Section 7.2 - Calculus of Parametric Curves

$$\boxed{\frac{dy}{dx} = \frac{y'(t)}{x'(t)}} = \frac{dy/dt}{dx/dt}$$

Proof Given $y = y(t)$ $\quad x = x(t)$

$$y = F(x) = F(x(t)) \qquad \text{take } \frac{d}{dt}:$$

$$\frac{d}{dt}[y] = \frac{d}{dt}[F(x(t))]$$

$$= F'(x(t)) \cdot \frac{d}{dt}[x(t)]$$

$$y'(t) = F'(x) \cdot x'(t)$$

$$\frac{y'(t)}{x'(t)} = F'(x) = \frac{dy}{dx}$$

Ex 1 $\quad x = t + 2 \quad (x = a + bt)$. line or

$y = 3 - 2t \quad (y = c + dt)$. line segment.

if $-\infty < t < \infty$

Problem Find $\frac{dy}{dx}$ at $t = 2$.

$x'(t) = 1 + 0 = 1$

$y'(t) = 0 - 2 = -2$

$$\frac{dy}{dx} = \frac{-2}{1} = -2$$

$$\left.\frac{dy}{dx}\right|_{t=2} = \boxed{-2} \checkmark$$

check:

$x = t + 2$

$x - 2 = t$ $\xrightarrow{\text{plug}}$ $y = 3 - 2t = 3 - 2(x-2)$

$y = 3 - 2x + 4 = -2x + 7$

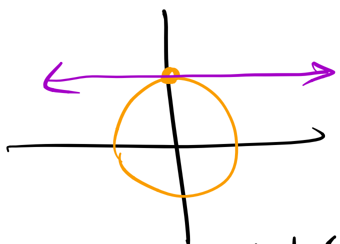
<u>Ex 1</u> $x = \cos t$ Find $\frac{dy}{dx}$ at $t = \frac{\pi}{2}$

$y = \sin t$

$$\left.\begin{aligned} x' &= -\sin t \\ y' &= \cos t \end{aligned}\right\} \rightarrow \frac{dy}{dx} = \frac{\cos t}{-\sin t}\Bigg|_{\frac{\pi}{2}} = \frac{0}{-1} = \boxed{0}$$

<u>Check</u> : Elim t :

$x^2 + y^2 = 1$



$m_{tan} = \frac{dy}{dx} = 0$

<u>Arc length Formula</u>

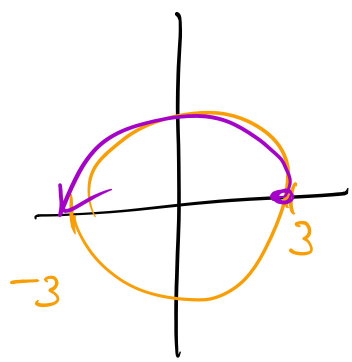
$a \le t \le b$

$$\int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$$

<u>Ex 2</u> $x = 3\cos t$

$y = 3\sin t$

$0 \le t \le \pi$



<u>Check</u> $\frac{\text{circumf}}{2} = \frac{\cancel{2}\pi(3)}{\cancel{2}} = 3\pi$

$x' = -3\sin t$

$y' = 3\cos t$

$$\int_0^{\pi} \sqrt{(-3\sin t)^2 + (3\cos t)^2}\, dt$$

$$\sqrt{9\sin^2 t + 9\cos^2 t}$$

$$\sqrt{9(\sin^2 t + \cos^2 t)}$$

$$\sqrt{9(1)}$$

$$= \int_0^{\pi} 3\, dt = 3t \Big|_0^{\pi} = 3(\pi - 0) = \boxed{3\pi}$$

The parametrized curve is the upper semicircle of the radius 3 circle at the origin. Its arc length is 3π.