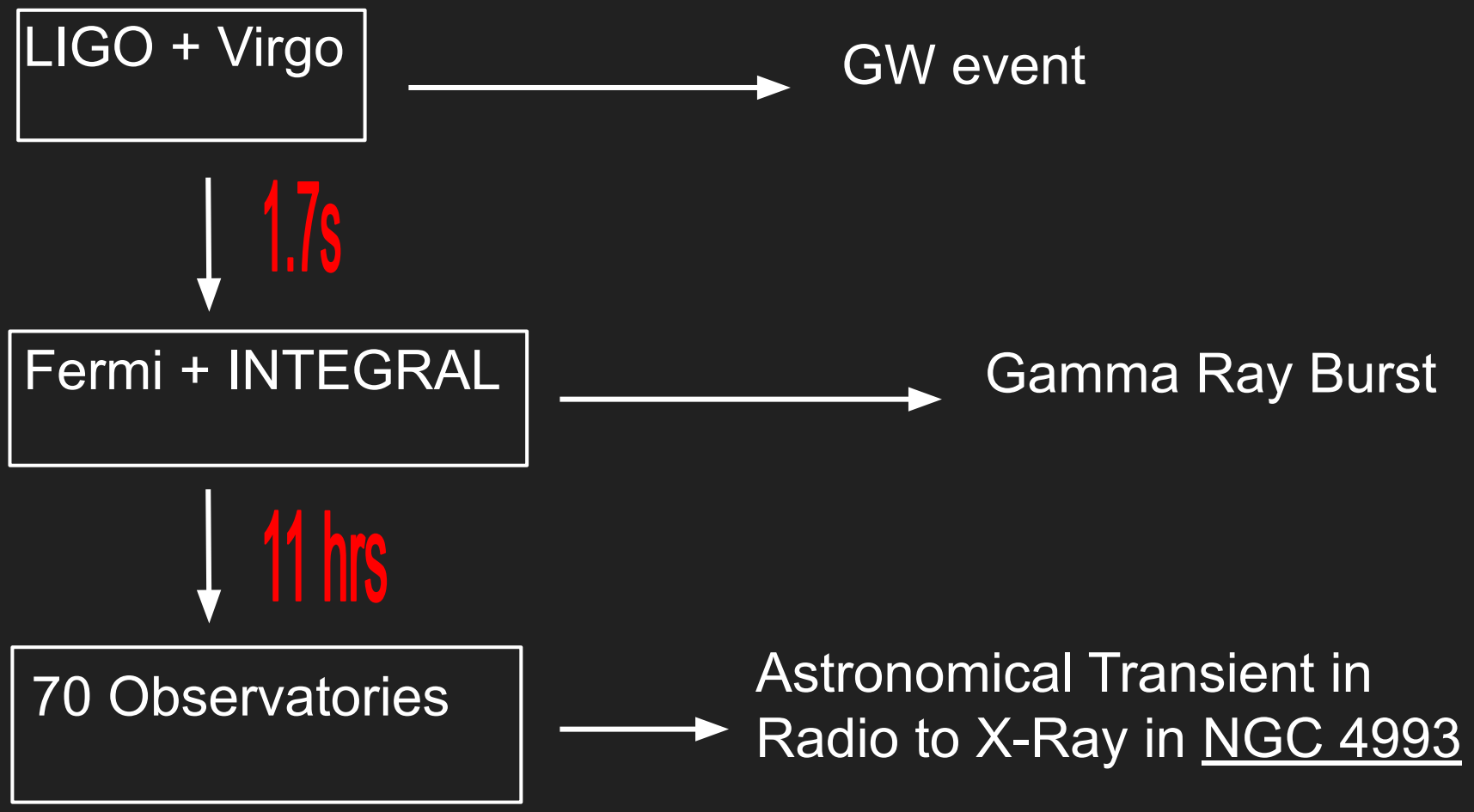
4 A gravitational wave affects the interferometer's arms differently; when one extends the other contracts as they are passed by the peaks and troughs of the gravitational waves.

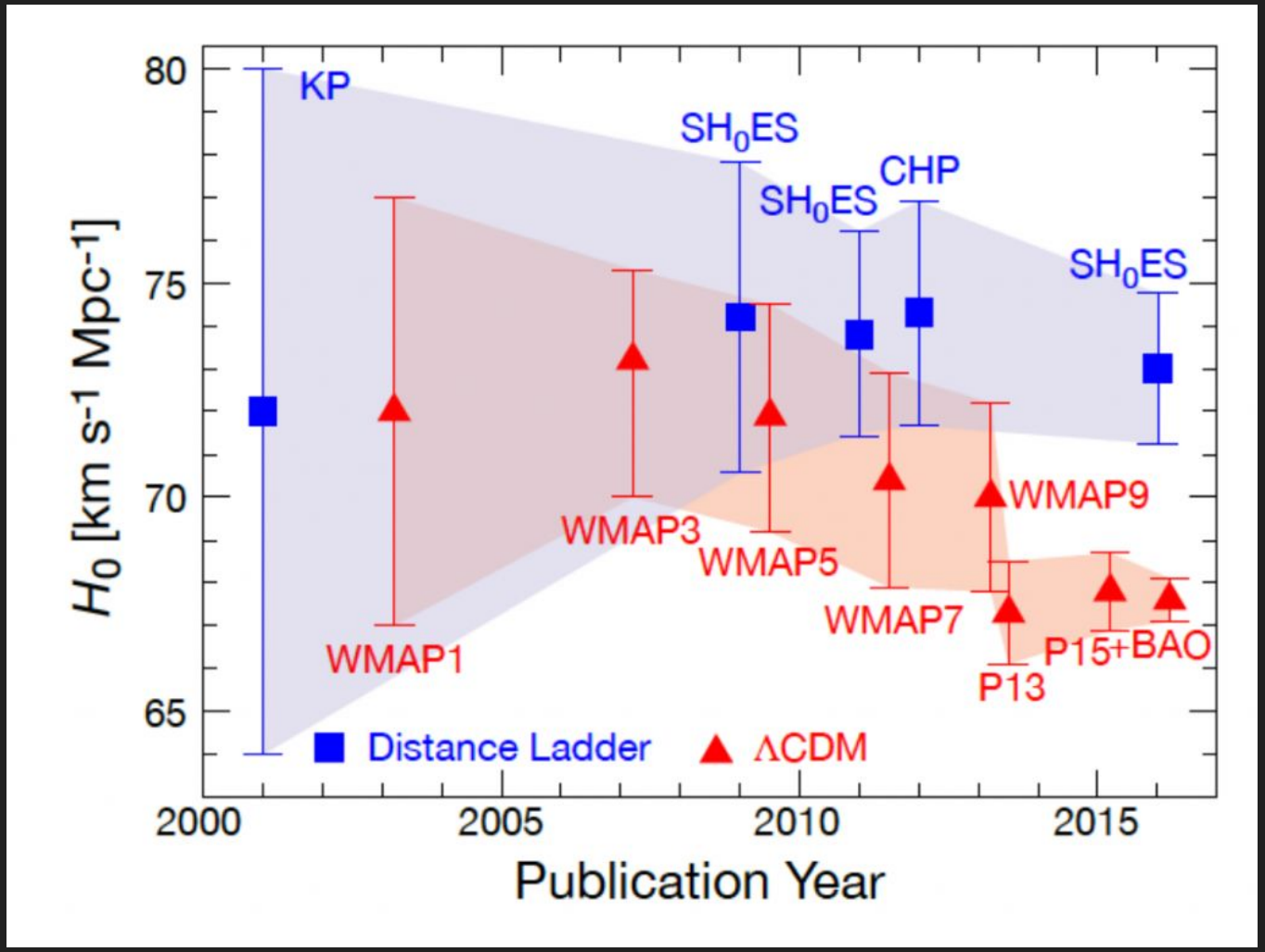
5 Normally, the light returns unchanged to the beam splitter from both arms and the light waves cancel each other out.

6 If the arms are disturbed by a gravitational wave, the light waves will have travelled different distances. Light then escapes through the splitter and hits the detector.

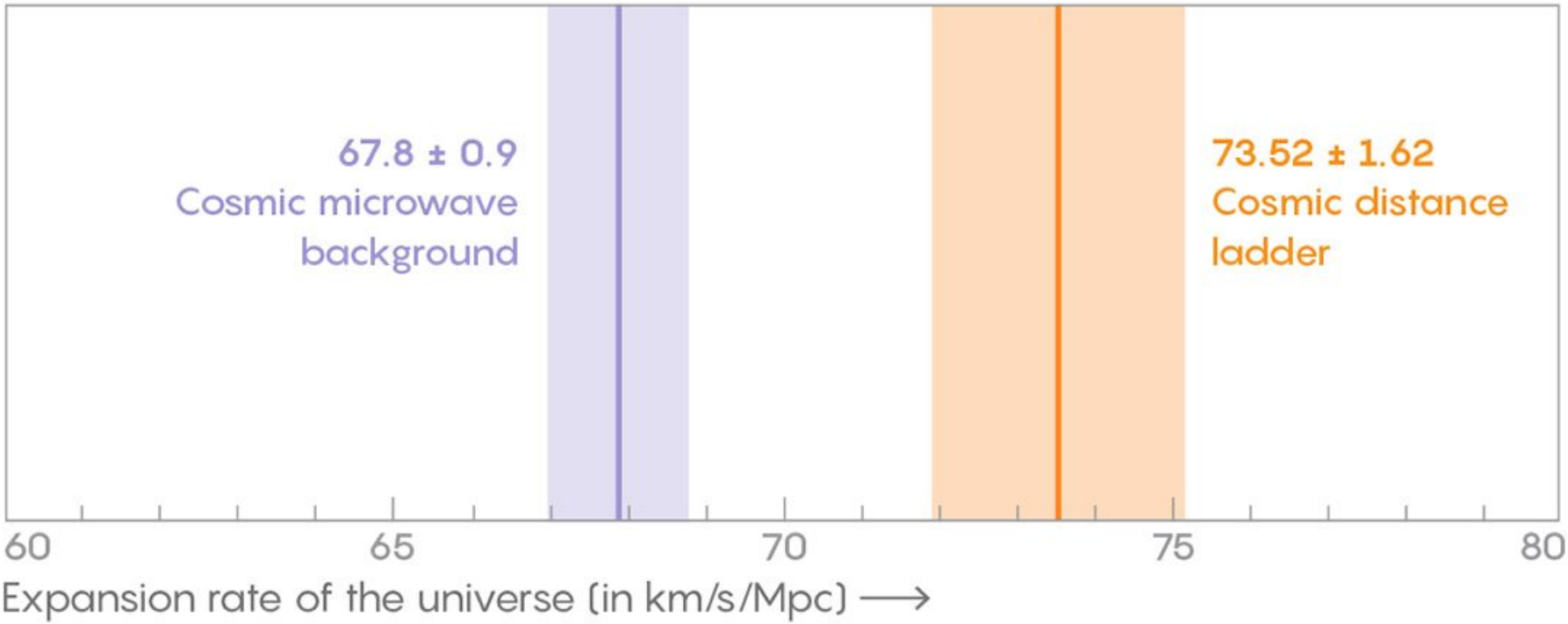


GW170817


LIGO + Virgo
GW event
1.7s
Fermi + INTEGRAL
Gamma Ray Burst
11 hrs
70 Observatories
Astronomical Transient in
Radio to X-Ray in NGC 4993


80
75
70
65
H_0 [km s^{-1} Mpc^{-1}]
2000
2005
2010
2015
Publication Year
KP
SH$_0$ES
SH$_0$ES
CHP
SH$_0$ES
WMAP1
WMAP3
WMAP5
WMAP7
WMAP9
P13
P15+BAO
Distance Ladder
ΛCDM

A Discrepancy in the Hubble Constant



GW 170817

Distance =
43.8 +2.9 - 6.9
between 16%
and 7%

NGC 4993

Redshift = 3017
+/- 166
5%

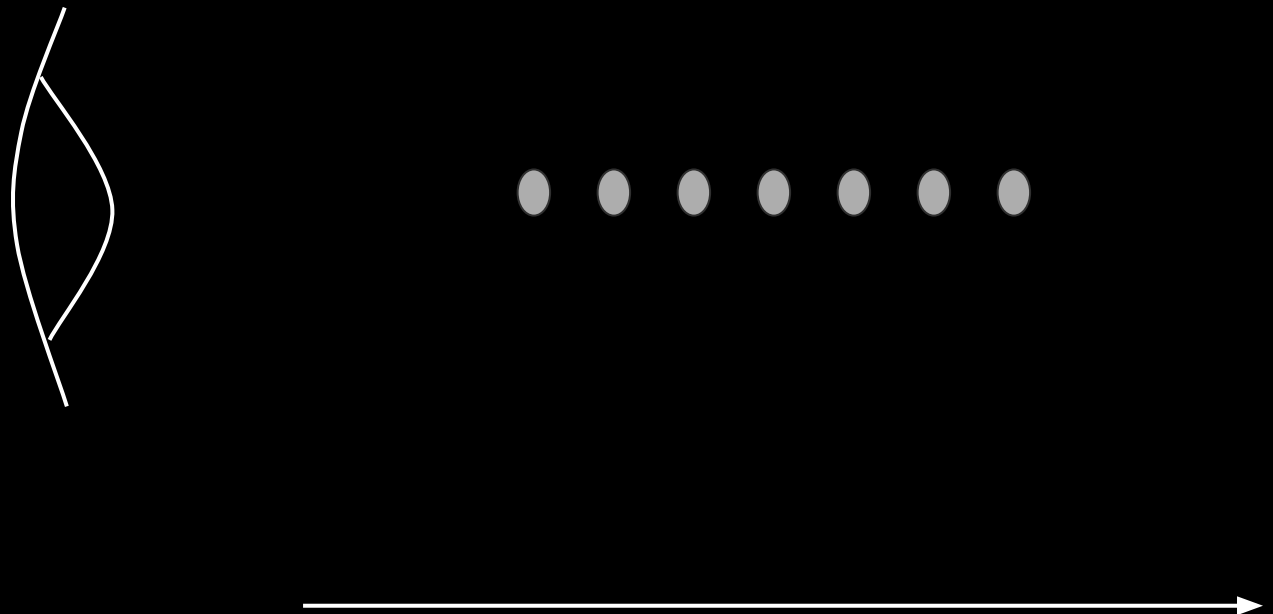
H0 = 70. +/-
+12., -8.

Observed Redshift

Hubble flow

Peculiar Velocity:

- Any velocity component that deviates from the Hubble flow
- Could be due to an interaction
- Motion towards a gravitational overdensity
- Motion in a gravitational potential



Motivating Questions

- ➔ How good are the assumptions? Will they create a systematic bias?
- ➔ What if they don't correct the recessional velocity? In how many detections will we approach H0?
- ➔ Is there scope to obtain better estimates of peculiar velocity contributions?

$v = H*d$

Expanding Space

GR

$v = v_H$ {velocity of expansion or Hubble velocity}

$H = H_0$ {rate of expansion at current epoch}

d {distance}

$$H = \frac{\Delta v}{\Delta d}$$

$$v\ (d = 0) = 0$$

$$\boxed{v = H * d}$$

Hubble Law

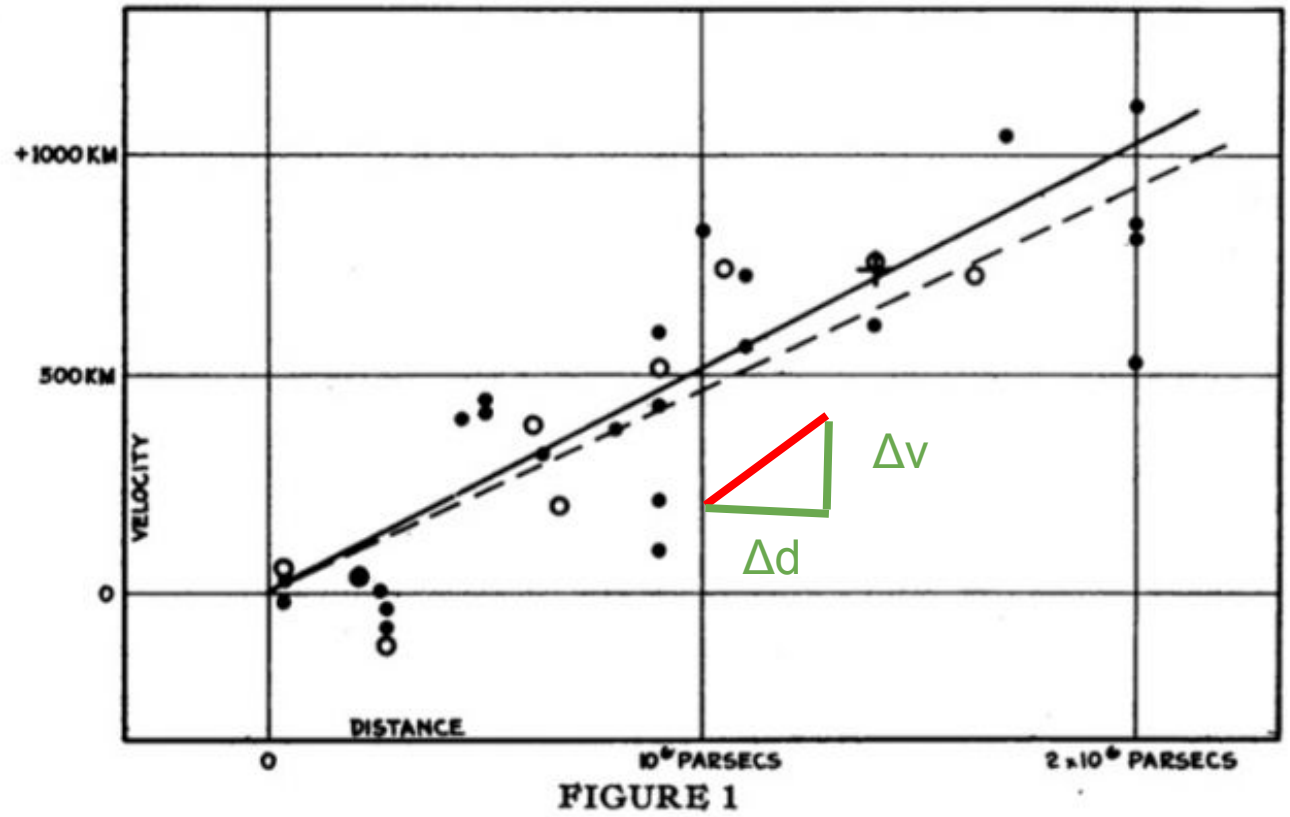
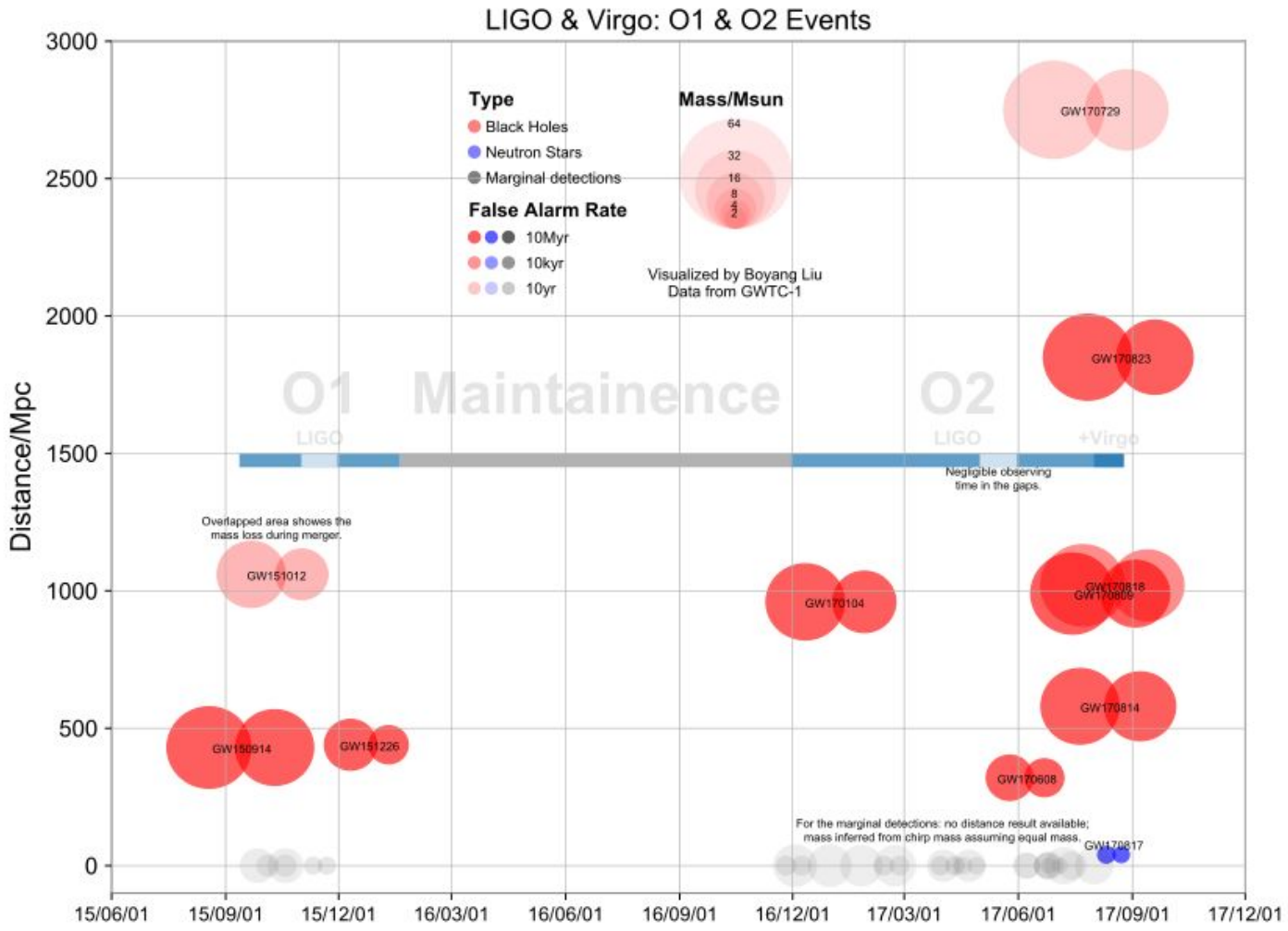
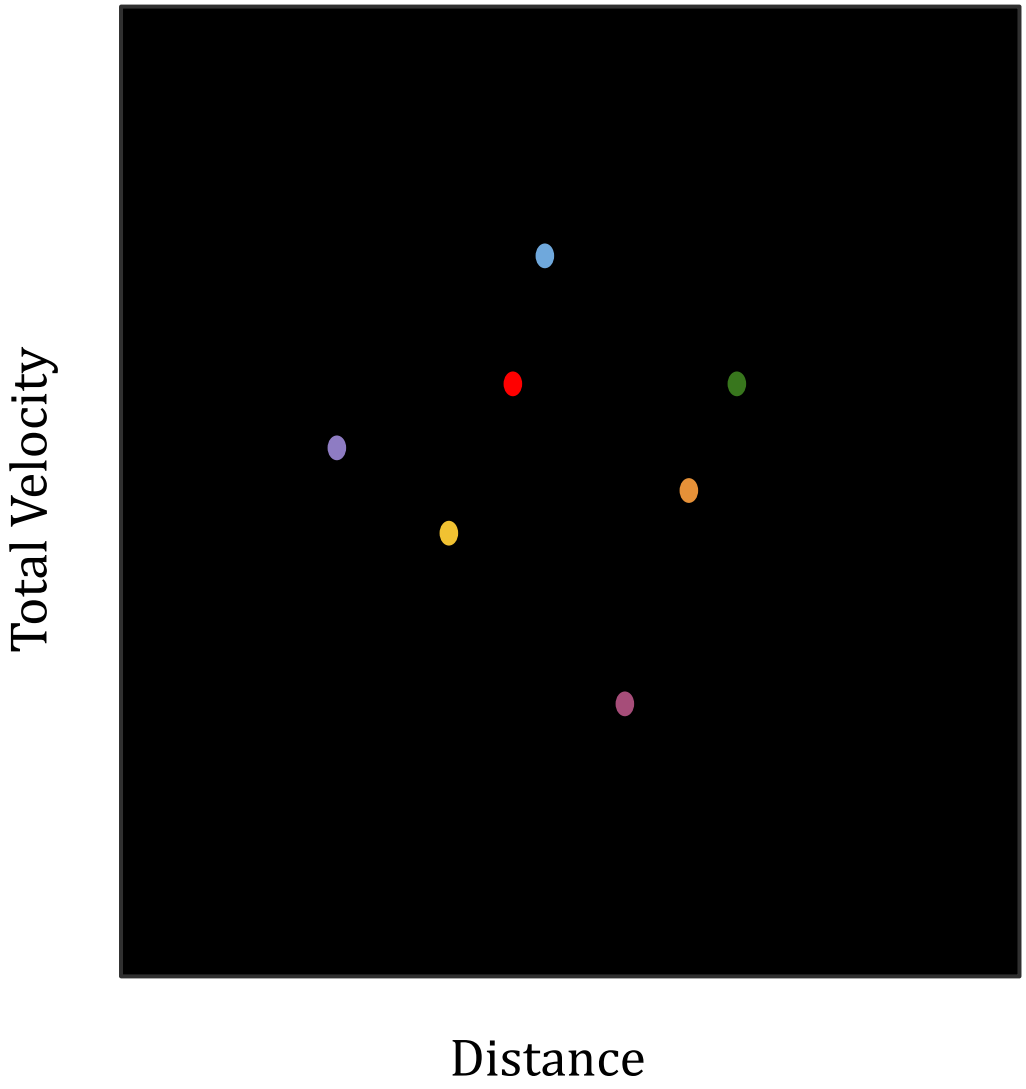


FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.


LIGO & Virgo: O1 & O2 Events
Distance/Mpc
3000
2500
2000
1500
1000
500
0
Type
Black Holes
Neutron Stars
Marginal detections
False Alarm Rate
10Myr
10kyr
10yr
Mass/Msun
64
32
16
8
4
2
Visualized by Boyang Liu
Data from GWTC-1
O1
Maintainence
O2
LIGO
LIGO
+Virgo
Negligible observing time in the gaps.
Overlapped area showes the mass loss during merger.
GW151012
GW150914
GW151226
GW170104
GW170729
GW170823
GW170818
GW170809
GW170814
GW170608
GW170817
For the marginal detections: no distance result available; mass inferred from chirp mass assuming equal mass.
15/06/01
15/09/01
15/12/01
16/03/01
16/06/01
16/09/01
16/12/01
17/03/01
17/06/01
17/09/01
17/12/01

I. Measuring the scatter (1 detection in 7 universes)



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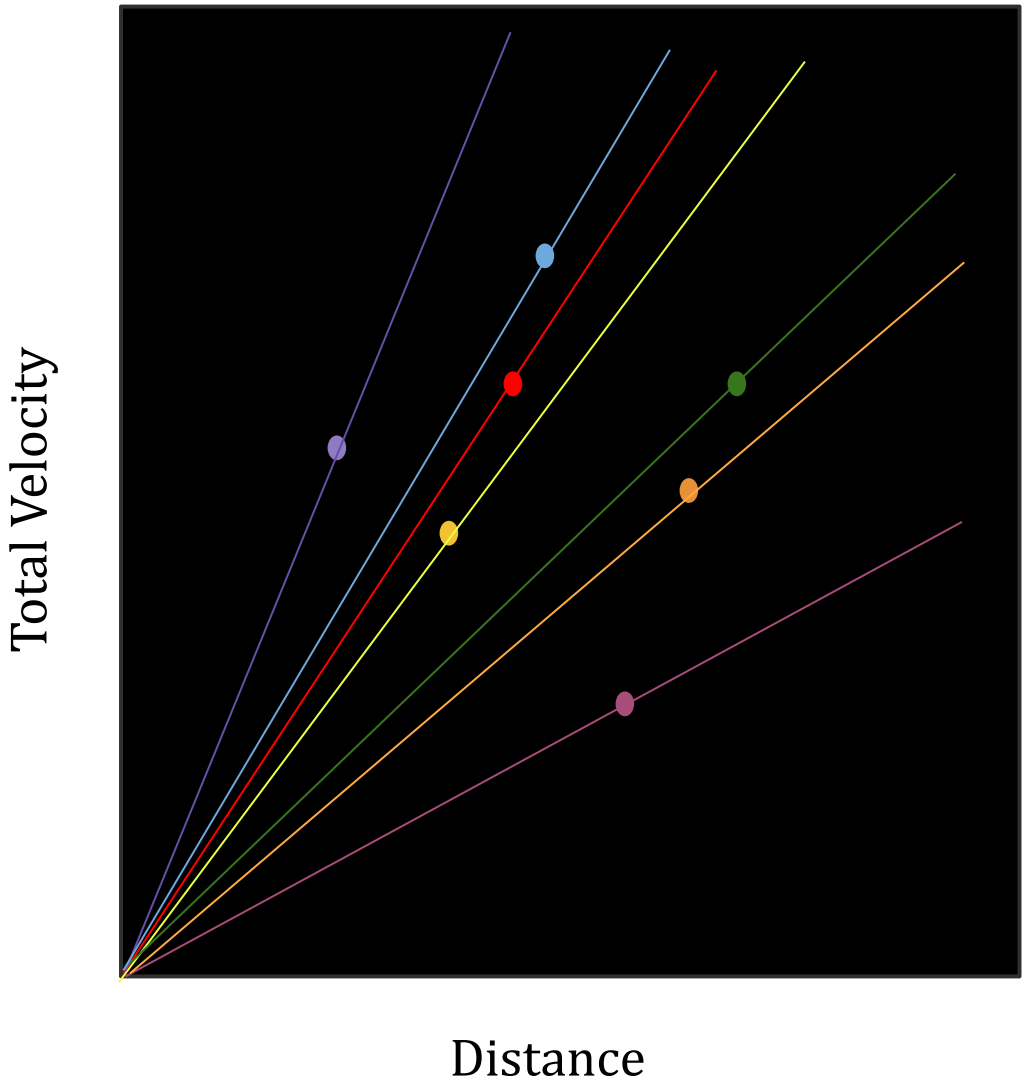

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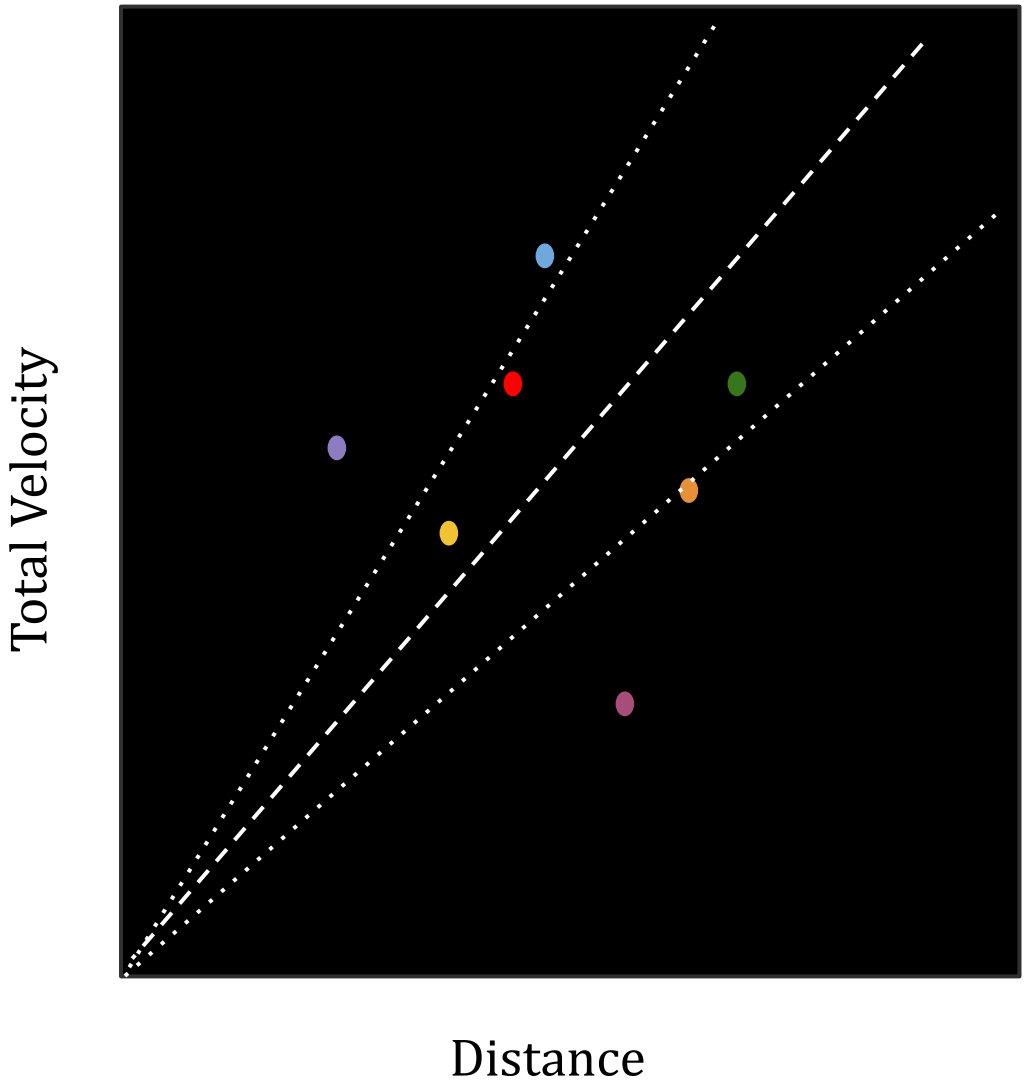

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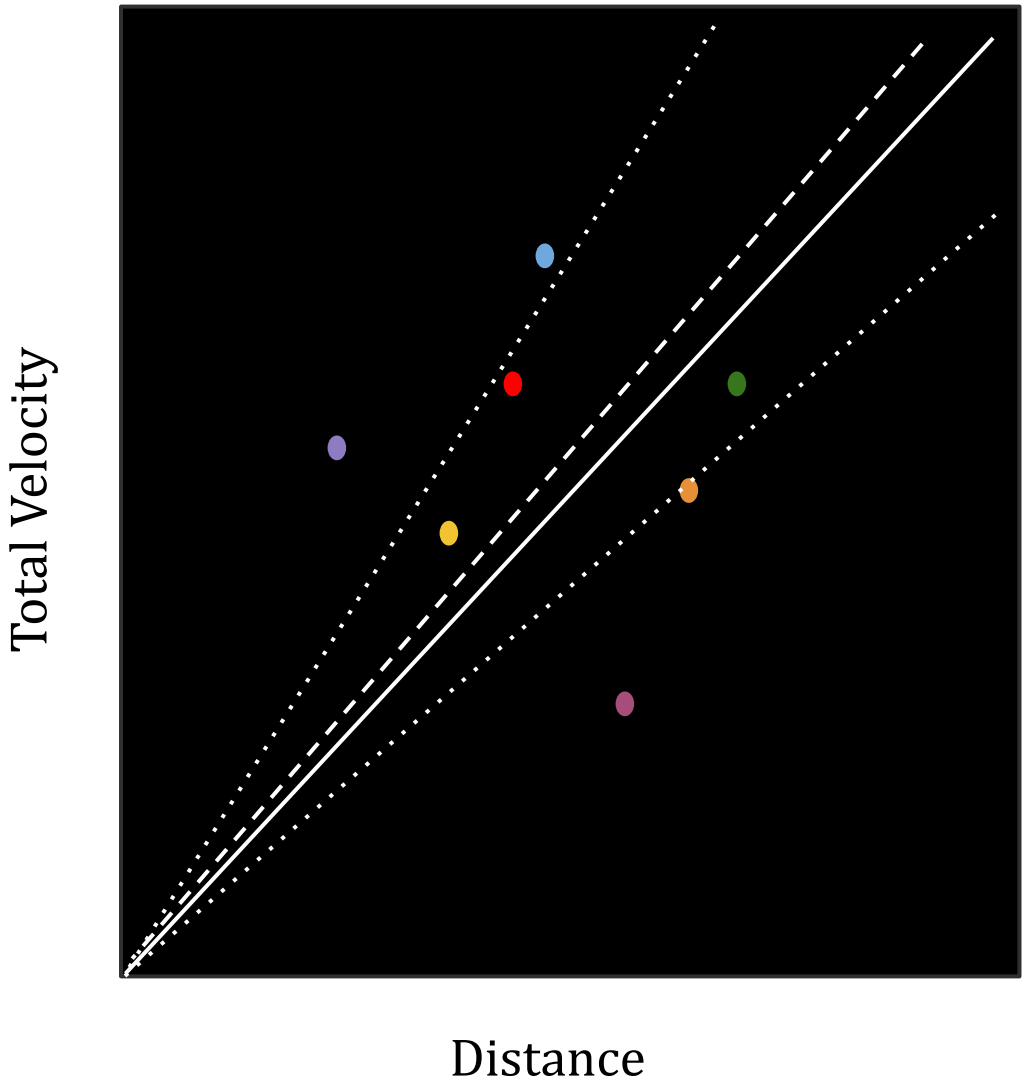

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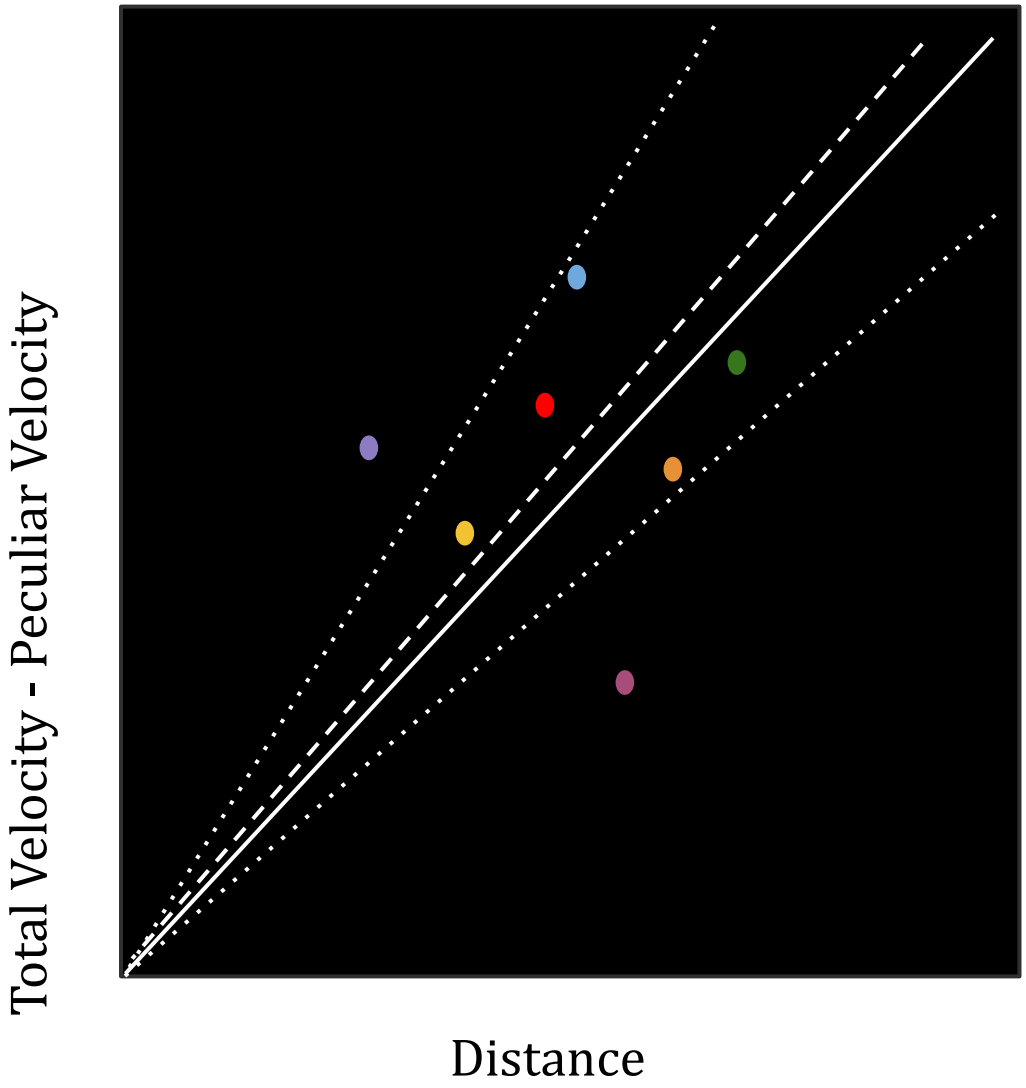

