SINGAPORE POLYTECHNIC
2022/2023 SEMESTER TWO EXAMINATION

Common Infocomm Technology Programme (CITP)
Diploma in Applied AI & Analytics (DAAA)
Diploma in Infocomm Security Management (DISM)
Diploma in Information Technology (DIT)
Diploma in Media, Arts & Design (DMAD)

MS0105 – Mathematics
MS0151 – Mathematics for Games

Time allowed: 2 hours 10 minutes

Instructions to Candidates

1. The SP examination rules are to be complied with.
   Any candidate who cheats or attempts to cheat will face disciplinary action.

2. This paper consists of **8** printed pages (including the cover page and formula sheet).

3. This paper consists of three sections (100 marks in total):

   Section A: 5 multiple-choice questions (10 marks)
   Answer all questions behind the cover page of the answer booklet.

   Section B: 7 structured questions (50 marks)
   The total mark of the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

   Section C: 3 structured questions (40 marks)
   Answer all questions.

4. Unless otherwise stated, all **non-exact** decimal answers should be rounded to **at least two** decimal places.

5. Except for sketches, graphs and diagrams, no solutions are to be written in pencil. Failure to comply may result in loss of marks.

SECTION A (10 marks)

Answer ALL **FIVE** questions. Each question carries 2 marks.
Tick the choice of answer for each question in the box of the MCQ answer sheet provided in the answer booklet.

A1. It is given that $\mathbf{A}=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$ and $\mathbf{B}=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$, and that $\mathbf{A}^{2023}+\mathbf{B}^{2023}=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$.

What is the value of $a+b+c+d$?

(a) 2023 (b) 2025 (c) 4046 (d) 4050

A2. Given any two sets A and B such that $|A|=|B|$, which one of the following statements is **always** true?

(a) $|A\cup B|=|A\cap B|$

(b) $|A\cup B|>|A\cap B|$

(c) $|A\cup B|+|A\cap B|=2|A|$

(d) $|A\cup B|-|A\cap B|<2|A|$

A3. What is a possible value of x such that the following proposition is **false**?

"If x is an integer or x is positive, then x is an integer."

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) -2

A4. There are 16 players participating in a table tennis league. The rule of the league states that every player must play exactly **two** matches against every other player. How many matches will be played in total during the league?

(a) 120 (b) 240 (c) 480 (d) 960

A5. You have four different cards with colours on both sides of the card, as follows:

- Card 1: Green and Green
- Card 2: Green and Red
- Card 3: Green and Blue
- Card 4: Blue and Blue

You randomly take one card and observe that one of the sides of the card facing you is green. What is the probability that the other side of the card is also green?

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

SECTION B (50 marks)

Each question carries 10 marks. The total mark of all the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

B1. Let $\mathbf{A} = \begin{bmatrix} 1 & 5 \\ -6 & 2 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} -2 & 1 \\ -4 & 3 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} -2 & 0 & -3 \\ -1 & 1 & -2 \\ 0 & 1 & 2 \end{bmatrix}$ and $\mathbf{D} = \begin{bmatrix} 4 & -3 & 3 \\ 2 & -4 & -1 \\ -1 & 2 & -2 \end{bmatrix}$.

(a) Evaluate the following:

(i) $\mathbf{A} - 2\mathbf{B}$

(ii) $\mathbf{A}\mathbf{B}^T$

(5 marks)

(b) Evaluate $\mathbf{CD}$. Hence, find $\mathbf{D}^{-1}$.

(5 marks)

B2. **Solve this question using homogeneous coordinates.**

A line segment $\mathbf{P}$ with coordinates $(-2, 2)$ and $(3, 1)$ undergoes the following sequence of transformations:

$\mathbf{T}_1$: shearing in the x-direction by a factor of 2, followed by

$\mathbf{T}_2$: translation 3 units to the left and 2 units upwards.

(a) Write down the transformation matrices $\mathbf{T}_1$ and $\mathbf{T}_2$. Hence, compute the composite matrix $\mathbf{C}$ for the above sequence of transformations.

