SINGAPORE POLYTECHNIC
2023/2024 SEMESTER ONE MID-SEMESTER TEST

Common Infocomm Technology Programme (CITP)
Diploma in Applied AI & Analytics (DAAA)
Diploma in Infocomm Security Management (DISM)
Diploma in Information Technology (DIT)
Diploma in Media, Arts & Design (DMAD)

MS0105 – Mathematics Time allowed: 1 hour 30 minutes
MS0151 – Mathematics for Games

Instructions to Candidates

1. The SP examination rules are to be complied with.
   Any candidate who cheats or attempts to cheat will face disciplinary action.

2. This paper consists of **4** printed pages (including the cover page).
   There are 4 questions (100 marks in total), and you are to answer all the questions.

3. Unless otherwise stated, all **non-exact** decimal answers should be rounded to **at least two** decimal places.

4. Except for sketches, graphs and diagrams, no solutions are to be written in pencil. Failure to comply may result in loss of marks.

Formula Sheet: Transformation Matrices

1. Reflection			3. Shearing	
a. about the y-axis	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		a. in the x-direction	$\begin{bmatrix} 1 & s_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
b. about the x-axis	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		b. in the y-direction	$\begin{bmatrix} 1 & 0 & 0 \\ s_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
c. about the line $y = x$	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		4. Rotation about the origin	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
2. Scaling relative to the origin	$\begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$		5. Translation	$\begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix}$

1. Given the following matrices:

$$\mathbf{A}=\begin{bmatrix}4 & -6 & -2\\ -5 & 2 & 0\end{bmatrix}\qquad \mathbf{B}=\begin{bmatrix}-5 & 3 & x-4\\ 3 & 8 & 6\\ y+2 & y-3 & 3\end{bmatrix}\qquad \mathbf{C}=\begin{bmatrix}2 & -3\\ -9 & 2\\ 4 & 1\end{bmatrix}$$

$$\mathbf{D}=\begin{bmatrix}-8 & 11\\ -4 & 4\end{bmatrix}\qquad \mathbf{E}=\begin{bmatrix}3 & 0 & 2\\ 2 & 0 & -2\\ 1 & 1 & 1\end{bmatrix}\qquad \mathbf{F}=\begin{bmatrix}2 & 2 & 0\\ -4 & 1 & 10\\ 2 & -3 & 0\end{bmatrix}$$

(a) State the order of $(\mathbf{AF})^{\mathbf{T}}$.

(2 marks)

(b) Find matrix $\mathbf{P}$ such that $\mathbf{3E}+\mathbf{P}^{\mathbf{T}}=-2\mathbf{CA}$.

(5 marks)

(c) Given that $\mathbf{B}$ is a symmetric matrix, solve for x and y.

(3 marks)

(d) Evaluate the following wherever possible.
State the reason(s) clearly if the expressions cannot be evaluated.

(i) $\mathbf{D}^2$

(ii) $\mathbf{B}-\mathbf{C}$

(iii) $\mathbf{FC}$

(iv) $(\mathbf{DC}^{\mathbf{T}}\mathbf{E})^3$

(8 marks)

(e) (i) Given that $\mathbf{EF}=k\mathbf{I}_3$, where k is an integer, find the value of k.

(ii) Hence, find matrix $\mathbf{G}^{-1}$ such that $10\mathbf{GF}=9\mathbf{E}$.

(12 marks)

2. Let the universal set $U = \{x \in \mathbb{Z} \mid -3 \le x \le 8\}$ and define the following sets within U :

$$A = \{x \text{ is an even number } \& \ x \in \mathbb{N} \mid x < 8\}$$

$$B = \{x \in \mathbb{N}_0 \mid 2^x \text{ -1 is a prime number}\}$$

$$C = \{x \in \mathbb{Z} \mid x \text{ is divisible by } 2\}$$

(a) Rewrite sets U , A, B and C by listing.

(5 marks)

(b) Find $B \cap \overline{C}$ and $C \cap (A \cup B)$.

(4 marks)

(c) Draw a Venn diagram showing the above sets U , A, B and C, indicating all the elements clearly.

