Discussion Notes 213-02

5.1 #13 $\left\{1, \frac{-1}{3}, \frac{1}{5}, \frac{-1}{7}, \ldots\right\}$ $\qquad a_n = f(n)$

$\uparrow$ $n=1$ $\quad\uparrow$ $n=2$ $\quad\uparrow$ $n=3$

$\{(-1)^n\} \;\; \{(-1)^1, (-1)^2, (-1)^3, \ldots\}$

$= \{-1, 1, -1, 1, \ldots\}$

alternating sign

$+/-$

OR

$-/+$

$\{(-1)(-1)^n\}$

$= \{(-1)^{n+1}\} \;\; \{1, -1, 1, -1, \ldots\} = \{(-1)^{n-1}\}$

$* \; (-1)^{n-1} = (-1)^{1-1}, (-1)^{(2-1)}, \ldots$

$= 1, -1, \ldots$

denom: $+2n$

$2n + a = 1 \qquad (n=1)$

$2(1) + a = 1$

$a = 1 - 2 = -1$

denom: $2n-1 = 1, 3, 5, \ldots$

$\uparrow$ $n=1$ $\quad\uparrow$ $n=2$ $\quad\uparrow$ $n=3$

Multiply together:

$$\boxed{a_n = \frac{(-1)^{n-1}}{2n-1}}$$

$\frac{1}{2(1)-1} = 1 \qquad \frac{(-1)^1}{4-1} = \frac{-1}{3} \qquad \frac{(-1)^{3-1}}{6-1} = \frac{1}{5}$ ✓

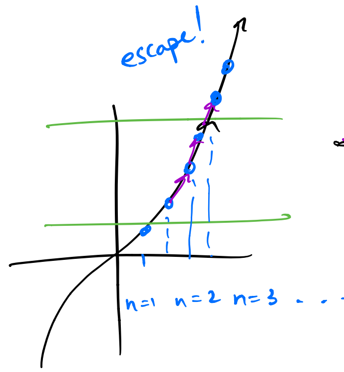
$\uparrow$ $n=1$ $\quad\uparrow$ $n=2$ $\quad\uparrow$ $n=3$

odd #s: $2k+1$, $2k-1$ ...

Review last time

$a_n = n^3 = f(n)$

$f(x) = x^3$



Questions

31

~~37 tan(n)~~

49

$a_n = n^3$

$\frac{n}{2^n}$

$\left(1-\frac{2}{n}\right)^n$

bounded? NO. $\Rightarrow$ DIVERGENT

monotone? YES. it is monotone increasing.

(a convergent sequence must be bounded)

#31 $a_n = \frac{n}{2^n}$

idea if $a_n = f(n)$ and $\lim_{x\to\infty} f(x) = L$ then $a_n \to L$.

$f(x) = \frac{x}{2^x}$

$\lim_{x\to\infty} \frac{x}{2^x}$ "$\frac{\infty}{\infty}$"

$\overset{LH}{=} \lim_{x\to\infty} \frac{\frac{d}{dx}[x]}{\frac{d}{dx}[2^x]}$

$= \lim_{x\to\infty} \frac{1}{2^x \cdot \ln 2}$ "$\frac{1}{\infty}$"

$= \boxed{0}$

So $a_n \to 0$.

$x \ll 2^x$ growth rates

log $\ll$ polynomial $\ll$ exp

can also use: bounded monotone sequences converge.

Ex

$a_n = \left(1 - \frac{1}{n}\right)^n$

$f(x) = \left(1 - \frac{1}{x}\right)^x = y$

$\ln\left(1 - \frac{1}{x}\right)^x = \ln y$

$x \cdot \ln\left(1 - \frac{1}{x}\right) = \ln y$ as $x \to \infty$

" $\infty \cdot \ln(1)$ "

" $\infty \cdot 0$ "

$\lim_{x \to \infty} \left(1 - \frac{1}{x}\right)^x$

$(1 - 0)^\infty = 1^\infty$

indeterminate

" $\frac{0}{0}$ " $\frac{\ln\left(1 - \frac{1}{x}\right)}{\left(\frac{1}{x}\right)} = \ln y$

Now can apply LH to $\ln y$:

$\lim_{x \to \infty} \frac{\ln\left(1 - \frac{1}{x}\right)}{\left(\frac{1}{x}\right)}$ " $\frac{0}{0}$ " $\leftarrow \ln\left(1 - \frac{1}{\infty}\right) = \ln(1 - 0) = 0$, $\leftarrow \frac{1}{\infty}$

#49: you have 2

$\overset{LH}{=} \lim_{x \to \infty} \frac{\frac{d}{dx}\left[\ln\left(1 - \frac{1}{x}\right)\right]}{\frac{d}{dx}\left[\frac{1}{x}\right]}$ ✓ ✓

#49: it's 2

$\frac{d}{dx}\left[1 - \frac{1}{x}\right] = (-)\frac{d}{dx}\left[\frac{1}{x}\right]$

$= (-)\left[-\frac{1}{x^2}\right]$

$= \frac{1}{x^2}$

$= \lim_{x \to \infty} \frac{\boxed{\frac{1}{1 - \frac{1}{x}}} - \cancel{\left(-\frac{1}{x^2}\right)}}{\cancel{\left(\frac{-1}{x^2}\right)}}$

$= \lim_{x \to \infty} \boxed{\frac{-1}{1 - \frac{1}{x} \to 0}} = \frac{-1}{1} = \ln y$

look for e^{-2}!!

$e^{-1} = e^{\ln y} = y$ limit is $\boxed{\frac{1}{e}}$

Maybe you have done $(1+\frac{1}{n})^n \to e$

Use this strategy on #49.

#49 $\quad \ln(x^3) = 3 \cdot \ln x = \frac{\ln x}{(\frac{1}{3})}$ ratio!

Further Questions

$y = (1-\frac{1}{x})^x$

want: $\lim_{x\to\infty} y$

got: $\lim_{x\to\infty} \ln y = -1$

$e \qquad e$

raise by e:

$$\lim_{x\to\infty} e^{\ln y} = \lim_{x\to\infty} y = e^{-1}$$

Qu $\quad \frac{d}{dx} \ln(\underbrace{1-\frac{1}{x}}_{u}) = \underbrace{\frac{1}{u}}_{\text{outside}} \cdot \underbrace{u'}_{\text{inside}}$ chain Rule.

$$\frac{d}{dx} \ln x = \frac{1}{x}$$