

PHASES OF THE HUBBARD AND SACHDEV YE KITAEV MODEL

Project Report

submitted in partial fulfillment of the requirement for the degree of

MASTER OF SCIENCE

IN

PHYSICS

by

ARYA ANTHERJANAM V

Reg No:16400216PH03

Under the guidance of

Dr.DEEPAK VAID



DEPARTMENT OF PHYSICS

NATIONAL INSTITUTE OF TECHNOLOGY

KARNATAKA,SURATHKAL,MANGALORE-575025

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DECLARATION

I hereby declare that the report of the P.G. Project Work entitled “**PHASES OF THE HUBBARD AND SACHDEV YE KITAEV MODEL**” which is submitted to National Institute of Technology Karnataka, Surathkal, in partial fulfillment of the requirements for the award of the Degree of Master of Science in the Department of Physics, is a bonafide report of the work carried out by me. The material contained in this report has not been submitted to any University or Institution for the award of any degree.

In keeping with the general practice in reporting scientific observations, due acknowledgement has been made whenever the work described is based on the findings of other investigators.

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CERTIFICATE

This is to certify that the project entitled “**PHASES OF THE HUBBARD AND SACHDEV YE KITAEV MODEL**” is an authenticated record of work carried out by **ARYA ANTHERJANAM V** ,Reg.No:16400216PH03 in partial fulfillment of the requirement for the award of the Degree of Master of Science in Physics which is submitted to Department of Physics, National Institute of Technology, Karnataka, during the period 2017-2018.

Chairman-DPGC

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ABSTRACT

An object that crosses the event horizon of a black hole is fully absorbed and hence the information contained in it is lost forever. But according to quantum mechanics, information can neither be created nor be destroyed. This information paradox could be solved if we consider that all the information about the matter and energy within the three dimensional volume of a black hole is encoded as a hologram upon its two dimensional surface, the event horizon. This is termed as the holographic principle. Sachdev Ye Kitaev model emerged as a model for holography. It deals with random infinite-range interactions including Majorana fermions. The model is studied in detail and the Green's function for the model is determined and analysed.

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Chapter 1

Introduction

1.1 The Many-Body Problem

The systems that we have in the physical world like electrons in a metal, molecules in a liquid, etc.,... comprise of many particles interacting with each other, giving rise to different phenomena. But, the particles in intricate motion are difficult to study. Hence, we make attempts to bring them to a single body problem just like two masses connected by a spring are treated as a single body by considering the centre of mass. But converting every physical problem to single body problem is practically impossible. So, we need to equip ourselves with techniques for handling many-body problems efficiently.

Many-body problem may be defined as the study of effects of interaction between bodies on the behaviour of a many-body system. It deals with general methods applicable to all many-body systems. To solve a many-body problem, we need to understand the problem first. The essential part is that there should be many bodies interacting with each other. A system of non-interacting bodies will not come under many body problem, rather it constitutes many one body problems. These interactions play a major role in determining the physical aspects of the system.

In many cases, many-body problems were solved by ignoring the interactions. Another approach used was the canonical transformation technique, which involves transforming the basic equations of the many-body system to a new set of coordinates in which the interaction term becomes small. These were to an extent, able to explain the behaviour of the system. But, the lack of a systematic method kept the many-body

problem in its infant stage well up into 1950s. Emergence of quantum fields theory opened new doors to deal with many-body problems.

1.2 Quasi Particles

Quantum field theory provided a new simple picture of matter in which systems of interacting real particles are described in terms of approximately non-interacting fictitious bodies called 'quasi particles' and 'collective excitations'. Consider two masses connected by a spring as shown in the figure.

