Chapter Four: Set Theory

Learning Objectives:
By the end of the chapter, students should be able to:
1. Explain basic terms used in set theory such as element, set, subset, null set, universal set and cardinality of a set.
2. Interpret sets described using the set builder notation and standard infinite sets such as natural numbers, whole numbers, integers, rational numbers and real numbers.
3. Evaluate set operations such as union, intersection, difference and complement of sets.
4. Draw Venn diagrams and shade the regions corresponding to a given set notation.

Introduction

Set theory is fundamental in the use of mathematics for Computing and Information Technology. In this topic, we will be concentrating on the basics of sets. The basics of set theory covered in this topic will also be used in subsequent topics in this module.

4.1 Definition of a Set

A **set** is a well-defined list or collection of objects. The objects in a set are called the **elements** or **members** of the set. These objects can be anything – numbers, people, letters, etc.

Sets are commonly specified in two ways:

1. By listing all its elements, in any order, within braces, if possible.
   Example: $A=\{1,3,5,7,9\}$, $A=\{3,5,9,7,1\}$, etc.
   If not possible to list all, but the elements have some defined patterns, use ellipsis ($\cdots$).
   Example: $B=\{1,3,5,7,9,\cdots\}$
2. By using **set builder notation** |.
   Example: $A=\{x\mid x \text{ is odd}, 1\le x\le 10\}$. This is read as " A is the set of all x such that x is an odd number between 1 and 10 (inclusive)." Note that this is another way of specifying the set $A=\{1,3,5,7,9\}$.

It is often helpful to assign a name to a set. We use uppercase letters for the names of sets and lowercase letters for their elements.

Example 4.1
List down the members of the following sets:

(a) $C = \{x \mid x \text{ is a vowel of the alphabet}\}$

(b) $D = \{x \mid x \text{ is a solution of the equation } x^2 - x - 2 = 0\}$

(c) $E = \{k \mid k \text{ is an even number between 0 and 10 inclusive}\}$

Example 4.2
Rewrite the following sets using set builder notation $|$, by specifying the common property of the elements in the sets.

(a) $F = \{\clubsuit, \diamondsuit, \heartsuit, \spadesuit\}$

(b) $G = \{3, 6, 9, 12\}$

(c) $H = \{1, 4, 9, 16, 25\}$

4.1.1 Membership of a Set

If an object x is a member of set A, or A contains x as one of its elements, then we say

$$x \in A$$

which can be read as "x is an element of A" or "x belongs to A" or "x is in A".

Otherwise, if an object x is **not** a member of set A, then we say $x \notin A$, which can be read as "x is not an element of A" or "x does not belong to A" or "x is not in A".

Example 4.3

1. Let $V = \{v \mid v \text{ is a vowel of the alphabet}\}$, then
   a ___ V; b ___ V; e ___ V; a, i, o ___ V; b, c, d ___ V

2. Let $E = \{x \mid x \text{ is an even number}\}$, then
   3 ___ E; 6 ___ E; $1, 3, 5$ ___ E; 0 ___ E; $-2, -4$ ___ E

3. Let $S = \{k \mid k \text{ is a perfect square}\}$, then
   1 ___ S; 3 ___ S; 4 ___ S; -4 ___ S; $0, 25$ ___ S

Some Important Terminologies

Positive, Zero and Negative

The *Law of Trichotomy* states that all real numbers are divided into 3 categories:

1. Positive
2. Zero
3. Negative

Thinking Point:

Is positive the same as non-negative?

Even and Odd

Even numbers are any integers that when divided by two, gives a remainder of 0.
Odd numbers are any integers that when divided by two, gives a remainder of 1.

Thinking Point:

Many of us know that $2,4,6,8,\cdots$ are even and that $1,3,5,7,\cdots$ are odd. Questions are:

- Is 0 even?
- Is -3 odd?
- Is -2 even?

4.1.2 Numerical Sets

There are common numerical sets that mathematicians use. These sets are denoted by special symbols, and they will be used throughout the rest of the module.

