Arrow of Time from Spontaneous Symmetry Breaking in LQG

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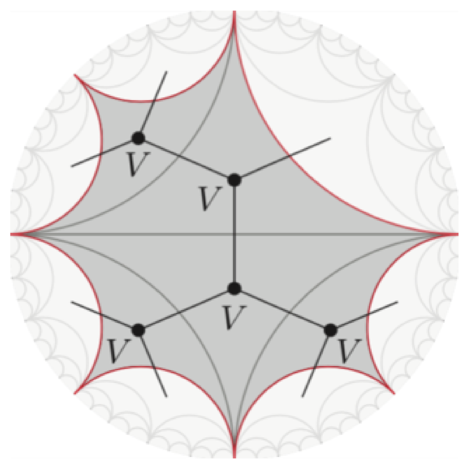


Abstract

The question of the origin time's arrow is a major outstanding problem in physics. Here we present a possible mechanism for generation of a cosmological arrow via spontaneous symmetry breaking in a theory of quantum gravity. In [1], Chen and Vishwanath have shown that a local notion of time-reversal symmetry can be encoded into a gauge field living on a tensor network. Using the fact that tensor networks can be used to describe the state space of Loop Quantum Gravity [2, 3] we show that a spontaneous symmetry breaking mechanism operating on such tensor networks can lead to the spontaneous generation of an arrow of time corresponding to the generation of non-zero order parameter for the Chen-Vishwanath gauge field.

Time Reversal in Quantum Mechanics

Operators:	$\hat{\boldsymbol{x}} \to \hat{\boldsymbol{x}}; \quad \hat{\boldsymbol{p}} \to -\hat{\boldsymbol{p}}$
Commutators:	$\left[\hat{x}_i, \hat{p}_j\right] = i\hbar\delta_{ij} \to \left[\hat{x}'_i, \hat{p}'_j\right] = -i\hbar\delta_{ij}$
Equation of motion	$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi \to i\hbar\frac{\partial\psi}{\partial t'} = -\hat{H}\psi$

where $t' = -t$. Define Wigner quantum time reversal operator:

$$\mathcal{T}(c_1\psi_1 + c_2\psi_2) = c_1^*\mathcal{T}\psi_1 + c_2^*\mathcal{T}\psi_2 \tag{1}$$

Such an operator is called *anti-linear* or *anti-unitary*. Acting with this operator on both sides of the Schrodinger equation we obtain:

$$\mathcal{T}\left(i\hbar\frac{\partial\psi}{\partial t}\right) = i\hbar\frac{\partial(\mathcal{T}\psi)}{\partial t'} = H(\mathcal{T}\psi) \tag{2}$$

Under $\mathcal{T}$, the quantum commutators no longer change sign:

$$\mathcal{T}\left(\left[\hat{x}_i, \hat{p}_j\right]\right)\mathcal{T}^{-1} = \mathcal{T}i\mathcal{T}^{-1}\hbar\delta_{ij} \to \left[\hat{x}'_i, \hat{p}'_j\right] = i\hbar\delta_{ij} \tag{3}$$

Time Reversal of Many Body State

$$|\psi\rangle = \sum_{i_1,i_2,\ldots,i_N} C_{i_1,i_2,\ldots,i_N}|i_1, i_2, \ldots, i_N\rangle \tag{4}$$

Co-efficients $C_{i_1,i_2,\ldots,i_N}$ are defined over the entire Hilbert space and cannot be decomposed into local contributions in any simple manner. Global action of time-reversal is well defined

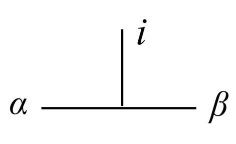
$$\mathcal{T}|\psi\rangle = \sum_{i_1,i_2,\ldots,i_N} C^*_{i_1,i_2,\ldots,i_N} U_1 \otimes \ldots \otimes U_N|i_1, i_2, \ldots, i_N\rangle \tag{5}$$

but local action is not clear, because $C_{i_1,i_2,\ldots,i_N}$ cannot be simply split into sum of local pieces. To do so, we need matrix product states.