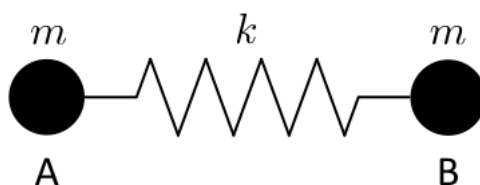


Figure 1.1: Two body system

Our system consists of two strongly coupled real bodies. The motion of each mass is complicated due to interaction. We break up the complicated motion into two parts: motion of the centre of mass and motion about centre of mass. Centre of mass is an independent body of mass $2m$, and the latter part has a reduced mass of $\frac{m}{2}$. Thus, the system acts as if it was composed of two non-interacting fictitious bodies: centre of mass body and the reduced mass body.

The concept of quasi particle arises from the fact that when a real particle moves through the system, it pushes or pulls on its neighbours and thus becomes surrounded by a 'cloud' of agitated particles. The real particle plus its cloud forms the quasi particle. The presence of cloud makes the properties of quasi particle different from that of a real particle. The quasi particle has an effective mass and a lifetime. It is in an excited energy level of the many-body system, hence referred to as 'elementary excitation' of the system.

A model used to describe many-body effects in metals is the electron gas model. It consists of a box containing a large number of electrons interacting with each other via Coulomb force. A uniform, fixed, positive charge background is present in the system

to keep the whole system electrically neutral. In the ground state, electrons are spread out uniformly. Suppose we shoot a single, well-localised electron into the electron gas. The coulombic repulsions keep the other electron away from the new electron such that an empty space is created near the extra electron. The empty space has positive charge due to the background charge. Thus, the extra electron has lifted out electrons from the charge distribution in its vicinity, thereby creating holes.

This gives a picture of an extra electron surrounded by a cloud of constantly changing holes and lifted out electrons. This combination is termed as quasi electron. The positive hole cloud shields the negative charge of the electron. Hence, two quasi electrons whose hole cloud do not overlap, interact weakly due to shielding. This is why metals generally behave as independent of electron-electron interaction.

Quasi particles have an energy different from that of real particle due to the cloud surrounding it.

$$E = \frac{p^2}{2m^*}$$

where m^* is the effective mass.

$$E_{quasi} - E_{real} = E_{self} \tag{1.1}$$

where E_{self} is the self energy, which is the energy that a particle has due to the changes it caused by itself in its environment.

Chapter 2

Scope and Objectives

2.1 Scope

It is believed that all information of an object that enters a black hole is lost forever. But, this is in contradiction with the basic law that information cannot be destroyed or created. This paradox could be solved by the holographic principle, which states that information in a higher dimension can be encoded as a hologram in lower dimension. A model for the same needed to be devised. But, in many theories that explain physical phenomena, we restrict the interactions to nearest neighbours. But it is not the case with the real systems. So, a model should be devised which can explain the system with long range interactions.

2.2 Objectives

- To understand the treatment of many-body problems.
- To acquire an in-depth knowledge about Green's function.
- To get familiar with diagrammatic methods.
- To study about the principle of holography.
- To study about the model and its hamiltonian.
- To find the Green's function for the model.

Chapter 3

Green's Function

3.1 Definition and basic properties

The Green's function, named after the English mathematician and physicist George Green, is a powerful mathematical tool to solve linear differential equation with constraining boundary conditions. Green's function method enables the solution of a differential equation containing an inhomogeneous term, often called a source term, to be related to an integral operator containing the source.

As an elementary example, consider the Poisson's equation, which gives the potential $\psi(\mathbf{r})$ generated by a charge distribution whose charge density is $\rho(\mathbf{r})$.

$$-\nabla^2\psi(\mathbf{r}) = \frac{1}{\epsilon_0}\rho(\mathbf{r}) \quad (3.1)$$

The solution gives the Coulombic potential

$$\psi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \quad (3.2)$$

The right hand side of the equation is an integral operator that converts ρ into ψ , and the kernel can be seen as the Green's function,

$$G(\mathbf{r}, \mathbf{r}') = \frac{1}{4\pi\epsilon} \frac{1}{|\mathbf{r} - \mathbf{r}'|} \quad (3.3)$$

i.e.,

$$\psi(\mathbf{r}) = \int d^3r' G(\mathbf{r}, \mathbf{r}')\rho(\mathbf{r}') \quad (3.4)$$

