Math 213-02 Spring 2020 Name: ______________________________

Final Exam

Show all your work for each problem. Answers with insufficient work shown will not receive full credit. There are six questions, and each question is out of 30 points.

Instructions: This test is open book and open notes and you may use a scientific calculator. You may not collaborate with any other person and you may not search for information online or in any other source. Your work must be completely justified and written in your own words.

Work that is not fully justified will not receive full credit. Papers with *unusually* identical language, or with language that matches with an online source, will be referred to the Office of Student Conduct for further investigation and may result in a zero for the assignment and/or further penalties.

1. **Parametric Equations.**

Consider the parametrization

$$x = 2\cos(2t), \quad y = 2\sin(2t); \quad 0 \le t \le \pi$$

(a) Sketch the parametrized curve using any method, but you must explain your thinking in clear sentences, or show mathematical work. Pay attention to the given domain.

(b) Choose **one** of (i) or (ii) below – only **one** will be graded.

   i. Set up and solve an integral for the arc length of this curve
   OR

   ii. Calculate the equation of the tangent line to this curve at some point (tell the reader which point you chose!) using methods of 7.2.

2. **Geometric Series.**

Consider $\sum_{n=1}^{\infty} \left(\frac{4}{5}\right)^n$

(a) Determine whether the series converges, and if it does, find the sum. Show all your work.

(b) What does S_N represent?

(c) Suppose that you are told that $S_N = \frac{(4/5)(1-(4/5)^N)}{1-(4/5)}$ for the series above. Determine whether the sequence $\{S_N\}$ converges or diverges. If it converges, find its limit. What is the relationship between the convergence of this sequence and the convergence of the series, above?

3. **Power Series.**

Consider the power series $\sum_{n=1}^{\infty} \frac{10^n (x-3)^n}{(2n)!}$.

(a) Find the Radius of Convergence.

(b) Does this series converge at $x = 4$? At $x = -4$? Explain your thinking.

4. **Series Convergence Tests for Positive Series**

Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^2+1}$. Use your choice of either the Integral Test or the Limit Comparison Test (with p-series) to determine whether the given series converges or diverges. You must show all your work.

5. **Alternating Series Test with Remainder**

The given series converges by Alternating Series Test. Use the estimate $|R_N| \leq b_{N+1}$ to find the least value of N that guarantees that the sum S_N differs from the infinite sum

$$\sum_{n=1}^{\infty} \left(-\frac{1}{11}\right)^{n-1}$$

by at most an error of 0.001.

Answer

(a) What is N?

(b) What is S_N and what is the actual sum S of the series?

(c) Is $|S - S_N| < 0.001$?

6. **Taylor Series.**

(a) Find the Taylor polynomial centered at the point $a = 1$ of degree four, $p_4(x)$, approximating $f(x) = e^x$.

(b) Calculate $p_4(0)$ (still centered at $a = 1$) and e^0 and calculate the absolute value of their difference (the absolute error).

(c) Calculate a bound on the absolute error in part (b) that is given by the Taylor Remainder Theorem. Show that this bound is greater than your answer in part (b).