

Bibliography

- [1] S. M. ALESSANDRINI, *A Motivational Example for the Numerical Solution of Two-Point Boundary-Value Problems*, SIAM Review, 37 (1995), pp. 423–427.
- [2] U. M. ASCHER, R. M. M. MATTHEIJ, AND R. D. RUSSELL, *Numerical Solution of Boundary Value Problems for Ordinary Differential Equations*, Prentice–Hall, Englewood Cliffs, NJ, 1988.
- [3] U. M. ASCHER AND L. R. PETZOLD, *The Numerical Solution of Delay-Differential-Algebraic Equations of Retarded and Neutral Type*, SIAM Journal on Numerical Analysis, 32 (1995), pp. 1635–1657.
- [4] C. ASHCRAFT, R. G. GRIMES, AND J. G. LEWIS, *Accurate Symmetric Indefinite Linear Equation Solvers*, SIAM Journal on Matrix Analysis and Applications, 20 (1998), pp. 513–561.
- [5] F. BASHFORTH AND J. C. ADAMS, *Theories of Capillary Action*, Cambridge University Press, New York, 1883.
- [6] R. H. BATTIN, *An Introduction to the Mathematics and Methods of Astrodynamics*, AIAA Education Series, American Institute of Aeronautics and Astronautics, Inc., 1633 Broadway, New York, NY 10019, 1987.
- [7] B. BAUMRUCKER, J. RENFRO, AND L. T. BIEGLER, *MPEC Problem Formulations and Solution Strategies with Chemical Engineering Applications*, Computers & Chemical Engineering, 32 (2008), pp. 2903–2913.
- [8] D. A. BENSON, *A Gauss Pseudospectral Transcription for Optimal Control*, PhD thesis, Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge, MA, 2004.
- [9] P. BERKMANN AND H. J. PESCH, *Abort Landing in Windshear: Optimal Control Problem with Third-Order State Constraint and Varied Switching Structure*, Journal of Optimization Theory and Applications, 85 (1995), pp. 21–57.
- [10] J. T. BETTS, *An Improved Penalty Function Method for Solving Constrained Parameter Optimization Problems*, Journal of Optimization Theory and Applications, 16 (1975), pp. 1–24.
- [11] ———, *Solving the Nonlinear Least Square Problem: Application of a General Method*, Journal of Optimization Theory and Applications, 18 (1976), pp. 469–483.

- [12] ———, *Optimal Three-Burn Orbit Transfer*, AIAA Journal, 15 (1977), pp. 861–864.
- [13] ———, *Using Sparse Nonlinear Programming to Compute Low Thrust Orbit Transfers*, The Journal of the Astronautical Sciences, 41 (1993), pp. 349–371.
- [14] ———, *The Application of Sparse Least Squares in Aerospace Design Problems*, in Optimal Design and Control, Proceedings of the Workshop on Optimal Design and Control, J. Borggaard, J. Burkardt, M. Gunzburger, and J. Peterson, eds., vol. 19 of Progress in Systems and Control Theory, Birkhäuser, Basel, 1994, pp. 81–96.
- [15] ———, *Optimal Interplanetary Orbit Transfers by Direct Transcription*, The Journal of the Astronautical Sciences, 42 (1994), pp. 247–268.
- [16] ———, *The Application of Optimization Techniques to Aerospace Systems*, in Fourth International Conference on Foundations of Computer-Aided Process Design, Proceedings of the Conference at Snowmass, Colorado, July 10–14, 1994, L. T. Biegler and M. F. Doherty, eds., CACHE, American Institute of Chemical Engineers, New York, 1995.
- [17] ———, *Experience with a Sparse Nonlinear Programming Algorithm*, in Large-Scale Optimization with Applications, L. T. Biegler, T. F. Coleman, A. R. Conn, and F. N. Santosa, eds., vol. 93 of The IMA Volumes in Mathematics and its Applications, Springer-Verlag, New York, 1997, pp. 53–72.
- [18] ———, *Survey of Numerical Methods for Trajectory Optimization*, AIAA Journal of Guidance, Control, and Dynamics, 21 (1998), pp. 193–207.
- [19] ———, *Very Low Thrust Trajectory Optimization*, in High Performance Scientific and Engineering Computing, Proceedings of the International FORTWIHR Conference on HPSEC, Munich, March 16–18, 1998, H.-J. Bungartz, F. Durst, and C. Zenger, eds., Springer-Verlag, Berlin, Heidelberg, 1999.
- [20] ———, *Very Low Thrust Trajectory Optimization Using a Direct SQP Method*, Journal of Computational and Applied Mathematics, 120 (2000), pp. 27–40.
- [21] ———, *Practical Methods for Optimal Control Using Nonlinear Programming*, SIAM, Philadelphia, PA, 2001.
- [22] ———, *Optimal Lunar Swingby Trajectories*, The Journal of the Astronautical Sciences, 55 (2007), pp. 349–371.
- [23] J. T. BETTS, N. BIEHN, AND S. L. CAMPBELL, *Convergence of Nonconvergent IRK Discretizations of Optimal Control Problems with State Inequality Constraints*, SIAM Journal on Scientific Computing, 23 (2002), pp. 1981–2007.
- [24] J. T. BETTS, N. BIEHN, S. L. CAMPBELL, AND W. P. HUFFMAN, *Exploiting Order Variation in Mesh Refinement for Direct Transcription Methods*, Tech. Report M&CT-TECH-99-022, Mathematics and Computing Technology, The Boeing Company, P.O. Box 3707, Seattle, WA 98124-2207, Oct. 1999.

