Zoom Notes 213-02

The parameter is any third variable θ so that $x = x(\theta)$

$y = y(\theta)$

#13 Ex4 in Lecture 7.1, add/subtract first.

isolate $(\cos\theta)^2 + (\sin\theta)^2 = 1$

$x = 3 - 2\cos\theta \Rightarrow x - 3 = -2\cos\theta$

$y = -5 + 3\sin\theta \Rightarrow y + 5 = 3\sin\theta$

$$\Rightarrow \left(\frac{x-3}{-2}\right)^2 = (\cos\theta)^2$$

$$\Rightarrow \left(\frac{y+5}{3}\right)^2 = (\sin\theta)^2$$

$+$

$$\boxed{\frac{(x-3)^2}{2^2} + \frac{(y+5)^2}{3^2} = 1}$$

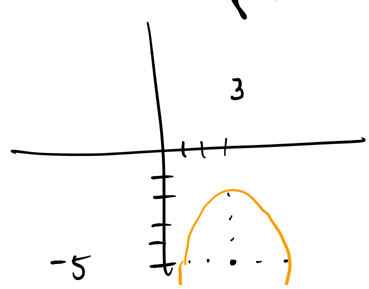
This is the Cartesian equation having eliminated the parameter θ

sketch: ellipse

x-radius: 2

y-radius: 3

center: $(3,-5)$



$$\frac{(x-a)^2}{r_1^2} + \frac{(y-b)^2}{r_2^2} = 1 \quad \text{ellipse}$$

Ex $\sqrt{x^2+y^2} = 3$ circle radius 3

ellipse form

$$x^2+y^2 = 3^2$$

$$\frac{x^2}{3^2} + \frac{y^2}{3^2} = 1$$

All HW, Quizzes due @ 11:59pm on due date.

7.2 $\boxed{\left.\frac{dy}{dx}\right|_{t_0} = \left.\frac{y'(t)}{x'(t)}\right|_{t_0}}$ if $x'(t) \neq 0$

Eqn of tangent line.

Eqn of a line:

$y - y_1 = m(x - x_1)$ $\quad m = m_{tan} = \frac{dy}{dx}$

At time t_0: $(x_1, y_1) = (x(t_0), y(t_0))$

Eqn of tangent line to the curve at $t=t_0$:

$$y - y(t_0) = \frac{y'(t_0)}{x'(t_0)}\left(x - x(t_0)\right)$$

Group work : Start 7.1 #43

7.2 #66

7.1 #10

don't cancel 3 too soon.

7.2 #66 $\rightarrow$ $y = 3\cos t \Rightarrow y'(t) = -3\sin t$

$x = 3\sin t \Rightarrow x'(t) = 3\cos t$

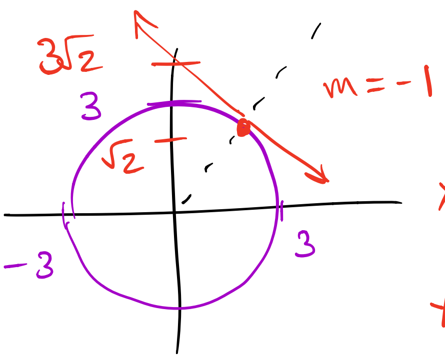
$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{-\cancel{3}\sin t}{\cancel{3}\cos t} = \left.\frac{-\sin t}{\cos t}\right|_{\frac{\pi}{4}} \quad \text{using } t = \frac{\pi}{4}$$

$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$$= \frac{-\sqrt{2}/2}{\sqrt{2}/2}$$

$$= -1$$



$x_1 = x\left(\frac{\pi}{4}\right) = 3\cdot\frac{\sqrt{2}}{2}$

$y_1 = y\left(\frac{\pi}{4}\right) = 3\cdot\frac{\sqrt{2}}{2}$

$\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$

Eqn : $\boxed{y - \frac{3\sqrt{2}}{2} = -1\left(x - \frac{3\sqrt{2}}{2}\right)}$ ✓

$y = -x + 3\sqrt{2}$

$y = -x + 3\sqrt{2}$

7.1 #10 with a table

t	$x = e^t$	$y = e^{2t} + 1$	
-1	e^{-1}	$(e^{-2}) + 1$	
0	1	$(1) + 1$	$\rightarrow (1, 2)$
1	e	$(e^2) + 1$	

Plot

eliminate the parameter : $t = \ln x$

$\ln x = t$

$y = e^{2(\ln x)} + 1$

take ln of both sides

$y = e^{\ln x^2} + 1$

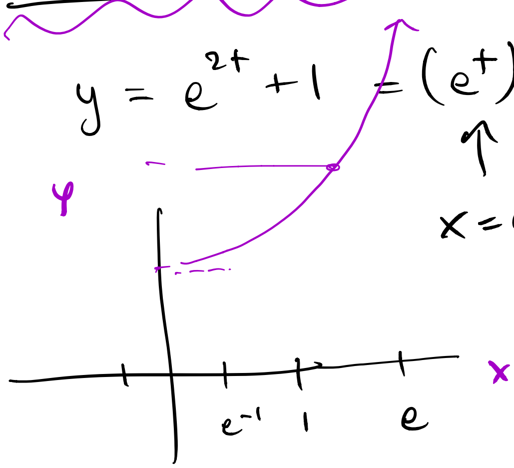
$2 \ln x = \ln(x^2)$

$\boxed{y = x^2 + 1}$

Alternatively

$y = e^{2t} + 1 = (e^t)^2 + 1 = (x)^2 + 1 = x^2 + 1$

$x = e^t$



log rules

$e^{a+b} = e^{a} \cdot e^{b}$

$2^{6} = 2^{2} \cdot 2^{4}$

$= 2 \cdot 2 \cdot (2 \cdot 2 \cdot 2 \cdot 2)$

let $a = \ln x$

$b = \ln y$

$e^{\ln x + \ln y} = e^{\ln x} \cdot e^{\ln y} = (x) \cdot (y) = e^{\ln(x \cdot y)}$

$\hookrightarrow \boxed{\ln x + \ln y = \ln(x \cdot y)}$ let $x = y$

$= \ln x + \ln x = \ln(x \cdot x) = \ln(x^{2})$

$2\ln x$