NASA/TM–20250002029

Process for Sun-Pointing Attitude in Earth Orbit

Kevin D Deighton
Amentum Services, Inc
NASA Johnson Space Center, Houston, TX
Astromaterials Research and Exploration Science

February 2025

NASA STI Program Report Series

Since its founding, NASA has been dedicated to the advancement of aeronautics and space science. The NASA scientific and technical information (STI) program plays a key part in helping NASA maintain this important role.

The NASA STI program operates under the auspices of the Agency Chief Information Officer. It collects, organizes, provides for archiving, and disseminates NASA's STI. The NASA STI program provides access to the NTRS Registered and its public interface, the NASA Technical Reports Server, thus providing one of the largest collections of aeronautical and space science STI in the world. Results are published in both non-NASA channels and by NASA in the NASA STI Report Series, which includes the following report types:

- TECHNICAL PUBLICATION. Reports of completed research or a major significant phase of research that present the results of NASA Programs and include extensive data or theoretical analysis. Includes compilations of significant scientific and technical data and information deemed to be of continuing reference value. NASA counterpart of peer-reviewed formal professional papers but has less stringent limitations on manuscript length and extent of graphic presentations.

- TECHNICAL MEMORANDUM. Scientific and technical findings that are preliminary or of specialized interest, e.g., quick release reports, working papers, and bibliographies that contain minimal annotation. Does not contain extensive analysis.

- CONTRACTOR REPORT. Scientific and technical findings by NASA-sponsored contractors and grantees.

- CONFERENCE PUBLICATION. Collected papers from scientific and technical conferences, symposia, seminars, or other meetings sponsored or co-sponsored by NASA.

- SPECIAL PUBLICATION. Scientific, technical, or historical information from NASA programs, projects, and missions, often concerned with subjects having substantial public interest.

- TECHNICAL TRANSLATION. English-language translations of foreign scientific and technical material pertinent to NASA's mission.

Specialized services also include organizing and publishing research results, distributing specialized research announcements and feeds, providing information desk and personal search support, and enabling data exchange services.

For more information about the NASA STI program, see the following:

- Access the NASA STI program home page at http://www.sti.nasa.gov

.

NASA/TM–20250002029

Process for Sun-Pointing Attitude in Earth Orbit

Kevin D Deighton
Amentum Services, Inc
NASA Johnson Space Center, Houston, TX
Astromaterials Research and Exploration Science

National Aeronautics and
Space Administration

NASA Johnson Space Center
Houston, TX

February 2025

Available from:

NASA STI Program / Mail Stop 050
NASA Langley Research Center
Hampton, VA 23681-2199

TABLE OF CONTENTS

TABLES

FIGURES

Acronyms

FEM	Finite Element Model
GMAT	General Mission Analysis Tool
HVIT	Hypervelocity Impact Technology
ISS	International Space Station
LEO	Low Earth Orbit
MEM	Meteoroid Engineering Model
ORDEM	Orbital Debris Engineering Model
RAAN	Right Ascension of the Ascending Node
STK	Systems Toolkit
UTC	Universal Coordinated Time
VNC	Velocity-Normal-Conormal

Introduction

In the Earth-centered Velocity-Normal-Conormal (VNC) coordinate system, the Sun is in constant motion. Therefore, a Sun-pointing spacecraft will be in constant rotation relative to the Earth-centered VNC coordinate system. Because an individual Bumper run requires a fixed attitude, analyzing an extended period of time in constant rotation results in a large number of attitudes to be individually run in Bumper. This creates a need for a defined process for selecting appropriate attitudes to assess.

An investigation into the mechanics driving attitude revealed patterns that can be leveraged to obtain the optimum attitudes to maximize fidelity while minimizing the number of Bumper runs. A key parameter to these patterns is Beta Angle, the angle between the orbit plane and Sun vector.

Attitude Calculation

The NASA Johnson Space Center's Hypervelocity Impact Technology (HVIT) Group internally uses an Attitude Calculator that uses data from a mission analysis tool, such as the General Mission Analysis Tool (GMAT) or Ansys Systems Toolkit (STK), to calculate the rotations around the fixed VNC (XYZ) coordinate system to enter into Bumper's Geometry module to achieve a desired pointing. The calculator will provide the rotations as a function of time using the timestep in which the analyst enters input data. It is necessary for a Bumper user looking to assess complex attitudes to obtain rotations as a function of time, likely by creating a similar tool. These rotations will constantly change over time; the purpose of this document is to provide guidance on how to bin this data into Bumper runs finely enough to capture the range of vehicle attitudes while keeping the number of Bumper runs manageable.

If an analyst were to gather spacecraft and Sun data across an entire year and enter it into the Attitude Calculator, two difficulties would be presented. As will be expanded upon in future sections, the spacecraft rotates over the course of an orbit and repeats these attitudes from one orbit to the next. If the mission were only sampled once per orbit, in the same position each time, all per-orbit rotation would be lost, and the analyst may come away with the erroneous impression of little to no rotation. Therefore, the timestep must be small enough to gather multiple points per orbit, resulting in tens of thousands of data rows to process. This causes the first challenge, which is that such large data sets can cause latency and crash Excel.

The second challenge is in turning a large volume of data into usable information. Figure 1 shows the rotations about Y (left) and Z (right) needed to point the +X axis of a spacecraft at the sun for an entire year with timesteps of just over 10 minutes (8 timesteps per orbit).

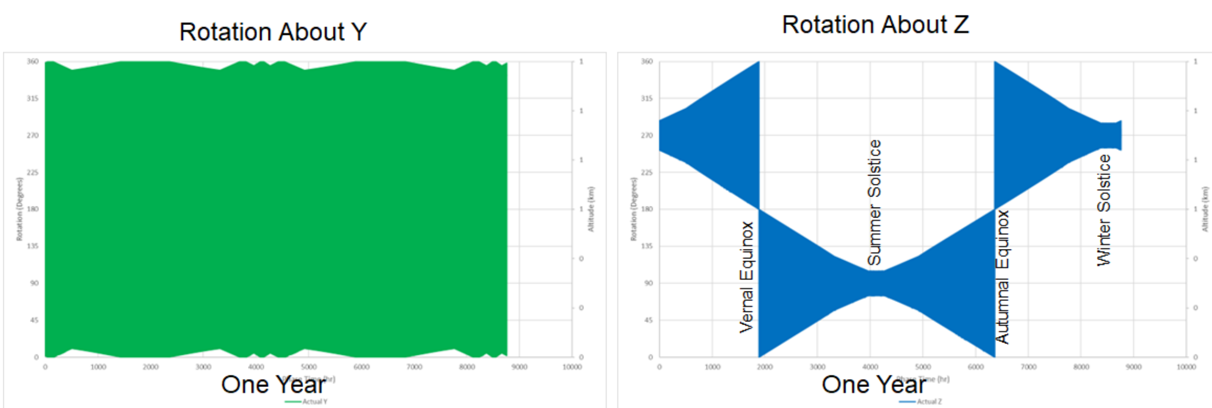


Figure 1: Example Sun Pointing Y & Z Rotations Across a Year

The left graph shows that Y rotation values can be nearly any value across the whole year, consistent with continuous rotation about that axis. The right graph shows that the Z rotation covers ranges that change with the seasons.

It is possible to treat all the timesteps/rotations as Big Data and bin rotations into arbitrarily small bins, weighting each bin by occurrence. However, this method does not provide enough insight into patterns of behavior to properly understand the vehicle attitude. Relatedly, it is difficult to determine from pure data what bin size is appropriate, and using one-size-fits-all bins may not capture the most common values.

It is therefore necessary to investigate the orbital mechanics involved to build a proper understanding of attitude patterns from which to derive the most effective analysis process, maximizing accuracy while minimizing Bumper run count.

Orbital Mechanics

A celestial coordinate system, whose axis directions are fixed relative to distant stars, is the basis for the J2000 coordinate system, upon which much of the ensuing logic is built. The Z axis of the J2000 coordinate system is normal to Earth's equatorial plane in the North Pole direction. The X axis is determined by the intersection between Earth's equatorial plane and the ecliptic plane in which the Earth revolves around the Sun; these planes are angled approximately 23.45° apart. The right side of Figure 2 depicts this intersection. As the Earth revolves around the Sun, carrying the Earth-centered coordinate frame with it, this intersection line intercepts the Sun twice per year: the Vernal (Spring) Equinox and Autumnal (Fall) Equinox. On these days, the Sun rises and sets due east and west, respectively, passing directly over the equator, the duration of day and night are equal worldwide, and the northern and southern hemispheres are equally illuminated. The positive X axis is defined as the direction on that intersection line that points from the Earth to the Sun at the Vernal Equinox. Therefore, this direction is also referred to at the Vernal Equinox. The Y axis follows from the X and Z axes using the right-hand rule.

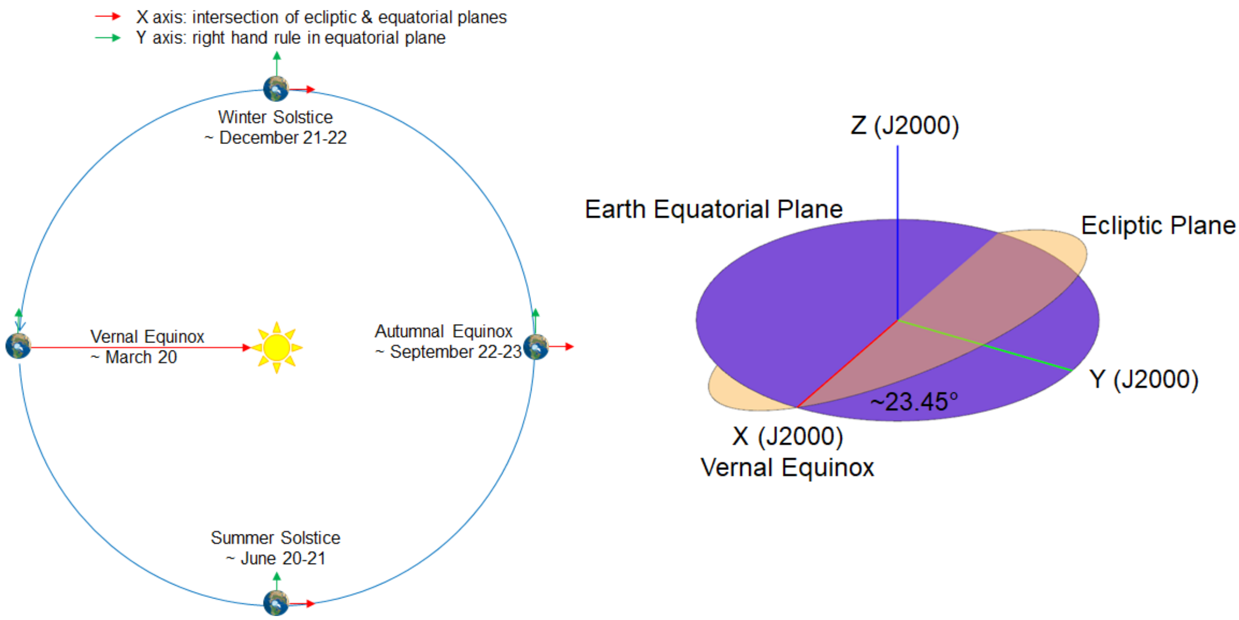


Figure 2: Vernal Equinox Definition

The modern view of the solar system is that the Earth revolves around the Sun, as depicted on the left in Figure 3. However, because all motion is relative, the Earth-centered view on the right of the same figure is also useful. In this frame of reference, the Sun appears to revolve around the Earth over the course of one year. The Sun's position may be tracked as the Ecliptic True Solar Longitude, the angular measurement of the Sun's position relative to the Vernal Equinox direction. The Ecliptic True Solar Longitude is 0° on the Vernal Equinox in late March, 90° on the Summer Solstice in late June, 180° on the Autumnal Equinox in late September, and 270° on the Winter Solstice in late December.