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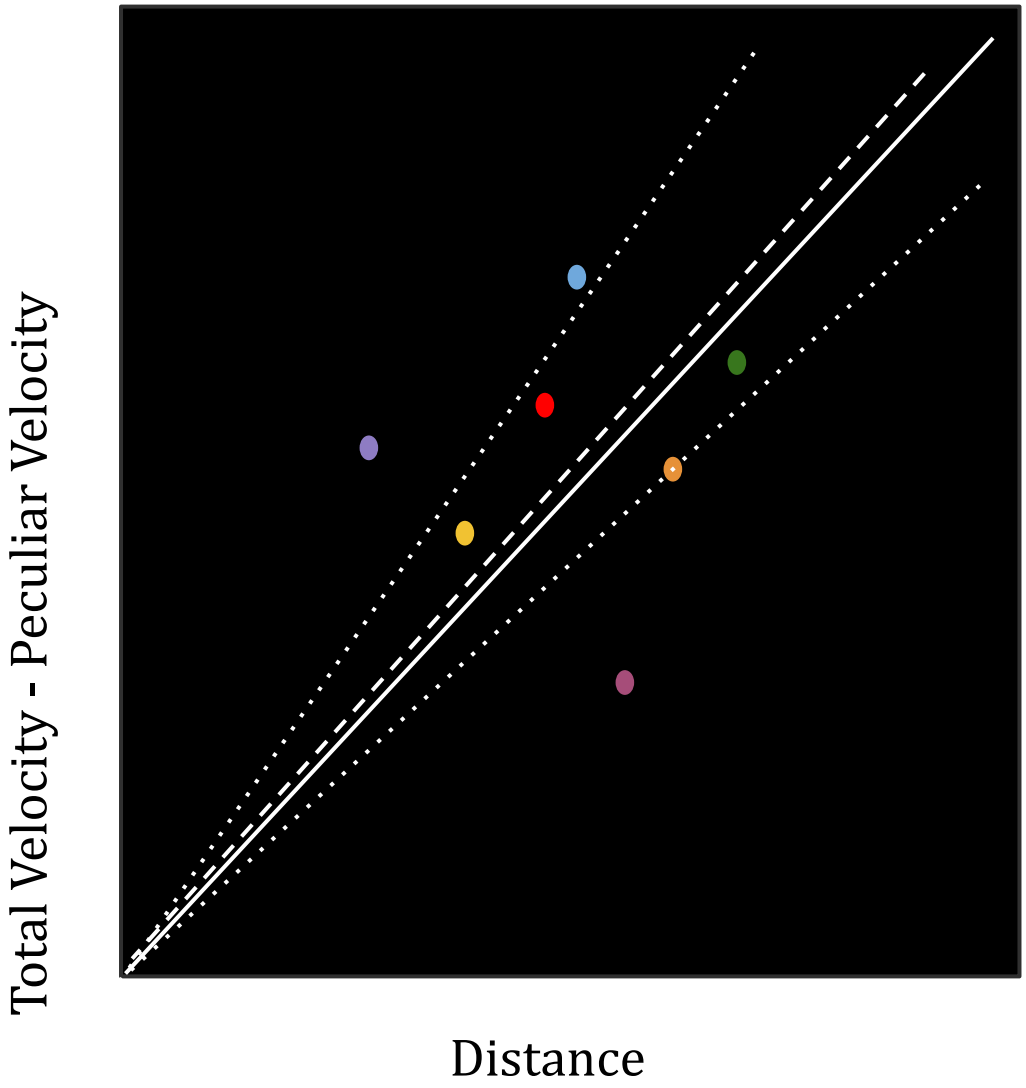
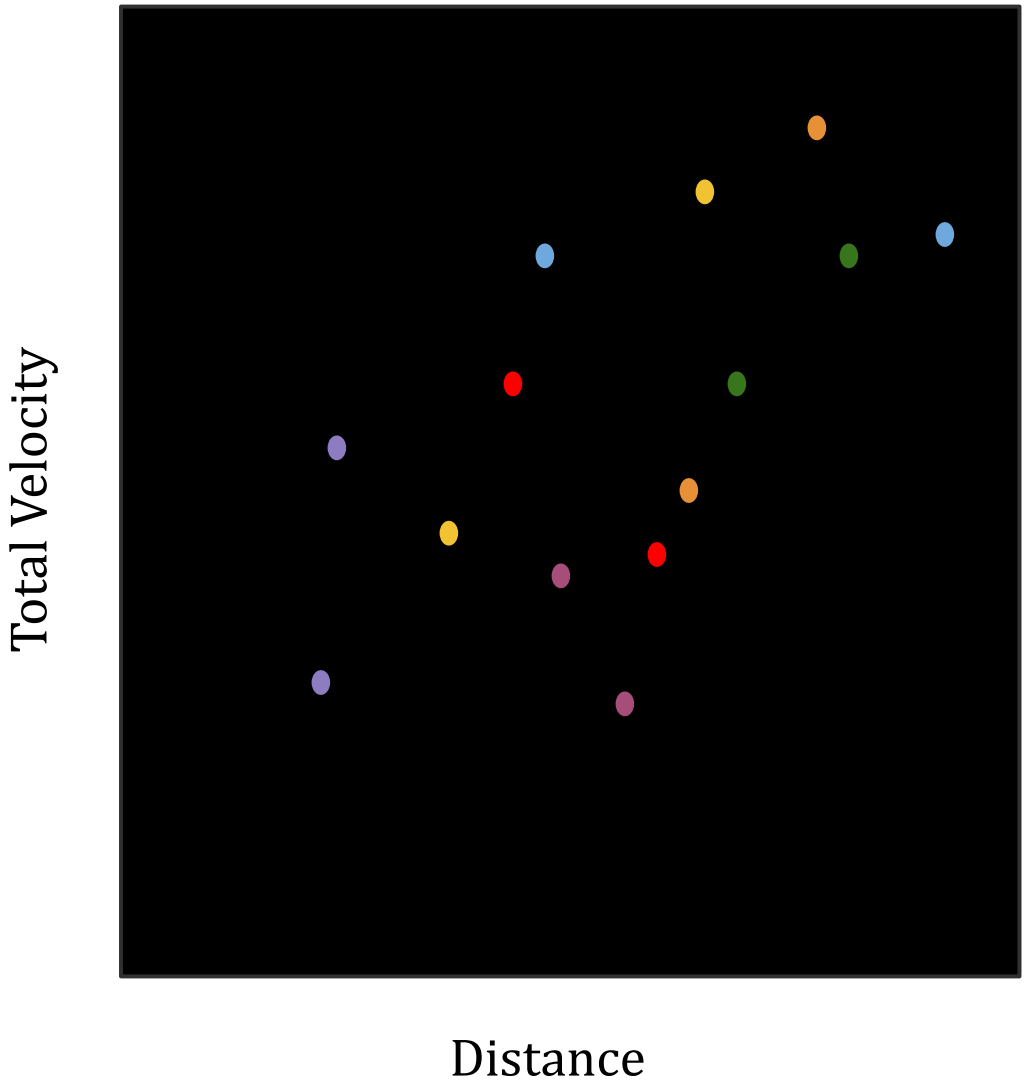
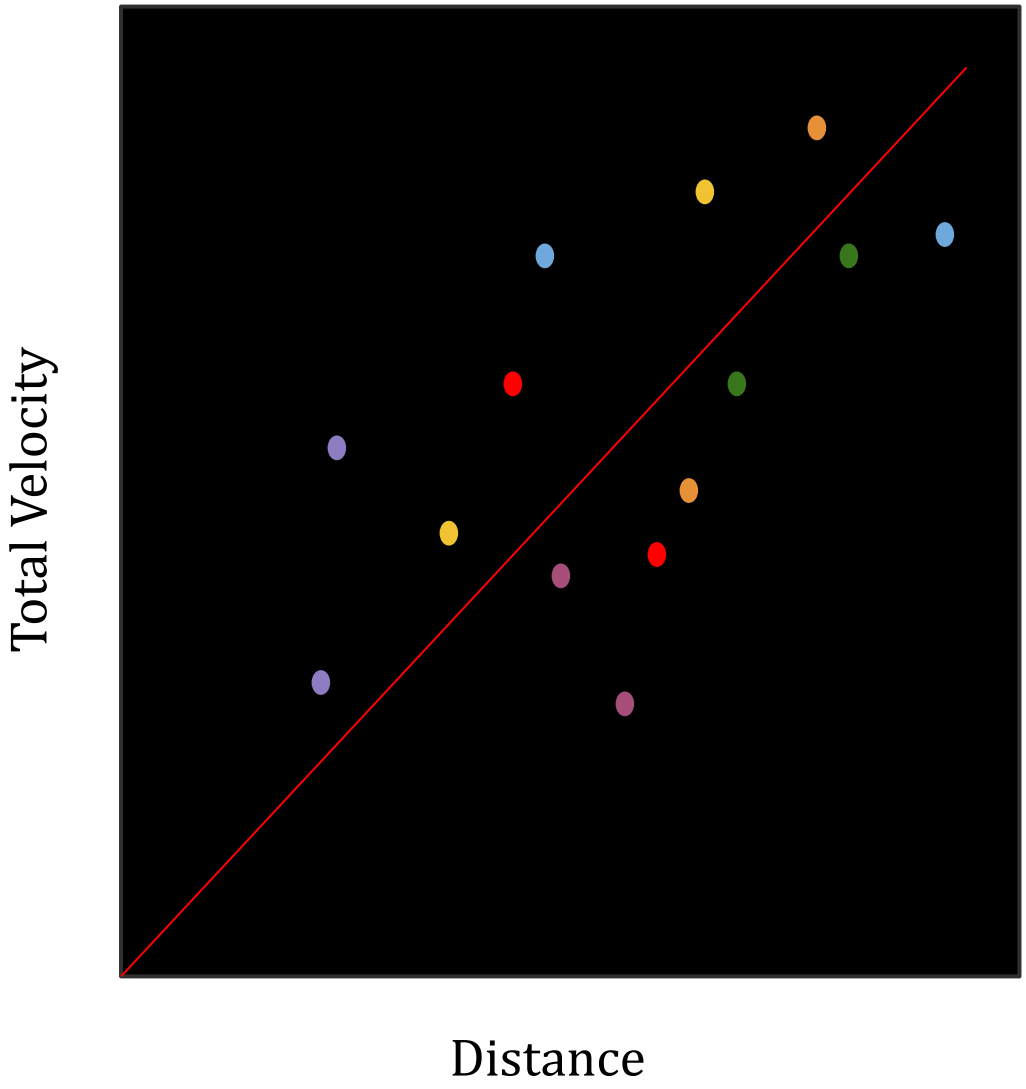


I. Measuring the scatter (2 detections in 7 universes)



I. Measuring the scatter (2 detections in 7 universes)



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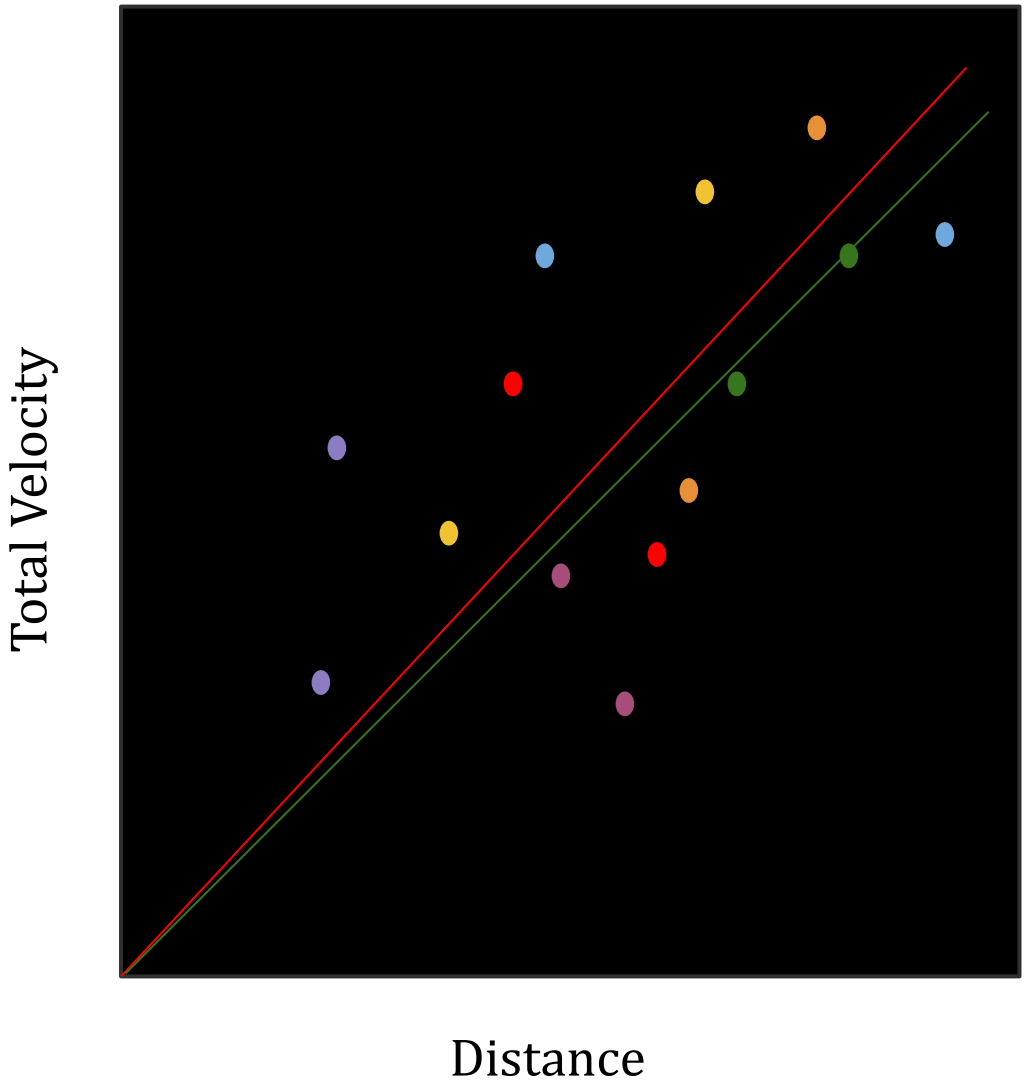

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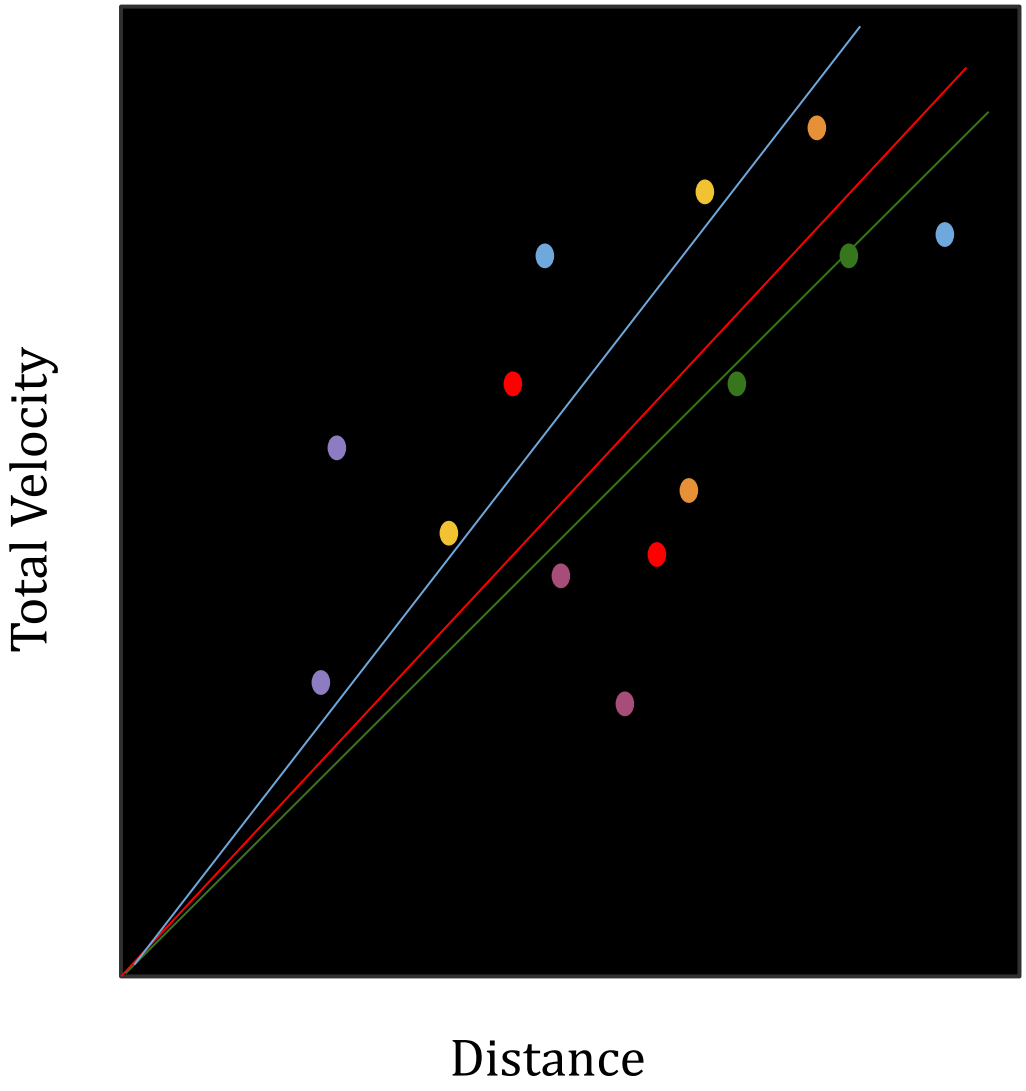

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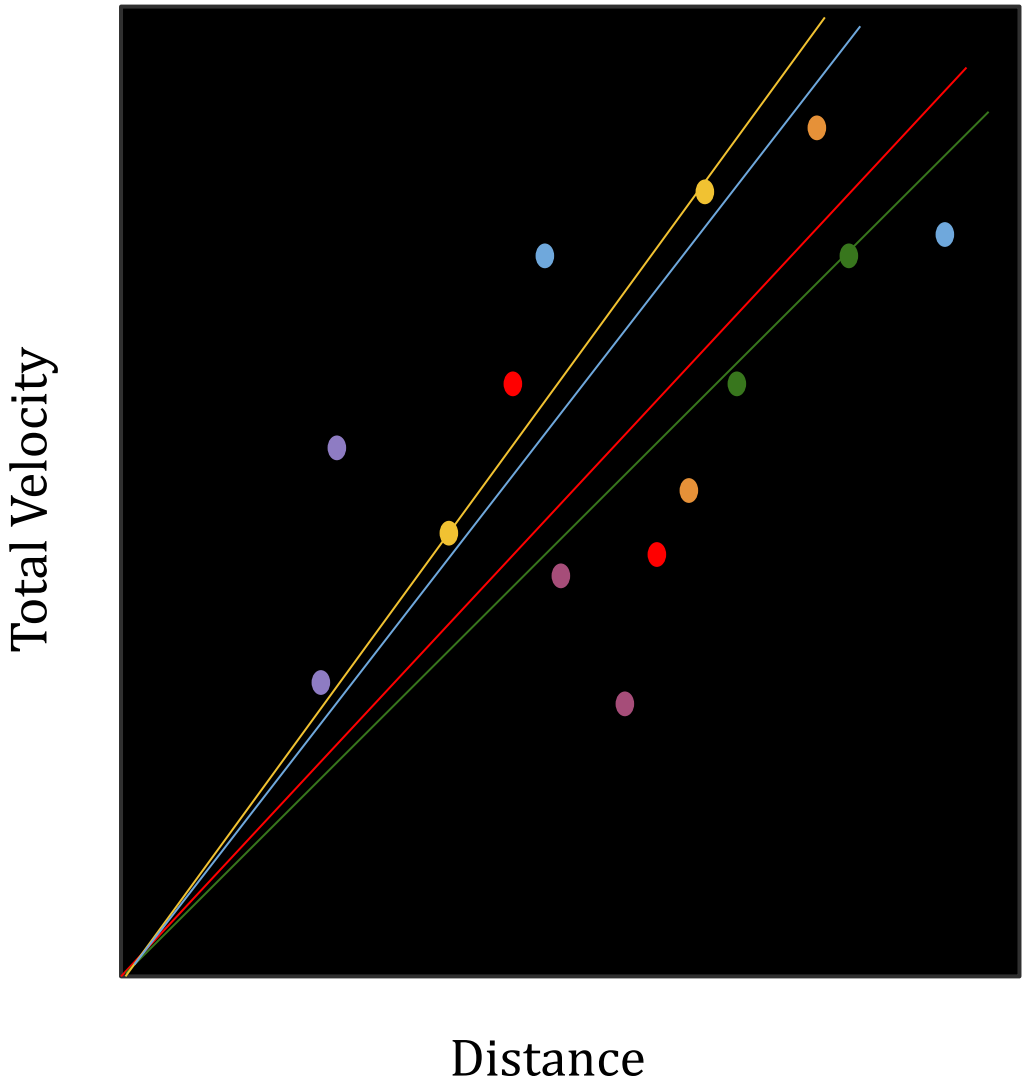

