SOLUTIONS

SINGAPORE POLYTECHNIC
2021/2022 Semester 1 Mid-Semester Test

No.	SOLUTION
1(a)	$\mathbf{A}_{13}=\mathbf{A}_{31}:\ b=2$ $\mathbf{A}_{12}=\mathbf{A}_{21}:\ 4a=b+2$ $4a=2+2=4$ $a=1$ $\mathbf{A}_{23}=\mathbf{A}_{32}:\ b+c=a-c$ $2c=a-b=1-2=-1$ $c=-\frac{1}{2}$
1(b)	$\mathbf{E}=3\mathbf{I}_2-2\mathbf{F}$ $\mathbf{F}=\frac{1}{2}\left(3\mathbf{I}_2-\mathbf{E}\right)=\frac{1}{2}\left(\begin{bmatrix}3&0\\0&3\end{bmatrix}-\begin{bmatrix}-1&6\\4&-3\end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix}4&-6\\-4&6\end{bmatrix}=\begin{bmatrix}2&-3\\-2&3\end{bmatrix}$
1(c)(i)	Not possible to evaluate, because $\mathbf{D}$ does not have the same number of rows and columns (or $\mathbf{D}$ is not a square matrix).
1(c)(ii)	$\mathbf{CD}^T=\begin{bmatrix}-2&-1&-6\\7&5&8\\8&2&9\end{bmatrix}\begin{bmatrix}3&-1\\-2&4\\1&2\end{bmatrix}=\begin{bmatrix}-10&-14\\19&29\\29&18\end{bmatrix}$
1(c)(iii)	$2\mathbf{E}-\mathbf{E}^T=\begin{bmatrix}-2&12\\8&-6\end{bmatrix}-\begin{bmatrix}-1&4\\6&-3\end{bmatrix}=\begin{bmatrix}-1&8\\2&-3\end{bmatrix}$
1(d)(i)	$\mathbf{C}-2\mathbf{I}_3=\begin{bmatrix}-2&-1&-6\\7&5&8\\8&2&9\end{bmatrix}-\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}=\begin{bmatrix}-4&-1&-6\\7&3&8\\8&2&7\end{bmatrix}$ $\mathbf{B}\left(\mathbf{C}-2\mathbf{I}_3\right)=\begin{bmatrix}1&-1&2\\3&4&-2\\-2&0&-1\end{bmatrix}\begin{bmatrix}-4&-1&-6\\7&3&8\\8&2&7\end{bmatrix}=\begin{bmatrix}5&0&0\\0&5&0\\0&0&5\end{bmatrix}$
1(d)(ii)	Given $\mathbf{GC}=\mathbf{C}+2\mathbf{G}$: $\mathbf{GC}-2\mathbf{G}=\mathbf{C}$ $\mathbf{G}\left(\mathbf{C}-2\mathbf{I}\right)=\mathbf{C}$ $\mathbf{G}\left(\mathbf{C}-2\mathbf{I}\right)\mathbf{B}=\mathbf{CB}$ $\mathbf{G}\left(5\mathbf{I}\right)=\mathbf{CB}$ $\mathbf{G}=\frac{1}{5}\mathbf{CB}=\frac{1}{5}\begin{bmatrix}-2&-1&-6\\7&5&8\\8&2&9\end{bmatrix}\begin{bmatrix}1&-1&2\\3&4&-2\\-2&0&-1\end{bmatrix}=\frac{1}{5}\begin{bmatrix}7&-2&4\\6&13&-4\\-4&0&3\end{bmatrix}$

