Practical Methods for Optimal Control and Estimation Using Nonlinear Programming

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SECOND EDITION

John T. Betts

Society for Industrial and Applied Mathematics
Philadelphia

10 9 8 7 6 5 4 3 2 1

Library of Congress Cataloging-in-Publication Data

Betts, John T. 1943-

Practical methods for optimal control and estimation using nonlinear programming / John T. Betts. — 2nd ed.

p. cm. — (Advances in design and control)

Includes bibliographical references and index.

ISBN 978-0-898716-88-7

1. Control theory. 2. Mathematical optimization. 3. Nonlinear programming. I. Title.

QA402.3.B47 2009

629.8’312—dc22

2009025106

For Theon and Dorothy

He Inspired Creativity
She Cherished Education

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Preface

Solving an optimal control or estimation problem is not easy. Pieces of the puzzle are found scattered throughout many different disciplines. Furthermore, the focus of this book is on *practical methods*, that is, methods that I have found actually work! In fact everything described in this book has been implemented in production software and used to solve real optimal control problems. Although the reader should be proficient in advanced mathematics, no theorems are presented.

Traditionally, there are two major parts of a successful optimal control or optimal estimation solution technique. The first part is the "optimization" method. The second part is the "differential equation" method. When faced with an optimal control or estimation problem it is tempting to simply "paste" together packages for optimization and numerical integration. While naive approaches such as this may be moderately successful, the goal of this book is to suggest that there is a better way! The methods used to solve the differential equations and optimize the functions are intimately related.

The first two chapters of this book focus on the optimization part of the problem. In Chapter 1 the important concepts of nonlinear programming for small dense applications are introduced. Chapter 2 extends the presentation to problems which are both large and sparse. Chapters 3 and 4 address the differential equation part of the problem. Chapter 3 introduces relevant material in the numerical solution of differential (and differential-algebraic) equations. Methods for solving the optimal control problem are treated in some detail in Chapter 4. Throughout the book the interaction between optimization and integration is emphasized. Chapter 5 describes how to solve optimal estimation problems. Chapter 6 presents a collection of examples that illustrate the various concepts and techniques. Real world problems often require solving a sequence of optimal control and/or optimization problems, and Chapter 7 describes a collection of these "advanced applications."

While the book incorporates a great deal of new material not covered in *Practical Methods for Optimal Control Using Nonlinear Programming* [21], it does not cover everything. Many important topics are simply not discussed in order to keep the overall presentation concise and focused. The discussion is general and presents a unified approach to solving optimal estimation and control problems. Most of the examples are drawn from my experience in the aerospace industry. Examples have been solved using a particular implementation called $\mathbb{SOCS}$. I have tried to adhere to notational conventions from both optimization and control theory whenever possible. Also, I have attempted to use consistent notation throughout the book.

The material presented here represents the collective contributions of many people. The nonlinear programming material draws heavily on the work of John Dennis, Roger Fletcher, Phillip Gill, Sven Leyffer, Walter Murray, Michael Saunders, and Margaret Wright. The material on differential-algebraic equations (DAEs) is drawn from the work of Uri Ascher, Kathy Brenan, and Linda Petzold. Ray Spiteri graciously shared his classroom notes on DAEs. I was introduced to optimal control by Stephen Citron, and I routinely refer to the text by Bryson and Ho [54]. Over the past 20 years I have been fortunate to participate in workshops at Oberwolfach, Munich, Minneapolis, Victoria, Banff, Lausanne, Griefswald, Stockholm, and Fraser Island. I've benefited immensely simply by talking with Larry Biegler, Hans Georg Bock, Roland Bulirsch, Rainer Callies, Kurt Chudej, Tim Kelley, Bernd Kugelmann, Helmut Maurer, Rainer Mehlhorn, Angelo Miele, Hans Josef Pesch, Ekkehard Sachs, Gottfried Sachs, Roger Sargent, Volker Schulz, Mark Steinbach, Oskar von Stryk, and Klaus Well.

Three colleagues deserve special thanks. Interaction with Steve Campbell and his students has inspired many new results and interesting topics. Paul Frank has played a major role in the implementation and testing of the large, sparse nonlinear programming methods described. Bill Huffman, my coauthor for many publications and the $\mathbb{SOCS}$ software, has been an invaluable sounding board over the last two decades. Finally, I thank Jennifer for her patience and understanding during the preparation of this book.

John T. Betts