- [25] ———, *Compensating for Order Variation in Mesh Refinement for Direct Transcription Methods*, Journal of Computational and Applied Mathematics, 125 (2000), pp. 147–158.
- [26] J. T. BETTS AND S. L. CAMPBELL, *Discretize then Optimize*, in Mathematics for Industry: Challenges and Frontiers, D. R. Ferguson and T. J. Peters, eds., SIAM Proceedings Series, SIAM, Philadelphia, PA, 2005, pp. 140–157.
- [27] J. T. BETTS, S. L. CAMPBELL, AND A. ENGELSONE, *Direct Transcription Solution of Inequality Constrained Optimal Control Problems*, in Proceedings, 2004 American Control Conference, Boston, MA, June 2004, pp. 1622–1626.
- [28] ———, *Direct Transcription Solution of Optimal Control Problems with Higher Order State Constraints: Theory vs Practice*, Optimization and Engineering, 8 (2007), pp. 1–19.
- [29] J. T. BETTS, M. J. CARTER, AND W. P. HUFFMAN, *Software for Nonlinear Optimization*, Mathematics and Engineering Analysis Library Report MEA-LR-83 R1, Boeing Information and Support Services, The Boeing Company, P.O. Box 3707, Seattle, WA 98124-2207, June 1997.
- [30] J. T. BETTS, S. K. ELDERSVELD, AND W. P. HUFFMAN, *A Performance Comparison of Nonlinear Programming Algorithms for Large Sparse Problems*, in Proceedings of the AIAA Guidance, Navigation, and Control Conference, AIAA-93-3751-CP, Monterey, CA, Aug. 1993, pp. 443–455.
- [31] J. T. BETTS AND S. O. ERB, *Optimal Low Thrust Trajectories to the Moon*, SIAM Journal on Applied Dynamical Systems, 2 (2003), pp. 144–170. <http://www.siam.org/journals/siads/2-2/40908.html>.
- [32] J. T. BETTS AND P. D. FRANK, *A Sparse Nonlinear Optimization Algorithm*, Journal of Optimization Theory and Applications, 82 (1994), pp. 519–541.
- [33] J. T. BETTS AND J. M. GABLONSKY, *A Comparison of Interior Point and SQP Methods on Optimal Control Problems*, Technical Document Series MCT-TECH-02-004, Mathematics and Engineering Analysis, The Boeing Company, P.O. Box 3707, Seattle, WA 98124-2207, Mar. 2002. <http://www.boeing.com/phantom/socs/benchmark/benchmark.pdf>.
- [34] J. T. BETTS AND W. P. HUFFMAN, *Trajectory Optimization on a Parallel Processor*, AIAA Journal of Guidance, Control, and Dynamics, 14 (1991), pp. 431–439.
- [35] ———, *Application of Sparse Nonlinear Programming to Trajectory Optimization*, AIAA Journal of Guidance, Control, and Dynamics, 15 (1992), pp. 198–206.
- [36] ———, *Path Constrained Trajectory Optimization Using Sparse Sequential Quadratic Programming*, AIAA Journal of Guidance, Control, and Dynamics, 16 (1993), pp. 59–68.
- [37] ———, *Sparse Nonlinear Programming Test Problems (Release 1.0)*, BCS Technology Technical Report BCSTech-93-047, Boeing Computer Services, The Boeing Company, P.O. Box 3707, Seattle, WA 98124-2207, 1993.

