SINGAPORE POLYTECHNIC
2020/2021 SEMESTER TWO MID-SEMESTER TEST

Common Infocomm Technology Programme (CITP)
Diploma in Information Technology (DIT)
Diploma in Game Design & Development (DGDD)

MS0105 – Mathematics
MS0151 – Mathematics for Games

Time allowed: 1.5 hours

Instructions to Candidates

1. The SP examination rules are to be complied with.
Any candidate who cheats or attempts to cheat will face disciplinary action.

2. This paper consists of **4** printed pages (including the cover page).
There are 4 questions (100 marks in total), and you are to answer all the questions.

3. Unless otherwise stated, all **non-exact** decimal answers should be rounded to **at least two** decimal places.

4. Except for sketches, graphs and diagrams, no solutions are to be written in pencil. Failure to do so will result in loss of marks.

Formula Sheet: Transformation Matrices

1. Reflection			3. Shearing	
a. about the y-axis	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		a. in the x-direction	$\begin{bmatrix} 1 & s_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
b. about the x-axis	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		b. in the y-direction	$\begin{bmatrix} 1 & 0 & 0 \\ s_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
c. about the line $y = x$	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		4. Rotation about the origin	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
2. Scaling relative to the origin	$\begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$		5. Translation	$\begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix}$

1. Given the following matrices:

$$\mathbf{A}=\begin{bmatrix}1 & 2a & 2\\ b & 1 & b\\ c & -c & 1\end{bmatrix} \qquad \mathbf{B}=\begin{bmatrix}2 & 1\\ 4 & 2\end{bmatrix} \qquad \mathbf{C}=\begin{bmatrix}1 & 3 & 2\\ 1 & 4 & 1\\ 0 & -3 & 2\end{bmatrix}$$

$$\mathbf{D}=\begin{bmatrix}1 & -1 & 4\\ 2 & -3 & 1\end{bmatrix} \qquad \mathbf{E}=\begin{bmatrix}-1 & 3\\ -2 & 2\end{bmatrix}$$

(a) If **A** is a symmetric matrix, find the values of a, b and c.

(4 marks)

(b) Find matrix $\mathbf{F}$ such that $2\mathbf{B}-3\mathbf{F}=4\mathbf{I}_2$, where $\mathbf{I}_2$ is the 2×2 identity matrix.

(6 marks)

(c) Evaluate the following wherever possible.
State the reason(s) clearly if the expressions cannot be evaluated.

(i) $\mathbf{BD}$

(ii) $(\mathbf{B}+\mathbf{E})^T$

(iii) $\mathbf{AD}^T\mathbf{C}$

(8 marks)

(d) (i) If $\mathbf{C}^{-1}=\begin{bmatrix}x & 12 & 5\\ 2 & y & -1\\ 3 & -3 & z\end{bmatrix}$, find the values of x, y and z.

(ii) Hence, find matrix $\mathbf{G}^{-1}$ such that $\mathbf{G}^T\mathbf{C}=\mathbf{C}^T$.

(12 marks)

2. (a) Let the universal set $U = \{x \in \mathbb{Z} \mid 1 \le x \le 10\}$ and define the following sets within U :

$$A = \{x \in \mathbb{N} \mid x \le 5\}$$

$$B = \{x + 6 \mid x \in U\}$$

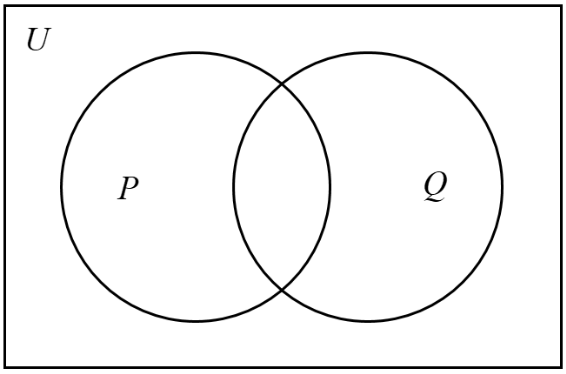
$$C = \{3x \mid (x-2)(x-3)(x-4) = 0\}$$

(i) Rewrite sets U , A, B and C by listing.

(ii) Find $|B \cap C|$, $B - C$, $A \cup \overline{B}$ and $\overline{A} \cap B$.

(14 marks)

(b) Use the Venn diagram below to answer the following parts.



(i) Redraw the above Venn diagram in your answer booklet, and shade the region $(P \cap Q) \cup \left(\overline{P \cup Q}\right)$.

(ii) Hence, rewrite $(P \cap Q) \cup \left(\overline{P \cup Q}\right)$ using only the **difference** set operator $(-)$.

(6 marks)

3. **For this question, show your working clearly. No marks will be awarded if the steps involved are not shown.**

(a) Convert 2570.35_{10} to its binary and hexadecimal representation.

Express your answers in **exact** form, showing the recursion clearly for the fractional part, if any.

(12 marks)

(b) What is the decimal representation of the **largest** 6-digit hexadecimal number that has twice as many integral digits as fractional digits?

Round your answer to four decimal places.

(4 marks)

(c) Given that 1855_{10} is the decimal representation for the base-k number 357_k , find the value of k .

(4 marks)

4. **Solve this question using homogeneous coordinates.**

A triangle ABC has vertices $A(2,4)$, $B(6,4)$ and $C(4,8)$.

(a) Triangle ABC undergoes the following sequence of transformations:

$\mathbf{T}_1$: translation 2 units to the left and 3 units upwards, followed by

$\mathbf{T}_2$: reflection about the line $y = x$, followed by

$\mathbf{T}_3$: shearing in the *y*-direction by a factor of 2.

(i) Express triangle ABC in homogeneous coordinates as matrix $\mathbf{P}$.

(ii) Write down the transformation matrices $\mathbf{T}_1$, $\mathbf{T}_2$ and $\mathbf{T}_3$.

(iii) Compute the composite matrix $\mathbf{C}$ for the above sequence of transformations.

(iv) After undergoing the sequence of transformations, triangle ABC is transformed to triangle $A'B'C'$. Find $\mathbf{P}'$, the image matrix representing triangle $A'B'C'$.

(v) Find $\mathbf{C}^{-1}$, the composite matrix that transforms triangle $A'B'C'$ back to triangle ABC.

(20 marks)

(b) Triangle ABC now undergoes another sequence of **two** simple transformations, and is subsequently transformed to an **equilateral** triangle $A''B''C''$ with vertices $A''(-2,0)$, $B''(6,0)$ and $C''(p,q)$, where $q > 0$.

(Note: An equilateral triangle is a triangle that has three equal sides.)

(i) Find the values of p and q.

(ii) Describe, in words, the two transformations required to transform triangle ABC to equilateral triangle $A''B''C''$, and write down the corresponding transformation matrices.

(iii) Hence, derive the composite matrix $\mathbf{T}$ for the above sequence of transformations and verify that $\mathbf{T}$ successfully transforms triangle ABC to equilateral triangle $A''B''C''$.

(10 marks)

******* END OF PAPER *******