

213-02 Discussion5.2 #126 (Example 2 in lecture.)

$n = \# \text{ of quarters}$

$a_n = \text{amount of \$ after } n^{\text{th}} \text{ quarter}$

1

$$d + r d = (1+r)d = \underbrace{1.01d}_{a_1}$$

2

$$\underbrace{d + 1.01d}_{(1+r)(d+1.01d)} + r \underbrace{(d+1.01d)}_{a_1} = \underbrace{1.01(d+1.01d)}_{a_2}$$

3

$$\underbrace{(a_2 + d) + r(a_2 + d)}_{(a_2 + d) + r(a_2 + d)} = 1.01(a_2 + d)$$

$d = 10$ (\\$ at beginning of each quarter)

$$r = \frac{0.4}{4} = .01 \quad (\text{monthly interest rate})$$

$$1+r = 1+.01 = 1.01$$

recursive pattern : $a_{n+1} = 1.01(d + a_n)$

explicit formula : $a_n = f(n)$

b_n = interest after n quarters.

$$b_1 = a_1 - d = 1.01d - d$$

$$b_2 = a_2 - 2d$$

$$b_n = a_n - n \cdot d$$

$$\begin{aligned}
 a_3 &= 1.01(a_2 + d) = 1.01(1.01(d + 1.01d) + d) \\
 &= 1.01d + 1.01^2d + 1.01^3d \\
 &\vdots \\
 &d \left(\frac{1.01^{n+1} - 1}{0.01} \right) \dots
 \end{aligned}$$

$$b_n = d \left(\frac{1.01^{n+1} - 1}{0.01} - (n+1) \right)$$

Compare the formulas:

$$\textcircled{1} \quad b_n = 10 \left(\frac{1.01^{n+1} - 1}{0.01} - (n+1) \right) \quad \left. \begin{array}{l} \text{explicit} \\ \text{formula} \end{array} \right\}$$

Sequence $(b_n, n, 1, 8)$

$$\textcircled{2} \quad b_n = a_n - n \cdot d \quad \text{where}$$

$$a_{n+1} = 1.01(a_n + 10) \quad \left. \begin{array}{l} \text{recursive} \end{array} \right\}$$

$a_1 = 10.1$

Iteration List $(f(x), 0, 8)$

$$f(x) = 1.01(x + 10)$$