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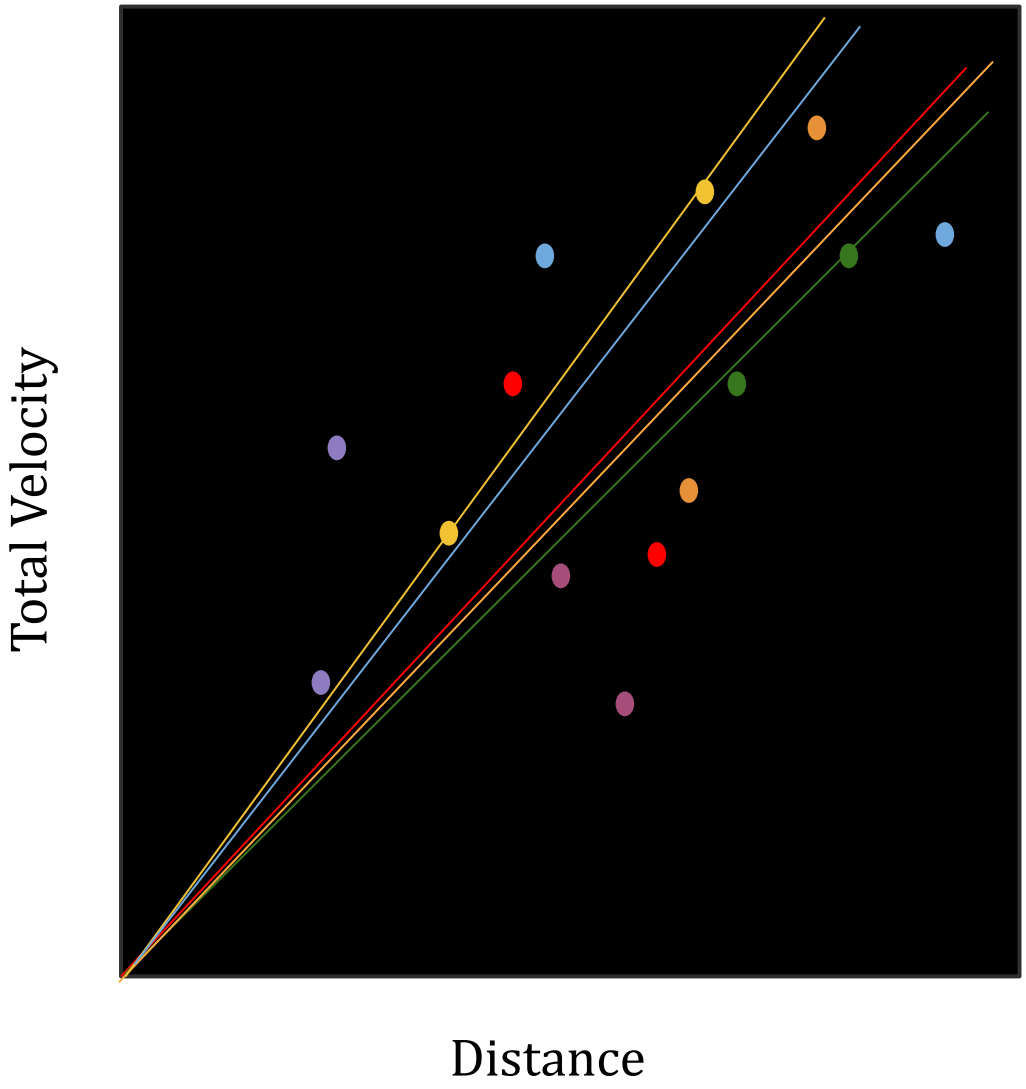

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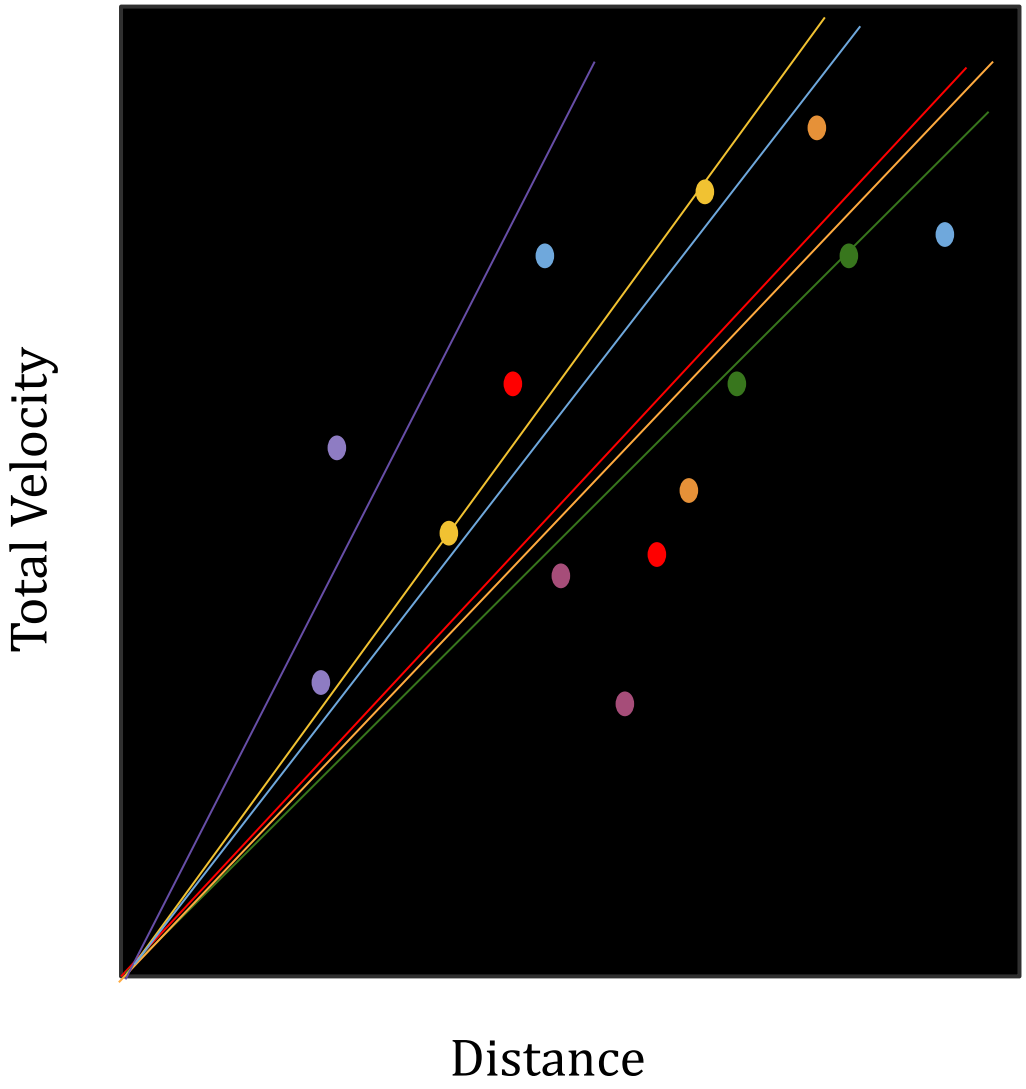

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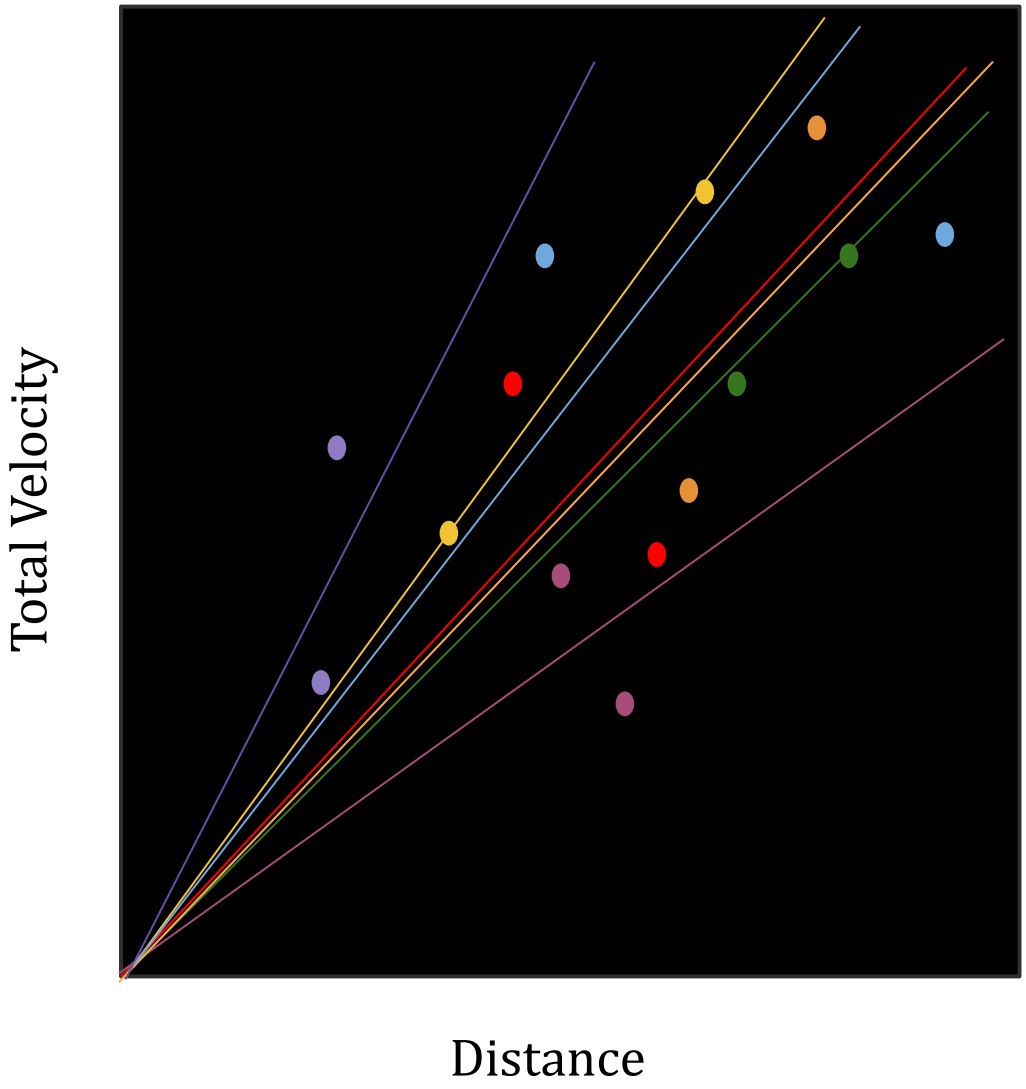

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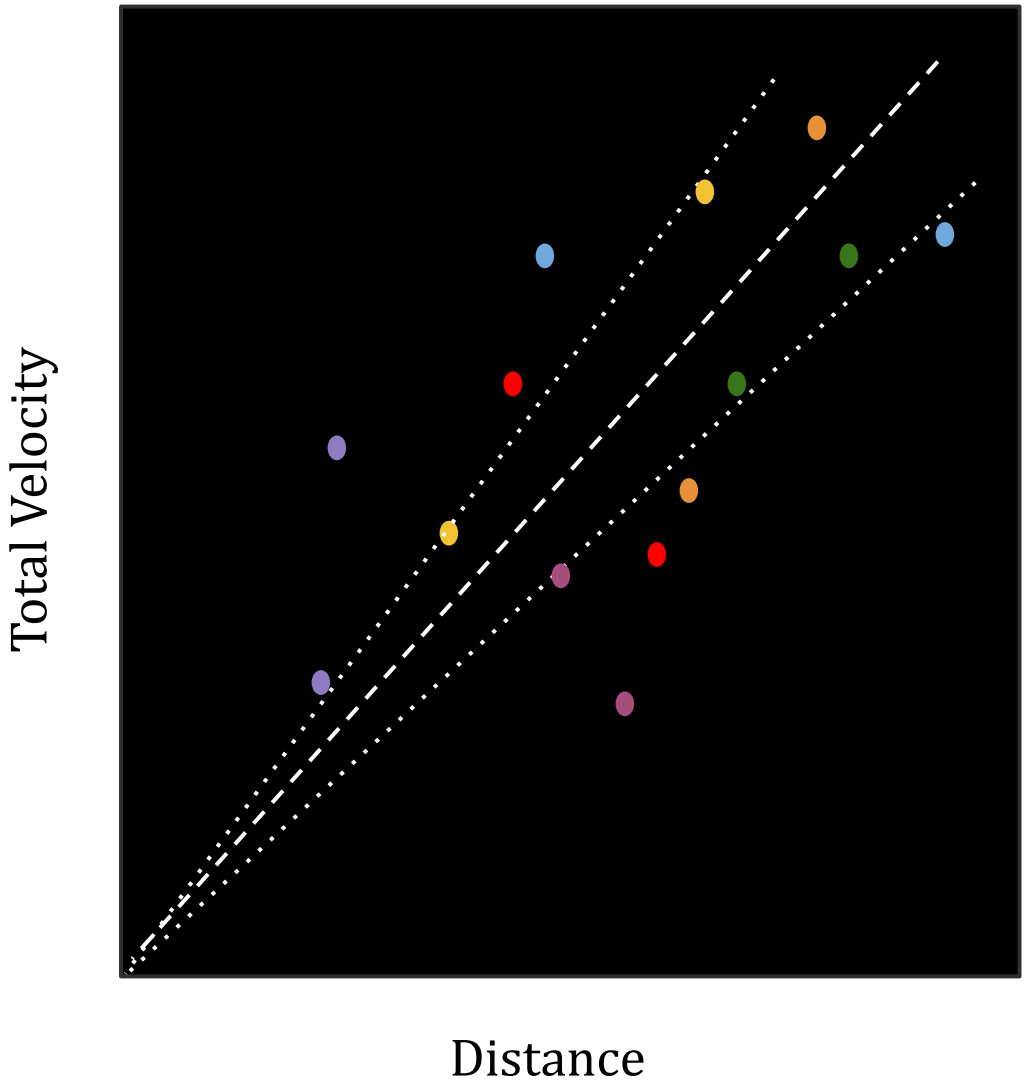

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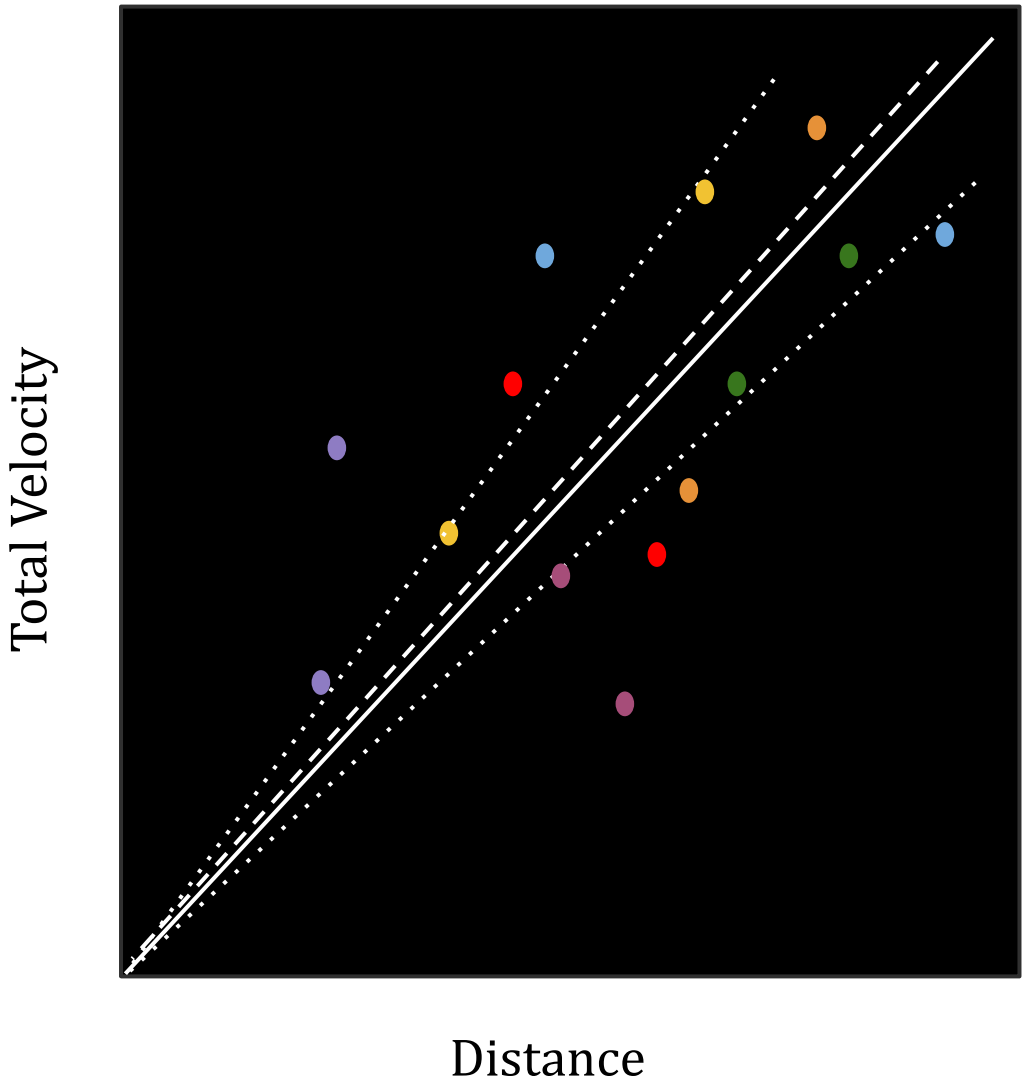

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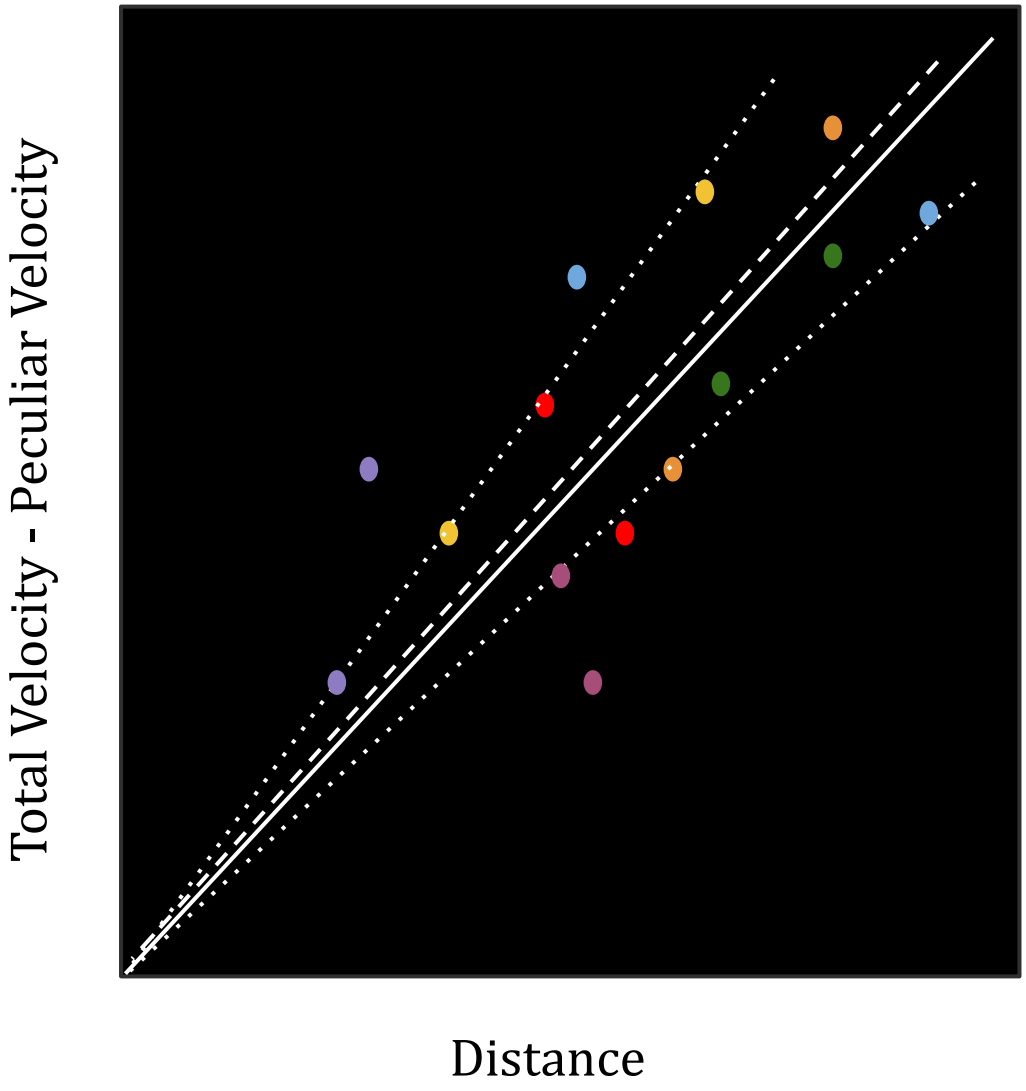

