SINGAPORE POLYTECHNIC
2023/2024 SEMESTER ONE EXAMINATION

Common Infocomm Technology Programme (DCITP)
Diploma in Applied AI & Analytics (DAAA)
Diploma in Infocomm Security Management (DISM)
Diploma in Information Technology (DIT)
Diploma in Media, Arts & Design (DMAD)

MS0105 – Mathematics Time allowed: 2 hours
MS0151 – Mathematics for Games

Instructions to Candidates

1. The SP examination rules are to be complied with.
   Any candidate who cheats or attempts to cheat will face disciplinary action.

2. This paper consists of **8** printed pages (including the cover page and formula sheet).

3. This paper consists of three sections (100 marks in total):

   Section A: 5 multiple-choice questions (10 marks)
   Answer all questions behind the cover page of the answer booklet.

   Section B: 7 structured questions (50 marks)
   The total mark of the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

   Section C: 3 structured questions (40 marks)
   Answer all questions.

4. Unless otherwise stated, all **non-exact** decimal answers should be rounded to **at least two** decimal places.

5. Except for sketches, graphs and diagrams, no solutions are to be written in pencil. Failure to comply may result in loss of marks.

SECTION A (10 marks)

Answer ALL **FIVE** questions. Each question carries 2 marks.
Tick the choice of answer for each question in the box of the MCQ answer sheet provided in the answer booklet.

A1. If Matrix **A** has more than 1 row, then $\mathbf{AA^T}$ will be

(a) a diagonal matrix.

(b) an identity matrix.

(c) a symmetric matrix.

(d) a zero matrix.

A2. For finite sets A and B, where $A \subset B$ but $B \not\subset A$, which of the following set theory expressions are correct?

I. $\overline{A} \cap B = \overline{A}$

II. $\overline{A} \cap B = B - A$

III. $\overline{A \cup B} = \overline{A}$

IV. $\overline{A} \cup \overline{B} = \overline{A}$

(a) I and III only

(b) I and IV only

(c) II and III only

(d) II and IV only

A3. Three individuals are seated in a row. Among them is an artist, another is a computer scientist, and the remaining one is an engineer. The following information are known:

- The 3 individuals are named Alice, Bob and Claire.
- Alice is the engineer.
- If Alice is sitting next to the artist, then Claire is not the artist.
- If Bob is sitting next to Alice, then Claire is not the computer scientist.

Which of the following statements about the middle seat is **true**?

(a) Alice cannot be seated in the middle.

(b) Bob cannot be seated in the middle.

(c) Claire cannot be seated in the middle.

(d) Anyone can be seated in the middle.

A4. Denise has 11 friends. 2 of her friends are married to each other and if they were to attend an event, they must attend it together. Find the number of ways Denise can invite 5 of her friends to an event.

(a) 84 (b) 126 (c) 210 (d) 462

A5. A manufacturing production line produces items of which 0.1% at random are defective. What is the maximum number of items that can be packed into a box while keeping the probability of one or more defectives in the box to be no more than 0.1?

(a) 10 (b) 105 (c) 1053 (d) 2301

SECTION B (50 marks)

Each question carries 10 marks. The total mark of all the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

B1 . (a) Find matrix **A** such that $2\mathbf{A}+\begin{bmatrix}4 & 3 & 2\\ 1 & 5 & 8\end{bmatrix}=3\begin{bmatrix}2 & 5 & 8\\ 7 & 3 & 6\end{bmatrix}$.

(3 marks)

(b) Find the values of k and n such that $\begin{bmatrix}5 & 0 & k+2\\ 0 & -1 & 3k\\ 2k-1 & n & 3\end{bmatrix}$ is a symmetric matrix.

