SOLUTIONS

SINGAPORE POLYTECHNIC
2023/2024 Semester 1 Mid-Semester Test

No.	SOLUTION
1(a)	Order of $(\mathbf{AF})^{\mathbf{T}}$: 3x2
1(b)	$3\mathbf{E}+\mathbf{P}^T=-2\mathbf{CA}$ $3\begin{bmatrix}3&0&2\\2&0&-2\\1&1&1\end{bmatrix}+\mathbf{P}^T=-2\begin{bmatrix}2&-3\\-9&2\\4&1\end{bmatrix}\begin{bmatrix}4&-6&-2\\-5&2&0\end{bmatrix}$ $\begin{bmatrix}9&0&6\\6&0&-6\\3&3&3\end{bmatrix}+\mathbf{P}^T=\begin{bmatrix}-46&36&8\\92&-116&-36\\-22&44&16\end{bmatrix}$ $\mathbf{P}^T=\begin{bmatrix}-46&36&8\\92&-116&-36\\-22&44&16\end{bmatrix}-\begin{bmatrix}9&0&6\\6&0&-6\\3&3&3\end{bmatrix}$ $=\begin{bmatrix}-55&36&2\\86&-116&-30\\-25&41&13\end{bmatrix}$ $\mathbf{P}=\begin{bmatrix}-55&86&-25\\36&-116&41\\2&-30&13\end{bmatrix}$
1(c)	Given that B is a symmetric matrix, $y-3=6$ $y=9$ $y+2=x-4$ sub $y=9$, $x=15$
1(d)(i)	$\mathbf{D}^2=\begin{bmatrix}-8&11\\-4&4\end{bmatrix}\begin{bmatrix}-8&11\\-4&4\end{bmatrix}=\begin{bmatrix}20&-44\\16&-28\end{bmatrix}$ or $4\begin{bmatrix}5&-11\\4&-7\end{bmatrix}$
1(d)(ii)	**B**-**C** cannot be evaluated because the order/size of **B** is not the same as **C**.
1(d)(iii)	$\mathbf{FC}=\begin{bmatrix}2&2&0\\-4&1&10\\2&-3&0\end{bmatrix}\begin{bmatrix}2&-3\\-9&2\\4&1\end{bmatrix}=\begin{bmatrix}-14&-2\\23&24\\31&-12\end{bmatrix}$

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1(d)(iv)	$(\mathbf{DC^TE})^3$ cannot be evaluated because $\mathbf{(DC^TE)}$ is not a square matrix / $\mathbf{(DC^TE)}$ is a 2x3 matrix which is not conformable with another 2x3 matrix. (Accept other valid reasons).
1(e)(i)	Given that $\mathbf{EF} = k\mathbf{I}_3$, $\begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 2 & 0 \\ -4 & 1 & 10 \\ 2 & -3 & 0 \end{bmatrix} = kI_3$ $\begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} = kI_3$ $10\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = kI_3$ $k = 10$

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1(e)(ii)	Given $10\mathbf{GF} = 9\mathbf{E}$, $10\mathbf{G}^{-1}\mathbf{GF} = 9\mathbf{G}^{-1}\mathbf{E}$ $10\mathbf{F} = 9\mathbf{G}^{-1}\mathbf{E}$ $10\mathbf{FE}^{-1} = 9\mathbf{G}^{-1}\mathbf{EE}^{-1}$ $10\mathbf{FE}^{-1} = 9\mathbf{G}^{-1}$ $\mathbf{G}^{-1} = \frac{10}{9}\mathbf{FE}^{-1}$ From part (e)(i), $\mathbf{EF} = 10\mathbf{I}_3$ $\mathbf{F} = 10\mathbf{E}^{-1}$ $\mathbf{E}^{-1} = \frac{1}{10}\mathbf{F}$ $\mathbf{G}^{-1} = \frac{10}{9}\mathbf{FE}^{-1}$ $= \frac{10}{9}\mathbf{F}.\frac{1}{10}\mathbf{F}$ $= \frac{1}{9}\mathbf{F}^2$ $= \frac{1}{9}\begin{bmatrix} 2 & 2 & 0 \\ -4 & 1 & 10 \\ 2 & -3 & 0 \end{bmatrix}\begin{bmatrix} 2 & 2 & 0 \\ -4 & 1 & 10 \\ 2 & -3 & 0 \end{bmatrix}$ $= \frac{1}{9}\begin{bmatrix} -4 & 6 & 20 \\ 8 & -37 & 10 \\ 16 & 1 & -30 \end{bmatrix}$