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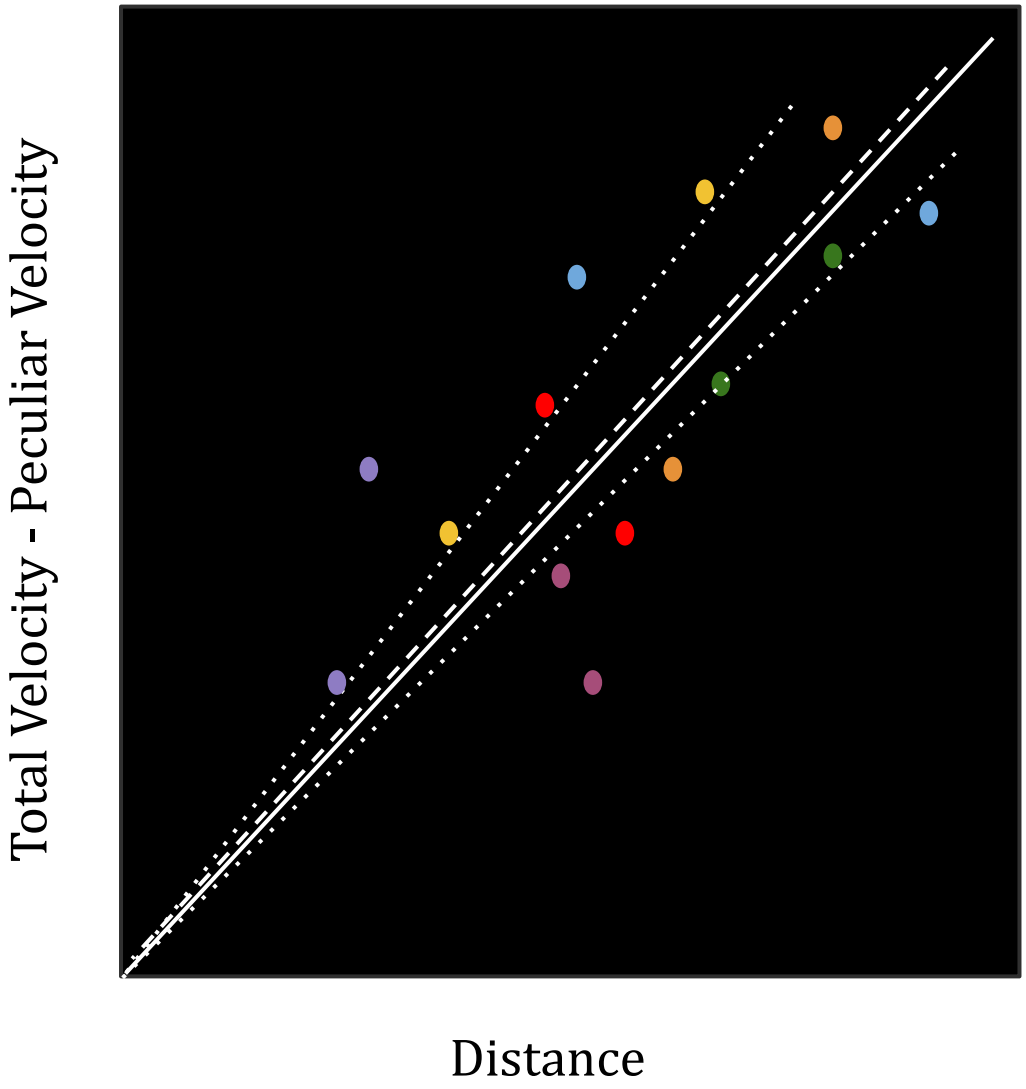

Using the Power of Simulations

AREPO

Constrained Initial Conditions

Using the Power of Simulations

AREPO

Constrained Initial Conditions

1. Exact position & peculiar velocity components
2. Above can be used to obtain the exact distance and hubble velocity.

Using the Power of Simulations

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Constrained Initial Conditions

1. Properties close to those observed in our universe.

Using the Power of Simulations

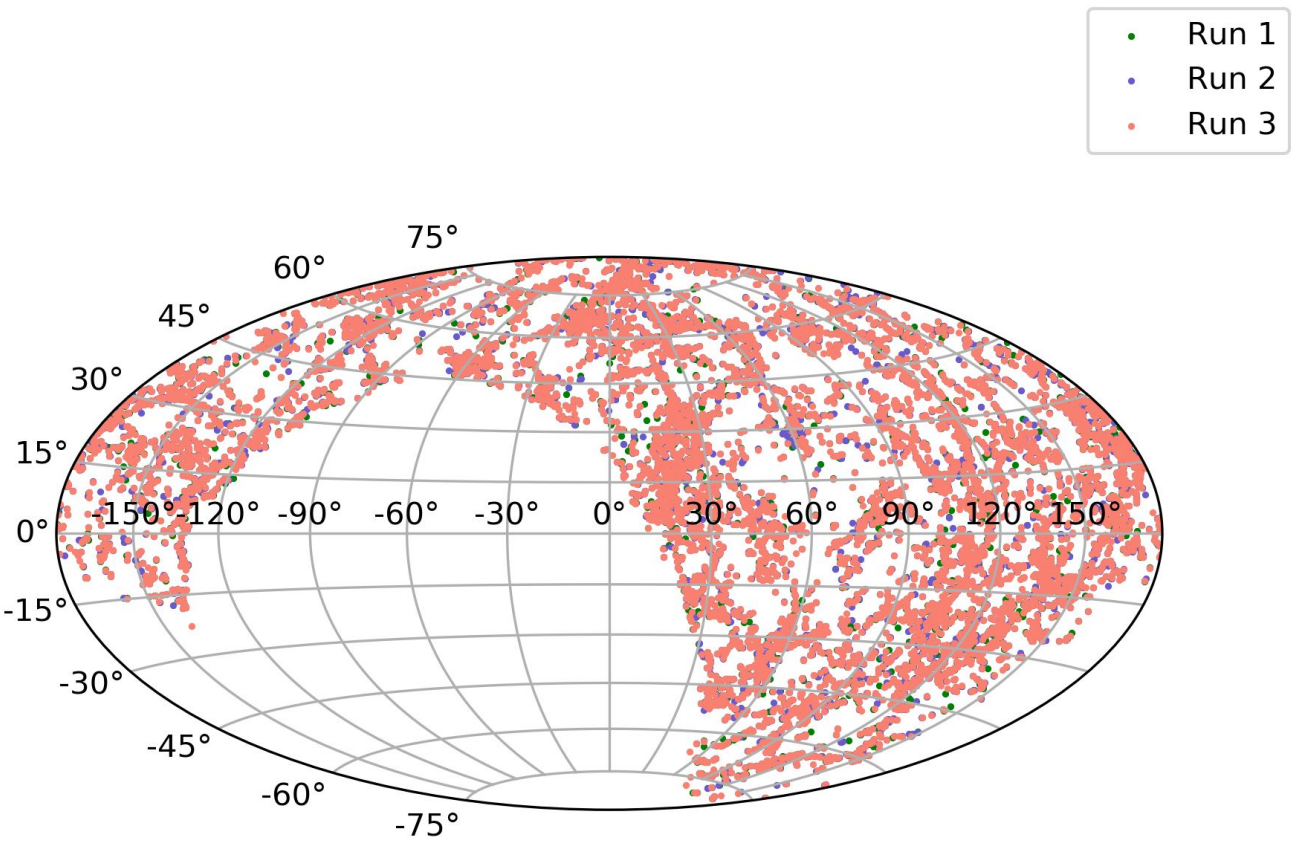
AREPO

1. Exact position & peculiar velocity components
2. Above can be used to obtain the exact distance and hubble velocity.

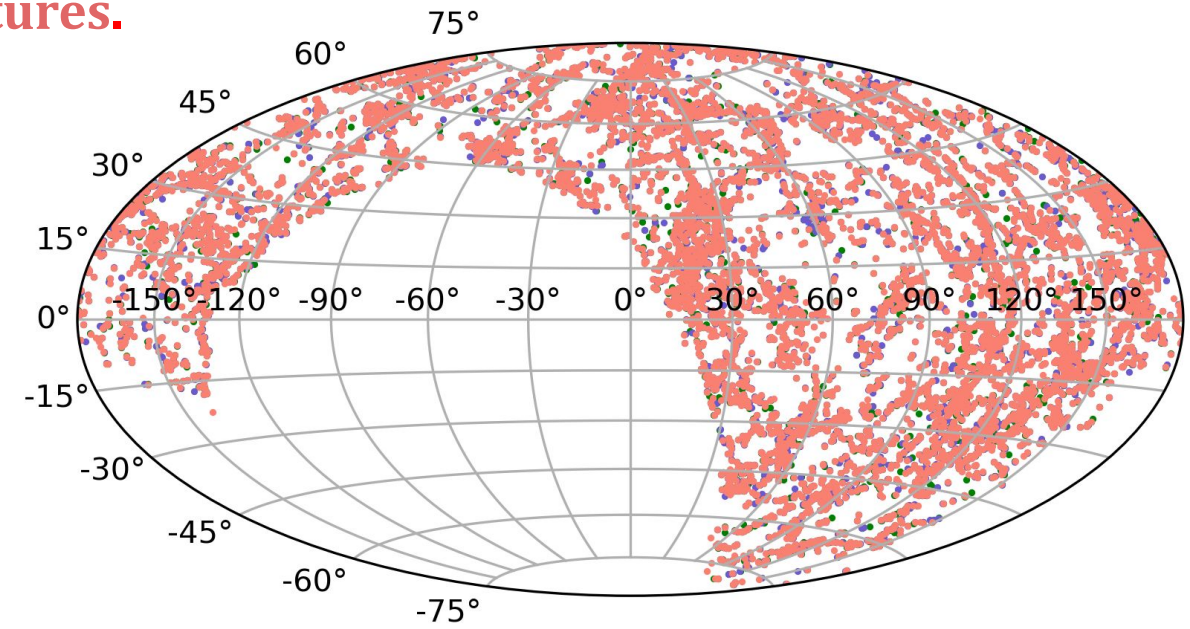
Constrained Initial Conditions

1. Properties close to those observed in our universe.

"Close" because only the large scale structures are constrained by SDSS D7 footprint.


Run 1
Run 2
Run 3
75°
60°
45°
30°
15°
0°
-15°
-30°
-45°
-60°
-75°
-150°
-120°
-90°
-60°
-30°
0°
30°
60°
90°
120°
150°

Pick a random and unique power for the density fluctuations of small-structures.



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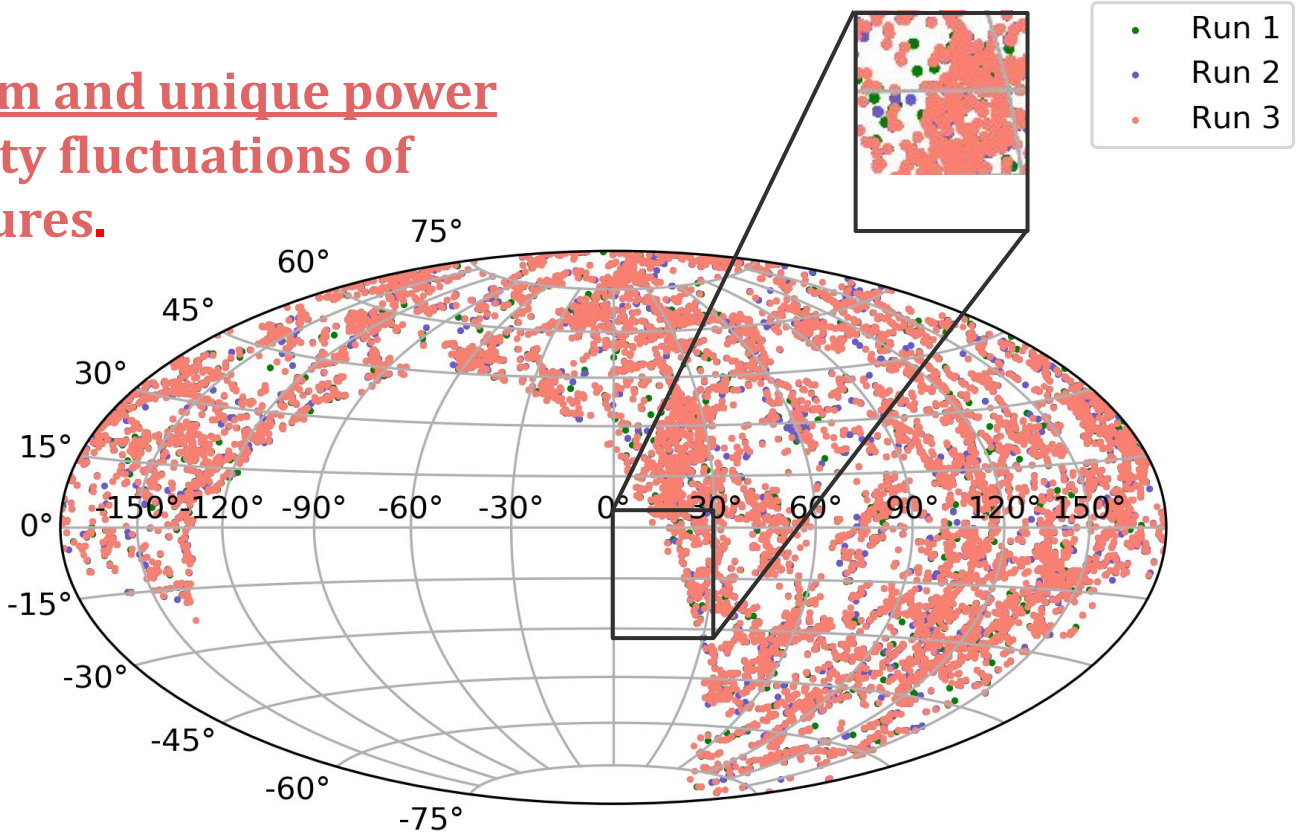
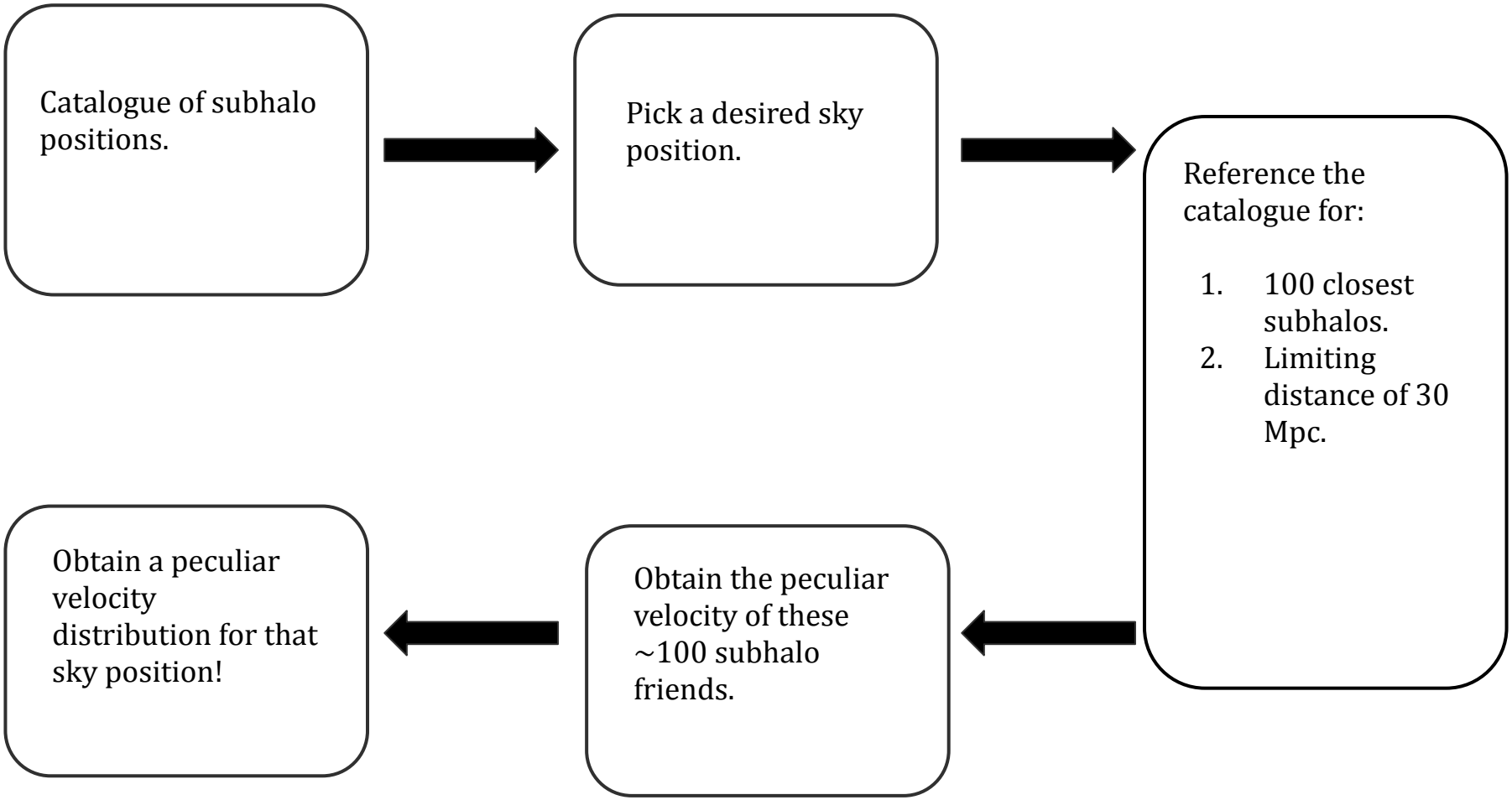
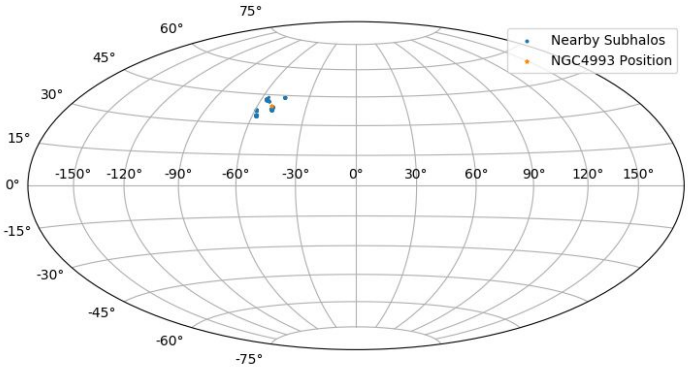
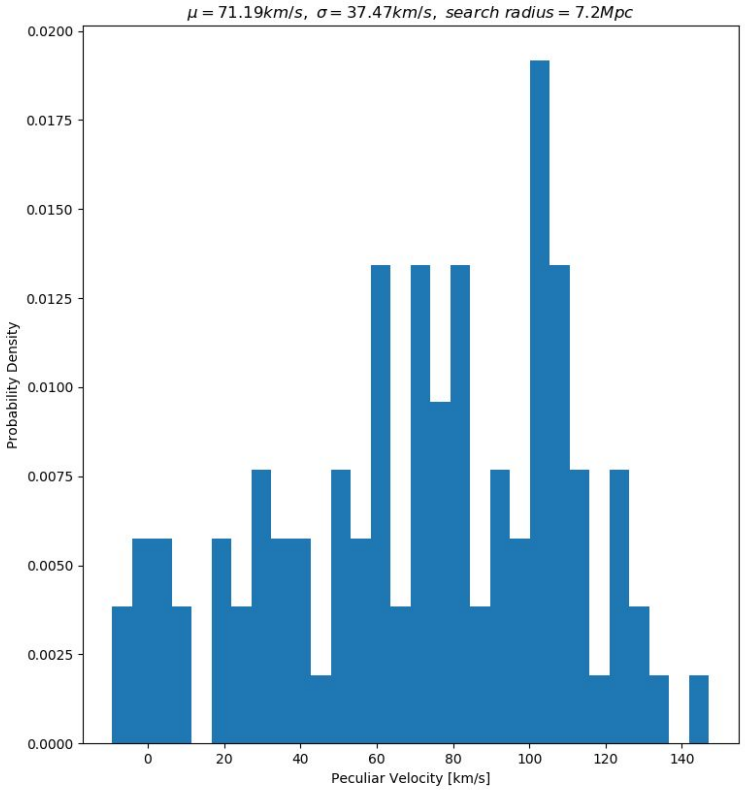


Peculiar Velocity Distribution







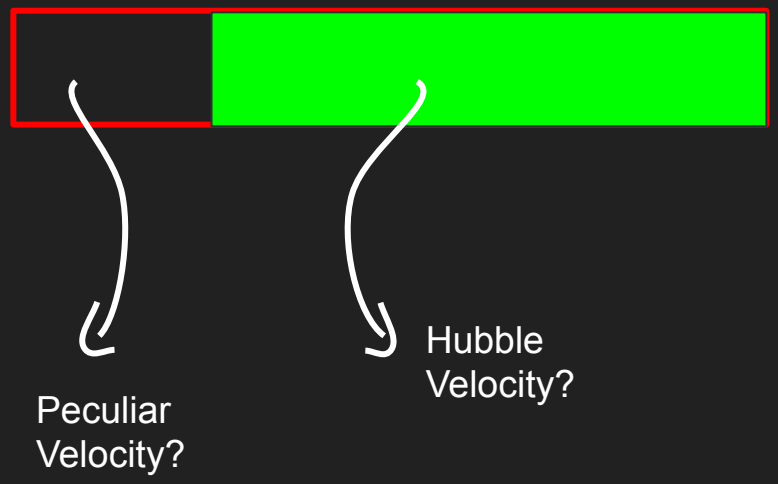
Peculiar Velocity Distribution at the position of NGC 4993
(EM counterpart of the first NS-NS merger)

Questions

1. How will correcting for peculiar velocity using simulations with constrained initial condition impact the convergence rate of H0? Will there be a systematic bias?