$G(\mathbf{r}, \mathbf{r}')$ gives the contribution to ψ at the point \mathbf{r} produced by a point source of unit magnitude at the point \mathbf{r}' . In general, if D is a linear operator, $f(x)$ is the desired solution and $g(x)$ is the homogeneity source, then

$$Df(x) = g(x)$$

then the Green's function is defined as the solution of a differential equation with a delta homogeneity:

$$DG(x, x') = \delta(x - x') \quad (3.5)$$

and

$$f(x) = \int dx' G(x, x') g(x') \quad (3.6)$$

There are two types of Green's function: advanced and retarded Green's functions. If the final state of a system is given, we can find the initial state by retarded Green's function. We can calculate the response of a system after it is perturbed. On the other hand, if the initial state of the system is known, the final state can be predicted using advanced Green's function.

Properties

When D and its boundary conditions define the eigenvalue problem $D\psi = \lambda\psi$ with eigenfunctions $\phi(\mathbf{r})$ and corresponding eigenvalues λ_n , then

- G is symmetric.

$$G(r, r') = G(r', r) \quad (3.7)$$

- G has the eigenfunction expansion:

$$G(r, r') = \sum_n \frac{\phi_n^*(r') \phi_n(r)}{\lambda_n}$$

- G is continuous at $r=r'$.

$$G|_{r=r'+} = G|_{r=r'-}$$

- Derivative of G experiences a specific jump discontinuity at $x=x'$.

$$\frac{dG}{dr} \Big|_{r=r'+} - \frac{dG}{dr} \Big|_{r=r'-} = \frac{-1}{p(r')}$$

3.2 Propagators in Quantum Mechanics

In the classical scenario, the detailed description of a many-body system requires the position of each particle as a function of time $r_1(t), r_2(t), \dots$. In quantum case, it depends on the time dependent wave function of the system $\psi(r_1, r_2, \dots, r_N, t)$. This is a difficult task to carry out. But it turns out that it is not necessary to know the detailed behaviour of each particle in order to find the important physical properties of the system, but rather just the average behaviour of one or two typical particles. The quantities that describe the average behaviour are called the propagators.

By looking at (3.6), we can say that the Green's function takes a function at some space and evolve it to some other space. In fact, we say that Green's function is a propagator which propagates the particle from one point to another. Propagators play an important role in the treatment of many body problem. The reasons for their immense use is attributed as follows. First, they yield in a direct way the most important physical properties of the system. Second, they have a simple physical interpretation. Third, they can be calculated in a highly systematic and automatic way and appeals to one's physical intuition.

The idea is simple. We start with a system in its ground state say $|0\rangle$. Then, we introduce (create) a particle in it which is used to probe the system. It makes interactions with the systems that might cause some excitations. Later, we remove (annihilate) the particle and check whether the system remains in ground state. Propagator is the probability amplitude that the system remains in ground state.

$$G(r_2, r_1) = \langle 0 | (particleannihilated)(particlecreated) | 0 \rangle$$

There are one particle, two particle and zero particle propagators. One particle propagator can be defined as the probability amplitude that a particle will be observed at the point r_2 at time t_2 if it was introduced into the system at a point r_1 in time t_1 . Two particle propagator is the probability amplitude for observing one particle at (r_2, t_2) and another at (r_4, t_4) if they were put into the system at (r_1, t_1) and (r_3, t_3) respectively. Zero propagator or vacuum amplitude is the probability that no particle emerges from the system at t_2 when we introduce no particle at time t_2 .

For instance, take the example of a metal that consists of a set of positively charged ions arranged so that they form a regular lattice or a lattice with some irregularities. An electron interacts with these ions by means of the Coulomb force. Here, the single particle propagator is the sum of the quantum mechanical probability amplitudes for all the possible ways the electron can propagate from point r_1 in the crystal to point r_2 . It can propagate freely, or can interact with various ions. The total probability will be a sum of all these individual properties.

$$P(r_2, r_1) = P_o(r_2, r_1) + P_o(r_A, r_1)P(A)P_o(r_2, r_A) + P_o(r_B, r_1)P(B)P(r_2, r_B) + \\ P_o(r_A, r_1)P(A)P_o(r_A, r_A)P(A)P_o(r_2, r_A) + \dots \quad (3.8)$$

where A,B, etc.. are the ions from which the electron interacts with, P_o denotes the free propagator and the term on left denotes the total propagator.

