SINGAPORE POLYTECHNIC
2022/2023 SEMESTER ONE MID-SEMESTER TEST

Common Infocomm Technology Programme (CITP)
Diploma in Applied AI & Analytics (DAAA)
Diploma in Infocomm Security Management (DISM)
Diploma in Information Technology (DIT)

MS0105 – Mathematics

Time allowed: 1 hour 40 minutes

Instructions to Candidates

1. The SP examination rules are to be complied with.
   Any candidate who cheats or attempts to cheat will face disciplinary action.

2. This paper consists of **4** printed pages (including the cover page).
   There are 4 questions (100 marks in total), and you are to answer all the questions.

3. Unless otherwise stated, all **non-exact** decimal answers should be rounded to **at least two** decimal places.

4. Except for sketches, graphs and diagrams, no solutions are to be written in pencil. Failure to comply may result in loss of marks.

Formula Sheet: Transformation Matrices

1. Reflection		3. Shearing	
a. about the y-axis	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	a. in the x-direction	$\begin{bmatrix} 1 & s_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
b. about the x-axis	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	b. in the y-direction	$\begin{bmatrix} 1 & 0 & 0 \\ s_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
c. about the line $y=x$	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	4. Rotation about the origin	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
2. Scaling relative to the origin	$\begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	5. Translation	$\begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix}$

1. Given the following matrices:

$$\mathbf{A}=\begin{bmatrix} 2 & 10 & 2a-3b \\ 5a & -3 & a+b+c \\ 7 & 5 & 8 \end{bmatrix} \qquad \mathbf{B}=\begin{bmatrix} 6 & 1 \\ -1 & 4 \\ 3 & -2 \end{bmatrix} \qquad \mathbf{C}=\begin{bmatrix} 4 & -1 & 2 \\ -3 & 1 & 0 \\ -2 & 5 & 8 \end{bmatrix}$$

$$\mathbf{D}=\begin{bmatrix} 3 & -5 \\ -2 & 7 \end{bmatrix} \qquad \mathbf{E}=\begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix}$$

(a) If **A** is a symmetric matrix, find the values of a, b and c.

(5 marks)

(b) Find matrix $\mathbf{X}$ such that $4\mathbf{X}^T+5\mathbf{D}=3\mathbf{E}$.

(5 marks)

(c) Evaluate the following wherever possible.
State the reason(s) clearly if the expressions cannot be evaluated.

(i) $\mathbf{BD}$

(ii) $\left(\mathbf{EB}^T\mathbf{C}\right)^2$

(6 marks)

(d) (i) Evaluate $\mathbf{E}^2-2\mathbf{E}-9\mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix.

(ii) The expression $\mathbf{E}^2-2\mathbf{E}-9\mathbf{I}$ can be expressed as $(\mathbf{E}+3\mathbf{I})(\mathbf{E}+p\mathbf{I})+q\mathbf{I}$, where p and q are constants to be determined. Find the values of p and q.

(iii) Hence, find $(\mathbf{E}+3\mathbf{I})^{-1}$.

(14 marks)

2. Let the universal set $U=\{x\in\mathbb{N} \mid x\le 12\}$ and define the following sets within U :

$$A=\{x\in\mathbb{Z} \mid 3\le x\le 7\}$$

$$B=\left\{\frac{x}{3} \,\middle|\, x\in U\right\}$$

$$C=\{x \mid x^2+1 \text{ is a multiple of } 5\}$$

(a) Rewrite sets U, A, B and C by listing.

(5 marks)

(b) Find $|A|$, $\overline{A}\cap B$ and $C-A$.

(5 marks)

(c) Draw a Venn diagram showing the above sets U, A, B and C, indicating all the elements clearly.

(5 marks)

(d) Another set D is defined within U, as follows:

$$D=\left\{x \,\middle|\, \frac{x}{y}\notin\mathbb{Z},\ y\in\left(A\cap(B\cup C)\right)\right\}$$

Rewrite set D by listing. Show your working clearly.

(5 marks)

3. **Show your working clearly for this question. No marks will be awarded if the steps involved are not shown.**

(a) Convert 1685.9_{10} to its binary and hexadecimal representations.

Express your answers in **exact** form, showing the recursion clearly for the fractional part, if any.

(12 marks)

(b) Convert $0.1\overline{10010}_2$ to its decimal representation **exactly**.

Express your answer as an **exact fraction**.

(Hint: For $0<x<1$, we have $1+x+x^2+\ldots=\frac{1}{1-x}$.)

