Nomenclature

$\overline{\mathbf{F}}$ = resultant force vector in body-fixed coordinate system
F_i = ith applied force vector
F_m = force produced by the moving mass
I = inertia matrix relative to center of mass in body-fixed coordinate system
$\hat{\mathbf{I}}$ = vector constructed from elements of I, $[I_x \quad I_y \quad I_z \quad -I_{xy} \quad -I_{xz} \quad -I_{yz}]$
J = quadratic cost function in batch least squares estimation
B = total applied torque vector
M = mass of the body
m = size of the moving mass
D = upper triangular matrix in decomposition of F
$\mathbf{R}$ = position vector in body-fixed coordinate system from origin to center of mass
$\mathbf{R}_m$ = contribution of moving mass to the location of the center of mass
$\mathbf{R}_M$ = contribution of body to the location of the center of mass
r_i = position vector from origin of body-fixed coordinate system to center of mass
r_M = location of center of body in body-fixed coordinate system
r_m = location of center of moving mass in body-fixed coordinate system
T_r = rotational kinetic energy
Y = output vector from n measurements
? = parameter vector
$?_{LS}$ = least squares estimation of parameter vector
$?_i$ = position vector from center of mass to point of application of F_i
F = regressor matrix obtained after n measurements
? = angular rate vector in body-fixed coordinate system

Background Information

The papers discussed below are concerned with on-orbit estimation algorithms of the dynamic parameters of a spacecraft. Dynamic parameters include the moments of inertia, thruster parameters, mass, etc.[1] The most important role of dynamic parameters is in the backup attitude control system which may be used in case of primary sensor failure.

The attitude control subsystem is of the utmost importance for success of a spacecraft mission. Many of the other subsystems are dependent upon the accuracy of the orientation of the spacecraft. The thermal and power budgets are heavily dependent upon the trajectory and orientation. Precise control of the attitude is also necessary for scientific objectives. Continued success of a mission even after primary sensor failure relies on accurate knowledge of the parameters in Euler's equations. Further motivation for on-orbit estimation of parameters is the increasing complexity of spacecraft. The space station has parts added to it, which changes the mass properties. Also, an extra-vehicular activity astronaut maneuvering while manipulating massive components requires knowledge of the inertia matrix.[3]

Many different models and algorithms have been developed to solve the problem of parameter estimation. Each solution has a unique method, but most employ the use of rate and attitude sensors. A conservation of energy method is used by Tanygin and Williams[3] to relate the inertia and angular velocity during a maneuver. Lee and Wertz[4] use the principal of conservation of angular momentum to determine the inertia tensor of the Cassini spacecraft. Once the dynamic equations are solved, a numerical technique is often necessary to optimize the answer. Extended Kalman filters, batch least squares estimation, or other cost function minimization techniques are used in estimating parameters.[1,5]

A paper by Mark Psiaki[2] presents an approach based on satisfying the Euler equation. The unknowns are constituted by the parameters being estimated while the attitude and rates are known. Linear and non-linear optimization techniques are used in conjunction with each other to yield the best guess at the parameters. Recursive linear techniques are used to obtain the estimates of the error. The iterative non-linear method is used to estimate the parameters. The physical model of the system includes gravity gradient torques, magnetic dipole moments, and an inertial impulse due to unmodeled torques. Psiaki's model is more complete than the others, but along with the completeness comes complexity.

A simpler approach is taken by Tanygin and Williams.[3] The equation of motion (equation 1) is used as the physical model.

$$I\dot{w} + w \times Iw = M + \sum_i r_i \times F_i \tag{1}$$

Motion is excited by applying external torques, *M*, and forces, *F*. The equation must be manipulated into a form suitable for a batch least squares estimation. The standard form consists of a regressor matrix, F, a parameter vector, ?, and the output vector, Y (equation 2).

$$\Phi q = Y \tag{2}$$

The time rate of change of rotational kinetic energy involves the inertia matrix and is used to convert the equations of motion into the necessary form. A vector of the unknown parameters, inertia matrix and location of the center of mass, is formed. A batch least squares estimation is used to minimize the quadratic cost function based on all the measurements available. Noise and disturbances are not estimated in this method resulting in a simpler, less reliable result. An alternative cost function is suggested by Clemen (equation 3), where y is the measured value, $\hat{y}$ is the simulated value, and $\overline{y}$ is the mean of the measured values.[1]

$$J = \frac{\sum_{i=1}^{N}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{N}(y_i - \overline{y}_i)^2} \tag{3}$$