Now, assume that there is a particular affinity towards A and also the free propagators are all equal, i.e.,

$$P_o(r_2, r_1) = P_o(r_A, r_1) = P_o(r_2, r_B) = \dots = c \quad (3.9)$$

Then,

$$P(r_2, r_1) = c + c^2P(A) + c^3P^2(A) + \dots \\ = c [1 + c P(A) + c^2P^2(A) + \dots] \\ = c [1 + c P(A) + (c P(A))^2 + (cP(A))^3 + \dots]$$

This is an infinite geometric progression whose sum which can be summed as

$$P(r_2, r_1) = c \left[\frac{1}{1 - cP(A)} \right] \quad (3.10)$$

3.3 Electron's Green's function

The single electron Green's function is defined as the statistical expectation value of the product of fermion operators at different positions l and n and at different times t

and t' .

$$G(t, t') = -i \langle \theta(t - t') [C_l(t) C_n^\dagger(t')] \rangle \quad (3.11)$$

where C_n^\dagger creates an electron at n^{th} site at time t' and C_l annihilates an electron on l^{th} site at time t ; and, $\theta(t - t')$ is the Heaviside step function which takes the value 1 if t is greater than t' , and 0 otherwise.

$$[C_l(t) C_n^\dagger(t')] = C_l(t) C_n^\dagger(t') - C_n^\dagger(t') C_l(t) \quad (3.12)$$

Then, (3.11) can be written as

$$G(t, t') = -i \theta(t - t') \langle \{C_l(t) C_n^\dagger(t')\} \rangle \quad (3.13)$$

where the operators obey anti commutation relation

$$\{C_l(t) C_n^\dagger(t')\} = C_l(t) C_n^\dagger(t') + C_n^\dagger(t') C_l(t) \quad (3.14)$$

3.4 Equation of Motion Technique

Equation of motion technique is used to find Green's function by determining its time evolution. Let us start with Green's function.

$$\begin{aligned} G(t, t') &= -i \theta(t - t') \langle \{C_l(t) C_n^\dagger(t')\} \rangle \\ i \delta_t G_{ij}(t, t') &= i(-i) \delta(t - t') \langle \{C_i(t) C_j^\dagger(t')\} \rangle - i \theta(t - t') \langle \{i \dot{C}_i(t) C_j^\dagger(t')\} \rangle \\ &= \delta(t - t') \delta_{ij} - i \theta(t - t') \langle \{[C_i, H](t), C_j^\dagger(t')\} \rangle \end{aligned} \quad (3.15)$$

This comes from the Heisenberg picture of operators.

$$i \frac{dC_i}{dt} = [C_i, H] + i \frac{\delta C_i}{\delta t} = [C_i, H] \quad (3.16)$$

Commutator in the LHS of (3.15) tells us that the dynamics of Green's function is fully determined by the Hamiltonian of the system. In this way, we will get a chain of Green's functions. However, for non-interacting systems, the commutator is a single fermion operator and ends at the first equation itself.

Chapter 4

Diagrammatic Methods

4.1 Feynman Diagrams

There are two methods for calculating propagators. One is to solve the differential equations and the other is to expand the propagator in an infinite series and evaluate the series approximately. This can be done in a systematic, picturesque way with the help of Feynman diagrams.

Equation (3.8) can be represented as a picture using Feynman diagrams.