- **Natural numbers:** $\mathbb{N} = \{1, 2, 3, 4, 5, \cdots\}$
- **Whole numbers:** $\mathbb{N}_0 = \{0, 1, 2, 3, 4, 5, \cdots\}$
- **Integers:** $\mathbb{Z} = \{\cdots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \cdots\}$
- **Rational numbers:** $\mathbb{Q} = \left\{ \frac{m}{n} \,\middle|\, m, n \in \mathbb{Z}, n \neq 0 \right\}$
- **Real numbers:** $\mathbb{R}$ $\rightarrow$ any numbers, rational or irrational, on a number line.

There are some common variations to the sets $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$:

$\mathbb{Z}^+ = \{x \mid x \in \mathbb{Z}, x > 0\}$	$\mathbb{Q}^+ = \{x \mid x \in \mathbb{Q}, x > 0\}$	$\mathbb{R}^+ = \{x \mid x \in \mathbb{R}, x > 0\}$
$\mathbb{Z}^- = \{x \mid x \in \mathbb{Z}, x < 0\}$	$\mathbb{Q}^- = \{x \mid x \in \mathbb{Q}, x < 0\}$	$\mathbb{R}^- = \{x \mid x \in \mathbb{R}, x < 0\}$
$\mathbb{Z}_0^+ = \{x \mid x \in \mathbb{Z}, x \geq 0\}$	$\mathbb{Q}_0^+ = \{x \mid x \in \mathbb{Q}, x \geq 0\}$	$\mathbb{R}_0^+ = \{x \mid x \in \mathbb{R}, x \geq 0\}$
$\mathbb{Z}_0^- = \{x \mid x \in \mathbb{Z}, x \leq 0\}$	$\mathbb{Q}_0^- = \{x \mid x \in \mathbb{Q}, x \leq 0\}$	$\mathbb{R}_0^- = \{x \mid x \in \mathbb{R}, x \leq 0\}$

Example 4.4

The following sets have been written using set builder notation |. Rewrite the sets by listing.

(a) $\{x \in \mathbb{N} \mid x^2 = 4\}$

(b) $\{x \in \mathbb{Z} \mid x^2 = 4\}$

(c) $\{x \in \mathbb{Q} \mid 3x = 4\}$

(d) $\{x \in \mathbb{R} \mid 3x^2 = 4\}$

(e) $\{x \in \mathbb{R}^- \mid 3x^2 = 4\}$

(f) $\{n \in \mathbb{Z} \mid n = 3k, k \in \{1, 2, 3\}\}$

(g) $\left\{n \in \mathbb{Q} \,\middle|\, n = \frac{k}{3}, k \in \{1, 2, 3\}\right\}$

4.1.3 Null Set

A set that contains no elements is called a **null set** or an **empty set**. It is denoted by $\varnothing$ or $\{\ \}$.

Example 4.5

Which of the following are null sets? Tick all that apply.

(a) $A=\{x\in\mathbb{Z} \mid x^2=3\}$

(b) $B=\{x\in\mathbb{Q} \mid x^2=3\}$

(c) $C=\{x\in\mathbb{R} \mid x^2=3\}$

(d) $D=\{k\in\mathbb{Z} \mid 3k=4\}$

(e) $E=\{x \mid x^2=4, x \text{ is odd}\}$

(a, b, d, e)

4.1.4 Universal Set

All the sets under investigation is called the **universal set** and denoted by U or ε.
In mathematics, common universal sets are $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$.

4.1.5 Subset

If **every** element of set A is also an element of set B, then A is called a **subset** of B.
We denote this as $A \subset B$, which is read as "A is contained in B", or "A is a subset of B".
We use the notation $\not\subset$ to indicate "not a subset of".

Example 4.6

(a) If $C = \{1,3,5\}$ and $D = \{1,2,3,4,5\}$, then C ___ D.

(b) If $C = \{1,3,5\}$ and $D = \{1,2,3,4\}$, then C ___ D.

