SOLUTIONS

SINGAPORE POLYTECHNIC
2022/2023 Semester 2 Mid-Semester Test

No.	SOLUTION
1(a)	For symmetric matrix, $\mathbf{A}_{ij}=\mathbf{A}_{ji}$. $\mathbf{A}_{23}=\mathbf{A}_{32}:\ 2c+3=7\Rightarrow c=2$ $\mathbf{A}_{12}=\mathbf{A}_{21}:\ a-c=1\Rightarrow a-2=1\Rightarrow a=3$ $\mathbf{A}_{13}=\mathbf{A}_{31}:\ 3b+c=-1\Rightarrow 3b+2=-1\Rightarrow b=-1$
1(b)	Given $\mathbf{E}^T=2\mathbf{F}-\mathbf{X}$: $\mathbf{X}=2\mathbf{F}-\mathbf{E}^T=\begin{bmatrix}4&6\\10&-2\end{bmatrix}-\begin{bmatrix}-3&5\\8&4\end{bmatrix}=\begin{bmatrix}7&1\\2&-6\end{bmatrix}$
1(c)(i)	$\mathbf{ED}=\begin{bmatrix}-3&8\\5&4\end{bmatrix}\begin{bmatrix}1&0&-4\\3&-1&2\end{bmatrix}=\begin{bmatrix}21&-8&28\\17&-4&-12\end{bmatrix}$
1(c)(ii)	$\mathbf{D}+\mathbf{E}+\mathbf{F}$ cannot be evaluated because the order of $\mathbf{D}$ is not the same as the order of $\mathbf{E}$ and $\mathbf{F}$.
1(c)(iii)	$\mathbf{F}^2-10\mathbf{I}=\begin{bmatrix}19&3\\5&16\end{bmatrix}-\begin{bmatrix}10&0\\0&10\end{bmatrix}=\begin{bmatrix}9&3\\5&6\end{bmatrix}$
1(d)(i)	$\mathbf{B}^T\mathbf{B}=\begin{bmatrix}1&4&8\\8&-4&1\\-4&-7&4\end{bmatrix}\begin{bmatrix}1&8&-4\\4&-4&-7\\8&1&4\end{bmatrix}=\begin{bmatrix}81&0&0\\0&81&0\\0&0&81\end{bmatrix}$
1(d)(ii)	Given $\mathbf{CY}=\mathbf{B}+\mathbf{Y}$: $\mathbf{CYY}^{-1}=\mathbf{BY}^{-1}+\mathbf{YY}^{-1}$ $\mathbf{C}=\mathbf{BY}^{-1}+\mathbf{I}$ $\mathbf{BY}^{-1}=\mathbf{C}-\mathbf{I}$ $\mathbf{B}^{-1}\mathbf{BY}^{-1}=\mathbf{B}^{-1}(\mathbf{C}-\mathbf{I})$ $\mathbf{Y}^{-1}=\mathbf{B}^{-1}(\mathbf{C}-\mathbf{I})$ From part (i), $\mathbf{B}^T\mathbf{B}=81\mathbf{I}\Rightarrow\mathbf{B}^{-1}=\frac{1}{81}\mathbf{B}^T$ Hence, $\mathbf{Y}^{-1}=\frac{1}{81}\mathbf{B}^T(\mathbf{C}-\mathbf{I})=\frac{1}{81}\begin{bmatrix}1&4&8\\8&-4&1\\-4&-7&4\end{bmatrix}\begin{bmatrix}7&2&-3\\5&-2&12\\3&9&3\end{bmatrix}=\frac{1}{27}\begin{bmatrix}17&22&23\\13&11&-23\\-17&14&-20\end{bmatrix}$

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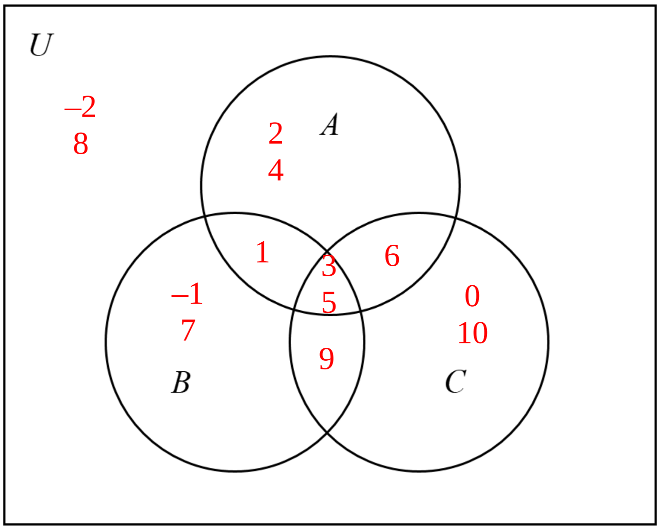
2(a)	$U=\{-2,-1,0,1,2,3,4,5,6,7,8,9,10\}$ $A=\{1,2,3,4,5,6\}$ $B=\{-1,1,3,5,7,9\}$ $C=\{0,3,5,6,9,10\}$
2(b)	$A-B=\{2,4,6\}$ $A\cap(B\cup C)=\{1,3,5,6\}$
2(c)	
2(d)(i)	$P=\{3,4,5,6,7,\cdots\}$ $Q=\{7,14,23,34,47,\cdots\}$
2(d)(ii)	By observing the pattern of the ordered elements in set Q, the formula for the n^{th} element in set Q is given by $(n+2)^2-2$. $\therefore$ 100th element in set $Q=(100+2)^2-2=10,402$