- [38] ———, *Sparse Optimal Control Software SOCS*, Mathematics and Engineering Analysis Technical Document MEA-LR-085, Boeing Information and Support Services, The Boeing Company, P.O. Box 3707, Seattle, WA 98124-2207, July 1997.
- [39] ———, *Mesh Refinement in Direct Transcription Methods for Optimal Control*, Optimal Control Applications and Methods, 19 (1998), pp. 1–21.
- [40] ———, *Exploiting Sparsity in the Direct Transcription Method for Optimal Control*, Computational Optimization and Applications, 14 (1999), pp. 179–201.
- [41] S. A. BHATT, *Optimal Reorientation of Spacecraft Using Only Control Moment Gyroscopes*, Master's thesis, Rice University, Houston, TX, May 2007.
- [42] G. A. BLISS, *Lectures on the Calculus of Variations*, University of Chicago Press, Chicago, IL, 1946.
- [43] H. G. BOCK, *Numerical Treatment of Inverse Problems in Chemical Reaction Kinetics*, in Modelling of Chemical Reaction Systems, K. H. Ebert, Peter Deuflhard, and W. Jäger, eds., vol. 18 of Springer Series in Chemical Physics, Springer-Verlag, New York, 1981, pp. 102–125.
- [44] ———, *Recent Advances in Parameter Identification Techniques for O.D.E.*, in Numerical Treatment of Inverse Problems in Differential and Integral Equations, P. Deuflhard and Ernst Hairer, eds., Birkhäuser Verlag, Heidelberg, 1982, pp. 95–121.
- [45] I. BONGARTZ, A. R. CONN, N. I. M. GOULD, AND P. L. TOINT, *CUTE: Constrained and Unconstrained Testing Environment*, ACM Transactions on Mathematical Software, 21 (1995), pp. 123–160.
- [46] K. E. BRENAN, *A Smooth Approximation to the GTS 1962 Standard Atmosphere Model*, Aerospace Technical Memorandum ATM 82-(2468-04)-7, The Aerospace Corporation, 2350 E. El Segundo Blvd., El Segundo, CA 90245-4691, May 1982.
- [47] ———, *Engineering Methods Report: The Design and Development of a Consistent Integrator/Interpolator For Optimization Problems*, Aerospace Technical Memorandum ATM 88(9975)-52, The Aerospace Corporation, 2350 E. El Segundo Blvd., El Segundo, CA 90245-4691, 1988.
- [48] K. E. BRENAN, S. L. CAMPBELL, AND L. R. PETZOLD, *Numerical Solution of Initial-Value Problems in Differential-Algebraic Equations*, vol. 14 of Classics in Applied Mathematics, SIAM, Philadelphia, PA, 1995.
- [49] R. A. BROUCKE AND P. J. CEFOLA, *On Equinoctial Orbit Elements*, Celestial Mechanics, 5 (1972), pp. 303–310.
- [50] C. G. BROYDEN, *A Class of Methods for Solving Nonlinear Simultaneous Equations*, Mathematics of Computation, 19 (1965), pp. 577–593.
- [51] ———, *The Convergence of a Class of Double-Rank Minimization Algorithms*, Journal of the Institute of Mathematics and Applications, 6 (1970), pp. 76–90.

- [52] A. E. BRYSON, JR., M. N. DENHAM, AND S. E. DREYFUS, *Optimal Programming Problems with Inequality Constraints I: Necessary Conditions for Extremal Solutions*, AIAA Journal, 1 (1963), pp. 2544–2550.
- [53] A. E. BRYSON, JR., M. N. DESAI, AND W. C. HOFFMAN, *Energy-State Approximation in Performance Optimization of Supersonic Aircraft*, Journal of Aircraft, 6 (1969), pp. 481–488.
- [54] A. E. BRYSON, JR. AND Y.-C. HO, *Applied Optimal Control*, John Wiley and Sons, New York, 1975.
- [55] R. BULIRSCH, *Die Mehrzielmethode zur numerischen Lösung von nichtlinearen Randwertproblemen und Aufgaben der optimalen Steuerung*, Report of the Carl-Cranz Gesellschaft, Carl-Cranz Gesellschaft, Oberpfaffenhofen, Germany, 1971.
- [56] R. BULIRSCH, F. MONTRONE, AND H. J. PESCH, *Abort Landing in the Presence of Windshear as a Minimax Optimal Control Problem, Part 1: Necessary Conditions*, Journal of Optimization Theory and Applications, 70 (1991), pp. 1–23.
- [57] ———, *Abort Landing in the Presence of Windshear as a Minimax Optimal Control Problem, Part 2: Multiple Shooting and Homotopy*, Journal of Optimization Theory and Applications, 70 (1991), pp. 223–253.
- [58] R. BULIRSCH, E. NERZ, H. J. PESCH, AND O. VON STRYK, *Combining Direct and Indirect Methods in Optimal Control: Range Maximization of a Hang Glider*, in Optimal Control, R. Bulirsch, A. Miele, J. Stoer, and K. H. Well, eds., vol. 111 of International Series of Numerical Mathematics, Birkhäuser Verlag, Basel, 1993, pp. 273–288.
- [59] R. H. BYRD, M. E. HRIBAR, AND J. NOCEDAL, *An Interior Point Algorithm for Large-Scale Nonlinear Programming*, SIAM Journal on Optimization, 9 (1999), pp. 877–900.
- [60] S. L. CAMPBELL, N. BIEHN, L. JAY, AND T. WESTBROOK, *Some Comments on DAE Theory for IRK Methods and Trajectory Optimization*, Journal of Computational and Applied Mathematics, 120 (2000), pp. 109–131.
- [61] M. D. CANON, C. D. CULLUM, AND E. POLAK, *Theory of Optimal Control and Mathematical Programming*, McGraw–Hill, New York, 1970.
- [62] M. CARACOTSIOS AND W. E. STEWART, *Sensitivity Analysis of Initial Value Problems with Mixed ODE's and Algebraic Equations*, Computers & Chemical Engineering, 9 (1985), pp. 359–365.
- [63] T. F. COLEMAN, B. S. GARBOW, AND J. J. MORÉ, *Software for Estimating Sparse Hessian Matrices*, ACM Transactions on Mathematical Software, 11 (1985), pp. 363–377.
- [64] T. F. COLEMAN AND J. J. MORÉ, *Estimation of Sparse Jacobian Matrices and Graph Coloring Problems*, SIAM Journal on Numerical Analysis, 20 (1983), pp. 187–209.

