

Note that for this analysis Squared Axes Ratio is mentioned as arxy (aspect ratio).

In [1]:

```
import pandas as pd

pd.set_option('display.max_columns', None)

pd.set_option('display.max_colwidth', None)

pd.set_option('display.max_rows', None)

ip08 = pd.read_csv('/kaggle/input/nfl-big-data-bowl-2026-analytics/114239_nfl_competition_files_published_analytics_final/train/input_2023_w08.csv')

op08 = pd.read_csv('/kaggle/input/nfl-big-data-bowl-2026-analytics/114239_nfl_competition_files_published_analytics_final/train/output_2023_w08.csv')

sup_data = pd.read_csv('/kaggle/input/nfl-big-data-bowl-2026-analytics/114239_nfl_competition_files_published_analytics_final/supplementary_data.csv')
```

/tmp/ipykernel\_39/2835549976.py:14: DtypeWarning: Columns (25) have mixed types. Specify dtype option on import or set low\_memory=False.

```
sup_data = pd.read_csv('/kaggle/input/nfl-big-data-bowl-2026-analytics/114239_nfl_competition_files_published_analytics_final/supplementary_data.csv')
```

In [2]:

```
import pandas as pd
import pywt
import numpy as np
import json

from tqdm import tqdm

# Load the datasets
# games = pd.read_csv('/kaggle/input/nfl-big-data-bowl-2025/games.csv')
# plays = pd.read_csv('/kaggle/input/nfl-big-data-bowl-2025/plays.csv')
# players = pd.read_csv('/kaggle/input/nfl-big-data-bowl-2025/players.csv')
# player_play = pd.read_csv('/kaggle/input/nfl-big-data-bowl-2025/player_play.csv')

# sup_data = pd.read_csv('/kaggle/input/nfl-big-data-bowl-2026-analytics/114239_nfl_competition_files_published_analytics_final/supplementary_data.csv')

tracking_data_dict_play = {}

# for week in tqdm(range(6, 8)):
#     tracking_week_i = pd.read_csv(f'/kaggle/input/nfl-big-data-bowl-2025/tracking_week_{week}.csv')

print("input files")
tracking_data_ip = pd.concat([
#     pd.read_csv(f'/kaggle/input/nfl-big-data-bowl-2025/tracking_week_{week}.csv') for week in tqdm(range(6, 8))
#     # pd.read_csv(f'/kaggle/input/nfl-big-data-bowl-2025/tracking_week_{week}.csv') for week in tqdm(range(1, 10))
#     pd.read_csv(f'/kaggle/input/nfl-big-data-bowl-2026-analytics/114239_nfl_competition_files_published_analytics_final/train/input_2023_w{week:02d}.csv') for week in tqdm(range(1, 19))
])

print("output files")
tracking_data_op = pd.concat([
#     pd.read_csv(f'/kaggle/input/nfl-big-data-bowl-2025/tracking_week_{week}.csv') for week in tqdm(range(6, 8))
#     # pd.read_csv(f'/kaggle/input/nfl-big-data-bowl-2025/tracking_week_{week}.csv') for week in tqdm(range(1, 10))
])
```

```
pd.read_csv(f'/kaggle/input/nfl-big-data-bowl-2026-analytics/114239_nfl_competition_files_published_analytics_final/train/output_2023_week{week:02d}.csv') for week in tqdm(range(1, 19))]
```

```
print("supplementary files")
sup_data = pd.read_csv('/kaggle/input/nfl-big-data-bowl-2026-analytics/114239_nfl_competition_files_published_analytics_final/supplementary_data.csv')
sup_data["play_action"] = sup_data["play_action"].astype("float64")
```

```
# # dtype_dict = {'passResult':'str', 'foulName1':'str', 'foulName2': 'str'}
```

```
# # for (gameId, playId), group_data in tracking_data.groupby(['gameId', 'playId']):
```

```
# for (gameId, playId), group_data in tqdm(tracking_data.groupby(['gameId', 'playId'])), total=tracking_data.groupby(['gameId', 'playId']).ngroups:
```

```
# #     print('>', end="")
```

```
#     if (gameId, playId) not in tracking_data_dict_play:
#         tracking_data_dict_play[(gameId, playId)] = {
#             'tracking_data': group_data,
#             'meta_data': {
#                 'gameId': gameId, 'playId': playId
#
#             }
#         }
#     else:
#         print('x', end="")
```

```
#         tracking_data_dict_play[(gameId, playId)]['tracking_data'] = group_data
#         tracking_data_dict_play[(gameId, playId)]['meta_data'] = {'gameId': gameId, 'playId': playId
#             }
```

input files

100% | ██████████ | 18/18 [00:19<00:00, 1.10s/it]

output files

100% | ██████████ | 18/18 [00:00<00:00, 32.63it/s]

supplementary files

```
/tmp/ipykernel_39/851042370.py:38: DtypeWarning: Columns (25) have
mixed types. Specify dtype option on import or set low_memory=False.
```

```
sup_data = pd.read_csv('/kaggle/input/nfl-big-data-bowl-2026-anal
ytics/114239_nfl_competition_files_published_analytics_final/supple
mentary_data.csv')
```

In [3]:

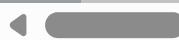
```
game_id = 2023091709 #2023102908      #2023122406
play_id = 1228 #596 # 1653
nfl_id = 44959

sup_data[(sup_data['game_id']==game_id) & (sup_data['play_id']==play_id)]
```

```
/usr/local/lib/python3.11/dist-packages/pandas/io/formats/format.p
y:1458: RuntimeWarning: invalid value encountered in greater
    has_large_values = (abs_vals > 1e6).any()
/usr/local/lib/python3.11/dist-packages/pandas/io/formats/format.p
y:1459: RuntimeWarning: invalid value encountered in less
    has_small_values = ((abs_vals < 10 ** (-self.digits)) & (abs_vals
> 0)).any()
/usr/local/lib/python3.11/dist-packages/pandas/io/formats/format.p
y:1459: RuntimeWarning: invalid value encountered in greater
    has_small_values = ((abs_vals < 10 ** (-self.digits)) & (abs_vals
> 0)).any()
```

Out[3]:

	game_id	season	week	game_date	game_time_eastern	home_team_abbr	visitor_tea
1385	2023091709	2023	2	09/17/2023	16:05:00	LA	SF



Adding player side

```
In [4]: ip_side = tracking_data_ip[['game_id', 'play_id', 'nfl_id', 'player_side']].drop_duplicates()

tracking_data_op = tracking_data_op.merge(
    ip_side,
    on=['game_id', 'play_id', 'nfl_id'],
    how='left'
)
```

```
In [5]: tracking_data_op['player_side'].isna().sum()
```

```
Out[5]: 0
```

```
In [6]: # tracking_data_op.head()
tracking_data_op['player_side'].value_counts()
```

```
Out[6]: player_side
Defense    402576
Offense    160360
Name: count, dtype: int64
```

```
In [7]: tracking_data_op.columns
```

```
Out[7]: Index(['game_id', 'play_id', 'nfl_id', 'frame_id', 'x', 'y', 'player_side'], dtype='object')
```

In [8]:

```
tracking_data_op.shape  
tracking_data_op.head()
```

Out[8]:

	game_id	play_id	nfl_id	frame_id	x	y	player_side
0	2023090700	101	46137	1	56.22	17.28	Defense
1	2023090700	101	46137	2	56.63	16.88	Defense
2	2023090700	101	46137	3	57.06	16.46	Defense
3	2023090700	101	46137	4	57.48	16.02	Defense
4	2023090700	101	46137	5	57.91	15.56	Defense

In [ ]:

In [9]:

```
tracking_data_ip['frame_id'].max() #123  
tracking_data_op['frame_id'].max() #94  
len(tracking_data_ip['frame_id']) #4880579  
len(tracking_data_op['frame_id']) #562936  
len(tracking_data_ip) #4880579  
len(tracking_data_op) #562936
```

Out[9]:

562936

In [10]:

```
# dtype_dict = {'passResult':'str', 'foulName1':'str', 'foulName2': 'str'}
```

```
ip_gb = tracking_data_ip.groupby(["game_id","play_id"], sort=False)
# # list(ip_gb.groups.keys())[10] # first 10 keys
# ip_gb = list(tracking_data_ip.groupby(["game_id","play_id"], sort=False).groups.keys())
```

```
op_gb = tracking_data_op.groupby(["game_id","play_id"], sort=False)
sup_gb = sup_data.groupby(["game_id","play_id"], sort=False)
```

```
tracking_data_dict_play = {}
```

```
from collections import defaultdict
from tqdm import tqdm
```

```
tracking_data_dict_play = defaultdict(lambda: {
    "input": None,
    "output": None,
    "supplementary": None,
    "meta_data": None,
})
```

```
# for key, grp in tqdm(ip_gb): #ip_grp:
#     if key not in tracking_data_dict_play:
#         tracking_data_dict_play[key]["input"] = grp
#         tracking_data_dict_play[key]["meta_data"] = {"gameId": key[0],
# "playId": key[1]}
#     else: print('x ip')
```

```
# INPUT
```

```
for key, grp in tqdm(ip_gb):
    d = tracking_data_dict_play[key] # auto-created if missing
    d["input"] = grp
    d["meta_data"] = {"gameId": key[0], "playId": key[1]}
```

```
# OUTPUT
```

```
for (game_id, play_id), grp in op_gb:
    d = tracking_data_dict_play[(game_id, play_id)]
    d["output"] = grp
    if d["meta_data"] is None:
        d["meta_data"] = {"gameId": game_id, "playId": play_id}

# SUPPLEMENTARY

for (game_id, play_id), grp in sup_gb:
    d = tracking_data_dict_play[(game_id, play_id)]
    d["supplementary"] = grp
    if d["meta_data"] is None:
        d["meta_data"] = {"gameId": game_id, "playId": play_id}

# for key, grp in op_gb:
#     if key not in tracking_data_dict_play:
#         tracking_data_dict_play[key]["output"] = grp
#         if tracking_data_dict_play[key]["meta_data"] is None:
#             tracking_data_dict_play[key]["meta_data"] = {"gameId": key[0], "playId": key[1]}
#     else: print('x op')

# for key, grp in sup_gb:
#     if key not in tracking_data_dict_play:
#         tracking_data_dict_play[key]["supplementary"] = grp # ALWAYS
# a DataFrame
#         if tracking_data_dict_play[key]["meta_data"] is None:
#             tracking_data_dict_play[key]["meta_data"] = {"gameId": key[0], "playId": key[1]}
#     else: print('x sup')

# for key, grp in ip_grp:
#     tracking_data_dict_play[key]["input"] = grp
#     tracking_data_dict_play[key]["meta_data"] = {"gameId": key[0], "playId": key[1]}

# for key, grp in op_gb:
#     tracking_data_dict_play[key]["output"] = grp
#     if tracking_data_dict_play[key]["meta_data"] is None:
#         tracking_data_dict_play[key]["meta_data"] = {"gameId": key[0], "playId": key[1]}
```

```
"playId": key[1]}

# for key, grp in sup_gb:
#     tracking_data_dict_play[key]["supplementary"] = grp # ALWAYS a Dataframe
#     if tracking_data_dict_play[key]["meta_data"] is None:
#         tracking_data_dict_play[key]["meta_data"] = {"gameId": key[0],
# "playId": key[1]}

# # for (gameId, playId), group_data in tracking_data.groupby(['gameId',
# 'playId']):
# for (gameId, playId), group_data_ip in tqdm(tracking_data_ip.groupby(
# ['gameId', 'playId']), total=tracking_data_ip.groupby(['gameId', 'playId']).ngroups):

# #     print('>', end="")

#     if (gameId, playId) not in tracking_data_dict_play:
#         tracking_data_dict_play[(gameId, playId)] = {
#             'tracking_data': group_data_ip,
#             'meta_data': {
#                 'gameId': gameId, 'playId': playId

#             }
#         }
#     else:
#         print('x', end="")

#         tracking_data_dict_play[(gameId, playId)]['tracking_data'] = group_data
#         tracking_data_dict_play[(gameId, playId)]['meta_data'] = {'gameId': gameId, 'playId': playId
#             }
```

100% |██████████| 14108/14108 [00:01<00:00, 7090.07it/s]

In [ 1]:

In [32]:

```
ip_keys = set(ip_gb.groups.keys())
op_keys = set(op_gb.groups.keys())
sup_keys = set(sup_gb.groups.keys())

# Are input and output keys identical?
print("ip == op:", ip_keys == op_keys)

# Are input and supplementary keys identical?
print("ip == sup:", ip_keys == sup_keys)

# Any common keys at all?
print("ip n op:", len(ip_keys & op_keys))
print("ip n sup:", len(ip_keys & sup_keys))
print("op n sup:", len(op_keys & sup_keys))

print(len(ip_keys), len(op_keys), len(sup_keys))
```

```
ip == op: True
ip == sup: False
ip n op: 14108
ip n sup: 14108
op n sup: 14108
14108 14108 18009
```

In [33]:

```
# KEYS ORDER

ip_list = list(ip_gb.groups.keys())
op_list = list(op_gb.groups.keys())

ip_list == op_list
```

Out[33]:

```
True
```

In [ ]:

In [34]:

```
ip_keys = set(ip_gb.groups.keys())
sup_keys = set(sup_gb.groups.keys())

extra_sup_keys = sup_keys - ip_keys
print("Rows in sup but not in ip:", len(extra_sup_keys))

mask = sup_data.apply(lambda row: (row["game_id"], row["play_id"]) in extra_sup_keys, axis=1)
sup_extra_rows = sup_data[mask]

sup_extra_rows.head()
```

Rows in sup but not in ip: 3901

```
/usr/local/lib/python3.11/dist-packages/pandas/io/formats/format.p
y:1458: RuntimeWarning: invalid value encountered in greater
    has_large_values = (abs_vals > 1e6).any()
/usr/local/lib/python3.11/dist-packages/pandas/io/formats/format.p
y:1459: RuntimeWarning: invalid value encountered in less
    has_small_values = ((abs_vals < 10 ** (-self.digits)) & (abs_vals
> 0)).any()
/usr/local/lib/python3.11/dist-packages/pandas/io/formats/format.p
y:1459: RuntimeWarning: invalid value encountered in greater
    has_small_values = ((abs_vals < 10 ** (-self.digits)) & (abs_vals
> 0)).any()
```

Out[34] :

	game_id	season	week	game_date	game_time_eastern	home_team_abbr	visitor_te
14108	2024120500	2024	14	12/05/2024	20:15:00	DET	GB
14109	2024120500	2024	14	12/05/2024	20:15:00	DET	GB
14110	2024120500	2024	14	12/05/2024	20:15:00	DET	GB
14111	2024120500	2024	14	12/05/2024	20:15:00	DET	GB
14112	2024120500	2024	14	12/05/2024	20:15:00	DET	GB

In [ ]:

In [ ]:

In [35]:

```
ip_gb3 = tracking_data_ip.groupby(["game_id", "play_id", 'nfl_id'], sort=False)
# # list(ip_gb.groups.keys())[:10]    # first 10 keys
# ip_gb = list(tracking_data_ip.groupby(["game_id", "play_id"], sort=False).groups.keys())

op_gb3 = tracking_data_op.groupby(["game_id", "play_id", "nfl_id"], sort=False)
# sup_gb3 = sup_data.groupby(["game_id", "play_id", 'nfl_id'], sort=False)
```

In [36]:

```
ip_keys3 = set(ip_gb3.groups.keys())
op_keys3 = set(op_gb3.groups.keys())
# sup_keys = set(sup_gb.groups.keys())

# Are input and output keys identical?
print("ip == op:", ip_keys3 == op_keys3)

# Are input and supplementary keys identical?
# print("ip == sup:", ip_keys == sup_keys)

# Any common keys at all?
print("ip n op:", len(ip_keys3 & op_keys3))
# print("ip n sup:", len(ip_keys & sup_keys))
# print("op n sup:", len(op_keys & sup_keys))

print(len(ip_keys3), len(op_keys3), len(sup_keys))
```

```
ip == op: False
ip n op: 4
173150 4
```

In [37]:

```
op_keys3.issubset(ip_keys3)
```

Out[37]:

```
True
```

In [ ]:

In [38]:

```
# sup_extra_rows = sup_data.merge(  
#     tracking_data_ip[["game_id", "play_id"]].drop_duplicates(),  
#     on=["game_id", "play_id"],  
#     how="left",  
#     indicator=True  
# ).query('_merge == "left_only"').drop(columns="_merge")  
  
# sup_extra_rows.head()
```

In [ ]:

In [39]:

```
game_id = 2023091709 #2023102908      #2023122406  
play_id = 1228 #596 # 1653  
nfl_id = 44959 #52433 #47834      #54475
```

In [40]:

```
tracking_data_ip[(tracking_data_ip['game_id'] == game_id) & (tracking_d  
ata_ip['play_id'] == play_id)]  
tracking_data_ip[(tracking_data_ip['game_id'] == game_id) & (tracking_d  
ata_ip['play_id'] == play_id)][['nfl_id']].unique()
```

