SINGAPORE POLYTECHNIC
2019/2020 SEMESTER ONE EXAMINATION

Common Infocomm Technology Programme (CITP)
Diploma in Infocomm Security Management (DISM)
Diploma in Information Technology (DIT)

MS0105 – Mathematics Time allowed: 2 hours

Instructions to Candidates

1. The examination rules set out on the last page of the answer booklet are to be complied with.

2. This paper consists of three sections:

   Section A: 5 multiple-choice questions (10 marks)
   Answer all questions on the back of the cover page of the answer booklet.

   Section B: 7 structured questions (50 marks)
   The total mark of the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

   Section C: 3 structured questions (40 marks)
   Answer all questions.

3. Unless otherwise stated, all decimal answers should be rounded to at least **two** decimal places.

4. This examination paper consists of 7 printed pages (including cover page and formula sheet).

SECTION A (10 marks)

Answer ALL **FIVE** questions. Each question carries 2 marks.
Tick the choice of answer for each question in the box of the MCQ answer sheet provided in the answer booklet.

A1. The coordinates of the image of point A after $45°$ anti-clockwise rotation about the origin is $A'\left(-\sqrt{2},\sqrt{2}\right)$. Which of the following are the coordinates of point A?

(a) $(0,2)$ (c) $\left(\sqrt{2},\sqrt{2}\right)$

(b) $(-2,0)$ (d) $\left(-\sqrt{2},-\sqrt{2}\right)$

A2. The largest four-digit hexadecimal number has a decimal value of __________ .

(a) $2^{16}-1$ (c) $2^{64}-1$

(b) 2^{16} (d) 2^{64}

A3. If the compound propositions $p \Leftrightarrow q$ and $r \Rightarrow q$ are both FALSE, which of the following compound propositions is TRUE ?

(a) $p \wedge q$ (c) $p \wedge r$

(b) $q \wedge r$ (d) $p \wedge q \wedge r$

A4. A committee of three is selected from five people: Alice, Bobby, Charlie, David and Eunice. What is the probability that Charlie is in the committee?

(a) 20% (c) 50%

(b) 40% (d) 60%

A5. If each permutation of the digits `1, 2, 3, 4, 5, 6` is listed in increasing order of magnitude, the 289th term will be:

(a) `251346` (c) `423156`

(b) `341256` (d) `512346`

<u>SECTION B</u> (50 marks)

Each question carries 10 marks. The total mark of the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

B1. Let $\mathbf{A} = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$.

(a) Show that $\mathbf{A}^2 = 7\mathbf{A} + 2\,\mathbf{I}$, where $\mathbf{I}$ is a 2×2 identity matrix.

(b) Hence, show that $\mathbf{A}^{-1} = \dfrac{1}{2}(\mathbf{A} - 7\mathbf{I})$.

B2. (Solve this question using homogeneous coordinates.)

A square $\mathbf{S}$ with vertices $(-2,2)$, $(2,2)$, $(2,-2)$ and $(-2,-2)$ undergoes two transformations: $\mathbf{T}_1$, followed by $\mathbf{T}_2$.

$\mathbf{T}_1$: expansion in the x-direction by 2 and contraction in the y-direction by $\frac{1}{2}$, relative to the origin

$\mathbf{T}_2$: rotation $90°$ anticlockwise about the origin

(a) Find $\mathbf{C}$, the composite matrix that effects the above transformations.

(b) Find the image matrix $\mathbf{S}'$ of the square after undergoing the above sequence of transformations.

(c) Find $\mathbf{C}^{-1}$, the transformation matrix that brings $\mathbf{S}'$ back to $\mathbf{S}$.

B3. (Show all the steps clearly. No marks awarded if only final answer is presented.)

(a) Convert 1142.125_{10} to its binary and hexadecimal representation.

(b) Convert A1F.7B_{16} to its decimal representation.

Round your answer to four decimal places.

B4. Let the universal set $U = \{x \in \mathbb{Z} \mid -5 \le x \le 8\}$, and define the following sets within U :

$$R = \{2x \mid x \in \mathbb{Z}, -3 < x \le 6\}$$

$$S = \{x \in \mathbb{N} \mid x \le 6\}$$

(a) Write down the sets U, R and S by listing.

(b) Find $R \cap S$, $R \cap \overline{S}$, $\overline{S} - R$ and $\left|\overline{S} - R\right|$.

B5. (Show all the intermediate steps clearly.)

Construct truth table for the compound proposition $(\neg p \vee (q \vee r)) \vee ((p \wedge q) \Rightarrow r)$, and explain whether it is a tautology, contradiction, or neither.

B6. How many ways can the letters of the word `COMPUTER` be arranged, if:

(a) there are no restrictions?

(b) the first three characters must be '`COM`'?

(c) the word formed does not contain the string '`COM`'?

(d) the first three characters must be vowels? (Vowels are letters `A`, `E`, `I`, `O`, `U`)

B7. A group of 80 students were from class 4A and 4B at Gardens Bay Secondary School. The following table shows the number of male and female students in the group:

	Class 4A	Class 4B	Total
Male	17	26	43
Female	22	15	37
Total	39	41	80

If a student is selected from the group at random, find the probability that the student is:

(a) a female.