(c) If $C = \{1,3,5\}$ and $E = \{5,3,1\}$, then C ___ E and E ___ C.

Note that:

1. Every set A is a subset of itself, i.e. $A \subset A$.
2. The null set $\varnothing$ is a subset of any set.
3. If $A \not\subset B$, then there is at least one element in A that is not an element of B.
4. If $A \subset B$ and $B \subset C$, then $A \subset C$.
5. If $A \subset B$ and $B \subset A$, then $A = B$.
   In other words, two sets are **equal** if and only if each is a subset of the other.
6. $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$.

4.1.6 Cardinality of a Set

The **number of elements** in set A is called the **cardinality** of set A, denoted by $|A|$ or $n(A)$.

Example 4.7

Find the cardinality of the following sets:

(a) W, where W is the set of days of the week.

(b) C, where C is the set of countries in the world today.

(c) $N = \{x \mid x \text{ is an even number and } 1 \le x \le 20\}$.

(7; 196; 10)

Example 4.8

Let $A=\{x\in\mathbb{N}\mid x=2y, y\in\mathbb{Z}, -1\le y\le 5\}$. List down the members in A, and find $|A|$.

(5)

If the cardinality of a set is a whole number, then the set is called a **finite** set. Otherwise, it is called an **infinite** set.

For example, let $P=\{r\mid r \text{ is a river on the earth}\}$. In this case, although it may be difficult to count the number of rivers on the earth, P is still a finite set.

Note that the set of natural numbers, integers, rational numbers and real numbers are all infinite. However, not all infinite sets are considered to be the same "size". The set of real numbers is considered to be a much larger set than the set of integers. In fact, this set is so large that we cannot possibly list all its elements in any organized manner the way the integers can be listed. We call a set like the real numbers that cannot be listed as *uncountable* and a set like integers that can be listed as *countable*.

4.2 Venn Diagrams

It is often very helpful to be able to visualize the relationship between sets. A very useful and simple device to do this is the *Venn diagram*. In a Venn diagram, the universal set U is represented by a rectangle, and circles within the rectangle represent any other subsets of U .

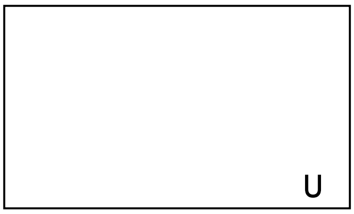


Figure 3.1 Universal Set, U

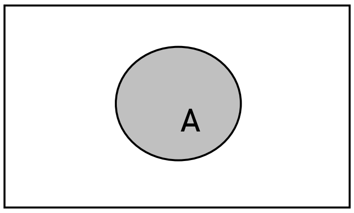


Figure 3.2 Set A

If $A \subset B$, then the circle representing A is entirely within the circle representing B as shown in Figure 3.3.

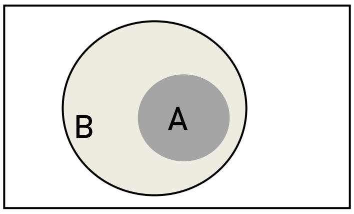


Figure 3.3 $A \subset B$

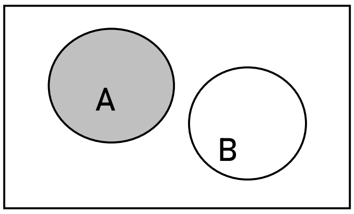


Figure 3.4 Disjoint Sets A and B

If A and B are disjoint, i.e. they have no element in common, then the circle representing A will be separated from the circle representing B as shown in Figure 3.4.

Example 4.9

Draw a Venn diagram of sets A , B and C where $A \subset B$, sets A and C are disjoint, but B and C have some elements in common.

4.3 Set Operations

4.3.1 Union

The *union* of sets A and B, denoted by $A \cup B$, is the set of all elements that belong to either A or B or both. In short,

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

$A \cup B$ is read as "A **union** B" or "A **or** B".

