SINGAPORE POLYTECHNIC
2021/2022 SEMESTER ONE MID-SEMESTER TEST

Common Infocomm Technology Programme (CITP)
Diploma in Applied AI & Analytics (DAAA)
Diploma in Infocomm Security Management (DISM)
Diploma in Information Technology (DIT)

MS0105 – Mathematics

Time allowed: 1.5 hours

Instructions to Candidates

1. The SP examination rules are to be complied with.
   Any candidate who cheats or attempts to cheat will face disciplinary action.

2. This paper consists of **4** printed pages (including the cover page).
   There are 4 questions (100 marks in total), and you are to answer all the questions.

3. Unless otherwise stated, all **non-exact** decimal answers should be rounded to **at least two** decimal places.

4. Except for sketches, graphs and diagrams, no solutions are to be written in pencil. Failure to comply may result in loss of marks.

Formula Sheet: Transformation Matrices

1. Reflection			3. Shearing	
a. about the y-axis	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		a. in the x-direction	$\begin{bmatrix} 1 & s_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
b. about the x-axis	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		b. in the y-direction	$\begin{bmatrix} 1 & 0 & 0 \\ s_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
c. about the line $y = x$	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		4. Rotation about the origin	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
2. Scaling relative to the origin	$\begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$		5. Translation	$\begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix}$

1. Given the following matrices:

$$\mathbf{A}=\begin{bmatrix} 1 & 4a & b \\ b+2 & 3 & b+c \\ 2 & a-c & -1 \end{bmatrix} \qquad \mathbf{B}=\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -2 \\ -2 & 0 & -1 \end{bmatrix} \qquad \mathbf{C}=\begin{bmatrix} -2 & -1 & -6 \\ 7 & 5 & 8 \\ 8 & 2 & 9 \end{bmatrix}$$

$$\mathbf{D}=\begin{bmatrix} 3 & -2 & 1 \\ -1 & 4 & 2 \end{bmatrix} \qquad \mathbf{E}=\begin{bmatrix} -1 & 6 \\ 4 & -3 \end{bmatrix}$$

(a) If **A** is a symmetric matrix, find the values of a, b and c.

(5 marks)

(b) Find matrix $\mathbf{F}$ such that $\mathbf{E}=3\mathbf{I}_2-2\mathbf{F}$, where $\mathbf{I}_2$ is the 2×2 identity matrix.

(5 marks)

(c) Evaluate the following wherever possible.
State the reason(s) clearly if the expressions cannot be evaluated.

(i) $\mathbf{D}^2$

(ii) $\mathbf{CD}^T$

(iii) $2\mathbf{E}-\mathbf{E}^T$

(8 marks)

(d) (i) Evaluate $\mathbf{B}(\mathbf{C}-2\mathbf{I}_3)$, where $\mathbf{I}_3$ is the 3×3 identity matrix.

(ii) Hence, find matrix $\mathbf{G}$ such that $\mathbf{GC}=\mathbf{C}+2\mathbf{G}$.

(12 marks)

2. (a) Let the universal set $U = \{x \in \mathbb{N} \mid x \leq 8\}$ and define the following sets within U :

$$A = \{x \mid x \text{ is even}\}$$

$$B = \{x \mid 2x \in A\}$$

$$C = \{2x \mid x \in A\}$$

(i) Rewrite sets U , A, B and C by listing.

(ii) Find $A - C$, $B \cap \overline{C}$ and $\overline{A \cup B}$.

(12 marks)

(b) Draw a Venn diagram containing three sets P , Q and R such that $P \subset Q$, $R \not\subset Q$, $P \cap R = \varnothing$ and $Q \cap R \neq \varnothing$.

(4 marks)

(c) Redraw your Venn diagram in part (b) and shade the region $(P \cup R) \cap \left(Q \cup \overline{R}\right)$.

