SINGAPORE POLYTECHNIC
2023/2024 SEMESTER TWO EXAMINATION

Common Infocomm Technology Programme (DCITP)
Diploma in Applied AI & Analytics (DAAA)
Diploma in Infocomm Security Management (DISM)
Diploma in Information Technology (DIT)
Diploma in Media, Arts & Design (DMAD)

MS0105 – Mathematics Time allowed: 2 hours
MS0151 – Mathematics for Games

Instructions to Candidates

1. The SP examination rules are to be complied with.
Any candidate who cheats or attempts to cheat will face disciplinary action.

2. This paper consists of **7** printed pages (including the cover page and formula sheet).

3. This paper consists of three sections (100 marks in total):

   Section A: 5 multiple-choice questions (10 marks)
   Answer all questions behind the cover page of the answer booklet.

   Section B: 7 structured questions (50 marks)
   The total mark of the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

   Section C: 3 structured questions (40 marks)
   Answer all questions.

4. Unless otherwise stated, all **non-exact** decimal answers should be rounded to **at least two** decimal places.

5. Except for sketches, graphs and diagrams, no solutions are to be written in pencil. Failure to comply may result in loss of marks.

SECTION A (10 marks)

Answer ALL **FIVE** questions. Each question carries 2 marks.
Tick the choice of answer for each question in the box of the MCQ answer sheet provided in the answer booklet.

A1. If Matrix **A** is a $m \times n$ matrix and Matrix **B** is a $n \times n$ matrix where $m \neq n$, which of the following products will be a square matrix?

(a) $\mathbf{AB}$ (b) $\mathbf{BA}^{\mathrm{T}}$ (c) $\mathbf{AB}^2$ (d) $\mathbf{AA}^{\mathrm{T}}$

A2. Sets *A*, *B* and *C* are defined as follows:

- $A = \{3, 6, 9\}$
- $A \cap B = B$ and $|A - B| = 1$
- $B \subset C$ and $C \cup A = A$

Which of the following statements is ALWAYS true?

(a) $A - C = \varnothing$ (b) $C \subset \{2n+1 \mid n \in \mathbb{Z}\}$

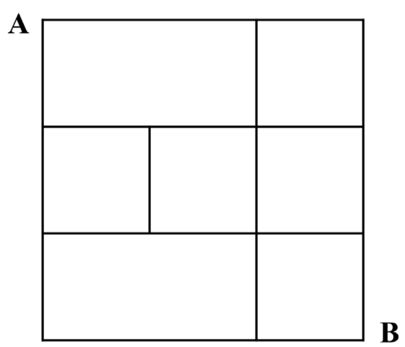
(c) $A \cap B \cap C = B$ (d) $A \cup B \cup C = C$

A3. If $A + B$ means A is the mother of B; $A - B$ means A is the brother of B; $A\%B$ means A is the father of B and $A \times B$ means A is the sister of B, which of the following shows that P is the maternal uncle (i.e. brother of one's mother) of Q?
(Note: M, N and S in the options below represent other family members of P and/or Q.)

(a) $Q - N + M \times P$ (b) $P + S \times N - Q$

(c) $P - M + N \times Q$ (d) $Q - S\%P$

A4. The figure below displays a map, and we are required to move from the upper-left corner A to the lower-right corner B. Given that we are restricted to move either rightwards or downwards, how many possible paths are there?



(a) 10 (b) 12 (c) 14 (d) 16

A5. A bag contains three coins. Two of the coins are fair while the third one is biased which lands on heads 70% of the time. A coin is randomly picked from the bag and flipped. If it lands on heads, what is the probability that it is the biased coin?

(a) 0.23 (b) 0.33 (c) 0.41 (d) 0.47

SECTION B (50 marks)

Each question carries 10 marks. The total mark of all the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

B1. Let $\mathbf{A}=\begin{bmatrix}3 & -2\\ 4 & 1\end{bmatrix}$, $\mathbf{B}=\begin{bmatrix}-2 & 3\\ 5 & -6\end{bmatrix}$, $\mathbf{C}=\begin{bmatrix}3 & 0 & 2\\ 2 & 0 & -2\\ 0 & 1 & 1\end{bmatrix}$ and $\mathbf{D}=\begin{bmatrix}1 & 1 & 0\\ -1 & \frac{3}{2} & 5\\ 1 & -\frac{3}{2} & 0\end{bmatrix}$

(a) Evaluate the following:
(i) $3\mathbf{A}+\mathbf{B}$
(ii) $\mathbf{BA}^{\mathrm{T}}$

(5 marks)

(b) Evaluate **CD**. Hence, find $\mathbf{C}^{-1}$.

