213-02 discussion OH @ 2:00pm

5.4 Given $\sum a_n$, $a_n \geq 0$

Comparison Test (CT) #202

* ① if $\boxed{0 \leq a_n \leq b_n}$ + $\sum b_n$ converges $\Rightarrow \sum a_n$ converges

② if $0 \leq b_n \leq a_n$ + $\sum b_n$ diverges $\Rightarrow \sum a_n$ diverges

Limit Comparison Test (LCT) #208

① if $\frac{a_n}{b_n} \to L \neq 0$, then $\sum a_n , \sum b_n$ have the same convergence behavior.

② if $\frac{a_n}{b_n} \to 0$ + $\sum b_n$ converges $\Rightarrow \sum a_n$ converges

③ if $\frac{a_n}{b_n} \to \infty$ + $\sum b_n$ diverges $\Rightarrow \sum a_n$ diverges

#202 $a_n = \frac{\sin^2 n}{n^2}$

$b_n = \frac{1}{n^2}$ $p = 2 > 1$ by p-series

* $\sum b_n$ converges

Since $-1 \leq \sin n \leq 1$

$0 \leq \sin^2 n \leq 1$

$\div n^2$ $\hookrightarrow a_n = \frac{\sin^2 n}{n^2} \leq \frac{1}{n^2} = b_n$

Therefore, by CT, $\sum a_n$ converges.

#208

$$a_n = \frac{\ln(n)}{n^{0.6}} = \frac{\ln^2(n)}{n^{1.2}} \sim \frac{1}{n^{1.2}} \; ?$$

$\sum$ conv $\quad \frac{a_n}{b_n} \to \infty$

$$b_n = \frac{1}{n^p} \qquad \text{choose } 0 < p < 1.2$$

like $p = 1.1$

$$\frac{a_n}{b_n} = \frac{\ln^2 n}{n^{1.2}} \times \frac{n^{1.1}}{1}$$

Since $p = 1.1 > 1$, series $\sum \frac{1}{n^{1.1}}$ converges.

$$= \frac{\ln^2 n}{n^{1.2-1.1}}$$

$$\frac{n^a}{n^b} = n^{a-b} = \frac{1}{n^{b-a}}$$

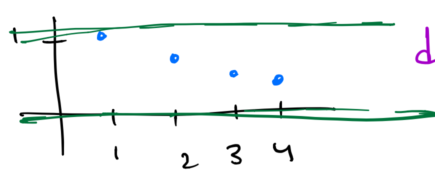
$$= \frac{\ln^2 n}{n^{0.1}} \to 0$$

log << poly. $\quad \frac{(\ln n)^\alpha}{n^\beta} \to 0$

$\alpha, \beta > 0$

Since $\sum \frac{1}{n^{1.1}}$ convs and $\frac{a_n}{b_n} \to 0$, by LCT $\sum a_n$ converges as well.

OH $\quad \left\{ \frac{n+1}{n^2+1} \right\}$



decreasing.

n	a_n
1	1
2	0.6
3	0.4
4	0.29

bounded? $\quad 0 \le \frac{n+1}{n^2+1}$ for $n \ge 1$

bc it's positive

show $\frac{n+1}{n^2+1} \le 1$

$n+1 \le n^2+1$

$\Downarrow$

$$f(x) = \frac{x+1}{x^2+1}$$

$n \le n^2$ ✓ true

yes

$0 \le a_n \le 1$

sequence is bounded

$$f'(x) = \frac{(x^2+1)(1) - (x+1)(2x)}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2-2x}{''}$$

$$= \frac{-x^2-2x+1}{(x^2+1)^2} < 0 \quad \text{for all big enough } x$$

so $f(x)$ is decreasing

so a_n is decreasing. (monotone ✓)

alt $\quad a_{n+1} \le a_n$

$$\frac{(n+1)+1}{(n+1)^2+1} \le \frac{n+1}{n^2+1}$$

cross multiply

show it is true.

Bounded, monotone sequences converge, so

$$\left\{\frac{n+1}{n^2+1}\right\} \text{ converges.}$$

$$\lim_{n\to\infty} \frac{n+1}{n^2+1}\left(\frac{\frac{1}{n^2}}{\frac{1}{n^2}}\right) = \lim_{n\to\infty} \frac{\cancel{\frac{1}{n}}^{0} + \cancel{\frac{1}{n^2}}^{0}}{1+\cancel{\frac{1}{n^2}}^{0}} = \frac{0}{1} = \boxed{0}$$

$a_n \longrightarrow 0$.

can you do $\left\{ \frac{n}{2^n} \right\}$ #31

#13 $\left\{ \frac{(-1)^{n-1}}{2n-1} \right\}_{n=1} = \left\{ 1, \frac{-1}{3}, \frac{1}{5}, \frac{-1}{7}, \ldots \right\}$

Example 5.3 a. b. c.

are good examples of $\{a_n\}$

a. $5 - \frac{3}{n^2}$

b. $\frac{3n^4 - 7n^2 + 5}{6 - 4n^4}$

c. $\frac{2^n}{n^2}$