SINGAPORE POLYTECHNIC
2022/2023 SEMESTER TWO MID-SEMESTER TEST

Common Infocomm Technology Programme (CITP)
Diploma in Applied AI & Analytics (DAAA)
Diploma in Infocomm Security Management (DISM)
Diploma in Information Technology (DIT)
Diploma in Media, Arts & Design (DMAD)

MS0105 – Mathematics Time allowed: 1 hour 40 minutes
MS0151 – Mathematics for Games

Instructions to Candidates

1. The SP examination rules are to be complied with.
   Any candidate who cheats or attempts to cheat will face disciplinary action.

2. This paper consists of **4** printed pages (including the cover page).
   There are 4 questions (100 marks in total), and you are to answer all the questions.

3. Unless otherwise stated, all **non-exact** decimal answers should be rounded to **at least two** decimal places.

4. Except for sketches, graphs and diagrams, no solutions are to be written in pencil. Failure to comply may result in loss of marks.

Formula Sheet: Transformation Matrices

1. Reflection			3. Shearing	
a. about the y-axis	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		a. in the x-direction	$\begin{bmatrix} 1 & s_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
b. about the x-axis	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		b. in the y-direction	$\begin{bmatrix} 1 & 0 & 0 \\ s_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
c. about the line $y = x$	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		4. Rotation about the origin	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
2. Scaling relative to the origin	$\begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$		5. Translation	$\begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix}$

1. Given the following matrices:

$$\mathbf{A}=\begin{bmatrix} -2 & 1 & 3b+c \\ a-c & 4 & 2c+3 \\ -1 & 7 & 5 \end{bmatrix} \qquad \mathbf{B}=\begin{bmatrix} 1 & 8 & -4 \\ 4 & -4 & -7 \\ 8 & 1 & 4 \end{bmatrix} \qquad \mathbf{C}=\begin{bmatrix} 8 & 2 & -3 \\ 5 & -1 & 12 \\ 3 & 9 & 4 \end{bmatrix}$$

$$\mathbf{D}=\begin{bmatrix} 1 & 0 & -4 \\ 3 & -1 & 2 \end{bmatrix} \qquad \mathbf{E}=\begin{bmatrix} -3 & 8 \\ 5 & 4 \end{bmatrix} \qquad \mathbf{F}=\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$$

(a) If **A** is a symmetric matrix, find the values of a, b and c.

(5 marks)

(b) Find matrix $\mathbf{X}$ such that $\mathbf{E}^T = 2\mathbf{F} - \mathbf{X}$.

(5 marks)

(c) Evaluate the following wherever possible.
State the reason(s) clearly if the expressions cannot be evaluated.

(i) $\mathbf{ED}$

(ii) $\mathbf{D}+\mathbf{E}+\mathbf{F}$

(iii) $\mathbf{F}^2 - 10\mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix.

(8 marks)

(d) (i) Evaluate $\mathbf{B}^T\mathbf{B}$.

(ii) Hence, find matrix $\mathbf{Y}^{-1}$ such that $\mathbf{CY} = \mathbf{B} + \mathbf{Y}$.

(12 marks)

2. (a) Let universal set $U = \{x \in \mathbb{Z} \mid -2 \le x \le 10\}$ and define the following sets within U:

$$A = \{x \in \mathbb{N} \mid x \le 6\}$$

$$B = \{2x - 3 \mid x \in \mathbb{N}\}$$

$$C = \{xy \mid x \in \mathbb{Z},\ y \in \{3,5\}\}$$

(i) Rewrite sets U, A, B and C by listing.

(ii) Find $A - B$ and $A \cap (B \cup C)$.

(iii) Draw a Venn diagram showing the above sets U, A, B and C, indicating all the elements clearly.

(14 marks)

(b) Sets P and Q are defined as follows:

$$P = \{x + 2 \mid x \in \mathbb{N}\}$$

$$Q = \{x \mid \sqrt{x+2} \in P\}$$

The elements in each of the sets P and Q are arranged in **ascending** order.

