Preface

This book constitutes the second volume in the *Elsevier Astrodynamics Book Series*, and is devoted to spacecraft formation flying. As opposed to Volume I in the series, titled *Modern Astrodynamics*, the current volume is a *textbook* rather than an edited volume.

Spacecraft formation flying is a vast subject, which can be studied from many different viewpoints. This book develops the theory from an astrodynamical viewpoint, emphasizing modeling, control and navigation of formation flying satellites on Earth orbits – mostly low Earth orbit (LEO) missions. Spacecraft formation flying in deep-space and on libration-point orbits is not covered in the current text; this subject deserves a stand-alone book.

We have attempted to create a coherent exposition of spacecraft relative motion, both in the unperturbed and perturbed settings, to discuss the main control approaches for regulating relative satellite dynamics, using both impulsive and continuous maneuvers, and to present the main constituents required for relative navigation. We also discuss relative attitude dynamics, and the coupling between translational and rotational dynamics.

This book is intended for graduate students and academic researchers, for university professors teaching courses on distributed space systems, aerospace and mechanical engineers, mathematicians, astronomers, and astrophysicists. The book assumes prior knowledge of basic astrodynamics, attitude dynamics and control theory. Nevertheless, we provide introductory chapters dedicated to orbital mechanics, perturbation methods, control and estimation. To illustrate the main developments, we solve a number of examples in each chapter, and provide a sample simulation of a formation flying mission involving high-fidelity modeling, control and relative navigation.

Notation

Relative motion between satellites or spacecraft involves a number of coordinate systems and a few notational conventions. Usually, we will refer to one spacecraft as the *chief*, and to the other one as the *deputy*. An alternative terminology designates one of the satellites as *leader*, and the other as *follower*. The chief/deputy notation implies a more general setup, in which the motion of a satellite is modeled with respect to a potentially *virtual* or *fictitious* satellite, or, in other words, with respect to a *reference point* or a *reference orbit*. The leader/follower notation is reserved for in-line motion or for particular formation control and/or navigation problems. In the context of orbital rendezvous, one of the satellites is usually referred to as *target*, and the other is called *chaser* or *pursuer*. While usually only a single satellite is designated as the chief, there could be many deputies. We will usually denote quantities referred to the chief, leader or target by $(\cdot)_0$, and those related to the deputy, follower or chaser by $(\cdot)_1$.

Exceptions are made for denoting the epoch t_0, and the mean anomaly at epoch of the chief, $(M_0)_0$. We assume that $t_0 = 0$ frequently, and denote the initial condition of some time-dependent vector $\mathbf{w}(t)$ by $\mathbf{w}(0)$. During the discussion involving a single satellite, as in Chapter 2, $(M_0)_0$ is replaced by M_0, but in subsequent chapters dealing with multiple satellites, we designate M_0 as the mean anomaly of the chief. In some of the expressions for the relative motion variables, which are functions of the chief's orbital elements, the subscript 0 is dropped to simplify the presentation of the equations. Another exception pertains to perturbed variables. For example, we designate the zero-order and first-order Hamiltonians, respectively, by $\mathcal{H}_0$ and $\mathcal{H}_1$.

Throughout this book, we will resolve some vector $\mathbf{w}$ in a particular coordinate system, $\mathscr{A}$, by writing $[\mathbf{w}]_{\mathscr{A}}$. This notation will be followed only when a number of coordinate systems are involved in the analysis, and will be omitted when the context permits no ambiguities. Unless otherwise stated, we assume that all vectors are column vectors. The time derivative of $\mathbf{w}$ in frame $\mathscr{A}$ will be denoted by $\mathrm{d}^{\mathscr{A}}\mathbf{w}/\mathrm{d}t$.

To denote the p-norm of $\mathbf{w}$, we will use the notation $\|\mathbf{w}\|_p$. By default, $\|\mathbf{w}\| \equiv \|\mathbf{w}\|_2$, viz. the Euclidean norm.

We will use calligraphic fonts to denote quantities measured in units of energy per unit mass; for example, $\mathcal{E}$ will denote energy, and $\mathcal{H}$ will denote a Hamiltonian. Calligraphic fonts will also be used for denoting cost functions (arising in optimization problems throughout the book), which are most often related to energy measures. In a frame of reference centered at one of the orbiting satellites, we distinguish between *radial*, *along-track* and *cross-track* components of the position vector. The latter component is also referred to as *out-of-plane* or *normal*.

Depending upon context, we will use $\bar{x}$ to denote a non-dimensional (normalized) x, and $\bar{x}'$ to denote differentiation of $\bar{x}$ with respect to a non-dimensional variable; alternatively, $\overline{\text{œ}}$ will denote the mean orbital element œ; for example, $\bar{e}$ is the mean value of the eccentricity, e.