(5 marks)

B2. **Solve this question using homogeneous coordinates.**

A triangle **P** with coordinates $(1,0)$, $(-1,1)$ and $(0,2)$ undergoes the following sequence of transformations:

$\mathbf{T_1}$: Shearing in the y-direction by a factor of -2, followed by

$\mathbf{T_2}$: Rotation 180° clockwise about the origin.

(a) Write down the transformation matrices $\mathbf{T_1}$ and $\mathbf{T_2}$. Hence, compute the composite matrix **C** for the above sequence of transformations.

(4 marks)

(b) Find $\mathbf{P}'$, the image matrix of triangle **P**, after undergoing the above sequence of transformations.

(2 marks)

(c) Write down the inverse transformation matrices $\mathbf{T}_1^{-1}$ and $\mathbf{T}_2^{-1}$. Hence, compute the composite matrix $\mathbf{C}^{-1}$ that transforms $\mathbf{P}'$ back to **P**.

(4 marks)

B3. **Show your working clearly for this question.**

(a) Convert 1100101001.101_2 to its hexadecimal and decimal representations.

(4 marks)

(b) Convert 264.425_{10} to its binary representation.
Express your answer in **exact** form, showing the recursion clearly for the fractional part, if any.

(6 marks)

B4. Let the universal set $U = \{x \in \mathbb{Z} \mid -4 \le x < 6\}$ and define the following sets within U :

$$A = \{x \in \mathbb{Z} \mid -3 < x < 4,\ x \neq 0\}$$

$$B = \left\{\frac{x^2}{2} \,\middle|\, x \in U\right\}$$

(a) Rewrite sets U, A and B by listing. (3 marks)

(b) Find $A \cap \overline{B}$ and $\overline{A \cup B}$. (4 marks)

(c) Draw a Venn diagram showing sets U, A and B, indicating all the elements clearly.

(3 marks)

B5. Let p, q and r be simple propositions.

(a) If the truth values of p, q and r are false, true and false respectively, determine the truth values of

(i) $(p \wedge \neg q) \Rightarrow r$

(ii) $(p \vee r) \Leftrightarrow q$

(4 marks)

(b) By constructing a truth table, determine whether the compound proposition $(\neg(p \wedge q)) \Leftrightarrow (\neg p \vee \neg q)$ is a tautology or contradiction.

(6 marks)

B6. (a) There are eight married couples, i.e., eight men and eight women. Find the number of ways they can stand in a line such that

(i) there are no restrictions;

(ii) the men are alternating and the couples are all standing together.

(4 marks)

(b) Find the number of integers between 150 and 800 inclusive that are

(i) divisible by 3.

(ii) divisible by 6.

(iii) divisible by 3 or 6.

(6 marks)

B7. A box contains four white and eight black balls of the same shape and size. Two of the white and six of the black balls have the number "3" printed on them. The remaining balls have the number "4" printed on them. Three balls are randomly drawn from the box without replacement. Find the probability that

(a) all balls drawn are white; (2 marks)

(b) at least one black ball is drawn; (2 marks)

(c) the sum of the numbers on the balls drawn is exactly 10; (3 marks)

(d) the sum of the number on the balls drawn is exactly 10 and at least one black ball is drawn. (Hint: these two events are *not* independent events.) (3 marks)

<u>SECTION C</u> (40 marks)

Answer ALL **THREE** questions.

C1. (a) You are tasked to design a recommendation system in a streaming platform that suggests content to users based on the following criteria:

- The content belongs to user's preferred genre OR
- The content has a high user rating AND is currently available for streaming.

The Boolean variables are defined as follows:

Inputs	x = genre (1 = preferred, 0 = not preferred) y = user rating (1 = high, 0 = low) z = availability (1 = available, 0 = not available)
Output	$f(x, y, z)$ = content label (1 = recommended, 0 = not recommended)

Construct a truth table for the design of the recommendation system and obtain its **product-of-sums** (POS) expression. *Do not simplify the expression.*

(6 marks)

(b) The *equivalence gate* is a Boolean operator denoted by '$\odot$' and is defined as $p \odot q = p \cdot q + \overline{p} \cdot \overline{q}$.

(i) Based on the information provided, construct a truth table for $p \odot q$.

(ii) Using Laws of Boolean Algebra, simplify $p \odot (p \odot q) \odot q$.

(<u>Note:</u> No marks will be awarded for using a truth table to simplify.)

(7 marks)

C2. (a) The expression $2x^2 + 5y^2 + 3z^2 + 2xy - 2xz + 8yz$ can be represented in matrix form as $\mathbf{PQP}^T$, where $\mathbf{P} = \begin{bmatrix} x & y & z \end{bmatrix}$ and $\mathbf{Q}$ is a symmetric matrix. Find $\mathbf{Q}$.

