SOLUTIONS

SINGAPORE POLYTECHNIC
2023/2024 Semester 2 Mid-Semester Test

No.	SOLUTION
1(a)	$(\mathbf{A}_{2\times2}\mathbf{D}_{2\times3}\mathbf{D}^{\mathbf{T}}{}_{3\times2})^3$ Order of $(\mathbf{ADD^T})^3$ is 2×2.
1(b)	As **G** is diagonal matrix, $x-1=0 \Rightarrow x=1$ $2x-y=0 \Rightarrow y=2$ $\sqrt{z}-4=0 \Rightarrow z=16$
1(c)	Given $\mathbf{AB^T} - 3\mathbf{C} = \mathbf{I}$: $\begin{bmatrix} a & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 3 & b \\ 0 & -4 \end{bmatrix} - \begin{bmatrix} 3 & 12 \\ 6 & 3c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 3a & ab+4 \\ 6 & 2b-4 \end{bmatrix} - \begin{bmatrix} 3 & 12 \\ 6 & 3c \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 3a-3 & ab-8 \\ 0 & 2b-3c-4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $3a-3=1 \Rightarrow a=\frac{4}{3}$ $ab-8=0 \Rightarrow b=6$ $2b-3c-4=1 \Rightarrow c=\frac{7}{3}$
1(d)(i)	$\mathbf{D}^2$ cannot be evaluated because $\mathbf{D}$ is not a square matrix (or no. of columns $\neq$ no. of rows).
1(d)(ii)	$\mathbf{D}-2\mathbf{E}$ cannot be evaluated because D and E are not in the same order.
1(d)(iii)	$\mathbf{DE} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 5 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 3 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 12 & 6 \\ 24 & 13 \end{bmatrix}$

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1(e)	Given $\mathbf{F}^3+2\mathbf{F}-4\mathbf{I}=3\mathbf{F}^2$: $\mathbf{F}^{-1}(\mathbf{F}^3+2\mathbf{F}-4\mathbf{I})=3\mathbf{F}^{-1}\mathbf{F}^2$ $\mathbf{F}^2+2\mathbf{I}-4\mathbf{F}^{-1}=3\mathbf{F}$ $4\mathbf{F}^{-1}=\mathbf{F}^2+2\mathbf{I}-3\mathbf{F}$ $\mathbf{F}^{-1}=\frac{1}{4}(\mathbf{F}^2+2\mathbf{I}-3\mathbf{F})$ $\mathbf{F}^{-1}=\frac{1}{4}(\begin{bmatrix}1&0&2\\2&1&1\\0&1&1\end{bmatrix}\begin{bmatrix}1&0&2\\2&1&1\\0&1&1\end{bmatrix}+2\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}-3\begin{bmatrix}1&0&2\\2&1&1\\0&1&1\end{bmatrix})$ $\mathbf{F}^{-1}=\frac{1}{4}(\begin{bmatrix}1&2&4\\4&2&6\\2&2&2\end{bmatrix}+\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}-\begin{bmatrix}3&0&6\\6&3&3\\0&3&3\end{bmatrix})$ $\mathbf{F}^{-1}=\frac{1}{4}\begin{bmatrix}0&2&-2\\-2&1&3\\2&-1&1\end{bmatrix}$
1(f)	$(2\mathbf{PQ})(3\mathbf{PQ})^{-1}$ $=2\mathbf{PQ}\cdot\frac{1}{3}\mathbf{Q}^{-1}\mathbf{P}^{-1}=\frac{2}{3}\mathbf{PIP}^{-1}=\frac{2}{3}\mathbf{I}$
2(a)	$U=\{-2,-1,0,1,2,3,4,5,6,7\}$ $A=\{1,3,5\}$ $B=\{-1,1,3,5,7\}$ $C=\{2,3\}$

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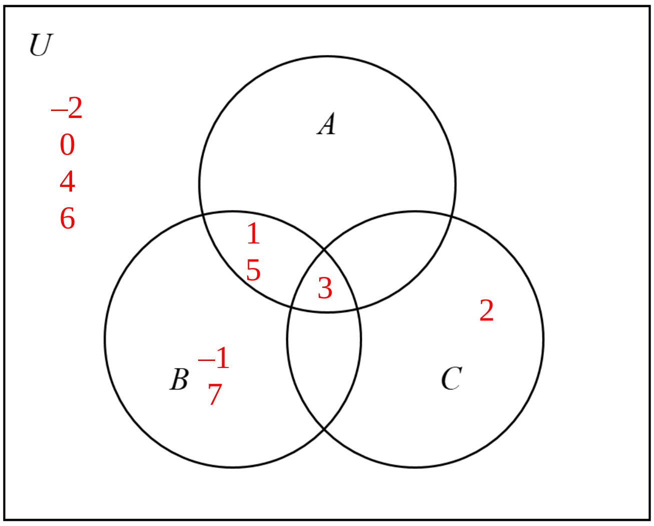
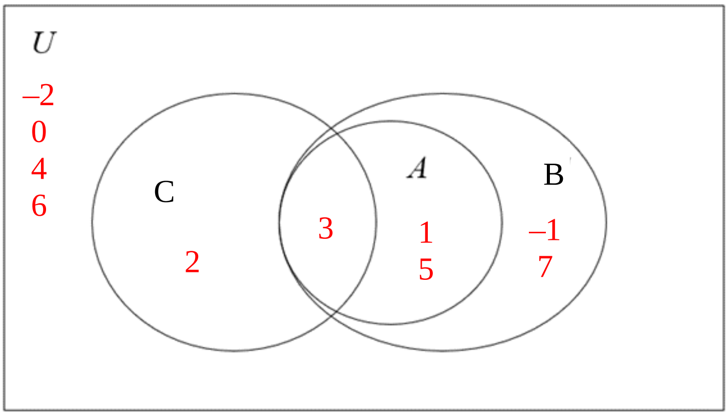
<table>
<tr><td>2(b)</td><td>