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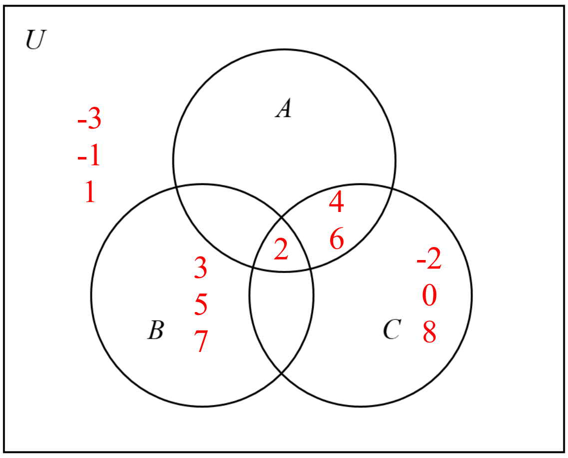
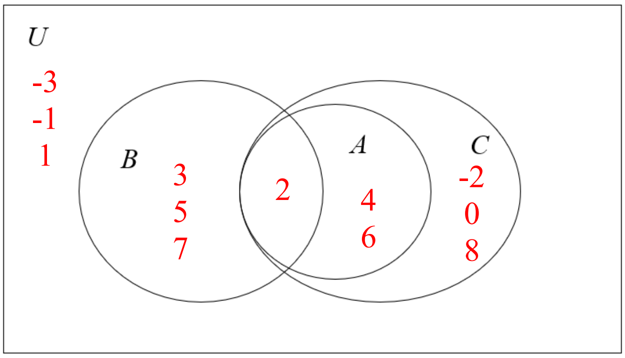
2(a)	$U=\{-3,-2,-1,0,1,2,3,4,5,6,7,8\}$ $A=\{2,4,6\}$ $B=\{2,3,5,7\}$ $C=\{-2,0,2,4,6,8\}$
2(b)	$B\cap\overline{C}=\{3,5,7\}$ $C\cap(A\cup B)=\{2,4,6\}$
2(c)	 Alternatively, 
2(d)(i)	$P(A)=\{A,\{2,4\},\{2,6\},\{4,6\},\{2\},\{4\},\{6\},\varnothing\}$ $P(B)=\left\{\begin{matrix}B,\{2,3,5\},\{2,3,7\},\{2,5,7\},\{3,5,7\},\{2,3\},\{2,5\},\\ \{2,7\},\{3,5\},\{3,7\},\{5,7\},\{2\},\{3\},\{5\},\{7\},\varnothing\end{matrix}\right\}$
2(d)(ii)	By observing number of elements in set $P(A)$ and $P(B)$, $\lvert A\rvert=3,\lvert P(A)\rvert=8=2^3$ $\lvert B\rvert=4,\lvert P(B)\rvert=16=2^4$ $\therefore$ no. of element in set $P(X)=2^{\lvert X\rvert}\ or\ 2^{n(X)}$

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3(a)

Integral part:		
2	712	
2	356	0
2	178	0
2	89	0
2	44	1
2	22	0
2	11	0
2	5	1
2	2	1
2	1	0
2	0	1

Fractional part:		
2x	0.525	
1.05	0.05	1
0.1	0.1	0
0.2	0.2	0
0.4	0.4 (rep)	0
0.8	0.8	0
1.6	0.6	1
1.2	0.2	1
0.4	0.4 (rep)	0
0.8	0.8	0
1.6	0.6	1
1.2	0.2	1

$\therefore 712.525_{10} = 10\ 1100\ 1000.100\overline{0011}_2 = 2C8.8\overline{6}_{16}$

3(b)

$255_{10} = 1111\ 1111_2$ (1 byte)

Any integer between 0 and 255 or any fraction that can be expressed in exactly 8 bits will result in zero truncation error - the smallest truncation error is 0.

The largest truncation error is

$= 0.0000\ 0000\ \bar{1}_2$

$= 1_{10} - 0.1111\ 1111_2$

$= 1 - (2^{-1} + 2^{-2} + 2^{-3} + 2^{-4} + 2^{-5} + 2^{-6} + 2^{-7} + 2^{-8})$

$= 1 - 0.99609375$

$= 0.00390625$

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4(a)(i)	$\mathbf{T}_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $\mathbf{T}_2 = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $\mathbf{T}_3 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
4(a)(ii)	$\mathbf{C} = \mathbf{T}_3\mathbf{T}_2\mathbf{T}_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} 0 & -0.5 & 0 \\ -0.5 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
4(a)(iii)	$\mathbf{P}' = \mathbf{CP} = \begin{bmatrix} 0 & -0.5 & 0 \\ -0.5 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$ $= \begin{bmatrix} -1 & 0 & 0.5 \\ -1 & 0.5 & -0.5 \\ 1 & 1 & 1 \end{bmatrix}$
4(a)(iv)	$\mathbf{T}_1^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $\mathbf{T}_2^{-1} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $\mathbf{T}_3^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
4(a)(v)	$\mathbf{C}^{-1} = \mathbf{T}_1^{-1}\mathbf{T}_2^{-1}\mathbf{T}_3^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} 0 & -2 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
4(b)(i)	$\mathbf{T}_a$: Reflection about the line $y = x$ $\mathbf{T}_b$: Shearing in the y-direction by a factor of -2 $\mathbf{T}_c$: Translation 3 units upwards $\mathbf{T}_a = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $\mathbf{T}_b = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $\mathbf{T}_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

4(b)(ii)	$\mathbf{T} = \mathbf{T}_c \mathbf{T}_b \mathbf{T}_a = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ $\mathbf{U}' = \mathbf{T}\mathbf{U} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 2 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ (verified)