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Lab 4 Answers

3.a. If A and B are Independent, then

$$P(A \cap B) = P(A)P(B)$$
$$P(A | B) = P(A)$$
$$P(B | A) = P(B)$$

$P(\bar{A}) = 1 - P(A)$

$P(\bar{B}) = 1 - P(B)$

substitute $P(A)$ for

$$P(A) = P(A \cap B) + P(A \cap \bar{B}) \Rightarrow$$
$$P(A) = P(A)P(B) + P(A \cap B')$$
$$P(A) - P(A)P(B) = P(A \cap B')$$
$$P(A)(1 - P(B)) = P(A \cap B')$$
$$P(A)P(\bar{B}) = P(A \cap B')$$

substitute $P(B)$ for

$$P(B) = P(B \cap A) + P(B \cap \bar{A}) \Rightarrow$$
$$P(B) = P(B)P(A) + P(B \cap \bar{A})$$
$$P(B) - P(B)P(A) = P(B \cap \bar{A})$$
$$P(B)(1 - P(A)) = P(B \cap \bar{A})$$
$$P(B)P(\bar{A}) = P(B \cap \bar{A})$$

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Continued...

3. a. continued...

start with $P(\bar{A} \cap \bar{B})$

Demorgans law $\Rightarrow P(A \cup B)'$

$$= 1 - P(A \cup B)$$

$$= 1 - (P(A) + P(B) - P(A \cap B))$$

$$= 1 - P(A) - P(B) + \underline{P(A \cap B)}$$ · i.e. independence assumed

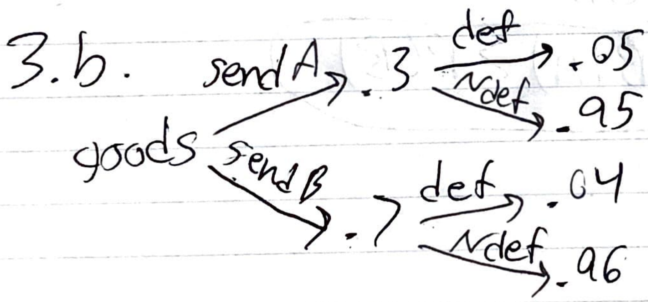
$$= 1 - P(A) - P(B) + P(A)P(B)$$

$$= (1 - P(A)) - P(B)(1 - P(A))$$

$$= (1 - P(A))(1 - P(B))$$ substituted

$$= P(A')P(B)$$

3.b.



i. P(sent to A ∩ defective) $= .3 \times .05 = .015$

ii. P(sent to A ∩ not defective) $= .3 \times .95 = .285$

iii. P(sent to B ∩ defective) $= .7 \times .04 = .028$

iv. P(sent to B ∩ not defective) $= .7 \times .96 = .672$

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Continued...

3.c. For events A and B,

$$P(A|B) > P(A) \Rightarrow P(B|A) > P(B)$$

$$P(A|B) = \frac{P(A\cap B)}{P(B)} \qquad P(B|A) = \frac{P(B\cap A)}{P(A)}$$

$$P(A\cap B) = P(A|B)\,P(B) \qquad P(B\cap A) = P(B|A)\,P(A)$$

$$P(A|B) > P(A)$$

$$\frac{P(A\cap B)}{P(B)} > P(A)$$

$$P(A\cap B) > P(A)\,P(B)$$

$$\frac{P(A\cap B)}{P(A)} > P(B)$$

$$P(B|A) > P(B)$$