(4 marks)

3. **For this question, show your working clearly. No marks will be awarded if the steps involved are not shown.**

(a) Convert the following numbers to their binary representations:

(i) 627.45_{10}

(ii) $9\text{E}4.\text{A}_{16}$

Express your answers in **exact** form, showing the recursion clearly for the fractional part, if any.

(10 marks)

(b) Let x be the **smallest** 12-bit binary number that has:

- twice as many 1's as 0's, and
- thrice as many integral bits as fractional bits.

What is the decimal representation of x ?

(4 marks)

(c) An arcade programmer designed a game machine that only allows customers to win a prize every 300 counts (in decimal). The machine started from an initial count of zero and generated a last count of 189C_{16} when the previous customer won a prize.

You decide to play the machine immediately afterwards and will only stop playing after you have won two prizes. What will be the count (**in hexadecimal**) generated by the machine when you win your second prize?

(6 marks)

4. **Solve this question using homogeneous coordinates.**

(a) A triangle $\mathbf{P}$ with vertices $(2,2)$, $(3,1)$ and $(1,4)$ undergoes the following sequence of transformations:

$\mathbf{T}_1$: translation 2 units to the left and 1 unit downwards, followed by

$\mathbf{T}_2$: reflection about the *y*-axis, followed by

$\mathbf{T}_3$: rotation 90° anticlockwise about the origin.

(i) Write down the transformation matrices $\mathbf{T}_1$, $\mathbf{T}_2$ and $\mathbf{T}_3$.

(ii) Compute the composite matrix $\mathbf{C}$ for the above sequence of transformations.

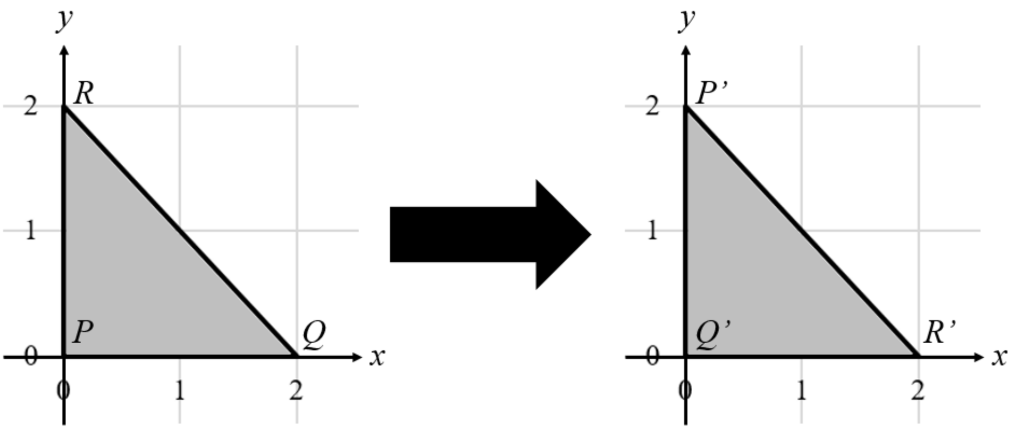
(iii) Find $\mathbf{P}'$, the image matrix of triangle $\mathbf{P}$ after undergoing the above sequence of transformations.

(iv) Write down the inverse transformation matrices $\mathbf{T}_1^{-1}$, $\mathbf{T}_2^{-1}$ and $\mathbf{T}_3^{-1}$.

(v) Compute the composite matrix $\mathbf{C}^{-1}$ that transforms $\mathbf{P}'$ back to $\mathbf{P}$.

(20 marks)

(b) In the figure below, triangle PQR is transformed to $P'Q'R'$ through a sequence of **three** simple transformations.



(i) Describe, in words, the three transformations needed to transform triangle PQR to $P'Q'R'$, and write down the corresponding transformation matrices.

(ii) Hence, derive the composite matrix **T** for the above sequence of transformations and verify that **T** successfully transforms triangle PQR to $P'Q'R'$.

(10 marks)

******* END OF PAPER *******