Finally, we elaborate below the main abbreviations used throughout this book:

BL = Baseline
CDGPS = Carrier-Phase Differential GPS
CLF = Control Lyapunov Function
CM = Center-of-Mass
CRD = Cartesian Rectangular Dextral
CW = Clohessy–Wiltshire

DCM = Directional Cosines Matrix
DLQR = Discrete-Time Linear Quadratic Regulator
DOF = Degrees-of-Freedom
ECEF = Earth-Centered, Earth-Fixed
ECI = Earth-Centered Inertial
EKF = Extended Kalman Filter
GA = Gim–Alfriend
GCO = General Circular Orbit
GM = Geometric Method
GVE = Gauss' Variational Equations
IC = Initial Condition
KF = Kalman Filter
LCJ = Lee–Cochran–Jo
LON = Line-of-Nodes
LP = Linear Programming
LPE = Lagrange's Planetary Equations
LQR = Linear Quadratic Regulator
LTI = Linear Time-Invariant
LVLH = Local-Vertical, Local–Horizontal
MPC = Model Predicative Control
PCO = Projected Circular Orbit
RAAN = Right Ascension of the Ascending Node
RHS = Right-Hand Side
STM = State Transition Matrix
TH = Tschauner–Hempel
UKF = Unscented Kalman Filter
YA = Yamanaka–Ankersen

Book Organization

This book's organization reflects our commitment to create a complete, standalone text. The book is divided into 14 chapters. Chapter 1 is an introduction, wherein we raise the fundamental questions, issues, and approaches relating to spacecraft formation flying; this chapter therefore serves to provide a broad overview of the topics covered in the book. Chapter 2 is devoted to some notions in orbital mechanics. In this chapter, we concisely explain some key coordinate systems and discuss the Keplerian two-body problem. Chapter 3 contains diverse topics in mechanics, optimization, control and estimation, including Lagrangian and Hamiltonian mechanics, a discussion of static optimization, feedback control, and filtering methods, which are all pertinent to the devolvement of relative spacecraft control, measurement and navigation methods. Chapters 1–3 are all introductory chapters, constituting the first part of the book, intended to create a foundation on which we construct methods and tools for the analysis and design of spacecraft formation flying.

Chapter 4 is the first of six chapters devoted to modeling relative spacecraft dynamics. This chapter presents commonly used nonlinear models for relative motion, for both perturbed and unperturbed motion. Chapter 5 presents a variety of linear differential equations for modeling relative motion under the two-body assumptions, including satellite rendezvous. The focus in this chapter is on the physical-coordinate description of relative motion; motion modeling using orbital elements is treated extensively in Chapter 6, where we present an important modification of relative spacecraft motion modeling – the use of orbital elements as constants of motion instead of the initial conditions. Chapter 7 builds on Chapter 6 and extends its results to include orbital perturbations using a number of approaches; methods to mitigate the effect of perturbations on relative motion is the subject of Chapter 8. Chapter 9, which concludes the second part of the book, is focused on the coupling between rotation and translation; it provides equations for relative spacecraft translation as well as translational equations taking into account kinematic coupling.

Chapter 10 opens the third part of the book, which is focused on the development of spacecraft formation controllers and their practical implementation. Chapter 10 contains a brief discussion of methods for continuous and impulsive formation control: establishment, maintenance, and reconfiguration. Chapter 11 is concerned with the impact of the maneuver acceleration implementation error on the performance of spacecraft formation flying, rendezvous and docking. It also explores how accelerometers can be used to improve performance by providing accurate measurements of the applied maneuver acceleration.

The final part of this book is devoted to relative navigation and high-fidelity simulation of spacecraft formations, incorporating actual relative navigation modeling. Chapter 12 discusses an application of various estimation algorithms to the relative orbital navigation problem. Chapter 13 illustrates most of the previously introduced material by performing a series of nonlinear simulations of portions of a formation flying reference mission.

Chapter 14 discusses some future prospects. It raises an important question: given the advantages that can be gained by flying spacecraft in formations, why are there no spacecraft formations on orbit currently?

Acknowledgments

Much of the impetus for research on perturbation mitigation, control, and relative navigation for formation flying began as a means to address the biggest challenge facing the TechSat21 program proposed by AFRL more than ten years ago: how to keep small satellites in close proximity in LEO with very little fuel. TechSat21 was the first proposed mission involving satellites in close proximity and in orbits with different inclinations. This challenge has mushroomed into a body of literature consisting of more than a hundred journal articles and doctoral dissertations. Innovative solutions for formation maintenance, fuel balancing, and relative navigation developed so far can directly or indirectly be attributed to the technical challenges set by the TechSat21 mission.

The authors were supported by grants from AFOSR, AFRL, NASA GSFC, the Galileo Supervisory Authority, the European Sixth Framework Program, Israel Ministry of Science and Technology, the Asher Space Research Institute, the Gutwirth Fund for the Promotion of Research, the Rosenthal Fund for

Aerospace Research, the National Science Foundation Graduate Research Fellowship Program, NASA Grants #NAG3-2839, #NAG5-10440, #NAG5-11349, #NCC5-704, and #NCC5-729, the National Defense Science and Engineering Graduate Fellowship, administered by the American Society for Engineering Education, and Mitsubishi Electric Corporation.

The authors wish to acknowledge Alok Das (AFRL), Mark Jacobs (AFOSR), Jesse Leitner (GSFC), Russell Carpenter (GSFC), and Richard Burns (GSFC), as well as the following individuals, without whose contributions this manuscript would not have formed: John L. Junkins, Oliver Montenbruck, Sesha S. Vaddi, Prasenjit Sengupta, Hui Yan, Hanspeter Schaub, Dario Izzo, Marco Sabatini, Dong-Woo Gim, Jeremy Kasdin, David Mishne, Konstantin V. Kholshevnikov, Shay Segal and Egemen Kolemen.

In particular, we wish to acknowledge the instrumental contributions of Matthew Jeffrey to Chapter 11 and Megan Mitchell to Chapter 12. We also want to thank Kim Luu, whose contribution to the preparation of the Appendices was invaluable.