Section 5.1 – 5.3 Applications

5.1 Ex 1 Suppose you take out \$180,000 for the purchase of a \$200,000 home. Suppose this loan has a 3% annual interest rate compounded monthly and that you pay \$1500 at the end of each month.

(1) How much do you still owe after 1 year.

(2) When are you done with payments?

$n = \#$ of months

$a_n =$ amount we still owe in \$

$$a_{n+1} = (1+r)a_n - 1500$$

monthly interest rate $r = \frac{.03}{12}$

$$f(x) = \left(1 + \frac{.03}{12}\right)x - 1500$$

Iteration $(f(x), 180\,000, 12)$ soln to (1)

\$167,225.30

Iteration $(f(x), 180000, n) < 0$ soln to (2)

what is $n =$? 143 $\frac{143}{12} = 11.92 \approx 12$ years.

(see 5.1 #61)

Ex 2 5.2 #126

$d = 10$ $n = \#$ of quarters

$r = \frac{.04}{4} = .01$ periodic (quarterly) interest rate

$n=1$ d amt at beginning of quarter

$n=2$ $(1+r)d + d$

$n=3$ $(1+r)((1+r)d + d) + d = (1+r)^2 d + (1+r)d + d$

$= d(1 + (1+r) + (1+r)^2)$

$= d \sum_{i=1}^{\cancel{3}\, n} (1+r)^{i-1}$

$$\sum_{i=1}^{n} s^{i-1} = 1 + s + \cdots + s^{n-1} = \frac{1-s^n}{(1-s)}$$

$= d\left(\frac{1-1.01^n}{1-1.01}\right) \frac{x-1}{x-1}$

$1+r = 1+0.01 = 1.01$

$= \frac{d(1.01^n - 1)}{.01}$

a. after n quarters is like the beginning of the $(n+1)$st quarter, except w/o that payment.

$$\frac{d(1.01^{n+1} - 1)}{.01} - \overset{(n+1)}{n} \cdot d$$

see problem 5.2 #126

Ex3   5.2 #127

Ibuprofen decreases by $\approx 29\%$ /hr in the body

A new does every $N = 8$ hours

How much drug is in the body after 25 hrs?

$n = 25 = 3 \cdot 8 + 1$ $(n = m \cdot N + k)$

$d = 400$mg / dose.

$A(n)$ = amount after n hours

$r = \cancel{0.29}$

$1 - 0.29 = 0.71$

	n (hours)	amount
	0	d
	1	dr
	$\vdots$	
	7	dr^7
	8	$dr^8 + d$
+1	9	$(dr^8 + d)r$
	$\vdots$	
+7	15	$(dr^8 + d)r^7$
	16	$(dr^8 + d)r^8 + d$
	17	$((dr^8 + d)r^8 + d)r$
$n =$	18	$((dr^8 + d)r^8 + d)r^2$

$$d\,r^2\left(1 + r^8 + (r^8)^2\right)$$

$$= d\,r^2\left(1 + s + s^2\right) \qquad s = r^8$$

$\underset{\downarrow}{r^2}$ +2 hrs. #3 #2 #1

$$n = 18 = 2 \cdot 8 + 2 = m \cdot N + k$$

$$A(n) = d \cdot r^k \left(1 + s + \dots + s^m\right)$$

$$= d \cdot r^k \frac{\left(1 - s^{m+1}\right)}{1 - s}, \quad s = r^8$$

if $k = 0$ $\quad \lim_{m \to \infty}$ $\quad d \cdot (1) \cdot \dfrac{(1 - 0)}{1 - s}$

$$= \frac{d}{1 - s}$$

$$= \frac{400}{1 - (.71)^8} \approx 427\,\text{mg}$$

steady state

(decreases until the next dose).

If $M = 2$

$k = 0$

$m = 12$

$d = 266\,\text{mg}$

steady state is 536mg if 266mg is dosed every 2 hr.

(see Mathematica).