$x^2-6x+25=0$

$x=\frac{6\pm 8i}{2}$

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H10.1 $B_1=\begin{pmatrix}3&-4\\4&3\end{pmatrix}$ $\quad xI_2-B_1=\begin{pmatrix}x-3&4\\-4&x-3\end{pmatrix}$ $\quad \chi_{B_1}=x^2-6x+9+16=(x-3-4i)(x-3+4i)$

die Eigenwerte sind $\lambda_1=3+4i$ $\quad\lambda_2=3-4i$

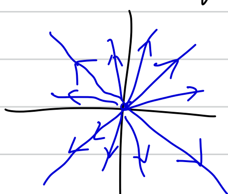
ein Eigenvektor $v_1$ zu $\lambda_1$ ist $\begin{pmatrix}-1\\i\end{pmatrix}$ weil $\begin{pmatrix}3&-4\\4&3\end{pmatrix}\begin{pmatrix}1\\-i\end{pmatrix}=\begin{pmatrix}3+4i\\4-3i\end{pmatrix}=(3+4i)\begin{pmatrix}1\\-i\end{pmatrix}$

ein Eigenvektor $v_2$ zu $\lambda_2$ ist $\begin{pmatrix}1\\i\end{pmatrix}$ weil $\begin{pmatrix}3&-4\\4&3\end{pmatrix}\begin{pmatrix}1\\i\end{pmatrix}=\begin{pmatrix}3-4i\\4+3i\end{pmatrix}=(3-4i)\begin{pmatrix}1\\i\end{pmatrix}$

$\begin{pmatrix}-1&1\\i&i\end{pmatrix}^{-1}=-\frac{1}{2i}\begin{pmatrix}i&-1\\-i&-1\end{pmatrix}=\begin{pmatrix}-\frac12&-\frac{i}{2}\\\frac12&-\frac{i}{2}\end{pmatrix}$ Also $B_1=\begin{pmatrix}-1&1\\i&i\end{pmatrix}\begin{pmatrix}3+4i&0\\0&3-4i\end{pmatrix}\begin{pmatrix}-\frac12&-\frac{i}{2}\\\frac12&-\frac{i}{2}\end{pmatrix}$

und damit $e^{tB_1}=\begin{pmatrix}-1&1\\i&i\end{pmatrix}\begin{pmatrix}e^{t(3+4i)}&0\\0&e^{t(3-4i)}\end{pmatrix}\begin{pmatrix}-\frac12&-\frac{i}{2}\\\frac12&-\frac{i}{2}\end{pmatrix}=e^{3t}\begin{pmatrix}-1&1\\i&i\end{pmatrix}\begin{pmatrix}e^{4i}&0\\0&e^{-4i}\end{pmatrix}\begin{pmatrix}-\frac12&-\frac{i}{2}\\\frac12&-\frac{i}{2}\end{pmatrix}$

$=e^{3t}\begin{pmatrix}\frac{e^{4i}+e^{-4i}}{2}&\frac{ie^{4i}-ie^{-4i}}{2}\\\frac{-ie^{4i}+ie^{-4i}}{2}&\frac{e^{4i}+e^{-4i}}{2}\end{pmatrix}=e^{3t}\begin{pmatrix}\cos(4t)&\cos(4t)\\-\cos(4t)&\cos(4t)\end{pmatrix}=e^{3t}\cos(4t)\begin{pmatrix}1&1\\-1&1\end{pmatrix}\Rightarrow\begin{pmatrix}x(t)\\y(t)\end{pmatrix}=e^{3t}\cos(4t)\begin{pmatrix}1&1\\-1&1\end{pmatrix}\begin{pmatrix}x_0\\y_0\end{pmatrix}$



$B_2=\begin{pmatrix}0&1\\-3&-4\end{pmatrix}$ $\quad xI_2-B_2=\begin{pmatrix}x&-1\\3&x+4\end{pmatrix}$ $\quad\chi_{B_2}=x^2+4x+3=(x+1)(x+3)$

