$\sqrt{h} \to \sqrt{h + g\Delta}$

and

$g \sim 1/(l_s/l_{pl})$

assuming $l_s \gg l_{pl}$

then $g \ll 1$.

expanding the square root:

$$\sqrt{h + g\Delta} \sim \sqrt{h}\left(1 + \frac{1}{2}\frac{g\Delta}{h} + \mathcal{O}(g^2/h^2)\right)$$

$$+\ \mathcal{O}(\ /n))$$

in limit of large n $\mathcal{O}(1/n^2)$ and higher terms would be negligible. Also the g^2 would suppress it even more.

$$\Rightarrow \sqrt{n+g\Delta} \sim \underbrace{\sqrt{n} + \frac{1}{2}\frac{g\Delta}{\sqrt{n}}}$$

we need an action with an inverse dependence with [illegible]

on the area !