- [65] A. R. CURTIS, M. J. D. POWELL, AND J. K. REID, *On the Estimation of Sparse Jacobian Matrices*, Journal of the Institute of Mathematics and Applications, 13 (1974), pp. 117–120.
- [66] G. DAHLQUIST AND AKE BJÖRK, *Numerical Methods*, Prentice–Hall, Englewood Cliffs, NJ, 1974.
- [67] G. B. DANTZIG, *Linear Programming and Extensions*, Princeton University Press, Princeton, NJ, 1963.
- [68] W. C. DAVIDON, *Variable Metric Methods For Minimization*, A. E. C. Res. and Develop. Report ANL-5900, Argonne National Laboratory, Argonne, IL, 1959.
- [69] C. DE BOOR, *A Practical Guide to Splines*, Springer-Verlag, New York, 1978.
- [70] J. E. DENNIS, JR., D. M. GAY, AND R. E. WELSCH, *An Adaptive Nonlinear Least-Squares Algorithm*, ACM Transactions on Mathematical Software, 7 (1981), pp. 348–368.
- [71] J. E. DENNIS, JR. AND R. B. SCHNABEL, *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*, Prentice–Hall, Englewood Cliffs, NJ, 1983.
- [72] V. DESHMUKH, H. MA, AND E. BUTCHER, *Optimal Control of Parametrically Excited Linear Delay Differential Systems via Chebyshev Polynomials*, in Proceedings of the American Control Conference, Denver, CO, June 2003.
- [73] E. D. DICKMANN, *Maximum Range Three-Dimensional Lifting Planetary Entry*, Tech. Report TR R-387, National Aeronautics and Space Administration, Washington, D.C., 1972.
- [74] T. N. EDELBAUM, L. L. SACKETT, AND H. L. MALCHOW, *Optimal Low Thrust Geocentric Transfer*, in AIAA 10th Electric Propulsion Conference, AIAA 73-1074, Lake Tahoe, NV, Oct.–Nov. 1973.
- [75] A. ENGELSONE AND S. L. CAMPBELL, *Adjoint Estimation Using Direct Transcription Multipliers: Compressed Trapezoidal Method*, Optimization and Engineering, 9 (2008), pp. 291–305.
- [76] P. J. ENRIGHT AND B. A. CONWAY, *Optimal Finite-Thrust Spacecraft Trajectories Using Collocation and Nonlinear Programming*, AIAA Journal of Guidance, Control, and Dynamics, 14 (1991), pp. 981–985.
- [77] ———, *Discrete Approximations to Optimal Trajectories Using Direct Transcription and Nonlinear Programming*, AIAA Journal of Guidance, Control, and Dynamics, 15 (1992), pp. 994–1002.
- [78] P. R. ESCOBAL, *Methods of Orbit Determination*, Second Edition, Robert E. Krieger Publishing Company, Malabar, FL, 1985.
- [79] A. V. FIACCO AND G. P. MCCORMICK, *Nonlinear Programming: Sequential Unconstrained Minimization Techniques*, John Wiley and Sons, New York, 1968.

