Notes from Office Hour 1: 10:40am

7.1 #39 $x = 3t + 4$

$y = 5t - 2$

Eliminate the parameter

line
circle
ellipse
parabola
⋮

Solve for t : $\frac{x-4}{3} = t$

plug in t to y : $y = 5\left(\frac{x-4}{3}\right) - 2$

$$y = \frac{5}{3}x - \frac{20}{3} - \frac{6}{3}$$

Cartesian equation for the curve parametrized by $x(t)$, $y(t)$.

$$\boxed{y = \frac{5}{3}x - \frac{26}{3}}$$

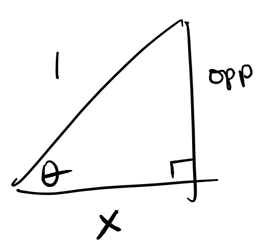
$y = mx + b$

Ex $x = \cos t$, $y = \sin t$ $\}$ $x^2 + y^2 = \cos^2 t + \sin^2 t = 1$

If you solve t $\cos^{-1}(x) = t$

put in y $y = \sin(\cos^{-1}(x)) = \sin\theta = \sqrt{1-x^2}$

$y = \sqrt{1-x^2}$



$\cos\theta = \frac{x}{1} = x$

$\theta = \cos^{-1}x$

$$x^2 + opp^2 = 1$$

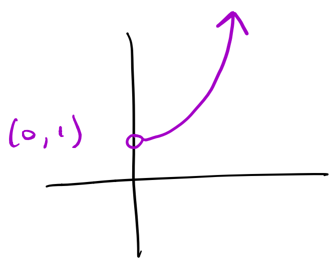
$$opp^2 = 1 - x^2$$

$$opp = \sqrt{1 - x^2}$$

If we can avoid solving for t to make it go away, we prefer it.

<u>#10</u> $x = e^t > 0$

$$\lim_{t \to -\infty} e^t = 0$$



$$\lim_{t \to -\infty} y = \lim_{t \to -\infty} e^{2t} + 1 = 1$$

$y = x^2 + 1$ domain: $x > 0$

<u>#59</u> $$x^2 + y^2 = t^2 \cos^2 t + t^2 \sin^2 t$$

$$= t^2(1)$$

$$= t^2$$

$$\sqrt{x^2 + y^2} = t \text{ (radius)}$$

~~How~~ to break this up into functions?

You don't have to eliminate the parameter to graph.

7.2 #65 use $\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$

$$\boxed{\text{arc length} = \int_a^b \sqrt{x'(t)^2 + y'(t)^2}\, dt,} \qquad a \le t \le b$$

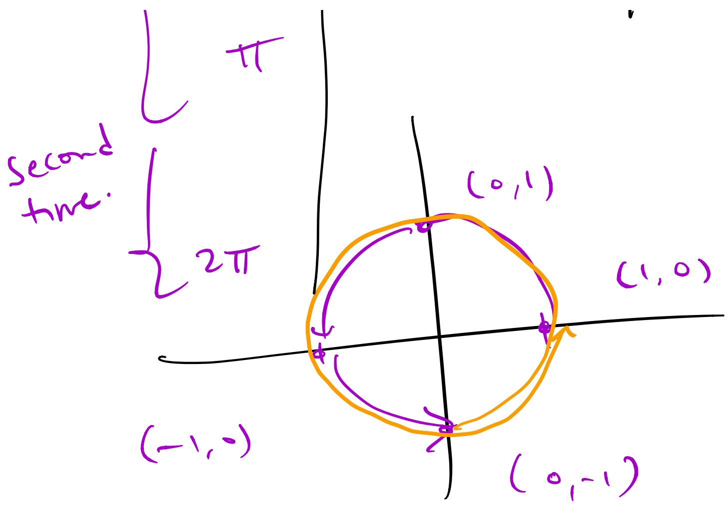
Qu 7.1 How many times do we go around the circle.

$$\left.\begin{array}{l} x = \cos t \\ y = \sin t \end{array}\right\} \begin{array}{l} 0 \le t \le 2\pi \\ \text{goes around once} \end{array}$$

$$\left.\begin{array}{l} x = \cos 2t \\ y = \sin 2t \end{array}\right\} \begin{array}{l} 0 \le t \le 2\pi \\ \text{goes around twice} \end{array}$$

t	$\cos 2t$	$\sin 2t$	
0	$\cos 0 = 1$	$\sin 0 = 0$	$(1, 0)$
$\frac{\pi}{4}$	$\cos \frac{\pi}{2} = 0$	$\sin \frac{\pi}{2} = 1$	$(0, 1)$
$\frac{\pi}{2}$	$\cos \pi = -1$	$\sin \pi = 0$	$(-1, 0)$
$\frac{3\pi}{4}$	$\cos \frac{3\pi}{2} = 0$	$\sin \frac{3\pi}{2} = -1$	$(0, -1)$

once



Notes from Office Hour 2: 1:20pm

#65 $\left.\begin{array}{l} x = -5t+7 \\ y = 3t-1 \end{array}\right\}$ to use the 7.1 method

solve for t : $x - 7 = -5t$

$$\frac{x-7}{-5} = t$$

plug t into y : $y = 3\left(\frac{x-7}{-5}\right) - 1$

$$y = \frac{3x}{-5} - \frac{21}{-5} - 1$$

$$y = -\frac{3}{5}x + \frac{21}{5} - \frac{5}{5}$$

$$\boxed{y = \frac{-3}{5}x + \frac{16}{5}}$$

$\checkmark\ m = \frac{-3}{5}$

So they are asking you actually to

<u>use 7.2</u>:

$$\text{slope } m = m_{tan} = \frac{dy}{dx} = \frac{y'(t)}{x'(t)}$$

$$\left.\begin{aligned} x &= a + bt \\ y &= c + dt \end{aligned}\right\} \text{ line}$$

$$\checkmark = \frac{(3)}{(-5)}$$

#66 $x = 3\sin t$ $\quad \frac{dy}{dx} = \frac{-3\sin t}{3\cos t} = \frac{-\sin t}{\cos t} = -\tan t$

$y = 3\cos t$

$$\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{4}} = -\tan\left(\frac{\pi}{4}\right) = -1 = m_{tan}$$

eqn of line slope m, point (a,b):

$$\boxed{y - b = m(x - a)}$$

<u>tangent line</u> set $m = m_{tan} = -1$

at the point $a = x\left(\frac{\pi}{4}\right) = 3\sin\frac{\pi}{4}$

$b = y\left(\frac{\pi}{4}\right) = 3\cos\frac{\pi}{4}$