(8 marks)

2. Let the universal set $U=\{x\in\mathbb{N}\mid x\le 12\}$ and define the following sets within U:

$$A=\{x\in\mathbb{Z}\mid 3\le x\le 7\}$$

$$B=\left\{\frac{x}{3}\,\middle|\, x\in U\right\}$$

$$C=\left\{x\mid x^2+1 \text{ is a multiple of } 5\right\}.$$

(a) Rewrite sets U, A, B and C by listing.

(5 marks)

(b) Find $|A|$, $\overline{A}\cap B$ and $C-A$.

(5 marks)

(c) Draw a Venn diagram showing the above sets U, A, B and C, indicating all the elements clearly.

(5 marks)

(d) Another set D is defined within U, as follows:

$$D=\left\{x\,\middle|\,\frac{x}{y}\notin\mathbb{Z},\, y\in\left(A\cap\left(B\cup C\right)\right)\right\}$$

Rewrite set D by listing. Show your working clearly.

(5 marks)

3. **Show your working clearly for this question. No marks will be awarded if the steps involved are not shown.**

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(Hint: For $0<x<1$, we have $1+x+x^2+\ldots=\dfrac{1}{1-x}$.)

(8 marks)

4. **Solve this question using homogeneous coordinates.**

(a) Triangle $\mathbf{P}$ with vertices $(1,0)$, $(-3,1)$ and $(4,2)$ undergoes the following sequence of transformations:

$\mathbf{T}_1$: rotation 180° about the origin, followed by

$\mathbf{T}_2$: shearing in the x-direction by a factor of 1, followed by

$\mathbf{T}_3$: translation 1 unit to the left and 2 units upwards.

(i) Write down the transformation matrices $\mathbf{T}_1$, $\mathbf{T}_2$ and $\mathbf{T}_3$.

(ii) Compute the composite matrix $\mathbf{C}$ for the above sequence of transformations.

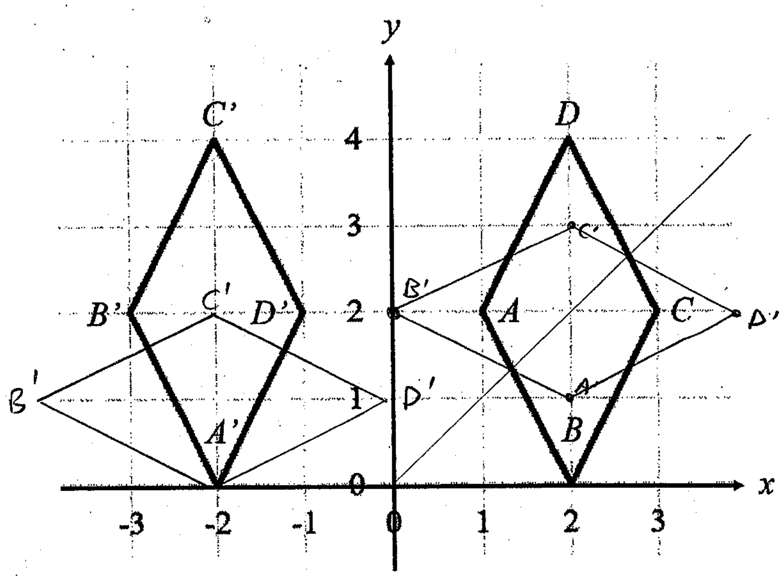
(iii) Find $\mathbf{P}'$, the image matrix of triangle $\mathbf{P}$ after undergoing the above sequence of transformations.

(iv) Write down the inverse transformation matrices $\mathbf{T}_1^{-1}$, $\mathbf{T}_2^{-1}$ and $\mathbf{T}_3^{-1}$.

(v) Compute the composite matrix $\mathbf{C}^{-1}$ that transforms $\mathbf{P}'$ back to $\mathbf{P}$.

(20 marks)

(b) In the diagram below, rhombus $ABCD$ is transformed to $A'B'C'D'$ after undergoing a sequence of **three** simple transformations.



(i) Describe, in words, the three transformations that will transform rhombus $ABCD$ to $A'B'C'D'$, and write down the corresponding transformation matrices.

(ii) Hence, derive the composite matrix $\mathbf{T}$ for the above sequence of transformations and verify that $\mathbf{T}$ successfully transforms rhombus $ABCD$ to $A'B'C'D'$.

(10 marks)

***** **END OF PAPER** *****