SINGAPORE POLYTECHNIC
2023/2024 SEMESTER TWO MID-SEMESTER TEST

Common Infocomm Technology Programme (DCITP)
Diploma in Applied AI & Analytics (DAAA)
Diploma in Infocomm Security Management (DISM)
Diploma in Information Technology (DIT)
Diploma in Media, Arts & Design (DMAD)

MS0105 – Mathematics Time allowed: 1 hour 30 minutes
MS0151 – Mathematics for Games

Instructions to Candidates

1. The SP examination rules are to be complied with.
   Any candidate who cheats or attempts to cheat will face disciplinary action.
2. This paper consists of **4** printed pages (including the cover page).
   There are 4 questions (100 marks in total), and you are to answer all the questions.
3. Unless otherwise stated, all **non-exact** decimal answers should be rounded to **at least two** decimal places.
4. Except for sketches, graphs and diagrams, no solutions are to be written in pencil. Failure to comply may result in loss of marks.

Formula Sheet: Transformation Matrices

1. Reflection			3. Shearing	
a. about the y-axis	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		a. in the x-direction	$\begin{bmatrix} 1 & s_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
b. about the x-axis	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		b. in the y-direction	$\begin{bmatrix} 1 & 0 & 0 \\ s_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
c. about the line $y = x$	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		4. Rotation about the origin	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
2. Scaling relative to the origin	$\begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$		5. Translation	$\begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix}$

1. Given the following matrices:

$$\mathbf{A}=\begin{bmatrix} a & -1 \\ 2 & 1 \end{bmatrix} \quad \mathbf{B}=\begin{bmatrix} 3 & 0 \\ b & -4 \end{bmatrix} \quad \mathbf{C}=\begin{bmatrix} 1 & 4 \\ 2 & c \end{bmatrix} \quad \mathbf{D}=\begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 5 \end{bmatrix}$$

$$\mathbf{E}=\begin{bmatrix} 3 & 1 \\ 3 & 1 \\ 3 & 2 \end{bmatrix} \quad \mathbf{F}=\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \quad \mathbf{G}=\begin{bmatrix} 2 & 0 & 0 \\ 2x-y & 3 & x-1 \\ 0 & \sqrt{z}-4 & y \end{bmatrix}$$

(a) State the order of $(\mathbf{ADD}^{\mathbf{T}})^3$.

(3 marks)

(b) Given that $\mathbf{G}$ is a diagonal matrix, solve for *x, y,* and *z*.

(4 marks)

(c) If $\mathbf{AB}^{\mathbf{T}} - 3\mathbf{C} = \mathbf{I}$, find the values of *a, b,* and *c*.

(7 marks)

(d) Evaluate the following wherever possible.
State the reason(s) clearly if the expressions cannot be evaluated.

(i) $\mathbf{D}^2$

(ii) $\mathbf{D}-2\mathbf{E}$

(iii) $\mathbf{DE}$

(6 marks)

(e) Given that $\mathbf{F}^3 + 2\mathbf{F} - 4\mathbf{I} = 3\mathbf{F}^2$, find $\mathbf{F}^{-1}$.

(7 marks)

(f) Given **P** and **Q** are invertible matrices, simplify the expression $(2\mathbf{PQ})(3\mathbf{PQ})^{-1}$.

(3 marks)

2. Let the universal set $U = \{x \in \mathbb{Z} \mid -2 \le x \le 7\}$ and define the following sets within U :

$$A = \{x \text{ is an odd number } \& \ x \in \mathbb{N} \mid x < 6 \}$$

$$B = \{x \in \mathbb{Z} \mid x = 2y - 1 \ \& \ y \in \mathbb{Z}\}$$

$$C = \{x \in \mathbb{R} \mid x^2 - 5x + 6 = 0\}$$

(a) Rewrite sets U , A, B and C by listing.

(5 marks)

(b) Draw a Venn diagram showing the above sets U , A, B and C, indicating all the elements clearly.