(4 marks)

(b) Find $\mathbf{P}'$, the image matrix of line segment $\mathbf{P}$ after undergoing the above sequence of transformations.

(2 marks)

(c) Write down the inverse transformation matrices $\mathbf{T}_1^{-1}$ and $\mathbf{T}_2^{-1}$. Hence, compute the composite matrix $\mathbf{C}^{-1}$ that transforms $\mathbf{P}'$ back to $\mathbf{P}$.

(4 marks)

B3. **Show your working clearly for this question.**

(a) Convert 1011110.01_2 to its hexadecimal and decimal representations.

(4 marks)

(b) Convert 632.95_{10} to its binary representation.
Express your answer in **exact** form, showing the recursion clearly for the fractional part, if any.

(6 marks)

B4. Let the universal set $U = \{x \in \mathbb{N} \mid x \leq 10\}$ and define the following sets within U :

$$A = \{x \in \mathbb{Z} \mid 2 \leq x \leq 6\}$$

$$B = \left\{x \,\middle|\, \frac{x}{3} \in \mathbb{Z}\right\}$$

(a) Rewrite sets U, A and B by listing.

(3 marks)

(b) Find $A \cap \overline{B}$ and $|A \cup B|$.

(4 marks)

(c) Draw a Venn diagram showing sets U, A and B, indicating all the elements clearly.

(3 marks)

B5. Propositions p , q and r are defined as:

p : John goes to university.

q : John's GPA is above 3.8.

r : John secures a full-time job.

(a) Write down the compound proposition $p \Leftrightarrow (q \wedge \neg r)$ in words.

(2 marks)

(b) Express the following statement in logical notation:

"If John's GPA is not above 3.8 or he secures a full-time job, then he does not go to university."

(2 marks)

(c) By constructing a truth table, determine whether the compound proposition $\neg(p \vee q) \wedge (p \wedge \neg q)$ is a tautology or a contradiction.

(6 marks)

B6. A student must choose 5 different subjects from the following 12 subjects:

- Accounting
- Biology
- Chemistry
- Computing
- Drama
- Economics
- Geography
- History
- Literature
- Mathematics
- Music
- Physics

How many choices does the student have, if

(a) there are no restrictions?

(2 marks)

(b) Computing must be among the chosen subjects?

(2 marks)

(c) Computing must **not** be among the chosen subjects?

(2 marks)

(d) Computing and Mathematics must be among the chosen subjects?

(2 marks)

(e) Computing must be among the chosen subjects, but Mathematics must **not** be among the chosen subjects?

(2 marks)

B7. Peter throws two darts at a target, one at a time. The probability that he hits the target on his first throw is 40%. If he hits the target on his first throw, the probability that he hits the target on his second throw is 70%. If he does not hit the target on his first throw, the probability that he hits the target on his second throw is 20%.

(a) Draw a probability tree diagram depicting all the possible outcomes in the scenario described above.

(4 marks)

(b) Hence, what is the probability that

(i) Peter hit the target exactly once out of his two throws?

(ii) Peter hit the target on his first throw, given that he hit the target on his second throw?

(6 marks)

SECTION C (40 marks)

Answer ALL **THREE** questions.

C1. (a) Let p, q and r be propositions. It is known that the truth value of the compound proposition $(p \vee \neg p) \Leftrightarrow (p \wedge q \wedge r)$ is **true**.

Explain clearly, **with reasons**, what the truth values of p, q and r are.

(3 marks)

(b) A committee of three people, Xavier (the chairman), Yvonne (the vice-chairman) and Zachary (the treasurer), vote for a proposal. Each person can vote either a *Yes* or a *No*. Let x, y and z represent Xavier's vote, Yvonne's vote and Zachary's vote respectively.

The proposal is accepted if
- Xavier votes a *Yes*, or
- both Yvonne and Zachary vote a *Yes*.

