213-02 notes

Quiz 3 is available after class.

Midterm will be similar in terms of submission

but timed during 1:20 – 2:30pm.

5.3 Integral Test

Ex 1

$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{n}$

$\frac{1}{n} = a_n \to 0$ seq. converges

$\sum a_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots$ series diverges

$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^2}$

$\frac{1}{n^2} = b_n \to 0$ seq. converges

$\sum b_n = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$

series converges

conclusion

$a_n \to 0$ is not a reliable test of whether the series converges. Sometimes it does, sometimes it doesn't because $a_n \to 0$ but not fast enough.

Divergence Test

if $a_n \not\to 0$, $\sum a_n$ diverges

if $a_n \to 0$, test gives no information

Ex 2 $\quad \sum_{n=1}^{\infty} 1 = 1+1+1+1+\cdots$

$a_n = 1 \longrightarrow 1$

$s_k = \sum_{n=1}^{k} 1 = \underbrace{1+1+\cdots+1}_{k} = k$

$\lim_{k\to\infty} S_k = \lim_{k\to\infty} \sum_{n=1}^{k} 1 = \sum_{n=1}^{\infty} 1 = \lim_{k\to\infty} k = \boxed{\infty}$

The series diverges and has no sum.

Answer using Divergence Test

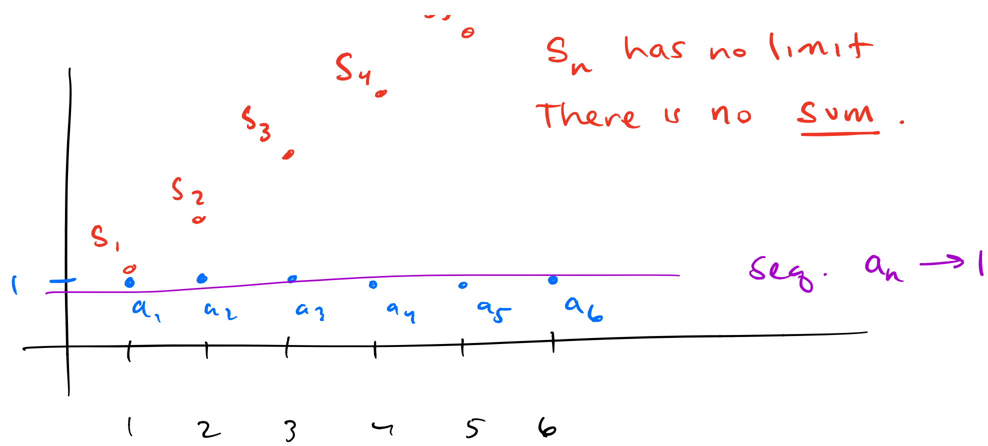
$a_n \to 1 \neq 0$, so $\sum a_n$ diverges, by the Divergence Test.

Also make a table $\quad$ Let $a_n = 1$

n	a_n	$S_n = a_1 + a_2 + \cdots + a_n$	
1	$1 = a_1$	1	$= 1$
2	$1 = a_2$	$1+1$	$= 2$
3	$1 = a_3$	$1+1+1$	$= 3$
4	1		4
5	1		5
6	1		6

S_6

S_5



The sequence $\{a_n\}$ converges to 1.

$\sum_{1} a_n$ diverges (to ∞).

#139, #141

#139 $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \dfrac{n}{5n^2-3}$ $\quad$ "$\dfrac{\infty}{\infty}$" L'Hôpital's

$$= \lim_{x\to\infty} \frac{\frac{d}{dx}[x]}{\frac{d}{dx}[5x^2-3]} = \lim_{x\to\infty} \frac{1}{10x} = \text{"}\frac{1}{\infty}\text{"} = \boxed{0}$$

$$\lim_{x\to\infty} \frac{p(x)}{q(x)} = \boxed{0}$$ $\longleftarrow$ short cut.

deg $q(x)$ > deg $p(x)$ *

Alternative

$$\lim_{n\to\infty} \frac{n}{5n^2-3} \frac{\left(\frac{1}{n^2}\right)}{\left(\frac{1}{n^2}\right)} = \lim_{n\to\infty} \frac{\frac{n}{n^2}}{\left(\frac{5n^2}{n^2} - \frac{3}{n^2}\right)}$$

$$= \lim_{n\to\infty} \frac{\cancel{\frac{1}{n}}^{\to 0}}{5 - \cancel{\frac{3}{n^2}}^{\to 0}} = \frac{0}{5-0} = \frac{0}{5} = \boxed{0} \checkmark$$

$a_n \to 0$ so test gives no info abt the series.

#141 $$\lim_{n\to\infty} \frac{(2n+1)(n+1)}{(n+1)^2} \sim \frac{2n^2}{n^2}$$

You can multiply this out and use either

(1) L'Hopitals

(2) dividing by heghest power of n.

In either case, bc $\deg p(x) = \deg q(x)$ $(=2)$ the limit is the ratio of the coefficients in the leading terms

$$\lim_{n\to\infty} \frac{(2n+1)(n-1)}{(n+1)^2} = \frac{2}{1} = 2$$

ratio of leading terms.

$a_n \to 2$

So series diverges (has no sum) by Divergence Test. ✓