LID DRIVEN CAVITY FLOW

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OUR TEAM

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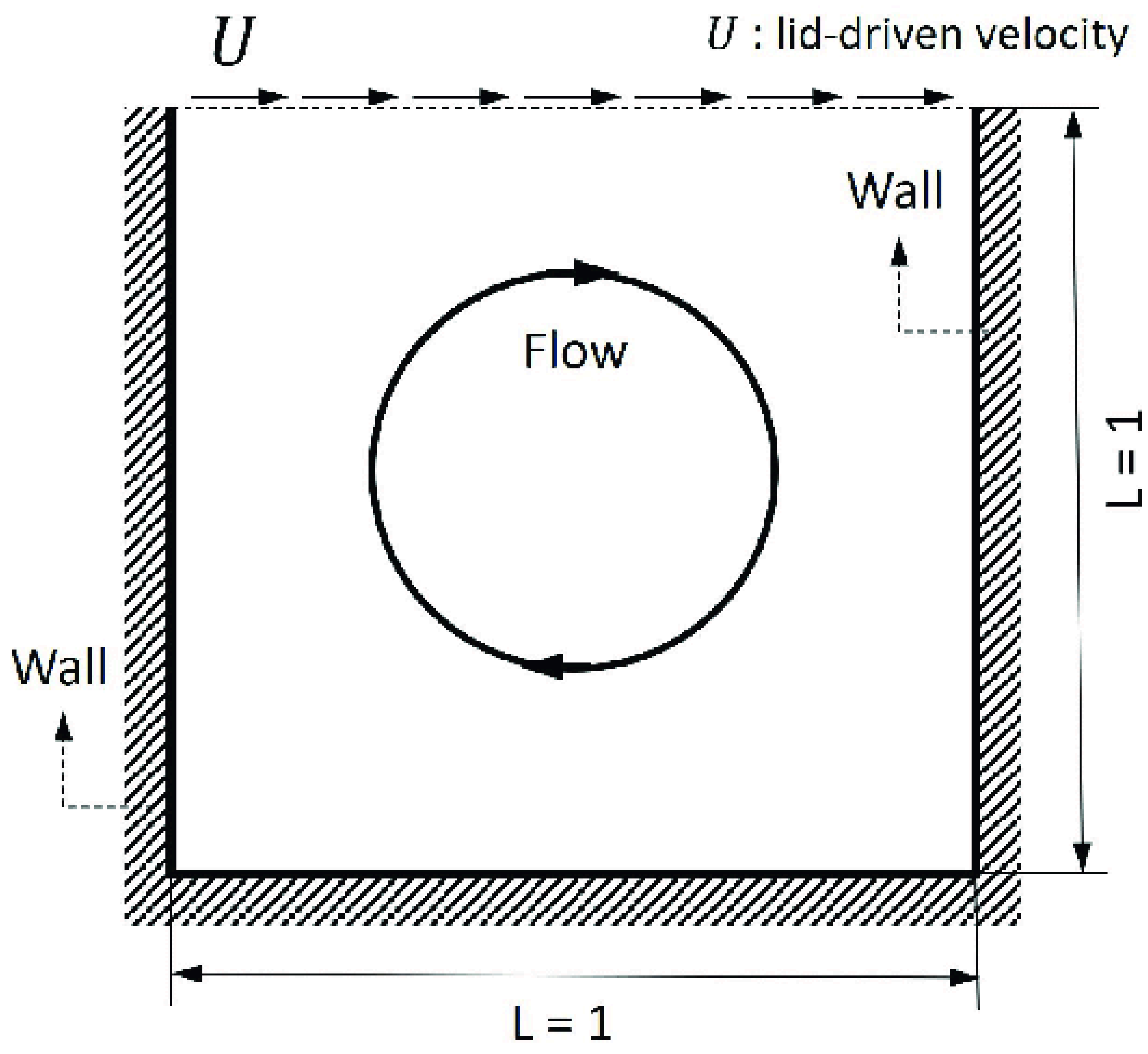
PROBLEM STATEMENT

- We have a lid (a square lid) in two dimensions and the top plate moves at some speed (say 1 m/s). The other three sides of the lid are stationary.
- The lid can be characterized in the xy plane with the origin at the bottom left.
- We wish to simulate this flow and study the properties.



U
U : lid-driven velocity
Wall
Flow
Wall
L = 1
L = 1

FORMULATION OF THE PROBLEM

- The momentum equation (differential approach) is:

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \vec{v}$$

FORMULATION OF THE PROBLEM

- We assume no body forces on the fluid. The above equation is the Navier Stokes' Equation.
- We get two equations for the x and y components of velocity and one equation for pressure.