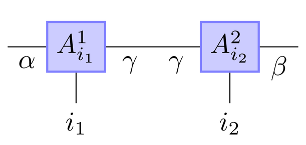
Matrix Product State

Following Bridgeman and Chubbs [4]:

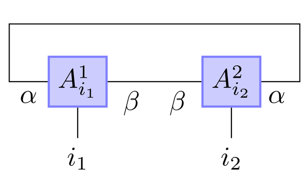
$$|\psi\rangle = \sum_{i_1,i_2,\ldots,i_N} \mathrm{Tr}\left[A^{(1)}_{i_1}A^{(2)}_{i_2}\ldots A^{(N)}_{i_N}\right]|i_1, i_2, \ldots, i_N\rangle \tag{6}$$



$A^k_{i_k;\alpha\beta}$: Single vertex of a matrix product state. Internal indices are denoted by α, β. Physical indices by $i, j, \ldots$

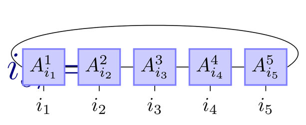


A connection between two adjacent vertices corresponds to multiplying the associated matrices.



To take the trace we complete the loop by connecting the first and last vertices

$$|\psi\rangle = \sum_{i_1,i_2,\ldots,i_5} \mathrm{Tr}\left[A^{(1)}_{i_1}A^{(2)}_{i_2}\ldots A^{(N)}_{i_5}\right]|i_1, i_2, \ldots, i_5\rangle$$



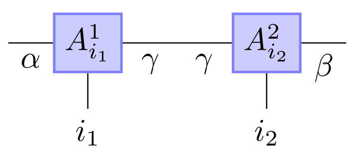
Tensor Network State on Arbitrary Graphs

Vertex Tensors	$T^v_{i_v;\alpha_1\ldots\alpha_{v_k}}$	v : vertex i_v: vertex state/physical index $\alpha_1..\alpha_k$: internal ("bond") indices
Edge Tensors	$g^e_{i_e;\alpha\beta}$	e: edge label i_e : edge state/physical index α, β: bonding indices

$$|\Psi\rangle = \sum_{\{i_1,\ldots,i_{n_v}\},\{j_1,\ldots,j_{n_e}\}} \mathrm{Tr}\left[\prod_{v\in\Gamma} T^v_{i_v}\prod_{e\in\Gamma} g^e_{j_e}\right]|i_1, \ldots, i_{n_v}; j_1, \ldots, j_{n_e}\rangle$$

Dangling Indices: Bulk vs Boundary d.o.f.

States with ***dangling indices***, *i.e.* physical degrees of freedom which are not traced over, represent physical states of quantum geometry. Consider the MPS for a two-site system as before:



$$|\Psi_{\alpha\beta}\rangle = \sum_{\{i_1,i_2\}} A^1_{i_1;\alpha\gamma}A^2_{i_2;\gamma\beta}|i_1, i_2\rangle$$

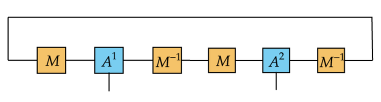
Now the state is a two-index object. Likewise if the graph has n dangling edges, then the resulting state will also have n indices of size equal to the bond dimension. Consider a graph Γ, which has a subset of n vertices which are connected to only a single edge, each. These vertices and their accompanying edges $\{v_1, \ldots, v_n; e_1, \ldots, e_n\}$ form a subset $\partial\Gamma \subset \Gamma$. Let $\bar{\Gamma} = \Gamma/\partial\Gamma$, be the "bulk" of the graph which does not have any dangling edges. Resulting state is n index object.

