

Midterm

Show all your work for each problem. Answers with insufficient work shown will not receive full credit.

Instructions: This test is open book and open notes and you may use a scientific calculator. You may not collaborate with any other person and you may not search for information online or in any other source. Your work must be completely justified and written in your own words.

Work that is not fully justified will not receive full credit. Papers with *unusually* identical language, or with language that matches with an online source, will be referred to the Office of Student Conduct for further investigation and may result in a zero for the assignment and/or further penalties.

1. (20 points, unless skipped) **Disks/Washers.**

CHOOSE either this OR Question #2 to solve.

If you choose to skip this question, write “SKIP #1” for this question.

- (a) Sketch the region R bounded by $y = x$, $x = 1$, and $y = 2$ and **shade** the region.
- (b) Find the volume of the solid obtained by rotating R around the x -axis by setting up the appropriate integral and solving.

2. (20 points, unless skipped) **Polar.**

CHOOSE either this OR Question #1 to solve.

If you choose to skip this question, write “SKIP #2” for this question.

(a) Convert $(x, y) = (-\sqrt{3}, -1)$ to polar coordinates (r, θ) such that $r \geq 0$.

(b) Convert the equation $y = x$ from rectangular/Cartesian form to polar form.

3. (20 points) **Parametric Equations.**

Sketch the parametrized curve by first eliminating the parameter to obtain an equation in x and y . Pay attention to the given domain. In your sketch, indicate at least two points on the curve.

$$x = \cos 2t, \quad y = \sin 2t; \quad 0 \leq t \leq \pi$$

Describe this curve in words here: _____

4. (30 points) **Sequences.**

- (a) Plot the first four terms of the sequence for each of (a) $\left\{\frac{n+1}{n}\right\}$ and (b) $\{2^n\}$ on two separate pairs of axes (label which sequence is which). Plot the n -values on the x -axis and the a_n -values on the y -axis, as we do in class.

[continued \Rightarrow]

- (b) Fill out the tables below. Choose answers from the bold-faced text. If you are not printing the exam, please draw tables like this on your paper.

sequence	bounded? Y/N	bounded by which lines (e.g., $y = 1$, $y = 2$) or why not bounded?
(a) $\left\{ \frac{n+1}{n} \right\}$		
(b) $\{2^n\}$		

sequence	mononote? Increasing/Decreasing/Neither	justification
(a) $\left\{ \frac{n+1}{n} \right\}$		
(b) $\{2^n\}$		

sequence	converges? Y/N	justification (and what is the limit L , if it exists?)
(a) $\left\{ \frac{n+1}{n} \right\}$		
(b) $\{2^n\}$		

5. (30 points) **Geometric Series.**

(a) Consider $\sum_{n=1}^{\infty} \left(-\frac{2}{3}\right)^{n-1}$

i. Calculate S_4 for the series above (in part(a)).

ii. Determine whether the series converges, and if it does, find the sum. Show all your work.

(b) Consider $\sum_{n=1}^{\infty} \left(\frac{9}{4}\right)^{n-1}$

i. Determine whether the series converges (in part (b)), and if it does, find the sum. Show all your work.

ii. Suppose that you are told that $S_N = \frac{1 - (9/4)^N}{1 - (9/4)}$ for the series in part 5b. Determine whether the sequence $\{S_N\}$ converges or diverges. If it converges, find its limit. What is the relationship between this question and question 5(b)i?