(3 marks)

(c) Matrices **B**, **C** and **D** are given below.

$$\mathbf{B}=\begin{bmatrix}2 & x\\ 3 & 1\end{bmatrix}, \qquad \mathbf{C}=\begin{bmatrix}2 & 1\\ 1 & 4\end{bmatrix}, \qquad \mathbf{D}=\begin{bmatrix}3x+2 & 7\\ 7-x & 7\end{bmatrix},$$

Find the value of x, given $\mathbf{BC} = \mathbf{D}^{\mathrm{T}}$.

(4 marks)

B2. **Solve this question using homogeneous coordinates.**

A triangle **P** with coordinates $(1,1)$, $(2,1)$ and $(2,3)$ undergoes the following sequence of transformations:

$\mathbf{T_1}$: Rotate 90° in anti-clockwise direction, followed by

$\mathbf{T_2}$: Expansion by 2 times in the x-direction and 3 times in the y-direction.

(a) Write down the transformation matrices $\mathbf{T_1}$ and $\mathbf{T_2}$. Hence, compute the composite matrix **C** for the above sequence of transformations.

(4 marks)

(b) Find **P'**, the image matrix of triangle **P**, after undergoing the above sequence of transformations.

(2 marks)

(c) Write down the inverse transformation matrices $\mathbf{T_1}^{-1}$ and $\mathbf{T_2}^{-1}$. Hence, compute the composite matrix $\mathbf{C}^{-1}$ that transforms **P'** back to **P**.

(4 marks)

B3. **Show your working clearly for this question.**

(a) Convert $8EF.A3_{16}$ to its binary and decimal representations.

(4 marks)

(b) Convert 125.3_{10} to its binary representation.
Express your answer in **exact** form, showing the recursion clearly for the fractional part, if any.

(6 marks)

B4. Let the universal set $U = \{x \in \mathbb{Z} \mid -3 < x \leq 4\}$ and define the following sets within U:

$$A = \{x \in \mathbb{N} \mid 3x - 7 < 0\}$$

$$B = \{x^2 \mid x \in \mathbb{Z}\}$$

(a) Rewrite sets U, A and B by listing.

(3 marks)

(b) Find $A \cup \overline{B}$ and $\overline{A - B}$.

(4 marks)

(c) Draw a Venn diagram showing sets U, A and B, indicating all the elements clearly.

(3 marks)

B5. (a) By constructing a truth table, determine whether the compound proposition $(p \wedge q) \wedge \neg(p \vee q)$ is a tautology or contradiction.

(5 marks)

(b) Simplify the Boolean expression $(\overline{x + \overline{yz}}) + (\overline{xy} + z)$.

(5 marks)

B6. Find the number of possible **6**-character passwords under the following restrictions:

(a) Only *lower-case letters* are allowed; repetition of letters is allowed.

(3 marks)

(b) Only *lower-case letters* are allowed; repetition of letters is NOT allowed.

(3 marks)

(c) All characters contain only *distinct lower-case letters and digits* arranged in *alternating* manner (e.g., “a1b8c5” or “1s2b3g” etc.).

(4 marks)

B7. Venti and Zhongli are playing a game. In each match, two fair six-sided dice are rolled. If the sum of both dice is a prime number (i.e., 2, 3, 5, 7 or 11), Venti wins. Otherwise, Zhongli wins.

(a) List the sample space of Venti winning a match. (3 marks)

(b) Find the probability of Venti winning a match. (2 marks)

(c) Three matches are played in succession. Find the probability of the following events:

(i) Venti wins all three matches.

(ii) Venti wins at least one match.

(5 marks)

SECTION C (40 marks)

Answer ALL **THREE** questions.

C1. Quine's Dagger is a logical operator denoted by the following symbol: $\downarrow$.

$x \downarrow y$ evaluates to true if and only if x and y are false. $x \downarrow y$ evaluates to false for all other truth-value combinations of x and y.

(a) Based on the information provided, construct the truth table for $x \downarrow y$.

(2 marks)

(b) Derive both the sum-of-products and the product-of-sums expressions for $x \downarrow y$.