2. What if we continue to follow LIGO's analysis? Will this introduce a systematic bias? What will the convergence rate be?

3. What is the best way to correct for peculiar velocity?



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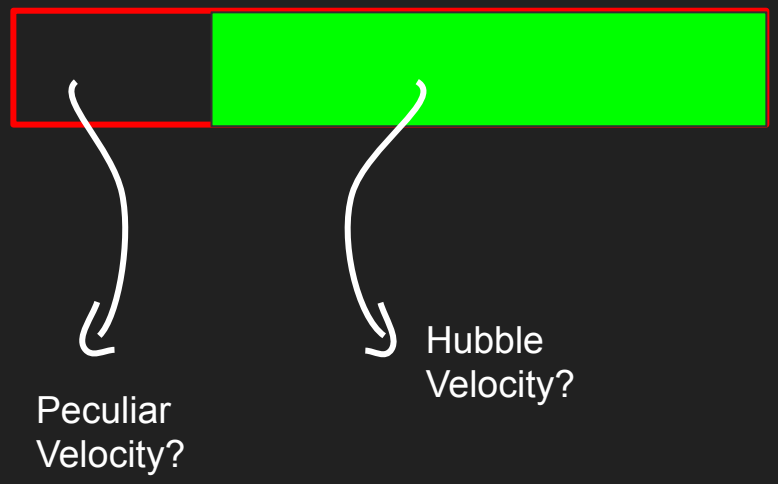


I. Measuring the scatter

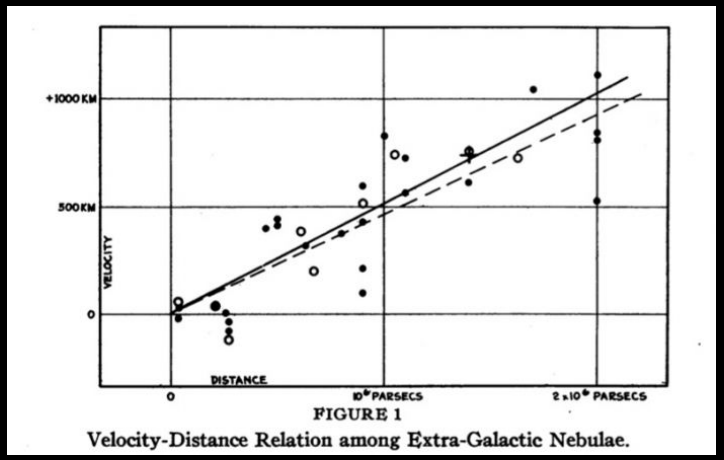


FIGURE 1
Velocity-Distance Relation among Extra-Galactic Nebulae.

Georges Lemaitre

Edwin Hubble

"A homogeneous Universe of constant mass and increasing radius accounting for the radial velocity of extra-galactic nebulae".
1927. *Annals of the Scientific Society of Brussels*

$$v = H_0 * d$$

"A relation between distance and radial velocity among extra-galactic nebulae".
1929. PNAS. 15 (3): 168–173.

Hubble-Lemaitre Law

Image Credit: wikipedia.com

$$v = H_0 * d$$

Hubble-Lemaitre Law

$v = v_H$ {velocity of expanding space or Hubble flow velocity}

$H = H_0$ {rate of expansion at current epoch}

d {distance}

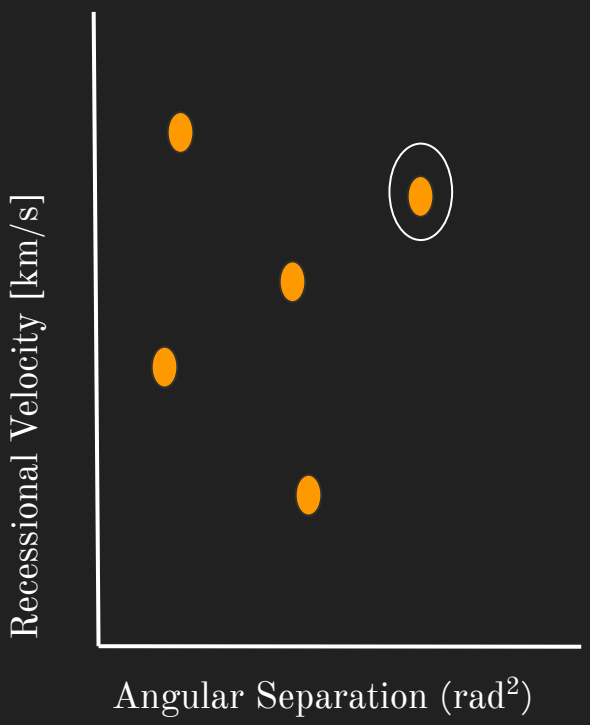
2MRS Galaxy Group HDC 763

Grouping by Crook et al. 2007's friends-of-friends algorithm

2MRS Galaxy Group HDC 763

Grouping by Crook et al. 2007's friends-of-friends algorithm

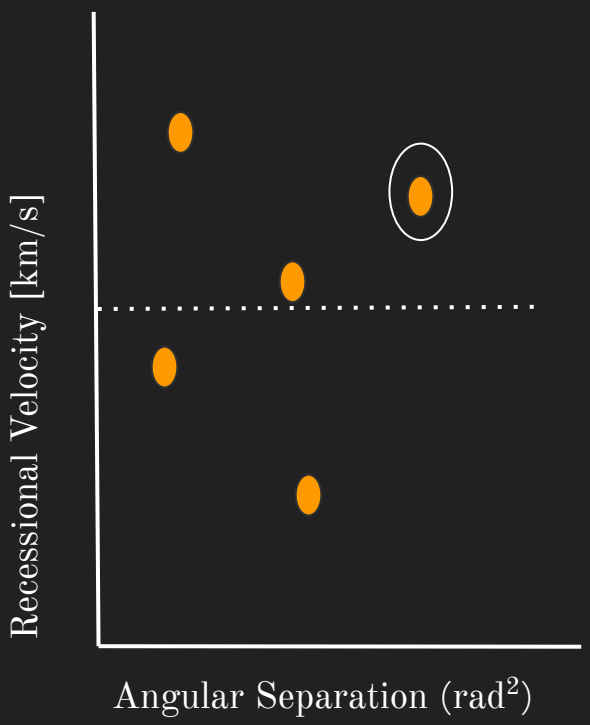
2MRS Galaxy Group HDC 763



Grouping by Crook et al. 2007's friends of friends of algorithm

Correct NGC 4993 recessional velocity to mass averaged group recessional velocity

2MRS Galaxy Group HDC 763



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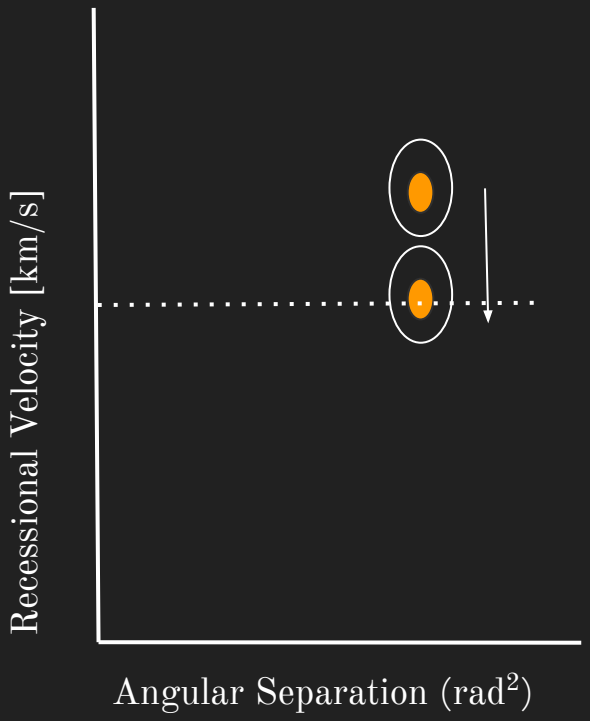
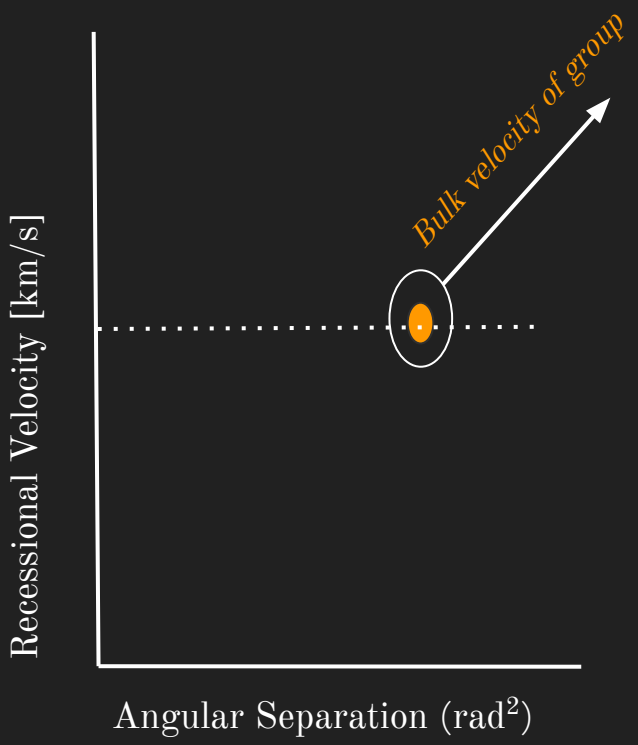
2MRS Galaxy Group HDC 763



Grouping by Crook et al. 2007's friends of friends of algorithm

Assign mass-weighted average of group recessional velocity

2MRS Galaxy Group HDC 763



Grouping by Crook et al. 2007's friends of friends of algorithm

Assign mass-weighted average of group recessional velocity

Correct for the bulk velocity of the group using peculiar velocity maps (Springob et al. 2014)

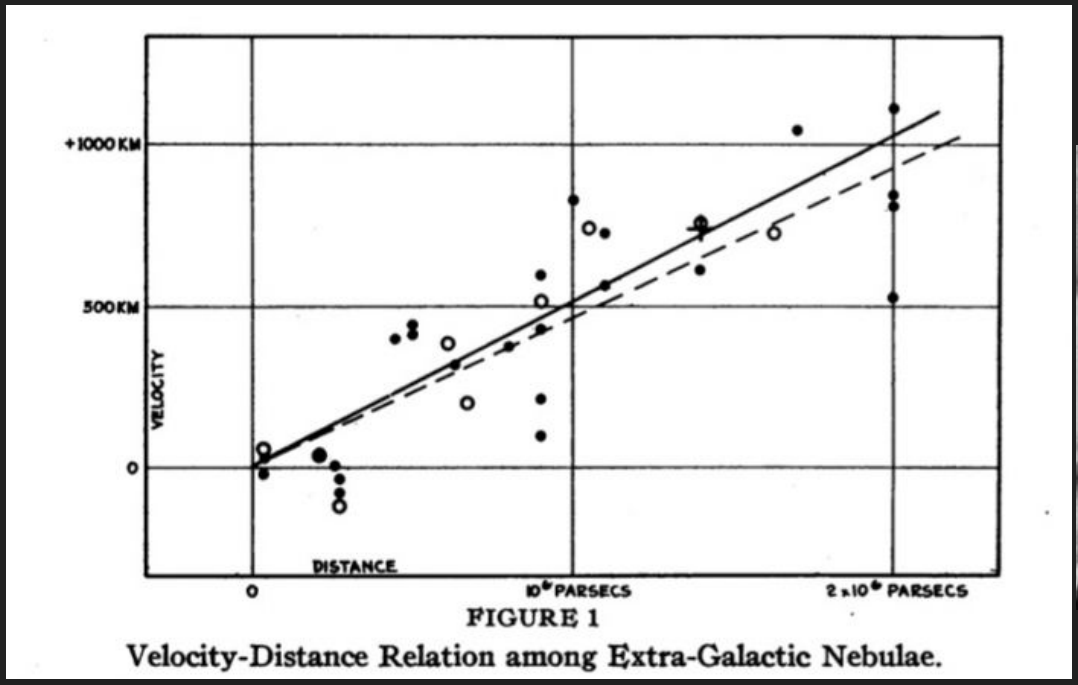


FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.

$$v_{recessional} = v_{H} + v_{peculiar}$$

$$v_{recessional} = H_0 \, d$$

$$v_{recessional} = v_H + v_{peculiar}$$

$$v_{recessional} - v_{peculiar} = H_0 \, d$$

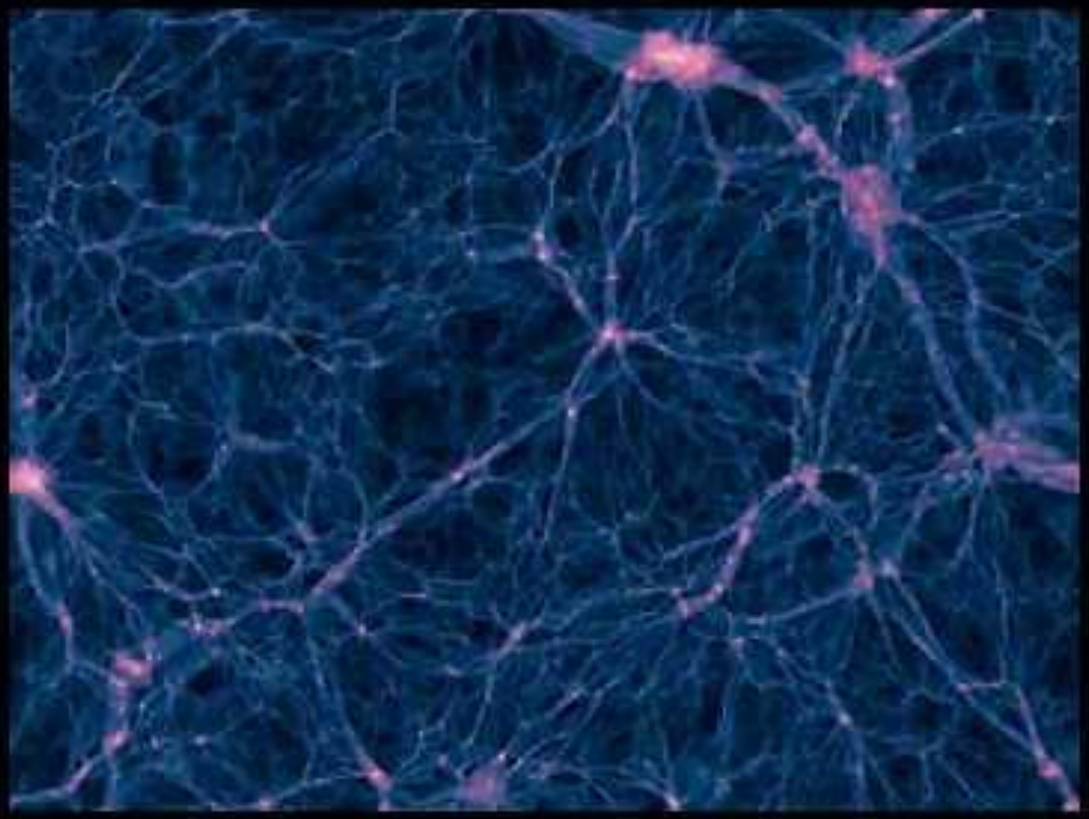
Hubble-Lemaitre Law

$$v_{recessional} = v_H + v_{peculiar}$$

$$v_{recessional} - v_{peculiar} = H_0 \, d$$

Hubble-Lemaitre Law

$$v_{recessional} = v_H + v_{peculiar}$$



- ➔ Assign galaxies to the dark matter halos using halo occupation distribution (HOD) models.
- ➔ Built a friends-of-friends algorithm
- ➔ Calibrate the friends-of-friends algorithm to the Crook et al. 2007 algorithm.
- ➔ Generate final groups!

- ➔ Not sure how I will go about this! :)

Assign mass-weighted average of group recessional velocity

Correct for the bulk velocity of the group using peculiar velocity maps (Springob et al. 2014)

LIGO's correction for Peculiar Velocity of NGC 4993

- ➔ NGC 4993 identified with HDC 763 group
  - ◆ Group catalog built using friends-of-friends algorithm
- ➔ Adopted the mass-weighted average group recessional velocity as the recessional velocity of the source

- ➔ Used peculiar velocity maps built using Fundamental Plane relation to correct for the bulk velocity of the group

LIGO's correction for Peculiar Velocity of NGC 4993

- ➔ NGC 4993 identified with HDC 763 group
  - ◆ Group catalog built using friends-of-friends algorithm
  - ◆ Recessional velocity ≠ Distance
- ➔ Adopted the mass-weighted average group recessional velocity as the recessional velocity of the source

- ➔ Used peculiar velocity maps built using Fundamental Plane relation to correct for the bulk velocity of the group

LIGO's correction for Peculiar Velocity of NGC 4993

- ➔ NGC 4993 identified with HDC 763 group
  - ◆ Group catalog built using friends-of-friends algorithm
  - ◆ Recessional velocity $\neq$ Distance
  - ◆ Contamination ratio? Completion ratio?
- ➔ Adopted the mass-weighted average group recessional velocity as the recessional velocity of the source
- ➔ Used peculiar velocity maps built using Fundamental Plane relation to correct for the bulk velocity of the group

LIGO's correction for Peculiar Velocity of NGC 4993

- ➔ NGC 4993 identified with HDC 763 group
  - ◆ Group catalog built using friends-of-friends algorithm
  - ◆ Recessional velocity $\neq$ Distance
  - ◆ Interlopers? Incomplete?
- ➔ Adopted the mass-weighted average group recessional velocity as the recessional velocity of the source
  - ◆ Could be adding an incorrect velocity component?
- ➔ Used peculiar velocity maps built using Fundamental Plane relation to correct for the bulk velocity of the group

LIGO's correction for Peculiar Velocity of NGC 4993

- ➔ NGC 4993 identified with HDC 763 group
  - ◆ Group catalog built using friends-of-friends algorithm
  - ◆ Recessional velocity $\neq$ Distance
  - ◆ Interlopers? Incomplete?
- ➔ Adopted the mass-weighted average group recessional velocity as the recessional velocity of the source
  - ◆ Could be adding an incorrect velocity component?
- ➔ Used peculiar velocity maps built using Fundamental Plane relation to correct for the bulk velocity of the group
  - ◆ Fundamental plane relationship applicable to elliptical galaxies

$$v_{hubble} = H_0 \, d$$

Hubble-Lemaitre Law

$$v_{recessional} - v_{peculiar} = H_0 \, d$$

Hubble-Lemaitre Law

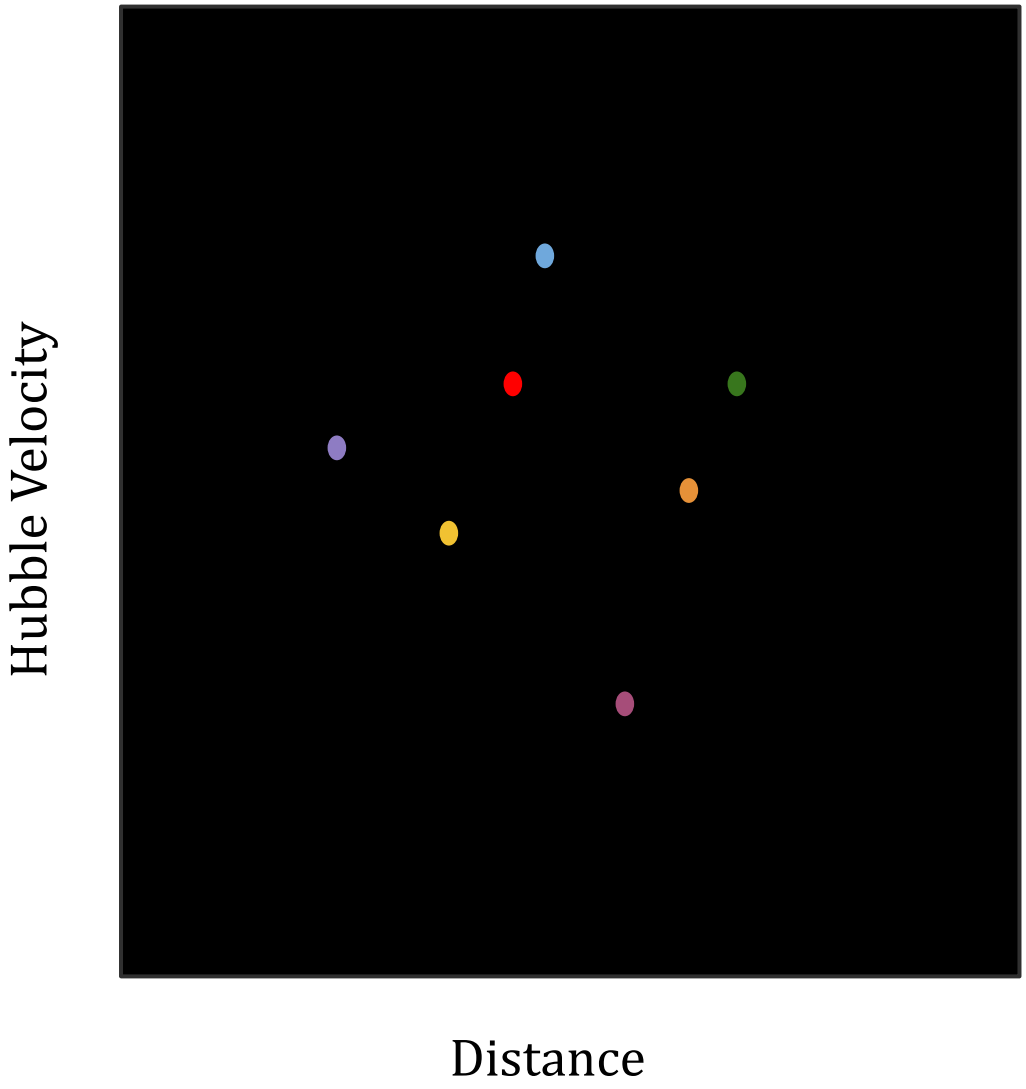
$$v_h = v_{recessional} + v_{peculiar}$$