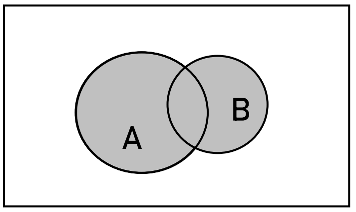


Figure 3.5 A ∪ B

Example 4.10

Find $A \cup B$, given the following definitions of A and B:

(a) $A = \{a, b, c, d\}$, $B = \{f, b, d, g\}$

(b) $A = \{x \in \mathbb{N} \mid x \text{ is odd}, x \le 10\}$, $B = \{x \in \mathbb{N} \mid x \text{ is prime}, x \le 10\}$

(c) $A = \mathbb{N}$, $B = \mathbb{Z}$

Note that:

- $A \cup B = B \cup A$
- $A \subset (A \cup B)$ and $B \subset (A \cup B)$

4.3.2 Intersection

The *intersection* of sets A and B, denoted by $A \cap B$, is the set of all elements that belong to both A and B. In short,

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

$A \cap B$ is read as "A **intersect** B" or "A **and** B".

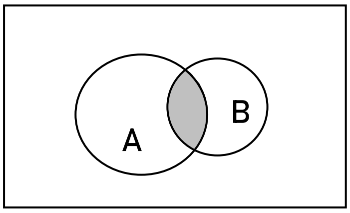


Figure 3.6 A ∩ B

Example 4.11

Find $A \cap B$, given the following definitions of A and B:

(a) $A = \{a, b, c, d\}$, $B = \{f, b, d, g\}$

(b) $A = \{x \in \mathbb{N} \mid x \text{ is odd}, x \leq 10\}$, $B = \{x \in \mathbb{N} \mid x \text{ is prime}, x \leq 10\}$

(c) $A = \mathbb{N}$, $B = \mathbb{Z}$

Note that:

- $A \cap B = B \cap A$
- $(A \cap B) \subset A$ and $(A \cap B) \subset B$
- If $A \cap B = \varnothing$, then sets A and B are said to be **disjoint**.
- If $A \cap B \neq \varnothing$, then sets A and B have some elements in common.

4.3.3 Complement

The complement of A, denoted by $\overline{A}$, A' or A^c, is the set of all elements not belonging to A. In short,

$$\overline{A} = \{x \mid x \in U \text{ and } x \notin A\}$$

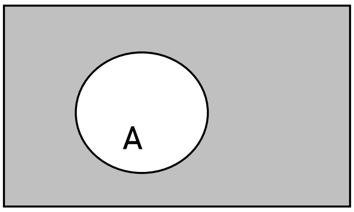


Figure 3.8 $\overline{A}$

$\overline{A}$ is read as "A **complement**", "**not in** A", "**outside** A", or simply "A **bar**".

Example 4.12

Let A be a set within the universal set U.

For the following definitions of A and U, find $\overline{A}$:

(a) $U = \{x \in \mathbb{N} \mid x \le 10\}$, $A = \{x \mid x \text{ is odd}\}$

(b) $U = \mathbb{N}$, $A = \{2, 4, 6, 8, 10, \cdots\}$

Note that:

- $A \cup \overline{A} = U$
- $A \cap \overline{A} = \varnothing$
- $\overline{U} = \varnothing$, $\overline{\varnothing} = U$
- $\overline{\overline{A}} = A$

4.3.4 Difference

The difference of sets A and B, denoted by $A-B$, is the set of all elements which belong to A but not to B. In short,

$$A-B=\{x \mid x\in A \text{ and } x\notin B\}$$

$A-B$ is read as "A **difference** B" or "A **minus** B".