- [80] A. FLEMING, P. SEKHAVAT, AND I. M. ROSS, *Minimum-Time Reorientation of an Asymmetric Rigid Body*, in AIAA Guidance, Navigation and Control Conference, AIAA 2008-7012, Honolulu, HI, Aug. 2008.
- [81] R. FLETCHER, *A New Approach to Variable Metric Algorithms*, Computer Journal, 13 (1970), pp. 317–322.
- [82] ———, *Practical Methods of Optimization, Vol. 2, Constrained Optimization*, John Wiley and Sons, New York, 1985.
- [83] R. FLETCHER, N. I. M. GOULD, S. LEYFFER, AND P. L. TOINT, *Global Convergence of Trust-Region SQP-Filter Algorithms for General Nonlinear Programming*, Tech. Report RAL-TR-1999-041, Rutherford Appleton Laboratory, White Horse, South Oxfordshire, England, 1999.
- [84] R. FLETCHER AND S. LEYFFER, *Nonlinear Programming without a Penalty Function*, University of Dundee Numerical Analysis Report NA/171, University of Dundee, Department of Mathematics, Dundee, Scotland, UK, 1997.
- [85] R. FLETCHER AND M. J. D. POWELL, *A Rapidly Convergent Descent Method for Minimization*, Computer Journal, 6 (1963), pp. 163–168.
- [86] A. FORSGREN, *On Warm Starts for Interior Methods*, in System Modeling and Optimization, IFIP International Federation for Information Processing, F. Ceragioli, A. Dontchev, H. Furuta, K. Marti, and L. Pandolfi, eds., vol. 199, Springer-Verlag, Boston, 2006, pp. 51–66.
- [87] A. FORSGREN AND P. E. GILL, *Primal-Dual Interior Methods for Nonconvex Nonlinear Programming*, SIAM Journal on Optimization, 8 (1998), pp. 1132–1152.
- [88] D. M. GAY, M. L. OVERTON, AND M. H. WRIGHT, *A Primal-Dual Interior Method for Nonconvex Nonlinear Programming*, in Advances in Nonlinear Programming, Y. Yuan, ed., Kluwer Academic Publishers, Dordrecht, The Netherlands, 1998, pp. 31–56.
- [89] C. W. GEAR, *Numerical Initial Value Problems in Ordinary Differential Equations*, Prentice-Hall, Englewood Cliffs, NJ, 1971.
- [90] ———, *The Simultaneous Numerical Solution of Differential-Algebraic Equations*, IEEE Transactions on Circuit Theory, CT-18 (1971), pp. 89–95.
- [91] C. W. GEAR AND T. V. VU, *Smooth Numerical Solutions of Ordinary Differential Equations*, in Proceedings of the Workshop on Numerical Treatment of Inverse Problems for Differential and Integral Equations, Heidelberg, 1982.
- [92] M. GERDTS, *Direct Shooting Method for the Numerical Solution of Higher-Index DAE Optimal Control Problems*, Journal of Optimization Theory and Applications, 117 (2003), pp. 267–294.
- [93] P. E. GILL, W. MURRAY, AND M. A. SAUNDERS, *SNOPT: An SQP Algorithm for Large-Scale Constrained Optimization*, Tech. Report SOL 97-3, Department of Operations Research, Stanford University, Stanford, CA, 1997.

- [94] ———, *SNOPT: An SQP Algorithm for Large-Scale Constrained Optimization*, SIAM Review, 47 (2005), pp. 99–131.
- [95] P. E. GILL, W. MURRAY, M. A. SAUNDERS, AND M. H. WRIGHT, *Some Theoretical Properties of an Augmented Lagrangian Merit Function*, Tech. Report SOL 86-6, Department of Operations Research, Stanford University, Stanford, CA, Apr. 1986.
- [96] ———, *User's Guide for NPSOL (Version 4.0): A Fortran Package for Nonlinear Programming*, Tech. Report SOL 86-2, Department of Operations Research, Stanford University, Stanford, CA, Jan. 1986.
- [97] ———, *A Schur-Complement Method for Sparse Quadratic Programming*, Tech. Report SOL 87-12, Department of Operations Research, Stanford University, Stanford, CA, Oct. 1987.
- [98] ———, *A Schur-Complement Method for Sparse Quadratic Programming*, in *Reliable Numerical Computation*, M. G. Cox and S. Hammarling, eds., Oxford University Press, Oxford, UK, 1990, pp. 113–138.
- [99] P. E. GILL, W. MURRAY, AND M. H. WRIGHT, *Practical Optimization*, Academic Press, London, 1981.
- [100] ———, *Numerical Linear Algebra and Optimization*, Addison–Wesley Publishing Company, Redwood City, CA, 1991.
- [101] G. B. GILYARD, *Development of a Real-Time Transport Performance Optimization Methodology*, Tech. Report NASA TM-4730, NASA Dryden Flight Research Center, Edwards, CA, Jan. 1996.
- [102] G. B. GILYARD, J. GEORGIE, AND J. S. BARNICKI, *Flight Test of an Adaptive Configuration Optimization System for Transport Aircraft*, Tech. Report NASA/TM-1999-206569, NASA Dryden Flight Research Center, Edwards, CA, Jan. 1999.
- [103] D. GOLDFARB, *A Family of Variable Metric Methods Derived by Variational Means*, *Mathematics of Computation*, 24 (1970), pp. 23–26.
- [104] N. I. M. GOULD, *On Practical Conditions for the Existence and Uniqueness of Solutions to the General Equality Quadratic Programming Problem*, *Mathematical Programming*, 32 (1985), pp. 90–99.
- [105] W. W. HAGER, *Rates of Convergence for Discrete Approximations to Unconstrained Optimal Control Problems*, *SIAM Journal on Numerical Analysis*, 13 (1976), pp. 449–472.
- [106] E. HAIRER, S. P. NORSETT, AND G. WANNER, *Solving Ordinary Differential Equations. I. Nonstiff Problems*, Springer-Verlag, New York, 1993.
- [107] E. HAIRER AND G. WANNER, *Solving Ordinary Differential Equations. II. Stiff and Differential-Algebraic Problems*, Springer-Verlag, New York, 1996.