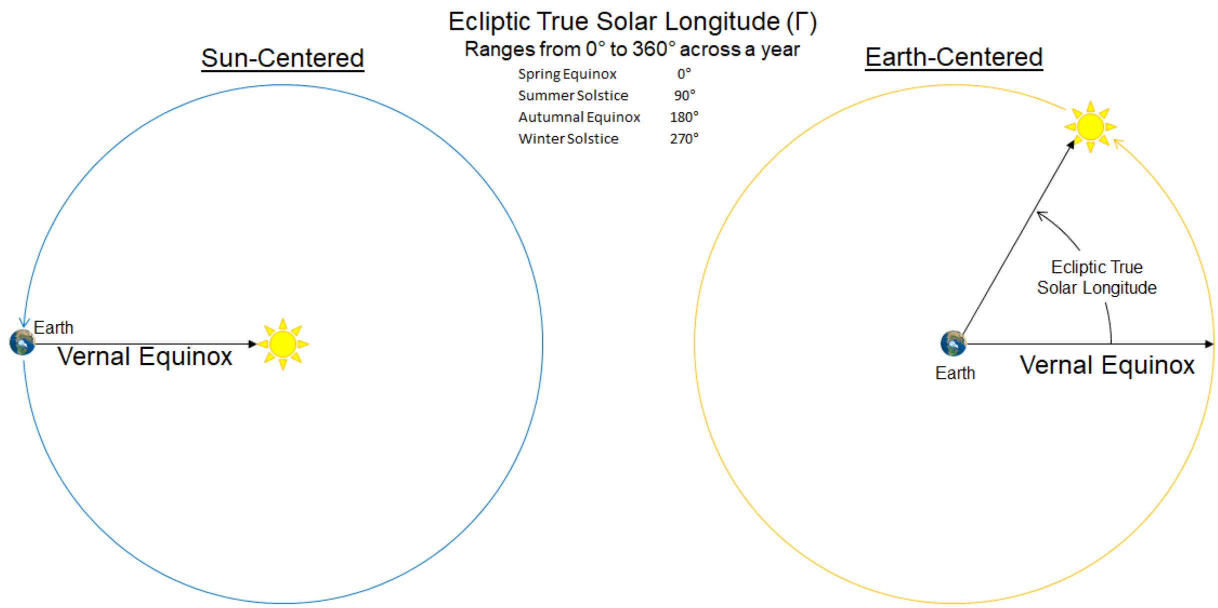


Figure 3: Ecliptic True Solar Longitude

Transitioning to a spacecraft in orbit around Earth, a Keplerian orbit is defined by five parameters, with a sixth locating the spacecraft's position in orbit:

- Semi-major Axis: defines the size of the orbit
- Eccentricity: defines the shape of the orbit
- Inclination: defines the orbit's plane relative to the equator
- Argument of Perigee: defines the orbit's perigee-apogee orientation relative to the equator
- Right Ascension of the Ascending Node (RAAN): defines the orbit's plane relative to the Vernal Equinox (the rotation of the orbit around the pole)
- True Anomaly: locates the spacecraft's position in orbit

The RAAN is measured from the Vernal Equinox to the Ascending Node (where the spacecraft crosses the Equator heading from south to north), as shown in Figure 4. This orbit parameter is initially determined by the longitude of the launch site at the time of launch. If a specific RAAN is desired, the launch window may be constrained to achieve the desired value. Ideally, RAAN would remain fixed once in orbit, however, realistically, it tends to change by a few degrees per day as orbital precession causes the Ascending Node to move eastward (for prograde spacecraft) at that rate.

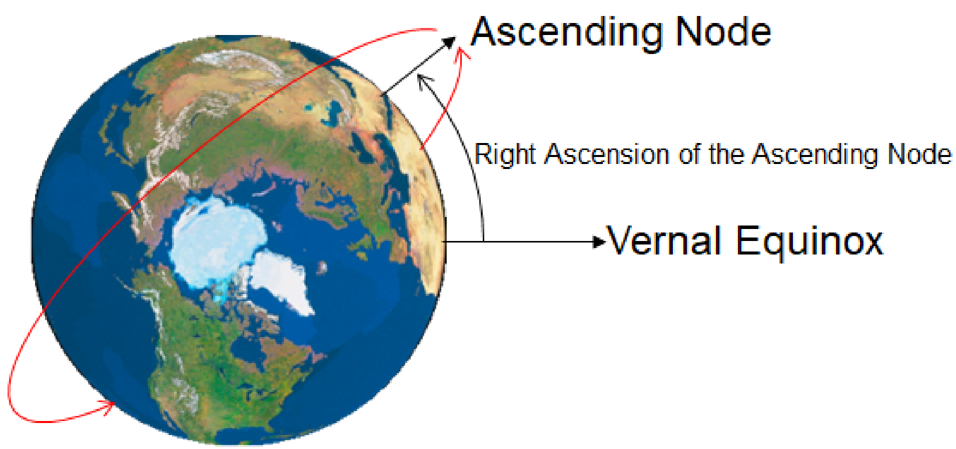


Figure 4: Right Ascension of the Ascending Node

Ecliptic True Solar Longitude and RAAN come together as the Beta Angle, which is the angle between the Sun and the orbital plane, as indicated in Figure 5.

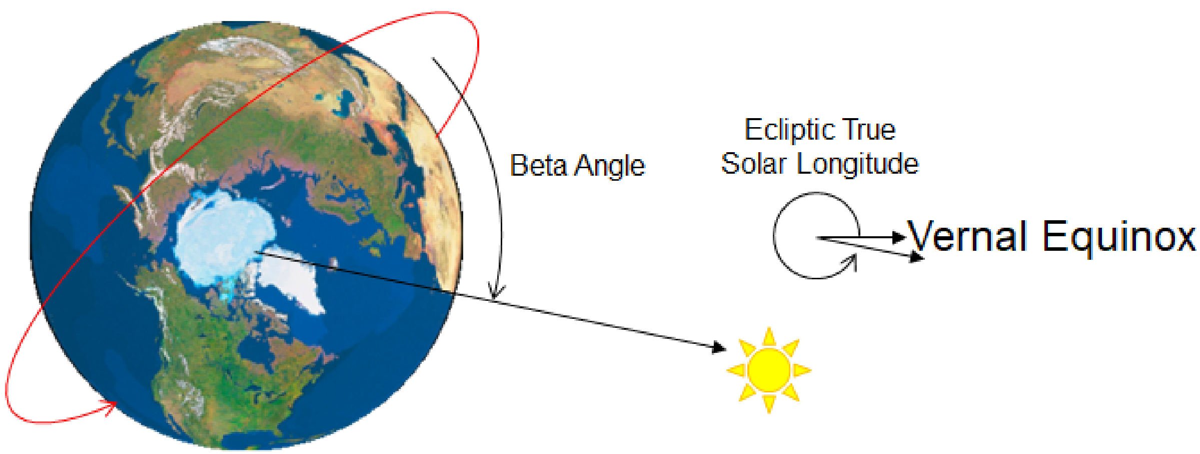


Figure 5: Beta Angle

Beta Angle is calculated from the angles discussed above by:

$$\sin\boldsymbol{\beta} = \cos\boldsymbol{\Gamma} * \sin\boldsymbol{\Omega} * \sin\boldsymbol{i} - \sin\boldsymbol{\Gamma} * \cos\boldsymbol{\epsilon} * \cos\boldsymbol{\Omega} * \sin\boldsymbol{i} + \sin\boldsymbol{\Gamma}\sin\boldsymbol{\epsilon} * \cos\boldsymbol{i} \tag{1}$$

β: Beta Angle
Γ: Ecliptic True Solar Longitude
Ω: Right Ascension of the Ascending Node
i: Inclination
ϵ: Obliquity of the Ecliptic ≈ 23.45°

With some assistance from the equation above, Figure 6 visually reasons Beta Angle as a function of RAAN for various times of the year. In the image, all compositions are views in the

Vernal Equinox direction with the Ecliptic plane as horizontal and north as up. When RAAN is equal to 0°, the orbital plane aligns with the Sun, making Beta = 0°, at the spring and fall equinoxes. At the summer and winter solstices, Beta reaches its extreme values, which is the difference between Inclination and the Obliquity of the Ecliptic due to these angles being measured in opposite directions. When RAAN is equal to 180°, similar trends occur, however, the extreme Beta Angles are the positive and negative summation of Inclination and the Obliquity of the Ecliptic, as these angles are then measured in the same direction. When RAAN is equal to 90° or 270°, the Beta is driven by the Inclination at the equinoxes and approaches zero around the solstices.

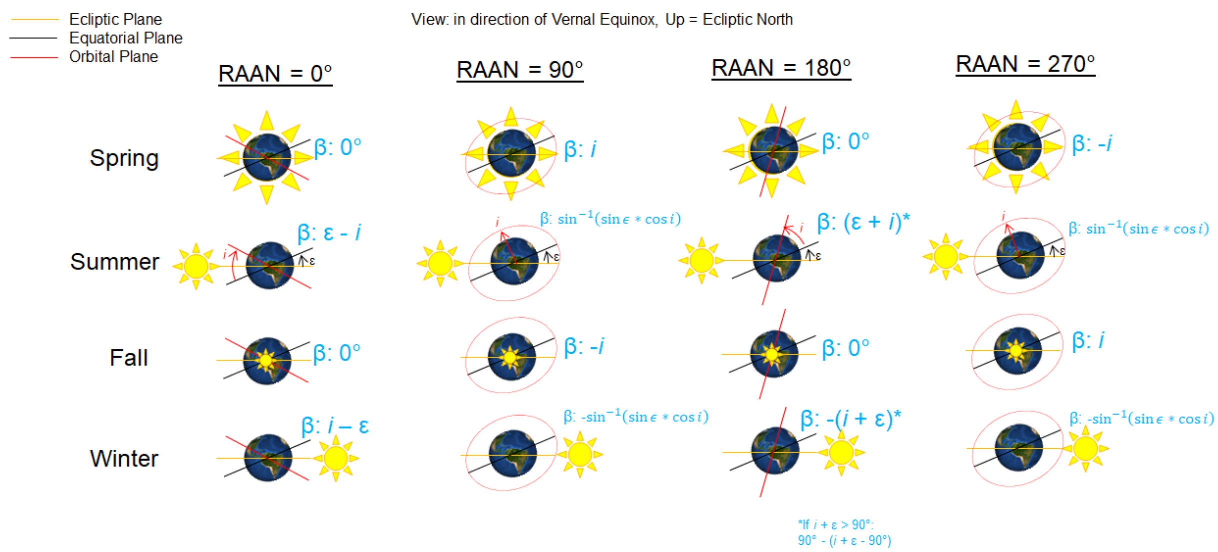


Figure 6: Beta Angle Variations

Figure 7 graphs Beta Angle as a function of Ecliptic True Solar Longitude for four values of RAAN and fixed Inclination of 51.6°, that of the International Space Station (ISS). This graph aligns with the visual reasoning above, with several points annotated. For inclinations up to 66.55° (90° - the Obliquity of the Ecliptic), Beta Angle experiences the widest possible range when RAAN = 180° and the narrowest possible range when RAAN = 0° due to these times resulting in Inclination and the Obliquity of the Ecliptic either complementing or offsetting each other, respectively. The summation of these two angles represents the greatest achievable value for Beta. For higher inclinations, there is a transition to Beta Angle experiencing the widest possible range when RAAN = 90° or 270°. The greatest achievable value for such an orbit is the inclination, up to 90°, which is the greatest Beta value physically possible.