I. Reliability

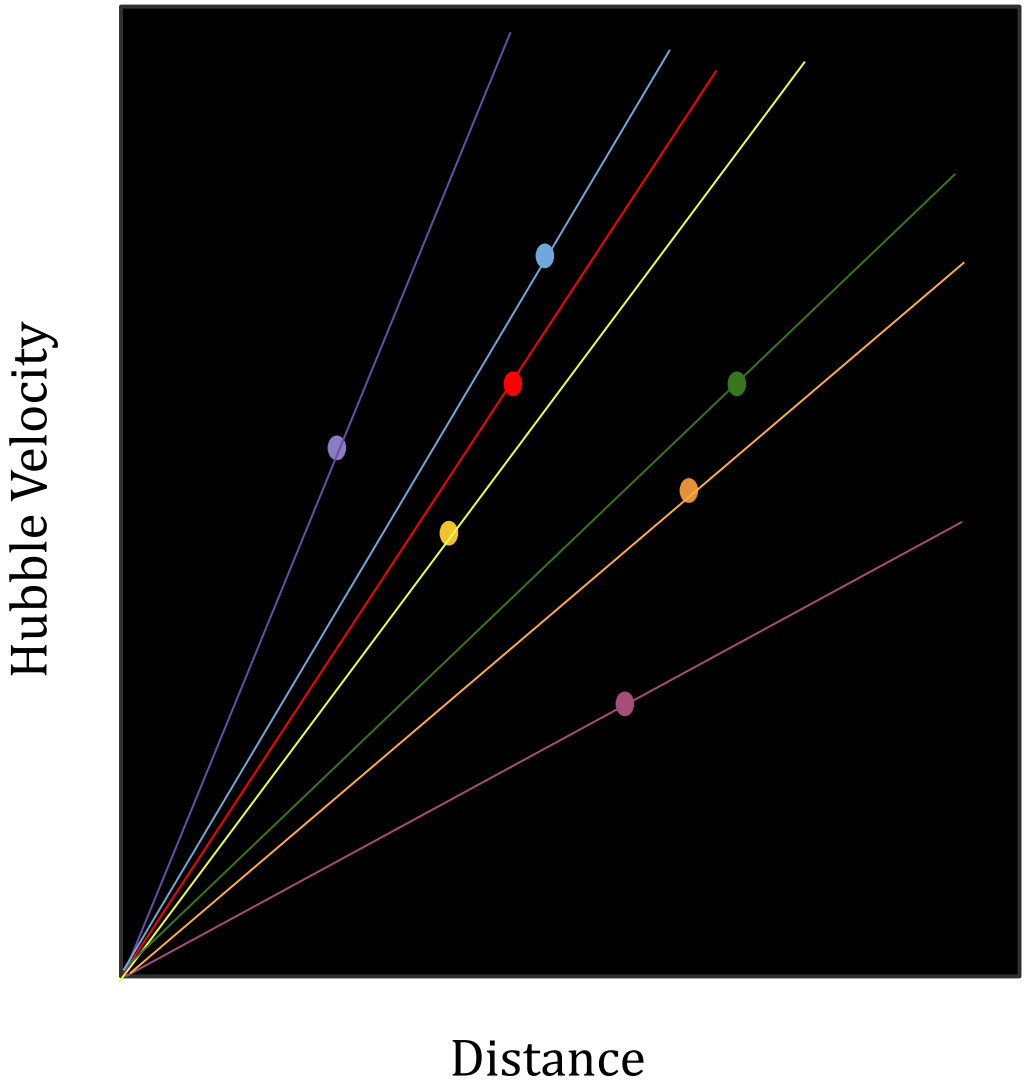
1. Contamination ratio of grouping algorithm
2. Completion ratio of grouping algorithm
3. Accuracy of peculiar velocity map

Scenario	Distance Measurements	Velocity Measurements
I.	$D_{LIGO} = (D_{true} + D_{random}) \pm 0.15*D_{true}$	$V_H = V_{Tot,true} \pm V_{pec,SD}$
II.	$D_{LIGO} = (D_{true} + D_{random}) \pm 0.15*D_{true}$	$V_H = (V_{Tot,true} - \langle V_{pec} \rangle) \pm V_{pec,SD}$

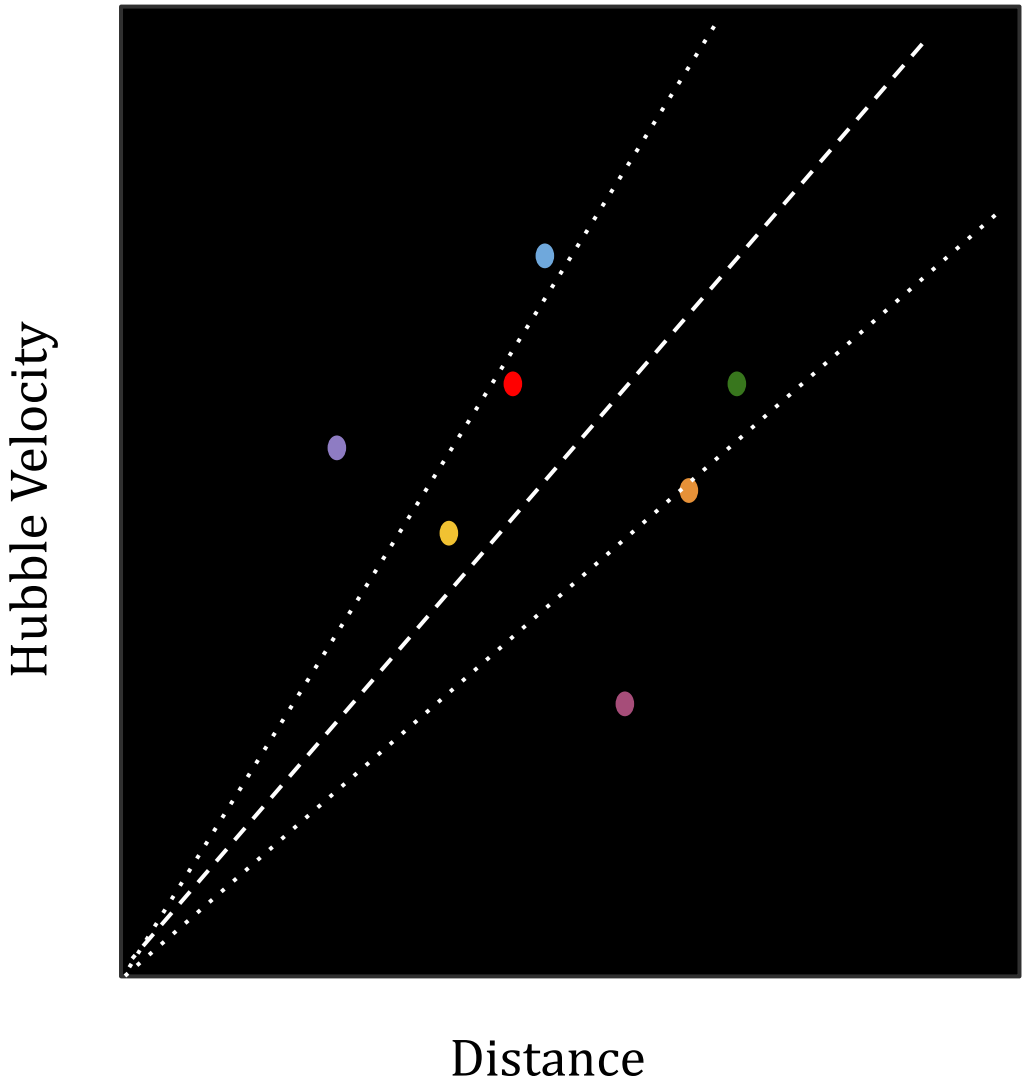
I. Measuring the scatter (1 detection in 7 universes)



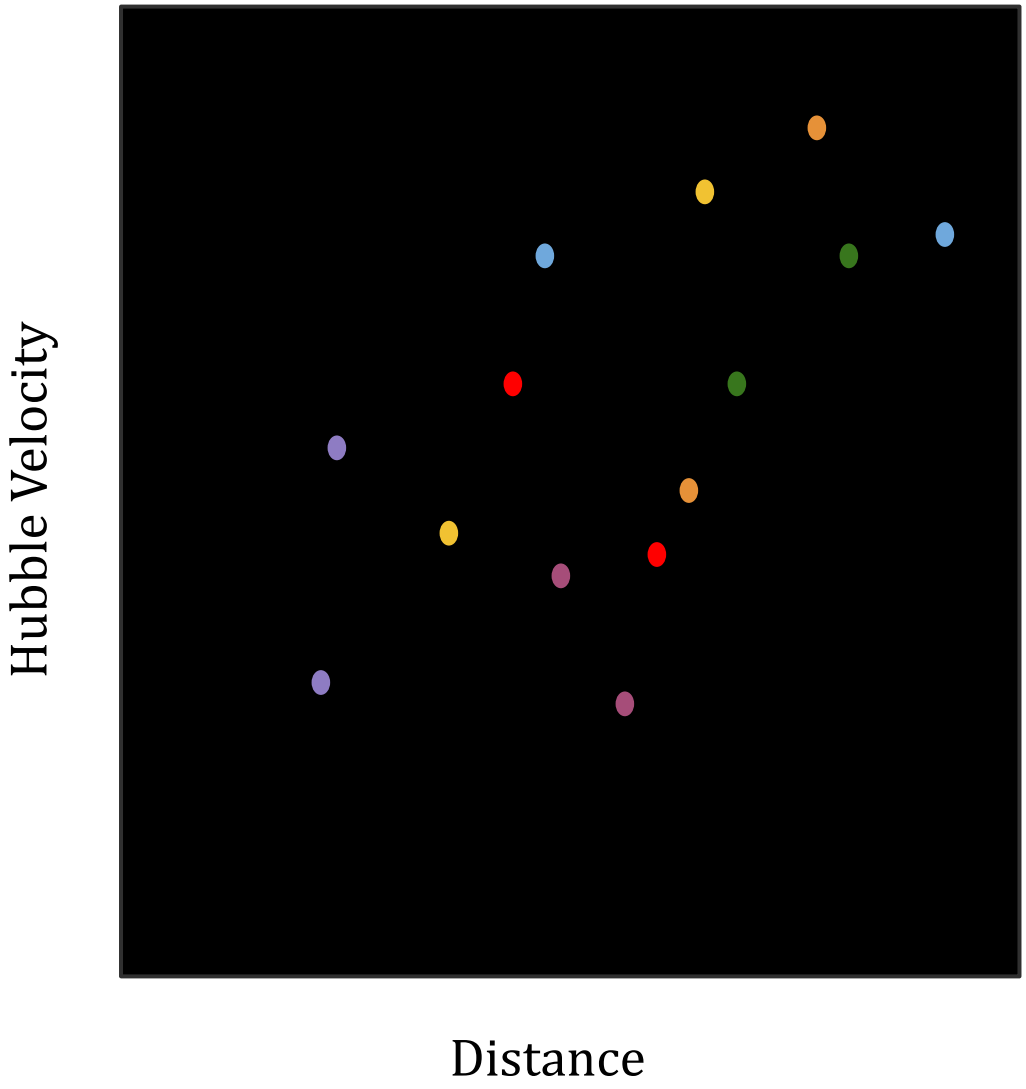
I. Measuring the scatter (1 detection in 7 universes)



I. Measuring the scatter (1 detection in 7 universes)



I. Measuring the scatter (2 detections in 7 universes)



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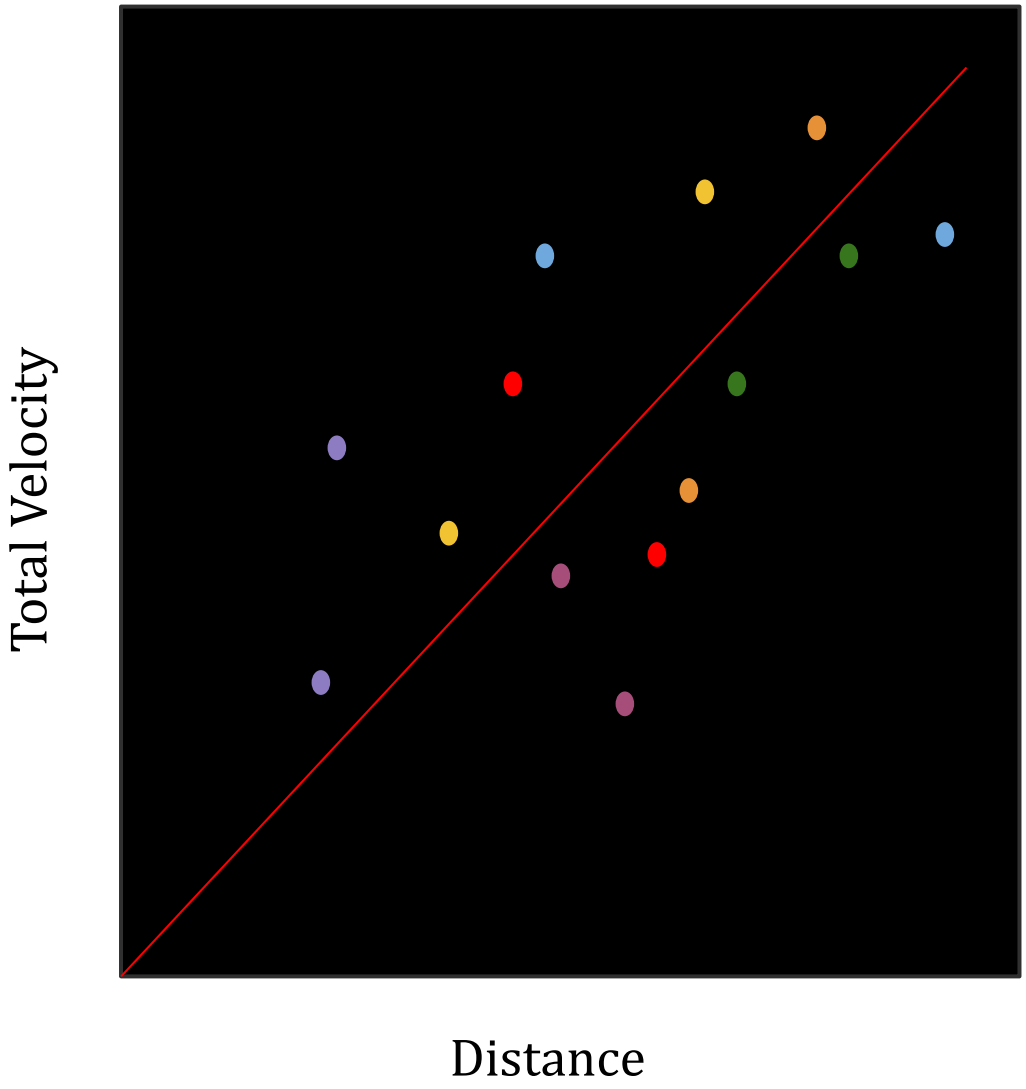

