SINGAPORE POLYTECHNIC
2019/2020 SEMESTER TWO EXAMINATION

Common Infocomm Technology Programme (CITP)
Diploma in Game Design & Development (DGDD)

MS0105 – Mathematics Time allowed: 2 hours
MS0151 – Mathematics for Games

Instructions to Candidates

1. The SP examination rules are to be complied with.
Any candidate who cheats or attempts to cheat will face disciplinary action.

2. This paper consists of 8 printed pages (including the cover page and formula sheet).

3. This paper consists of three sections (100 marks in total):

Section A: 5 multiple-choice questions (10 marks)
Answer all questions behind the cover page of the answer booklet.

Section B: 7 structured questions (50 marks)
The total mark of the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

Section C: 3 structured questions (40 marks)
Answer all questions.

4. Unless otherwise stated, all **non-exact** decimal answers should be rounded to **at least two** decimal places.

5. Except for sketches, graphs and diagrams, no solutions are to be written in pencil. Failure to do so will result in loss of marks.

SECTION A (10 marks)

Answer ALL **FIVE** questions. Each question carries 2 marks.
Tick the choice of answer for each question in the box of the MCQ answer sheet provided in the answer booklet.

A1. The next hexadecimal number after 5FF is

(a) 6FF (b) 5FF1 (c) 600 (d) 6F0

A2. Which of the following statements is/are true of $\mathbf{T}=\begin{bmatrix}-1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{bmatrix}$?

I. $\mathbf{T}$ represents rotation by 180° about the origin.

II. $\mathbf{T}^n=\mathbf{I}_3$, where n is even.

III. The inverse of $\mathbf{T}$ is $\begin{bmatrix}0 & -1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$.

(a) I only (b) II only (c) I and II only (d) I, II and III

A3. Which of the following pairs of sets A and B fulfills the condition that $A-B=\varnothing$?

(a) $A=\{x|x\in\mathbb{Z}\}$ and $B=\{x|x\in\mathbb{N}\}$

(b) $A=\{x|x\in\mathbb{R}\}$ and $B=\{x|x\in\mathbb{Q}\}$

(c) $A=\{x|x \text{ is even integer}\}$ and $B=\{2x|x\in\mathbb{N}\}$

(d) $A=\{x^2|x\in\mathbb{N}\}$ and $B=\{x|x\in\mathbb{N}\}$

A4. Given the propositions p: $2.5\in\mathbb{Z}$ and q: $0\in\mathbb{N}$, which of the following is a TRUE compound proposition?

(a) $\neg p\vee q$ (b) $\neg p\Rightarrow q$ (c) $p\wedge\neg q$ (d) $\neg p\Leftrightarrow q$

A5. A survey was administered to find out SP students' opinions of Food Courts 3 and 4. Results showed that 80% of the students like Food Court 3, 60% like Food Court 4 and x% like both food courts. What are the minimum and maximum possible values of x?

(Note: Liking Food Court 3 or 4 includes all those who indicated that they like Food Court 3 or 4 respectively, regardless of whether they like the other food court or not.)

(a) Minimum = 40 and maximum = 60

(b) Minimum = 40 and maximum = 80

(c) Minimum = 60 and maximum = 80

(d) Minimum = 20 and maximum = 60

SECTION B (50 marks)

Each question carries 10 marks. The total mark of the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

B1. Let $\mathbf{A} = \begin{bmatrix} 0 & -3 \\ -2 & 1 \end{bmatrix}$.

(a) Find $\mathbf{A}^2 - \mathbf{A}$.

(4 marks)

(b) Solve for the value of n, if it is given that $\mathbf{A}^2 - \mathbf{A} = n\mathbf{I}_2$.

(1 mark)

(c) Hence, find $\mathbf{A}^{-1}$.

(5 marks)

B2. **Solve this question using homogeneous coordinates.**

A point at coordinates $(-2, 3)$ undergoes the following sequence of transformations:

$\mathbf{T}_1$: translation 2 units to the right and 3 units downwards, followed by

$\mathbf{T}_2$: shearing in the x-direction by a factor of –1.

(a) Write down matrix **P** which represents the object in homogeneous coordinates.

(1 mark)

(b) Find the composite matrix $\mathbf{C}$ for the above sequence of transformations.