(7 marks)

(b) Four boys (Jack, Zack, Nick, and Victor) and two girls (Emma and Amanda) played escape room, and all six of them managed to escape the room eventually (one person at a time). The table below provides some clues about the sequence of their escape:

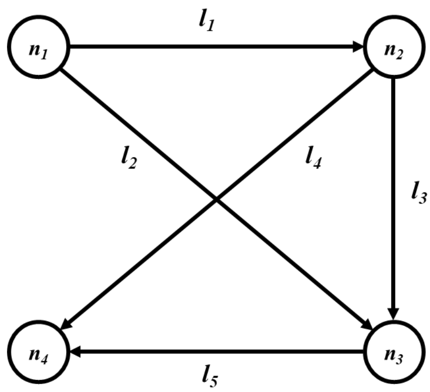
No.	Clues
1	Twice as many boys as girls managed to escape before Jack did.
2	An equal number of boys and girls escaped after Jack did.
3	One additional boy than girls escaped before Zack did.
4	Amanda escaped before Emma did.
5	The genders of those who escaped first and last are different.
6	Nick escaped before Zack.
7	The individuals who escaped right before and right after Victor are both boys.

Based on the clues provided, deduce the sequence of the escape of the six people. Justify your answer. (6 marks)

C3. (a) Xavier and Yasir play a game to take turns picking a ball at random from an urn containing five green balls and seven black balls. The balls are picked out of the urn without replacement and who first picks a green ball is the winner. Xavier is the first person to start. What is the probability that Yasir will win the game?

(7 marks)

(b) A communication network (refer to the diagram below) has four nodes n_1, n_2, n_3 and n_4 and five directed links between the nodes: $l_1 = (n_1, n_2)$, $l_2 = (n_1, n_3)$, $l_3 = (n_2, n_3)$, $l_4 = (n_2, n_4)$ and $l_5 = (n_3, n_4)$. A message is to be sent from the source node n_1 to the destination node n_4 through all possible paths simultaneously, with each path comprising of a series of independent links that connect n_1 to n_4. Let p_i represent the probability that the corresponding link l_i is functioning, where $i = 1, 2, \ldots, 5$. A path from node n_1 to node n_4 is only functioning if each of its links is functioning, and the message is only successfully sent through the network if at least one path is functioning.



(i) Derive the expression of the probability that the message is successfully sent through the network. Express your answer in terms of p_i where $i = 1, 2, \ldots, 5$.

(ii) Hence, calculate the probability that the message is successfully sent through the network when $p_i = 0.65$ for all i.

(7 marks)

***** **END OF PAPER** *****

Formula Sheet

Transformation Matrices

Reflection	about the y-axis	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	about the x-axis	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	about the line $y = x$	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Scaling	relative to the origin	$\begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Shearing	in the x-direction	$\begin{bmatrix} 1 & s_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	in the y-direction	$\begin{bmatrix} 1 & 0 & 0 \\ s_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Rotation	about the origin	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Translation		$\begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix}$

Boolean Algebra

Name	**Identity**
Commutative Laws	$x \bullet y = y \bullet x$ $x + y = y + x$
Associative Laws	$x \bullet (y \bullet z) = (x \bullet y) \bullet z = x \bullet y \bullet z$ $x + (y+z) = (x+y) + z = x + y + z$
Distributive Laws	$x \bullet (y+z) = (x \bullet y) + (x \bullet z)$ $x + (y \bullet z) = (x+y) \bullet (x+z)$
Identity Laws	$x \bullet 1 = x$ $x + 0 = x$
Complement Laws	$x \bullet \overline{x} = 0$ $x + \overline{x} = 1$
Involution Law	$\overline{\overline{x}} = x$
Idempotent Laws	$x \bullet x = x$ $x + x = x$
Bound Laws	$x \bullet 0 = 0$ $x + 1 = 1$
De Morgan's Laws	$\overline{x \bullet y} = \overline{x} + \overline{y}$ $\overline{x + y} = \overline{x} \bullet \overline{y}$
Absorption Laws	$x \bullet (x + y) = x$ $x + (x \bullet y) = x$ $x \bullet (\overline{x} + y) = x \bullet y$ $x + (\overline{x} \bullet y) = x + y$

Probability Rules

Addition	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Subtraction	$P(\overline{A}) = 1 - P(A)$
Multiplication	$P(A \cap B) = P(A)P(B)$ if A and B are independent events
Conditional Probability	$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$