213-01 discussion: 5.2 HW problems.

7.3 $(3,-\sqrt{3}) = (x,y)$

#141

$$x^2+y^2 = 3^2+(-\sqrt{3})^2 = 9+3 = 12$$

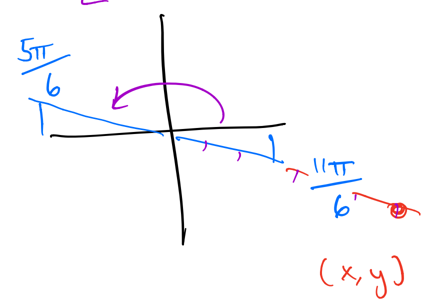
$$r = \sqrt{x^2+y^2} = \sqrt{12} = 2\sqrt{3} \quad (\text{if } r>0)$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-\sqrt{3}}{3}\right)$$

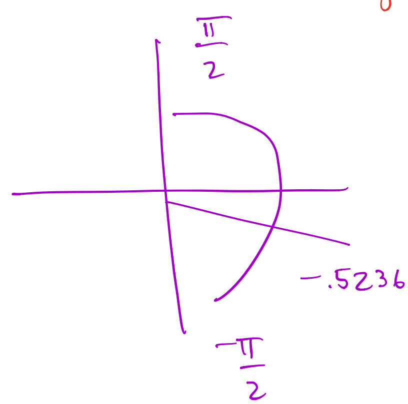
$$= \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \tan^{-1}\left(\frac{-1/2}{\sqrt{3}/2}\right)$$

$$\left(2\sqrt{3}, \frac{11\pi}{6}\right)$$

$$\left(-2\sqrt{3}, \frac{5\pi}{6}\right)$$



shouldn't θ in $(0, 2\pi)$ *



$$\frac{-.5236}{\pi} = -0.1666\ldots = \frac{-1}{6}$$

$$-.5236 = -\frac{\pi}{6} \quad +2\pi$$

Instead write $\frac{11\pi}{6}$

5.2 Group work! #69, #75

#69 $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$ alternate +/− sign
denomenator grows +1

$a_1 + a_2 + a_3 + a_4 + \cdots$

$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ ✓

$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ ✓

1	$a_1 = 1$	1	$n=1$
-1	$a_2 = -\frac{1}{2}$	$\frac{1}{2}$	$n=2$
1	$a_3 = \frac{1}{3}$	$\frac{1}{3}$	$n=3$
-1	$a_4 = -\frac{1}{4}$	$\frac{1}{4}$	$n=4$

$= (-1)^n$ or $(-1)^{n-1}$ $\qquad = \frac{1}{n}$

write in terms of n.

<u>Alt</u> sub $k+1$ for n, start at $k=0$.

$$\sum_{k=0}^{\infty} \frac{(-1)^{(k+1)-1}}{k+1} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k+1} = \underset{k=0}{\frac{1}{1+0}} - \underset{k=1}{\frac{1}{1+1}} + \underset{k=2}{\frac{1}{2+1}} \cdots$$

RE-INDEXING

$$= 1 - \frac{1}{2} + \frac{1}{3}$$

Suggestion $\sum_{n=1}^{\infty} \underbrace{1 + \frac{1}{n}}_{a_n} = a_1 + a_2 + a_3 + \cdots$

$$= (1+1) + (1 + \tfrac{1}{2}) + (1 + \tfrac{1}{3}) + \cdots$$

$a_1 = 1 + \frac{1}{1}$

$a_2 = 1 + \frac{1}{2}$

$a_3 = 1 + \frac{1}{3}$ too many 1's . . .

update : $1 + \sum_{n=\cancel{1}\,2}^{\infty} \frac{1}{n}$

$1 + \left(\cancel{\frac{1}{1}} + \frac{1}{2} + \frac{1}{3} + \cdots\right)$

#75 $S_n = 1 - \frac{1}{n}$ $n \geq 2$ what is $\begin{cases} S\,? \\ \boxed{a_n\,?} \end{cases}$

n	a_n	$S_n = 1 - \frac{1}{n}$		
~~1~~	~~a_1~~	~~$S_1 = a_1$~~	$a_2 = S_2 - S_1$	~~$1 - \frac{1}{1} = 0$~~
2	a_2	$S_2 = a_1 + a_2$		$1 - \frac{1}{2} = \frac{1}{2}$
3	a_3	$S_3 = a_1 + a_2 + a_3$	$a_3 = S_3 - S_2$	$1 - \frac{1}{3} = \frac{2}{3}$
4	a_4	$S_4 = a_1 + a_2 + a_3 + a_4$		$1 - \frac{1}{4} = \frac{3}{4}$

$\downarrow$ is the limit ∞?

$a_n = S_n - S_{n-1} = \left(1 - \frac{1}{n}\right) - \left(1 - \frac{1}{n-1}\right)$

$= \cancel{1} - \frac{1}{n} - \cancel{1} + \frac{1}{n-1}$

$$\boxed{a_n = \frac{1}{n-1} - \frac{1}{n}}$$

series $\sum_{n=2}^{\infty} a_n = \sum_{n=2}^{\infty} \left(\frac{1}{n-1} - \frac{1}{n}\right)$

$= a_2 + a_3 + a_4 + \cdots$

$= \left(\frac{1}{2-1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{3-1}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{4-1}} - \frac{1}{4}\right) + \cdots$

$S_n = 1 \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad - \frac{1}{n}$

$$\lim_{n\to\infty}\left(1-\frac{1}{n}\right) = \lim 1 - \cancel{\lim \frac{1}{n}} \to 0$$

$$= 1 - 0$$

$$= \boxed{1}$$

The sequence of partial sums $\{S_n\} = \{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots\}$ is increasing, but bounded above by $y = 1$

$1 - \frac{1}{n} \le 1$

So the sequence converges to the sum of the series, i.e. $\boxed{\sum_{n=2}^{\infty} \frac{1}{n-1} - \frac{1}{n} = 1}$ $= \lim S_n$

$$\neq \lim a_n = \lim_{n\to\infty} \cancel{\frac{1}{n-1}} - \cancel{\frac{1}{n}} = 0 - 0 = 0$$