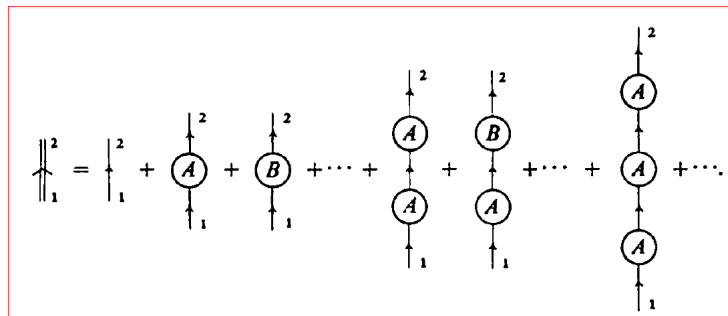


Figure 4.1: Pictorial representation of infinite series

Assuming that A dominates, the series can be approximated using partial summation.

$$\begin{aligned}
\parallel &= \uparrow \times \left\{ 1 + \text{circle with } A \text{ and } \uparrow + \left(\text{circle with } A \text{ and } \uparrow \right)^2 + \left(\text{circle with } A \text{ and } \uparrow \right)^3 + \dots \right\} \\
&= \uparrow \times \left(\frac{1}{1 - \text{circle with } A \text{ and } \uparrow} \right) = \frac{1}{\uparrow^{-1} - \text{circle with } A \text{ and } \uparrow}
\end{aligned}$$

4.2 Single particle propagator

Consider a system of many particles into which a particle is introduced at (r_1, t_1) . The single particle propagator is the sum of the probability amplitudes for all the ways the particle can travel through the system from (r_1, t_1) to (r_2, t_2) . It can have free propagation without interaction or it can emerge from the system after one or more interactions.

For first-order interactions, it follows the following steps:

- An extra particle enters the system at time t_1 .
- It interacts with a particle at point r_2 and changes place with it.
- Extra particle leaves at time t_2

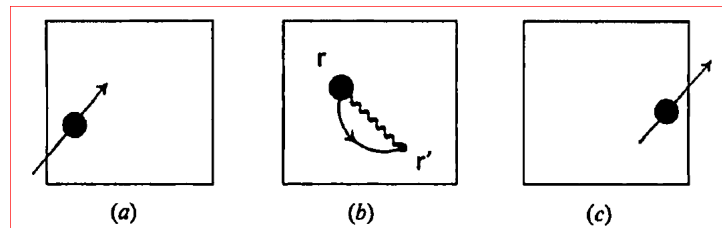


Figure 4.2: First order interaction

Second-order interaction follows the following steps:

- An extra particle enters the system at t_1 .
- It interacts with a particle in the system at time t , lifting it out of its place, thus creating a hole in the system.

- The extra particle plus the 'hole' and the 'lifted-out particle' ('particle-hole pair') propagate through the system.
- The extra particle interacts with the lifted-out particle at time t' , knocking it back into the hole, thus destroying the particle-hole pair.
- The extra particle moves out of the system at time t_2 .

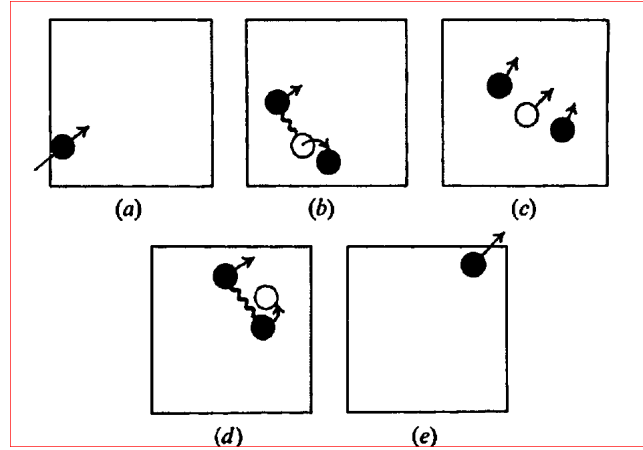


Figure 4.3: Second order interaction

To represent these diagrammatically, assume that the time increases in the upward direction.