Alternatively,



</td></tr>
<tr><td>2(c)
(i)</td><td>$|A\cup B|=5$</td></tr>
<tr><td>2(c)
(ii)</td><td>$B-A=\{-1,7\}$</td></tr>
<tr><td>2(c)
(iii)</td><td>$\overline{A\cap B\cap \overline{C}}=\{-2,-1,0,2,3,4,6,7\}$</td></tr>
<tr><td>2(d)</td><td>

$M=\{1,2,3...8,9,10\}$

The following numbers are divisible by 16 within [1, 100]: 16, 32, 48, 64, 80, 96

$16=4\times 4, 2\times 8, 8\times 2$

$32=4\times 8, 8\times 4$

$48=6\times 8, 8\times 6$

$64=8\times 8$

$80=10\times 8, 8\times 10$

As a≠b, 4×4 & 8×8 are rejected.

$\therefore |N|=10-2=8$
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3(a)

Integral part:		
2	863	
2	431	1
2	215	1
2	107	1
2	53	1
2	26	1
2	13	0
2	6	1
2	3	0
2	1	1
	0	1

Fractional part:		
2	0.55	
2	0.1	1
2	0.2	0
2	0.4	0
2	0.8	0
2	0.6	1
2	0.2	1
2	0.4 (rep)	0
2	0.8	0

$$\boxed{\therefore 863.55_{10} = 1101011111.10\overline{0011}_2 = 35F.8\overline{C}_{16}}$$

3(b)

Largest 8 digit octal number = 777777.777_8

$$777777.777_8 = 7\times(8^5+8^4+8^3+8^2+8^1+8^0+8^{-1}+8^{-2}+8^{-3})$$
$$= 262143.998$$
$$\approx 262144$$

3(c)

$$x = x^2 - 12 \Rightarrow (x+3)(x-4) = 0 \Rightarrow x = 4 \text{ (reject -tive ans)}$$
$$\tfrac{2y^2}{5} - 3 = y + 2 \Rightarrow (2y+5)(y-5) = 0 \Rightarrow y = 5 \text{ (reject -tive ans)}$$

Base $x + y = 9$

$$\therefore 4774_9 = 4\times9^3 + 7\times9^2 + 7\times9^1 + 4\times9^0 = 3550_{10}$$

4(a) (i)

$$\mathbf{T}_1 = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \;;\; \mathbf{T}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \;;\; \mathbf{T}_3 = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

4(a) (ii)

$$\mathbf{C} = \mathbf{T}_3\mathbf{T}_2\mathbf{T}_1 = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{1}{2} & 1 & -3 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

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4(a) (iii)	$\mathbf{P}' = \mathbf{CP} = \begin{bmatrix} \frac{1}{2} & 1 & -3 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 4 \\ 2 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & -2 & 1 \\ 2 & 3 & 5 \\ 1 & 1 & 1 \end{bmatrix}$
4(a) (iv)	$\mathbf{T}_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $\mathbf{T}_2^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $\mathbf{T}_3^{-1} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$
4(a) (v)	$\mathbf{C}^{-1} = \mathbf{T}_1^{-1}\mathbf{T}_2^{-1}\mathbf{T}_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} 0 & 1 & -1 \\ 1 & -\frac{1}{2} & \frac{7}{2} \\ 0 & 0 & 1 \end{bmatrix}$
4(b) (i)	$\mathbf{T}_a$: anti-clockwise rotation 135° about the origin $\mathbf{T}_b$: scaling in the x-direction & y-direction by a factor of $\sqrt{2}$ $\mathbf{T}_c$: Translation 8 units to the right and 1 unit upwards $\mathbf{T}_a = \begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $\mathbf{T}_b = \begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $\mathbf{T}_c = \begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$
4(b) (ii)	$\mathbf{T} = \mathbf{T}_c\mathbf{T}_b\mathbf{T}_a = \begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 8 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $\mathbf{U}' = \mathbf{TU} = \begin{bmatrix} -1 & -1 & 8 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 & 3 \\ 2 & 3 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 2 & 4 \\ 1 & 1 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ (verified)