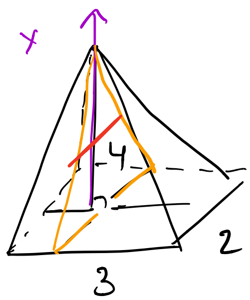
Section 2.2 213-02

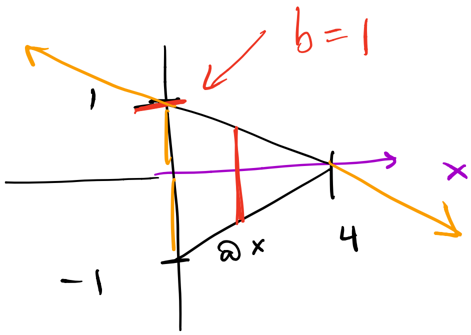
Talk about
- ✓ # 83
- ✓ # 95
- # 167 Arc length

In 2.2 First lecture



Formula for volume =

$\frac{1}{3}(2 \cdot 3) \cdot 4 = 8 \text{ un}^3$



points
$x_1\ y_1$
$(0,1)$
$(4,0)$
$x_2\ y_2$

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0-1}{4-0} = \frac{-1}{4}$

$b = 1$

slope intercept

$y = mx + b$

$y = \frac{-1}{4}x + 1$

$\left(\frac{1}{4}x + y = 1\right) \times 4$

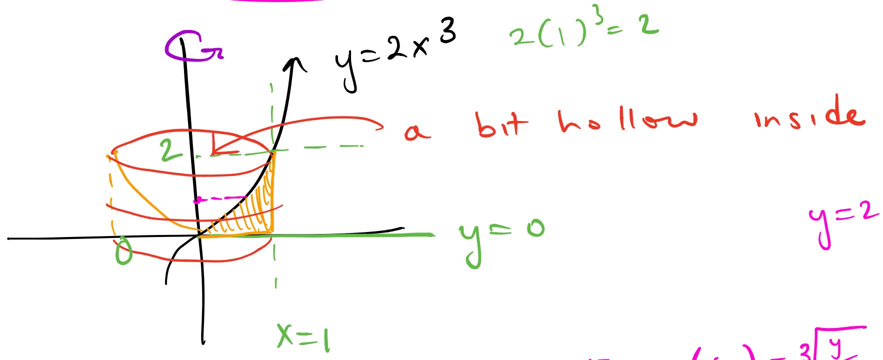
$x + 4y = 4$

point slope

$y - y_1 = m(x - x_1)$

$\boxed{y - 1 = -\frac{1}{4}(x - 0)}$

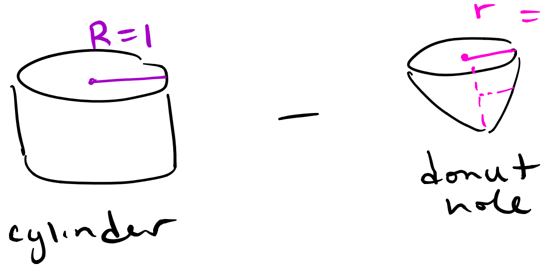
#83 $y = 2x^3$, $x = 0$, $x = 1$, $y = 0$



$y = 2x^3 \longrightarrow \frac{y}{2} = x^3$

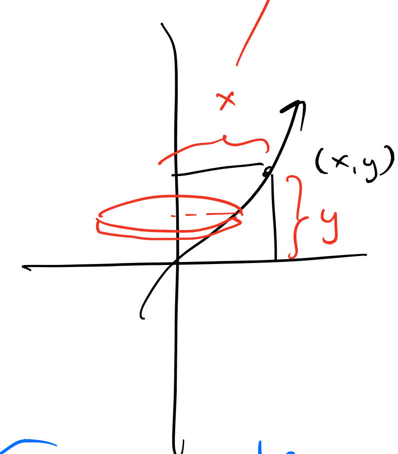
$\longrightarrow \sqrt[3]{\frac{y}{2}} = x$

$r = r(y) = \sqrt[3]{\frac{y}{2}}$



volume: $\pi R^2 \quad - \quad \pi r^2$

$$\int_0^2 \pi(1)^2 - \pi\left(\sqrt[3]{\frac{y}{2}}\right)^2 dy$$



alternate

$x = g_2(y) \quad > \quad x = g_1(y)$

(furthest right) (furthest left)

