Introducing Algorithm Performance The "Maximum Subsequence Sum Problem"

KV for CSE 330, Jan 2020

Based on Weiss, DSAAC++, Chapter 2

These notes do not strictly follow the order of presentation in the textbook ...

Three algorithms are given and implemented. Here are runs for Algorithm1, Algorithm2, and Algorithm4. We will focus on Alg1 and Alg4 ...

```
Vector size: 10
vector: 21 -45 12 -45 -6 29 42 25 -9 -23
max_sub_seq_alg1: 96(220 additions)
max_sub_seq_alg2: 96(55 additions)
max_sub_seq_alg4: 96(10 additions)
Another run? [y/n] y
Vector size: 25
vector: 21 -45 12 -45 -6 29 42 25 -9 -23 1 35 4 -30 12 1 -39 13 -17 24 47 -18 -2
8 23 27 32 9 25 38 22 -15 -47 -43 4 39
max_sub_seq_alg1: 245(7770 additions)
max_sub_seq_alg2: 245(630 additions)
max_sub_seq_alg4: 245(35 additions)
Another run? [y/n] y
Vector size: 100
        cor: 21 -45 12 -45 -6 29 42 25 -9 -23 1 35 4 -30 12 1 -39 13 -17 : 27 32 9 25 38 22 -15 -47 -43 4 39 50 22 45 24 42 -36 1 -11 20 -49 31 15 42 -43 50 5 -34 7 15 6 -39 1 49 -7 43 4 38 24 41 20 -9 42 7 -13 -20 45 41 27 29 6 -35 33 5 -32 36 13 7 40 40 22 39 -32 35 26 -33 5 24 3 27 29 -27 44 36 44 -15 -23 18 19 -23 -23 34 30 : 50 7 15 2 36 -7 14
max_sub_seq_alg1: 1456(419220 additions)
max_sub_seq_alg2: 1456(9180 additions)
max_sub_seq_alg4: 1456(135 additions)
Another run? [y/n] n
Press any key to continue . . .
```

	#Adds	#Adds	#Adds
Vector Size	Alg1	Alg2	Alg3
10	220	55	10
25	7770	630	35
100	419220	9180	135

```
// adopted from Weiss, DSAAC++, p. 61, Algorithm1
int max_subseq_sum_alg1(const vector<int>& vec, int& ops)
     int maxSum = 0;
     for (int i = 0; i < vec.size(); i++)</pre>
          for(int j = i; j < vec.size(); j++)</pre>
               int thisSum = 0;
               for (int k = i; k <= j; k++)
                     thisSum += vec[k];
               if (thisSum > maxSum)
                    maxSum = thisSum;
     return maxSum;
}
// adopted from Weiss, DSAAC++, p. 62, Algorithm2
int max_subseq_sum_alg2(const vector<int>& vec, int& ops)
{
     // see book
```

We are skipping Algorithm3 ...

```
// adopted from Weiss, DSAAC++, p. 66, Algorithm4
int max_subseq_sum_alg4(const vector<int>& vec, int& ops)
{
    int maxSum = 0;
    int thisSum = 0;
    ops = 0;

    for (int i = 0; i < vec.size(); i++)
    {
        thisSum += vec[i];
        ops += 1;

        if (thisSum > maxSum)
            maxSum = thisSum;
        else if (thisSum < 0)
            thisSum = 0;
    }
    return maxSum; // WHY DOES THIS WORK????
}</pre>
```

All three algorithms solve the problem, i.e., produce the correct solution.

The algorithms solve the problem at different cost:

Cost as:

- # of operations (e.g., additions)
- Run time (proportional to # ops)
- memory space

Algorithm performance is measured <u>not</u> in terms of exact numbers, but in terms of the order of magnitude of the performance trend when problems get biggger.

Size of the problem: vector of size 10 – small problem vector of size 100 – larger problem

vector of size 100K - very large problem

etc.

For the order of magnitude performances of Algorithms1,2,4, we write:

Algorithm1: cost of run time is O(n³), n being the size of problem ("cubic")

With increasingly larger problems (e.g., ones involving an increasingly large data set), the run time of the algorithm follows a cubic trend relative to problem size n.

Algorithm2: cost is O(n2) ("quadratic")

With increasingly larger problems (size = n), the run time of the algorithm grows following a quadratic trend.

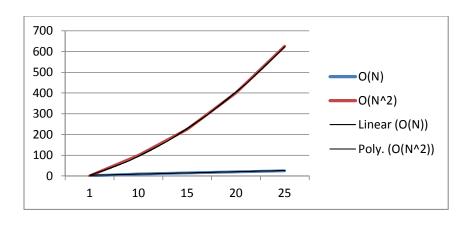
Algorithm4: cost is O(n) ("linear")

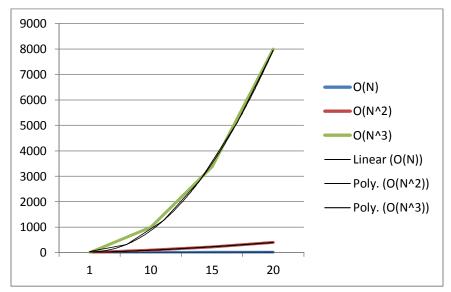
With increasingly larger problems (size = n), the run time of the algorithm grows following a linear trend.

SMALLER is BETTER ("more efficient"), and "linear" is a very desirable trend.

Is there better than LINEAR? – YES Is there worse than CUBIC? – YES

Graphically





Formally:

(a preview of CSE 431 Algorithm Analysis)

Let the performance of your program follow some function T(N), where n is the size of the problem.

Then $T(N) = O(N^2)$ means that there are positive constants c and k such that $0 \le T(N) \le cN^2$ for all $n \ge k$.

General:

T(N) = O(g(N)) means that there are positive constants c and k such that $0 \le T(N) \le cg(N)$ for all $n \ge k$.