SETTING UP THE EQUATIONS

- The following are the equations we are going to use:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = -\rho\left(\frac{\partial u}{\partial x}\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}\frac{\partial v}{\partial y}\right)$$

INITIAL AND BOUNDARY CONDITIONS

The initial condition is $u, v, p = 0$ everywhere, and the boundary conditions are:

- $u = 1$ at $y = 2$ (the "lid");
- $u, v = 0$ on the other boundaries;
- $\frac{\partial p}{\partial y} = 0$ at $y = 0$;
- $p = 0$ at $y = 2$;
- $\frac{\partial p}{\partial x} = 0$ at $x = 0, 2$.

THE STEPS

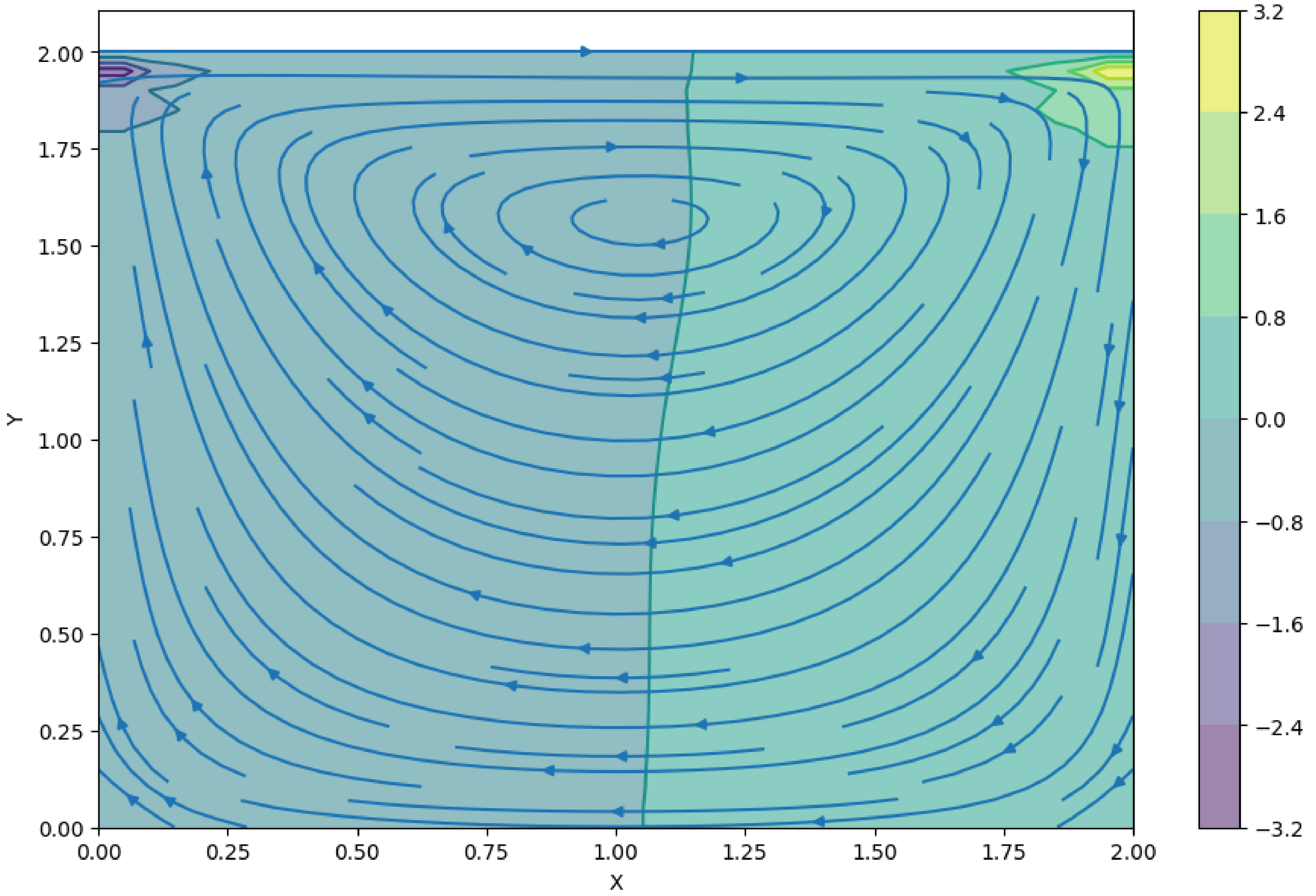
- We discretize these quasi-linear Partial Differential Equations over a 2x2 grid
- The discretized equations are then implemented in Python, where we iteratively compute the values for indices i and j across the grid.
- We plot the results and also try varying the initial velocity and the dimensions of the grid to observe different results

