Math 213 Spring 2020 Name: Colin Morris-Moncada

Quiz 4

1. Determine whether the series converges, and if it converges, find the sum. Justify your answer completely.

(a) $\displaystyle\sum_{n=1}^{\infty}\left(-\frac{1}{5}\right)^{n-1}$

$|r|<1$, so series converges

$$S_\infty = \frac{a}{1-r} = \frac{1}{1+\frac{1}{5}} = \frac{1}{\frac{5}{5}+\frac{1}{5}} = \frac{1}{\frac{6}{5}} = \frac{1}{1}\cdot\frac{5}{6} \rightarrow \boxed{=\frac{5}{6}}$$

Infinite terms $S_\infty = \{1 - \frac{1}{5} + \frac{1}{25} - \frac{1}{125} + \dots\}$

Finite # of terms $S_k = \{1 - \frac{1}{5} + \frac{1}{25} - \frac{1}{125} + \frac{1}{625}\}$

$\rightarrow S_1, S_2, S_3, S_4, S_5$

(b) $\displaystyle\sum_{n=1}^{\infty}\left(-\frac{4}{5}\right)^{n}$

$|r|<1$, so series converges

$$S_\infty = \frac{a}{1-r} = \frac{-\frac{4}{5}}{1+\frac{4}{5}} = \frac{-\frac{4}{5}}{\frac{9}{5}} \rightarrow = -\frac{4}{5}\cdot\frac{5}{9}\quad \boxed{=-\frac{4}{9}}$$

$S_\infty = \{-\frac{4}{5} + \frac{16}{25} - \frac{256}{625} + \dots\}$

$S_k = \{-\frac{4}{5} + \frac{16}{25} - \frac{256}{625} + \frac{65{,}536}{390{,}625}\}$

$\rightarrow S_1, S_2, S_3, S_4$

(c) $\displaystyle\sum_{n=2}^{\infty}\left(\frac{4}{5}\right)^{n+3}$ [Hint: write out the first three terms of the series, then find a and r as in the Definition of geometric series on p. 459 in Section 5.2.]

$|r|<1$, so series converges

$$S_\infty = \frac{a}{1-r} = \frac{\frac{1024}{3125}}{1-\frac{4}{5}} = \frac{\frac{1024}{3125}}{\frac{1}{5}} = \frac{1024}{3125}\cdot\frac{5}{1} = \frac{5120}{3125} \rightarrow \boxed{=\frac{1024}{625}}$$

$= \{\frac{1024}{3125} + \frac{1024}{3125}\left(\frac{4}{5}\right)^1 + \frac{1024}{3125}\left(\frac{4}{5}\right)^2\}$

$S_k = \{\frac{1024}{3125} + \frac{4096}{15625} + \frac{16384}{78125}\}$

$\displaystyle\sum_{n=1}^{\infty} ar^{n-1} = a + ar^1 + ar^2 + ar^3 + \dots$

$\boxed{\text{So, } a = \frac{1024}{3125}}$

$\boxed{r = \frac{4}{5}}$