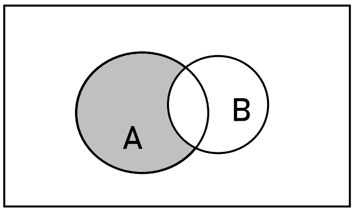


Figure 3.7 A − B

Example 4.13

Find $A-B$, given the following definitions of A and B:

(a) $A=\{a,b,c,d\}$, $B=\{f,b,d,g\}$

(b) $A=\{x\in\mathbb{N} \mid x \text{ is odd}, x\le 10\}$, $B=\{x\in\mathbb{N} \mid x \text{ is prime}, x\le 10\}$

(c) $A=\mathbb{N}$, $B=\mathbb{Z}$

Note that:

- $\overline{A}=U-A$
- $(A-B)\subset A$
- $A-B$, $A\cap B$ and $B-A$ are **mutually disjoint**. That is, the intersection of any two of the above three sets is the null set.
- $A-B=A\cap\overline{B}$

4.4 Shading Venn Diagrams

Example 4.14

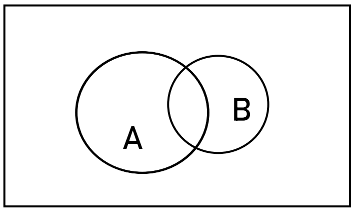
Redraw the following Venn diagram and shade the areas the belong to the following set notations:

(a) $A \cap B$

(b) $A \cap \overline{B}$

(c) $(A \cup B) \cap (B \cap \overline{A})$

(d) $(A \cup B) - (A \cap B)$



Example 4.15 (De Morgan's Laws for set theory)

De Morgan's laws are named after Augustus De Morgan, a 19th century British mathematician. The laws state that:

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

Prove these laws by shading Venn diagrams.

Example 4.16

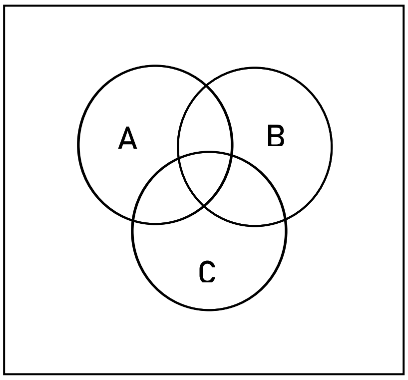
Redraw the following Venn diagram and shade the areas the belong to the following set notations:

(a) $(A \cup B) - C$

(b) $A \cap \left(B \cup \overline{C}\right)$

(c) $(A \cup B) - (A \cap C)$

(d) $\overline{A} \cup \overline{B \cap \overline{C}}$



Tutorial 4 – Set Theory

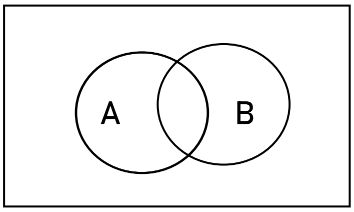
Section A (Basic)

1. Describe the following sets in words:
   (a) $A=\{♣, ♦, ♥, ♠\}$
   (b) $B=\{x\in\mathbb{N} \mid x\le 50\}$
   (c) $C=\{y\in\mathbb{Z} \mid -5\le y\le 5\}$

2. Let $U=\{1,2,3,4,5,6,7,8,9,10\}$, $A=\{1,3,5,7\}$, $B=\{4,5,6,7,8\}$ and $C=\{5,6,7,8,9,10\}$.
   Find the following sets:
   (a) $A\cup B$
   (b) $B\cap C$
   (c) $\overline{A}$
   (d) $A\cap\overline{C}$
   (e) $A-C$
   (f) $A\cap(B\cup C)$
   (g) $A\cup(B\cap C)$

3. List down the numbers in the following sets:
   (a) $A=\{x\in\mathbb{Z} \mid x^2=9\}$
   (b) $B=\{x\in\mathbb{Z} \mid x^2=3\}$
   (c) $C=\{x\in\mathbb{Q} \mid (3x-2)(x-1)=0\}$
   (d) $D=\{x\in\mathbb{R} \mid 2x^2-3x-4=0\}$
   (e) $E=\{x\in\mathbb{Q} \mid 2x^2-3x-4=0\}$
   (f) $F=\{x\in\mathbb{N} \mid y\in G,\ x=y^2\}$ where $G=\{-3,-2,-1,0,1,2,3\}$