- [108] S. M. HAMMES, *Optimization Test Problems*, Aerospace Technical Memorandum ATM 89(4464-06)-12, The Aerospace Corporation, 2350 E. El Segundo Blvd., El Segundo, CA 90245-4691, 1989.
- [109] C. R. HARGRAVES AND S. W. PARIS, *Direct Trajectory Optimization Using Non-linear Programming and Collocation*, AIAA Journal of Guidance, Control, and Dynamics, 10 (1987), pp. 338–342.
- [110] M. T. HEATH, *Numerical Methods for Large Sparse Linear Least Squares Problems*, SIAM Journal on Scientific and Statistical Computing, 5 (1984), pp. 497–513.
- [111] M. HEINKENSCHLOSS, *Projected Sequential Quadratic Programming Methods*, SIAM Journal on Optimization, 6 (1996), pp. 373–417.
- [112] A. L. HERMAN AND B. A. CONWAY, *Direct Optimization Using Collocation Based on High-Order Gauss-Lobatto Quadrature Rules*, AIAA Journal of Guidance, Control, and Dynamics, 19 (1996), pp. 592–599.
- [113] A. C. HINDMARSH, *ODEPACK, A Systematized Collection of ODE Solvers*, in Scientific Computing, R. S. Stepleman et al., eds., North-Holland, Amsterdam, 1983, pp. 55–64.
- [114] W. HOCK AND K. SCHITTKOWSKI, *Test Examples for Nonlinear Programming Codes*, Springer-Verlag, New York, 1981.
- [115] W. P. HUFFMAN, *An Analytic Propagation Method for Low Earth Orbits*, Internal Document, The Aerospace Corporation, 2350 E. El Segundo Blvd., El Segundo, CA 90245-4691, Nov. 1981.
- [116] P. C. HUGHES, *Spacecraft Attitude Dynamics*, Dover Publications, Inc., Mineola, NY, 2004.
- [117] D. H. JACOBSEN, M. M. LELE, AND J. L. SPEYER, *New Necessary Conditions of Optimality for Control Problems with State Variable Inequality Constraints*, Journal of Mathematical Analysis and Applications, 35 (1971), pp. 255–284.
- [118] L. JAY, *Convergence of a Class of Runge-Kutta Methods for Differential-Algebraic Systems of Index 2*, BIT, 33 (1993), pp. 137–150.
- [119] E. JUNG, S. LENHART, AND Z. FENG, *Optimal Control of Treatments in a Two-Strain Tuberculosis Model*, Discrete and Continuous Dynamical Systems—Series B, 2 (2002), pp. 479–482.
- [120] S. K. KAMESWARAN AND L. T. BIEGLER, *A Further Analysis of the Betts and Campbell Heat Problem*, Tech. Report, Chemical Engineering Department, Carnegie Mellon University, Pittsburgh, PA, 2004.
- [121] J. A. KECHICHIAN, *Equinoctial Orbit Elements: Application to Optimal Transfer Problems*, in AIAA/AAS Astrodynamics Specialist Conference, AIAA 90-2976, Portland, OR, Aug. 1990.

- [122] ———, *Trajectory Optimization with a Modified Set of Equinoctial Orbit Elements*, in AIAA/AAS Astrodynamics Specialist Conference, AAS 91-524, Durango, CO, Aug. 1991.
- [123] H. B. KELLER, *Numerical Methods for Two-Point Boundary Value Problems*, Waltham: Blaisdell, London, 1968.
- [124] C. T. KELLEY AND E. W. SACHS, *A Pointwise Quasi-Newton Method for Unconstrained Optimal Control Problems*, *Numerische Mathematik*, 55 (1989), pp. 159–176.
- [125] ———, *Pointwise Broyden Methods*, *SIAM Journal on Optimization*, 3 (1993), pp. 423–441.
- [126] H. W. KUHN AND A. W. TUCKER, *Non-linear Programming*, in Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability, University of California Press, Berkeley, CA, 1951, pp. 481–493.
- [127] C. L. LAWSON AND R. J. HANSON, *Solving Least Squares Problems*, Prentice–Hall, Englewood Cliffs, NJ, 1974.
- [128] U. LEDZEWICZ AND H. SCHÄTTLER, *Analysis of Optimal Controls for a Mathematical Model of Tumour Anti-angiogenesis*, *Optimal Control Applications and Methods*, 29 (2008), pp. 41–57.
- [129] T. LEE, D. DUNHAM, S. HSU, AND C. ROBERTS, *Large Inclination Change Using Lunar-Swingby Techniques*, in AIAA/AAS Astrodynamics Conference, AIAA 1988-4290, Minneapolis, MN, Aug. 1988.
- [130] D. B. LEINWEBER, *Efficient Reduced SQP Methods for the Optimization of Chemical Processes Described by Large Sparse DAE Models*, PhD thesis, Interdisziplinäres Zentrum für Wissenschaftliches Rechnen (IWR), Universität Heidelberg, 1998.
- [131] K. LEVENBERG, *A Method for the Solution of Certain Problems in Least Squares*, *Quarterly of Applied Mathematics*, 2 (1944), pp. 164–168.
- [132] A. L. MARTINS AND F. M. CATALANO, *Drag Optimization for Transport Aircraft Mission Adaptive Wing*, in 38th Aerospace Sciences Meeting and Exhibit, AIAA-2000-0648, Reno, NV, Jan. 2000.
- [133] H. MAURER AND D. AUGUSTIN, *Sensitivity Analysis and Real-Time Control of Parametric Optimal Control Problems Using Boundary Value Methods*, in Online Optimization of Large Scale Systems, M. Grötschel, S. O. Krumke, and J. Rambau, eds., Springer-Verlag, Berlin, 2001, pp. 17–55.
- [134] A. MIELE, T. WANG, AND W. W. MELVIN, *Optimal Abort Landing Trajectories in the Presence of Windshear*, *Journal of Optimization Theory and Applications*, 55 (1987), pp. 165–202.
- [135] F. R. MOULTON, *New Methods in Exterior Ballistics*, University of Chicago Press, Chicago, IL, 1926.