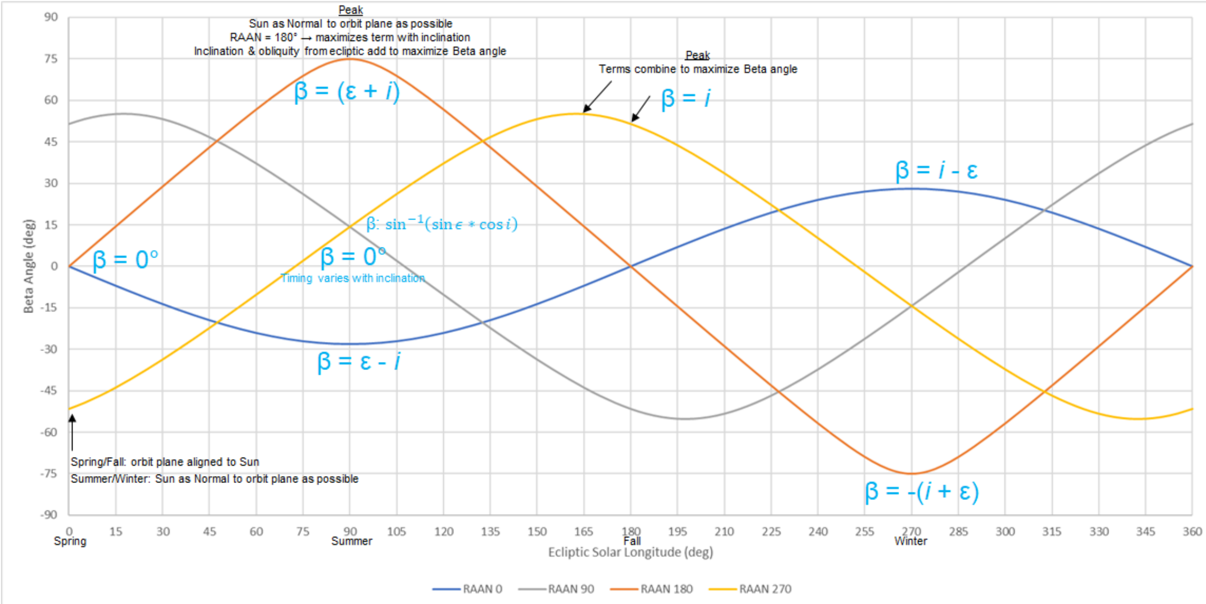


Figure 7: Beta Angle as a Function of Ecliptic True Solar Longitude, Inclination = 51.6°

Single Orbit Attitudes

Two rules fully define an attitude:

1. Primary / Align: exactly points a part of the vehicle at a direction in space
    a. For the purpose of this memo, this will always be pointing at the Sun
        i. The part of the vehicle being pointed will most likely be associated with solar arrays and defined by an axis in the vehicle's coordinate frame (geometry Finite Element Model (FEM) being run in Bumper)
        ii. +X axis being pointed at the Sun will be used in all examples herein
2. Secondary / Constraint: restricts rotation around the pointing vector, generally by pointing a second part of the spacecraft toward a second direction in space
    a. Will only be exact pointing if the desired vector is orthogonal to the Primary pointing vector
        i. Need not be exact
    b. +Z axis toward Earth will be used in all examples herein

Sun-pointing vehicle attitude in the Earth-centered VNC coordinate system is primarily a function of Beta Angle. This section will visually reason how vehicle attitude changes across a single orbit for different Beta Angles.

Figure 8 shows how a spacecraft pointing its solar arrays at the Sun moves when Beta Angle = 90°. On the left is an Earth-fixed view, in which the spacecraft and its VNC frame orbit the Earth; on the right, the same positions are shown reoriented to a fixed VNC frame. This is the simplest case because the Sun never moves in the sky—it is always in the Normal direction. Therefore, the VNC rotation is always the same and a single VNC rotation can potentially cover the entire orbit. The secondary rotation about the Sun-Vector may require different VNC rotations at different points in the orbit. Among possible secondary constraints:

- Single VNC rotation: point at velocity vector or Earth/zenith
    - Point to Earth shown
- Multiple VNC rotations: point at equatorial or ecliptic north
- Invalid: point at orbit normal (a different axis is already fixed in that direction)

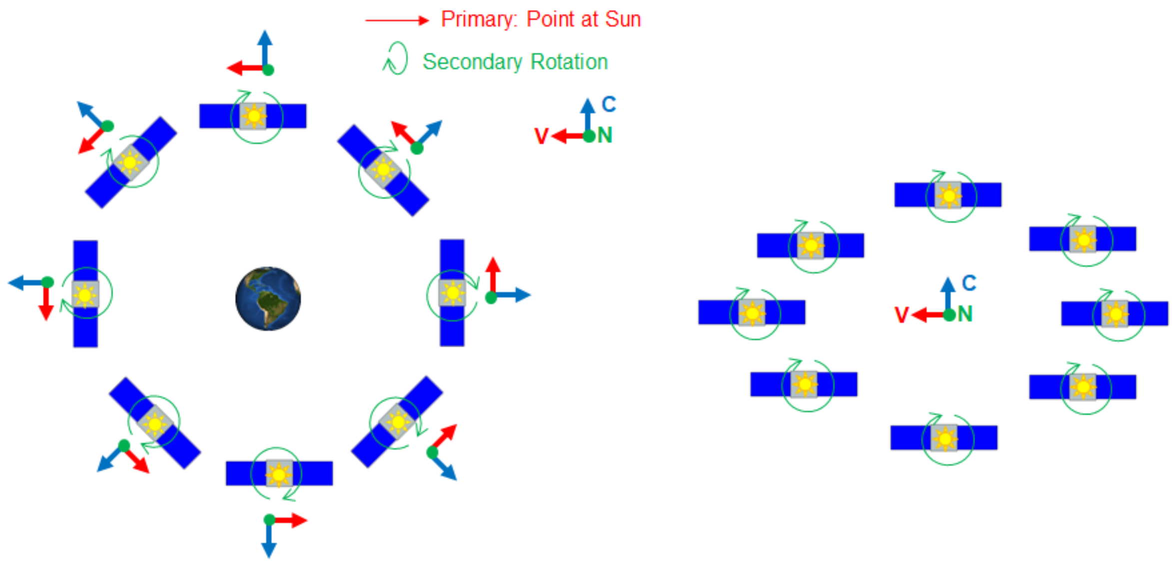


Figure 8: Vehicle Attitude, Single Orbit, Beta Angle = 90°

Figure 9 shows how a spacecraft pointing its solar arrays at the Sun moves when Beta Angle = 0°. Viewed in the VNC frame (right), the Sun revolves around the spacecraft in the V-C plane. Therefore, a continuous 360° rotation is needed to point at the Sun; this rotation must be discretized for Bumper. Among possible secondary constraints, pointing at the velocity vector or Earth/zenith are invalid for moments of the orbit but pointing at the orbit normal is always valid. The secondary constraint must be evaluated across the orbit and included in the total rotation for each discretized step of Sun pointing but is unlikely to dictate additional division of Bumper runs beyond the discretization required for Sun pointing.

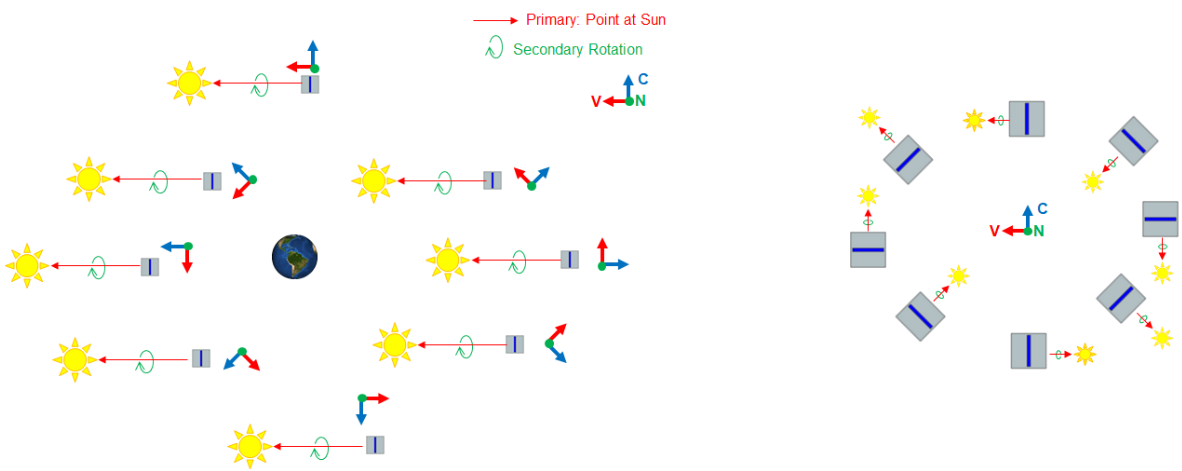


Figure 9: Vehicle Attitude, Single Orbit, Beta Angle = 0°

Figure 10 shows how a spacecraft pointing its solar arrays at the Sun moves when Beta Angle is somewhere between 0° and 90°. Generally similar to Beta Angle = 0°, the Sun revolves around the spacecraft such that the Sun vector makes a loop around the normal direction. The higher the Beta Angle, the tighter the Sun loop, which results in less rotation and fewer Bumper runs.