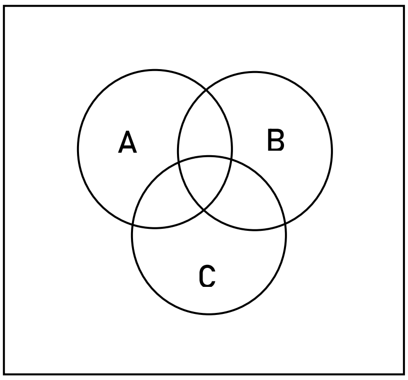
For Questions 4 and 5, make use of the Venn diagram below:



4. Shade the following region:
   (a) $\overline{A}$
   (b) $A\cap\overline{B}$
   (c) $A\cup\overline{B}$
   (d) $\overline{B-A}$

5. Draw Venn diagrams for $\overline{A\cup B}$, $\overline{A\cap B}$, $\overline{A}\cup\overline{B}$ and $\overline{A}\cap\overline{B}$. Which of these are the same?

6. In the Venn diagram below, shade the following:
   (a) $\overline{A} \cap (B \cup C)$ (b) $A \cup (B \cap \overline{C})$



7. List down the numbers in the following sets:
   (a) $A = \{x \in \mathbb{N} \mid x^2 < 100, x > 4\}$
   (b) $B = \{x \in \mathbb{Z} \mid x^2 < 10\}$
   (c) $C = \{x \in \mathbb{N} \mid x = y^2, y \in B\}$
   (d) $D = \{x \in \mathbb{N}_0 \mid x = 2y, y \in B\}$
   (e) $E = \left\{x \in \mathbb{Z} \,\middle|\, x = \dfrac{y}{2}, y \in B\right\}$
   (f) $F = \left\{x \in \mathbb{Q} \,\middle|\, x = \dfrac{y}{2}, y \in B\right\}$
   (g) $G = \{x \in \mathbb{R}^+ \mid x = \sqrt{y}, y \in B\}$

8. Which of the following sets are equal to the set of all integers that are multiples of 3?
   (a) $\{3, 6, 9, 12, \cdots\}$
   (b) $\{0, 3, 6, 9, 12, \cdots\}$
   (c) $\{\cdots, -9, -6, -3, 0, 3, 6, 9, 12, \cdots\}$
   (d) $\{3n \mid n \in \mathbb{Z}\}$
   (e) $\{3n \mid n \in \mathbb{R}\}$
   (f) $\{n \in \mathbb{Z} \mid n = 3k, k \in \mathbb{Z}\}$
   (g) $\{n \in \mathbb{N} \mid n = 3k, k \in \mathbb{Z}\}$
   (h) $\{n \in \mathbb{Z} \mid n = 3k, k \in \mathbb{N}\}$

9. Find the cardinality of the following sets, and hence classify them as finite or infinite.
   (a) $A = \{x \in \mathbb{Z} \mid 1 < x < 10\}$
   (b) $B = \{x \in \mathbb{R} \mid 1 < x < 10\}$
   (c) $C = \{x \in \mathbb{R} \mid 3 < x < \pi\}$

Section B (Intermediate/Challenging)

10. It is given that the universal set $U = \{n \in \mathbb{Z} \mid n^2 \le 36\}$, $A = \{x \in \mathbb{N} \mid 3x \le 16\}$ and $B = \{2y \mid y \in \mathbb{Z}\}$ such that both A and B are within U.
    (a) Rewrite sets U, A and B by listing.
    (b) Find $B - A$ and $\overline{A \cap B}$.
    (c) Find $A \cap \overline{B}$. Hence, determine $\left|A \cap \overline{B}\right|$.