Out[40]:

```
array([44878, 55942, 52844, 54677, 53532, 52607, 52835, 54718, 4482  
0,  
     47819, 40078, 56118, 54727, 44959])
```

In [41]:

```
# tracking_data_ip[(tracking_data_ip['game_id'] == game_id) & (tracking_
data_ip['play_id'] == play_id)]
tracking_data_op[(tracking_data_op['game_id'] == game_id) & (tracking_d
ata_op['play_id'] == play_id)]
# sup_data[(sup_data['game_id'] == game_id) & (sup_data['play_id'] == pl
ay_id)] # and sup_data['game_id'] == n_id
tracking_data_op[(tracking_data_op['game_id'] == game_id) & (tracking_d
ata_op['play_id'] == play_id)]['nfl_id'].unique()
```

Out[41]:

array([], dtype=int64)

In [42]:

```
tracking_data_ip.shape
# tracking_data_op.shape
```

Out[42]:

(4880579, 23)

In [43]:

```
print("output files")
tracking_data_op = pd.concat([
    pd.read_csv(f'/kaggle/input/nfl-big-data-bowl-2025/tracking_week_{week}.csv') for week in tqdm(range(6, 8))
    # pd.read_csv(f'/kaggle/input/nfl-big-data-bowl-2025/tracking_week_{week}.csv') for week in tqdm(range(1, 10))
    pd.read_csv(f'/kaggle/input/nfl-big-data-bowl-2026-analytics/114239_nfl_competition_files_published_analytics_final/train/output_2023_w{we
ek:02d}.csv') for week in tqdm(range(1, 19))
])

tracking_data_op.shape
```

output files

100%|██████████| 18/18 [00:00&lt;00:00, 52.35it/s]

Out[43]:

(562936, 6)

In [ ]:

In [ ]:

In [44]:

```
# tracking_data_ip['nfl_id'].value_counts(dropna=False)
# ball_rows = tracking_data_ip[tracking_data_ip['nfl_id'].isna()]
# ball_rows.head()

# ball_rows = tracking_data_op[tracking_data_ip['nfl_id'].isna()]
# ball_rows.head()

# ball_rows = sup_data[tracking_data_ip['nfl_id'].isna()]
# ball_rows.head()
```

In [ ]:

In [ ]:

## Functions

In [45]:

```
import numpy as np

# def calculate_eccentricity(A, B, C):
def calculate_eccentricity(A, B, C, s, acc, dis, o, dir_, theta_) :
    # Step 1: Calculate eigenvalues for semi-major and semi-minor axis lengths
    eig_val1 = 0.5 * (A + C + np.sqrt((A - C)**2 + B**2))
    eig_val2 = 0.5 * (A + C - np.sqrt((A - C)**2 + B**2))
    a = np.sqrt(1 / np.abs(eig_val1)) # Major axis length
    b = np.sqrt(1 / np.abs(eig_val2)) # Minor axis length
    # a, b = sorted([a, b], reverse=True) # Ensure a >= b

    # Step 2: Calculate the orientation angle of the principal axes
    # theta = 0.5 * np.arctan2(B, A - C)
    # theta_deg = np.degrees(theta_)
    # theta_deg = torch.degrees(theta_)
    theta_deg = torch.rad2deg(theta_)

    # Return eccentricity and orientation information
    # eccentricity = np.sqrt(1 - (b**2 / a**2)) if B**2 - 4 * A * C < 0
    # else np.sqrt(1 + (b**2 / a**2))
    # eccentricity =

    # Determine if alignment is close to horizontal or vertical
    # is_horizontal = np.isclose(theta % (np.pi / 2), 0, atol=1e-3)
    # Determine if close to horizontal or vertical
    # if np.isclose(theta_deg % 180, 0, atol=5): # e.g., -5° to +5°
    if torch.isclose((theta_deg % 180), torch.tensor(0.0, device=theta_deg.device), atol=5.0):
        orientation = "horizontal"
        eccentricity = a**2/b**2
    # elif np.isclose(theta_deg % 180, 90, atol=5): # e.g., 85° to 95°
    elif torch.isclose((theta_deg % 180), torch.tensor(90.0, device=theta_deg.device), atol=5.0):
        orientation = "vertical"
        eccentricity = b**2/a**2
```

```
else:  
    orientation = "diagonal" # if not close to 0° or 90°  
    eccentricity = b**2/a**2  
#        eccentricity = a**2/b**2  
  
#    # Return eccentricity and orientation information  
#    eccentricity = np.sqrt(1 - (b**2 / a**2)) if B**2 - 4 * A * C < 0  
else np.sqrt(1 + (b**2 / a**2))  
return eccentricity#, is_horizontal, theta
```

In [46]:

```
import torch  
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")  
device  
  
# device(type='cuda')
```

Out[46]:

```
device(type='cpu')
```

In [47]:

```
import pandas as pd
import numpy as np
from tqdm import tqdm
from scipy.optimize import minimize

# # Objective function to minimize
# def objective_fn(params, x, y):
#     return np.sum(conics(params, x, y)**2)

# Objective function to minimize
def objective_fn(params, x, y):
    return torch.sum(conics(params, x, y)**2)

import torch
from torch.optim import LBFGS

# # Define the closure function for optimization
# def closure():
#     optimizer.zero_grad() # Clear previous gradients
#     loss = objective_fn(params, sx, sy) # Compute loss using objective_fn
#     loss.backward() # Backpropagate the gradients
#     return loss

# def closure():
def closure(optimiser, sx, sy, params):
    optimizer.zero_grad()
    # Split params into sx and sy (assuming sx and sy are of the same size)
    #     sx = params[:len(sx)] # First part of params for sx
    #     sy = params[len(sx):] # Second part of params for sy

    # Now call the objective function with sx and sy
    loss = objective_fn(params, sx, sy)
    loss.backward()
    return loss

# Define conic equation
```

```
def conics(params, x, y):
    a, b, c, d, e, f = params
    return a * x**2 + b * x * y + c * y**2 + d * x + e * y + f

# # Objective function to minimize
# def objective_fn(params, x, y):
#     return np.sum(conics(params, x, y)**2)

def curvature_conic(A, B, C, D, E, x, y):
    # First derivative (dy/dx)
    numerator_first = -2*A*x - D
    denominator_first = B*x + 2*C*y + E
    dy_dx = numerator_first / denominator_first

    # Second derivative (d^2y/dx^2)
    # Derivatives of the numerator and denominator
    d_numerator_first_dx = -2*A
    d_denominator_first_dx = B

    # Using the quotient rule for the second derivative
    numerator_second = (d_numerator_first_dx * denominator_first) - (numerator_first * d_denominator_first_dx)
    denominator_second = denominator_first**2

    d2y_dx2 = numerator_second / denominator_second

    # Curvature calculation
    curvature = d2y_dx2 / (1 + dy_dx**2)**(3/2)

    return curvature

# def average_curvature(A, B, C, D, E, x_start, x_end, num_points=100):
#     x_values = np.linspace(x_start, x_end, num_points)
#     curvature_values = []

#     for x in x_values:
#         # To find corresponding y values, we would typically need to solve the conic equation.
#         # For simplicity, we can use a fixed value of y for average calculation, but this may not be accurate.
#         # Here, just for demonstration, we are considering y = 0, which might not always lie on the conic.
#         y = 0
#         curv = curvature_conic(A, B, C, D, E, x, y)
```

```

#         curvature_values.append(curv)

#     # Calculate average curvature
#     average_curv = np.mean(curvature_values)
#     return average_curv

def average_curvature(A, B, C, D, E, x_start, x_end, num_points=100):
    # Create a tensor of x values, using torch.linspace
    x_values = torch.linspace(x_start, x_end, num_points, device='cuda')
if torch.cuda.is_available() else 'cpu')

    # Initialize the curvature_values tensor
    curvature_values = torch.zeros_like(x_values)

    # Compute the curvature for each x value
    for i, x in enumerate(x_values):
        y = 0 # Fixed y value for the calculation (this can be modified
        based on specific requirements)
        curvature_values[i] = curvature_conic(A, B, C, D, E, x, y)

    # Calculate the average curvature using torch.mean
    average_curv = torch.mean(curvature_values)

return average_curv

# #hyperbolic only
# def calculate_eccentricity(A, B, C, s, acc, dis, o, dir_) :
def calculate_eccentricity_fx(A, B, C, s, acc, dis, o, dir_, theta_) :
    eig_val1 = 0.5 * (A + C + np.sqrt((A - C)**2 + B**2))
    eig_val2 = 0.5 * (A + C - np.sqrt((A - C)**2 + B**2))
    a = np.sqrt(1 / np.abs(eig_val1))
    b = np.sqrt(1 / np.abs(eig_val2))
    #    a, b = sorted([a, b], key=abs)
    #    a, b = sorted([a, b], key=abs, reverse=True)
    #    a, b = sorted([a, b])
    a, b = sorted([a, b], reverse=True)

    #        eccentricity = np.sqrt(1 - (b**2 / a**2))
    #        eccentricity = np.sqrt(1 + (b**2 / a**2))

```

```
#      return np.sqrt(1 + (b**2 / a**2)) #
#      return (b**2 / a**2)
#      return np.log(b**2 / a**2)
#      return (a / b) #aspect ratio
#      return (a**2/b**2) # not ^
#      return 2*b**2/a #latus rectum lr
#      return np.abs(theta_)
#      return B**2 - 4 * A * C #dis
#      return (b**2 / a**2) #aspect ratio
#      return np.sqrt((b**2 / a**2))
#else return np.nan

# Initial guess for the conic parameters
# p0 = [1, 1, 1, 1, 1, 1]
p0 = torch.tensor([1, 1, 1, 1, 1, 1], dtype=torch.float32, requires_grad=True, device=device)
```

In [ ]:

```
import pandas as pd
import numpy as np
from tqdm import tqdm
from scipy.optimize import minimize

# # Objective function to minimize
# def objective_fn(params, x, y):
#     return np.sum(conics(params, x, y)**2)

# Objective function to minimize
def objective_fn(params, x, y):
    return torch.sum(conics(params, x, y)**2)

import torch
from torch.optim import LBFGS

# # Define the closure function for optimization
# def closure():
#     optimizer.zero_grad() # Clear previous gradients
#     loss = objective_fn(params, sx, sy) # Compute loss using objective_fn
#     loss.backward() # Backpropagate the gradients
#     return loss

# def closure():
def closure(optimiser, sx, sy, params):
    optimizer.zero_grad()
    # Split params into sx and sy (assuming sx and sy are of the same size)
    #     sx = params[:len(sx)] # First part of params for sx
    #     sy = params[len(sx):] # Second part of params for sy

    # Now call the objective function with sx and sy
    loss = objective_fn(params, sx, sy)
    loss.backward()
    return loss

# Define conic equation
```

```
def conics(params, x, y):
    a, b, c, d, e, f = params
    return a * x**2 + b * x * y + c * y**2 + d * x + e * y + f

# # Objective function to minimize
# def objective_fn(params, x, y):
#     return np.sum(conics(params, x, y)**2)

def curvature_conic(A, B, C, D, E, x, y):
    # First derivative (dy/dx)
    numerator_first = -2*A*x - D
    denominator_first = B*x + 2*C*y + E
    dy_dx = numerator_first / denominator_first

    # Second derivative (d^2y/dx^2)
    # Derivatives of the numerator and denominator
    d_numerator_first_dx = -2*A
    d_denominator_first_dx = B

    # Using the quotient rule for the second derivative
    numerator_second = (d_numerator_first_dx * denominator_first) - (numerator_first * d_denominator_first_dx)
    denominator_second = denominator_first**2

    d2y_dx2 = numerator_second / denominator_second

    # Curvature calculation
    curvature = d2y_dx2 / (1 + dy_dx**2)**(3/2)

    return curvature

# def average_curvature(A, B, C, D, E, x_start, x_end, num_points=100):
#     x_values = np.linspace(x_start, x_end, num_points)
#     curvature_values = []

#     for x in x_values:
#         # To find corresponding y values, we would typically need to solve the conic equation.
#         # For simplicity, we can use a fixed value of y for average calculation, but this may not be accurate.
#         # Here, just for demonstration, we are considering y = 0, which might not always lie on the conic.
#         y = 0
#         curv = curvature_conic(A, B, C, D, E, x, y)
```

```

#         curvature_values.append(curv)

#     # Calculate average curvature
#     average_curv = np.mean(curvature_values)
#     return average_curv

def average_curvature(A, B, C, D, E, x_start, x_end, num_points=100):
    # Create a tensor of x values, using torch.linspace
    x_values = torch.linspace(x_start, x_end, num_points, device='cuda')
    if torch.cuda.is_available() else 'cpu')

    # Initialize the curvature_values tensor
    curvature_values = torch.zeros_like(x_values)

    # Compute the curvature for each x value
    for i, x in enumerate(x_values):
        y = 0 # Fixed y value for the calculation (this can be modified
        based on specific requirements)
        curvature_values[i] = curvature_conic(A, B, C, D, E, x, y)

    # Calculate the average curvature using torch.mean
    average_curv = torch.mean(curvature_values)

    return average_curv

# #hyperbolic only
# def calculate_eccentricity(A, B, C, s, acc, dis, o, dir_) :
def calculate_eccentricity_fx(A, B, C, s, acc, dis, o, dir_, theta_) :
    eig_val1 = 0.5 * (A + C + np.sqrt((A - C)**2 + B**2))
    eig_val2 = 0.5 * (A + C - np.sqrt((A - C)**2 + B**2))
    a = np.sqrt(1 / np.abs(eig_val1))
    b = np.sqrt(1 / np.abs(eig_val2))
    #     a, b = sorted([a, b], key=abs)
    #     a, b = sorted([a, b], key=abs, reverse=True)
    #     a, b = sorted([a, b])
    a, b = sorted([a, b], reverse=True)

    #         eccentricity = np.sqrt(1 - (b**2 / a**2))
    #         eccentricity = np.sqrt(1 + (b**2 / a**2))