(3 marks)

(c) Using the laws of Boolean algebra, show that the sum-of-products and the product-of-sums expressions from part (b) are equivalent.

(Note: No marks will be awarded for using a truth table to verify the equivalence.)

(3 marks)

(d) Express $p + q$ using only Quine's Dagger and no other operators.

(Hint: Make use of the sum-of-products expression from part (b).)

(5 marks)

C2. (a) Show that if n is an odd integer, i.e., $n = 2k+1$, where $k \in \mathbb{Z}$, then:

$$\left\lfloor \frac{n^2}{4} \right\rfloor = \left(\frac{n+1}{2}\right)\left(\frac{n-1}{2}\right)$$

(4 marks)

(b) There are 20 equally spaced dots distributed evenly along the x-axis and y-axis, as shown in the diagram below.

(i) How many triangles can be formed if all vertices of the triangles must lie on the dots in the diagram?

(ii) How many quadrilaterals (i.e. four-sided figures) can be formed if all vertices of the quadrilaterals must lie on the dots in the diagram?

(Note: Vertices are points where two or more lines meet.)

(8 marks)

C3. (a) Peter has a fair m-sided die (where each side is numbered from $1,2,\ldots,m$) and a fair n-sided die (where each side is numbered from $1,2,\ldots,n$).

A die roll is said to be a *low roll* if its outcome is a '1' or '2'. When Peter rolls his two dice simultaneously:

- The probability that **only** the m-sided die results in a *low roll* is $\dfrac{1}{8}$.
- The probability that **only** the n-sided die results in a *low roll* is $\dfrac{5}{24}$.

What are the values of m and n?

(7 marks)

(b) Inside a control room, a switchboard panel has three lights: red, yellow and green. The red light, yellow light and green light blink at exact regular intervals of once every 3 minutes, once every 4 minutes and once every 5 minutes respectively.

One day, you enter the control room, and none of the three lights are blinking at that instant. You have no idea what time the lights have blinked previously, but you know that the lights will blink independently of each other. What is the probability that the red light is the next to blink?

(8 marks)

******* END OF PAPER *******

Formula Sheet

Transformation Matrices

Reflection	about the y-axis	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	about the x-axis	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	about the line $y = x$	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Scaling	relative to the origin	$\begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Shearing	in the x-direction	$\begin{bmatrix} 1 & s_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	in the y-direction	$\begin{bmatrix} 1 & 0 & 0 \\ s_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Rotation	about the origin	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Translation		$\begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix}$

Boolean Algebra

Name	**Identity**
Commutative Laws	$x \bullet y = y \bullet x$ $x + y = y + x$
Associative Laws	$x \bullet (y \bullet z) = (x \bullet y) \bullet z = x \bullet y \bullet z$ $x + (y+z) = (x+y) + z = x + y + z$
Distributive Laws	$x \bullet (y+z) = (x \bullet y) + (x \bullet z)$ $x + (y \bullet z) = (x+y) \bullet (x+z)$
Identity Laws	$x \bullet 1 = x$ $x + 0 = x$
Complement Laws	$x \bullet \overline{x} = 0$ $x + \overline{x} = 1$
Involution Law	$\overline{\overline{x}} = x$
Idempotent Laws	$x \bullet x = x$ $x + x = x$
Bound Laws	$x \bullet 0 = 0$ $x + 1 = 1$
De Morgan's Laws	$\overline{x \bullet y} = \overline{x} + \overline{y}$ $\overline{x + y} = \overline{x} \bullet \overline{y}$
Absorption Laws	$x \bullet (x + y) = x$ $x + (x \bullet y) = x$ $x \bullet (\overline{x} + y) = x \bullet y$ $x + (\overline{x} \bullet y) = x + y$

Probability Rules

Addition	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Subtraction	$P(\overline{A}) = 1 - P(A)$
Multiplication	$P(A \cap B) = P(A)P(B)$ if A and B are independent events
Conditional Probability	$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$