(b) a male from class 4B.

(c) a male, or from class 4B.

(d) from class 4A, given that the student is female.

<u>SECTION C</u> (40 marks)

Answer ALL **THREE** questions.

C1. (a) Let p, q, r and s be propositions. It is known that the truth value of the compound proposition $(\neg p \wedge q) \Rightarrow (r \vee \neg s)$ is false.

Explain clearly, **with reasons**, what the truth values of p, q, r and s are.

(b) Simplify the following Boolean expression:

$$\overline{a}(a+b)+(a+c)(a+\overline{b})$$

(c) A large room has three doors, and a switch near each door (namely a, b and c) controls a light in the center of the room. The light is turned on or off by changing the state of any one of the switches, more specifically in the following manner:

- When all three switches are open, the light is off.
- Closing any one switch will turn the light on.
- Closing any two switches will turn the light off.
- Closing all three switches will turn the light on.

Use the following convention:

	'0'	'1'
Input	Switch is open	Switch is closed
Output	Light is off	Light is on

(i) Construct a complete truth table to depict the switch and light status.

(ii) By using the **sum-of-products** method, obtain the Boolean function $f(a,b,c)$ for the light system. You do **not** need to simplify the expression obtained.

(13 marks)

C2. A Racquet Sports Committee comprising of 10 members is to be formed from 5 squash players, 6 tennis players and 7 badminton players.

How many ways can this committee be formed, if the committee must include:

(a) at least 4 squash players.

(b) only squash and badminton players.

(c) at least 1 player from each sport.

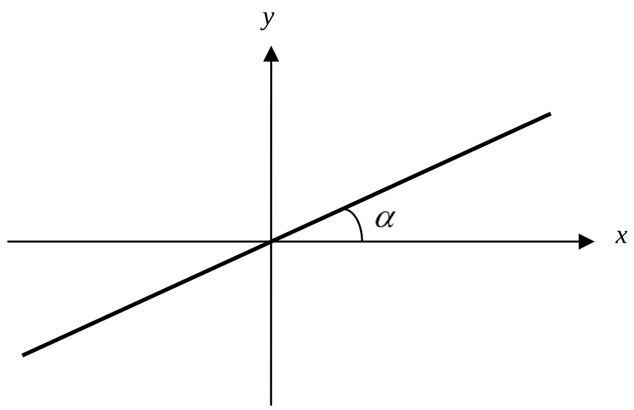
(12 marks)

C3. Solve the following question <u>without</u> using homogeneous coordinates.

Hints:

$$\cos(\pm x) = \cos x$$
$$\sin(\pm x) = \pm \sin x$$
$$\sin 2x = 2\sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

(a)



Show that the transformation matrix $\mathbf{R}_\alpha$ that effects a reflection about a line inclined at an angle of α with respect to the x-axis, is given by

$$\mathbf{R}_\alpha = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{bmatrix}.$$

(b) Hence, show that a reflection about a line inclined at an angle of α with respect to the x-axis, followed by a reflection about a line inclined at an angle of β with respect to the x-axis, is equivalent to a rotation of angle $2\beta - 2\alpha$ anti-clockwise about the origin.

(c) An object is reflected about the line $y = x$, then reflected again about the line $y = -x$. This is equivalent to rotating the object with angle θ anti-clockwise about the origin. State the value of θ.

(15 marks)

***** END OF PAPER *****

Formula Sheet

Transformation Matrices

Reflection	in the y-axis	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	in the x-axis	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	in the line $y = x$	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Scaling	relative to origin	$\begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Shearing	in the x-direction	$\begin{bmatrix} 1 & s_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	in the y-direction	$\begin{bmatrix} 1 & 0 & 0 \\ s_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Rotation	about the origin	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Translation		$\begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix}$

Boolean Algebra

Name	Identity
Commutative Laws	$x \cdot y = y \cdot x$ $x + y = y + x$
Associative Laws	$x \cdot (y \cdot z) = (x \cdot y) \cdot z = x \cdot y \cdot z$ $x + (y + z) = (x + y) + z = x + y + z$
Distributive Laws	$x + (y \cdot z) = (x + y) \cdot (x + z)$ $x \cdot (y + z) = (x \cdot y) + (x \cdot z)$
Identity Laws	$x \cdot 1 = x$ $x + 0 = x$
Complement Laws	$x \cdot \overline{x} = 0$ $x + \overline{x} = 1$
Involution law	$\overline{\overline{x}} = x$
Idempotent Laws	$x \cdot x = x$ $x + x = x$
Bound Laws	$x \cdot 0 = 0$ $x + 1 = 1$
DeMorgan's Laws	$(\overline{x \cdot y}) = \overline{x} + \overline{y}$ $(\overline{x + y}) = \overline{x} \cdot \overline{y}$
Absorption Laws	$x \cdot (x + y) = x$ $x + (x \cdot y) = x$ $x \cdot (\overline{x} + y) = x \cdot y$ $x + (\overline{x} \cdot y) = x + y$

Probability Rules

Addition	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Subtraction	$P(\overline{A}) = 1 - P(A)$
Multiplication	$P(A \cap B) = P(A)\,P(B)$ if A and B are independent events
Conditional Probability	$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$