```

```
#      return np.sqrt(1 + (b**2 / a**2)) #
#      return (b**2 / a**2)
#      return np.log(b**2 / a**2)
#      return (a / b) #aspect ratio
#      return (a**2/b**2) # not ^
#      return 2*b**2/a #latus rectum lr
#      return np.abs(theta_)
#      return B**2 - 4 * A * C #dis
#      return (b**2 / a**2) #aspect ratio
#      return np.sqrt((b**2 / a**2))
#else return np.nan

# Initial guess for the conic parameters
# p0 = [1, 1, 1, 1, 1, 1]
p0 = torch.tensor([1, 1, 1, 1, 1, 1], dtype=torch.float32, requires_grad=True, device=device)

# Initialize the main dictionary to store no transforms
no_transforms = {}

# Summary stats for tracking processing
play_count_with_snap_event = 0
play_count_without_snap_event = 0
events_without_snap = {}

n = 200 #500 #Subset for testing
# tracking_data_dict_play_subset = dict(list(tracking_data_dict_play.items())[:n])
tracking_data_dict_play_subset = dict(list(tracking_data_dict_play.items()))

# Device setup: Use GPU if available
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")

for (gameId, playId), group_data in tqdm(tracking_data_dict_play_subset.items()):
    # Initialize gameId in no_transforms if not already present
    if gameId not in no_transforms:
```

```
no_transforms[gameId] = {}

# Initialize playId within the gameId if not already present
if playId not in no_transforms[gameId]:
    no_transforms[gameId][playId] = {
        'offense': {},
        'defense': {},
    }

# offensiveTeamAbbr = plays[(plays['gameId'] == gameId) & (plays['playId'] == playId)].iloc[0]['possessionTeam']
# defensiveTeamAbbr = plays[(plays['gameId'] == gameId) & (plays['playId'] == playId)].iloc[0]['defensiveTeam']
offensiveTeamAbbr = sup_data[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId)].iloc[0]['possession_team']
defensiveTeamAbbr = sup_data[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId)].iloc[0]['defensive_team']

# Retrieve tracking data for the current game/play
# tracking_data = group_data['tracking_data']
tracking_data = group_data['input']
tracking_data_op = group_data['output']

# Process tracking data for each player (nflId)
for nflId, player_data in tracking_data.groupby('nfl_id'):
    if pd.isna(nflId): # Skip if it's NaN (ball)
        continue

    # ball_snap_idx = player_data[player_data['event'] == 'ball_snap'].index.min()
    # snap_direct_idx = player_data[player_data['event'] == 'snap_direct'].index.min()

    # snap_event_idx = min(ball_snap_idx, snap_direct_idx) if pd.notna(snap_direct_idx) and pd.notna(ball_snap_idx) else (ball_snap_idx if pd.notna(ball_snap_idx) else snap_direct_idx)

    # Assuming player_data is sorted by time, so that the last row represents the latest event
```

```
#         last_event_idx = player_data.groupby('player_id')['event'].idx
max() # Adjust to get the last event by time
#         last_event = player_data.loc[last_event_idx]

snap_event_idx = 30
# player_data_truncated = player_data

if pd.notna(snap_event_idx):
    # player_data_truncated = player_data.loc[:snap_event_idx]
    player_data_truncated = player_data
    # player_data_truncated = player_data["input"]
    # play_count_with_snap_event += 1
else :
    #     print("No")
#     player_data_truncated = None

# print(player_data.columns)

#         x_ = player_data_truncated['x'].values
x_ = torch.tensor(player_data_truncated['x'].values, device=device)
y_ = torch.tensor(player_data_truncated['y'].values, device=device)
s_ = torch.tensor(player_data_truncated['s'].values, device=device)
a_ = torch.tensor(player_data_truncated['a'].values, device=device)
# dis_ = torch.tensor(player_data_truncated['dis'].values, device=device)
dir_ = torch.tensor(player_data_truncated['dir'].values, device=device)

o_ = torch.tensor(player_data_truncated['o'].values, device=device)

#
# #
#         disp = np.sum(dis_) #$disall = np.sum(dis_)
#         disp = player_data_truncated['dis'].sum() # Instead of n
```

```
p.sum  
#           xl = x_[-1]  
#           yl = y_[-1]  
# #           x_mean = np.mean(x_)  
# #           y_mean = np.mean(y_)  
#           x_mean = player_data_truncated['x'].mean()  
#           y_mean = player_data_truncated['y'].mean()  
  
#           # Sum of 'dis' (already in CUDA if player_data_truncated  
['dis'] is transferred as tensor)  
#           disp = dis_.sum() # PyTorch automatically performs the su  
m on GPU  
  
#  
#           # Last elements of x_ and y_ (directly available in GPU te  
nsors)  
#           xl = x_[-1]  
#           yl = y_[-1]  
  
#           # Mean calculations using PyTorch  
#           x_mean = x_.mean() # Mean calculated on GPU  
#           y_mean = y_.mean()  
  
#  
#optimized version  
#           disp = dis_.sum() # Sum of 'dis' directly on GPU  
xl = x_[-1]           # Last x-coordinate  
yl = y_[-1]           # Last y-coordinate  
x_mean = x_.mean() # Mean of x-coordinates on GPU  
y_mean = y_.mean() # Mean of y-coordinates on GPU  
  
#  
#           dir_rad = np.radians(dir_)  
#           sx = s_*np.sin(dir_rad) #np.mean(s_*np.sin())  
#           sy = s_*np.cos(dir_rad)  
#           ax = a_*np.sin(dir_rad)  
#           ay = a_*np.cos(dir_rad)  
#           param_s = minimize(objective_fn, p0, args=(sx, sy), method  
='L-BFGS-B') #xs  
#           param_a = minimize(objective_fn, p0, args=(sy, ay), method  
='L-BFGS-B')  
#           sa,sb,sc,sd,se,sf = param_s.x #$params_s.
```

```

#           aa, ab, ac, ad, ae, af = param_a.x
#           theta_s = np.arctan2(sb, sa-sc)/2
#           theta_a = np.arctan2(ab, aa-ac)/2

#
#           # Convert dir_ to radians
#           dir_rad = torch.radians(dir_)

#
#           # Trigonometric operations (PyTorch versions)
#           sx = s_ * torch.sin(dir_rad) # PyTorch handles this on GPU
U
#
#           sy = s_ * torch.cos(dir_rad)
#           ax = a_ * torch.sin(dir_rad)
#           ay = a_ * torch.cos(dir_rad)
#           param_s = minimize(objective_fn, p0, args=(sx.cpu().numpy(),
#           (), sy.cpu().numpy()), method='L-BFGS-B')
#           param_a = minimize(objective_fn, p0, args=(sy.cpu().numpy(),
#           (), ay.cpu().numpy()), method='L-BFGS-B')

#
#           # Extract parameters
#           sa, sb, sc, sd, se, sf = param_s.x
#           aa, ab, ac, ad, ae, af = param_a.x

#
#           # Trigonometric calculations
#           theta_s = torch.arctan2(torch.tensor(sb), torch.tensor(sa
- sc)) / 2
#           theta_a = torch.arctan2(torch.tensor(ab), torch.tensor(aa
- ac)) / 2

dir_rad = torch.deg2rad(dir_) #not radians
sx = s_ * torch.sin(dir_rad) # PyTorch handles this on GPU
sy = s_ * torch.cos(dir_rad)
ax = a_ * torch.sin(dir_rad)
ay = a_ * torch.cos(dir_rad)

# Define initial parameters for optimization
#           params = torch.tensor(p0, requires_grad=True, device=device)
params = p0.clone().detach().requires_grad_(True).to(device)

```

```
# Create LBFGS optimizer
optimizer = LBFGS([params])

#           # Step through the optimizer
#           optimizer.step(closure)

# Use closure for (sx, sy)
optimizer.step(lambda: closure(optimizer, sx, sy, params))
#           optimizer.step(lambda: closure(optimizer, sx, sy))

# After optimization, the parameters are updated. You can now access the optimized values:
optimized_params = params.detach() # Detach the tensor from the computation graph

# Extract the optimized parameters (sa, sb, sc, sd, se, sf) for further use
sa, sb, sc, sd, se, sf = optimized_params.tolist() # Convert tensor to list for easy usage

# Perform any further calculations or use these parameters in your code
#           theta_s = torch.arctan2(sb, sa - sc) / 2
theta_s = torch.arctan2(torch.tensor(sb, device=device), torch.tensor(sa - sc, device=device)) / 2

#           params = torch.tensor(p0, requires_grad=True, device=device)
params = p0.clone().detach().requires_grad_(True).to(device)
optimizer = LBFGS([params])
optimizer.step(lambda: closure(optimizer, ax, ay, params))
optimized_params = params.detach()
aa, ab, ac, ad, ae, af = optimized_params.tolist()
#           theta_a = torch.arctan2(ab, aa - ac) / 2
theta_a = torch.arctan2(torch.tensor(ab, device=device), torch.tensor(aa - ac, device=device)) / 2
```

```

#           params = torch.tensor(p0, requires_grad=True, device=device)
#           params = p0.clone().detach().requires_grad_(True).to(device)
#           optimizer = LBFGS([params])
#           optimizer.step(lambda: closure(optimizer, x_, y_, params))
# xx or xa
#           optimized_params = params.detach()
#           a,b,c,d,e,f = optimized_params.tolist() # aa bb cc ...
#           theta_ = torch.arctan2(ab, aa - ac) / 2
#           theta_ = torch.arctan2(torch.tensor(b, device=device), torch.tensor(a - c, device=device)) / 2
#           cos_theta_, sin_theta_ = torch.cos(theta_), torch.sin(theta_)

#           params = minimize(objective_fn, p0, args=(x_, y_), method='L-BFGS-B')
#           a,b,c,d,e,f = params.x

#           theta_ = np.arctan2(b,a-c)/2
#           cos_theta_, sin_theta_ = np.cos(theta_), np.sin(theta_)

#####
#           a_prime = a*cos_theta_*2 + b*cos_theta_*sin_theta_ + c*sin_theta_*2
#           b_prime = 2*(c-a)*cos_theta_*sin_theta_ + b*(cos_theta_*2 - sin_theta_*2)
#           c_prime = a*sin_theta_*2 - b*cos_theta_*sin_theta_ + c*cos_theta_*2

#           params = torch.tensor(p0, requires_grad=True, device=device)
#           optimizer = LBFGS([params])
#           optimizer.step(closure)
#           optimized_params = params.detach()
#           sa, sb, sc, sd, se, sf = optimized_params.tolist()
#           theta_s = torch.arctan2(sb, sa - sc) / 2
#           theta_a = torch.arctan2(ab, aa - ac) / 2

```

```

#           eccentricity = calculate_eccentricity(a, b, c, np.mean(s_),
#                                           np.mean(a_), np.mean(dis_), np.mean(o_), np.mean(dir_), theta_)
#           eccentricity = calculate_eccentricity(a, b, c, torch.mean(s_),
#                                           torch.mean(a_), torch.mean(dis_), torch.mean(o_), torch.mean(dir_),
#                                           theta_)
# %%%%%%%%%%%%%%%%
#           curvature = curvature_conic(a,b,c,d,e, np.mean(x_), np.mean(y_) )
#           curvature = curvature_conic(a, b, c, d, e, torch.mean(x_), torch.mean(y_))
# %%%%%%%%%%%%%%%%
#           avg_cuvx = average_curvature(a,b,c,d,e, x_[0], x_[-1] )
#           avg_cuvy = average_curvature(a,b,c,d,e, y_[0], y_[-1] )
#           discriminant = b**2 - 4 * a * c
# %%%%%%%%%%%%%%%%
#           eig_val1 = 0.5 * (a + c + np.sqrt((a - c)**2 + b**2))
#           eig_val2 = 0.5 * (a + c - np.sqrt((a - c)**2 + b**2))
#           al = np.sqrt(1 / np.abs(eig_val1))
#           bl = np.sqrt(1 / np.abs(eig_val2))
#           al, bl = sorted([al, bl], reverse=True)
# %%%%%%%%%%%%%%%%
#           arxy = calculate_eccentricity_fx(a, b, c, np.mean(s_), np.mean(a_), np.mean(dis_), np.mean(o_), np.mean(dir_), theta_) #aspect ratio
#           ars = calculate_eccentricity_fx(sa, sb, sc, np.mean(s_), np.mean(a_), np.mean(dis_), np.mean(o_), np.mean(dir_), theta_)
#           ara = calculate_eccentricity_fx(aa, ab, ac, np.mean(s_), np.mean(a_), np.mean(dis_), np.mean(o_), np.mean(dir_), theta_)
#           arxy = calculate_eccentricity_fx(a, b, c, torch.mean(s_), torch.mean(a_), torch.mean(dis_), torch.mean(o_), torch.mean(dir_), theta_) #aspect ratio
#           ars = calculate_eccentricity_fx(sa, sb, sc, torch.mean(s_), torch.mean(a_), torch.mean(dis_), torch.mean(o_), torch.mean(dir_), theta_)
#           ara = calculate_eccentricity_fx(aa, ab, ac, torch.mean(s_), torch.mean(a_), torch.mean(dis_), torch.mean(o_), torch.mean(dir_), theta_)
# %%%%%%%%%%%%%%%%

```

```

# if player_data['club'].iloc[0] == offensiveTeamAbbr:
if player_data['player_side'].iloc[0] == "Offense":
#           no_transforms[gameId][playId]['offense'][nflId] = {'ec
centricity': eccentricity, 'disp': disp, 'theta': theta_, 'cuv': curvatur
e, 'cuvx': avg_cuvx, 'cuvy': avg_cuvy, 'disc': discriminant, 'major': al,
'minor': bl, 'arxy': arxy, 'ars': ars, 'ara': ara, 'theta_s': theta_s, 'the
ta_a': theta_a, 'xl': xl, 'yl': yl, 'x_mean': x_mean, 'y_mean': y_mean, 'game
id': group_data['meta_data']['gameId'], 'playid': group_data['meta_dat
a']['playId'], 'a': a, 'b': b, 'c': c, 'd': d, 'e': e, 'f': f, 'sa': sa, 'sb': sb,
'sc': sc, 's_sd': sd, 'se': se, 'sf': sf, 'aa': aa, 'ab': ab, 'ac': ac, 'ad': ad,
'ae': ae, 'af': af}

#           no_transforms[gameId][playId]['offense'][nflId] =
{'a': a, 'b': b, 'c': c, 'd': d, 'e': e, 'f': f, 'sa': sa, 'sb': sb, 'sc': sc, 's
d': sd, 'se': se, 'sf': sf, 'aa': aa, 'ab': ab, 'ac': ac, 'ad': ad, 'a
e': ae, 'af': af, 'xl': xl, 'yl': yl, 'x_mean': x_mean, 'y_mean': y_mean,
'theta_': theta_, 'theta_s': theta_s, 'theta_a': theta_a, 'gameid': group_
data['meta_data']['gameId'], 'playid': group_data['meta_data']['playI
d']} #, 'last_event'=last_event}
# 'disp': disp,

# elif player_data['club'].iloc[0] == defensiveTeamAbbr:
elif player_data['player_side'].iloc[0] == "Defense":
#           no_transforms[gameId][playId]['defense'][nflId] = {'ec
centricity': eccentricity, 'disp': disp, 'theta': theta_, 'cuv': curvatur
e, 'cuvx': avg_cuvx, 'cuvy': avg_cuvy, 'disc': discriminant, 'major': al,
'minor': bl, 'arxy': arxy, 'ars': ars, 'ara': ara, 'theta_s': theta_s, 'the
ta_a': theta_a, 'xl': xl, 'yl': yl, 'x_mean': x_mean, 'y_mean': y_mean, 'game
id': group_data['meta_data']['gameId'], 'playid': group_data['meta_dat
a']['playId']} }

no_transforms[gameId][playId]['defense'][nflId] = {'a': a, 'b': b, 'c': c, 'd': d, 'e': e, 'f': f, 'sa': sa, 'sb': sb, 'sc': sc, 's
d': sd, 'se': se, 'sf': sf, 'aa': aa, 'ab': ab, 'ac': ac, 'ad': ad, 'a
e': ae, 'af': af, 'xl': xl, 'yl': yl, 'x_mean': x_mean, 'y_mean': y_mean,
'theta_': theta_, 'theta_s': theta_s, 'theta_a': theta_a, 'gameid': group_
data['meta_data']['gameId'], 'playid': group_data['meta_data']['playI
d']} #, 'last_event'=last_event}
# 'disp': disp,


else:

```

```
play_count_without_snap_event += 1
# events_without_snap[(gameId, playId)] = player_data['event'].unique() # Store unique events

# Final summary
print(f"Total plays processed: {len(tracking_data_dict_play_subset)}")
print(f"Plays with 'ball_snap' or 'snap_direct' event: {play_count_with_snap_event}")
print(f"Plays without 'ball_snap' or 'snap_direct' event: {play_count_without_snap_event}")

# torch.save(no_transforms, 'no_transforms[all].pth')

# 12%|██████| 402/3347 [14:22<1:43:40, 2.11s/it]j
# 0%|          | 13/3362 [01:03<4:29:05, 4.82s/it] #gpu

# 7%|███| 1070/16124 [25:11<5:35:34, 1.34s/it]
# 51%|██████████| 8165/16124 [3:20:31<3:22:41, 1.53s/it]
```