(5 marks)

(c) Find the following:

(i) $|A \cup B|$

(ii) $B - A$

(iii) $\overline{A \cap B \cap \overline{C}}$

(6 marks)

(d) Define the sets M and N as follows:

$$M = \{\text{first ten natural numbers}\}$$

$$N = \{(a,b) \mid a, b \in M, a \ne b, \text{and } ab \text{ is divisible by } 16\}$$

What is the cardinality of the set N ?

(4 marks)

3. **Show your working clearly for this question. No marks will be awarded if the steps involved are not shown.**

(a) Convert 863.55_{10} to its binary and hexadecimal representations.

Express your answers in **exact** form, showing the recursion clearly for the fractional part, if any.

(12 marks)

(b) What is the decimal representation of the largest 9-digit octal number (base-8) that has twice as many integral digits as fractional digits? Round your answer to the nearest whole number.

(4 marks)

(c) A palindrome is a number that reads the same backward or forward, such as 88_{10} and 101_2. A 4-digit base-(x+y) number is defined as $[x][\frac{2y^2}{5} - 3][y+2][x^2 - 12]_{x+y}$, where each […] indicates one digit. Given that this 4-digit number is a palindrome, what is its decimal representation?

(4 marks)

4. **Solve this question using homogeneous coordinates.**

(a) Triangle $\mathbf{P}$ with vertices $(1,2)$, $(2,0)$ and $(4,2)$ undergoes the following sequence of transformations:

$\mathbf{T}_1$: shearing in the y-direction by a factor of 0.5, followed by

$\mathbf{T}_2$: reflection about the line $y = x$, followed by

$\mathbf{T}_3$: translation 3 units to the left and 1 unit upwards.

(i) Write down the transformation matrices $\mathbf{T}_1$, $\mathbf{T}_2$ and $\mathbf{T}_3$.

(ii) Compute the composite matrix $\mathbf{C}$ for the above sequence of transformations.

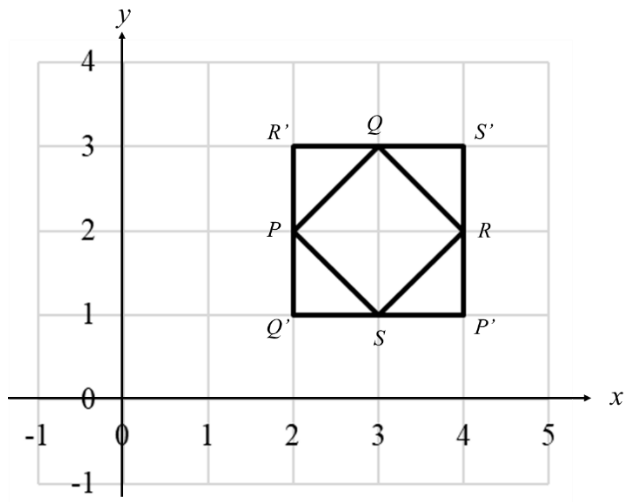
(iii) Find $\mathbf{P}'$, the image matrix of triangle $\mathbf{P}$ after undergoing the above sequence of transformations.

(iv) Write down the inverse transformation matrices $\mathbf{T}_1^{-1}$, $\mathbf{T}_2^{-1}$ and $\mathbf{T}_3^{-1}$.

(v) Compute the composite matrix $\mathbf{C}^{-1}$ that transforms $\mathbf{P}'$ back to $\mathbf{P}$.

(20 marks)

(b) In the diagram shown below, parallelogram $PQRS$ is transformed to $P'Q'R'S'$ after undergoing a sequence of **three** simple transformations listed in the Formula Sheet (on Page 1).



(i) Describe, in words, the three transformations that will transform parallelogram $PQRS$ to $P'Q'R'S'$ and write down the corresponding transformation matrices.

(ii) Hence, derive the composite matrix $\mathbf{T}$ for the above sequence of transformations and verify that $\mathbf{T}$ successfully transforms parallelogram $PQRS$ to parallelogram $P'Q'R'S'$.

(10 marks)

******* END OF PAPER *******