$$\boxed{|\Psi_{\alpha_1,\ldots,\alpha_n}\rangle = |\Psi_{\text{bulk}}\rangle|\alpha_1, \ldots, \alpha_n\rangle}$$

where:

$$\boxed{|\Psi_{\text{bulk}}\rangle = \sum_{\{i_v,j_e\}\in\bar{\Gamma}} \left[\prod_{v\in\bar{\Gamma}} T^v_{i_v}\prod_{e\in\bar{\Gamma}} g^e_{j_e}\right]|\{i_v\}, \{j_e\}\rangle}$$

Gauge Transformations



Global gauge rotation of two-site MPS state

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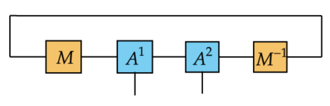

All the rotations in the interior cancel out and we are left with only two occurrences at the either end of the chain.

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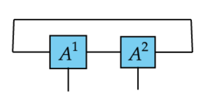
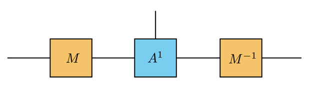
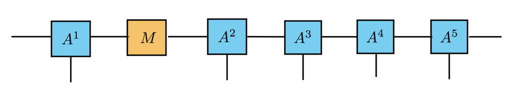


On taking the trace, the remaining two matrices cancel, leaving us the original state.



Action of gauge rotation on single site of MPS: $A^1 \to M(A^1)M^{-1}$



Insertion of a single symmetry flux: $A^2 \to M(A^2)$

Gauging Time Reversal Symmetry

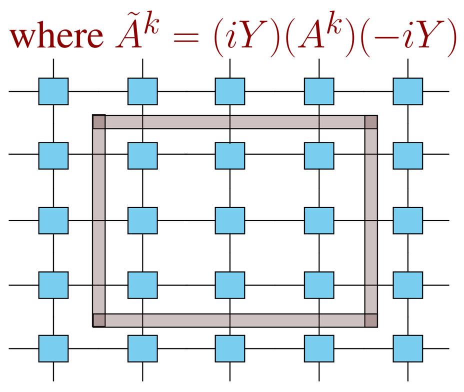
Action of time-reversal on a *single* system: $\mathcal{T} = UK$, where K is complex conjugation, and U is some unitary.

For a single spin $U = i\sigma_y$. State $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \to i\sigma_y|\psi\rangle^* = \alpha^*|1\rangle - \beta^*|0\rangle$

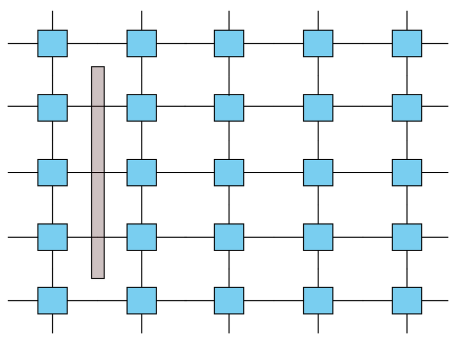
Local TRS action on k^{th} site of MPS:

$$\boxed{\mathcal{T}_k|\Psi\rangle_{MPS} = \sum_{i_1,i_2,\ldots,i_N} \mathrm{Tr}\left[A^1_{i_1}, \ldots, \tilde{A}^k, \ldots, A^N_{i_N}\right]|i_1, \ldots, (i\sigma_y)i_k, \ldots, i_N\rangle}$$

where $\tilde{A}^k = (iY)(A^k)(-iY)$



Applying constant gauge transformation to given region, leaves symmetry only along flux insertions along boundary of region.



