SOLUTIONS

SINGAPORE POLYTECHNIC
2020/2021 Semester 2 Examination

No.	SOLUTION
A1	C
A2	D
A3	C
A4	B
A5	D
B1a	$3\mathbf{A}+2\mathbf{B}^T=3\begin{bmatrix}2&1&-1\\3&1&-2\\-3&4&-1\end{bmatrix}+2\begin{bmatrix}7&-3&-1\\9&-5&1\\15&-11&-1\end{bmatrix}^T$ $=\begin{bmatrix}6&3&-3\\9&3&-6\\-9&12&-3\end{bmatrix}+\begin{bmatrix}14&18&30\\-6&-10&-22\\-2&2&-2\end{bmatrix}$ $=\begin{bmatrix}20&21&27\\3&-7&-28\\-11&14&-5\end{bmatrix}$
B1b	$\mathbf{AB}=\begin{bmatrix}2&1&-1\\3&1&-2\\-3&4&-1\end{bmatrix}\begin{bmatrix}7&-3&-1\\9&-5&1\\15&-11&-1\end{bmatrix}=\begin{bmatrix}8&0&0\\0&8&0\\0&0&8\end{bmatrix}=8\mathbf{I}$ $\mathbf{A}^{-1}\mathbf{AB}=8\mathbf{A}^{-1}\mathbf{I}$ $\Rightarrow \mathbf{B}=8\mathbf{A}^{-1}$ $\therefore \mathbf{A}^{-1}=\frac{1}{8}\mathbf{B}=\frac{1}{8}\begin{bmatrix}7&-3&-1\\9&-5&1\\15&-11&-1\end{bmatrix}$
B2a	$\mathbf{T}_1=\begin{bmatrix}-1&0&0\\0&1&0\\0&0&1\end{bmatrix}$; $\mathbf{T}_2=\begin{bmatrix}1&0&-3\\0&1&4\\0&0&1\end{bmatrix}$ $\mathbf{C}=\mathbf{T}_2\mathbf{T}_1=\begin{bmatrix}1&0&-3\\0&1&4\\0&0&1\end{bmatrix}\begin{bmatrix}-1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}-1&0&-3\\0&1&4\\0&0&1\end{bmatrix}$
B2b	$\mathbf{P}'=\mathbf{CP}=\begin{bmatrix}-1&0&-3\\0&1&4\\0&0&1\end{bmatrix}\begin{bmatrix}4&-3\\-1&2\\1&1\end{bmatrix}=\begin{bmatrix}-7&0\\3&6\\1&1\end{bmatrix}$

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B2c

$$\mathbf{T}_1^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \mathbf{T}_2^{-1} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{C}^{-1} = \mathbf{T}_1^{-1}\mathbf{T}_2^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & -3 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

B3a

Convert to decimal:

$$\begin{aligned} 5C.2A_{16} &= 5\times16^1 + 12\times16^0 + 2\times16^{-1} + 10\times16^{-2} \\ &= 92.1640625_{10} \end{aligned}$$

Convert to binary:

$$\begin{aligned} 5C.2A_{16} &= (0101)(1100).(0010)(1010)_2 \\ &= 1011100.0010101_2 \end{aligned}$$