In [ ]:

```
import pandas as pd
import numpy as np
from tqdm import tqdm
from scipy.optimize import minimize

# Objective function to minimize
def objective_fn(params, x, y):
    return torch.sum(conics(params, x, y)**2)

import torch
from torch.optim import LBFGS

# # Define the closure function for optimization

# def closure():
def closure(optimiser, sx ,sy, params ):
    optimizer.zero_grad()
    # Split params into sx and sy (assuming sx and sy are of the same size)

    # Now call the objective function with sx and sy
    loss = objective_fn(params, sx, sy)
    loss.backward()
    return loss

# Define conic equation
def conics(params, x, y):
    a, b, c, d, e, f = params
    return a * x**2 + b * x * y + c * y**2 + d * x + e * y + f

def curvature_conic(A, B, C, D, E, x, y):
    # First derivative (dy/dx)
    numerator_first = -2*A*x - D
    denominator_first = B*x + 2*C*y + E
    dy_dx = numerator_first / denominator_first
```

```
# Second derivative ( $d^2y/dx^2$ )
# Derivatives of the numerator and denominator
d_numerator_first_dx = -2*A
d_denominator_first_dx = B

# Using the quotient rule for the second derivative
numerator_second = (d_numerator_first_dx * denominator_first) - (numerator_first * d_denominator_first_dx)
denominator_second = denominator_first**2

d2y_dx2 = numerator_second / denominator_second

# Curvature calculation
curvature = d2y_dx2 / (1 + dy_dx**2)**(3/2)

return curvature

def average_curvature(A, B, C, D, E, x_start, x_end, num_points=100):
    # Create a tensor of x values, using torch.linspace
    x_values = torch.linspace(x_start, x_end, num_points, device='cuda')
    if torch.cuda.is_available() else 'cpu')

    # Initialize the curvature_values tensor
    curvature_values = torch.zeros_like(x_values)

    # Compute the curvature for each x value
    for i, x in enumerate(x_values):
        y = 0 # Fixed y value for the calculation (this can be modified
               based on specific requirements)
        curvature_values[i] = curvature_conic(A, B, C, D, E, x, y)

    # Calculate the average curvature using torch.mean
    average_curv = torch.mean(curvature_values)

    return average_curv

# #hyperbolic only
```

```
# def calculate_eccentricity(A, B, C, s, acc, dis, o, dir_) :
def calculate_eccentricity_fx(A, B, C, s, acc, dis, o, dir_, theta_) :
    eig_val1 = 0.5 * (A + C + np.sqrt((A - C)**2 + B**2))
    eig_val2 = 0.5 * (A + C - np.sqrt((A - C)**2 + B**2))
    a = np.sqrt(1 / np.abs(eig_val1))
    b = np.sqrt(1 / np.abs(eig_val2))

    a, b = sorted([a, b], reverse=True)

return (a**2/b**2)

# Initial guess for the conic parameters
# p0 = [1, 1, 1, 1, 1, 1]
p0 = torch.tensor([1, 1, 1, 1, 1, 1], dtype=torch.float32, requires_grad=True, device=device)

# Initialize the main dictionary to store no transforms
no_transforms = {}

# Summary stats for tracking processing
play_count_with_snap_event = 0
play_count_without_snap_event = 0
events_without_snap = {}

n = 200 #500 # Subset for testing
# tracking_data_dict_play_subset = dict(list(tracking_data_dict_play.items())[:n])
tracking_data_dict_play_subset = dict(list(tracking_data_dict_play.items()))

# Device setup: Use GPU if available
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")
print(device)

for (gameId, playId), group_data in tqdm(tracking_data_dict_play_subset.items()):
    # Initialize gameId in no_transforms if not already present
```

```
if gameId not in no_transforms:
    no_transforms[gameId] = {}

# Initialize playId within the gameId if not already present
if playId not in no_transforms[gameId]:
    no_transforms[gameId][playId] = {
        'offense': {},
        'defense': {},
    }

# offensiveTeamAbbr = plays[(plays['gameId'] == gameId) & (plays['playId'] == playId)].iloc[0]['possessionTeam']
# defensiveTeamAbbr = plays[(plays['gameId'] == gameId) & (plays['playId'] == playId)].iloc[0]['defensiveTeam']
offensiveTeamAbbr = sup_data[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId)].iloc[0]['possession_team']
defensiveTeamAbbr = sup_data[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId)].iloc[0]['defensive_team']

# Retrieve tracking data for the current game/play
# tracking_data = group_data['tracking_data']
tracking_data = group_data['input']
tracking_data_op = group_data['output']

# Process tracking data for each player (nflId)
for nflId, player_data in tracking_data.groupby('nfl_id'):
    if pd.isna(nflId): # Skip if it's NaN (ball)
        continue

    snap_event_idx = 30
    # player_data_truncated = player_data

    if pd.notna(snap_event_idx):
        # player_data_truncated = player_data.loc[:snap_event_idx]
        player_data_truncated = player_data
```

```
#           x_ = player_data_truncated['x'].values
#           x_ = torch.tensor(player_data_truncated['x'].values, device
# =device)
#           y_ = torch.tensor(player_data_truncated['y'].values, device
# =device)
#           s_ = torch.tensor(player_data_truncated['s'].values, device
# =device)
#           a_ = torch.tensor(player_data_truncated['a'].values, device
# =device)
#           # dis_ = torch.tensor(player_data_truncated['dis'].values, d
# evice=device)
#           dir_ = torch.tensor(player_data_truncated['dir'].values, de
# vice=device)

#           o_ = torch.tensor(player_data_truncated['o'].values, device
# =device)

#
# #           disp = np.sum(dis_) #$disall = np.sum(dis_)
#           disp = player_data_truncated['dis'].sum() # Instead of n
# p.sum
#           xl = x_[-1]
#           yl = y_[-1]
# #           x_mean = np.mean(x_)
# #           y_mean = np.mean(y_)
#           x_mean = player_data_truncated['x'].mean()
#           y_mean = player_data_truncated['y'].mean()

#           # Sum of 'dis' (already in CUDA if player_data_truncated
# ['dis'] is transferred as tensor)
#           disp = dis_.sum() # PyTorch automatically performs the su
# m on GPU

#
#           # Last elements of x_ and y_ (directly available in GPU te
# nsors)
#           xl = x_[-1]
#           yl = y_[-1]

#
#           # Mean calculations using PyTorch
#           x_mean = x_.mean() # Mean calculated on GPU
#           y_mean = y_.mean()
```

```
#  
#optimized version  
# disp = dis_.sum() # Sum of 'dis' directly on GPU  
x_l = x_[-1] # Last x-coordinate  
y_l = y_[-1] # Last y-coordinate  
x_mean = x_.mean() # Mean of x-coordinates on GPU  
y_mean = y_.mean() # Mean of y-coordinates on GPU  
  
#  
# dir_rad = np.radians(dir_)  
# sx = s_*np.sin(dir_rad) #np.mean(s_*np.sin())  
# sy = s_*np.cos(dir_rad)  
# ax = a_*np.sin(dir_rad)  
# ay = a_*np.cos(dir_rad)  
# param_s = minimize(objective_fn, p0, args=(sx, sy), method='L-BFGS-B') #xs  
# param_a = minimize(objective_fn, p0, args=(sy, ay), method='L-BFGS-B')  
# sa, sb, sc, sd, se, sf = param_s.x #$params_s.  
# aa, ab, ac, ad, ae, af = param_a.x  
# theta_s = np.arctan2(sb,sa-sc)/2  
# theta_a = np.arctan2(ab,aa-ac)/2  
  
#  
# # Convert dir_ to radians  
# dir_rad = torch.radians(dir_)  
  
# # Trigonometric operations (PyTorch versions)  
# sx = s_ * torch.sin(dir_rad) # PyTorch handles this on GP  
U  
# sy = s_ * torch.cos(dir_rad)  
# ax = a_ * torch.sin(dir_rad)  
# ay = a_ * torch.cos(dir_rad)  
# param_s = minimize(objective_fn, p0, args=(sx.cpu().numpy(), sy.cpu().numpy()), method='L-BFGS-B')  
# param_a = minimize(objective_fn, p0, args=(sy.cpu().numpy(), ay.cpu().numpy()), method='L-BFGS-B')  
  
# # Extract parameters  
# sa, sb, sc, sd, se, sf = param_s.x  
# aa, ab, ac, ad, ae, af = param_a.x
```

```
#           # Trigonometric calculations
#           theta_s = torch.arctan2(torch.tensor(sb), torch.tensor(sa
- sc)) / 2
#           theta_a = torch.arctan2(torch.tensor(ab), torch.tensor(aa
- ac)) / 2

dir_rad = torch.deg2rad(dir_) #not radians
sx = s_ * torch.sin(dir_rad) # PyTorch handles this on GPU
sy = s_ * torch.cos(dir_rad)
ax = a_ * torch.sin(dir_rad)
ay = a_ * torch.cos(dir_rad)

# Define initial parameters for optimization
#           params = torch.tensor(p0, requires_grad=True, device=device)
params = p0.clone().detach().requires_grad_(True).to(device)

# Create LBFGS optimizer
optimizer = LBFGS([params])

#           # Step through the optimizer
#           optimizer.step(closure)

#           # Use closure for (sx, sy)
optimizer.step(lambda: closure(optimizer, sx, sy, params))
#           optimizer.step(lambda: closure(optimizer, sx, sy))

# After optimization, the parameters are updated. You can now access the optimized values:
optimized_params = params.detach() # Detach the tensor from the computation graph

# Extract the optimized parameters (sa, sb, sc, sd, se, sf) for further use
sa, sb, sc, sd, se, sf = optimized_params.tolist() # Convert tensor to list for easy usage

# Perform any further calculations or use these parameters in your code
```

```
#           theta_s = torch.arctan2(sb, sa - sc) / 2
#           theta_s = torch.arctan2(torch.tensor(sb, device=device), torch.tensor(sa - sc, device=device)) / 2

#
#           params = torch.tensor(p0, requires_grad=True, device=device)
#           params = p0.clone().detach().requires_grad_(True).to(device)
#           optimizer = LBFGS([params])
#           optimizer.step(lambda: closure(optimizer, ax, ay, params))
#           optimized_params = params.detach()
#           aa, ab, ac, ad, ae, af = optimized_params.tolist()
#
#           theta_a = torch.arctan2(ab, aa - ac) / 2
#           theta_a = torch.arctan2(torch.tensor(ab, device=device), torch.tensor(aa - ac, device=device)) / 2

#
#           params = torch.tensor(p0, requires_grad=True, device=device)
#           params = p0.clone().detach().requires_grad_(True).to(device)
#           optimizer = LBFGS([params])
#           optimizer.step(lambda: closure(optimizer, x_, y_, params))
# xx or xa
#           optimized_params = params.detach()
#           a,b,c,d,e,f = optimized_params.tolist() # aa bb cc ...
#
#           theta_ = torch.arctan2(ab, aa - ac) / 2
#           theta_ = torch.arctan2(torch.tensor(b, device=device), torch.tensor(a - c, device=device)) / 2
#           cos_theta_, sin_theta_ = torch.cos(theta_), torch.sin(theta_)

#
#           arxy = calculate_eccentricity_fx(a, b, c, np.mean(s_), np.mean(a_), np.mean(dis_), np.mean(o_), np.mean(dir_), theta_) #aspect ratio
```

```

# if player_data['club'].iloc[0] == offensiveTeamAbbr:
    if player_data['player_side'].iloc[0] == "Offense":
        #
            no_transforms[gameId][playId]['offense'][nflId] = {'eccentricity': eccentricity, 'disp': disp, 'theta': theta_, 'cuv': curvature, 'cuvx': avg_cuvx, 'cuvy': avg_cuvy, 'disc': discriminant, 'major': al, 'minor': bl, 'arxy': arxy, 'ars': ars, 'ara': ara, 'theta_s': theta_s, 'theta_a': theta_a, 'xl': xl, 'yl': yl, 'x_mean': x_mean, 'y_mean': y_mean, 'gameid': group_data['meta_data']['gameId'], 'playid': group_data['meta_data']['playId'], 'a': a, 'b': b, 'c': c, 'd': d, 'e': e, 'f': f, 'sa': sa, 'sb': sb, 'sc': sc, 'sd': sd, 'se': se, 'sf': sf, 'aa': aa, 'ab': ab, 'ac': ac, 'ad': ad, 'ae': ae, 'af': af}
        #
            no_transforms[gameId][playId]['offense'][nflId] =
{'a': a, 'b': b, 'c': c, 'd': d, 'e': e, 'f': f, 'sa': sa, 'sb': sb, 'sc': sc, 'sd': sd, 'se': se, 'sf': sf, 'aa': aa, 'ab': ab, 'ac': ac, 'ad': ad, 'ae': ae, 'af': af}
            no_transforms[gameId][playId]['offense'][nflId] = {'a': a, 'b': b, 'c': c, 'd': d, 'e': e, 'f': f, 'sa': sa, 'sb': sb, 'sc': sc, 'sd': sd, 'se': se, 'sf': sf, 'aa': aa, 'ab': ab, 'ac': ac, 'ad': ad, 'ae': ae, 'af': af, 'theta_': theta_, 'theta_s': theta_s, 'theta_a': theta_a, 'gameid': group_data['meta_data']['gameId'], 'playid': group_data['meta_data']['playId']} #, 'last_event'=last_event}
                #
                    # 'disp': disp,
                    # elif player_data['club'].iloc[0] == defensiveTeamAbbr:
                        elif player_data['player_side'].iloc[0] == "Defense":
                            #
                                no_transforms[gameId][playId]['defense'][nflId] = {'eccentricity': eccentricity, 'disp': disp, 'theta': theta_, 'cuv': curvature, 'cuvx': avg_cuvx, 'cuvy': avg_cuvy, 'disc': discriminant, 'major': al, 'minor': bl, 'arxy': arxy, 'ars': ars, 'ara': ara, 'theta_s': theta_s, 'theta_a': theta_a, 'xl': xl, 'yl': yl, 'x_mean': x_mean, 'y_mean': y_mean, 'gameid': group_data['meta_data']['gameId'], 'playid': group_data['meta_data']['playId']} 
                                no_transforms[gameId][playId]['defense'][nflId] = {'a': a, 'b': b, 'c': c, 'd': d, 'e': e, 'f': f, 'sa': sa, 'sb': sb, 'sc': sc, 'sd': sd, 'se': se, 'sf': sf, 'aa': aa, 'ab': ab, 'ac': ac, 'ad': ad, 'ae': ae, 'af': af, 'theta_': theta_, 'theta_s': theta_s, 'theta_a': theta_a, 'gameid': group_data['meta_data']['gameId'], 'playid': group_data['meta_data']['playId']} #, 'last_event'=last_event}
                                    #
                                        # 'disp': disp,
                                        else:
                                            play_count_without_snap_event += 1

```

```
# events_without_snap[(gameId, playId)] = player_data['event'].unique() # Store unique events

# Final summary
print(f"Total plays processed: {len(tracking_data_dict_play_subset)}")
print(f"Plays with 'ball_snap' or 'snap_direct' event: {play_count_with_snap_event}")
print(f"Plays without 'ball_snap' or 'snap_direct' event: {play_count_without_snap_event}")

# torch.save(no_transforms, 'no_transforms[all].pth')
```

In [50]:

```
# tracking_data_dict_play_subset[2023120306][1620]
# tracking_data_dict_play_subset.keys()
# tracking_data_dict_play_subset[(2023120306, 1620)]
key_ = (2023120306, 1620)
data_ = tracking_data_dict_play_subset[(2023120306, 1620)]
# dict_gp = dict(key_:data_)
dict_gp = {key_: data_}

tracking_data_dict_play_subset[(2023120306, 1620)] == dict_gp
dict_gp[(2023120306, 1620)] is tracking_data_dict_play_subset[(2023120306, 1620)]
```

Out[50]:

True

In [ ]:

```
import pandas as pd
import numpy as np
from tqdm import tqdm
from scipy.optimize import minimize

# Objective function to minimize
def objective_fn(params, x, y):
    return torch.sum(conics(params, x, y)**2)

import torch
from torch.optim import LBFGS

# # Define the closure function for optimization

# def closure():
def closure(optimiser, sx ,sy, params ):
    optimizer.zero_grad()
    # Split params into sx and sy (assuming sx and sy are of the same size)

    # Now call the objective function with sx and sy
    loss = objective_fn(params, sx, sy)
    loss.backward()
    return loss

# Define conic equation
def conics(params, x, y):
    a, b, c, d, e, f = params
    return a * x**2 + b * x * y + c * y**2 + d * x + e * y + f

def curvature_conic(A, B, C, D, E, x, y):
    # First derivative (dy/dx)
    numerator_first = -2*A*x - D
    denominator_first = B*x + 2*C*y + E
    dy_dx = numerator_first / denominator_first
```

```
# Second derivative ( $d^2y/dx^2$ )
# Derivatives of the numerator and denominator
d_numerator_first_dx = -2*A
d_denominator_first_dx = B

# Using the quotient rule for the second derivative
numerator_second = (d_numerator_first_dx * denominator_first) - (numerator_first * d_denominator_first_dx)
denominator_second = denominator_first**2

d2y_dx2 = numerator_second / denominator_second

# Curvature calculation
curvature = d2y_dx2 / (1 + dy_dx**2)**(3/2)

return curvature

def average_curvature(A, B, C, D, E, x_start, x_end, num_points=100):
    # Create a tensor of x values, using torch.linspace
    x_values = torch.linspace(x_start, x_end, num_points, device='cuda')
    if torch.cuda.is_available() else 'cpu')

    # Initialize the curvature_values tensor
    curvature_values = torch.zeros_like(x_values)

    # Compute the curvature for each x value
    for i, x in enumerate(x_values):
        y = 0 # Fixed y value for the calculation (this can be modified
               based on specific requirements)
        curvature_values[i] = curvature_conic(A, B, C, D, E, x, y)

    # Calculate the average curvature using torch.mean
    average_curv = torch.mean(curvature_values)

    return average_curv

# #hyperbolic only
```

```
# def calculate_eccentricity(A, B, C, s, acc, dis, o, dir_) :
def calculate_eccentricity_fx(A, B, C, s, acc, dis, o, dir_, theta_) :
    eig_val1 = 0.5 * (A + C + np.sqrt((A - C)**2 + B**2))
    eig_val2 = 0.5 * (A + C - np.sqrt((A - C)**2 + B**2))
    a = np.sqrt(1 / np.abs(eig_val1))
    b = np.sqrt(1 / np.abs(eig_val2))

    a, b = sorted([a, b], reverse=True)

return (a**2/b**2)

# Initial guess for the conic parameters
# p0 = [1, 1, 1, 1, 1, 1]
p0 = torch.tensor([1, 1, 1, 1, 1, 1], dtype=torch.float32, requires_grad=True, device=device)

# Initialize the main dictionary to store no transforms
no_transforms = {}

# Summary stats for tracking processing
play_count_with_snap_event = 0
play_count_without_snap_event = 0
events_without_snap = {}

n = 200 #50 #500 # Subset for testing
# tracking_data_dict_play_subset = dict(list(tracking_data_dict_play.items())[:n])
tracking_data_dict_play_subset = dict(list(tracking_data_dict_play.items()))

###NEWLY
tracking_data_dict_play_subset = dict_gp

# Device setup: Use GPU if available
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")

for (gameId, playId), group_data in tqdm(tracking_data_dict_play_subset.items()):
```

```
# Initialize gameId in no_transforms if not already present
if gameId not in no_transforms:
    no_transforms[gameId] = {}

# Initialize playId within the gameId if not already present
if playId not in no_transforms[gameId]:
    no_transforms[gameId][playId] = {
        'offense': {},
        'defense': {},
    }

# offensiveTeamAbbr = plays[(plays['gameId'] == gameId) & (plays['playId'] == playId)].iloc[0]['possessionTeam']
# defensiveTeamAbbr = plays[(plays['gameId'] == gameId) & (plays['playId'] == playId)].iloc[0]['defensiveTeam']
offensiveTeamAbbr = sup_data[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId)].iloc[0]['possession_team']
defensiveTeamAbbr = sup_data[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId)].iloc[0]['defensive_team']

# Retrieve tracking data for the current game/play
# tracking_data = group_data['tracking_data']
tracking_data = group_data['input']
tracking_data_op = group_data['output']

# Process tracking data for each player (nflId)
for nflId, player_data in tracking_data.groupby('nfl_id'):
    if pd.isna(nflId): # Skip if it's NaN (ball)
        continue

    snap_event_idx = 30
    # player_data_truncated = player_data

    if pd.notna(snap_event_idx):
        # player_data_truncated = player_data.loc[:snap_event_idx]
        player_data_truncated = player_data
```

```
#           x_ = player_data_truncated['x'].values
#           x_ = torch.tensor(player_data_truncated['x'].values, device
# =device)
#           y_ = torch.tensor(player_data_truncated['y'].values, device
# =device)

#####
# s_ = torch.tensor(player_data_truncated['s'].values, device=device)
# a_ = torch.tensor(player_data_truncated['a'].values, device=device)
# # dis_ = torch.tensor(player_data_truncated['dis'].values, device=device)
# dir_ = torch.tensor(player_data_truncated['dir'].values, device=device)

# o_ = torch.tensor(player_data_truncated['o'].values, device=device)

#####
# # disp = np.sum(dis_) #$disall = np.sum(dis_)
#           disp = player_data_truncated['dis'].sum() # Instead of np.sum
#           xl = x_[-1]
#           yl = y_[-1]
# #           x_mean = np.mean(x_)
# #           y_mean = np.mean(y_)
#           x_mean = player_data_truncated['x'].mean()
#           y_mean = player_data_truncated['y'].mean()

#           # Sum of 'dis' (already in CUDA if player_data_truncated
#           # ['dis'] is transferred as tensor)
#           disp = dis_.sum() # PyTorch automatically performs the sum on GPU
```

```
#  
#           # Last elements of x_ and y_ (directly available in GPU tensors)  
#           xl = x_[-1]  
#           yl = y_[-1]  
  
#           # Mean calculations using PyTorch  
#           x_mean = x_.mean() # Mean calculated on GPU  
#           y_mean = y_.mean()  
  
#           #optimized version  
#           disp = dis_.sum() # Sum of 'dis' directly on GPU  
xl = x_[-1]           # Last x-coordinate  
yl = y_[-1]           # Last y-coordinate  
x_mean = x_.mean() # Mean of x-coordinates on GPU  
y_mean = y_.mean() # Mean of y-coordinates on GPU  
  
#  
#           dir_rad = np.radians(dir_)  
#           sx = s_*np.sin(dir_rad) #np.mean(s_*np.sin())  
#           sy = s_*np.cos(dir_rad)  
#           ax = a_*np.sin(dir_rad)  
#           ay = a_*np.cos(dir_rad)  
#           param_s = minimize(objective_fn, p0, args=(sx, sy), method  
='L-BFGS-B') #xs  
#           param_a = minimize(objective_fn, p0, args=(sy, ay), method  
='L-BFGS-B')  
#           sa, sb, sc, sd, se, sf = param_s.x #$params_s.  
#           aa, ab, ac, ad, ae, af = param_a.x  
#           theta_s = np.arctan2(sb,sa-sc)/2  
#           theta_a = np.arctan2(ab,aa-ac)/2  
  
#           # Convert dir_ to radians  
#           dir_rad = torch.radians(dir_)  
  
#           # Trigonometric operations (PyTorch versions)  
#           sx = s_ * torch.sin(dir_rad) # PyTorch handles this on GP  
U  
#           sy = s_ * torch.cos(dir_rad)
```

```

#           ax = a_ * torch.sin(dir_rad)
#           ay = a_ * torch.cos(dir_rad)
#           param_s = minimize(objective_fn, p0, args=(sx.cpu().numpy(),
#           sy.cpu().numpy()), method='L-BFGS-B')
#           param_a = minimize(objective_fn, p0, args=(sy.cpu().numpy(),
#           ay.cpu().numpy()), method='L-BFGS-B')

#           # Extract parameters
#           sa, sb, sc, sd, se, sf = param_s.x
#           aa, ab, ac, ad, ae, af = param_a.x

#           # Trigonometric calculations
#           theta_s = torch.arctan2(torch.tensor(sb), torch.tensor(sa
#           - sc)) / 2
#           theta_a = torch.arctan2(torch.tensor(ab), torch.tensor(aa
#           - ac)) / 2

#####
dir_rad = torch.deg2rad(dir_) #not radians
sx = s_ * torch.sin(dir_rad) # PyTorch handles this on GPU
sy = s_ * torch.cos(dir_rad)
ax = a_ * torch.sin(dir_rad)
ay = a_ * torch.cos(dir_rad)

# Define initial parameters for optimization
# params = torch.tensor(p0, requires_grad=True, device=device)
params = p0.clone().detach().requires_grad_(True).to(device)

# Create LBFGS optimizer
optimizer = LBFGS([params])

#           # Step through the optimizer
#           optimizer.step(closure)

#           # Use closure for (sx, sy)
optimizer.step(lambda: closure(optimizer, sx, sy, params))
#           optimizer.step(lambda: closure(optimizer, sx, sy))

#           # After optimization, the parameters are updated. You can no

```

```
w access the optimized values:  
    optimized_params = params.detach() # Detach the tensor from  
m the computation graph  
  
        # Extract the optimized parameters (sa, sb, sc, sd, se, sf)  
for further use  
        sa, sb, sc, sd, se, sf = optimized_params.tolist() # Convert tensor to list for easy usage  
  
        # Perform any further calculations or use these parameters i  
n your code  
#  
#           theta_s = torch.arctan2(sb, sa - sc) / 2  
        theta_s = torch.arctan2(torch.tensor(sb, device=device), to  
rch.tensor(sa - sc, device=device)) / 2  
  
  
#  
#           params = torch.tensor(p0, requires_grad=True, device=devic  
e)  
        params = p0.clone().detach().requires_grad_(True).to(devic  
e)  
        optimizer = LBFGS([params])  
        optimizer.step(lambda: closure(optimizer, ax, ay, params))  
        optimized_params = params.detach()  
        aa, ab, ac, ad, ae, af = optimized_params.tolist()  
#  
#           theta_a = torch.arctan2(ab, aa - ac) / 2  
        theta_a = torch.arctan2(torch.tensor(ab, device=device), to  
rch.tensor(aa - ac, device=device)) / 2  
  
  
#  
#           params = torch.tensor(p0, requires_grad=True, device=devic  
e)  
        params = p0.clone().detach().requires_grad_(True).to(devic  
e)  
        optimizer = LBFGS([params])  
        optimizer.step(lambda: closure(optimizer, x_, y_, params))  
# xx or xa  
        optimized_params = params.detach()  
        a,b,c,d,e,f = optimized_params.tolist() # aa bb cc ..  
#  
#           theta_ = torch.arctan2(ab, aa - ac) / 2  
        theta_ = torch.arctan2(torch.tensor(b, device=device), torc
```

```

h.tensor(a - c, device=device)) / 2
    cos_theta_, sin_theta_ = torch.cos(theta_), torch.sin(theta_)

#####
#      arxy = calculate_eccentricity_fx(a, b, c, np.mean(s_), np.
#      mean(a_), np.mean(dis_), np.mean(o_), np.mean(dir_), theta_) #aspect rat
#      io

# if player_data['club'].iloc[0] == offensiveTeamAbbr:
if player_data['player_side'].iloc[0] == "Offense":
#           no_transforms[gameId][playId]['offense'][nflId] = {'ec
#           centricity': eccentricity, 'disp':disp, 'theta': theta_, 'cuv':curvatur
#           e, 'cuvx': avg_cuvx, 'cuvy': avg_cuvy, 'disc':discriminant, 'major':al,
#           'minor':bl, 'arxy':arxy, 'ars':ars, 'ara': ara, 'theta_s': theta_s, 'the
#           ta_a':theta_a, 'xl':xl, 'yl':yl, 'x_mean':x_mean, 'y_mean':y_mean, 'game
#           id': group_data['meta_data']['gameId'], 'playid': group_data['meta_dat
#           a']['playId'], 'a'=a, 'b'= b, 'c'=c, 'd'=d, 'e'=e, 'f'=f, 'sa'=sa, 'sb'=sb,
#           'sc'=sc, 's'sd'=d, 'se'=se, 'sf'sf, 'aa'=aa, 'ab'=ab, 'ac'= ac, 'ad'=ad,
#           'ae'=ae, 'af'=af}
#           no_transforms[gameId][playId]['offense'][nflId] =
#           {'a'=a, 'b'= b, 'c'=c, 'd'=d, 'e'=e, 'f'=f, 'sa'=sa, 'sb'=sb, 'sc'=sc, s'sd'=
#           d, 'se'=se, 'sf'sf, 'aa'=aa, 'ab'=ab, 'ac'= ac, 'ad'=ad, 'ae'=ae, 'af'=a
#           f}
#           no_transforms[gameId][playId]['offense'][nflId] = {'a':
#           a, 'b': b, 'c': c, 'd': d, 'e': e, 'f': f, 'sa': sa, 'sb': sb, 'sc':sc, 's
#           d': sd, 'se':se, 'sf': sf, 'aa': aa, 'ab': ab, 'ac': ac, 'ad': ad, 'a
#           e': ae, 'af': af, 'xl':xl, 'yl':yl, 'x_mean':x_mean, 'y_mean':y_mean,
#           'theta_':theta_, 'theta_s':theta_s, 'theta_a':theta_a, 'gameid': group_
#           data['meta_data']['gameId'], 'playid': group_data['meta_data']['playI
#           d']} #, 'last_event'=last_event}
#           # 'disp':disp,

# elif player_data['club'].iloc[0] == defensiveTeamAbbr:
elif player_data['player_side'].iloc[0] == "Defense":
#           no_transforms[gameId][playId]['defense'][nflId] = {'ec
#           centricity': eccentricity, 'disp':disp, 'theta': theta_, 'cuv':curvatur
#           e, 'cuvx': avg_cuvx, 'cuvy': avg_cuvy, 'disc':discriminant, 'major':al,
#           'minor':bl, 'arxy':arxy, 'ars':ars, 'ara': ara, 'theta_s': theta_s, 'the
#           ta_a':theta_a, 'xl':xl, 'yl':yl, 'x_mean':x_mean, 'y_mean':y_mean, 'game
#           id': group_data['meta_data']['gameId'], 'playid': group_data['meta_dat
#           a']['playId']}

```

```
no_transforms[gameId][playId]['defense'][nflId] = {'a':  
    a, 'b': b, 'c': c, 'd': d, 'e': e, 'f': f, 'sa': sa, 'sb': sb, 'sc': sc, 's  
    d': sd, 'se': se, 'sf': sf, 'aa': aa, 'ab': ab, 'ac': ac, 'ad': ad, 'a  
    e': ae, 'af': af, 'xl': xl, 'yl': yl, 'x_mean': x_mean, 'y_mean': y_mean,  
    'theta_': theta_, 'theta_s': theta_s, 'theta_a': theta_a, 'gameid': group_  
    data['meta_data'][['gameId']], 'playid': group_data['meta_data'][['playI  
    d']]}, 'last_event'=last_event}  
    # 'disp': disp,  
  
else:  
    play_count_without_snap_event += 1  
    # events_without_snap[(gameId, playId)] = player_data['even  
t'].unique() # Store unique events  
  
# Final summary  
print(f"Total plays processed: {len(tracking_data_dict_play_subset)}")  
print(f"Plays with 'ball_snap' or 'snap_direct' event: {play_count_with  
_snap_event}")  
print(f"Plays without 'ball_snap' or 'snap_direct' event: {play_count_w  
ithout_snap_event}")  
  
# torch.save(no_transforms, 'no_transforms[all].pth')
```

In [ ]:

```
import pandas as pd
import numpy as np
from tqdm import tqdm
from scipy.optimize import minimize

# Objective function to minimize
def objective_fn(params, x, y):
    return torch.sum(conics(params, x, y)**2)

import torch
from torch.optim import LBFGS

# # Define the closure function for optimization

# def closure():
def closure(optimiser, sx ,sy, params ) :
    optimizer.zero_grad()
    # Split params into sx and sy (assuming sx and sy are of the same size)
    # Now call the objective function with sx and sy
    loss = objective_fn(params, sx, sy)
    loss.backward()
    return loss

# Define conic equation
def conics(params, x, y):
    a, b, c, d, e, f = params
    return a * x**2 + b * x * y + c * y**2 + d * x + e * y + f

# #hyperbolic only
# def calculate_eccentricity(A, B, C, s, acc, dis, o, dir_) :
def calculate_eccentricity_fx(A, B, C, s, acc, dis, o, dir_, theta_) :
    eig_val1 = 0.5 * (A + C + np.sqrt((A - C)**2 + B**2))
    eig_val2 = 0.5 * (A + C - np.sqrt((A - C)**2 + B**2))
```

```
a = np.sqrt(1 / np.abs(eig_val1))
b = np.sqrt(1 / np.abs(eig_val2))

a, b = sorted([a, b], reverse=True)

return (a**2/b**2)

# Initial guess for the conic parameters
# p0 = [1, 1, 1, 1, 1, 1]
p0 = torch.tensor([1, 1, 1, 1, 1, 1], dtype=torch.float32, requires_grad=True, device=device)

# Initialize the main dictionary to store no transforms
no_transforms = {}

# Summary stats for tracking processing
play_count_with_snap_event = 0
play_count_without_snap_event = 0
events_without_snap = {}

n = 200 #500 # Subset for testing
# tracking_data_dict_play_subset = dict(list(tracking_data_dict_play.items())[:n])
tracking_data_dict_play_subset = dict(list(tracking_data_dict_play.items()))

###NEWLY
tracking_data_dict_play_subset = dict_gp

# Device setup: Use GPU if available
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")

for (gameId, playId), group_data in tqdm(tracking_data_dict_play_subset.items()):
    # Initialize gameId in no_transforms if not already present
    if gameId not in no_transforms:
        no_transforms[gameId] = {}
```

```
# Initialize playId within the gameId if not already present
if playId not in no_transforms[gameId]:
    no_transforms[gameId][playId] = {
        'offense': {},
        'defense': {},
    }

# offensiveTeamAbbr = plays[(plays['gameId'] == gameId) & (plays['playId'] == playId)].iloc[0]['possessionTeam']
# defensiveTeamAbbr = plays[(plays['gameId'] == gameId) & (plays['playId'] == playId)].iloc[0]['defensiveTeam']
offensiveTeamAbbr = sup_data[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId)].iloc[0]['possession_team']
defensiveTeamAbbr = sup_data[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId)].iloc[0]['defensive_team']

# Retrieve tracking data for the current game/play
# tracking_data = group_data['tracking_data']
tracking_data = group_data['input']
tracking_data_op = group_data['output']

# Process tracking data for each player (nflId)
for nflId, player_data in tracking_data.groupby('nfl_id'):
    if pd.isna(nflId): # Skip if it's NaN (ball)
        continue

    snap_event_idx = 30
    # player_data_truncated = player_data

    if pd.notna(snap_event_idx):
        # player_data_truncated = player_data.loc[:snap_event_idx]
        player_data_truncated = player_data
```

```

#           x_ = player_data_truncated['x'].values
#           x_ = torch.tensor(player_data_truncated['x'].values, device
# =device)
#           y_ = torch.tensor(player_data_truncated['y'].values, device
# =device)
#           s_ = torch.tensor(player_data_truncated['s'].values, device
# =device)
#           a_ = torch.tensor(player_data_truncated['a'].values, device
# =device)
#           # dis_ = torch.tensor(player_data_truncated['dis'].values, d
# evice=device)
#           dir_ = torch.tensor(player_data_truncated['dir'].values, de
# vice=device)

#           o_ = torch.tensor(player_data_truncated['o'].values, device
# =device)

#           #
#           #optimized version
#           # disp = dis_.sum() # Sum of 'dis' directly on GPU
#           xl = x_[-1]          # Last x-coordinate
#           yl = y_[-1]          # Last y-coordinate
#           x_mean = x_.mean() # Mean of x-coordinates on GPU
#           y_mean = y_.mean() # Mean of y-coordinates on GPU

#           # print('x_',x_)
#           # print('y_',y_)
#           # print(len(x_), len(y_))

p0 = torch.tensor([1, 1, 1, 1, 1, 1], dtype=torch.float32,
requires_grad=True, device=device)
#           params = torch.tensor(p0, requires_grad=True, device=devic
e)
params = p0.clone().detach().requires_grad_(True).to(devic
e)
#           # print(params)
optimizer = LBFGS([params])
#           # print(optimizer)