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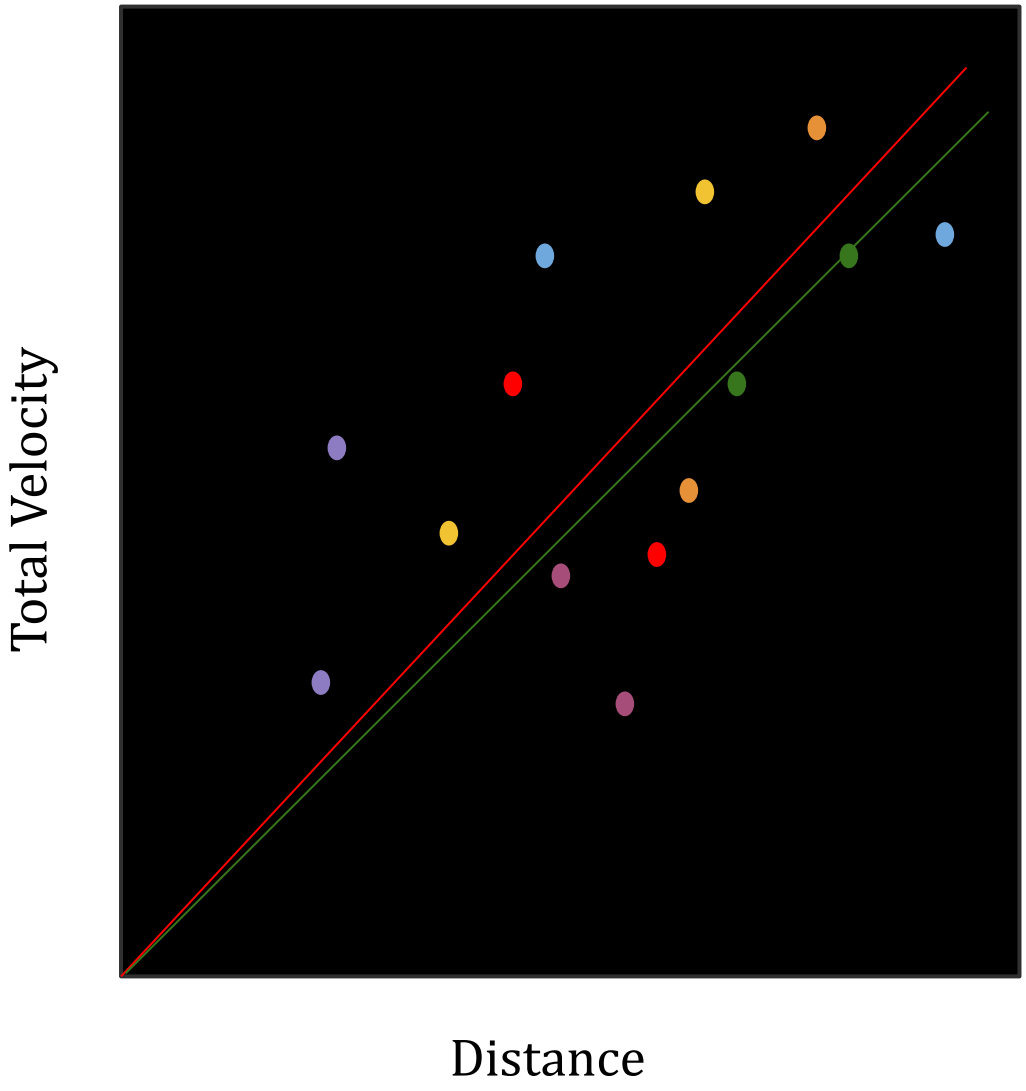
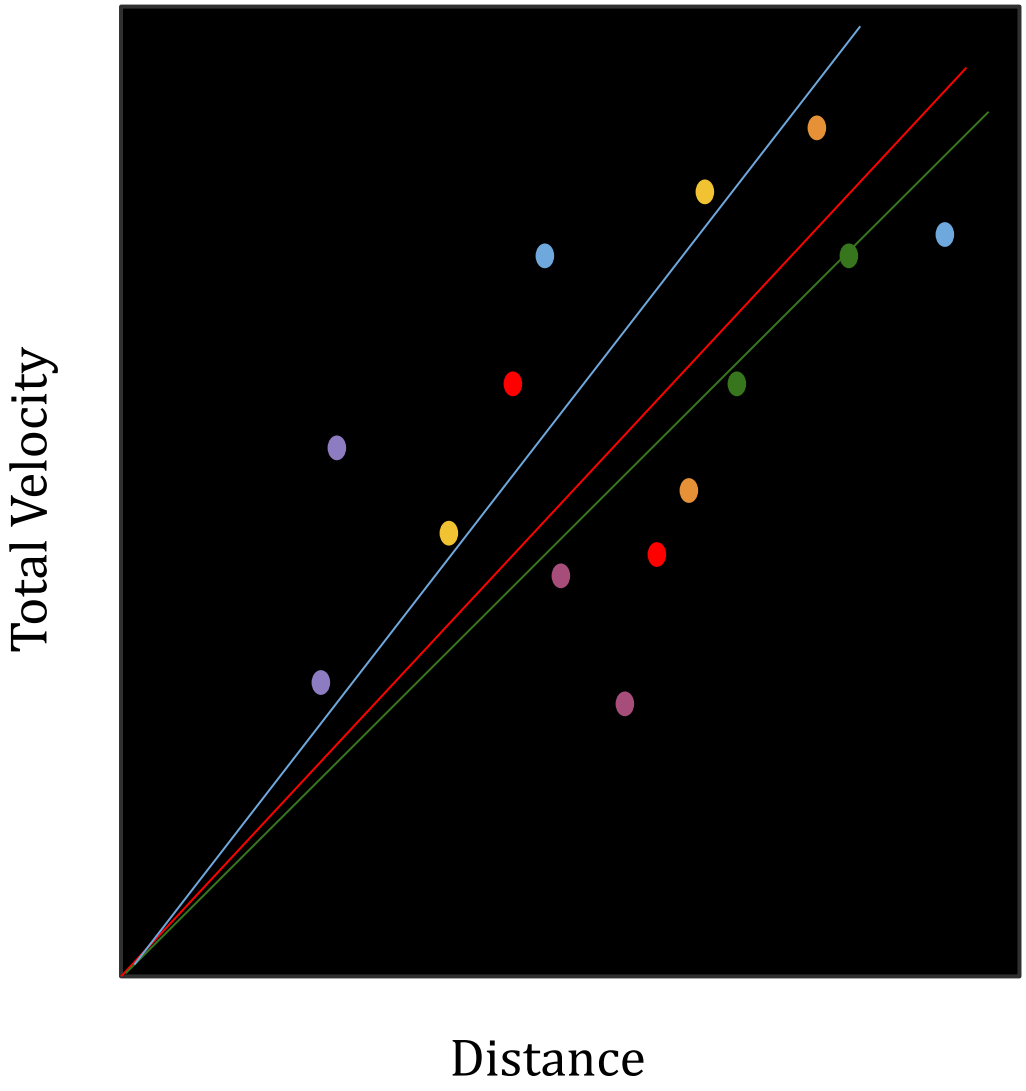
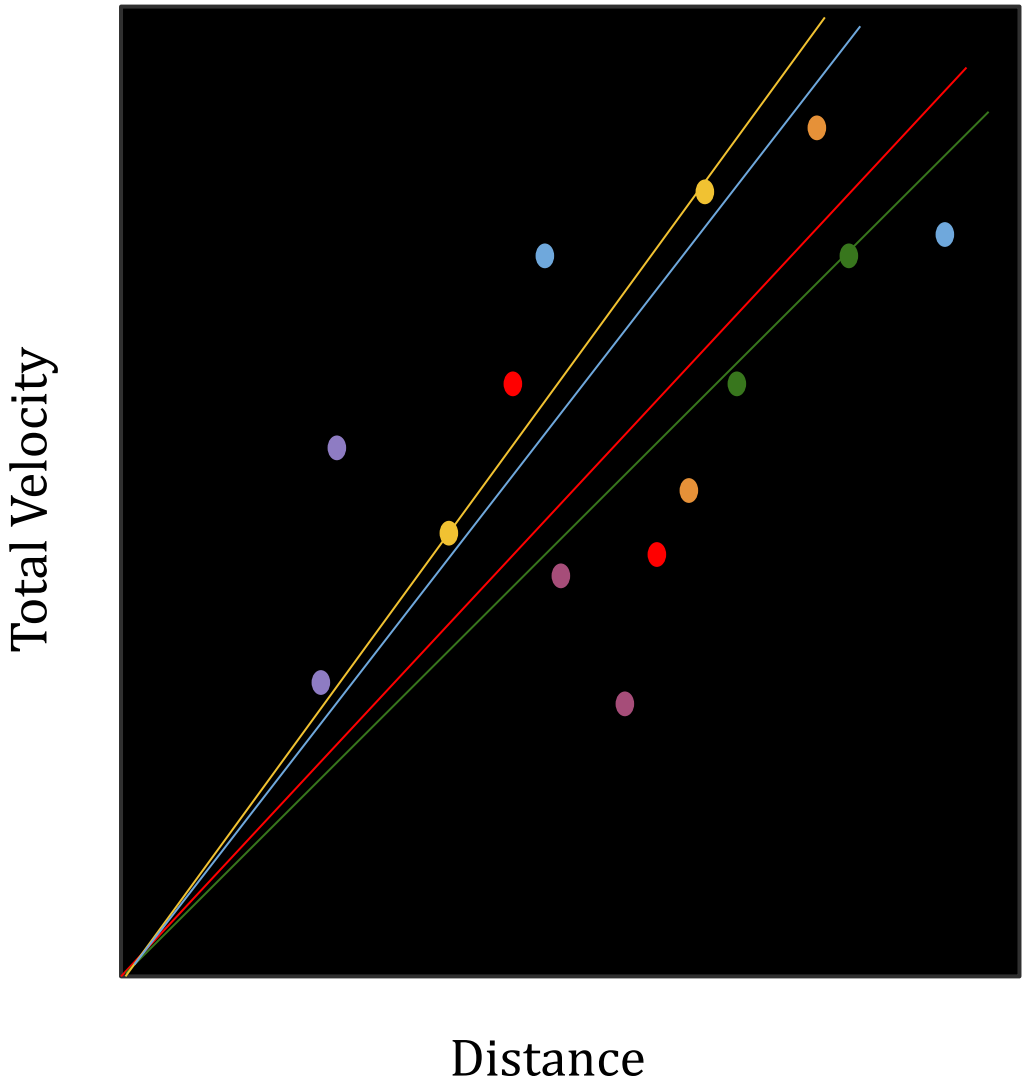


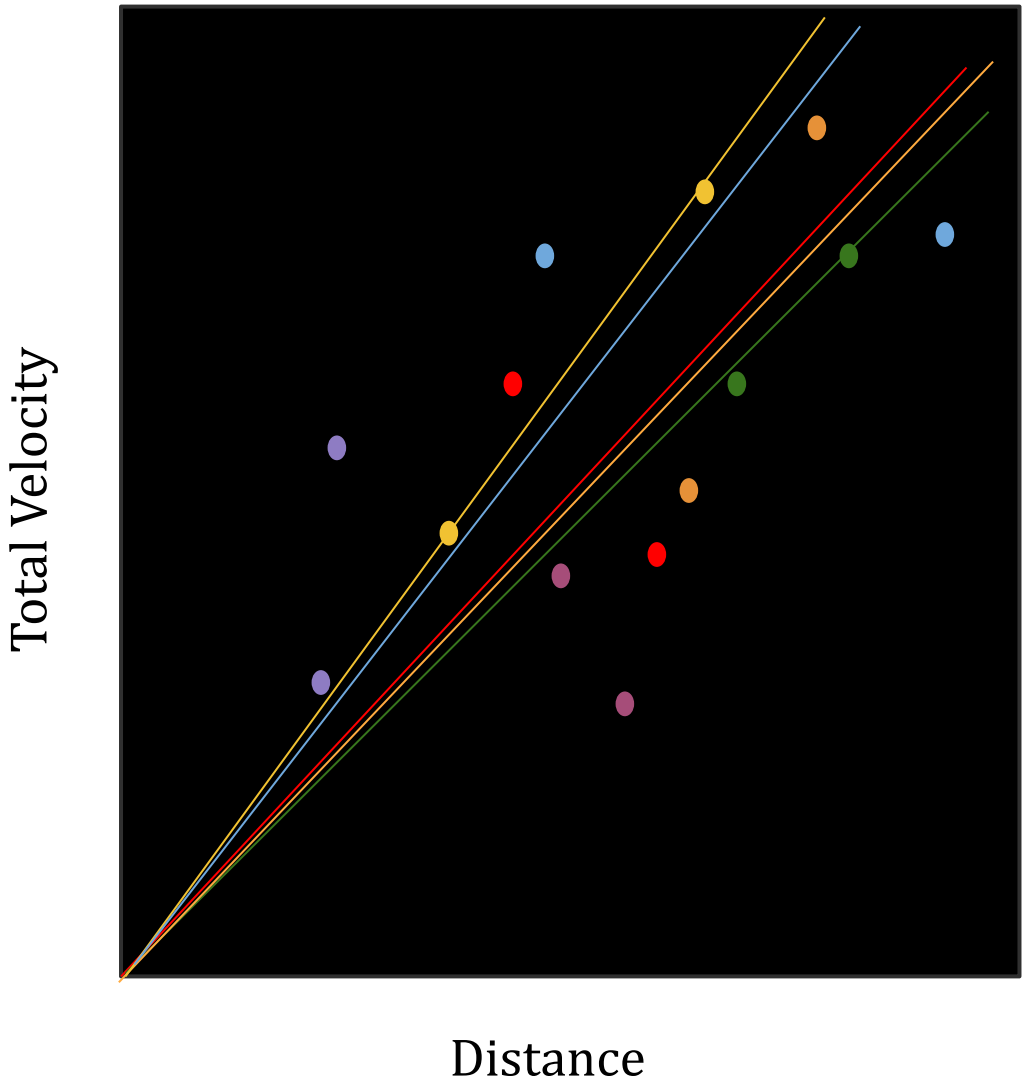
I. Measuring the scatter (2 detections in 7 universes)



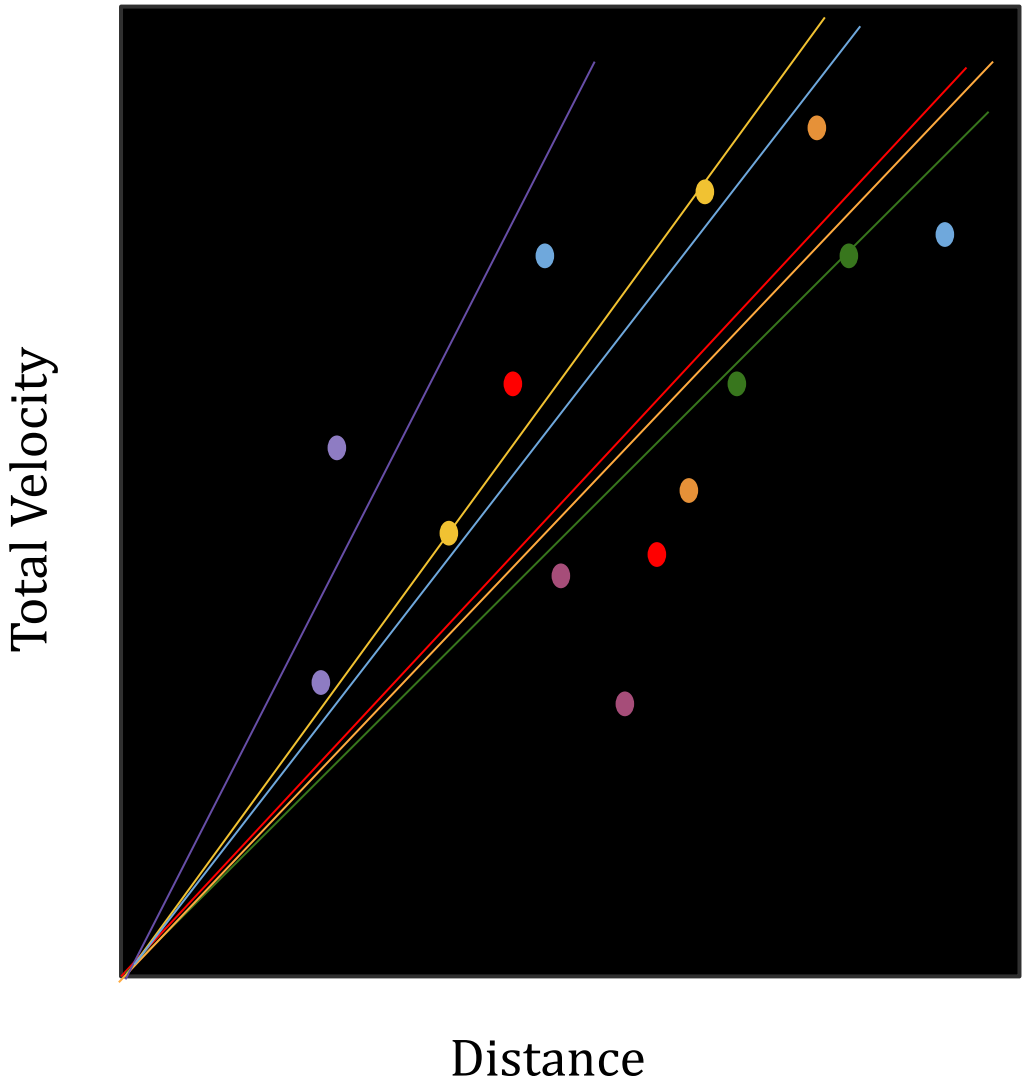
I. Measuring the scatter (2 detections in 7 universes)



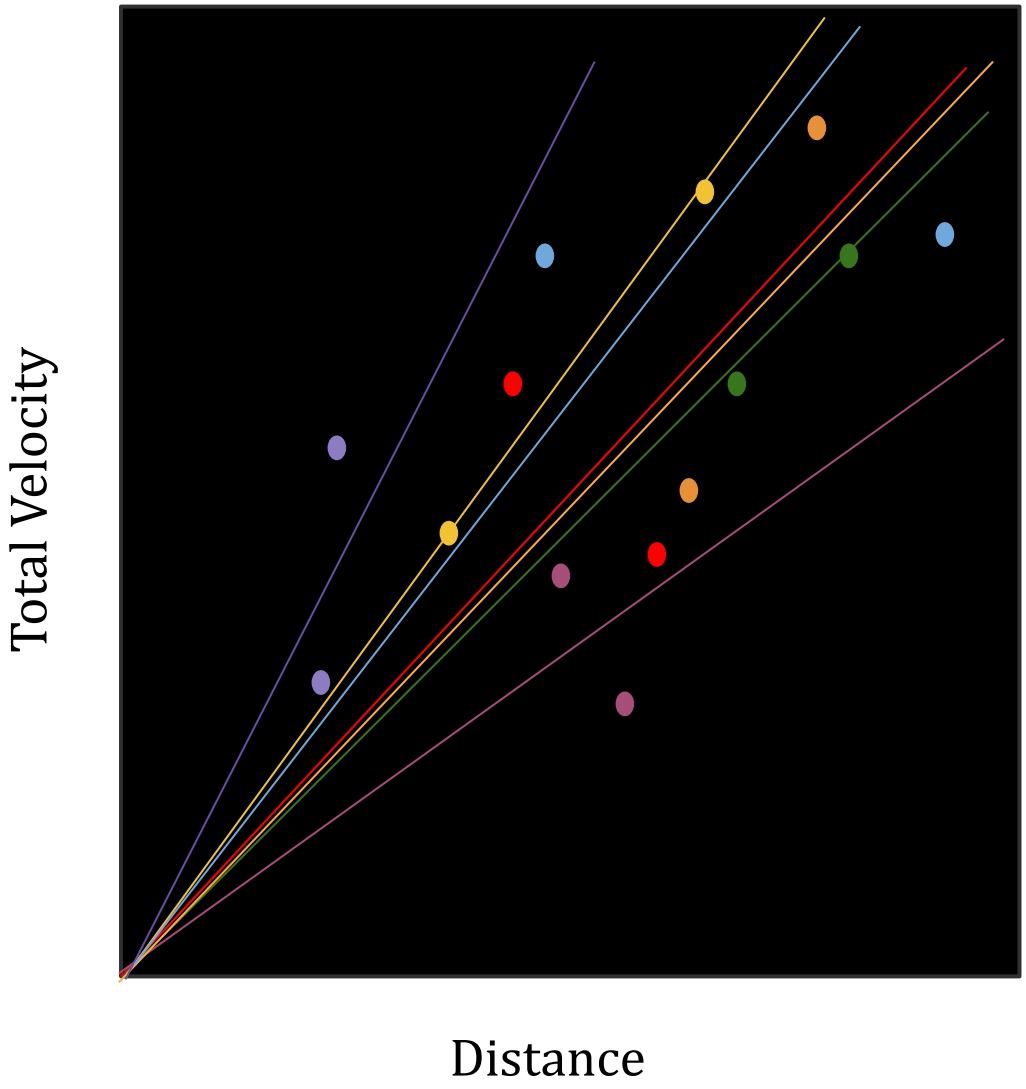
I. Measuring the scatter (2 detections in 7 universes)



I. Measuring the scatter (2 detections in 7 universes)



I. Measuring the scatter (2 detections in 7 universes)



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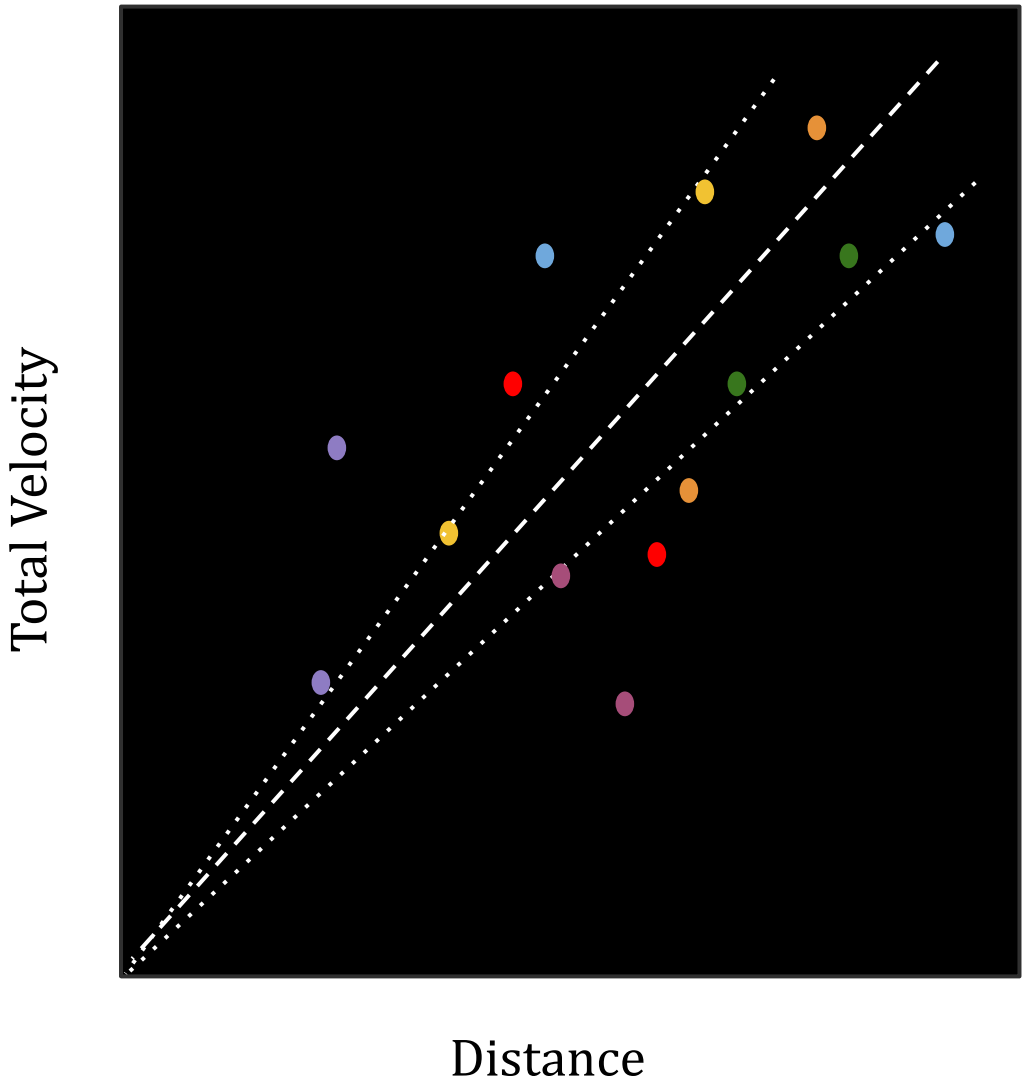

Evolution of H_0 distribution

Fiducial Model: $V_H = V_{Tot,true} \pm V_{pec,SD}$

Velocity Correction Model: $V_H = (V_{Tot,true} - \langle V_{pec} \rangle) \pm V_{pec,SD}$