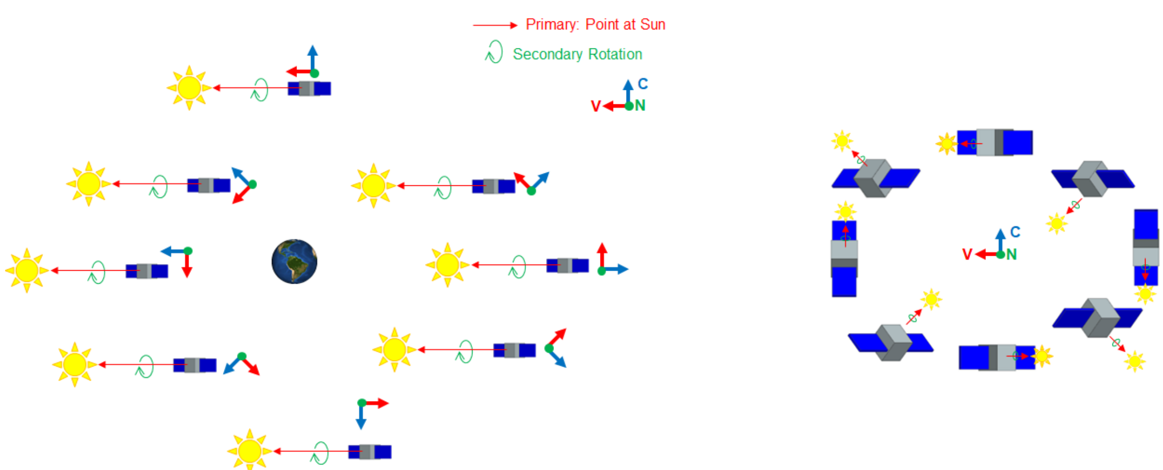


Figure 10: Vehicle Attitude, Single Orbit, 0° < Beta Angle < 90°

The internal HVIT Attitude Calculator can be used to quantify the specific rotation about each axis as a function of time needed to actualize the logic described above. Note that the calculator is set up to always use three rotations, one about each axis, and the analyst can choose the order in which the rotations are performed from a menu. While any order of rotations will result in the same spacecraft attitude, rotating around the axis being pointed in the primary direction (at the Sun) is recommended to be the first rotation; this tends to achieve the smoothest rotations when they change with time.

Figure 11 graphs the rotations needed to track the Sun when Beta Angle = 0°. Note that in the world of rotation, 0° = 360°, and a large rotation can also be interpreted as a negative rotation (i.e. 345° = -15°). The green line is the rotation about the orbit normal direction while the blue line is the rotation about the Conormal; the red is the rotation about the axis being pointed at the sun, and its fixed value in this example is not relevant to Sun tracking. The green and blue rotations can be visualized as a person tracking the sun with her eyes. At the beginning of the graph, the Sun is on the horizon directly in front of her, corresponding to the top image on the right of Figure 9; in this analogy, the VNC coordinate system corresponds to the human face, left arm, and head directions, respectively. As time moves to the right in the graph below, the figure above steps through the images counterclockwise. The Sun tracker will rotate her head upward, which is a negative rotation about the normal / left arm (green line). A human cannot rotate her head in a single axis infinitely, thus, when the Sun is directly overhead, she will turn around (180° rotation in blue) and follow the Sun's downward arc to the opposite horizon facing in the opposite direction from her starting point. She can hypothetically continue to track the Sun downward until it is below her feet, at which point, she will turn back around to face her original direction and raise her head with the Sun until it completes its circuit at the horizon.

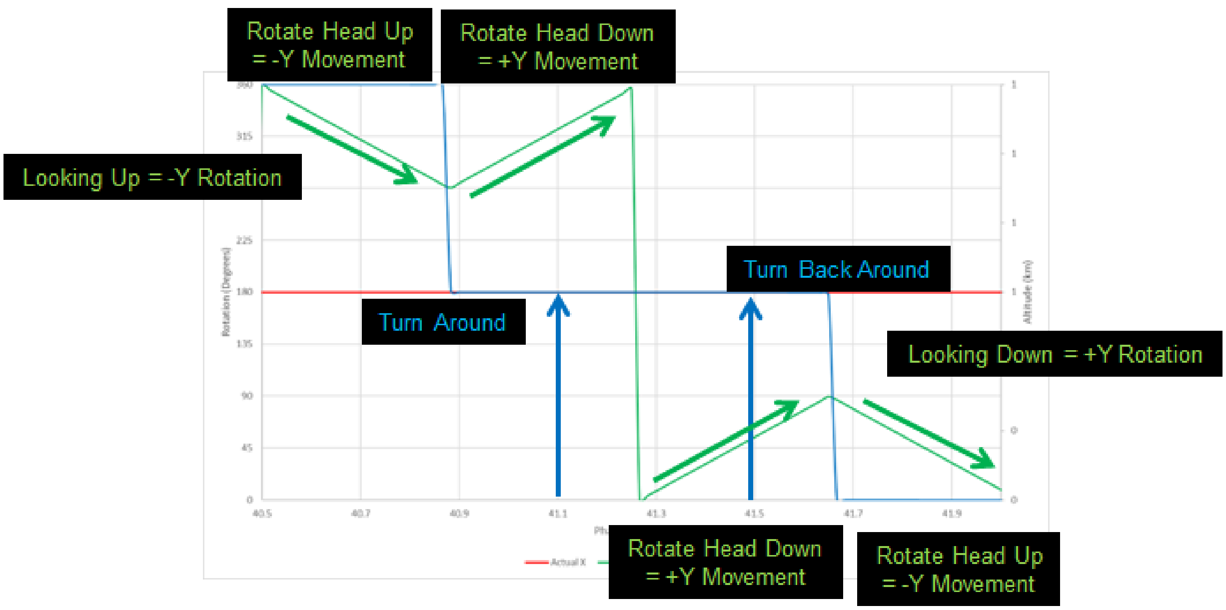


Figure 11: Attitude Rotations, Single Orbit, Beta Angle = 0°

The rotations graphed are the inherent output of the mathematics programmed into the Attitude Calculator. The use of both axes to track the Sun is a numerical consequence of the trigonometry used to convert a quaternion rotation into Euler angles. Unlike a human head, a spacecraft is able to rotate continuously. However, the combination of axes reduces the amount of rotation around the normal vector for higher Beta Angles, making the unintended outcome convenient.

This can be seen in Figure 12, which is the same style of graph for a Beta Angle of 57°. In this case, the Sun circles off to the side rather than directly overhead. Using the same human analogy, the person will start standing turned to the side, facing partly forward with her head level to view the Sun on the horizon. As the Sun rises, she will move her head up and turn her body until the Sun reaches its peak, at which point she will be facing exactly to the side and looking slightly upward. Continuing to turn her body and head in unison, she will face partly to the rear for the middle two quarters of the Sun's path, then return to facing partly forward for the final quarter. As the Sun no longer gets as high overhead, head movements are greatly reduced.

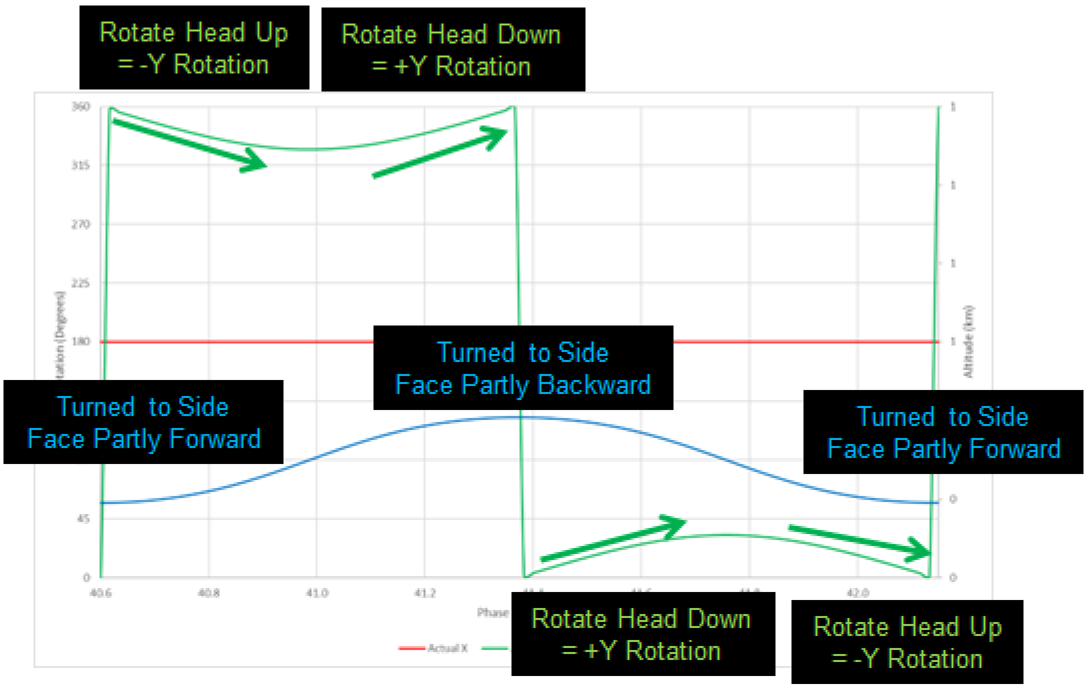


Figure 12: Attitude Rotations, Single Orbit, Beta Angle = 57°

Figure 13 shows these single orbit vehicle attitude graphs for a variety of Beta Angles. As Beta Angle increases, the Normal/Y and Conormal/Z rotations coalesce around values of 0° and 90°, respectively. This represents the logical conclusion that when Beta Angle = 90°, the Sun is fixed on the horizon off to the side, and the person will stand with her head level and motionless facing exactly to her left. The maximum difference from this value that each rotation attains is labeled Dispersion.

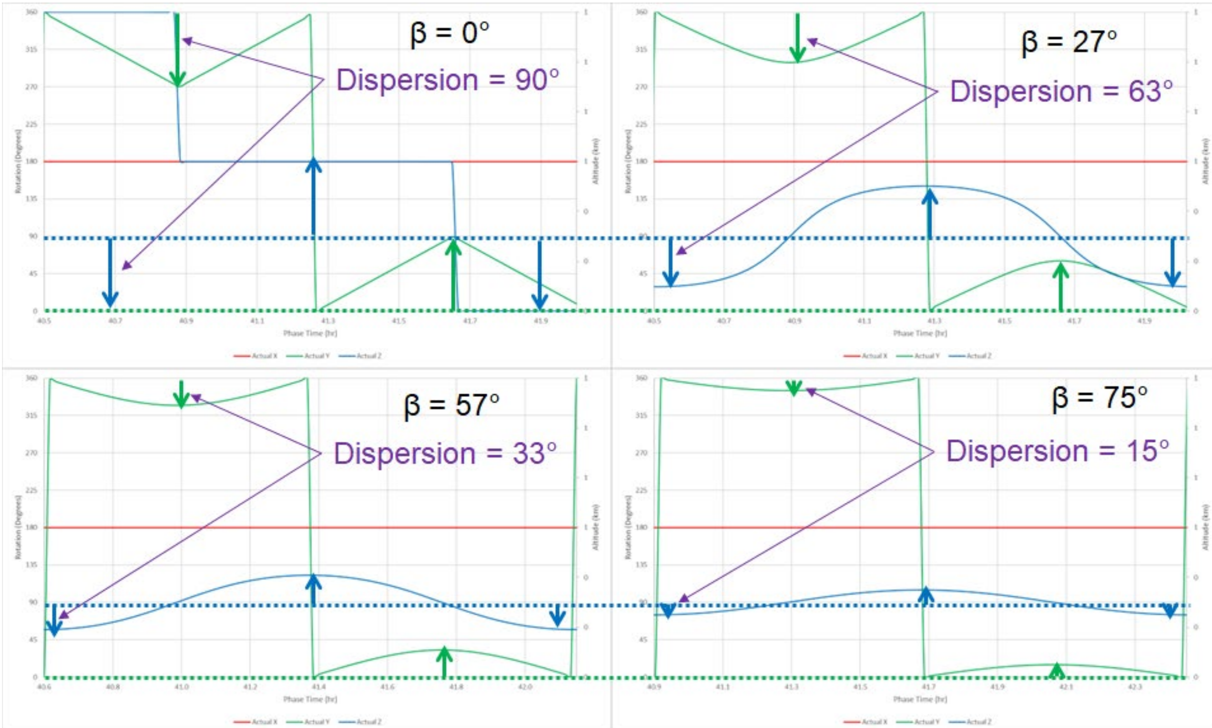


Figure 13: Attitude Rotations, Single Orbit, Beta Angles = 0°, 27°, 57°, 75°

It may be observed that the Dispersion is the complement of the Beta Angle. This is the conclusion of a deeper study of Dispersion as a function of Beta Angle whose results are graphed in Figure 14.

