SOLUTIONS

SINGAPORE POLYTECHNIC
AY2019/20 Semester 2 Examination

No.	SOLUTION
A1	C
A2	C
A3	D
A4	A
A5	A
B1a	$\mathbf{A}^2 = \begin{bmatrix} 0 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 0 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 6 & -3 \\ -2 & 7 \end{bmatrix}$ $\mathbf{A}^2 - \mathbf{A} = \begin{bmatrix} 6 & -3 \\ -2 & 7 \end{bmatrix} - \begin{bmatrix} 0 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$
B1b	$n = 6$
B1a	$\mathbf{A}^2 - \mathbf{A} = 6\mathbf{I}_2$ Multiply by $\mathbf{A}^{-1}$ throughout: $\mathbf{A} - \mathbf{I}_2 = 6\mathbf{A}^{-1}$ $\mathbf{A}^{-1} = \frac{1}{6}\left(\begin{bmatrix} 0 & -3 \\ -2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$ $= \frac{1}{6}\begin{bmatrix} -1 & -3 \\ -2 & 0 \end{bmatrix}$
B2a	$\mathbf{P} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$
B2b	$\mathbf{T}_1 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$ and $\mathbf{T}_2 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\mathbf{C} = \mathbf{T}_2\mathbf{T}_1 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$

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B2c	$\mathbf{P}' = \mathbf{CP} = \begin{bmatrix} 1 & -1 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$
B2d	$\mathbf{T}_1^{-1} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ and $\mathbf{T}_2^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\mathbf{C}^{-1} = \mathbf{T}_1^{-1}\mathbf{T}_2^{-1} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$
B3a	To convert $FAB.9_{16}$ to decimal: $FAB.9_{16} = 15\times16^2 + 10\times16^1 + 11\times16^0 + 9\times16^{-1}$ $= 4011.5625_{10}$ To convert $FAB.9_{16}$ to binary: $FAB.9_{16} = (1111)_2(1010)_2(1011)_2.(1001)_2$ $= 111110101011.1001_2$
B3b	Integral part: $67 = 2(33)+1$ $33 = 2(16)+1$ $16 = 2(8)+0$ $8 = 2(4)+0$ $4 = 2(2)+0$ $2 = 2(1)+0$ $1 = 2(0)+1$ $\Rightarrow 67_{10} = 1000011_2$ Fractional part: $0.875\times2 = 0.75+1$ $0.75\times2 = 0.5+1$ $0.5\times2 = 0+1$ $\Rightarrow 0.875_{10} = 0.111_2$ Hence, $67.875_{10} = 1000011.111_2$
B4a	$U = \{-2,-1,0,1,2,3,4,5\}$ $A = \{1,3,5\}$ $B = \{1,2,3\}$
B4b (i)	$\overline{B} = \{-2,-1,0,4,5\}$

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B4b (ii)	$A-B=\{5\}$
B4b (iii)	$A\cup B=\{1,2,3,5\}$ $\lvert A\cup B\rvert=4$
B5a (i)	$\neg(p\vee q)=\neg(T\vee T)=F$
B5a (ii)	$(p\wedge q)\Rightarrow r$ $=(T\wedge T)\Rightarrow F$ $=T\Rightarrow F$ $=F$
B5b	$\overline{\overline{x}+y}+xy$ $=x\overline{y}+xy$ $=x\left(\overline{y}+y\right)$ $=x$
B6a	$6!=720$
B6b	$2\times 5!=240$
B6c	${}^{4}P_{3}\times 3!=144$
B6d	$5!\times 2!=240$
B7a	$\frac{3}{11}\times\frac{5}{13}=\frac{15}{143}$
B7b	$\frac{3}{11}\times\frac{8}{13}+\frac{8}{11}\times\frac{5}{13}=\frac{64}{143}$
B7c	$\frac{8}{11}\times\frac{8}{13}=\frac{64}{143}$
B7d	$1-\frac{3}{11}\times\frac{5}{13}=\frac{128}{143}$ or $\frac{64}{143}+\frac{64}{143}=\frac{128}{143}$

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C1a	see table below

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$(p \Rightarrow q) \vee (q \Rightarrow p)$
F	F	T	T	T
F	T	T	F	T
T	F	F	T	T
T	T	T	T	T

C1b (i)

Converse: If $x = 3$, then $x^2 = 9$.

Inverse: If $x^2 \neq 9$, then $x \neq 3$.

Contrapositive: If $x \neq 3$, then $x^2 \neq 9$.

C1b (ii)

Implication: F

Converse: T

Inverse: T

Contrapositive: F

C2a

Dual: $xy + \bar{x}z + yz = xy + \bar{x}z$

Verify the dual:

$$\begin{aligned}
\text{RHS} &= xy + \bar{x}z \\
&= xy + \bar{x}z + xyz && \text{Absorption Law} \\
&= xy + z\left(\bar{x} + xy\right) && \text{Distributive Law} \\
&= xy + z\left(\bar{x} + y\right) && \text{Absorption Law} \\
&= xy + \bar{x}z + yz && \text{Distributive Law} \\
&= \text{LHS}
\end{aligned}$$

C2b (i)

Let p, q and r represent votes from Albert, Ben and Carol respectively.

p	q	r	$f(p,q,r)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

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C2b (ii)	$f(p,q,r)=(p+q+r)\bullet(p+q+\bar{r})\bullet(p+\bar{q}+r)$ $=\left[(p+q)+(r\bullet\bar{r})\right]\bullet(p+\bar{q}+r)$ $=(p+q)\bullet(p+\bar{q}+r)$ $=p+\left[q\bullet(\bar{q}+r)\right]$ $=p+\left[q\bullet\bar{q}+q\bullet r\right]$ $=p+qr$
C3a	Area of PQRS = x^2 Area of RXY = $\frac{1}{2}\left(\frac{1}{2}x\right)\left(\frac{1}{2}x\right)=\frac{1}{8}x^2$ Probability $=\frac{\text{Area of RXY}}{\text{Area of PQRS}}=\frac{\frac{1}{8}x^2}{x^2}=\frac{1}{8}$
C3b (i)	1. “up, up, right” 2. “up, right, up”
C3b (ii)	We require exactly 10 ups and 10 rights from a total of 20 moves, and order does not matter. Hence, $\frac{20!}{10!10!}=184756$
C3b (iii)	$\frac{(p+q)!}{p!q!}$