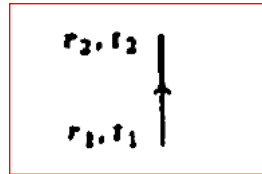


Figure 4.4: Propagator for particle moving freely from (r_1, t_1) to (r_2, t_2) .

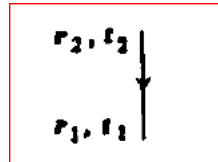


Figure 4.5: Propagator for hole moving freely from (r_1, t_1) to (r_2, t_2)

So, the total single particle propagator can be represented as the sum of all these Feynman diagrams.

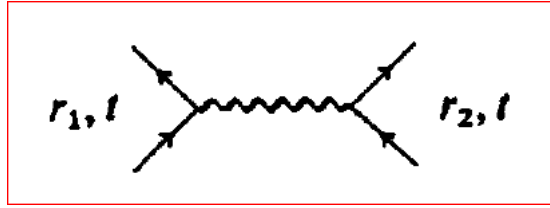


Figure 4.6: Propagator for a particle at r_1 interacting with a particle at r_2

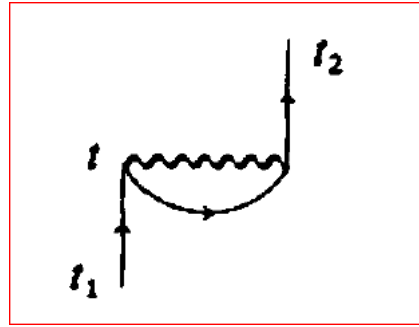


Figure 4.7: Open oyster diagram for first-order interaction

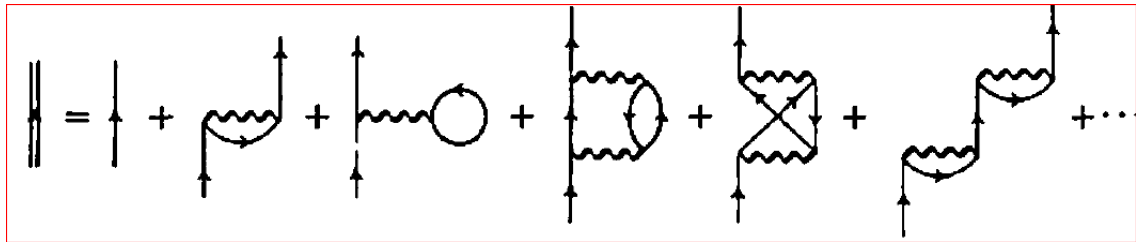


Figure 4.8: Feynman diagram for single particle propagator

4.3 Two particle propagator

Two particle propagator is the sum of probability amplitudes for all the possible ways two particles can enter the system, interact with it and emerge at a later time.

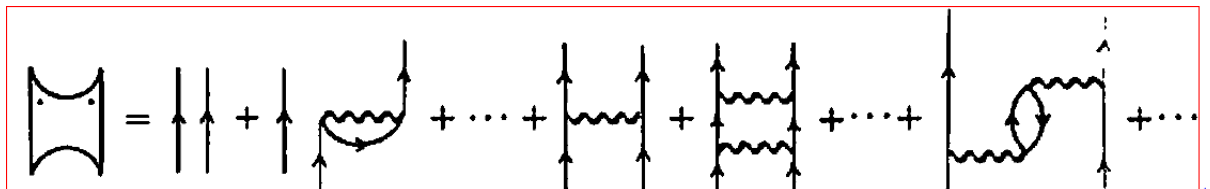


Figure 4.9: Feynman diagram for a two particle propagator

4.4 No particle propagator

This deals with the situation where a system has no extra particle in it. Interactions take place between particles inside the system. Finally, no particle emerges out of the system. It also goes by the term vacuum amplitude. It can be summarised in following steps:

- Vacuum.
- At t_1 , interaction between two particles in the system cause them to be lifted out, forming holes.
- The two particle-hole pairs move freely through the system.
- Both pairs annihilate at t_2 .
- Vacuum.

