

213-02 Discussion

5.2 #126 (Example 2 in lecture.)

$n = \#$ of quarters

$a_n = \text{amount of \$ after } n^{\text{th}} \text{ quarter}$

1

$$d + rd = (1+r)d = \underbrace{1.01d}_{a_1}$$

2

$$\underbrace{d + 1.01d}_{(1+r)(d+1.01d)} + r(d+1.01d) = \underbrace{1.01(d + \underbrace{1.01d}_{a_1})}_{a_2}$$

3

$$\underbrace{(a_2 + d) + r(a_2 + d)}_{1.01(a_2 + d)} = 1.01(a_2 + d)$$

$d = 10$ (\$ at beginning of each quarter)

$$r = \frac{.04}{4} = .01 \quad (\text{monthly interest rate})$$

$$1+r = 1+.01 = 1.01$$

recursive pattern : $a_{n+1} = 1.01(d + a_n)$

explicit formula : $a_n = f(n)$

$b_n = \text{interest after } n \text{ quarters.}$

$$b_1 = a_1 - d = 1.01d - d$$

$$b_2 = a_2 - 2d$$

$$\boxed{b_n = a_n - n \cdot d}^*$$

$$a_3 = 1.01(\underline{a_2} + d) = 1.01(\underbrace{1.01(d + 1.01d)} + d)$$

$$= 1.01d + 1.01^2d + 1.01^3d$$

$$\vdots$$

$$d \left(\frac{1.01^{n+1} - 1}{0.01} \right) \dots$$

$$b_n = d \left(\frac{1.01^{n+1} - 1}{0.01} - (n+1) \right)$$

Compare the formulas:

$$\textcircled{1} \quad b_n = 10 \left(\frac{1.01^{n+1} - 1}{0.01} - (n+1) \right) \quad \left. \vphantom{\frac{1.01^{n+1} - 1}{0.01}} \right\} \text{explicit formula}$$

Sequence $(b_n, n, 1, 8)$

$$\textcircled{2} \quad b_n = a_n - n \cdot d \quad \text{where}$$

$$a_{n+1} = 1.01(a_n + 10) \quad \left. \vphantom{a_{n+1}} \right\} \text{recursive}$$

$$a_1 = 10.1$$

Iteration List $(f(x), 0, 8)$

$$\underbrace{f(x) = 1.01(x + 10)}$$