11. Let $A = \{x \in \mathbb{Z} \mid -2 \le x \le 2\}$. List the members of the following sets.
    (a) $B = \{x \in \mathbb{Z} \mid 2x \in A\}$
    (b) $C = \{x \in \mathbb{Q} \mid 2x \in A\}$
    (c) $D = \{x \in A \mid \frac{x}{2} \in \mathbb{Z}\}$
    (d) $E = \{x \in \mathbb{R}^+ \mid \frac{2}{3}x \in A\}$
    (e) $F = \{x \in \mathbb{Q}^- \mid x^2 \in A\}$
    (f) $G = \{x \in \mathbb{R}^- \mid x^2 \in A\}$
    (g) $H = \{x \in \mathbb{R} \mid x^2 \in A\}$

Section C (MCQ)

12. **(1718S1/A3)** Which of the following two sets A and B have the relationship $A \subset B$?
    (a) $A = \{x \mid x \text{ is multiple of } 2\}$, $B = \{x \mid x \text{ is multiple of } 3\}$
    (b) $A = \{x \mid x \text{ is prime number}\}$, $B = \{x \mid x \text{ is odd number}\}$
    (c) $A = \{x \mid x \text{ is even number}\}$, $B = \{x \mid x \text{ is positive integer}\}$
    (d) $A = \{x \mid x \text{ is integer}\}$, $B = \{x \mid x \text{ is rational number}\}$

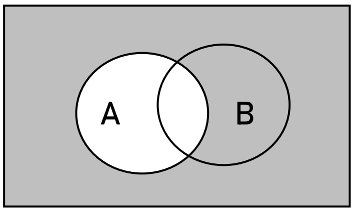
13. **(1718S2/A2)** Suppose we need to represent the following set of numbers in binary:

$$\left\{ \frac{k}{4} \,\middle|\, k \in \mathbb{Z}, 0 < k \le 10000 \right\}$$

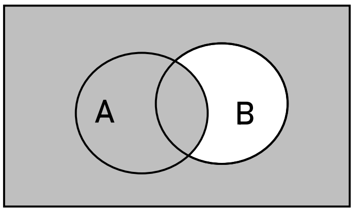
    The minimum number of bits required to represent this set of numbers is
    (a) 10 (b) 12 (c) 14 (d) 16

Tutorial 4 – Answers

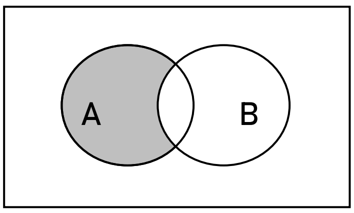
1. (a) A is the set of suits in poker cards
   (b) B is the set of natural numbers 50 or below
   (c) C is the set of integers between -5 and 5 inclusive
2. (a) $A \cup B = \{1,3,4,5,6,7,8\}$
   (b) $B \cap C = \{5,6,7,8\}$
   (c) $\overline{A} = \{2,4,6,8,9,10\}$
   (d) $A \cap \overline{C} = \{1,3\}$
   (e) $A - C = \{1,3\}$
   (f) $A \cap (B \cup C) = \{5,7\}$
   (g) $A \cup (B \cap C) = \{1,3,5,6,7,8\}$
3. (a) $A = \{-3,3\}$
   (b) $B = \varnothing$
   (c) $C = \left\{\frac{2}{3},1\right\}$
   (d) $D = \left\{\frac{3+\sqrt{41}}{4},\frac{3-\sqrt{41}}{4}\right\}$
   (e) $E = \varnothing$
   (f) $F = \{1,4,9\}$
4. (a) $\overline{A}$



