SINGAPORE POLYTECHNIC
2020/2021 SEMESTER ONE EXAMINATION

Common Infocomm Technology Programme (CITP)
Diploma in Applied AI & Analytics (DAAA)
Diploma in Infocomm Security Management (DISM)
Diploma in Information Technology (DIT)
Diploma in Game Design & Development (DGDD)

MS0105 – Mathematics Time allowed: 2 hours
MS0151 – Mathematics for Games

Instructions to Candidates

1. The SP examination rules are to be complied with.
Any candidate who cheats or attempts to cheat will face disciplinary action.

2. This paper consists of **9** printed pages (including the cover page and formula sheet).

3. This paper consists of three sections (100 marks in total):

Section A: 5 multiple-choice questions (10 marks)
Answer all questions behind the cover page of the answer booklet.

Section B: 7 structured questions (50 marks)
The total mark of the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

Section C: 3 structured questions (40 marks)
Answer all questions.

4. Unless otherwise stated, all **non-exact** decimal answers should be rounded to **at least two** decimal places.

5. Except for sketches, graphs and diagrams, no solutions are to be written in pencil. Failure to do so will result in loss of marks.

SECTION A (10 marks)

Answer ALL **FIVE** questions. Each question carries 2 marks.
Tick the choice of answer for each question in the box of the MCQ answer sheet provided in the answer booklet.

A1. Given that $\mathbf{A}$ is a square matrix, which of the following statements is **always** true?

(a) $\mathbf{A}+\mathbf{A}^T$ is a diagonal matrix.

(b) $\mathbf{A}+\mathbf{A}^T$ is a symmetric matrix.

(c) $\mathbf{A}-\mathbf{A}^T$ is a diagonal matrix.

(d) $\mathbf{A}-\mathbf{A}^T$ is a symmetric matrix.

A2. Two sets A and B are defined as $A=\{x\in\mathbb{Z} \mid 1\le x\le 4\}$ and $B=\left\{x\in A \,\middle|\, \sqrt{x^3}\in\mathbb{N}\right\}$.

What is the cardinality of set B?

(a) 0 (b) 1 (c) 2 (d) 3

A3. Which of the following statements is the **negation** of the following proposition?

I enjoy eating ice-cream or yoghurt, but I do not enjoy eating salad.

(a) I do not enjoy eating ice-cream or yoghurt, but I enjoy eating salad.

(b) I do not enjoy eating ice-cream or yoghurt, or I enjoy eating salad.

(c) I do not enjoy eating ice-cream and yoghurt, but I enjoy eating salad.

(d) I do not enjoy eating ice-cream and yoghurt, or I enjoy eating salad.

A4. At a party of 40 people, everyone shakes everyone else's hand exactly once. How many different handshakes will there be in total after the party?

(a) 760 (b) 780 (c) 800 (d) 820

A5. When John plays chess against his favourite computer program, he wins with probability 0.6 and loses with probability 0.3. All the other times, the game results in a draw. John plays with the computer five consecutive times.

What is the probability that John's **first win** occurs when he plays his **third** game?

(a) 0.054 (b) 0.096 (c) 0.144 (d) 0.6

SECTION B (50 marks)

Each question carries 10 marks. The total mark of all the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

B1. Let $\mathbf{A} = \begin{bmatrix} 1 & 4 \\ -2 & -3 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} -3 & -4 \\ 2 & 1 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 5 & -1 & 1 \\ 0 & 3 & -2 \end{bmatrix}$.

(a) Evaluate the following wherever possible.
State the reason(s) clearly if the expressions cannot be evaluated.

(i) $\mathbf{A} - 2\mathbf{B}$

(ii) $\mathbf{BC}^T$

(5 marks)

(b) Evaluate $\mathbf{AB}$. Hence, find $\mathbf{A}^{-1}$.

(5 marks)

B2. **Solve this question using homogeneous coordinates.**

A line segment $\mathbf{P}$ with coordinates $(2,-1)$ and $(1,3)$ undergoes the following sequence of transformations:

$\mathbf{T}_1$: shearing in the x-direction by a factor of 2, followed by

$\mathbf{T}_2$: reflection about the line $y = x$.

(a) Write down the transformation matrices $\mathbf{T}_1$ and $\mathbf{T}_2$. Hence, compute the composite matrix $\mathbf{C}$ for the above sequence of transformations.