$\Rightarrow\lambda_1=-1\quad\lambda_2=-3$

$\begin{pmatrix}0&1\\-3&-4\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}=\begin{pmatrix}-1\\1\end{pmatrix}=-\begin{pmatrix}1\\-1\end{pmatrix}$ also $\begin{pmatrix}1\\-1\end{pmatrix}$ ist Eigenvektor zu $\lambda_1$ $\quad\begin{pmatrix}1&1\\-1&-3\end{pmatrix}^{-1}=\frac{1}{-3+1}\begin{pmatrix}-3&-1\\1&1\end{pmatrix}=\begin{pmatrix}\frac32&\frac12\\-\frac12&-\frac12\end{pmatrix}$

$\begin{pmatrix}0&1\\-3&-4\end{pmatrix}\begin{pmatrix}1\\-3\end{pmatrix}=\begin{pmatrix}-3\\9\end{pmatrix}=-3\begin{pmatrix}1\\-3\end{pmatrix}$ also $\begin{pmatrix}1\\-3\end{pmatrix}$ ist Eigenvektor zu $\lambda_2$ $\quad$ Also $e^{tB_2}=\begin{pmatrix}1&1\\-1&-3\end{pmatrix}\begin{pmatrix}e^{-t}&0\\0&e^{-3t}\end{pmatrix}\begin{pmatrix}\frac32&\frac12\\-\frac12&-\frac12\end{pmatrix}$

$=\begin{pmatrix}\frac{3e^{-t}-e^{-3t}}{2}&\frac{e^{-t}-e^{-3t}}{2}\\\frac{-3e^{-t}+3e^{-3t}}{2}&\frac{-e^{-t}+3e^{-3t}}{2}\end{pmatrix}\Rightarrow\begin{pmatrix}x(t)\\y(t)\end{pmatrix}=\begin{pmatrix}\frac{3e^{-t}-e^{-3t}}{2}&\frac{e^{-t}-e^{-3t}}{2}\\\frac{-3e^{-t}+3e^{-3t}}{2}&\frac{-e^{-t}+3e^{-3t}}{2}\end{pmatrix}\begin{pmatrix}x_0\\y_0\end{pmatrix}$

$B_3=\begin{pmatrix}0&1\\-1&-2\end{pmatrix}$ $\quad xI_2-B_3=\begin{pmatrix}x&-1\\1&x+2\end{pmatrix}$ $\quad\chi_{B_3}=x^2+2x+1=(x+1)^2$ also einzige Eigenwert $\lambda=-1$

Wir wählen irgendeine Vektor $v_1$ außer $E_{-1}$: $\begin{pmatrix}1\\0\end{pmatrix}$. $\quad v_2=Av_1+v_1=\begin{pmatrix}0\\-1\end{pmatrix}+\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}1\\-1\end{pmatrix}$

$\begin{pmatrix}1&1\\-1&0\end{pmatrix}^{-1}=\begin{pmatrix}0&-1\\1&1\end{pmatrix}$ $\quad$ Also $B_3=\begin{pmatrix}1&1\\-1&0\end{pmatrix}\begin{pmatrix}-1&1\\0&-1\end{pmatrix}\begin{pmatrix}0&-1\\1&1\end{pmatrix}\Rightarrow e^{tB_3}=\begin{pmatrix}1&1\\-1&0\end{pmatrix}e^{-t}\begin{pmatrix}1&t\\0&1\end{pmatrix}$

$=\begin{pmatrix}e^{-t}&te^{-t}+e^{-t}\\-e^{-t}&-te^{-t}\end{pmatrix}$

$\Rightarrow\begin{pmatrix}x(t)\\y(t)\end{pmatrix}=\begin{pmatrix}e^{-t}&te^{-t}+e^{-t}\\-e^{-t}&-te^{-t}\end{pmatrix}\begin{pmatrix}x_0\\y_0\end{pmatrix}$