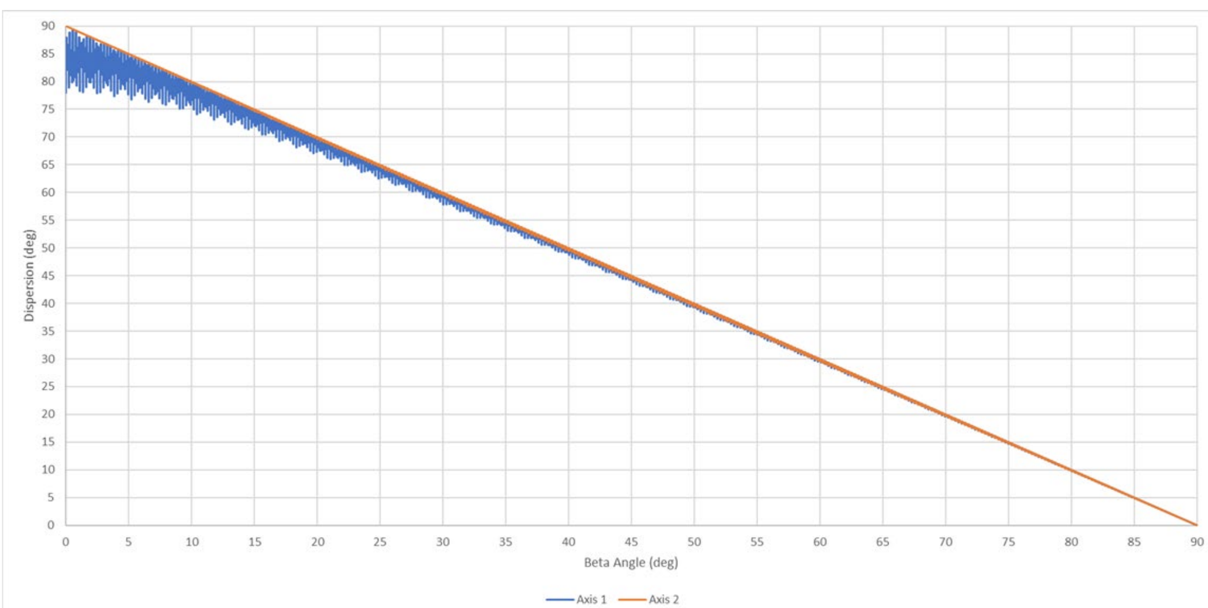


Figure 14: Dispersion

The patterns observed above may be generalized to all cases of Sun-pointing. Which two axes are involved and what their coalescence values will be will depend on which axis is being pointed at the Sun and whether the Beta Angle is positive or negative. Additionally, the secondary constraint must also be considered.

With the pattern understood, a best practice can then be developed. Each axis has regions above and below its coalescence line in four total combinations. Thus, the orbit can be split into Quads as shown in Figure 15.

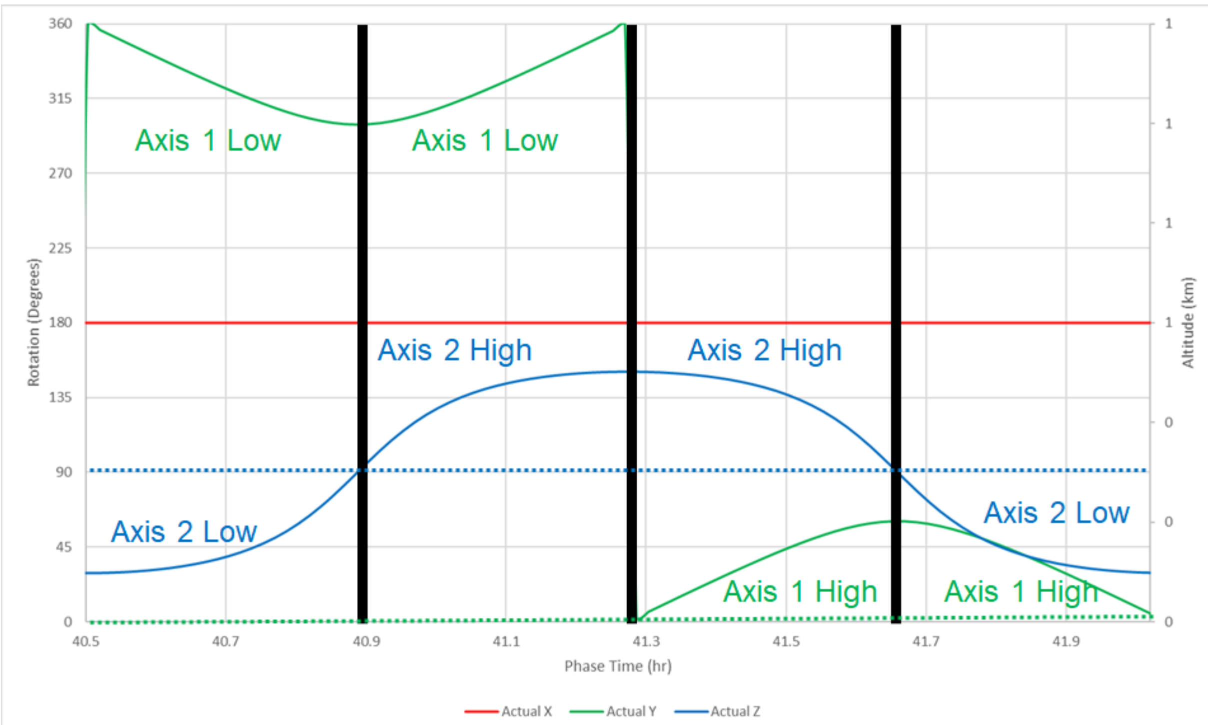


Figure 15: Splitting a Single Orbit into Quads

From there, it is necessary to decide how much rotation can be accepted within a single run. To guide this decision, a quick study was performed. A Gateway module was chosen for the study because the FEM was readily available. The cylindrical sides give the model a large area with a single shield configuration, however, several sensitive external items give the module risk a sensitivity to environment directionality and orientation. The model was rotated around the orbit normal in 15° steps in a simplified Sun tracking attitude set (Beta = 0°). Figure 16 describes the model and rotation.

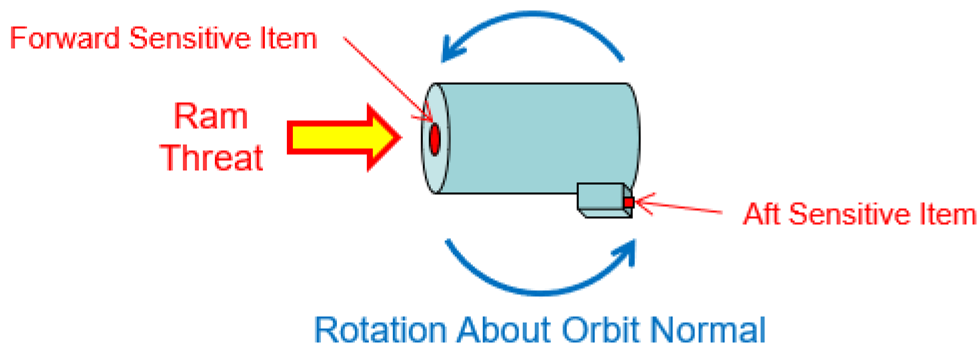


Figure 16: Rotation Study Description

Figure 17 graphs the risk to this test module as a function of the rotation about the orbit normal, starting from the position implied in the figure above. MMOD risk is highest at rotation angles around 345° and 150°, which place the forward and aft sensitive items into the ram direction, respectively. This demonstrates that MMOD risk varies with rotation by a substantial amount—more than a factor of three between the minimum and maximum cases.

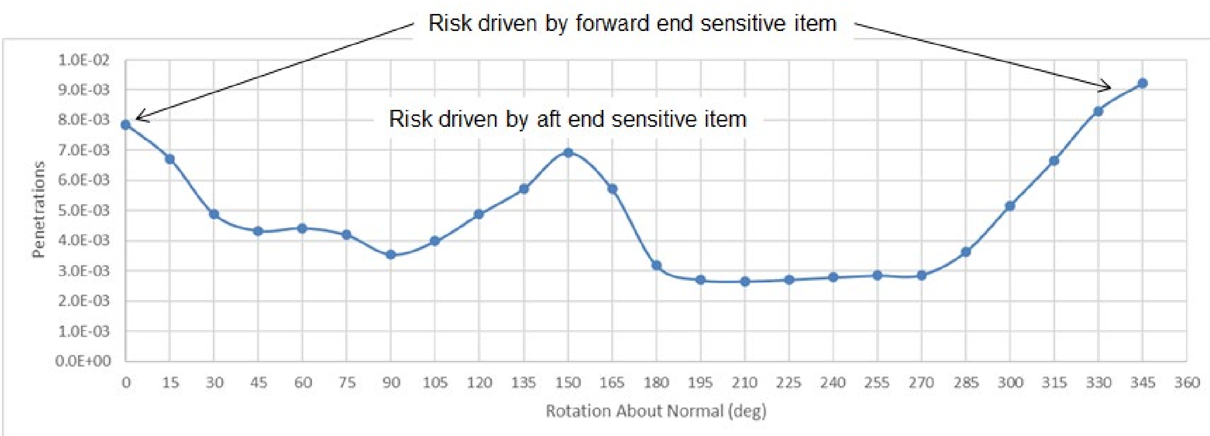


Figure 17: Rotation Study Risk by Rotation Angle

The question to be answered is how binning this smooth 360° rotation affects the outcome. The finest binning considered is 15°, in which all of the cases run were averaged to get a total for the orbit. Coarser binning options were evaluated by selecting only multiples of 30°, 45°, and 90° to be averaged together. All binning cases assumed equal weighting to each orientation included in that case per a smooth rotation in time. Table 1 gives the full orbit risk for each bin size option. The 30° bin size case matched the 15° bin size basis to within 1%, suggesting that 30° bins are within a bin size regime that has converged on the fine-bin solution. Larger bin sizes of 45° and 90° result in differences of 4.5% and 9.6%, respectively, which suggest a roughly linear phase of departing from the converged solution.