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3(a)

Integral part:		
2	978	
2	489	0
2	244	1
2	122	0
2	61	0
2	30	1
2	15	0
2	7	1
2	3	1
2	1	1
	0	1

Fractional part:		
2	0.425	
2	0.85	0
2	0.7	1
2	0.4	1
2	0.8	0
2	0.6	1
2	0.2	1
2	0.4	0
2	0.8 (rep)	0

$\therefore 978.425_{10} = 1111010010.011\overline{0110}_2 = 3D2.6\overline{C}_{16}$

3(b)

Convert x, y and z to decimal:

$x = 222.2_4 = 2\times\left(4^2 + 4^1 + 4^0 + 4^{-1}\right) = 42.5_{10}$

$y = 222.2_8 = 2\times\left(8^2 + 8^1 + 8^0 + 8^{-1}\right) = 146.25_{10}$

$z = 222.2_{16} = 2\times\left(16^2 + 16^1 + 16^0 + 16^{-1}\right) = 546.125_{10}$

Calculate the sum of x, y and z in decimal:

$w = x + y + z = 42.5_{10} + 146.25_{10} + 546.125_{10} = 734.875_{10}$

Convert w from decimal to base-32:

Integral part:		
32	734	
32	22	30
	0	22

Fractional part:		
32	0.875	
	0	28

$\therefore w = \mathrm{MU}.\mathrm{S}_{32}$

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4(a)(i)	$\mathbf{T}_1=\begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix}$; $\mathbf{T}_2=\begin{bmatrix}1&0&2\\0&1&1\\0&0&1\end{bmatrix}$; $\mathbf{T}_3=\begin{bmatrix}2&0&0\\0&3&0\\0&0&1\end{bmatrix}$
4(a)(ii)	$\mathbf{C}=\mathbf{T}_3\mathbf{T}_2\mathbf{T}_1=\begin{bmatrix}2&0&0\\0&3&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&2\\0&1&1\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}2&0&4\\0&-3&3\\0&0&1\end{bmatrix}$
4(a)(iii)	$\mathbf{P}'=\mathbf{CP}=\begin{bmatrix}2&0&4\\0&-3&3\\0&0&1\end{bmatrix}\begin{bmatrix}2&-1&1\\1&0&3\\1&1&1\end{bmatrix}=\begin{bmatrix}8&2&6\\0&3&-6\\1&1&1\end{bmatrix}$
4(a)(iv)	$\mathbf{T}_1^{-1}=\begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix}$; $\mathbf{T}_2^{-1}=\begin{bmatrix}1&0&-2\\0&1&-1\\0&0&1\end{bmatrix}$; $\mathbf{T}_3^{-1}=\begin{bmatrix}\frac{1}{2}&0&0\\0&\frac{1}{3}&0\\0&0&1\end{bmatrix}$
4(a)(v)	$\mathbf{C}^{-1}=\mathbf{T}_1^{-1}\mathbf{T}_2^{-1}\mathbf{T}_3^{-1}=\begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&-2\\0&1&-1\\0&0&1\end{bmatrix}\begin{bmatrix}\frac{1}{2}&0&0\\0&\frac{1}{3}&0\\0&0&1\end{bmatrix}=\begin{bmatrix}\frac{1}{2}&0&-2\\0&-\frac{1}{3}&1\\0&0&1\end{bmatrix}$
4(b)(i)	$\mathbf{T}_a$: Reflection about the y-axis $\mathbf{T}_b$: Shearing in the x-direction by a factor of 1 $\mathbf{T}_c$: Translation 3 units to the right $\mathbf{T}_a=\begin{bmatrix}-1&0&0\\0&1&0\\0&0&1\end{bmatrix}$; $\mathbf{T}_b=\begin{bmatrix}1&1&0\\0&1&0\\0&0&1\end{bmatrix}$; $\mathbf{T}_c=\begin{bmatrix}1&0&3\\0&1&0\\0&0&1\end{bmatrix}$
4(b)(ii)	$\mathbf{T}=\mathbf{T}_c\mathbf{T}_b\mathbf{T}_a=\begin{bmatrix}1&0&3\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&1&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}-1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}-1&1&3\\0&1&0\\0&0&1\end{bmatrix}$ $\mathbf{U}'=\mathbf{TU}=\begin{bmatrix}-1&1&3\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&3&4&2\\1&1&3&3\\1&1&1&1\end{bmatrix}=\begin{bmatrix}3&1&2&4\\1&1&3&3\\1&1&1&1\end{bmatrix}$ (verified)