(4 marks)

(c) Find the image matrix after undergoing the sequence of transformations.

(1 mark)

(d) Find $\mathbf{C}^{-1}$, the inverse of the composite matrix.

(4 marks)

B3. **Show your working clearly for this question.**

(a) Convert $FAB.9_{16}$ to its decimal and binary representation.

(5 marks)

(b) Convert 67.875_{10} to its binary representation.

(5 marks)

B4. Let the universal set $U = \{x \mid x \in \mathbb{Z}, -2 \le x \le 5\}$ and define the following sets within U :

$$A = \{x \mid x \text{ is odd}, x > 0\}$$

$$B = \{x \mid x \in \mathbb{N}, x \le 3\}$$

(a) Rewrite sets *U*, *A* and *B* by listing.

(3 marks)

(b) Find the following.

(i) $\overline{B}$

(ii) $A - B$

(iii) $A \cup B$. Hence, state the value of $|A \cup B|$.

(7 marks)

B5. (a) Given that the truth values for p, q and r are T, T and F respectively, determine the truth values of

(i) $\neg(p \vee q)$

(ii) $(p \wedge q) \Rightarrow r$

(5 marks)

(b) Simplify the Boolean expression $\overline{\overline{x} + y + xy}$.

(5 marks)

B6. A small carpark contains 6 parking lots arranged in a straight row. There are 2 distinct Toyota cars and 4 distinct Honda cars in the carpark, and each car will occupy one of the parking lots.

Find the number of ways the 6 cars can be parked if

(a) there are no restrictions.

(2 marks)

(b) the first parking lot must be occupied by a Toyota car.

(2 marks)

(c) the first three parking lots must be occupied by Honda cars.

(3 marks)

(d) all the Toyota cars must be parked together beside each other.

(3 marks)

B7. Brian and David are taking their Mathematics examination in Singapore Polytechnic. Brian answers an average of 8 questions correctly out of 11, and David answers an average of 8 questions correctly out of 13. Both of them attempt one same question independently.

What is the probability that

(a) none of them answers the question correctly?

(2 marks)

(b) only one of them answers the question correctly?

(3 marks)

(c) both answer the question correctly?

(2 marks)

(d) at least one of them answer the question correctly?

(3 marks)

SECTION C (40 marks)

Answer ALL **THREE** questions.

C1. (a) A tautology is a propositional formula that is always TRUE for all possible truth values of its propositional variables. Given propositions *p* and *q*, use a truth table to prove that $(p \Rightarrow q) \vee (q \Rightarrow p)$ is a tautology.

(4 marks)

(b) A proposition is given as "If $x^2 = 9$, then $x = 3$".

(i) Write down its converse, inverse and contrapositive in words.

(ii) State the respective truth values of the original proposition and its converse, inverse and contrapositive.

(7 marks)

C2. (a) Write down the dual of the following Boolean expression.

$$(x+y)(\overline{x}+z)(y+z)=(x+y)(\overline{x}+z)$$

Verify the dual, stating clearly the laws used.

(6 marks)

(b) A committee of three people, Albert, Ben and Carol, vote for a proposal. Each person can vote either a *Yes* or a *No*. The proposal is passed if

- Albert (the chairman) votes a *Yes*, regardless of what the others vote; OR
- both Ben and Carol vote *Yes*.

Otherwise, the proposal will be rejected.

Let a *Yes* vote be represented by 1 and a *No* vote be represented by 0. Let also a passed proposal be represented by 1 and a rejected proposal be represented by 0.

(i) Construct a truth table to depict the above scenario of voting. Define the Boolean variables that represent each person's vote.

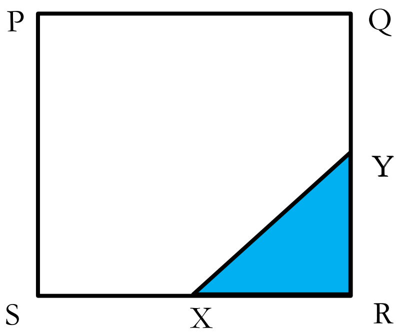
(ii) Use the **product-of-sums** method to derive the Boolean function that represents the above scenario of voting, and express your answer in its **simplest form**.