(i) Rewrite sets P and Q by listing the first **five** elements in each set, followed by the ellipsis ($\cdots$).

(ii) Hence, determine the 100^{th} element in set Q.

(6 marks)

3. **Show your working clearly for this question. No marks will be awarded if the steps involved are not shown.**

(a) Convert 978.425_{10} to its binary and hexadecimal representations.

Express your answers in **exact** form, showing the recursion clearly for the fractional part, if any.

(12 marks)

(b) A function defined within a certain code takes in three user inputs, x (in base-4), y (in base-8) and z (in base-16), and returns their sum as output w (in base-32).

If the user enters `222.2` for each of the inputs x, y and z, what will the output w be?

(8 marks)

4. **Solve this question using homogeneous coordinates.**

(a) Triangle $\mathbf{P}$ with vertices $(2,1)$, $(-1,0)$ and $(1,3)$ undergoes the following sequence of transformations:

$\mathbf{T}_1$: reflection about the x-axis, followed by

$\mathbf{T}_2$: translation 2 units to the right and 1 unit upwards, followed by

$\mathbf{T}_3$: scaling relative to the origin by a factor of 2 in the x-direction and a factor of 3 in the y-direction.

(i) Write down the transformation matrices $\mathbf{T}_1$, $\mathbf{T}_2$ and $\mathbf{T}_3$.

(ii) Compute the composite matrix $\mathbf{C}$ for the above sequence of transformations.

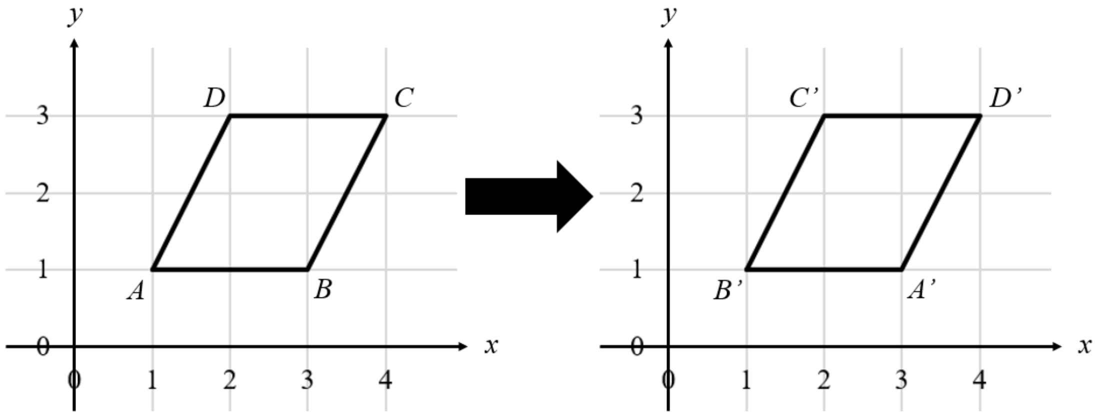
(iii) Find $\mathbf{P}'$, the image matrix of triangle $\mathbf{P}$ after undergoing the above sequence of transformations.

(iv) Write down the inverse transformation matrices $\mathbf{T}_1^{-1}$, $\mathbf{T}_2^{-1}$ and $\mathbf{T}_3^{-1}$.

(v) Compute the composite matrix $\mathbf{C}^{-1}$ that transforms $\mathbf{P}'$ back to $\mathbf{P}$.

(20 marks)

(b) In the diagram shown below, parallelogram $ABCD$ is transformed to $A'B'C'D'$ after undergoing a sequence of **three** simple transformations.



(i) Describe, in words, the three transformations that will transform parallelogram $ABCD$ to $A'B'C'D'$, and write down the corresponding transformation matrices.

(ii) Hence, derive the composite matrix $\mathbf{T}$ for the above sequence of transformations and verify that $\mathbf{T}$ successfully transforms parallelogram $ABCD$ to $A'B'C'D'$.

(10 marks)

******* END OF PAPER *******