Wed 4/15 213-02 discussion

#171, 165, 167

#91 $y=x^2$, $y=x+2$ draw region bounded by the curves, rotate about x-axis and find volume.

solution:

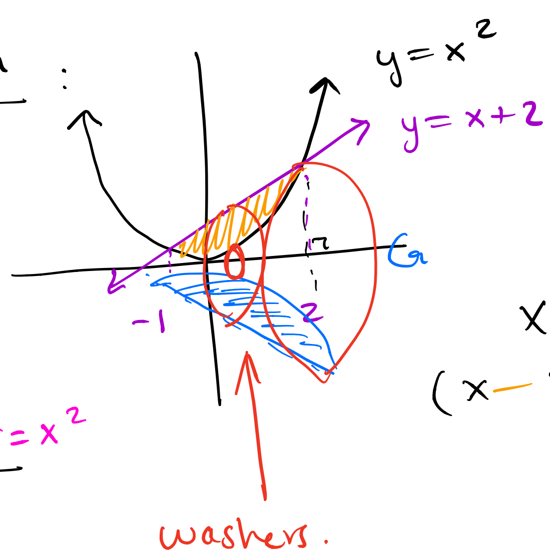
where do they intersect? set the ys equal:

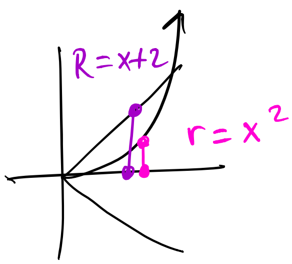
$$x^2 = x+2$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2, -1$$





Formula

$$\int_a^b \pi (R^2 - r^2)\, dx$$

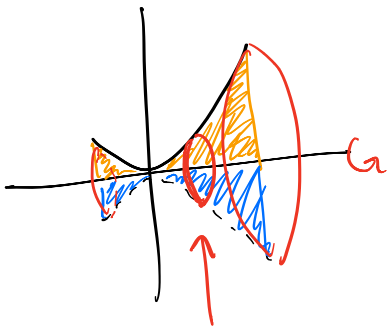
if rotating about the x-axis.

$$\pi \int_{-1}^{2} (x+2)^2 - (x^2)^2 \, dx$$

[if rotate about y-axis, use dy, and $R = x = g(y)$]

Alternative problem $y=x^2$ and $y=0$, $-1 \le x \le 2$

There's no space between the shaded region and the



axis of rotation, the x-axis. Use disks.

Formula : $\int_a^b \pi R^2 \, dx$

$= \int_{-1}^{2} \pi (x^2)^2 \, dx$

Don't need to set up $A(x) =$ area of cross-sections in disks/washers bc that work is done in the formula :

$$\text{vol} = \int_a^b A(x) \, dx = \int_a^b \underbrace{\pi R^2}_{A(x)} \, dx = \int_a^b \pi [f(x)]^2 dx$$

disks

$$= \int_a^b \underbrace{(\pi R^2 - \pi r^2)}_{A(x)} \, dx = \int_a^b \pi [f(x)^2 - g(x)^2] \, dx$$

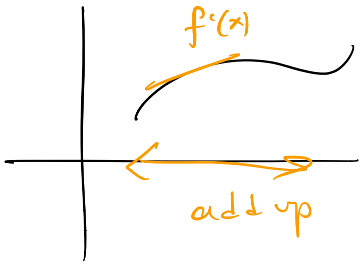
washers

2.4 work

#165 $y = 5x$ one function, not two

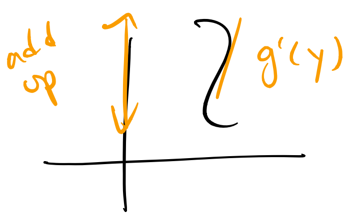
word "length", not "volume"

This is not 2.2 work, it requires p.172 $L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$



#165 $y = f(x)$ $0 \le x \le 2$

If you do $x = g(y)$ $-1 \le y \le 1$ # 167



$L = \int_c^d \sqrt{1 + [g'(y)]^2}\, dy$

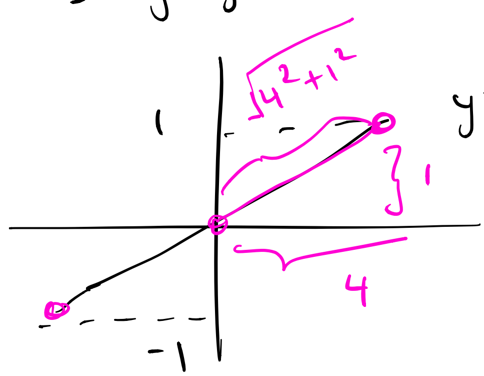
#167

$L = \int_{-1}^{1} \sqrt{1 + (4)^2}\, dy = \sqrt{17}\,(1 - (-1))$
$= 2\sqrt{17}$

$x = g(y) = 4y \longrightarrow g'(y) = 4$

$x = 4y$

$\frac{x}{4} = y$



$y = \frac{x}{4}$

Times 2: $L = 2\sqrt{17}$ ✓

#171 $y = x^{3/2} = f(x) \qquad 0 \le x \le 1$ OR

$g(y) = y^{2/3} = x \qquad 0 \le y \le 1$

$$f'(x) = \frac{3}{2} x^{\frac{3}{2}-\frac{2}{2}} = \frac{3}{2} x^{\frac{1}{2}}$$

$$[f'(x)]^2 = \left(\frac{3}{2}\right)^2 \cdot \left(x^{\frac{1}{2}}\right)^2 = \frac{9}{4} x$$

$$\sqrt{1 + [f'(x)]^2} = \sqrt{1 + \frac{9}{4} x}$$

integrate

$$L = \int_0^1 \sqrt{1 + \frac{9}{4} x} \, dx$$

$u = 1 + \frac{9}{4} x$

$du = \frac{9}{4} dx$

$\frac{4}{9} du = dx$

$$\int \sqrt{u} \, \frac{4}{9} du$$

$$\frac{4}{9} \int \sqrt{u} \, du$$

$$\frac{4}{9} \int u^{1/2} du = \frac{4}{9} \cdot \frac{u^{3/2}}{3/2} = \frac{4}{9} \cdot \frac{2}{3} \cdot u^{3/2}$$

$= \frac{8}{27} \left(1 + \frac{9}{4} x\right)^{3/2}$ is antiderivative.

$$L = \frac{8}{27} \left(1 + \frac{9}{4} x\right)^{3/2} \Big|_0^1$$

$$= \frac{8}{27}\left(\left(1+\frac{9}{4}(1)\right)^{3/2} - \left(1+\frac{9}{4}\cdot 0\right)^{3/2}\right)$$

$$= \frac{8}{27}\left(\left(\frac{13}{4}\right)^{3/2} - 1\right) \longleftarrow \text{preferred.}$$

ans : $\frac{13\sqrt{13} - 8}{27}$

$$a^{3/2} = a \cdot \sqrt{a}$$

$$4^{3/2} = 4\sqrt{4}$$
$$= 4 \cdot 2 = 8$$

$$\frac{\cancel{8}}{27}\left(\frac{13\sqrt{13}}{\cancel{8}} - 1 \times 8\right)$$

$$= \frac{1}{27}\left(13\sqrt{13} - 8\right) \checkmark$$