H10.2 (a) mit $y_1=z$, $y_2=z'$, $(y_0)_1=z(0)=0$ $\quad(y_0)_2=z'(0)=0\Rightarrow y'=\underbrace{\begin{pmatrix}0&1\\-1&0\end{pmatrix}}_{=:A}y+\begin{pmatrix}0\\F_0\cos(t)\end{pmatrix}$

(b) $y(t)=e^{At}\left(y_0+\int_0^t\underbrace{e^{-As}\begin{pmatrix}0\\F_0\cos(s)\end{pmatrix}}ds\right)=\begin{pmatrix}\cos(t)&\sin(t)\\-\sin(t)&\cos(t)\end{pmatrix}\left(y_0+F_0\begin{pmatrix}-\frac{\sin^2(t)}{2}\\\frac{t}{2}+\frac14\sin(2t)\end{pmatrix}\right)$

$=\begin{pmatrix}\cos(s)&-\sin(s)\\\sin(s)&\cos(s)\end{pmatrix}\begin{pmatrix}0\\F_0\cos(s)\end{pmatrix}=\begin{pmatrix}-F_0\sin(s)\cos(s)\\F_0\cos^2(s)\end{pmatrix}$

H10.3 (a) Das ist genau die Lösung von $\begin{pmatrix}y\\-cy-x-x^3\end{pmatrix}=0\Rightarrow y=0$, also $x(x^2+1)=0\Rightarrow x=0\quad\forall t\in[0,\infty)$

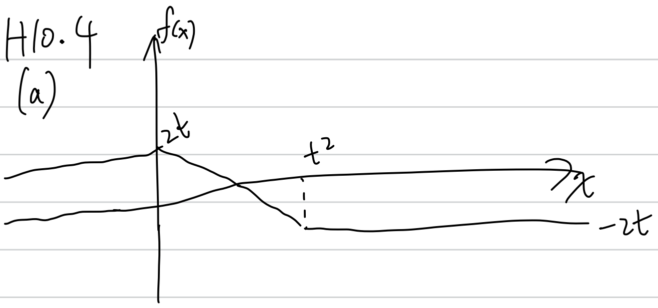
(b) $\frac{dH}{dt}=y\cdot y'+x\cdot x'+x^3\cdot x'=-cy^2-xy-x^3y+xy+x^3y=-cy^2\le 0$ weil $c\ge0, y^2\le0$

(c) Sei $H(x(0),y(0))=\frac12x(0)^2+\frac12y(0)^2+\frac14x(0)^4<\frac{\varepsilon^2}{2}=\delta\xRightarrow{(b)}\frac12x(t)^2+\frac12y(t)^2<\frac{\varepsilon^2}{2}\ \forall t\in[0,\infty)\Rightarrow\sqrt{x(t)^2+y(t)^2}<\varepsilon\ \forall t$

$\Rightarrow|x(t)+y(t)|<\varepsilon\ \forall t\in[0,\infty)$

H10.4

(a)




(b) $\frac{\partial f}{\partial t}(t,x) = \begin{cases} 2 & \text{für } x<0 \quad \text{stetig} \\ 2 + \frac{4x}{t^2}\cdot x' & \text{für } 0 \le x < t^2 \quad \text{stetig} \\ -2 & \text{für } x \ge t^2 \quad \text{stetig} \end{cases}$

$2 + \frac{4x\cdot x'(t)}{t^2} \to 2$ für $x \to 0$ $\forall t \Rightarrow f$ stetig für $x=0$ $\forall t$

$2 + \frac{4x\cdot x'(t)}{t^2} \to 2+4x'$ für $x \to t^2$

(c) $x'(t) = \frac{2}{3}t$ $\quad x(t) = \frac{1}{3}t^2 \Rightarrow x(t) \ge 0$ und $x(t) < t^2$ $\forall t \neq 0 \Rightarrow x'(t) = 2t - \frac{4x}{t} = 2t - \frac{\frac{4}{3}t^2}{t} = \frac{2}{3}t$

für $t=0$ ist $x(0)=0$ und $x'(0) = 0 = 2\cdot t = 2\cdot 0$

(d)