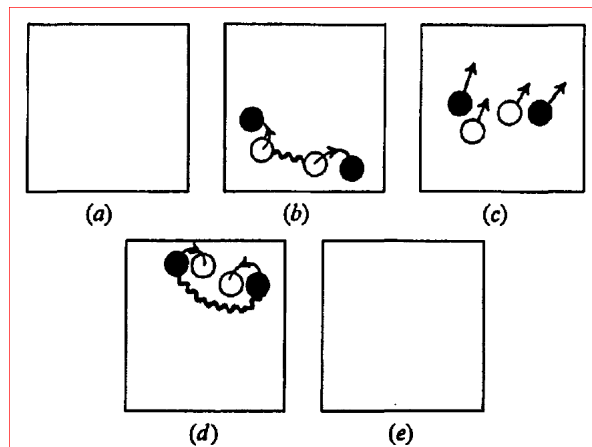


Figure 4.10: Representation of no particle propagator

Chapter 5

A Brief Introduction to Holography and Sachdev Ye Kitaev (SYK) Model

5.1 Black hole and Information Paradox

Black hole is a region of space-time from which gravity prevents anything, including light from escaping. The theory of general relativity predicts that a sufficiently compact mass will deform space-time to form a black hole. It absorbs all the light that falls into it and reflects nothing, just like a perfect black body. Around a black hole, there is a mathematically defined surface called an *event horizon* that marks the point of no return.

In 1970s, Stephen Hawking stated that when an object crosses the event horizon of a black hole, it is continuously dragged to the centre of the blackhole where it gets crushed. Every object contains a specific arrangement of matter, and hence some kind of information. When it is crushed, the information is destroyed.

If a black hole can evaporate, a portion of the information it contains is lost forever. The information contained in thermal radiation emitted by a black hole is degraded; it does not recapitulate information about matter previously swallowed by the black hole. The irretrievable loss of information conflicts with one of the basic postulates of quantum mechanics. According to Schrodinger equation, physical systems that change over

time cannot create or destroy information, a property known as '*unitarity*'. This apparent contradiction between general relativity and quantum mechanics is the information paradox.

A black hole in two dimensional space-time is given by 1 time and 1 space dimensions. It is a strongly chaotic system and should be described by a Hamiltonian. Its time evolution is given by a matrix. To resolve the information paradox, holographic principle has been introduced.

5.2 Holography

Studies showed that information paradox can be explained by holographic principle or holography. It states all the things falling inside a black hole were somehow captured in the horizon itself. So, the information was not lost, it is encoded on the surface of the black hole like a hologram. The information can be later retrieved in a chaotic form through the radiation released during quantum evaporation of the black hole.

The basic idea is that information carried by all the 3D objects in the world would be carried by some 2D surface that surround us; and we are just the holographic projection of that. The information is stored on some kind of holographic film on the edge of the universe. Indeed, it is now believed that all of the information contained within our Universe is nothing more than a 3D projection of the information stored on a 2D membrane at the edge of the cosmos.

The information contained in a black hole is not proportional to its volume, but its surface area. If we add one bit of information, the black hole grows by one square of planck unit. Holography is a convenient mathematical tool for us to solve higher dimensional problems by reducing it to lower dimensions. This principle helps in explaining quantum gravity. If you make mapping from the three-dimensional Universe to two-dimensional surface, it was found that the gravity disappears. So, our 3D Universe with gravity might be equal to a 2D Universe without gravity. So, if we could do calculations on the 2D structure which are normalisable and do not lead to infinity, then we can map it back to the 3D structure to explain about our Universe.

5.3 Motivation: SY model

The model for holography is greatly inspired by Sachdev Ye (SY) model, a random quantum spin system originally introduced to describe a quantum Heisenberg magnet with random infinite-range interactions. The original SY model is given by the Hamiltonian

$$H = \frac{1}{\sqrt{M}} \sum_{j,k=1}^N J_{j,k} \mathbf{S}_j \cdot \mathbf{S}_k \quad (5.1)$$

where the couplings $J_{j,k}$ are independent random variables drawn from a Gaussian distribution, each with the same variance and a mean zero. \mathbf{S}_j s are spin operators at lattice sites which may take the value $\frac{1}{2}, 1, \frac{3}{2}, 2, \dots$. The coupling constant J decreases as the distance between spins increases, but is a constant for nearest neighbours.