Table 1: Rotational Convergence

Bin Size (°)	Penetrations	Difference
15	4.8257E-03	Basis
30	4.7859E-03	-0.8%
45	4.6081E-03	-4.5%
90	4.3634E-03	-9.6%

It is therefore recommended that analysts default to a limit of 30° of rotation within a single Bumper run for each axis. An analyst may choose a tighter limit for an analysis requiring greater precision or may choose a looser limit to reduce scope if time/resources are limited.

To obtain attitudes for Bumper runs, the Quads must be subdivided based on the amount of rotation that occurs, which is defined by Dispersion. This effectively results in three regimes:

- Dispersion ≤ Rotation Limit / 2: the entire orbit can be accommodated within the limit and can be assessed with a single attitude (not even dividing between Quads)
    - The rotations to be entered into Bumper will be those of the coalescence lines
    - See β=75° (upper left) in Figure 18
- Rotation Limit / 2 < Dispersion ≤ Rotation Limit: each Quad should be a Bumper attitude; four attitudes)
    - See β=60° (upper right) in Figure 18
- Dispersion > Rotation Limit: each Quad needs to be divided into two or more Bumper attitudes; number of attitudes per Quad = CEILING(Dispersion / Rotation Limit)
    - See β=40° & β=20° (lower) in Figure 18

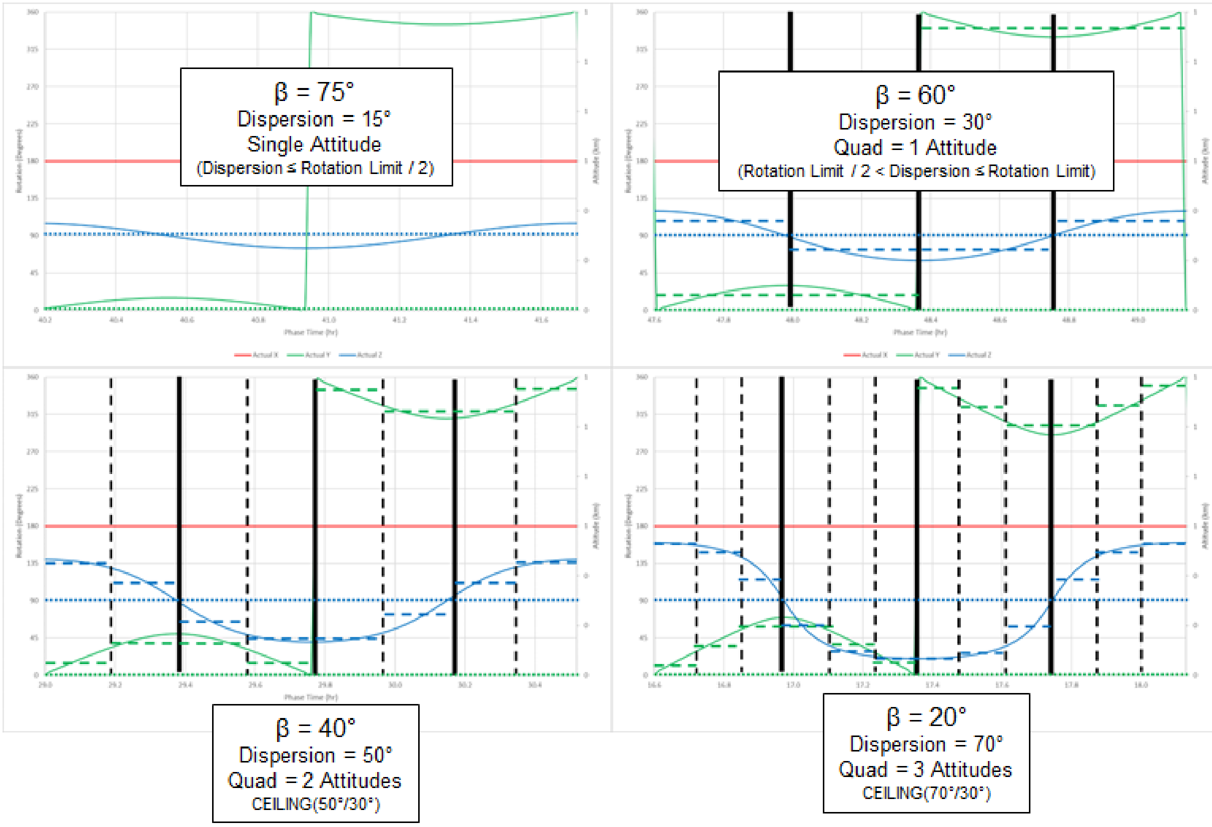


Figure 18: Dispersion & Subdivisions

It would be ideal to subdivide Quads into regions of equal angular rotation such that each Bumper run has an equal and minimum amount of actual rotation being averaged into a single bin. This is impractical, however, because the two axes do not see rotations at a correlated rate. As pictured in Figure 19, the rotation around the Y axis in the example given is more rapid near the start/end and halfway boundaries, marking times the Sun is near horizon level and a viewer's head is moving most rapidly. By contrast, the rotation around the Z axis is more rapid near the first and third quarter boundaries, marking the points on time in which the viewer most quickly turns around to face the opposite direction. Therefore, the recommended compromise is to subdivide the Quads into equal-time segments, accepting that some will slightly exceed the intended rotation limit on one axis. For each segment, the average rotation across that segment should be used for each axis. Subdividing into equal time has the ancillary benefit of allowing the subdivisions to be weighted equally, thus reducing complexity compared to needing to include unique time-based weighting factors on each subdivision.

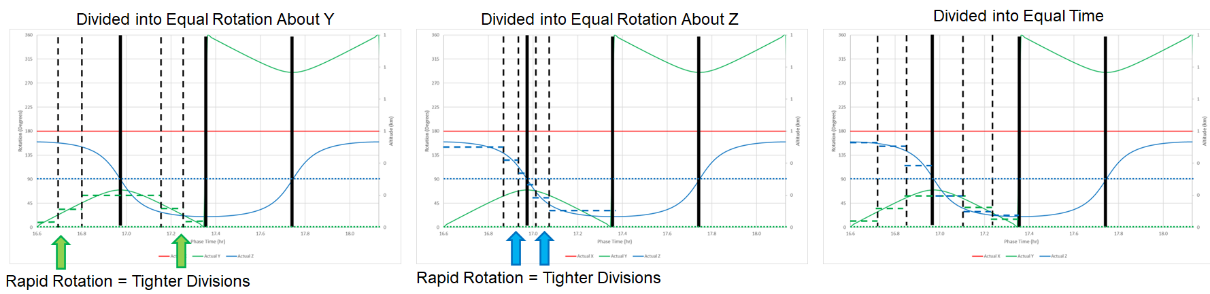


Figure 19: Subdivision by Angle vs. Time

Full Mission Attitudes

Beta Angle changes continuously at rates of no more than 1° per day, often much less depending on orbit parameters. This represents less than 0.1° per orbit. Therefore, consecutive orbits repeat the same attitudes with variations that are negligible from one orbit to the next. Thus, a single orbit as analyzed in the previous section represents a given slice of time.

However, the gradual Beta Angle changes build over long periods of time, potentially spanning 2*($\epsilon + i$) (up to 180°) during a year. Therefore, these long periods of time must be discretized into bins, and a single orbit in each bin will represent that bin. The entire geometry repeats on an annual basis, thus, only a single year needs to be discretized into attitudes. Additional years in a Bumper analysis can repeat the same set of attitudes.

Similar to the rotation limit applied to a single orbit, a Beta Angle bin size must be determined. The same Gateway module used above was used in a study to inform Beta Angle bin sizing. Beta Angles from -90° to +90° were assessed in 5° increments. A rotation limit of 45° was used because this study predates the study that resulted in the 30° limit recommendation. This resulted in Beta Angles up to ±40° being composed of eight runs, from ±45° to ±65° being composed of four runs, and ±70° and beyond being a single run. To expedite the study, attitude rotations were estimated from the patterns discussed above by using the known coalescence lines and adding plus or minus half the dispersion for the four-run cases and plus or minus one-quarter and three-quarters of the dispersion for the eight-run cases, all in the proper combinations per the patterns discussed above. These approximations are likely to vary from the averages the Attitude Calculator will produce, but good enough for this study's purposes.

Figure 20 graphs the MMOD risk the module would experience as a function of Beta Angle if that Beta Angle were the only Beta Angle for the entire year. The regions between 45° and 65° are not mirrored between the positive and negative Beta Angles due to the aft end sensitive item being canted off-center such that the relevant rotations orient this sensitive item relative to key threat directions differently. The cases from 70° to 90° are identical because they all use a single run with the coalescence lines defining the attitude because since is insufficient to separate the cases into multiple runs; again, the canting of the aft end sensitive item results in non-mirrored behavior between positive and negative Beta Angles at each end. This demonstrates that MMOD risk does change with Beta Angle, but at a 27% difference between the minimum and maximum cases, this variation is much less than the variation due to rotation angle within an orbit. These numbers are for a specific test case and will vary for other spacecraft, however, a comprehensive study of the interplay between spacecraft geometry and Beta Angle is beyond the scope of this effort.

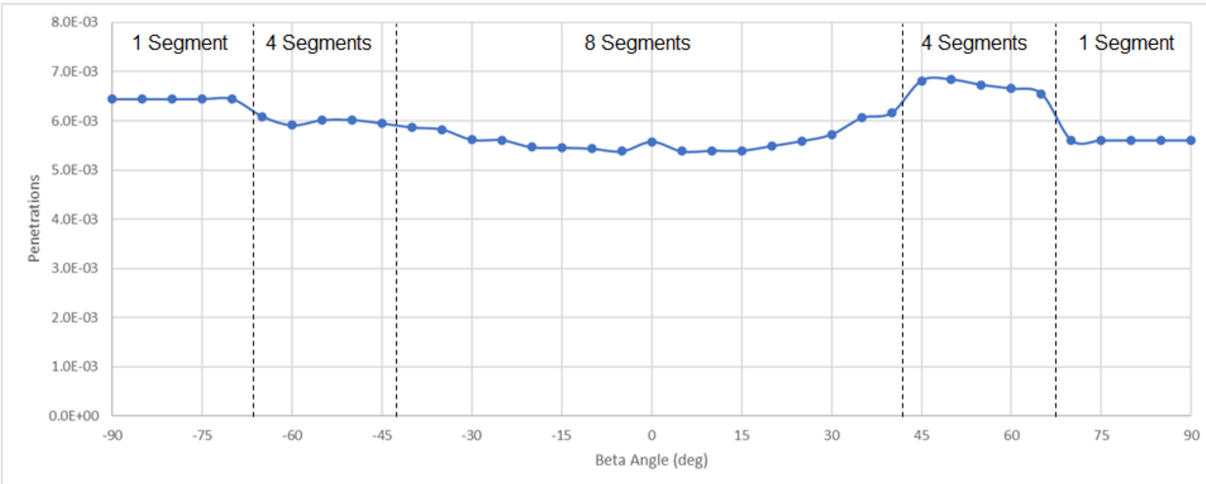


Figure 20: Beta Angle Binning Risk Distribution

The results by Beta Angle were combined with each case weighted by its number of occurrences, as will be discussed below, in various bin increments, with the results reported in Table 2.