Otherwise, the proposal is rejected.

Use the following convention to answer this part:

	'0'	'1'
Person's vote	*No*	*Yes*
Proposal outcome	Rejected	Accepted

(i) Construct a truth table for the proposal voting system.

(ii) Hence, obtain the **product-of-sums** (POS) expression for the proposal voting system and simplify the expression to its **simplest form**.

(10 marks)

C2. (a) How many distinct 4-letter words can be formed from the word `QUEENSTOWN`?

(5 marks)

(b) An urn contains N distinct marbles in total. Each marble in the urn is either black or white in colour. There is at least one marble of each colour in the urn. Also, there are more black marbles than white marbles in the urn.

If two marbles are drawn from the urn at random and **without replacement**, the probability of obtaining two marbles of the same colour is equal to the probability of obtaining two marbles of different colours.

(i) Find, in terms of N, the number of marbles of each colour in the urn.

(ii) Hence, write down the set of values that N can possibly take.

(9 marks)

C3. **Solve this question using homogeneous coordinates.**

Hints:

$$\sin\alpha = \frac{\text{opp}}{\text{hyp}} \;;\; \cos\alpha = \frac{\text{adj}}{\text{hyp}} \;;\; \tan\alpha = \frac{\text{opp}}{\text{adj}}$$

$$\sin(-\alpha) = -\sin\alpha$$

$$\cos(-\alpha) = \cos\alpha$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

(a) Show that the transformation matrix $\mathbf{T}_a$ that effects a reflection about a line inclined at an angle of α with respect to the positive x-axis is given by:

$$\mathbf{T}_a = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha & 0 \\ \sin 2\alpha & -\cos 2\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(6 marks)

(b) Hence, show that the transformation matrix $\mathbf{T}_b$ that effects a reflection about the line $y = mx + c$ is given by:

$$\mathbf{T}_b = \frac{1}{1+m^2}\begin{bmatrix} 1-m^2 & 2m & -2mc \\ 2m & m^2-1 & 2c \\ 0 & 0 & 1+m^2 \end{bmatrix}$$

(7 marks)

******* END OF PAPER *******

Formula Sheet

Transformation Matrices

Reflection	about the y-axis	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	about the x-axis	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	about the line $y = x$	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Scaling	relative to the origin	$\begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Shearing	in the x-direction	$\begin{bmatrix} 1 & s_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	in the y-direction	$\begin{bmatrix} 1 & 0 & 0 \\ s_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Rotation	about the origin	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Translation		$\begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix}$

Boolean Algebra

Name	**Identity**
Commutative Laws	$x \bullet y = y \bullet x$ $x + y = y + x$
Associative Laws	$x \bullet (y \bullet z) = (x \bullet y) \bullet z = x \bullet y \bullet z$ $x + (y+z) = (x+y) + z = x + y + z$
Distributive Laws	$x \bullet (y+z) = (x \bullet y) + (x \bullet z)$ $x + (y \bullet z) = (x+y) \bullet (x+z)$
Identity Laws	$x \bullet 1 = x$ $x + 0 = x$
Complement Laws	$x \bullet \overline{x} = 0$ $x + \overline{x} = 1$
Involution Law	$\overline{\overline{x}} = x$
Idempotent Laws	$x \bullet x = x$ $x + x = x$
Bound Laws	$x \bullet 0 = 0$ $x + 1 = 1$
De Morgan's Laws	$\overline{x \bullet y} = \overline{x} + \overline{y}$ $\overline{x + y} = \overline{x} \bullet \overline{y}$
Absorption Laws	$x \bullet (x + y) = x$ $x + (x \bullet y) = x$ $x \bullet (\overline{x} + y) = x \bullet y$ $x + (\overline{x} \bullet y) = x + y$

Probability Rules

Addition	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Subtraction	$P(\overline{A}) = 1 - P(A)$
Multiplication	$P(A \cap B) = P(A)P(B)$ if A and B are independent events
Conditional Probability	$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$