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Quiz 3

1. Determine whether the sequence converges or diverges. If the sequence converges, find the limit. Show all of your work.

(a) $\left\{\left(-\frac{1}{5}\right)^n\right\}$

$a_1=\left(-\frac{1}{5}\right)^1\ a_2=\left(-\frac{1}{5}\right)^2\ a_3=\left(-\frac{1}{5}\right)^3\ a_4=\left(-\frac{1}{5}\right)^4$

$a_n=\left\{\left(-\frac{1}{5}\right)^1,\left(-\frac{1}{5}\right)^2,\left(-\frac{1}{5}\right)^3,\left(-\frac{1}{5}\right)^4,\dots\right\}$

$=\left\{-\frac{1}{5},\frac{1}{25},-\frac{1}{125},\frac{1}{625},\dots\right\}$ → Neither increasing nor decreasing (oscillates between neg and pos values)

Bounded below at $y=-\frac{1}{5}$

Bounded above at $y=\frac{1}{25}$

Absolute Value Theorem

$\lim_{n\to\infty}|a_n|=\phi$ then, $\lim_{n\to\infty}a_n=0$

$\lim_{n\to\infty}\left|\left(-\frac{1}{5}\right)^n\right|=\phi$

thus, $\boxed{\lim_{n\to\infty}\left(-\frac{1}{5}\right)^n=\phi}$ → Converges to ϕ

(b) $\left\{\frac{2^n}{1+n}\right\}$

$a_1=\frac{2^1}{1+1}=\frac{2}{2}=1\quad a_2=\frac{2^2}{1+2}=\frac{4}{3}\quad a_3=\frac{2^3}{1+3}=\frac{8}{4}\quad a_4=\frac{2^4}{1+4}=\frac{16}{5}\quad a_5=\frac{2^5}{1+5}=\frac{32}{6}\quad a_6=\frac{2^6}{1+6}=\frac{64}{7}$

$a_n=\left\{1,\frac{4}{3},\frac{8}{4},\frac{16}{5},\frac{32}{6},\frac{64}{7},\dots\right\}$

→ seems to keep getting infinetly larger without approaching any number

• Not Bounded

$2^n>0$

$n+1>0$

as $n\to\infty$

$\lim_{n\to\infty}\frac{2^n}{1+n}=\infty$

$\boxed{\text{Thus, } a_n \text{ is divergent}}$

Limit D.N.E.

2. Convert the equations from polar form to rectangular/Cartesian form, or vice versa.

(a) $\theta=\pi/4$ (convert to rectangular/Cartesian)

$x=r\cos\theta$
$y=r\sin\theta$

$(r,\frac{\pi}{4})$

$x=r\cos(\frac{\pi}{4})$ $=(x,y)$

$y=r\sin(\frac{\pi}{4})$ $\boxed{=(r\cos(\frac{\pi}{4}),r\sin(\frac{\pi}{4}))}$

(b) $x^2+y^2=9$ (convert to polar)

$x=r\cos\theta$
$y=r\sin\theta$

$x^2+y^2=r^2\cos^2\theta+r^2\sin^2\theta$

$x^2+y^2=r^2(\cos^2\theta+\sin^2\theta)$

$x^2+y^2=r^2(1)$

$x^2+y^2=r^2$

$x^2+y^2=3^2$

$\boxed{\text{Thus, } r=3}$