SINGAPORE POLYTECHNIC
2021/2022 SEMESTER TWO MID-SEMESTER TEST

Common Infocomm Technology Programme (CITP)
Diploma in Applied AI & Analytics (DAAA)
Diploma in Infocomm Security Management (DISM)
Diploma in Information Technology (DIT)
Diploma in Game Design & Development (DGDD)

MS0105 – Mathematics Time allowed: 1 hour 40 minutes
MS0151 – Mathematics for Games

Instructions to Candidates

1. The SP examination rules are to be complied with.
   Any candidate who cheats or attempts to cheat will face disciplinary action.
2. This paper consists of **4** printed pages (including the cover page).
   There are 4 questions (100 marks in total), and you are to answer all the questions.
3. Unless otherwise stated, all **non-exact** decimal answers should be rounded to **at least two** decimal places.
4. Except for sketches, graphs and diagrams, no solutions are to be written in pencil. Failure to comply may result in loss of marks.

Formula Sheet: Transformation Matrices

1. Reflection			3. Shearing	
a. about the y-axis	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		a. in the x-direction	$\begin{bmatrix} 1 & s_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
b. about the x-axis	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		b. in the y-direction	$\begin{bmatrix} 1 & 0 & 0 \\ s_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
c. about the line $y = x$	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		4. Rotation about the origin	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
2. Scaling relative to the origin	$\begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$		5. Translation	$\begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix}$

1. Given the following matrices:

$$\mathbf{A}=\begin{bmatrix}-1 & 2a+1 & -2\\ 5 & -3 & 2c\\ b & 3a+4b & 1\end{bmatrix}\qquad \mathbf{B}=\begin{bmatrix}2 & -1 & 1\\ -5 & 3 & 2\\ 4 & -3 & 1\end{bmatrix}\qquad \mathbf{C}=\begin{bmatrix}9 & -2 & -5\\ 13 & -2 & -9\\ 3 & 2 & 1\end{bmatrix}$$

$$\mathbf{D}=\begin{bmatrix}-5 & 1 & 3 & -2\\ 2 & -6 & -1 & 4\end{bmatrix}\qquad \mathbf{E}=\begin{bmatrix}3 & -1\\ 7 & 4\end{bmatrix}\qquad \mathbf{F}=\begin{bmatrix}3 & 2\\ 4 & -1\end{bmatrix}$$

(a) If **A** is a symmetric matrix, find the values of a, b and c.

(5 marks)

(b) Find matrix $\mathbf{X}$ such that $\mathbf{E}^T+3\mathbf{X}=2\mathbf{F}$.

(6 marks)

(c) Evaluate the following wherever possible.
State the reason(s) clearly if the expressions cannot be evaluated.

(i) **ED**

(ii) $\mathbf{F}^T-4\mathbf{I}_2$, where $\mathbf{I}_2$ is the 2×2 identity matrix.

(6 marks)

(d) (i) Evaluate $\mathbf{BC}$.

(ii) Hence, find matrix $\mathbf{Y}^{-1}$ such that $(\mathbf{C}+\mathbf{Y})(\mathbf{CY}-\mathbf{I}_3)=\mathbf{YCY}$, where $\mathbf{I}_3$ is the 3×3 identity matrix.

(13 marks)

2. (a) Let the universal set $U = \{x \in \mathbb{Z} \mid -4 \le x \le 8\}$ and define the following sets within U:

$$A = \{x \in \mathbb{N} \mid x \le 6\}$$

$$B = \{3x \mid x \in \mathbb{Z}, -2 \le x \le 2\}$$

$$C = \left\{x \,\middle|\, \frac{x}{2} \in \mathbb{Z},\ \frac{x}{4} \notin \mathbb{N}\right\}$$

(i) Rewrite sets U, A, B and C by listing.

(ii) Find $A \cap \overline{B}$ and $\overline{A} \cup C$.

(iii) Draw a Venn diagram showing the above sets U, A, B and C, indicating all the elements clearly.

(14 marks)

(b) Sets P and Q are defined as follows:

$$P = \{1, 2, 3, 4, 5\}$$

$$Q = \{4, 5, 6, 7, 8\}$$

Another set X is defined such that $X \subset \mathbb{Z}$, $X \cap \overline{P} = \varnothing$ and $X \cup (P - Q) = X$.

List down all possible sets of X.

(6 marks)

3. **Show your working clearly for this question. No marks will be awarded if the steps involved are not shown.**

(a) Convert the decimal number 718.65 to its binary and hexadecimal representations.

Express your answers in **exact** form, showing the recursion clearly for the fractional part, if any.

(12 marks)

(b) The decimal number 718.65 in part (a) is stored using 16 bits of storage space, where 12 bits are integral bits and 4 bits are fractional bits.

The number is stored in such a way that only the four most significant fractional bits are kept, and the remaining fractional bits are truncated.

What is the error (in decimal) caused by this truncation?

(4 marks)

(c) A base-4 register can display up to 2 fractional digits in the base-4 number system.

List down all numbers (in decimal) between 0.625_{10} and 0.875_{10} that can be exactly displayed in the base-4 register.

(4 marks)

4. **Solve this question using homogeneous coordinates.**

(a) Triangle $\mathbf{P}$ with vertices $(1,-1)$, $(2,3)$ and $(4,1)$ undergoes the following sequence of transformations:

$\mathbf{T}_1$: shearing in the y-direction by a factor of -1, followed by

$\mathbf{T}_2$: translation 2 units to the right and 3 units downwards, followed by

$\mathbf{T}_3$: scaling in the x-direction by a factor of 2 relative to the origin.

(i) Write down the transformation matrices $\mathbf{T}_1$, $\mathbf{T}_2$ and $\mathbf{T}_3$.

(ii) Compute the composite matrix $\mathbf{C}$ for the above sequence of transformations.

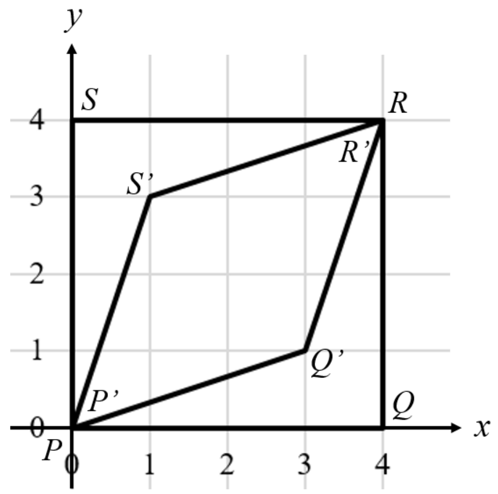
(iii) Find $\mathbf{P}'$, the image matrix of triangle $\mathbf{P}$ after undergoing the above sequence of transformations.

(iv) Write down the inverse transformation matrices $\mathbf{T}_1^{-1}$, $\mathbf{T}_2^{-1}$ and $\mathbf{T}_3^{-1}$.

(v) Compute the composite matrix $\mathbf{C}^{-1}$ that transforms $\mathbf{P}'$ back to $\mathbf{P}$.

(20 marks)

(b) In the diagram below, square $PQRS$ is transformed to rhombus $P'Q'R'S'$ through a sequence of **three** simple transformations.



(i) Describe, in words, the three transformations needed to transform square $PQRS$ to rhombus $P'Q'R'S'$, and write down the corresponding transformation matrices.

(ii) Hence, derive the composite matrix $\mathbf{T}$ for the above sequence of transformations and verify that $\mathbf{T}$ successfully transforms square $PQRS$ to rhombus $P'Q'R'S'$.

(10 marks)

******* END OF PAPER *******