Mathjax custom macros

Quantum Random Walks

Triviality of Unitary Random Walk

Reference: Ambainis, Quantum Walks and their Algorithmic Applications

In the simple 1D classical random walk, at each time step the walker moves either to left or to the right, depending on the result of a coin toss. The quantum analog of this would be a process where the n^{th} site on the lattice is identified with a basis state $|n\rangle$ and at each time step, the state would evolve under a unitary transformation as:

$$|n\rangle \rightarrow a|n-1\rangle + b|n\rangle + c|n+1\rangle$$

However, as shown by \cite{Meyer1996From}, the above transformation is unitary iff one of the coefficients is equal to ± 1 and other two are equal to zero. In other words **any** unitary transformation corresponding to the above process is necessarily trivial.

"Coin" States for Non-Trivial Walks

In order to be able to generate a non-trivial quantum random walk the introduction of an extra "spin" degree of freedom at each lattice site is required. Each basis state is therefore enlarged from a single spin $1/2$ Hilbert space (the two allowed states corresponding to whether the site is empty or occupied) to a two spin state:

$$|n\rangle \rightarrow \{|n,0\rangle, |n,1\rangle\}$$

Random Walker State Space

Formally speaking the walker is represented by a state in a Hilbert space $\mathcal{H}_p$, which has a *canonical* or *computational* basis given by a set of vectors $\{|n\rangle\}$, where $n = 0 \ldots (N-1)$. These basis states are the *position* eigenstates. A general state of the walker is given by:

$$|\Psi\rangle = \sum_{0}^{N-1} a_n |n\rangle$$

A state with all coefficients $a_n = 0$, except for $n = i$, corresponds to a walker localized at the i^{th} site.

Coin State Space

The *coin* is described by a single spin Hilbert space $\mathcal{H}_c$, with basis states $|0\rangle, |1\rangle$.