Table 2: Beta Angle Binning Convergence

Increments (°)	Risk	Difference
5	5.9265E-03	Basis
10	5.9125E-03	-0.2%
15	5.9452E-03	0.3%
30	5.9477E-03	0.4%
45	6.1640E-03	4.0%
90	5.8735E-03	-0.9%

Bin sizes up to 30° produce converged results, varying by less than 0.5%. At a bin size of 45°, the risk result begins to markedly depart from the converged solution. The result at 90° matches the converged solution to within 1%, which is likely a chance occurrence given that using so few data points misses so much behavior. Therefore, when setting a Beta Angle bin size, the general recommendation is 30°. An analyst may choose a finer bin size for an analysis requiring greater precision or may choose a coarser bin size to reduce scope if time/resources are limited.

When assessing a full mission, the number of single orbits that need to be included depends on maximum Beta Angle, again in three regimes:

- Max Beta Angle ≤ Bin Size / 2: a single orbit at Beta Angle = 0° represents the mission
- Bin Size / 2 < Max Beta Angle ≤ Bin Size: three orbits → Beta Angle = 0° and the positive/negative maximum Beta Angles
- Max Beta Angle > Bin Size: five or more orbits → Beta Angle = 0°, the positive/negative maximum Beta Angles, and one or more intermediate positive/negative Beta Angle cases between 0° and maximum
    - Number of orbits = 3+2*(CEILING(Max Beta Angle / Bin Size)-1)

Table 3 uses the rules above to specify the required number of single orbits for each maximum Beta Angle, assuming a bin size of 30°. The ISS inclination results in a maximum Beta Angle of 75° as annotated, which requires seven single orbits.

Table 3: Required Number of Single Orbits, Bin Size = 30°

Maximum Beta (°)	# Orbits	Comment
0	1	0° Only
5	1	0° Only
10	1	0° Only
15	1	0° Only
20	3	0° & ± Max Beta
25	3	0° & ± Max Beta
30	3	0° & ± Max Beta
35	5	0°, ± Max Beta, ± 1 intermediate point
40	5	0°, ± Max Beta, ± 1 intermediate point
45	5	0°, ± Max Beta, ± 1 intermediate point
50	5	0°, ± Max Beta, ± 1 intermediate point
55	5	0°, ± Max Beta, ± 1 intermediate point
60	5	0°, ± Max Beta, ± 1 intermediate point
65	7	0°, ± Max Beta, ± 2 intermediate points
70	7	0°, ± Max Beta, ± 2 intermediate points
75	7	0°, ± Max Beta, ± 2 intermediate points
80	7	0°, ± Max Beta, ± 2 intermediate points
85	7	0°, ± Max Beta, ± 2 intermediate points
90	7	0°, ± Max Beta, ± 2 intermediate points

ISS Inclination

The total number of attitudes needing to be assessed in Bumper is the summation of the attitudes needed for each single orbit being considered. Table 4 shows an example summation for the ISS orbit, assuming RAAN = 180° to maximize Beta Angle range, and using a rotation limit and Beta Angle bin size both equal to 30°. This example calls for assessing seven orbits discretized into 1, 4, 8, or 12 attitude segments each, totaling 46 Bumper runs.

Table 4: Required Number of Bumper Runs, Rotation Limit = 30°, Bin Size = 30°, ISS Orbit, RAAN = 180°

Beta (°)	Segments
75.05	1
56.79	8
28.89	8
0	12
-28.89	8
-56.79	8
-75.05	1
	46

It may be noticed that the Beta Angles in the table above are not evenly spaced by value in degrees. Instead, they are evenly spaced in time. As recorded in Table 5, dates corresponding to the minimum, maximum, and zero Beta Angle cases were found via orbital mechanics. Two evenly spaced times between these points were then identified and their corresponding Beta Angles were calculated.

Table 5: Example Chosen Beta Angles

Beta (°)	Date	Case	Comment
75.05	6/19/2024 19:30	Max	Found by Orbit Mechanics
56.79	7/20/2024 6:00	Intermediate +2	Even Spacing in Time
28.89	8/19/2024 16:30	Intermediate +1	Even Spacing in Time
0	9/19/2024 3:00	Zero	Found by Orbit Mechanics
-28.89	10/19/2024 13:30	Intermediate -1	Even Spacing in Time
-56.79	11/19/2024 0:00	Intermediate -2	Even Spacing in Time
-75.05	12/19/2024 10:30	Min	Found by Orbit Mechanics

During the course of a year, each minimum and maximum Beta Angle case will only happen once, while all other cases, including Beta Angle = 0°, will happen twice. Therefore, Bumper output from each single orbit analysis must be weighted accordingly. Table 6 displays the logic in list format as well as the resulting scaling factors.

Table 6: Beta Angle Factors

Full Year	
Beta	**Date**
Zero	Spring Equinox
Intermediate +1	
Intermediate +2	
Max	Summer Solstice
Intermediate +2	
Intermediate +1	
Zero	Fall Equinox
Intermediate -1	
Intermediate -2	
Min	Winter Solstice
Intermediate -2	
Intermediate -1	

Beta	Occurences	Factor
Zero	2	16.67%
Min	1	8.33%
Max	1	8.33%
Intermediate +1	2	16.67%
Intermediate +2	2	16.67%
Intermediate -1	2	16.67%
Intermediate -2	2	16.67%

Variable Right Ascension of the Ascending Node

The preceding logic assumes that Right Ascension of the Ascending Node is a known and fixed value. As previously noted, however, RAAN tends to change at a rate of a few degrees per day due to orbital precession. This allows RAAN to change across the full 360° range of possible values in less than three months, potentially five complete cycles per year.

Therefore, unless directed otherwise by a customer, RAAN should be considered variable and the impact of varying RAAN must be considered.

The main effect of varying RAAN is altering the distribution of time among different Beta Angle cases. For inclinations up to 66.55° (90° - the Obliquity of the Ecliptic), the maximum Beta Angle possible for that orbit ($\epsilon + i$) occurs when RAAN = 180°. The extreme Beta Angle for any given RAAN must be solved iteratively for an exact value, but may be estimated as:

$$\boldsymbol{\beta_{extremes}} = \pm(\mathbf{sin}(\boldsymbol{\Omega} - \mathbf{90°}) * \boldsymbol{\epsilon} + \boldsymbol{i}) \tag{2}$$

For higher inclinations up to 90°, the extreme value can be estimated as the inclination angle, and from 90° to 113.45° as (180° - Inclination).

Because a variable RAAN analysis involves collating a variety of Beta Angle ranges, it is recommended to shrink the Beta Angle bin size from 30° to 15°; this will provide more fidelity at the low end of Beta Angles, where the majority of occurrences happen. Table 7 counts the number of occurrences of each Beta Angle case, one for the min/max and two for all points between, across the full 360° range of possible RAAN values, considering the estimated maximum Beta Angle for each RAAN. The approximation places the Max Beta in the nearest bin, either above or below that Max Beta value.

Table 7: Distribution of Beta Angle Cases, Bin Size = 15°, ISS Orbit, Variable RAAN (15° Bins)

Inclination (°):	51.6											
		Beta Cases										
RAAN	Max Beta	-75°	-60°	-45°	-30°	-15°	0°	15°	30°	45°	60°	75°
0	28.15	-	-	-	1	2	2	2	1	-	-	-
15	28.95	-	-	-	1	2	2	2	1	-	-	-
30	31.29	-	-	-	1	2	2	2	1	-	-	-
45	35.02	-	-	-	1	2	2	2	1	-	-	-
60	39.88	-	-	1	2	2	2	2	2	1	-	-
75	45.53	-	-	1	2	2	2	2	2	1	-	-
90	51.60	-	-	1	2	2	2	2	2	1	-	-
105	57.67	-	1	2	2	2	2	2	2	2	1	-
120	63.33	-	1	2	2	2	2	2	2	2	1	-
135	68.18	1	2	2	2	2	2	2	2	2	2	1
150	71.91	1	2	2	2	2	2	2	2	2	2	1
165	74.25	1	2	2	2	2	2	2	2	2	2	1
180	75.05	1	2	2	2	2	2	2	2	2	2	1
195	74.25	1	2	2	2	2	2	2	2	2	2	1
210	71.91	1	2	2	2	2	2	2	2	2	2	1
225	68.18	1	2	2	2	2	2	2	2	2	2	1
240	63.33	-	1	2	2	2	2	2	2	2	1	-
255	57.67	-	1	2	2	2	2	2	2	2	1	-
270	51.60	-	-	1	2	2	2	2	2	1	-	-
285	45.53	-	-	1	2	2	2	2	2	1	-	-
300	39.88	-	-	1	1	2	2	2	1	1	-	-
315	35.02	-	-	-	1	2	2	2	1	-	-	-
330	31.29	-	-	-	1	2	2	2	1	-	-	-
345	28.95	-	-	-	1	2	2	2	1	-	-	-
Total		7	18	28	40	48	48	48	40	28	18	7
Factor		2%	5%	8%	12%	15%	15%	15%	12%	8%	5%	2%

From this, for the example ISS orbit, eleven single orbits with Beta Angles from -75° to +75° in 15° bins should be assessed. Bumper results from each should be weighted per the factors in the bottom row of Table 7 based on the occurrence of each Beta Angle across a year considering the effect of varying RAAN, which concentrates Beta Angles among values closer to zero. Table 8 sums the 78 Bumper runs this will require, assuming a rotation limit of 30°.

Table 8: Required Number of Bumper Runs, Rotation Limit = 30°, Bin Size = 15°, ISS Orbit, Variable RAAN

Beta (°)	Segments
75	1
60	4
45	8
30	8
15	12
0	12
-15	12
-30	8
-45	8
-60	4
-75	1
	78

Analysis Process

This process is applicable when a spacecraft points to the Sun, the analysis covers a long duration, and an Earth-centered VNC coordinate system must be used. If the Sun-pointing spacecraft is in Earth orbit for a short duration, perform explicit attitude calculations to determine the actual rotations to apply to that specific trajectory. The definition for "short duration" for the purpose of this process is a trajectory that can reasonably be explicitly divided into a number of runs compatible with the time and computing resources available. The ensuing assessment may require ORDEM Conics. If the Sun-pointing spacecraft is beyond Earth orbit and only MEM 3 is needed, use MEM 3 with the Sun as the central body—this will result in a well-behaved attitude.

Step 1: Right Ascension of the Ascending Node

If the customer specifies a singular fixed value for the RAAN, that value shall be used. If the customer does not specify a value to use, then RAAN shall be treated as varying across the range from 0° to 360°.