- [136] W. MURRAY AND M. H. WRIGHT, *Line Search Procedures for the Logarithmic Barrier Function*, SIAM Journal on Optimization, 4 (1994), pp. 229–246.
- [137] M. OKAMOTO AND K. HAYASHI, *Frequency Conversion Mechanism in Enzymatic Feedback Systems*, Journal of Theoretical Biology, 108 (1984), pp. 529–537.
- [138] M. OTTER AND S. TÜRK, *The DFVLR Models 1 and 2 of the Manutec r3 Robot*, DFVLR-Mitt. 88-3, Institut für Dynamik der Flugsysteme, Oberpfaffenhoffen, Germany, 1988.
- [139] H. J. PESCH, *A Practical Guide to the Solution of Real-Life Optimal Control Problems*, Control and Cybernetics, 23 (1994), pp. 7–60.
- [140] L. PETZOLD, *Differential/Algebraic Equations Are Not ODEs*, SIAM Journal on Scientific and Statistical Computing, 3 (1982), pp. 367–384.
- [141] ———, *A Description of DASSL: A Differential/Algebraic System Solver*, in Scientific Computing, R. S. Stepleman et al., eds., North-Holland, Amsterdam, 1983, pp. 65–68.
- [142] J. A. PIETZ, *Pseudospectral Collocation Methods for the Direct Transcription of Optimal Control Problems*, Master's thesis, Rice University, Houston, TX, Apr. 2003.
- [143] E. POLAK, *Computational Methods in Optimization*, Academic Press, New York, 1971.
- [144] ———, *Optimization: Algorithms and Consistent Approximations*, Springer-Verlag, New York, 1997.
- [145] L. S. PONTRYAGIN, *The Mathematical Theory of Optimal Processes*, Wiley-Interscience, New York, 1962.
- [146] M. J. D. POWELL, *Log Barrier Methods for Semi-infinite Programming Calculations*, Tech. Report DAMTP 1992/NA11, University of Cambridge, Department of Applied Mathematics and Theoretical Physics, Silver Street, Cambridge CB3 9EW, England, Dec. 1992.
- [147] A. V. RAO, *User's Manual for GPOCS Version 1.1, A MATLAB Implementation of the Gauss Pseudospectral Method for Solving Multiple-Phase Optimal Control Problems*, Tech. Report, University of Florida, Gainesville, FL, Aug. 2007.
- [148] A. V. RAO AND K. D. MEASE, *Eigenvector Approximate Dichotomic Basis Method for Solving Hyper-Sensitive Optimal Control Problems*, Optimal Control Applications and Methods, 20 (1999), pp. 59–77.
- [149] A. V. RAO, S. TANG, AND W. P. HALLMAN, *Numerical Optimization Study of Multiple-Pass Aeroassisted Orbital Transfer*, Optimal Control Applications and Methods, 23 (2002), pp. 215–238.
- [150] D. REDDING AND J. V. BREAKWELL, *Optimal Low-Thrust Transfers to Synchronous Orbit*, AIAA Journal of Guidance, Control, and Dynamics, 7 (1984), pp. 148–155.

