Math 213-02 Monday discussion of 5.1

<u>key words</u>: convergent, divergent
bounded, monotone

<u>Ex</u> $a_n = \frac{e^n}{n}$ $a_n = f(n)$ $f(x) = \frac{e^x}{x}$

(compare to $f(x) = x^{1/x}$ in Ex5 of lec. 1)

Is this sequence increasing / decreasing?

show slope = (+)

$f'(x) > 0$

OR
table

n	e^n/n
1	$e/1 = 2.72$
2	$e^2/2 = 3.69$
3	$e^3/3 \approx 6.7$

getting bigger.

$$f'(x) = \frac{d}{dx}\left[\frac{e^x}{x}\right] = \frac{x \cdot \frac{d}{dx}[e^x] - e^x \frac{d}{dx}[x]}{x^2}$$

quotient rule

$$= \frac{x \cdot e^x - e^x(1)}{x^2}$$

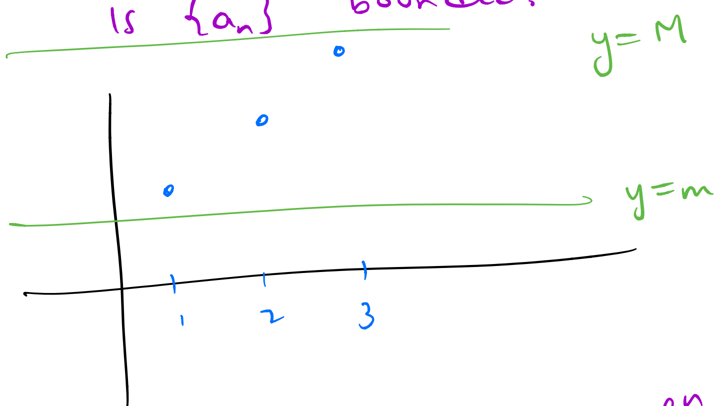
$$f'(x) = \frac{\underbrace{e^x}_{>0}\ \overset{(2)-1}{(x-1)}}{\underbrace{x^2}_{>0}} > 0, \text{ for } x > 1.$$

Since $f'(x) > 0$, slope $= \oplus$, function is increasing.

In Ex 5, $f'(x) < 0$, so slope $= \ominus$, function decreasing.
(we had to use logarithmic differentiation)

Because $f(x)$ is increasing, $\{a_n\}$ is increasing.

Is $\{a_n\}$ bounded?



There is no ceiling to $\frac{e^n}{n}$, so sequence is not bounded.

Try $\lim\limits_{x\to\infty} \frac{e^x}{x}$ "$\frac{\infty}{\infty}$"

$\overset{LH}{=} \lim\limits_{x\to\infty} \frac{\frac{d}{dx}[e^x]}{\frac{d}{dx}[x]}$

Show

$\frac{e^n}{n} > 1000$

try plugging in

$n = 1000$

$\cancel{\frac{e^{1000}}{1000}}$

$$= \lim_{x\to\infty} \frac{e^x}{1} \quad \text{"}\frac{\infty}{1}\text{"}$$

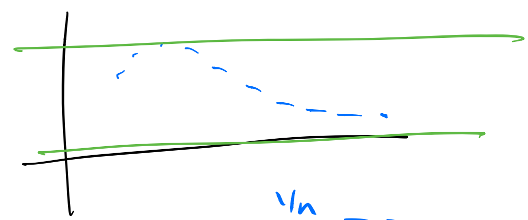
$$= \infty$$

So $\{a_n\}$ diverges (to ∞)

$n = 10$

$\frac{e^{10}}{10} = 2202.65 > 1000$

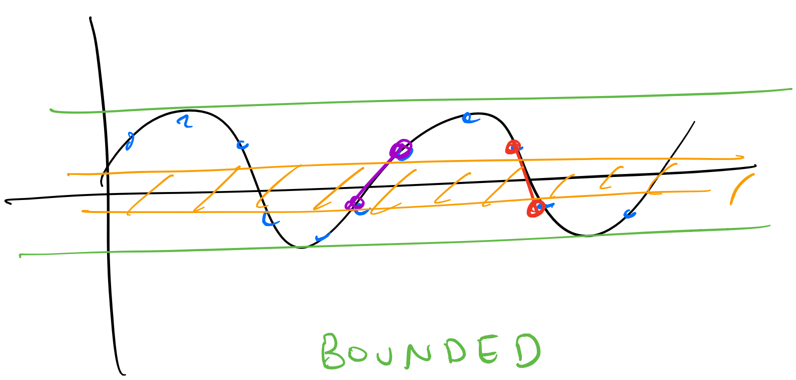
Typically to show something is bounded e.g. show it is bounded below (by zero) and decreasing.



$a_n = n^{1/n} > 0$

Ex 2

We worked on sin(n) in class



BOUNDED

not monotone. sometimes $a_n \le a_{n+1}$

sometimes $a_n > a_{n+1}$

DIVERGENT = it does not stay in a small

Stop. NO LIMIT.

#49
#13

#37

next time.
#31
n^3

2.2

#71 cross sections are squares.