Step 2: Beta Angle Limits

Determine the extremes of Beta Angle that will be considered. For a single, fixed RAAN value, evaluate the prescribed RAAN value using Equation 2 and its surrounding paragraphs. For the full spectrum of RAAN values, the extreme values shall be $\pm(\epsilon + i)$, up to ±90°.

Step 3: Beta Angle Bin Size

Determine the size of Beta Angle bins that the mission will be discretized into. Bins should not be larger than 30° unless there is a severe limitation on time or computing resources. If a single, fixed RAAN value has been designated, then 30° bins should be sufficient. If the full spectrum of RAAN values is being considered, then the bin size should be reduced to no more than 15° for better resolution.

Considering the uncertainty caused by the limited insight provided by the study described above, an analyst may choose finer bins for higher confidence if the time and computing resources are available. For mathematical convenience, consider a bin size that the maximum Beta Angle is divisible by (i.e. max Beta = 72°, bin size = 12°).

Step 4: Beta Angle Cases

Select the specific Beta Angles at which to assess a single orbit.

If a single, fixed RAAN value has been designated, use the bulleted regimes on Page 23 to determine how many single orbits to assess, including if intermediate cases between zero and the extremes are necessary. If intermediate cases are necessary, space them out evenly in time similar to Table 5.

In determining time, assume that the Vernal Equinox, $\Gamma = 0°$, occurs at noon, Universal Coordinated Time (UTC), on March 20. The actual time varies by approximately six hours each year until being reset by a leap year because a year is 365.25 days long, not 365 days. An analyst may consult an almanac for a specific year if greater precision is warranted. From this, the date and Ecliptic True Solar Longitude are related by:

$$\boldsymbol{UTC\ Date(\Gamma) = 20\ March\ 12{:}00 + 365.25 * \frac{\Gamma}{360°}} \qquad 3$$

If the full spectrum of RAAN values is being considered, then use Beta Angles that are multiples, positive and negative, of the bin size selected in Step 3 up to the limits derived in Step 2, including Beta = 0°.

Step 5: Beta Angle Weighting Factors

Determine weighting factors for each of the single orbits being considered.

If a single, fixed RAAN value has been designated and all single orbits are evenly spaced in time, then count one occurrence of the positive and negative extreme values and two occurrences of all other cases. Assign weighting factors for each case as a percentage of the sum of occurrences, similar to Table 6.

If the cases are not evenly spaced in time, then list out every occurrence of the cases being assessed with dates—similar to Table 5, only covering the entire mission duration, repeating cases as they reoccur—assume dividing lines halfway between each date, apportion time to each case according to these divisions, and assign weighting factors for each case as a percentage of the duration.

If the full spectrum of RAAN values is being considered, then create a table similar to Table 7. Rows shall be RAAN values covering the range from 0° to 360° in increments of no more than 15°; because this step is purely a spreadsheet task and more RAAN values will not increase the downstream workload, RAAN increments should be smaller, preferably 5° or even 1°. Columns shall be the Beta Angles chosen in Step 4. For each RAAN, estimate the extreme Beta Angles using Equation 2 and its surrounding paragraphs. Assign the positive and negative Beta Angle bins closest to the extreme values (above or below) a single occurrence each and assign two occurrences to each Beta Angle between these extremes. Sum the occurrences of each Beta Angle across each column. Assign weighting factors for each Beta Angle as a percentage of the sum of occurrences.

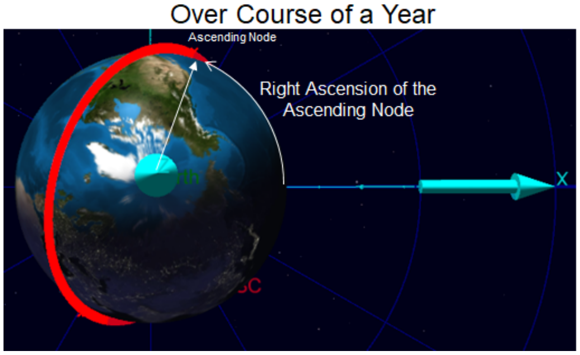
Step 6: Generate Mission Analysis Tool Data

For each Beta Angle case from Step 4, generate the data required for attitude calculation.

Using GMAT, it is likely most effective to create a separate script-based scenario for each Beta Angle case to focus computations and outputs in the desired regions of time. Using STK, it may be more efficient to create a single scenario of the desired orbit for a full year because it is possible to limit the output reports to desired time ranges.

Using GMAT, it is recommended to design an orbit with a fixed RAAN value, as seen on the left in Figure 21, thus guaranteeing predictable Beta Angle results. If Keplerian parameters are input into GMAT, the orbit will precess as seen on the right, in which the example orbit cycled through all possibles values of RAAN five times in a year. While realistic, this will confound the process of obtaining a single orbit at a desired Beta Angle. STK is not subject to this phenomenon, provided a basic propagator, such as TwoBody or J4Perturbation, is used.

Ephemeris Creator: Fixed RAAN Over Course of a Year



Keplerian Entry: Varying RAAN Over Course of a Year

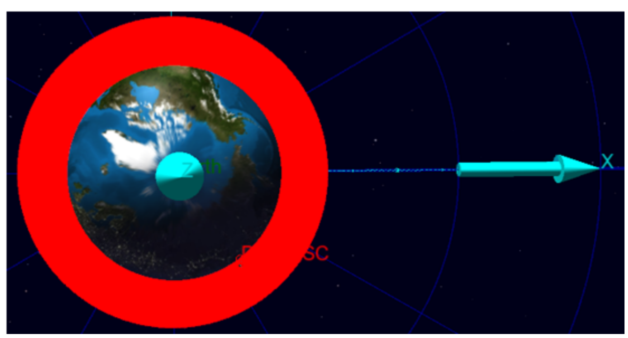


Figure 21: RAAN Outcomes in GMAT

For each Beta Angle case, find the date at which that Beta Angle occurs. Use Equation 1 to determine the Ecliptic True Solar Longitude that results in the desired Beta Angle and use Equation 3 to convert that Ecliptic True Solar Longitude into a date. If a single, fixed RAAN value has been designated, then use that value of RAAN in both the scenario and equation. If the full spectrum of RAAN values is being considered, then use the value of RAAN that produces the most extreme Beta Angles in both the scenario and equation. For inclinations up to 66.55° (90° - the Obliquity of the Ecliptic), this is RAAN = 180°; for higher inclinations, this is RAAN = 90°. Attitudes at a given Beta Angle do not change with RAAN, therefore, a single orbit with the most extreme RAAN can be used to generate all Beta Angle cases, and there is no need to separately consider the same Beta Angle under different RAAN values.

With the date at which each Beta Angle occurs known, obtain input data from the mission analysis tool for a time period surrounding that date. It is recommended to use a time period stretching at least one full day on either side of the date to ensure the desired Beta Angle is captured. Use timesteps small enough to resolve rotations across an orbit in detail, with one-minute intervals being recommended.

Step 7: Generate Continuous Rotations

Use the data generated in Step 6 to perform attitude calculations. The rotation angles should bear some resemblance to Figure 12, with two axes cycling above and below a coalescence line. The third axis may exhibit other behavior based on the secondary attitude constraint. This single-orbit time period will be the focus of the following step. This will be done for each Beta Angle.

Step 8: Discretize Continuous Rotations

For each Beta Angle, calculate the Dispersion as 90° - Beta Angle. Based on that Dispersion, determine the necessary number of segments per the regimes on Page 19. If needed, first divide the time period into Quads based on when the two axes involved in Sun tracking cross their respective coalescence lines. Then subdivide the Quads into the requisite number of equal-time segments. Record the percentage of the orbit that each segment represents.

This step will be performed separately for each Beta Angle being considered. Positive and negative Beta Angles of the same magnitude must be considered separately as they will have different—but related—attitudes.

Step 9: Environments and Bumper

Create environment files and run Bumper for each segment of each Beta Angle case.

For ORDEM, it will be most common for the parameters necessary for a Low Earth Orbit (LEO) analysis (perigee/apogee or semi-major axis/eccentricity and inclination) to be known and fixed. Therefore, a single ORDEM environment file pair using these orbit parameters will apply to all segments of all Beta Angles.

For MEM, a Sun-pointing spacecraft will remain in a fixed attitude relative to the Helion and Anti-Helion threat directions. Therefore, it would be most accurate to create a unique MEM environment file pair for each segment of each Beta Angle case using state vectors associated with the time period used to define that segment. A single MEM environment file pair encompassing the entire orbit would smear the Sun, and its corresponding threat directions, all over the sky such that the threats are not aligned with the Sun-pointing sides of the spacecraft as they realistically should be.

In LEO, orbital debris is the dominant source of risk, making maximum detail in the micrometeoroid analysis unnecessary. Therefore, it is acceptable to create a single MEM environment file pair to use with all segments of all Beta Angles.

In intermediate altitudes where both environments contribute risk, such as between 2,000 km and 10,000 km altitude, or at Geostationary orbit, where micrometeoroids are the dominant risk source but orbital debris remains of interest, then a more accurate approach is warranted. Two options are available to the analyst.

First, a unique MEM environment file pair may be created for each segment of each Beta Angle case using state vectors associated with the time period used to define that segment. This will entail performing separate ORDEM and MEM Bumper runs for each segment, and both environments will use the same Earth-centered attitude unique to that segment.

Second, the mission may be reconsidered for MEM 3 using the Sun as the central body. Sun-centered attitudes are expected to be well-behaved such that the entire year can be assessed with a single run or a small number of runs depending on the secondary rotation constraint. This will entail ORDEM and MEM using separate and unrelated discretization of the mission and attitudes in their respective Bumper runs.

In any case, Bumper must be run for each segment the mission in discretized into for each of ORDEM and MEM. The Geometry module must be run for each unique attitude; it is recommended for each run to provide Bumper the same FEM and enter the rotations unique to that segment. The Response module run is likely to be common to all segments for a given environment type. The Shield module must be run for each segment using that segment's Geometry output and the common Response output.

An alternative method exists to compact all rotations into a single Bumper run for each environment type. HVIT has a spreadsheet Igloo Rotation Macro for each environment model (ORDEM 3.2 and MEM 3), which performs this function. For each segment, a tool performing this function will rotate the environment igloo opposite to the entered vehicle rotation, which is equivalent to rotating the vehicle in a fixed igloo. The tool will then need to combine the rotated igloos for all the segments, each weighted by that segment's time factor. The environment file output by such a tool can then be run in the Shield module with an unrotated FEM (in the Geometry module) producing risk results for the entire mission in a single Bumper run for each environment type.

Step 10: Post-Processing

When running Bumper separately for each segment, each risk result must be factored by:

- that segment's percentage of the single orbit recorded in Step 8, and
- that orbit/Beta Angle's weighting factor derived in Step 5

The risk must also account for the total duration of the mission.

It is considered best practice to assign ORDEM a duration of one year in Procloo and MEM a duration of one year in Bumper, then apply the actual duration in years as a factor and both process-unique factors above in post-processing. Alternately, the actual duration and both factors may be used to determine the duration of each segment, which can be entered into Procloo and Bumper, but this is not recommended.

The total mission risk is the summation of the factored risk results across all segments for both ORDEM and MEM.