Quantum magnets are spin systems in which the spins interact via quantum mechanical exchange interactions. There are two possible ground states of a random quantum magnet. One is called spin-glass, which is a state with magnetic long range order with $\langle S_i \rangle \neq 0$. Another one is called spin fluid which is a quantum disordered state with $\langle S_i \rangle = 0$. At zero temperature, there is a transition from spin fluid to the magnetically ordered spin glass phase.

5.4 SYK model

5.4.1 Hamiltonian

Sachdev Ye Kitaev (SYK) model is a model for holography which relates quantum mechanical systems to quantum gravity systems. So, what we consider is a quantum mechanical system living in 0 space and 1 time and is related to quantum gravity problem in 1 space and 1 time. This model is for the systems that share some properties with black holes. SYK model deals with the quantum mechanics of N fermions, which satisfy commutation relation.

$$\{\chi_i, \chi_j\} = \delta_{ij} \quad (5.2)$$

This algebra can be realised in a space of dimension $L = 2^{\frac{N}{2}}$. χ_i s are $L \times L$ matrices and are called Majorana fermions. To formulate the Hamiltonian for the model, we build

a matrix as a sum over all possible products of four of these fermion operators.

$$H = \frac{1}{4!} \sum_{i < j < k < l} J_{ijkl} \chi_i \chi_j \chi_k \chi_l \quad (5.3)$$

where J_{ijkl} are couplings drawn from a random Gaussian distribution.

$$P(J_{ijkl}) = \exp\left(-\frac{N^3 J_{ijkl}^2}{12J^2}\right) \quad (5.4)$$

J defines the ensemble with mean

$$\langle J \rangle = 0$$

and the variance

$$\langle J^2 \rangle = \frac{3!J^2}{N^3} \quad (5.5)$$

Kitaev argued that it behaved in a way that was characteristic of systems which are holographic to quantum gravity.

Based on the similarities between the two and four point functions of 1 + 1 dimensional Schwarzschild black hole and those of the SYK model, this model has been proposed as a holographic dual to Schwarzschild black hole.

5.4.2 Majorana Fermions

All the matter in the Universe are made up of particles called fermions. Physicists know three kinds of fermions: Dirac, Weyl and Majorana fermions. Dirac fermions have mass, and is the class to which most fermions that make up matter belong. Weyl fermions are fermions that have no mass, and have only been found in lurking in solids with strange properties. A Majorana fermion is its own anti particle, i.e., when one Majorana fermion bumps into another Majorana fermion, they will both vanish.

Majorana fermions are electrically neutral elementary particles of matter first theorized by Ettore Majorana in 1937. Since they are their own anti particles, researchers believe that Majorana fermions may be useful in creating quantum computers because when moved, they are believed to have a property which allows them to remember their former position.

The difference between Dirac fermion and Majorana fermion can be mathematically expressed using creation and annihilation operators of second quantisation. The creation

operator creates a fermion in the site, whereas, an annihilation operator annihilates it. For Dirac fermion, the fermion operators are different, while, its same for Majorana fermions. Since particles and anti particles have opposite conserved charges, Majorana fermions have zero charge. They cannot possess intrinsic electric or magnetic moments, only toroidal moments

Chapter 6

Summary and Conclusion

This project started with addressing many body problems and finding a way to handle them easily via Feynman diagrams. Green's function is discussed in detail and the idea of propagators is also studied. Information paradox is reviewed and holographic principle is put forward as a solution. Sachdev Ye Kitaev model, which is proposed as a model for holography is studied in detail. Its Hamiltonian and the operators included are discussed in detail. Green's function for the model is also determined.

References