B3b

Integral part:		
16	26958	
16	1684	E
16	105	4
16	6	9
	0	6

Fractional part:		
16	0.15625	
16	0.5	2
	0	8

$\therefore 26958.15625_{10} = 694E.28_{16}$

B4a

$U = \{0,1,2,3,4,5,6\}$

$A = \{1,2,3,4\}$

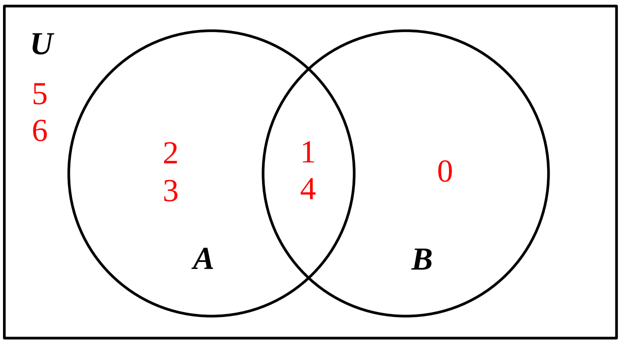
$B = \{0,1,4\}$

B4b

$A - B = \{2,3\}$

$A \cup \overline{B} = \{1,2,3,4,5,6\}$

B4c



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B5a	If you work hard and you do not get distracted, then you can finish the job.
B5b	$\neg p \wedge q \wedge r$
B5c	(see truth table below)
B6a	$\left\lvert A_3 \cap A_5 \right\rvert = \left\lvert A_{15} \right\rvert = \left\lfloor \dfrac{5000}{15} \right\rfloor = 333$
B6b	$\left\lvert A_3 \cup A_5 \right\rvert = \left\lvert A_3 \right\rvert + \left\lvert A_5 \right\rvert - \left\lvert A_3 \cap A_5 \right\rvert = \left\lfloor \dfrac{5000}{3} \right\rfloor + \left\lfloor \dfrac{5000}{5} \right\rfloor - \left\lfloor \dfrac{5000}{15} \right\rfloor = 2333$
B6c	$\left\lvert \overline{A_5 \cup A_{10}} \right\rvert = \left\lvert \overline{A_5} \right\rvert = \left\lvert U \right\rvert - \left\lvert A_5 \right\rvert = \left\lfloor \dfrac{5000}{1} \right\rfloor - \left\lfloor \dfrac{5000}{5} \right\rfloor = 4000$
B6d	$\left\lvert A_5 \cap A_{10} \cap \overline{A_{12}} \right\rvert = \left\lvert A_{10} \cap \overline{A_{12}} \right\rvert = \left\lvert A_{10} \right\rvert - \left\lvert A_{60} \right\rvert = \left\lfloor \dfrac{5000}{10} \right\rfloor - \left\lfloor \dfrac{5000}{60} \right\rfloor = 417$
B7a	$P(\text{chocolate}) = \dfrac{44+34}{200} = \dfrac{39}{100}$
B7b	$P(\text{almond}) = \dfrac{44+28+26}{200} = \dfrac{49}{100}$
B7c	$P(\text{strawberry} \cup \text{hazelnut}) = \dfrac{28+34+36+32}{200} = \dfrac{13}{20}$
B7d	$P(\text{vanilla} \mid \text{hazelnut}) = \dfrac{32}{34+36+32} = \dfrac{16}{51}$

B5c:

p	q	r	$p \wedge r$	$q \vee r$	$(p \wedge r) \Rightarrow (q \vee r)$
T	T	T	T	T	T
T	T	F	F	T	T
T	F	T	T	T	T
T	F	F	F	F	T
F	T	T	F	T	T
F	T	F	F	T	T
F	F	T	F	T	T
F	F	F	F	F	T

$\therefore (p \wedge r) \Rightarrow (q \vee r)$ is a **tautology**.

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C1a

x	y	z	$f(x,y,z)$
1	1	1	0
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	0

Sum-of-products expression: $f(x,y,z) = xy\bar{z} + x\bar{y}z$

C1b

$$
\begin{aligned}
\overline{f(x,y,z)} &= \overline{xy\bar{z} + x\bar{y}z} \\
&= \overline{x\left(y\bar{z} + \bar{y}z\right)} \\
&= \bar{x} + \overline{y\bar{z} + \bar{y}z} \\
&= \bar{x} + \left(\bar{y} + z\right)\left(y + \bar{z}\right) \\
&= \bar{x} + y\bar{y} + \bar{y}\bar{z} + yz + z\bar{z} \\
&= \bar{x} + yz + \bar{y}\bar{z}
\end{aligned}
$$

C2a

$^{10}C_6 = 210$

C2b

$^{5}C_3 \times {}^{5}C_3 = 100$

C2c

There are two possible cases:

Case 1: first and last bits are both ‘1’

Case 2: first and last bits are both ‘0’

$\therefore {}^{8}C_4 + {}^{8}C_6 = 98$

C2d

$540_{10} = 1000011100_2$

For the decimal equivalent of the binary number to be greater than 540_{10}, only the first bit of the binary number must be ‘1’.

$\therefore {}^{9}C_5 = 126$

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C3a	Since $\tan\alpha = \frac{\text{opp}}{\text{adj}} = \frac{1}{p}$, we thus have $\sin\alpha = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{p^2+1}}$ and $\cos\alpha = \frac{\text{adj}}{\text{hyp}} = \frac{p}{\sqrt{p^2+1}}$. $\mathbf{T}_1 = \begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) \\ \sin(-\alpha) & \cos(-\alpha) \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \frac{1}{\sqrt{p^2+1}} \begin{bmatrix} p & 1 \\ -1 & p \end{bmatrix}$ $\mathbf{T}_2 = \begin{bmatrix} p & 0 \\ 0 & \frac{1}{p} \end{bmatrix}$ $\mathbf{T}_3 = \begin{bmatrix} \cos(90°-\alpha) & -\sin(90°-\alpha) \\ \sin(90°-\alpha) & \cos(90°-\alpha) \end{bmatrix} = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix} = \frac{1}{\sqrt{p^2+1}} \begin{bmatrix} 1 & -p \\ p & 1 \end{bmatrix}$ $\mathbf{C} = \mathbf{T}_3\mathbf{T}_2\mathbf{T}_1 = \frac{1}{p^2+1} \begin{bmatrix} 1 & -p \\ p & 1 \end{bmatrix} \begin{bmatrix} p & 0 \\ 0 & \frac{1}{p} \end{bmatrix} \begin{bmatrix} p & 1 \\ -1 & p \end{bmatrix}$ $= \frac{1}{p^2+1} \begin{bmatrix} p^2+1 & 0 \\ p^3-\frac{1}{p} & p^2+1 \end{bmatrix}$ $= \begin{bmatrix} 1 & 0 \\ \frac{p^4-1}{p(p^2+1)} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{(p^2-1)(p^2+1)}{p(p^2+1)} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{p^2-1}{p} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ p-\frac{1}{p} & 1 \end{bmatrix}$ (shown)
C3b	$\tan\alpha = \frac{1}{p} \Rightarrow p = \frac{1}{\tan 60°} = \frac{1}{\sqrt{3}}$ Shear factor $= p - \frac{1}{p} = \frac{1}{\sqrt{3}} - \sqrt{3} = -\frac{2}{\sqrt{3}}$