(4 marks)

(b) Find $\mathbf{P}'$, the image matrix of line segment $\mathbf{P}$ after undergoing the above sequence of transformations.

(2 marks)

(c) Write down the inverse transformation matrices $\mathbf{T}_1^{-1}$ and $\mathbf{T}_2^{-1}$. Hence, compute the composite matrix $\mathbf{C}^{-1}$ that transforms $\mathbf{P}'$ back to $\mathbf{P}$.

(4 marks)

B3. **Show your working clearly for this question.**

(a) Convert 1011.101_2 to its decimal and hexadecimal representation.

(4 marks)

(b) Convert 43.7_{10} to its binary representation.
Express your answer in **exact** form, showing the recursion clearly for the fractional part, if any.

(6 marks)

B4. Let the universal set $U = \{x \mid x \in \mathbb{Z}, 1 \le x \le 9\}$ and define the following sets within U :

$$A = \{x \mid x \in \mathbb{N}, x \le 6\}$$

$$B = \left\{ x \,\middle|\, \frac{x}{3} \in \mathbb{Z} \right\}$$

(a) Rewrite sets U, A and B by listing.

(3 marks)

(b) Find $\overline{A}$, $A \cap B$ and $|A \cup B|$.

(4 marks)

(c) Draw a Venn diagram showing sets U, A and B, indicating all the elements clearly.

(3 marks)

B5. Let p and q be simple propositions.

(a) Given that the truth values of p and q are T and F respectively, determine the truth values of

(i) $\neg(p \wedge q)$

(ii) $(p \Rightarrow q) \vee p$

(4 marks)

(b) Two compound propositions M and N are defined as follows:

$$M = (\neg p \vee q) \wedge p$$

$$N = (\neg p \wedge q) \vee p$$

By constructing a truth table, determine whether or not $M \equiv N$.

(6 marks)

B6. How many four-digit numbers can be formed from the digits 1, 3, 4, 6, 7, 9 if

(a) the digits can be repeated?

(2 marks)

(b) the digits cannot be repeated?

(2 marks)

(c) the digits cannot be repeated, and the four-digit number must be an odd number?

(3 marks)

(d) the four-digit number must contain exactly two 1's with no other repeated digits?

(3 marks)

B7. Two fair six-sided dice are rolled simultaneously. What is the probability that

(a) both dice show the same number?

(2 marks)

(b) both dice show different numbers?

(2 marks)

(c) the sum of the numbers on both dice is 10 or 11?

(3 marks)

(d) the absolute difference between the numbers on both dice is 4?

(3 marks)

SECTION C (40 marks)

Answer ALL **THREE** questions.

C1. David wants to design a digital device that will help him manage the amount of time he spends on playing computer games during his school term. The device will comprise of three components x, y and z, as described in the table below.

Component	Description
x	x will light up if David's computer has any game client that is running continuously for more than 2 hours. Otherwise, x will not light up.
y	y will light up if the computer game is useful in helping David improve his proficiency in mathematics. Otherwise, y will not light up.
z	z will light up if the date of David's mathematics exam is less than one week from the current date. Otherwise, z will not light up.

The device will send a signal to David to remind him to stop playing computer games when:

- z is lighted up; or
- x is lighted up and y is not lighted up.

Otherwise, the device will not send a signal to David.

Use the following convention to answer this question.

	'0'	'1'
Component	Not lighted up	Lighted up
Device	Signal is not sent	Signal is sent

(a) Construct a truth table to depict the above scenario for the digital device.

(4 marks)

(b) By using the **product-of-sums** method, obtain the Boolean function $f(x, y, z)$ for the digital device and simplify the expression to its **simplest form**.

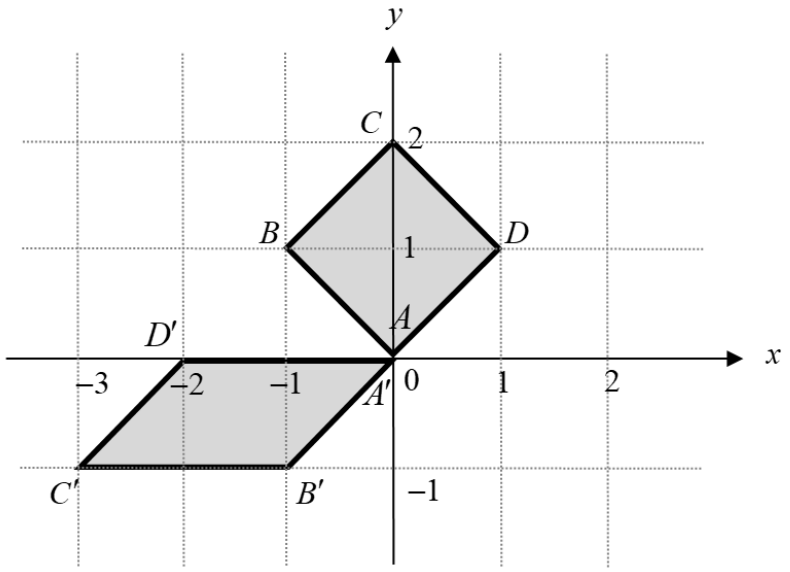
(6 marks)