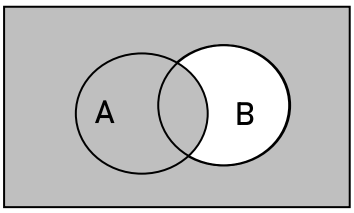
(c) $A \cup \overline{B}$



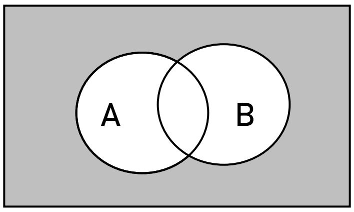
(b) $A \cap \overline{B}$



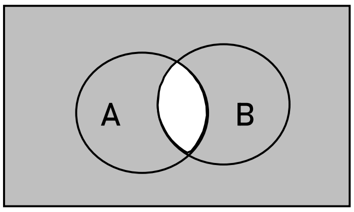
(d) $\overline{B - A}$



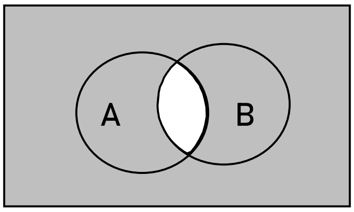
5. $\overline{A \cup B}$



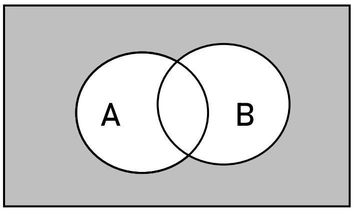
$\overline{A \cap B}$



$\overline{A} \cup \overline{B}$

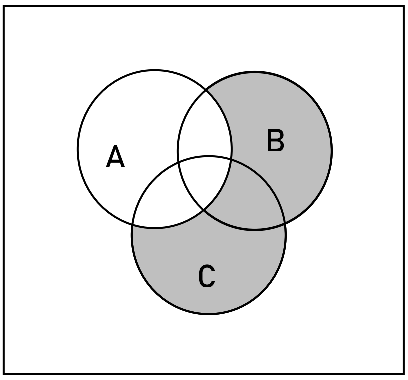


$\overline{A} \cap \overline{B}$

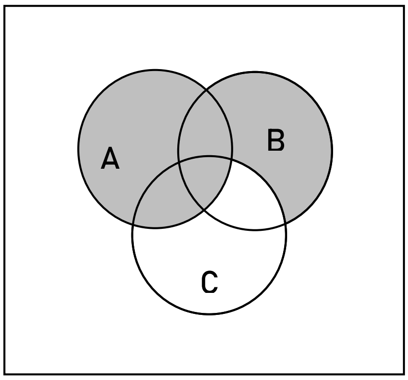


$\overline{A \cup B} = \overline{A} \cap \overline{B}$; $\overline{A \cap B} = \overline{A} \cup \overline{B}$ (This is called De Morgan's laws for set theory; we will revisit De Morgan's laws again in Chapter 5 and 6)

6. (a) $\overline{A} \cap (B \cup C)$



(b) $A \cup (B \cap \overline{C})$



7. (a) $A = \{5, 6, 7, 8, 9\}$
   (b) $B = \{-3, -2, -1, 0, 1, 2, 3\}$
   (c) $C = \{1, 4, 9\}$
   (d) $D = \{0, 2, 4, 6\}$
   (e) $E = \{-1, 0, 1\}$
   (f) $F = \{-\frac{3}{2}, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}\}$
   (g) $G = \{1, \sqrt{2}, \sqrt{3}\}$
8. Sets (c), (d) and (f) only.
9. (a) 8; Finite set (b) ∞; Infinite set (c) ∞; Infinite set

10. (a) $U = \{-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$; $A = \{1, 2, 3, 4, 5\}$; $B = \{-6, -4, -2, 0, 2, 4, 6\}$

(b) $B - A = \{-6, -4, -2, 0, 6\}$; $\overline{A \cap B} = \{-6, -5, -4, -3, -2, -1, 0, 1, 3, 5, 6\}$

(c) $A \cap \overline{B} = \{1, 3, 5\}$; $\left|A \cap \overline{B}\right| = 3$

11. (a) $B = \{-1, 0, 1\}$

(b) $C = \left\{-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\right\}$

(c) $D = \{-2, 0, 2\}$

(d) $E = \left\{\frac{3}{2}, 3\right\}$

(e) $F = \{-1\}$

(f) $G = \left\{-1, -\sqrt{2}\right\}$

(g) $H = \left\{-\sqrt{2}, -1, 0, 1, \sqrt{2}\right\}$

12. d 13. c