Insertion of a symmetry flux twist only along one line

Spin Networks

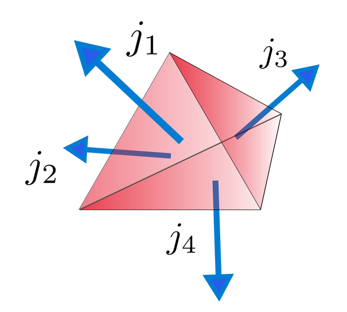


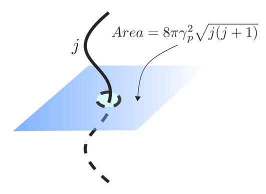
Illustration of a four-valent spin-network vertex.
Each edge is labelled with a representation j_k of $SU(2)$.
Face area: $8\pi\gamma l_p^2\sqrt{j_k(j_k+1)}$.
Vertex label: $I_v \in \mathrm{Inv}\left[\bigotimes_{i=1}^4 \mathcal{H}^{j_k}\right] \equiv \mathcal{H}_{\text{Inv}}$

Spin Network State on Arbitrary Graphs

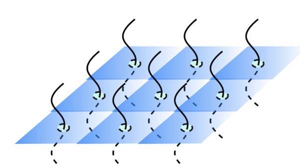
Vertex Tensors	$I^v_{i_v;\alpha_1\ldots\alpha_{v_k}}$	v : vertex i_v: vertex state/physical index $\alpha_i \in \{1, .., d_i\}$; $d_i = 2j_i + 1$: contracts with the state on the i^{th} edge joined to the vertex.
Edge Tensors	$D^e_{j_e;\alpha\beta}$	e: edge j_e : $SU(2)$ label of edge $\alpha, \beta \in \{1, \ldots, (2j_e + 1)\}$: bonding indices

$$|\Psi\rangle = \sum_{\{i_v\},\{j_e\}} \mathrm{Tr}\left[\prod_{v\in\Gamma} I^v_{i_v}\prod_{e\in\Gamma} D^e_{j_e}\right]|i_1, \ldots, i_{n_v}; j_1, \ldots, j_{n_e}\rangle$$

Geometry from Spin Networks

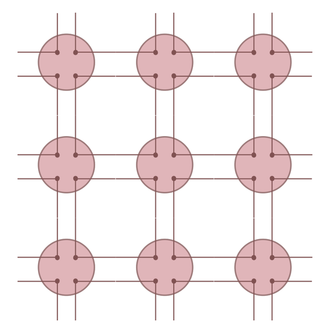


Quantum of area from single puncture

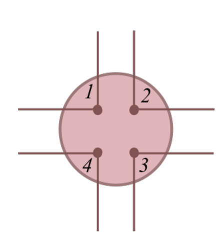


Many punctures generate macroscopic areas

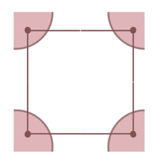
CZX State - SPT with Z_2 Order in 2D



CZX model [5, 6] is example of 2D SPT phase with Z_2 symmetry.
4 spins at each site; 4 neighboring spins form a plaquette.
State is invariant under ***local*** action of of U_{CZX} at each vertex



$$U_{CZX} = U_XU_{CZ}$$
$$U_X = X_1 \otimes X_2 \otimes X_3 \otimes X_4$$
$$U_{CZ} = (CZ)_{12}(CZ)_{23}(CZ)_{34}(CZ)_{41}$$
$$CZ = |00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| - |11\rangle\langle 11|$$



Each plaquette is in an entangled state: $|0000\rangle + |1111\rangle$

Prescription for Z_2 SPT State for Quantum Geometry

1. Join spin-network 4-valent vertices to make a square lattice.
2. Enforce U_{CZX} symmetry at each site to generate SPT Z_2 state.
3. Check that resulting state satisfies LQG constraints.

Discussion

1. Time-reversal symmetry breaking happens due to formation of Z_2 domains in spin-networks.
2. At "high temperatures", state will be disordered with no emergent macroscopic geometry.
3. As temperature is lowered, macroscopic volume and area states emerge
4. Accompanied by the formation of Z_2 domains which violate time-reversal symmetry.
5. To identify semiclassical field which leads to Z_2 symmetry in GR.

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