SINGAPORE POLYTECHNIC
2020/2021 SEMESTER TWO EXAMINATION

Common Infocomm Technology Programme (CITP)
Diploma in Information Technology (DIT)
Diploma in Game Design & Development (DGDD)

MS0105 – Mathematics Time allowed: 2 hours
MS0151 – Mathematics for Games

Instructions to Candidates

1. The SP examination rules are to be complied with.
Any candidate who cheats or attempts to cheat will face disciplinary action.

2. This paper consists of **8** printed pages (including the cover page and formula sheet).

3. This paper consists of three sections (100 marks in total):

Section A: 5 multiple-choice questions (10 marks)
Answer all questions behind the cover page of the answer booklet.

Section B: 7 structured questions (50 marks)
The total mark of the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

Section C: 3 structured questions (40 marks)
Answer all questions.

4. Unless otherwise stated, all **non-exact** decimal answers should be rounded to **at least two** decimal places.

5. Except for sketches, graphs and diagrams, no solutions are to be written in pencil. Failure to do so will result in loss of marks.

SECTION A (10 marks)

Answer ALL **FIVE** questions. Each question carries 2 marks.
Tick the choice of answer for each question in the box of the MCQ answer sheet provided in the answer booklet.

A1. If $\mathbf{A}$ is an invertible square matrix, which of the following statements is/are **always** true?

I. $\left(\mathbf{A}^2\right)^{-1} = \left(\mathbf{A}^{-1}\right)^2$

II. $\left(\mathbf{A}^T\right)^{-1} = \left(\mathbf{A}^{-1}\right)^T$

(a) I only
(b) II only
(c) Both I and II
(d) Neither I nor II

A2. How many subsets does the set $\{a, b, c\}$ have in total?

(a) 5 (b) 6 (c) 7 (d) 8

A3. Which of the following statements is the **contrapositive** of the following proposition?

I will use my umbrella if it is raining heavily.

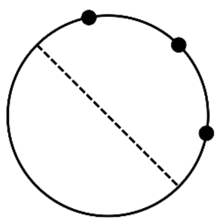
(a) If I use my umbrella, then it is raining heavily.
(b) If it is raining heavily, then I will use my umbrella.
(c) If I do not use my umbrella, then it is not raining heavily.
(d) If it is not raining heavily, then I will not use my umbrella.

A4. How many numbers can be formed using the digits 1, 2, 3, 4, 5, 6 if repetition is **not** allowed and each number formed must be less than 600?

(a) 100 (b) 136 (c) 156 (d) 180

A5. Three points are randomly chosen on the circumference of a circle. What is the probability that all the three points lie on a common semicircle?

(Hint: The figure on the right shows an example where all the three points lie on a common semicircle.)



(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

SECTION B (50 marks)

Each question carries 10 marks. The total mark of all the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

B1. Let $\mathbf{A}=\begin{bmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \\ -3 & 4 & -1 \end{bmatrix}$ and $\mathbf{B}=\begin{bmatrix} 7 & -3 & -1 \\ 9 & -5 & 1 \\ 15 & -11 & -1 \end{bmatrix}$.

(a) Evaluate $3\mathbf{A}+2\mathbf{B}^T$.

(5 marks)

(b) Evaluate $\mathbf{AB}$. Hence, find $\mathbf{A}^{-1}$.

(5 marks)

B2. **Solve this question using homogeneous coordinates.**

A line segment $\mathbf{P}$ with coordinates $(4,-1)$ and $(-3,2)$ undergoes the following sequence of transformations:

$\mathbf{T}_1$: reflection about the y-axis, followed by

$\mathbf{T}_2$: translation 3 units to the left and 4 units upwards.

(a) Write down the transformation matrices $\mathbf{T}_1$ and $\mathbf{T}_2$. Hence, compute the composite matrix $\mathbf{C}$ for the above sequence of transformations.

(4 marks)

(b) Find $\mathbf{P}'$, the image matrix of line segment $\mathbf{P}$ after undergoing the above sequence of transformations.

(2 marks)

(c) Write down the inverse transformation matrices $\mathbf{T}_1^{-1}$ and $\mathbf{T}_2^{-1}$. Hence, compute the composite matrix $\mathbf{C}^{-1}$ that transforms $\mathbf{P}'$ back to $\mathbf{P}$.

(4 marks)

B3. **Show your working clearly for this question.**

(a) Convert $5C.2A_{16}$ to its decimal and binary representation.

(5 marks)

(b) Convert 26958.15625_{10} to its hexadecimal representation.

(5 marks)

B4. Let the universal set $U = \{x \in \mathbb{Z} \mid 0 \le x \le 6\}$ and define the following sets within U :

$$A = \{x \in \mathbb{N} \mid x \le 4\}$$

$$B = \{(x-1)^2 \mid x \in A\}$$

(a) Rewrite sets U, A and B by listing.

(3 marks)

(b) Find $A - B$ and $A \cup \overline{B}$.

(4 marks)

(c) Draw a Venn diagram showing sets U, A and B, indicating all the elements clearly.

(3 marks)

B5. Propositions p, q and r are defined as:

p : You work hard.

q : You get distracted.

r : You can finish the job.

(a) Write down the compound proposition $(p \wedge \neg q) \Rightarrow r$ in words.

(2 marks)

(b) Express the following statement in logical notation:

You do not work hard and you get distracted, but you can finish the job.

(2 marks)

(c) By constructing a truth table, determine whether the compound proposition $(p \wedge r) \Rightarrow (q \vee r)$ is a tautology or a contradiction.

(6 marks)

B6. Find the number of integers between 1 and 5000 inclusive that are

(a) divisible by 3 and 5.