```

```

optimizer.step(lambda: closure(optimizer, x_, y_, params))

# xx or xa
    optimized_params = params.detach()
    a,b,c,d,e,f = optimized_params.tolist() # aa bb cc ...
    print(nflId, a,b,c,d,e,f)
#
    theta_ = torch.arctan2(ab, aa - ac) / 2
    theta_ = torch.arctan2(torch.tensor(b, device=device), torch.
    tensor(a - c, device=device)) / 2
    cos_theta_, sin_theta_ = torch.cos(theta_), torch.sin(theta_
    -)

#####
# arxy = calculate_eccentricity_fx(a, b, c, np.mean(s_), np.
mean(a_), np.mean(dis_), np.mean(o_), np.mean(dir_), theta_) #aspect ratio

# if player_data['club'].iloc[0] == offensiveTeamAbbr:
if player_data['player_side'].iloc[0] == "Offense":
#
    no_transforms[gameId][playId]['offense'][nflId] = {'ec
centricity': eccentricity, 'disp':disp, 'theta': theta_, 'cuv':curvatur
e, 'cuvx': avg_cuvx, 'cuvy': avg_cuvy, 'disc':discriminant, 'major':al,
'minor':bl, 'arxy':arxy, 'ars':ars, 'ara': ara, 'theta_s': theta_s, 'the
ta_a':theta_a, 'xl':xl, 'yl':yl, 'x_mean':x_mean, 'y_mean':y_mean, 'game
id': group_data['meta_data']['gameId'], 'playid': group_data['meta_dat
a']['playId'], 'a'=a, 'b'= b, 'c'=c, 'd'=d, 'e'=e, 'f'=f, 'sa'=sa, 'sb'=sb,
'sc'=sc, 's'sd'=d, 'se'=se, 'sf'sf, 'aa'=aa, 'ab'=ab, 'ac'= ac, 'ad'=ad,
'ae'=ae, 'af'=af}
#
    no_transforms[gameId][playId]['offense'][nflId] =
{'a'=a, 'b'= b, 'c'=c, 'd'=d, 'e'=e, 'f'=f, 'sa'=sa, 'sb'=sb, 'sc'=sc, s'sd'=
d, 'se'=se, 'sf'sf, 'aa'=aa, 'ab'=ab, 'ac'= ac, 'ad'=ad, 'ae'=ae, 'af'=a
f}
    no_transforms[gameId][playId]['offense'][nflId] = {'a':
a, 'b': b, 'c': c, 'd': d, 'e': e, 'f': f, 'sa': sa, 'sb': sb, 'sc':sc, 's
d': sd, 'se':se, 'sf': sf, 'aa': aa, 'ab': ab, 'ac': ac, 'ad': ad, 'a
e': ae, 'af': af, 'xl':xl, 'yl':yl, 'x_mean':x_mean, 'y_mean':y_mean,
'theta_':theta_, 'theta_s':theta_s, 'theta_a':theta_a, 'gameid': group_
data['meta_data']['gameId'], 'playid': group_data['meta_data']['playI
d']} #, 'last_event'=last_event}
    # 'disp':disp,

# elif player_data['club'].iloc[0] == defensiveTeamAbbr:
elif player_data['player_side'].iloc[0] == "Defense":
```

```
# no_transforms[gameId][playId]['defense'][nflId] = {'eccentricity': eccentricity, 'disp':disp, 'theta': theta_, 'cuv':curvature, 'cuvx': avg_cuvx, 'cuvy': avg_cuvy, 'disc':discriminant, 'major':al, 'minor':bl, 'arxy':arxy, 'ars':ars, 'ara': ara, 'theta_s': theta_s, 'theta_a':theta_a, 'xl':xl, 'yl':yl, 'x_mean':x_mean, 'y_mean':y_mean, 'game_id': group_data['meta_data']['gameId'], 'playid': group_data['meta_data']['playId']}  
no_transforms[gameId][playId]['defense'][nflId] = {'a': a, 'b': b, 'c': c, 'd': d, 'e': e, 'f': f, 'sa': sa, 'sb': sb, 'sc':sc, 'sd': sd, 'se':se, 'sf': sf, 'aa': aa, 'ab': ab, 'ac': ac, 'ad': ad, 'ae': ae, 'af': af, 'xl':xl, 'yl':yl, 'x_mean':x_mean, 'y_mean':y_mean, 'theta_':theta_, 'theta_s':theta_s, 'theta_a':theta_a, 'gameid': group_data['meta_data']['gameId'], 'playid': group_data['meta_data']['playId']} #, 'last_event'=last_event}  
# 'disp':disp,  
  
else:  
    play_count_without_snap_event += 1  
    # events_without_snap[(gameId, playId)] = player_data['event'].unique() # Store unique events  
  
# Final summary  
print(f"Total plays processed: {len(tracking_data_dict_play_subset)}")  
print(f"Plays with 'ball_snap' or 'snap_direct' event: {play_count_with_snap_event}")  
print(f"Plays without 'ball_snap' or 'snap_direct' event: {play_count_without_snap_event}")  
  
# torch.save(no_transforms, 'no_transforms[all].pth')
```

In [ ]:

```
tracking_data_dict_play_subset = dict_gp

for (gameId, playId), group_data in tqdm(tracking_data_dict_play_subset.items()) :

    tracking_data = group_data['input']

    for nflId, player_data in tracking_data.groupby('nfl_id'):

        player_data_truncated = player_data

        p0 = torch.tensor([1, 1, 1, 1, 1, 1], dtype=torch.float32, requires_grad=True, device=device)

        x_ = torch.tensor(player_data_truncated['x'].values, device=device)
        y_ = torch.tensor(player_data_truncated['y'].values, device=device)

        # x_ = x_t
        # y_ = y_t
        # x_ = torch.tensor(x_t)
        # y_ = torch.tensor(y_t)

        params = p0.clone().detach().requires_grad_(True).to(device)
        optimizer = LBFGS([params])
        optimizer.step(lambda: closure(optimizer, x_, y_, params)) # xx or xa

        optimized_params = params.detach()
        a,b,c,d,e,f = optimized_params.tolist() # aa bb cc ..
        print(nflId, a,b,c,d,e,f)
```

In [ ]:

```
tracking_data_dict_play_subset = dict_gp

for (gameId, playId), group_data in tqdm(tracking_data_dict_play_subset.items()) :

    tracking_data = group_data['input']

    for nflId, player_data in tracking_data.groupby('nfl_id'):

        if nflId == 54481 :

            player_data_truncated = player_data

            p0 = torch.tensor([1, 1, 1, 1, 1, 1], dtype=torch.float32,
                             requires_grad=True, device=device)

            x_ = torch.tensor(player_data_truncated['x'].values, device
                             =device)
            y_ = torch.tensor(player_data_truncated['y'].values, device
                             =device)

            print(x_)
            print(y_)
            print(len(x_))
            # x_ = x_t
            # y_ = y_t
            x_ = torch.tensor(x_t)
            y_ = torch.tensor(y_t)
            print()
            # print(x_==torch.tensor(x_t))
            print(y_==torch.tensor(y_t))
            print(x_)
            print(y_)
            print(len(x_))

            params = p0.clone().detach().requires_grad_(True).to(device)
            optimizer = LBFGS([params])
            optimizer.step(lambda: closure(optimizer, x_, y_, params))
            # xx or xa
            optimized_params = params.detach()
```

```
a,b,c,d,e,f = optimized_params.tolist() # aa bb cc ..  
print(nflId, a,b,c,d,e,f)
```

In [54]:

```
print(device)
```

cpu

```
In [ ]:  
x_t = tracking_data_ip[(tracking_data_ip['game_id'] == 2023120306) & (tracking_data_ip['play_id'] == 1620) & (tracking_data_ip['nfl_id'] == 54481)][‘x’].to_list()  
y_t = tracking_data_ip[(tracking_data_ip['game_id'] == 2023120306) & (tracking_data_ip['play_id'] == 1620) & (tracking_data_ip['nfl_id'] == 54481)][‘y’].to_list()  
  
tracking_data_dict_play_subset = dict_gp  
  
for (gameId, playId), group_data in tqdm(tracking_data_dict_play_subset.items()):  
  
    tracking_data = group_data[‘input’]  
  
    for nflId, player_data in tracking_data.groupby('nfl_id'):  
  
        if nflId == 54481:  
  
            player_data_truncated = player_data  
  
            p0 = torch.tensor([1, 1, 1, 1, 1, 1], dtype=torch.float32,  
                             requires_grad=True, device=device)  
  
            x_ = torch.tensor(player_data_truncated[‘x’].values, device  
                             =device)  
            y_ = torch.tensor(player_data_truncated[‘y’].values, device  
                             =device)  
  
            # x_ = torch.tensor(player_data_truncated[‘x’].numpy(), device  
            #                   =device)  
            # y_ = torch.tensor(player_data_truncated[‘y’].numpy(), device  
            #                   =device)  
  
            # x_ = torch.tensor(list(player_data_truncated[‘x’].values),  
            #                   device=device)  
            # y_ = torch.tensor(list(player_data_truncated[‘y’].values),  
            #                   device=device)  
  
            x_ = torch.tensor(player_data_truncated[‘x’].to_list(), device  
                             =device)  
            y_ = torch.tensor(player_data_truncated[‘y’].to_list(), device  
                             =device)
```

```
ice=device)

    print(x_)
    print(y_)

    params = p0.clone().detach().requires_grad_(True).to(device)
    optimizer = LBFGS([params])
    optimizer.step(lambda: closure(optimizer, x_, y_, params))

# xx or xa

    optimized_params = params.detach()
    a,b,c,d,e,f = optimized_params.tolist() # aa bb cc ...
    print(nflId, a,b,c,d,e,f)

print(type(x_t))

# x_t = pd.Series(x_t)
# y_t = pd.Series(y_t)
x_ = torch.tensor(x_t)
y_ = torch.tensor(y_t)
x_ = torch.tensor(x_t, device=device)
y_ = torch.tensor(y_t, device=device)

print(x_)
print(y_)

    params = p0.clone().detach().requires_grad_(True).to(device)
    optimizer = LBFGS([params])
    optimizer.step(lambda: closure(optimizer, x_, y_, params))

# xx or xa

    optimized_params = params.detach()
    a,b,c,d,e,f = optimized_params.tolist() # aa bb cc ...
    print(nflId, a,b,c,d,e,f)
```

```
In [ ]:  
DTYPE = torch.float32  
DEVICE = device # "cpu" if CPU-only  
  
def to_t(x):  
    # list/np/pandas/tensor -> torch tensor on DEVICE with DTTYPE  
    if isinstance(x, torch.Tensor):  
        return x.to(device=DEVICE, dtype=DTYPE)  
    return torch.as_tensor(x, device=DEVICE, dtype=DTYPE)  
  
tracking_data_dict_play_subset = dict_gp  
  
for (gameId, playId), group_data in tqdm(tracking_data_dict_play_subset.items()):  
    tracking_data = group_data['input']  
  
    for nflId, player_data in tracking_data.groupby('nfl_id'):   
        if nflId != 54481:  
            continue  
  
        p0 = torch.tensor([1,1,1,1,1,1], dtype=DTYPE, device=DEVICE, requires_grad=True)  
  
        # --- Fit 1: DataFrame values (force dtype/device) ---  
        x_ = to_t(player_data['x'].to_numpy())  
        y_ = to_t(player_data['y'].to_numpy())  
        print(x_); print(y_)  
  
        params = p0.detach().clone().requires_grad_(True)  
        opt = LBFGS([params])  
  
        def closure():  
            opt.zero_grad(set_to_none=True)  
            loss = objective_fn(params, x_, y_) # must return scalar tensor  
            loss.backward()  
            return loss  
  
        opt.step(closure)  
        a,b,c,d,e,f = params.unbind()  
        print(nflId, *(t.item() for t in (a,b,c,d,e,f)))  
  
        # --- Fit 2: lists x_t / y_t (force dtype/device once) ---  
        x_ = to_t(x_t)
```

```
y_ = to_t(y_t)
print(x_); print(y_)

params = p0.detach().clone().requires_grad_(True)
opt = LBFGS([params])

def closure2():
    opt.zero_grad(set_to_none=True)
    loss = objective_fn(params, x_, y_)
    loss.backward()
    return loss

opt.step(closure2)
a,b,c,d,e,f = params.unbind()
print(nflId, *(t.item() for t in (a,b,c,d,e,f)))
```

In [ ]:

```
len(no_transforms)
```

In [57]:

```
no_transforms[2023120306][1620]['offense'][54481]
```

Out[57]:

```
{'a': -0.003792740870267153,
 'b': 0.004493914544582367,
 'c': -0.008404567837715149,
 'd': -0.027560919523239136,
 'e': 0.3873665928840637,
 'f': 0.9618589878082275,
 'sa': 0.3807755708694458,
 'sb': -0.0560823455452919,
 'sc': -0.013180343434214592,
 'sd': -0.0820808932185173,
 'se': -0.01174800843000412,
 'sf': -0.002267630770802498,
 'aa': 1.083538336388301e-06,
 'ab': -7.991233360371552e-06,
 'ac': 2.6366751626483165e-07,
 'ad': -5.041834356234176e-06,
 'ae': -1.1503550467750756e-06,
 'af': 1.3299326383275911e-05,
 'xl': tensor(48.7800, dtype=torch.float64),
 'yl': tensor(21.0300, dtype=torch.float64),
 'x_mean': tensor(51.8264, dtype=torch.float64),
 'y_mean': tensor(28.7071, dtype=torch.float64),
 'theta_': tensor(0.3862),
 'theta_s': tensor(-0.0707),
 'theta_a': tensor(-0.7343),
 'gameid': 2023120306,
 'playid': 1620}
```

In [58]:

```
# optimizer = LBFGS([params], lr=1.0, max_iter=20, line_search_fn='strong_wolfe')
```

```
In [ ]:  
x_ = torch.tensor(x_t)  
y_ = torch.tensor(y_t)  
p0 = torch.tensor([1, 1, 1, 1, 1, 1], dtype=torch.float32, requires_grad=True, device=device)  
  
params = p0.clone().detach().requires_grad_(True).to(device)  
print(params)  
optimizer = LBFGS([params])  
# optimizer = LBFGS([params], lr=1.0, max_iter=20, line_search_fn='strong_wolfe')  
  
print(optimizer)  
optimizer.step(lambda: closure(optimizer, x_, y_, params)) # xx or xa  
  
optimized_params = params.detach()  
a,b,c,d,e,f = optimized_params.tolist() # aa bb cc ..  
print( a,b,c,d,e,f)
```

In [ ]:

In [ ]:

In [60]:

```

x_t = tracking_data_ip[(tracking_data_ip['game_id'] == 2023120306) & (tracking_data_ip['play_id'] == 1620) & (tracking_data_ip['nfl_id'] == 54481)]['x'].to_list()
y_t = tracking_data_ip[(tracking_data_ip['game_id'] == 2023120306) & (tracking_data_ip['play_id'] == 1620) & (tracking_data_ip['nfl_id'] == 54481)]['y'].to_list()

# x_t = tracking_data_ip[(tracking_data_ip['game_id'] == 2023120306) & (tracking_data_ip['play_id'] == 1620) & (tracking_data_ip['nfl_id'] == 41349 )]['x'].to_list()
# y_t = tracking_data_ip[(tracking_data_ip['game_id'] == 2023120306) & (tracking_data_ip['play_id'] == 1620) & (tracking_data_ip['nfl_id'] == 41349 )]['y'].to_list()

# x_t = tracking_data_ip[(tracking_data_ip['game_id'] == 2023120306) & (tracking_data_ip['play_id'] == 1620) & (tracking_data_ip['nfl_id'] == 44887 )]['x'].to_list()
# y_t = tracking_data_ip[(tracking_data_ip['game_id'] == 2023120306) & (tracking_data_ip['play_id'] == 1620) & (tracking_data_ip['nfl_id'] == 44887 )]['y'].to_list()

print(x_t)
print()
print(y_t)
# y_t
# y_t.shape, x_t.shape #yt xt
len(y_t), len(x_t)

```

[54.93, 54.91, 54.86, 54.76, 54.61, 54.41, 54.16, 53.87, 53.54, 53.19, 52.83, 52.47, 52.11, 51.76, 51.42, 51.11, 50.82, 50.55, 50.3, 50.07, 49.87, 49.69, 49.52, 49.37, 49.22, 49.08, 48.93, 48.78]