Can make a formula :

$$\text{vol} = \int_c^d [g_2(y)]^2 - [g_1(y)]^2 \, dy$$

Solve some integrals:

$$\pi \int_0^2 1 - \frac{y^{2/3}}{2^{2/3}} \, dy$$

$$= \pi \left[y - \frac{1}{2^{2/3}} \cdot \frac{y^{5/3}}{\left(\frac{5}{3}\right)} \right] \Big|_0^2$$

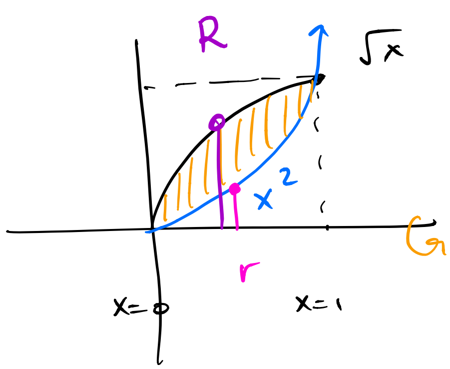
$$= \pi \left[\left(2 - \frac{3}{5 \cdot 2^{2/3}} \cdot 2^{5/3}\right) - (0 - 0) \right]$$

$$= \pi \left[2 \left(1 - \frac{3 \cdot \cancel{2^{2/3}}}{5 \cdot \cancel{2^{2/3}}}\right) \right] \quad \Big\} \; 2^{5/3} = 2^{3/3} \cdot 2^{2/3}$$

$$= \pi \left[2 \left(1 - \frac{3}{5}\right) \right]$$

$$= \boxed{\pi \cdot \frac{4}{5}} \quad \checkmark \; un^3$$

#95 $y = \sqrt{x}$, $y = x^2$, around x-axis.



$(r = f_1(x) < f_2(x) = R)$

$(\sqrt{x})^2 = (x^2)^2$ intersect.

$x = x^4$ $\qquad x \geq 0$

$0 = x^4 - x$

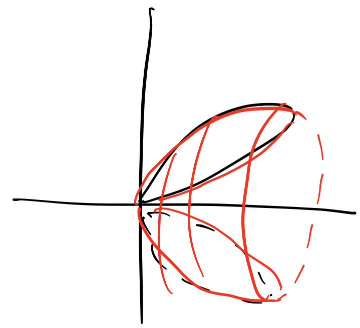
$0 = x(x^3 - 1)$

$\boxed{x = 0, \; x = 1}$

Try $x = \frac{1}{4}$ $\left(0 < \frac{1}{4} < 1\right)$:

$$\sqrt{\frac{1}{4}} = \frac{1}{2} > \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

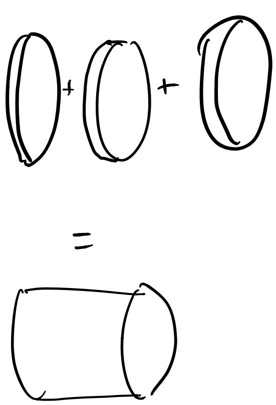
$$\sqrt{x} > x^2$$

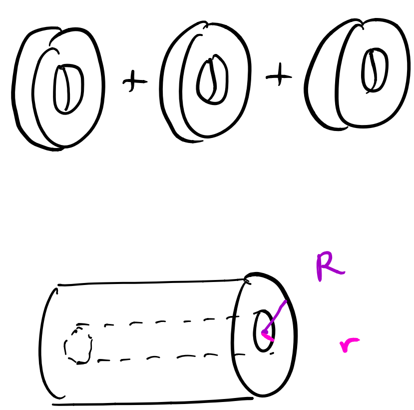


Take a break for polls

Poll #1 if there is a hole/hollow

disks or washers ?





slicing a carrot

slicing toilet paper!

$$\pi R^2 - \pi r^2$$

$$\int \pi (R^2 - r^2)\, dx$$

(about the x-axis, cross sections are $\perp$ to x-axis)

$$\int_0^1 \pi\left((\sqrt{x})^2 - (x^2)^2\right) dx$$

$$\pi \int_0^1 (x - x^4)\, dx$$

$$= \pi \left(\frac{x^2}{2} - \frac{x^5}{5} \right) \Big|_0^1$$

$$= \pi \left[\left(\frac{1}{2} - \frac{1}{5} \right) - (0 - 0) \right]$$

$$= \boxed{\pi \frac{3}{10}}$$ ✓ un^3