(c) Suppose the date of David's mathematics exam is 20 days from the current date and his computer has a game client that is running continuously for t hours. If the device sent a signal to David, determine the range of possible values for t.

(2 marks)

C2. **Solve this question using homogeneous coordinates.**

Square $ABCD$ in the figure below is transformed to parallelogram $A'B'C'D'$ through a sequence of **three** simple transformations.



(a) Describe, in words, the three transformations needed to transform square $ABCD$ to parallelogram $A'B'C'D'$, and write down the corresponding transformation matrices. (6 marks)

(b) Hence, derive the composite matrix **T** for the above sequence of transformations and verify that **T** successfully transforms square $ABCD$ to parallelogram $A'B'C'D'$. (4 marks)

(c) Parallelogram $A'B'C'D'$ now undergoes scaling in the y-direction by a factor of k relative to the origin, and is subsequently transformed to a rhombus. Determine the possible value(s) of k.

(<u>Note:</u> A rhombus is a four-sided object whose four sides are of equal length.) (3 marks)

C3. **Explain your working clearly for this question.**

Six people, Alice, Bob, Charlie, Dylan, Elise and Fiona, are participating in an activity called ‘pass the baton’. The rules of the activity are as follows:

- Each person can pass the baton to any person other than himself or herself.
- Each time a person passes the baton to another person, it counts as one pass.
- The activity ends after the baton goes through five passes.

Given that **Alice is always holding on to the baton initially**, how many ways can the six people pass the baton around such that at the end of the activity,

(a) each person held on to the baton exactly once?

(2 marks)

(b) Bob held on to the baton at most twice?

(4 marks)

(c) exactly three of the six people held on to the baton?

(4 marks)

(d) Alice is the last person holding on to the baton?

(5 marks)

******* END OF PAPER *******

Formula Sheet

Transformation Matrices

Reflection	about the y-axis	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	about the x-axis	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	about the line $y = x$	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Scaling	relative to the origin	$\begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Shearing	in the x-direction	$\begin{bmatrix} 1 & s_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	in the y-direction	$\begin{bmatrix} 1 & 0 & 0 \\ s_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Rotation	about the origin	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Translation		$\begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix}$

Boolean Algebra

Name	**Identity**
Commutative Laws	$x \bullet y = y \bullet x$ $x + y = y + x$
Associative Laws	$x \bullet (y \bullet z) = (x \bullet y) \bullet z = x \bullet y \bullet z$ $x + (y + z) = (x + y) + z = x + y + z$
Distributive Laws	$x \bullet (y + z) = (x \bullet y) + (x \bullet z)$ $x + (y \bullet z) = (x + y) \bullet (x + z)$
Identity Laws	$x \bullet 1 = x$ $x + 0 = x$
Complement Laws	$x \bullet \overline{x} = 0$ $x + \overline{x} = 1$
Involution Law	$\overline{\overline{x}} = x$
Idempotent Laws	$x \bullet x = x$ $x + x = x$
Bound Laws	$x \bullet 0 = 0$ $x + 1 = 1$
De Morgan's Laws	$\overline{x \bullet y} = \overline{x} + \overline{y}$ $\overline{x + y} = \overline{x} \bullet \overline{y}$
Absorption Laws	$x \bullet (x + y) = x$ $x + (x \bullet y) = x$ $x \bullet (\overline{x} + y) = x \bullet y$ $x + (\overline{x} \bullet y) = x + y$

Probability Rules

Addition	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Subtraction	$P(\overline{A}) = 1 - P(A)$
Multiplication	$P(A \cap B) = P(A)P(B)$ if A and B are independent events
Conditional Probability	$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$