(10 marks)

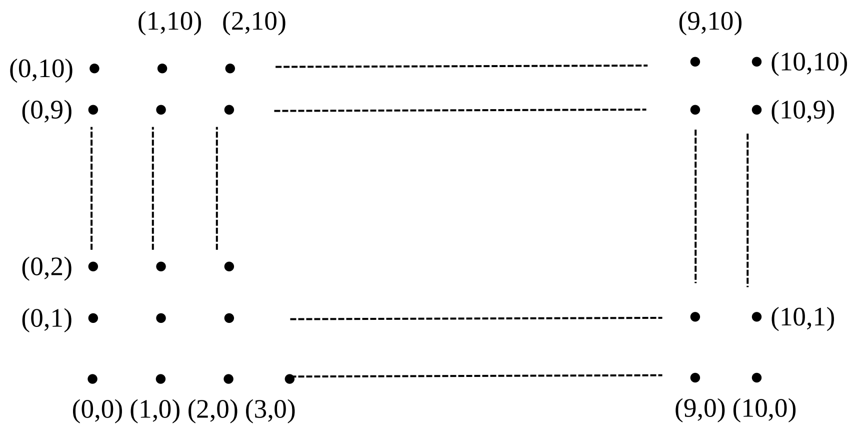
C3. **For this question, show your working or reasoning clearly.**

(a) PQRS is a square. Y is the midpoint of QR and X is the midpoint of SR. Suppose that a point is selected randomly inside the square, what is the probability that it lies inside the triangle XYR?

(4 marks)



(b) A wireless sensor grid consists of $11 \times 11 = 121$ sensor nodes. The nodes are located at points (i, j) of the plane such that $i \in \{0,1,2,...,10\}$ and $j \in \{0,1,2,...,10\}$, as shown in the diagram below. A message can only be sent from one sensor to a neighboring sensor that is located above or to the right of it. For example, the node $(4,1)$ can only send a message to $(4,2)$ or $(5,1)$.



(i) One of the paths of sending a message from $(0,0)$ to $(1,2)$ is "right, up, up". List the other two possible paths in a similar manner.

(ii) Without listing, determine the number of possible paths there are to send a message from $(0,0)$ to $(10,10)$.

(iii) Hence, determine the number of possible paths there are to send a message from $(0,0)$ to (p,q). Express your answer in terms of p and q.

(9 marks)

***** END OF PAPER *****

Formula Sheet

Transformation Matrices

Reflection	about the y-axis	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	about the x-axis	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	about the line $y = x$	$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Scaling	relative to the origin	$\begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Shearing	in the x-direction	$\begin{bmatrix} 1 & s_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
	in the y-direction	$\begin{bmatrix} 1 & 0 & 0 \\ s_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Rotation	about the origin	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Translation		$\begin{bmatrix} 1 & 0 & d_x \\ 0 & 1 & d_y \\ 0 & 0 & 1 \end{bmatrix}$

Boolean Algebra

Name	**Identity**
Commutative Laws	$x \bullet y = y \bullet x$ $x + y = y + x$
Associative Laws	$x \bullet (y \bullet z) = (x \bullet y) \bullet z = x \bullet y \bullet z$ $x + (y+z) = (x+y) + z = x + y + z$
Distributive Laws	$x \bullet (y+z) = (x \bullet y) + (x \bullet z)$ $x + (y \bullet z) = (x+y) \bullet (x+z)$
Identity Laws	$x \bullet 1 = x$ $x + 0 = x$
Complement Laws	$x \bullet \bar{x} = 0$ $x + \bar{x} = 1$
Involution Law	$\bar{\bar{x}} = x$
Idempotent Laws	$x \bullet x = x$ $x + x = x$
Bound Laws	$x \bullet 0 = 0$ $x + 1 = 1$
De Morgan's Laws	$\overline{x \bullet y} = \bar{x} + \bar{y}$ $\overline{x + y} = \bar{x} \bullet \bar{y}$
Absorption Laws	$x \bullet (x + y) = x$ $x + (x \bullet y) = x$ $x \bullet (\bar{x} + y) = x \bullet y$ $x + (\bar{x} \bullet y) = x + y$

Probability Rules

Addition	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Subtraction	$P(\bar{A}) = 1 - P(A)$
Multiplication	$P(A \cap B) = P(A)P(B)$ if A and B are independent events
Conditional Probability	$P(A\|B) = \dfrac{P(A \cap B)}{P(B)}$