[32.26, 32.26, 32.25, 32.24, 32.21, 32.15, 32.06, 31.95, 31.79, 31.59, 31.34, 31.03, 30.67, 30.26, 29.8, 29.29, 28.75, 28.17, 27.55, 26.9, 26.22, 25.52, 24.79, 24.06, 23.31, 22.55, 21.8, 21.03]

Out[60]:

(28, 28)

In [61]:

```
#      -0.003793      0.004494      -0.008405      -0.027561
0.387367      0.961859

print(' -0.003793      0.004494      -0.008405      -0.027561
0.387367      0.961859')

# dir_ = torch.tensor([0.0,0.0])
dis_ = np.array([0.0,0.0])
s_ = np.array([0.0,0.0])
a_ = np.array([0.0,0.0])
o_ = np.array([0.0,0.0])
dir_ = np.array([0.0,0.0])

# #hyperbolic only
# def calculate_eccentricity(A, B, C, s, acc, dis, o, dir_) :
def calculate_eccentricity_fx(A, B, C, s, acc, dis, o, dir_, theta_) :
    eig_val1 = 0.5 * (A + C + np.sqrt((A - C)**2 + B**2))
    eig_val2 = 0.5 * (A + C - np.sqrt((A - C)**2 + B**2))
    a = np.sqrt(1 / np.abs(eig_val1))
    b = np.sqrt(1 / np.abs(eig_val2))
    #     a, b = sorted([a, b], key=abs)
    #     a, b = sorted([a, b], key=abs, reverse=True)
    #     a, b = sorted([a, b])
    a, b = sorted([a, b], reverse=True)

    #     eccentricity = np.sqrt(1 - (b**2 / a**2))
    #     eccentricity = np.sqrt(1 + (b**2 / a**2))
    #     return np.sqrt(1 + (b**2 / a**2)) #
    #     return (b**2 / a**2)
    #     return np.log(b**2 / a**2)
    #     return (a / b) #asect ratio
    return (a**2/b**2) # not ^
    #     return 2*b**2/a #latus rectum lr
    #     return np.abs(theta_)
    #     return B**2 - 4 * A * C #dis
    #     return (b**2 / a**2) #asect ratio
    #     return np.sqrt((b**2 / a**2))
# else return np.nan
```

```
import pandas as pd
import numpy as np
from tqdm import tqdm
from scipy.optimize import minimize

# # Objective function to minimize
# def objective_fn(params, x, y):
#     return np.sum(conics(params, x, y)**2)

# Objective function to minimize
def objective_fn(params, x, y):
    return torch.sum(conics(params, x, y)**2)

import torch
from torch.optim import LBFGS

# # Define the closure function for optimization
# def closure():
#     optimizer.zero_grad() # Clear previous gradients
#     loss = objective_fn(params, sx, sy) # Compute loss using objective_fn
#     loss.backward() # Backpropagate the gradients
#     return loss

# def closure():
def closure(optimiser, sx, sy, params):
    optimizer.zero_grad()
    # Split params into sx and sy (assuming sx and sy are of the same size)
    # sx = params[:len(sx)] # First part of params for sx
    # sy = params[len(sx):] # Second part of params for sy

    # Now call the objective function with sx and sy
    loss = objective_fn(params, sx, sy)
    loss.backward()
    return loss
```

```

# Define conic equation
def conics(params, x, y):
    a, b, c, d, e, f = params
    return a * x**2 + b * x * y + c * y**2 + d * x + e * y + f

# # Objective function to minimize
# def objective_fn(params, x, y):
#     return np.sum(conics(params, x

# Initial guess for the conic parameters
# p0 = [1, 1, 1, 1, 1, 1]
p0 = torch.tensor([1, 1, 1, 1, 1, 1], dtype=torch.float32, requires_grad=True, device=device)

# x_ = x_t
# y_ = y_t
x_ = torch.tensor(x_t)
y_ = torch.tensor(y_t)

params = p0.clone().detach().requires_grad_(True).to(device)
optimizer = LBFGS([params])
optimizer.step(lambda: closure(optimizer, x_, y_, params)) # xx or xa
optimized_params = params.detach()
a,b,c,d,e,f = optimized_params.tolist() # aa bb cc ..
print(a,b,c,d,e,f)
#           theta_ = torch.arctan2(ab, aa - ac) / 2
theta_ = torch.arctan2(torch.tensor(b, device=device), torch.tensor(a - c, device=device)) / 2
cos_theta_, sin_theta_ = torch.cos(theta_), torch.sin(theta_)

```

```

-0.003793      0.004494      -0.008405      -0.027561
0.387367      0.961859
0.017734315246343613 -0.16258327662944794 0.1559484302997589 0.7624
068260192871 0.8475053906440735 0.9914431571960449

```

In [ ]:

In [62]:

```

x_t = tracking_data_ip[(tracking_data_ip['game_id'] == 2023111910) & (tracking_data_ip['play_id'] == 3746) & (tracking_data_ip['nfl_id'] == 54054)]['x'].to_list()
y_t = tracking_data_ip[(tracking_data_ip['game_id'] == 2023111910) & (tracking_data_ip['play_id'] == 3746) & (tracking_data_ip['nfl_id'] == 54054)]['y'].to_list()

# x_t = tracking_data_ip[(tracking_data_ip['game_id'] == 2023120306) & (tracking_data_ip['play_id'] == 1620) & (tracking_data_ip['nfl_id'] == 41349 )]['x'].to_list()
# y_t = tracking_data_ip[(tracking_data_ip['game_id'] == 2023120306) & (tracking_data_ip['play_id'] == 1620) & (tracking_data_ip['nfl_id'] == 41349 )]['y'].to_list()

# x_t = tracking_data_ip[(tracking_data_ip['game_id'] == 2023120306) & (tracking_data_ip['play_id'] == 1620) & (tracking_data_ip['nfl_id'] == 44887 )]['x'].to_list()
# y_t = tracking_data_ip[(tracking_data_ip['game_id'] == 2023120306) & (tracking_data_ip['play_id'] == 1620) & (tracking_data_ip['nfl_id'] == 44887 )]['y'].to_list()

print(x_t)
print()
print(y_t)
# y_t
# y_t.shape, x_t.shape #yt xt
len(y_t), len(x_t)

#      -0.003793      0.004494      -0.008405      -0.027561
0.387367      0.961859

print(' -0.003793      0.004494      -0.008405      -0.027561
0.387367      0.961859')

# dir_ = torch.tensor([0.0,0.0])
dis_ = np.array([0.0,0.0])
s_ = np.array([0.0,0.0])

```

```
a_ = np.array([0.0,0.0])
o_ = np.array([0.0,0.0])
dir_ = np.array([0.0,0.0])

# #hyperbolic only
# def calculate_eccentricity(A, B, C, s, acc, dis, o, dir_) :
def calculate_eccentricity_fx(A, B, C, s, acc, dis, o, dir_, theta_) :
    eig_val1 = 0.5 * (A + C + np.sqrt((A - C)**2 + B**2))
    eig_val2 = 0.5 * (A + C - np.sqrt((A - C)**2 + B**2))
    a = np.sqrt(1 / np.abs(eig_val1))
    b = np.sqrt(1 / np.abs(eig_val2))
    #     a, b = sorted([a, b], key=abs)
    #     a, b = sorted([a, b], key=abs, reverse=True)
    #     a, b = sorted([a, b])
    a, b = sorted([a, b], reverse=True)

    #         eccentricity = np.sqrt(1 - (b**2 / a**2))
    # eccentricity = np.sqrt(1 + (b**2 / a**2))
    # return np.sqrt(1 + (b**2 / a**2)) #
    # return (b**2 / a**2)
    # return np.log(b**2 / a**2)
    # return (a / b) #asect ratio
    return (a**2/b**2) # not ^
    #     return 2*b**2/a #latus rectum lr
    #     return np.abs(theta_)
    #     return B**2 - 4 * A * C #dis
    #     return (b**2 / a**2) #asect ratio
    #     return np.sqrt((b**2 / a**2))
#else return np.nan

import pandas as pd
import numpy as np
from tqdm import tqdm
from scipy.optimize import minimize

# # Objective function to minimize
# def objective_fn(params, x, y):
#     return np.sum(conics(params, x, y)**2)
```

```
# Objective function to minimize
def objective_fn(params, x, y):
    return torch.sum(conics(params, x, y)**2)

import torch
from torch.optim import LBFGS

# # Define the closure function for optimization
# def closure():
#     optimizer.zero_grad() # Clear previous gradients
#     loss = objective_fn(params, sx, sy) # Compute loss using objective_fn
#     loss.backward() # Backpropagate the gradients
#     return loss

# def closure():
def closure(optimiser, sx, sy, params):
    optimizer.zero_grad()
    # Split params into sx and sy (assuming sx and sy are of the same size)
    #     sx = params[:len(sx)] # First part of params for sx
    #     sy = params[len(sx):] # Second part of params for sy

    # Now call the objective function with sx and sy
    loss = objective_fn(params, sx, sy)
    loss.backward()
    return loss

# Define conic equation
def conics(params, x, y):
    a, b, c, d, e, f = params
    return a * x**2 + b * x * y + c * y**2 + d * x + e * y + f

# # Objective function to minimize
# def objective_fn(params, x, y):
#     return np.sum(conics(params, x
```

```
# Initial guess for the conic parameters
# p0 = [1, 1, 1, 1, 1, 1]
p0 = torch.tensor([1, 1, 1, 1, 1, 1], dtype=torch.float32, requires_grad=True, device=device)

# x_ = x_t
# y_ = y_t
x_ = torch.tensor(x_t)
y_ = torch.tensor(y_t)

params = p0.clone().detach().requires_grad_(True).to(device)
optimizer = LBFGS([params])
optimizer.step(lambda: closure(optimizer, x_, y_, params)) # xx or xa
optimized_params = params.detach()
a,b,c,d,e,f = optimized_params.tolist() # aa bb cc ..
print(a,b,c,d,e,f)
#           theta_ = torch.arctan2(ab, aa - ac) / 2
theta_ = torch.arctan2(torch.tensor(b, device=device), torch.tensor(a - c, device=device)) / 2
cos_theta_, sin_theta_ = torch.cos(theta_), torch.sin(theta_)
```

```
[59.55, 59.55, 59.55, 59.56, 59.57, 59.6, 59.63, 59.68, 59.74, 59.8, 59.86, 59.92, 59.97, 59.99, 59.98, 59.95, 59.88, 59.78, 59.64, 59.49, 59.32, 59.13, 58.94]
```

```
[24.24, 24.24, 24.26, 24.28, 24.31, 24.38, 24.5, 24.71, 24.97, 25.29, 25.68, 26.11, 26.59, 27.1, 27.64, 28.2, 28.78, 29.39, 30.02, 30.65, 31.3, 31.96, 32.64]
```

```
-0.003793      0.004494      -0.008405      -0.027561
0.387367      0.961859
-0.027767855674028397 0.033033911138772964 -0.053708042949438095 0.
968994677066803 0.9769294261932373 0.9992486238479614
```

In [63]:

```
# calculate_eccentricity_fx(  
#                 -0.027767855674028397, 0.033033911138772964, -0.053708042  
949438095,  
#                 0.0, 0.0, 0.0, 0.0, 0.0,  
#                 0.0  
#             )  
  
calculate_eccentricity_fx(  
    a, b, c,  
    0.0, 0.0, 0.0, 0.0, 0.0,  
    0.0  
)
```

Out[63]:

3.128045700576517

In [ ]:

In [ ]:

In [ ]:

In [64]:

len(tracking\_data\_dict\_play)

Out[64]:

18009

In [66]:

```
# Initial guess for the conic parameters
# pθ = [1, 1, 1, 1, 1, 1]
pθ = torch.tensor([1, 1, 1, 1, 1, 1], dtype=torch.float32, requires_grad=True, device=device)

# Initialize the main dictionary to store no transforms
no_transforms = {}

# Summary stats for tracking processing
play_count_with_snap_event = 0
play_count_without_snap_event = 0
events_without_snap = {}

n = 200 #50 #500 # Subset for testing
# tracking_data_dict_play_subset = dict(list(tracking_data_dict_play.items())[:n])
tracking_data_dict_play_subset = dict(list(tracking_data_dict_play.items()))

# Device setup: Use GPU if available
device = torch.device("cuda" if torch.cuda.is_available() else "cpu")

# for (gameId, playId), group_data in tqdm(tracking_data_dict_play_subset.items()):
items = list(tracking_data_dict_play_subset.items())
# for (gameId, playId), group_data in tqdm(items[13000:], total=len(items)-14000, initial=14000):
z = 14108

# for (gameId, playId), group_data in tqdm(items[z:], total=len(items)-z, initial=z):
for (gameId, playId), group_data in tqdm(items[z:]): #, total=len(items)-z, initial=z):

# for (gameId, playId), group_data in tqdm(items[14200:], total=len(items)-14200, initial=14200):

    # print(group_data['nfl_id'])
    # print(tracking_data['nfl_id'])
    # process
```

```
...  
  
# Initialize gameId in no_transforms if not already present  
if gameId not in no_transforms:  
    no_transforms[gameId] = {}  
  
# Initialize playId within the gameId if not already present  
if playId not in no_transforms[gameId]:  
    no_transforms[gameId][playId] = {  
        'offense': {},  
        'defense': {},  
    }  
  
# offensiveTeamAbbr = plays[(plays['gameId'] == gameId) & (plays['playId'] == playId)].iloc[0]['possessionTeam']  
# defensiveTeamAbbr = plays[(plays['gameId'] == gameId) & (plays['playId'] == playId)].iloc[0]['defensiveTeam']  
offensiveTeamAbbr = sup_data[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId)].iloc[0]['possession_team']  
defensiveTeamAbbr = sup_data[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId)].iloc[0]['defensive_team']  
  
# Retrieve tracking data for the current game/play  
# tracking_data = group_data['tracking_data']  
tracking_data = group_data['input']  
tracking_data_op = group_data['output']  
  
# print(tracking_data, tracking_data_op)  
# # Process tracking data for each player (nflId)  
# for nflId, player_data in tracking_data.groupby('nfl_id'):  
#     if pd.isna(nflId): # Skip if it's NaN (ball)  
#         continue  
  
#     snap_event_idx = 30  
#     # player_data_truncated = player_data  
  
#     if pd.notna(snap_event_idx):  
#         # player_data_truncated = player_data.loc[:snap_event_idx]  
#         player_data_truncated = player_data
```

```
# Final summary
print(f"Total plays processed: {len(tracking_data_dict_play_subset)}")
print(f"Plays with 'ball_snap' or 'snap_direct' event: {play_count_with_snap_event}")
print(f"Plays without 'ball_snap' or 'snap_direct' event: {play_count_without_snap_event}")

# None None
```

100% |██████████| 3901/3901 [00:05<00:00, 754.54it/s]

```
Total plays processed: 18009
Plays with 'ball_snap' or 'snap_direct' event: 0
Plays without 'ball_snap' or 'snap_direct' event: 0
```

In [69]:

```
items = list(tracking_data_dict_play_subset.items())
# row_key, row_value = items[14000]
# row_key, row_value = items[14108]
row_key, row_value = items[14107]

# print("GameId, PlayId:", row_key)
# print("Group Data:", row_value)
```

In [ ]:

In [ ]:

```
len(no_transforms)
```

In [ ]:

```
# torch.save(no_transforms, 'no_transforms[all].pth')
```

In [75]:

```
no_transforms = torch.load("no_transforms[all].pth", weights_only=False)
```

In [76]:

```
type(no_transforms)
```

Out[76]:

```
dict
```

In [77]:

```
len(no_transforms)
```

Out[77]:

```
273
```

In [ ]:

In [78]:

no\_transforms[2023120306][1620]['offense'][54481]

Out[78]:

```
{'a': -0.003792740870267153,
 'b': 0.004493914544582367,
 'c': -0.008404567837715149,
 'd': -0.027560919523239136,
 'e': 0.3873665928840637,
 'f': 0.9618589878082275,
 'sa': 0.11079810559749603,
 'sb': 0.17107662558555603,
 'sc': 0.11496926844120026,
 'sd': 0.3724491596221924,
 'se': 1.1378397941589355,
 'sf': 0.35767635703086853,
 'aa': 0.057990431785583496,
 'ab': 0.08438724279403687,
 'ac': 0.1850249022245407,
 'ad': 0.3267730474472046,
 'ae': 0.702515184879303,
 'af': 0.5347175002098083,
 'xl': tensor(48.7800, dtype=torch.float64),
 'yl': tensor(21.0300, dtype=torch.float64),
 'x_mean': tensor(51.8264, dtype=torch.float64),
 'y_mean': tensor(28.7071, dtype=torch.float64),
 'theta_': tensor(0.3862),
 'theta_s': tensor(0.7976),
 'theta_a': tensor(1.2776),
 'gameid': 2023120306,
 'playid': 1620}
```