CODE IMPLEMENTATION

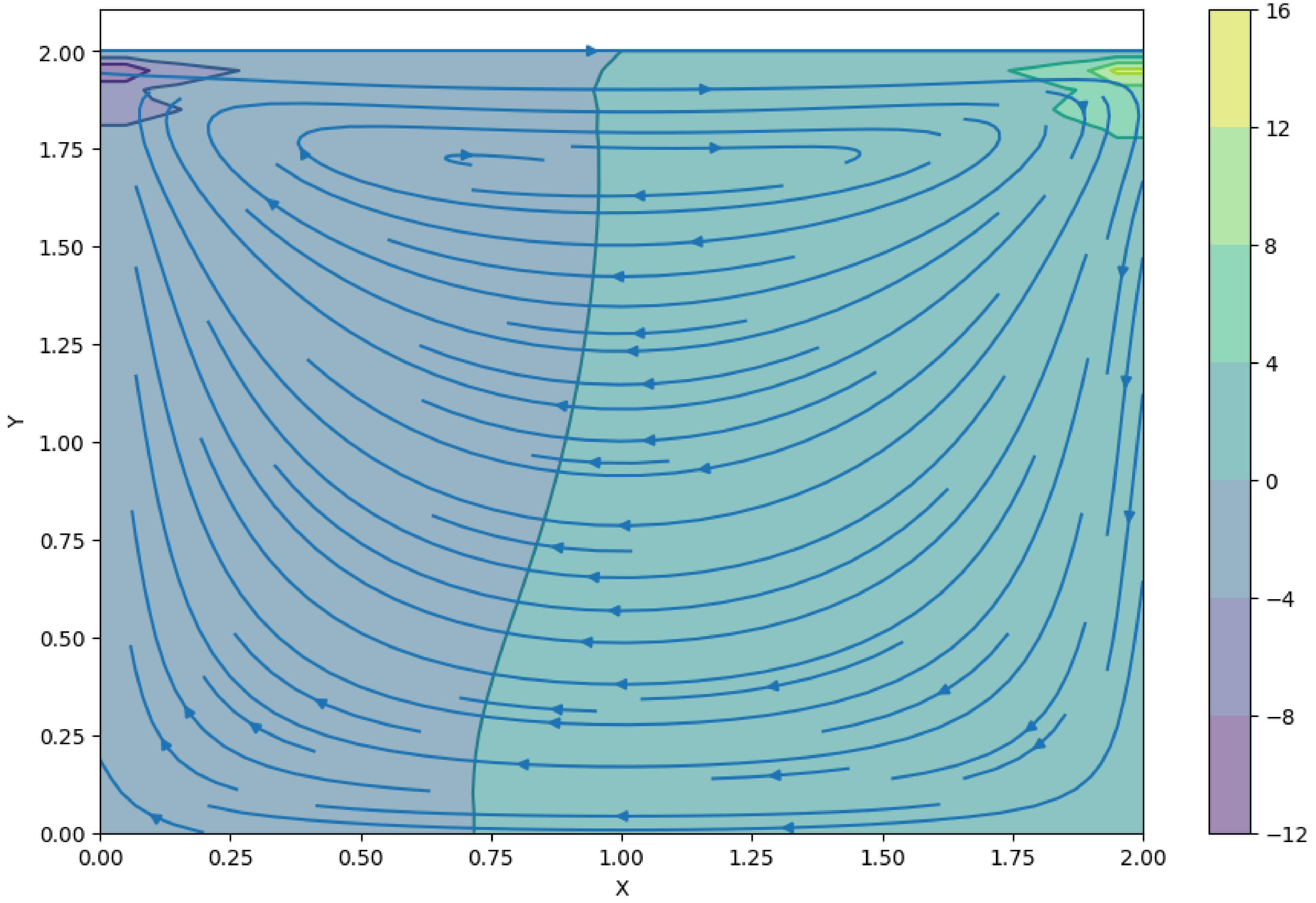
link to the code ;)

RESULTS

nt = 700 : U = 1



nt = 100 : U = 4



IMPROVEMENTS AND ALTERNATIVES

- The problem of lid-driven cavity flow can also be implemented using Physics-Informed-Neural-Networks or PINNs which offers the following advantages.
- **Mesh-free Solutions** - No need for mesh generation or grid discretization.
- **Flexibility -** Can handle complex geometries and boundary conditions.
- **Generatlization** - Potential to generalize the solution beyond the training data.

THE END

THANK YOU