Math 213 Spring 2020 Name: _____

Practice for Final Exam

How to use this study aid:

- Try to do all the problems. We can go over your questions on Monday, but I will ask for students to contribute answers for us to discuss.
- For each problem on the test, go to the recommended study sections and try to write **one more** example problem and solve.
- If you are having trouble with the specific problems, go to the HWs, Quizzes and Midterm to see if you can find similar problems. Do those problems again on a **blank sheet**. Check your work with the answer keys or ask me if you have questions.
- The test is written for **2 hours** with some time to upload/download. You are advised to spend no more than 15-20 min on each problem so that you have time to go back and check your work. If you are not doing your quizzes in 15 minutes, then you should go back and study until you can solve each quiz in 15 minutes. Again, try writing a new problem for each quiz and solve that problem.
- Write enough, but not too much. This is an important skill in math. Please check my comments on your previous work as well as answer keys. Ask if you have questions.
- This is an open book test, but it is **not enough** to just have the book open. You need to have had **good practice** before the test to be able to do the problems in 2 hours.

Show all your work for each problem. Answers with insufficient work shown will not receive full credit.

Instructions: This test is open book and open notes and you may use a scientific calculator. You may not collaborate with any other person and you may not search for information online or in any other source. Your work must be completely justified and written in your own words.

Work that is not fully justified will not receive full credit. Papers with *unusually* identical language, or with language that matches with an online source, will be referred to the Office of Student Conduct for further investigation and may result in a zero for the assignment and/or further penalties.

1. (20 points) **Parametric Equations.**

   Study: 7.1-7.2, especially 7.1 #39-46

   Consider the parametrization

$$x = \cos 2t, \quad y = \sin 2t; \quad 0 \leq t \leq \pi/2$$

   (a) Sketch the parametrized curve using any method, but you must explain your thinking in clear sentences, or show mathematical work. Pay attention to the given domain.

   (b) Choose **one** of (i) or (ii) below – only **one** will be graded.

      i. Set up and solve an integral for the arc length of this curve
      OR

      ii. Calculate the equation of the tangent line to this curve at some point using methods of 7.2.

2. (30 points) **Geometric Series.**

Study: p. 442 in 5.1 discussion of geometric sequences $\{r^n\}$

pp. 458-460 in 5.2 discussion of geometric series $\sum r^n$

Consider $\displaystyle\sum_{n=1}^{\infty} \left(\frac{9}{4}\right)^{n+1}$

(a) Determine whether the series converges, and if it does, find the sum. Show all your work.

(b) What does S_N represent?

(c) Suppose that you are told that $S_N = \dfrac{1-(9/4)^N}{1-(9/4)}$ for the series above. Determine whether the sequence $\{S_N\}$ converges or diverges. If it converges, find its limit. What is the relationship between the convergence of this sequence and the convergence of the series, above?

3. **Power Series.**

Study: 6.1

Consider the power series $\sum_{n=1}^{\infty} \frac{(n!)^2(x-1)^n}{(2n)!}$.

(a) Find the Radius of Convergence.

(b) Does this series converge at $x = 4$? At $x = -4$? Explain your thinking.

4. **Series Convergence Tests for Positive Series**

Study: 5.3, 5.4

Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$. Use your choice of either the Integral Test or the Limit Comparison Test (with p-series) to determine whether the given series converges or diverges.

5. **Alternating Series Test with Remainder**

Study: Ch. 5 5.2 #93-96, 5.5 280-285

The given series converges by Alternating Series Test. Use the estimate $|R_N| \leq b_{N+1}$ to find the least value of N that guarantees that the sum S_N differs from the infinite sum

$$\sum_{n=1}^{\infty} \left(-\frac{1}{5}\right)^n$$

by at most an error of 0.01.

Answer

(a) What is N?

(b) What is S_N and what is the actual sum S of the series?

(c) Is $|S - S_N| < 0.01$?

6. (20 points) **Taylor Series.**

Study: 6.3

(a) Find the Taylor polynomial centered at the point $a = \pi$ of degree three, $p_3(x)$, approximating $f(x) = \sin x$.

(b) Calculate $p_3(2\pi)$ (still centered at $a = \pi$) and $\sin 2\pi$ and calculate the absolute value of their difference (the absolute error).

(c) Calculate a bound on the absolute error in part (b) that is given by the Taylor Remainder Theorem.