In [ ]:

```
data_list = []

for gameId in tqdm(no_transforms, desc="Games", unit="game"):
#     print('YG')
# for gameId in no_transforms:
    for playId in no_transforms[gameId]: #2022101607 1127

        try :

            route_ran = sup_data.loc[(sup_data['game_id'] == gameId) &
(sup_data['play_id'] == playId), 'route_of_targeted_receiver'].values[0]

            pass_result = sup_data.loc[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId) , 'pass_result'].values[0]

            pass_length = sup_data.loc[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId) , 'pass_length'].values[0]

            offenseFormation = sup_data.loc[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId) , 'offenseFormation'].values[0]

            receiver_alignment = sup_data.loc[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId) , 'receiver_alignment'].values[0]

            play_action = sup_data.loc[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId) , 'play_action'].values[0]

            dropback_type = sup_data.loc[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId) , 'dropback_type'].values[0]

            dropback_distance = sup_data.loc[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId) , 'dropback_distance'].values[0]

            pass_location_type = sup_data.loc[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId) , 'pass_location_type'].values[0]

            defenders_in_the_box = sup_data.loc[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId) , 'defenders_in_the_box'].val
```

```
ues[0]

    team_coverage_man_zone = sup_data.loc[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId), 'team_coverage_man_zone'].values[0]

    team_coverage_type = sup_data.loc[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId), 'team_coverage_type'].values[0]

    penalty_yards = sup_data.loc[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId), 'penalty_yards'].values[0]

    pre_penalty_yards_gained = sup_data.loc[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId), 'pre_penalty_yards_gained'].values[0]

    yards_gained = sup_data.loc[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId), 'yards_gained'].values[0]

    w = sup_data.loc[sup_data['game_id'] == gameId, 'week'].values[0]#,
    
    q = sup_data.loc[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId), 'quarter'].values[0]#,
    
    d = sup_data.loc[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId), 'down'].values[0]

    possession_team = sup_data.loc[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId), 'possession_team'].values[0]#,
    
    defensive_team = sup_data.loc[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId), 'defensive_team'].values[0]

    home_team = sup_data.loc[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId), 'home_team_abbr'].values[0]#,
```

```
visitor_team = sup_data.loc[(sup_data['game_id'] == gameId) & (sup_data['play_id'] == playId), 'visitor_team_abbr'].values[0]

except IndexError:
    print('x', gameId, playId)
    route_ran = None # In case routeRan is missing

hadRushAttempt_ = None
rushingYards_ = None

pos = None

w = None
q = None
# d = None
# arxy = None
# ars = None
# ara = None

#     print('YP')
#     for team_type in ['home', 'away', 'offense', 'defense']:
for team_type in ['offense', 'defense']:
#         print('OD')
        for nflId, player_data in no_transforms[gameId][playId][team_type].items():
#             print(nflId)
#             if 'eccentricity' in player_data:
#                 print(player_data)

            try:
#                 route_ran = sup_data.loc[(player_play['gameId'] == gameId) &
#                                         # (player_play['playId'] ==
```

```

== playId) &
        #
        (player_play['nflId']
== nflId), 'route_of_targeted_receiver'].values[0]

        # hadRushAttempt_ = player_play.loc[(player_play['gameId'] == gameId) &
        #
        (player_play['playId'])

== playId) &
        #
        (player_play['nflId']
== nflId), 'hadRushAttempt'].values[0]
        # rushingYards_ = player_play.loc[(player_play['gameId'] == gameId) &
        #
        (player_play['playId'])

== playId) &
        #
        (player_play['nflId']
== nflId), 'rushingYards'].values[0]

        pos = tracking_data_ip.loc[(tracking_data_ip['nfl_id'] == nflId) & (tracking_data_ip['game_id'] == gameId) & (tracking_data_ip['play_id'] == playId), 'player_position'].values[0] #, # cant speak why becomes lal on coment
        frame = tracking_data_ip.loc[(tracking_data_ip['nfl_id'] == nflId) & (tracking_data_ip['game_id'] == gameId) & (tracking_data_ip['play_id'] == playId), 'frame_id'].values[0]
        # print(frame)
        # pos = tracking_data_ip.loc[(tracking_data_ip['nfl_id'] == nflId) & (tracking_data_ip['game_id'] == playId) & (tracking_data_ip['play_id'] == playId), 'position'].values[0]
        player_side = tracking_data_ip.loc[(tracking_data_ip['nfl_id'] == nflId) & (tracking_data_ip['game_id'] == gameId) & (tracking_data_ip['play_id'] == playId), 'player_side'].values[0]
        player_role = tracking_data_ip.loc[(tracking_data_ip['nfl_id'] == nflId) & (tracking_data_ip['game_id'] == gameId) & (tracking_data_ip['play_id'] == playId), 'player_role'].values[0]
        play_direction = tracking_data_ip.loc[(tracking_data_ip['nfl_id'] == nflId) & (tracking_data_ip['game_id'] == gameId) & (tracking_data_ip['play_id'] == playId), 'play_direction'].values[0]

        # eccentricity = calculate_eccentricity(a, b, c, torch.mean(s_), torch.mean(a_), torch.mean(dis_), torch.mean(o_), torch.mean(dir_), theta_)
        # eccentricity_s =
        # eccentricity_a =

```

```

#week quarter

# w = games.loc[games['gameId'] == gameId, 'week'].values[0]#, 

# q = plays.loc[(plays['gameId'] == gameId) &
#                 (plays['playId'] == playId), 'quarter'].values[0]#, 

# d = plays.loc[(plays['gameId'] == gameId) &
#                 (plays['playId'] == playId), 'down'].values[0]

# dis_ = np.array([0,0])
dis_ = torch.tensor([0.0,0.0])

# eccentricity
# arxy = calculate_eccentricity_fx(player_data['a'], player_data['b'], player_data['c'], torch.mean(s_), torch.mean(a_), torch.mean(dis_), torch.mean(o_), torch.mean(dir_), theta_)

# ars = calculate_eccentricity_fx(player_data['sa'], player_data['sb'], player_data['sc'], torch.mean(s_), torch.mean(a_), torch.mean(dis_), torch.mean(o_), torch.mean(dir_), theta_)

# ara = calculate_eccentricity_fx(player_data['aa'], player_data['ab'], player_data['ac'], torch.mean(s_), torch.mean(a_), torch.mean(dis_), torch.mean(o_), torch.mean(dir_), theta_)

# arxy = calculate_eccentricity_fx(player_data['a'], player_data['b'], player_data['c'], torch.mean(s_), torch.mean(a_), torch.mean(dis_), torch.mean(o_), torch.mean(dir_), theta_)

# ars = calculate_eccentricity_fx(player_data['sa'], player_data['sb'], player_data['sc'], torch.mean(s_), torch.mean(a_), torch.mean(dis_), torch.mean(o_), torch.mean(dir_), theta_)

# ara = calculate_eccentricity_fx(player_data['aa'], player_data['ab'], player_data['ac'], torch.mean(s_), torch.mean(a_), torch.mean(dis_), torch.mean(o_), torch.mean(dir_), theta_)

arxy = calculate_eccentricity_fx(player_data['a'], player_data['b'], player_data['c'], torch.mean(s_), torch.mean(a_), torch.mean(dis_), torch.mean(o_), torch.mean(dir_), theta_)

ars = calculate_eccentricity_fx(player_data['sa'], player_data['sb'], player_data['sc'], torch.mean(s_), torch.mean(a_), torch.mean(dis_), torch.mean(o_), torch.mean(dir_), theta_)

ara = calculate_eccentricity_fx(player_data['aa'], player_data['ab'], player_data['ac'], torch.mean(s_), torch.mean(a_), torch.mean(dis_), torch.mean(o_), torch.mean(dir_), theta_)

eccentricity = calculate_eccentricity(player_data

```

```
[ 'a' ], player_data[ 'b' ], player_data[ 'c' ], torch.mean(s_), torch.mean(a_), torch.mean(dis_), torch.mean(o_), torch.mean(dir_), theta_)

    except IndexError:
        print('x', gameId, playId)
        route_ran = None # In case routeRan is missing

        hadRushAttempt_ = None
        rushingYards_ = None

        pos = None

        w = None
        q = None
        d = None
        arxy = None
        ars = None
        ara = None

    data_list.append({
        'game_id': gameId,
        'play_id': playId,
        'nfl_id': nflId,
        '# 'eccentricity': player_data['eccentricity'],
        '# 'routeRan': route_ran,
        '# 'rushingYards': rushingYards_,

        '# 'hadRushAttempt': hadRushAttempt_,

        '# 'dis': player_data['dis'].mean()
        '# 'disp': player_data['disp'], #,
        'week': w,
        'quarter': q,
        'down': d,
        'arxy': arxy,
        'ars': ars,
        'ara': ara,
        'pos': pos,
        'route_ran': route_ran,
        'pass_result': pass_result,
```

```

'pass_length' : pass_length,
'offense_formation' : offense_formation,
'receiver_alignment' : receiver_alignment,
'play_action' : play_action,
'dropback_type' : dropback_type,
'dropback_distance' : dropback_distance,
'pass_location_type' : pass_location_type,
'defenders_in_the_box' : defenders_in_the_box,
'team_coverage_man_zone' : team_coverage_man_zone,
'team_coverage_type' : team_coverage_type,
'penalty_yards' : penalty_yards,
'pre_penalty_yards_gained' : pre_penalty_yards_gained

ed,
'yards_gained' : yards_gained,
# 'w' = w,
# 'q' = q,
# 'd' = d,
'possession_team' : possession_team,
'defensive_team' : defensive_team,
'home_team' : home_team,
'veisitor_team' : visitor_team,

'pos' : pos,
'frame' : frame,
'player_side' : player_side,
'player_role' : player_role,
'play_direction' : play_direction,

'eccentricity' : eccentricity

#           'quarter' :
#           #'theta': player_data['theta']
#           'theta': np.degrees(player_data['theta']),
#           'theta_abs': np.abs(np.degrees(player_data['theta'])),
#           'curvature' : player_data['cuv'],
#           'curvatureex' : player_data['cuvx'],
#           'curvatureey' : player_data['cuvy'],
#           'curvatureexabs' : np.abs(player_data['cuvx']),
#           'discriminant' : player_data['disc'],

```

```
#             #'rushingYards' : rushingYards_, 'disp':disp, 'wee  
k': w, 'quarter':q, 'theta': theta_, 'cuv':curvature, 'cuvx': avg_cuvx,  
'cuvy': avg_cuvy, 'disc':discriminant,  
#             'major':player_data['major'], 'minor':player_data  
['minor'], 'arxy':player_data['arxy'], 'ars':player_data['ars'], 'ara':  
player_data['ara'], 'theta_s': np.degrees(player_data['theta_s']), 'the  
t_a_a':np.degrees(player_data['theta_a']), 'xl':player_data['xl'], 'yl':pl  
ayer_data['yl'], 'x_mean':player_data['x_mean'], 'y_mean':player_data['y  
_mean']  
  
#             'dir'  
})  
  
# Create a DataFrame from the list  
eccentricity_df = pd.DataFrame(data_list)  
eccentricity_df.head()  
  
# Games: 65%/███████ | 88/136 [40:30<21:52, 27.34s/game]  
# 2022101607 1127  
# Games: 100%/██████████ | 136/136 [1:02:39<00:00, 27.64s/game]
```

In [ ]:

```
# torch.save(eccentricity_df, 'eccentricity_df.pth')
```

In [79]:

```
import pandas as pd
import pywt
import numpy as np
import json

from tqdm import tqdm

import torch

pd.set_option('display.max_columns', None)
pd.set_option('display.max_colwidth', None)
pd.set_option('display.max_rows', None)
```

In [80]:

```
# eccentricity_df_loaded = torch.load("eccentricity_df.pth")
# eccentricity_df = torch.load("eccentricity_df.pth")
eccentricity_df = torch.load("eccentricity_df.pth", weights_only=False)
```

In [81]:

```
eccentricity_df.shape
```

Out[81]:

```
(173150, 34)
```

In [82]:

```
eccentricity_df.head()
```

```
/usr/local/lib/python3.11/dist-packages/pandas/io/format/format.p
y:1458: RuntimeWarning: invalid value encountered in greater
    has_large_values = (abs_vals > 1e6).any()
/usr/local/lib/python3.11/dist-packages/pandas/io/format/format.p
y:1459: RuntimeWarning: invalid value encountered in less
    has_small_values = ((abs_vals < 10 ** (-self.digits)) & (abs_vals
> 0)).any()
/usr/local/lib/python3.11/dist-packages/pandas/io/format/format.p
y:1459: RuntimeWarning: invalid value encountered in greater
    has_small_values = ((abs_vals < 10 ** (-self.digits)) & (abs_vals
> 0)).any()
```

Out[82]:

	game_id	play_id	nfl_id	week	quarter	down	arxy	ars	ara
0	2023090700	101	43290	1	1	3	4.285046	17.689530	13.986375
1	2023090700	101	44930	1	1	3	1272.317907	29.623264	10.836651
2	2023090700	101	53541	1	1	3	7.995704	16.799922	5.577154
3	2023090700	101	53959	1	1	3	15.961717	21.050669	1.979233
4	2023090700	101	46137	1	1	3	8.025947	24.178478	77.014893



In [ ]:

In [ ]:

## Adding features

In [83]:

```
data_list2 = []

for gameId in tqdm(no_transforms, desc="Games", unit="game"):
#     print('YG')
# for gameId in no_transforms:
    for playId in no_transforms[gameId]: #2022101607 1127

        try :

            w = sup_data.loc[sup_data['game_id'] == gameId, 'week'].values[0]#,
            q = sup_data.loc[(sup_data['game_id'] == gameId) &
                             (sup_data['play_id'] == playId), 'quarter'].values[0]#,

        except IndexError:
            print('x', gameId, playId)

        # w = None
        # q = None

        for team_type in ['offense', 'defense']:

            for nflId, player_data in no_transforms[gameId][playId][team_type].items():

                try:
                    # pos = tracking_data_ip.loc[(tracking_data_ip['nfl_id'] == nflId) & (tracking_data_ip['game_id'] == gameId) & (tracking_data_ip['play_id'] == playId), 'player_position'].values[0] #, # cant speak why becomes lal on coment

                    # eccentricity = calculate_eccentricity(player_data['a'], player_data['b'], player_data['c'], torch.mean(s_), torch.mean(a_), torch.mean(dis_), torch.mean(o_), torch.mean(dir_), theta_)

                    a = player_data['a']
                    b = player_data['b']
                    c = player_data['c']
```

```
d = player_data['d']
e = player_data['e']
f = player_data['f']

except IndexError:

    a = None
    b = None
    c = None
    d = None
    e = None
    f = None

    data_list2.append({
        'game_id': gameId,
        'play_id': playId,
        'nfl_id': nflId,

        'a': a,
        'b': b,
        'c': c,
        'd': d,
        'e': e,
        'f': f

    })

# Create a DataFrame from the list
eccentricity_df2 = pd.DataFrame(data_list2)
eccentricity_df2.head()

# torch.save(eccentricity_df2, 'eccentricity_df2.pth')
# eccdf.to_parquet('eccentricity_df.parquet', index=False) # when you
next save
```

Games: 100% |  | 273/273 [00:11<00:00, 24.25game/s]

Out[83]:

	game_id	play_id	nfl_id	a	b	c	d	e	f
0	2023090700	101	43290	0.001110	-0.035468	-0.030741	0.965663	0.965806	0.998
1	2023090700	101	44930	0.035347	-0.229525	0.376368	-0.223143	0.618138	0.945
2	2023090700	101	53541	-0.036359	0.088478	-0.173440	0.887024	0.932098	0.995
3	2023090700	101	53959	-0.115785	0.279340	-0.223708	0.888024	0.887742	0.994
4	2023090700	101	46137	-0.028223	0.066825	-0.148180	0.903952	0.943100	0.996



In [84]:

```
eccentricity_df2 = torch.load("eccentricity_df2.pth", weights_only=False)
```

In [85]:

```
# len(eccentricity_df2)
```

In [86]:

```
# Reload your dataframe
# eccdf = torch.load("eccentricity_df.pth")
eccentricity_dfx = eccentricity_df.copy()
eccentricity_dfy = eccentricity_df2.copy()

# Add week and quarter from sup_data
# sup_add = sup_data[['game_id', 'play_id', 'week', 'quarter']].drop_duplicates(['game_id', 'play_id'])

# eccdf = eccdf.merge(sup_add, on=['game_id', 'play_id'], how='left')
eccentricity_dfz = eccentricity_dfx.merge(eccentricity_dfy, on=['game_id', 'play_id', 'nfl_id'], how='inner')
```

In [ ]:

In [87]:

```
torch.save(eccentricity_dfz, 'eccentricity_dfz.pth')
```

In [88]:

```
torch.save(eccentricity_dfz, 'eccentricity_dfz.pth')
eccentricity_dfz = torch.load("eccentricity_dfz.pth", weights_only=False)
eccentricity_dfz.shape
```

Out[88]:

(173150, 40)

In [ ]:

In [ ]:

In [89]:

eccentricity\_dfz.shape

Out[89]:

(173150, 40)