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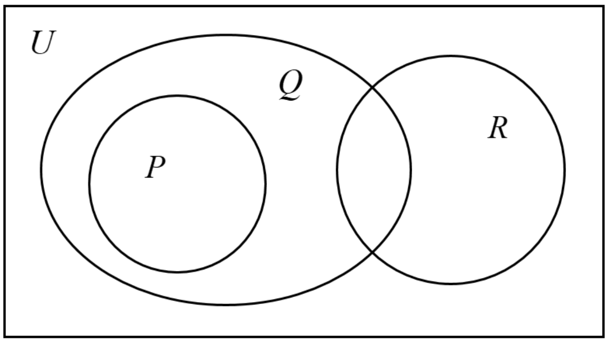
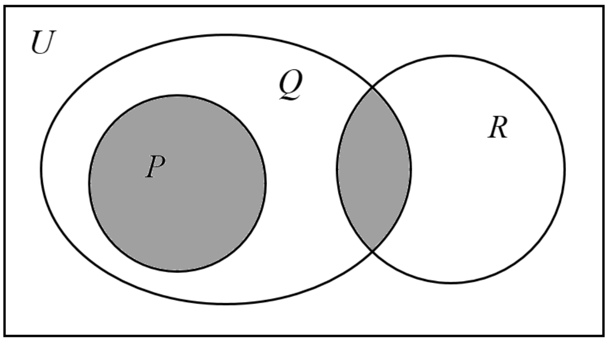
2(a) (i)	$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ $A = \{2, 4, 6, 8\}$ $B = \{1, 2, 3, 4\}$ $C = \{4, 8\}$
2(a) (ii)	$A - C = \{2, 6\}$ $B \cap \overline{C} = \{1, 2, 3\}$ $\overline{A \cup B} = \{5, 7\}$
2(b)	
2(c)	

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3(a)(i)

Integral part:		
2	627	
2	313	1
2	156	1
2	78	0
2	39	0
2	19	1
2	9	1
2	4	1
2	2	0
2	1	0
	0	1

Fractional part:		
2	0.45	
2	0.9	0
2	0.8	1
2	0.6	1
2	0.2	1
2	0.4	0
2	0.8	0
2	0.6	1
2	0.2	1
2	0.4	0
2	0.8	0
2	0.6 (rep)	1

$\therefore 627.45_{10} = 10011100\,11.01\overline{1100}_2$

3(a)(ii)

$9E4.A_{16} = (1001)(1110)(0100).(1010)_2 = 10011110\,0100.101_2$

3(b)

Smallest 12-bit binary number $= 000011111.111_2$

$11111.111_2 = 2^4 + 2^3 + 2^2 + 2^1 + 2^0 + 2^{-1} + 2^{-2} + 2^{-3} = 31.875_{10}$

3(c)

$189C_{16} = 1\times16^3 + 8\times16^2 + 9\times16^1 + 12\times16^0 = 6300_{10}$

After winning two prizes, the count will be: $6300 + 300\times2 = 6900$

Convert 6900_{10} to hexadecimal:

$6900 = 431(16) + 4$

$431 = 26(16) + 15$

$26 = 1(16) + 10$

$1 = 0(16) + 1$

Hence, $6900_{10} = 1AF4_{16}$

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4(a) (i)	$\mathbf{T}_1 = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$; $\mathbf{T}_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $\mathbf{T}_3 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
4(a) (ii)	$\mathbf{C} = \mathbf{T}_3\mathbf{T}_2\mathbf{T}_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$
4(a) (iii)	$\mathbf{P}' = \mathbf{CP} = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 & 1 \\ 2 & 1 & 4 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & -3 \\ 0 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
4(a) (iv)	$\mathbf{T}_1^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$; $\mathbf{T}_2^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $\mathbf{T}_3^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
4(a) (v)	$\mathbf{C}^{-1} = \mathbf{T}_1^{-1}\mathbf{T}_2^{-1}\mathbf{T}_3^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 2 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$
4(b) (i)	$\mathbf{T}_a$: Shearing in the *x*-direction by a factor of 1 $\mathbf{T}_b$: Rotation 90° clockwise about the origin $\mathbf{T}_c$: Translation 2 units upwards $\mathbf{T}_a = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $\mathbf{T}_b = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $\mathbf{T}_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$
4(b) (ii)	$\mathbf{T} = \mathbf{T}_c\mathbf{T}_b\mathbf{T}_a = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ $\mathbf{U}' = \mathbf{TU} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ (verified)