Name: Colin Morris-Moncada

Quiz 5: Section 5.4-5.5

1. Consider the power series $\sum_{n=1}^{\infty} (-1)^{n+1} \dfrac{n}{n^2+1}$. Does the series converge absolutely, condition-ally, or not at all? Answers must be fully justified. If you think a sequence $\{b_n\}$ is decreasing, this must be shown algebraically, or using calculus to show that $f'(x) < 0$ where $f(n) = a_n$. If you calculate a limit, do not just say what the answer is, show all your work.

A.S.T.

[Hint: You can solve this problem using only the Alternating Series Test in 5.5, the Limit Comparison Test in 5.4, and the p-Series Test in 5.3.]

$$f(x) = \frac{x}{x^2+1}$$

$$f'(x) = \frac{1(x^2+1) - (x(2x))}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2}{(x^2+1)^2}$$

$$= \frac{1-x^2}{(x^2+1)^2}$$

DECREASING IN X>2

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+1} = \left\{ \frac{1}{2} - \frac{2}{5} + \frac{3}{10} - \frac{4}{17} + \cdots \right\}$$

$$\lim_{n\to\infty} \frac{n}{n^2+1} = \frac{\frac{n}{n^2}}{\frac{n^2}{n^2}+\frac{1}{n^2}}$$

$$= \frac{\frac{1}{n}}{1+\frac{1}{n^2}}$$

$$= \frac{\lim_{n\to\infty} \frac{1}{n}}{\lim_{n\to\infty} 1+\frac{1}{n^2}}$$

$$= 0$$

$$\frac{\lim_{n\to\infty} 1 + \lim_{n\to\infty} \frac{1}{n^2}}{=}$$

$$= \frac{0}{1+0}$$

$$= \frac{0}{1}$$

$$= 0$$ passes divergent test

Compare to $\sum_{n=1}^{\infty} \frac{n}{n^2}$

REWRITE $\dfrac{n}{n^2} = n^{1-2} = n^{-1} = \dfrac{1}{n}$

p-series $p=1 \leq 1$

HOWEVER, WE KNOW ALTERNATING HARMONIC CONVERGES CONDITIONALLY

Conditionally CONVERGES because of A.S.T. and p-series test