- [151] R. T. ROCKAFELLAR, *The Multiplier Method of Hestenes and Powell Applied to Convex Programming*, Journal of Optimization Theory and Applications, 12 (1973), pp. 555–562.
- [152] G. SACHS AND K. LESCH, *Periodic Maximum Range Cruise with Singular Control*, AIAA Journal of Guidance, Control, and Dynamics, 16 (1993), pp. 790–793.
- [153] Y. SAKAWA, *Trajectory Planning of a Free-Flying Robot by Using the Optimal Control*, Optimal Control Applications and Methods, 20 (1999), pp. 235–248.
- [154] W. SCHIEHLEN, *Multibody Systems Handbook*, Springer-Verlag, Berlin, Heidelberg, 1990.
- [155] K. SCHITTKOWSKI, *Parameter Estimation in Dynamic Systems*, in Progress in Optimization, X. Yang, ed., Kluwer Academic Publishers, Dordrecht, The Netherlands, 2000, pp. 183–204.
- [156] H. SEYWALD AND E. M. CLIFF, *On the Existence of Touch Points for First-Order State Inequality Constraints*, Optimal Control Applications and Methods, 17 (1996), pp. 357–366.
- [157] L. F. SHAMPINE AND P. GAHINET, *Delay-Differential-Algebraic Equations in Control Theory*, Applied Numerical Analysis, 56 (2005), pp. 574–588.
- [158] L. F. SHAMPINE AND M. K. GORDON, *Computer Solution of Ordinary Differential Equations: The Initial Value Problem*, W. H. Freeman and Co., San Francisco, CA, 1975.
- [159] D. F. SHANNO, *Conditioning of Quasi-Newton Methods for Function Minimization*, Mathematics of Computation, 24 (1970), pp. 647–657.
- [160] J. L. SPEYER, *Periodic Optimal Flight*, AIAA Journal of Guidance, Control, and Dynamics, 19 (1996), pp. 745–755.
- [161] E. M. STANDISH, *JPL Planetary and Lunar Ephemerides, DE405/LE405*, Interoffice Memorandum IOM 312.F - 98 -048, Jet Propulsion Laboratory, Pasadena, CA, Aug. 1998.
- [162] D. STEWART AND M. ANITESCU, *Optimal Control of Systems with Discontinuous Differential Equations*, Tech. Report ANL/MCS-P1258-0605, Mathematics and Computer Science Division, Argonne National Laboratory, Argonne, IL, June 2005.
- [163] J. STOER AND R. BULIRSCH, *Introduction to Numerical Analysis*, Springer-Verlag, New York, Berlin, Heidelberg, 1980.
- [164] D. TABAK AND B. C. KUO, *Optimal Control by Mathematical Programming*, Prentice-Hall, Englewood Cliffs, NJ, 1971.
- [165] H. S. TSIEN AND R. C. EVANS, *Optimum Thrust Programming for a Sounding Rocket*, Journal of the American Rocket Society, 21 (1951), pp. 99–107.

- [166] R. J. VANDERBEI AND D. F. SHANNO, *An Interior-Point Method for Nonconvex Nonlinear Programming*, Computational Optimization and Applications, 13 (1999), pp. 231–252.
- [167] O. VON STRYK, *Numerical Solution of Optimal Control Problems by Direct Collocation*, in Optimal Control, R. Bulirsch, A. Miele, J. Stoer, and K. H. Well, eds., vol. 111 of International Series of Numerical Mathematics, 1993, Birkhäuser Verlag, Basel, pp. 129–143.
- [168] O. VON STRYK AND R. BULIRSCH, *Direct and Indirect Methods for Trajectory Optimization*, Annals of Operations Research, 37 (1992), pp. 357–373.
- [169] O. VON STRYK AND M. SCHLEMMER, *Optimal Control of the Industrial Robot Manutec r3*, in Computational Optimal Control, R. Bulirsch and D. Kraft, eds., vol. 115 of International Series of Numerical Mathematics, Birkhäuser Verlag, Basel, 1994, pp. 367–382.
- [170] T. V. VU, *Numerical Methods for Smooth Solutions of Ordinary Differential Equations*, Tech. Report UIUCDCS-R-83-1130, Department of Computer Science, University of Illinois, Urbana, IL, May 1983.
- [171] M. J. H. WALKER, B. IRELAND, AND J. OWENS, *A Set of Modified Equinoctial Orbit Elements*, Celestial Mechanics, 36 (1985), pp. 409–419.
- [172] R. S. WILSON AND K. C. HOWELL, *Trajectory Design in the Sun-Earth-Moon System Using Lunar Gravity Assists*, Journal of Spacecraft and Rockets, 35 (1998), pp. 191–198.
- [173] S. J. WRIGHT, *Primal-Dual Interior-Point Methods*, SIAM, Philadelphia, PA, 1997.
- [174] T. YEE AND J. A. KECHICHIAN, *On the Dynamic Modeling in Optimal Low-Thrust Orbit Transfer*, in AAS/AIAA Spaceflight Mechanics Meeting, AAS 92-177, Colorado Springs, CO, Feb. 1992.
- [175] K. P. ZONDERVAN, T. P. BAUER, J. T. BETTS, AND W. P. HUFFMAN, *Solving the Optimal Control Problem Using a Nonlinear Programming Technique Part 3: Optimal Shuttle Reentry Trajectories*, in Proceedings of the AIAA/AAS Astrodynamics Conference, AIAA-84-2039, Seattle, WA, Aug. 1984.
- [176] K. P. ZONDERVAN, L. J. WOOD, AND T. K. CAUGHY, *Optimal Low-Thrust, Three-Burn Transfers with Large Plane Changes*, The Journal of the Astronautical Sciences, 32 (1984), pp. 407–427.

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