(2 marks)

(b) divisible by 3 or 5.

(2 marks)

(c) not divisible by 5 or 10.

(3 marks)

(d) divisible by 5 and 10, but not 12.

(3 marks)

B7. There are 200 tubs of ice cream in a large freezer. Each tub contains either the chocolate ice cream, strawberry ice cream or vanilla ice cream. In addition, each tub also contains either the almond topping or hazelnut topping. The table below shows the total number of tubs in the freezer, categorised by the ice cream flavour and topping.

	Chocolate ice cream	Strawberry ice cream	Vanilla ice cream
Almond topping	44	28	26
Hazelnut topping	34	36	32

What is the probability that a randomly selected tub

(a) contains chocolate ice cream?

(2 marks)

(b) contains the almond topping?

(2 marks)

(c) contains strawberry ice cream or the hazelnut topping?

(3 marks)

(d) contains vanilla ice cream, given that it contains the hazelnut topping?

(3 marks)

<u>SECTION C</u> (40 marks)

Answer ALL **THREE** questions.

C1. A student in Singapore Polytechnic designs a mask vending machine that can dispense two different colours of reusable masks – red mask and blue mask.

The mask vending machine has three input components, which are defined as follows:

x : identity card (IC) scanner that verifies the user's identity

y : red button for dispensing the red mask

z : blue button for dispensing the blue mask

Use the following convention to answer this question.

	'0'	'1'
IC scanner	User's identity is not verified	User's identity is verified
Red button	Button is not pressed	Button is pressed
Blue button	Button is not pressed	Button is pressed
Dispenser	Mask is not dispensed	Mask is dispensed

The mask vending machine is designed such that it will only dispense a mask when the user's identity is verified **AND exactly one** of the two buttons is pressed.

(a) Construct a truth table for the mask vending machine system, and obtain its **sum-of-products** expression. **Do not simplify** the expression obtained.

(6 marks)

(b) Write down the **complement** of the expression obtained in part (a), and simplify the resulting expression to its **simplest form**.

(6 marks)

C2. How many different 10-bit binary numbers can be formed using six 1's and four 0's if

(a) there are no restrictions?

(2 marks)

(b) there are exactly three 1's in the first five bits of the binary number?

(3 marks)

(c) the first and last bits of the binary number must be the same as each other?

(4 marks)

(d) the decimal equivalent of the binary number must be greater than 540_{10} ?

(4 marks)

C3. **Solve this question <u>without</u> using homogeneous coordinates.**

> Hints:
>
> $$\sin\alpha = \frac{\text{opp}}{\text{hyp}}\ ;\ \cos\alpha = \frac{\text{adj}}{\text{hyp}}\ ;\ \tan\alpha = \frac{\text{opp}}{\text{adj}}$$
> $$\sin(-\alpha) = -\sin\alpha$$
> $$\cos(-\alpha) = \cos\alpha$$
> $$\sin(90^\circ - \alpha) = \cos\alpha$$
> $$\cos(90^\circ - \alpha) = \sin\alpha$$

A sequence of three transformations is defined as follows:

$\mathbf{T}_1$: rotation by angle α clockwise about the origin, followed by

$\mathbf{T}_2$: scaling relative to the origin by a factor of p in the *x*-direction and a factor of $\frac{1}{p}$ in the *y*-direction, followed by

$\mathbf{T}_3$: rotation by angle $90^\circ - \alpha$ anticlockwise about the origin.

It is given that α is an acute angle ($0^\circ < \alpha < 90^\circ$) and that $\tan\alpha = \frac{1}{p}$.

(a) Show that the composition of the above sequence of transformations is equivalent to shearing in the *y*-direction by a factor of $p - \frac{1}{p}$.

(12 marks)

(b) Hence, find the shear factor in the *y*-direction if $\alpha = 60^\circ$.

(3 marks)

******* END OF PAPER *******

Formula Sheet

Transformation Matrices

Reflection	about the y-axis	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	about the x-axis	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	about the line $y = x$	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Scaling	relative to the origin	$\begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Shearing	in the x-direction	$\begin{bmatrix} 1 & s_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	in the y-direction	$\begin{bmatrix} 1 & 0 & 0 \\ s_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Rotation	about the origin	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Translation		$\begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix}$

Boolean Algebra

Name	**Identity**
Commutative Laws	$x \bullet y = y \bullet x$ $x + y = y + x$
Associative Laws	$x \bullet (y \bullet z) = (x \bullet y) \bullet z = x \bullet y \bullet z$ $x + (y + z) = (x + y) + z = x + y + z$
Distributive Laws	$x \bullet (y + z) = (x \bullet y) + (x \bullet z)$ $x + (y \bullet z) = (x + y) \bullet (x + z)$
Identity Laws	$x \bullet 1 = x$ $x + 0 = x$
Complement Laws	$x \bullet \overline{x} = 0$ $x + \overline{x} = 1$
Involution Law	$\overline{\overline{x}} = x$
Idempotent Laws	$x \bullet x = x$ $x + x = x$
Bound Laws	$x \bullet 0 = 0$ $x + 1 = 1$
De Morgan's Laws	$\overline{x \bullet y} = \overline{x} + \overline{y}$ $\overline{x + y} = \overline{x} \bullet \overline{y}$
Absorption Laws	$x \bullet (x + y) = x$ $x + (x \bullet y) = x$ $x \bullet (\overline{x} + y) = x \bullet y$ $x + (\overline{x} \bullet y) = x + y$

Probability Rules

Addition	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Subtraction	$P(\overline{A}) = 1 - P(A)$
Multiplication	$P(A \cap B) = P(A)P(B)$ if A and B are independent events
Conditional Probability	$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$