(5 marks)

(d) A **power set** of Set X , denoted as $P(X)$, is defined as the set of all subsets of Set X including the set itself and the null set.

For example, for Set $M = \{a,b\}$, the power set $P(M) = \{M, \{a\}, \{b\}, \varnothing\}$.

(i) Rewrite $P(A)$ and $P(B)$ by listing.

(ii) Hence, for any given set X with finite elements, deduce the number of elements in its power set.

(6 marks)

3. **Show your working clearly for this question. No marks will be awarded if the steps involved are not shown.**

(a) Convert 712.525_{10} to its binary and hexadecimal representations.

Express your answers in **exact** form, showing the recursion clearly for the fractional part, if any.

(12 marks)

(b) A system uses decimal numbers that range from 0 to 255 (inclusive). The system stores the numbers in binary format using one byte for the integer part and one byte for the fractional part such that any remaining bits not stored will be truncated.

(i) State the smallest truncation error and justify your answer.

(ii) Find the largest truncation error in decimal representation.

Hint: A byte is a group of 8 bits.

(8 marks)

4. **Solve this question using homogeneous coordinates.**

(a) Triangle $\mathbf{P}$ with vertices $(2,2)$, $(-1,0)$ and $(1,-1)$ undergoes the following sequence of transformations:

$\mathbf{T}_1$: reflection about the y-axis, followed by

$\mathbf{T}_2$: scaling relative to the origin by a factor of 0.5 in the x-direction and by a factor of 0.5 in the y-direction, followed by

$\mathbf{T}_3$: anti-clockwise rotation by 90° about the origin.

(i) Write down the transformation matrices $\mathbf{T}_1$, $\mathbf{T}_2$ and $\mathbf{T}_3$.

(ii) Compute the composite matrix $\mathbf{C}$ for the above sequence of transformations.

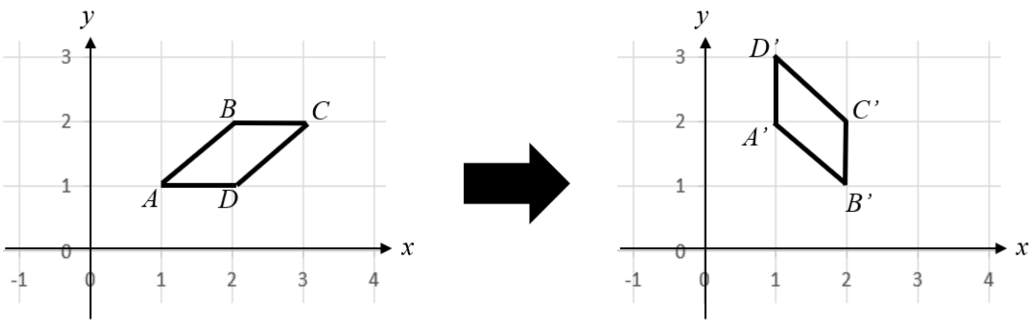
(iii) Find $\mathbf{P}'$, the image matrix of triangle $\mathbf{P}$ after undergoing the above sequence of transformations.

(iv) Write down the inverse transformation matrices $\mathbf{T}_1^{-1}$, $\mathbf{T}_2^{-1}$ and $\mathbf{T}_3^{-1}$.

(v) Compute the composite matrix $\mathbf{C}^{-1}$ that transforms $\mathbf{P}'$ back to $\mathbf{P}$.

(20 marks)

(b) In the diagram shown below, parallelogram $ABCD$ is transformed to $A'B'C'D'$ after undergoing a sequence of **three** simple transformations listed in the Formula Sheet (on Page 1).



(i) Describe, in words, the three transformations that will transform parallelogram $ABCD$ to $A'B'C'D'$ and write down the corresponding transformation matrices.

(ii) Hence, derive the composite matrix $\mathbf{T}$ for the above sequence of transformations and verify that $\mathbf{T}$ successfully transforms parallelogram $ABCD$ to parallelogram $A'B'C'D'$.

(10 marks)

******* END OF PAPER *******