An example of actual parameter estimation was done on-board the Cassini spacecraft which will arrive at Saturn in July, 2004. The inertia tensor is used by the attitude-control fault protection algorithms, attitude estimator, thruster vector control algorithms, and the reaction wheel actuator.[2] Prior to launch an estimation of the inertia tensor was made. The inertia of each component was determined and the location of the system center of mass was estimated. The overall system inertia tensor was then found to be $\begin{bmatrix} 8810.8 & -136.8 & 115.3 \\ -136.8 & 8157.3 & 156.4 \\ 115.3 & 156.4 & 4721.8 \end{bmatrix}$ kg-m^2.[2] An on-board algorithm involving the conservation of angular momentum was used to make further estimations of the inertia tensor. A maneuver consisting of a Y-axis slew, followed by a X-axis slew, another Y-axis slew, a Z-axis slew, and finally another Y-axis slew was performed. The angular rates, reaction wheel spin rates, quaternion, reaction wheel inertias, and location of the reaction wheels are all assumed to be accurately known. Equating the initial angular momentum to the current angular momentum at each time step provides an explicit equation for the inertia tensor. The angular momentum is conserved by neglecting the effects of external torques for the duration of the maneuver. The resulting estimate for the inertia tensor was $\begin{bmatrix} 8655.2 & -144 & 132.1 \\ -144 & 7922.7 & 192.1 \\ 132.1 & 192.1 & 4586.2 \end{bmatrix}$ kg-m^2.[2]

Many of the algorithms are developed for space missions; however, the application to spacecraft simulators is of interest. Spacecraft simulators are typically used by small satellite builders. Small satellite programs are limited in time, financial, and manpower resources.[6] Simulators are used to test the attitude determination and control system before sending the satellite into orbit. The size and shape of spacecraft simulators limit the possibilities of directly determining the mass properties. Therefore, similar methods as those used for on-board parameter estimation must be used on simulators. The major difference between on-board and simulator estimations is the small gravity effect due to the inevitable misalignment of the center of mass with the ball center.[6] An algorithm based solely on rate gyro measurements was developed by Kim and Lee[6]. The conservation of angular momentum is used in the same way as the method developed by Lee and Wertz. However, the gravity effect must be accounted for since it is an external torque. A function is developed (equation 4) to allow for the external torque, T_g.

$$f = h - h_0 + \int (w \times h - T_g) \tag{4}$$

Imbedded within the torque are the Euler angles between the body frame and inertial frame. If attitude sensors are not available, the rate data must be integrated to yield values for the angles. The numerical integration to find the Euler angles and f leads to erroneous results.[6]

Many techniques have been developed for on-orbit parameter estimation. There are advantages and disadvantages associated with each algorithm. The complete models are able to provide better estimates, but at the cost of more computation time. Computation time onboard spacecraft is typically limited. The simpler algorithms are able to provide useful data if information such as the attitude *and* rates are known. Parameter estimation is limited for spacecraft simulators due to the lack of attitude sensors. The technique chosen for on-orbit parameter estimation is unique for each spacecraft. The choice should be based upon the available sensors, computation time, and accuracy needed.

Abstract

This paper considers the problem of estimating the mass properties of a rotating rigid body, namely an air-bearing spacecraft simulator. These properties include the inertia matrix and location of the center of mass. An estimation procedure involving a least squares fit will be used. The equations which are derived will be tested using simulated data from the integration of the equation of motion. Several different input scenarios will be explored.

Introduction

Spacecraft simulators have become an important part of the design process. The simulators are used to test the attitude determination and control system (ADCS). The identification of problems can be done on the ground, before millions of dollars are spent to place the spacecraft in orbit. The spacecraft simulators modeled in this paper are air-bearing systems. A spherical ball is placed in a hemispherical cup which acts as a spherical air table. Compressed air is pumped through the cup to support the air bearing and the attached equipment. Knowledge of mass properties such as the inertia matrix and location of center of mass are necessary for the testing of the ADCS. The properties are always present as parameters in the equations of motion. The size and shape of the simulators often prohibit the direct measurement of these properties. Determination of these properties by another means is therefore necessary.

Algorithms have been developed for on-board estimation of mass properties. These algorithms observe the effect of external torques on the attitude and angular velocity. Knowledge of the strength of the external torques, along with the measurements of attitude and angular rates, leaves only the mass properties as unknowns in the equations of motion. The algorithm for parameter estimation developed by Tanygin and Williams[3] is outlined in the analysis and estimation sections of the paper. The algorithm was also extended to include the situation of time variant mass properties.

Analysis

The spacecraft simulator is modeled as a rigid body. The appropriate rotational equation of motion is given as equation 1.

$$I\dot{w} + w \times Iw = B + \sum_{i} r_i \times F_i \tag{1}$$

Mass property estimation requires a different form in which the elements of the inertia matrix and the location of the center of mass form a vector of unknown parameters. The external torques and measured angular rates are combined to form a regressor matrix and output vector. The motivation for forming the vectors and matrix is to fit the equation of motion into the standard regression model (equation 2).

$$\Phi q = Y \tag{2}$$

The kth row of F, f_k^T, and kth element of Y, y_k, are formed from the data obtained during the kth measurement. Standard batch least squares estimation (LSE) is used to find an estimation of the parameter vector, $\hat{q}_{LS}$ (equation 4). The LSE determines the value of the parameter vector that minimizes the quadratic cost function, J (equation 3), based on all the measurements.

$$J = \sum_{k} \left(y_k - f_k^T \hat{q}_{LS} \right)^2 \tag{3}$$

$$\hat{q}_{LS} = \left(D^T D \right)^{-1} \Phi^T Y \tag{4}$$

The estimated parameter vector is not necessarily a good approximation of the real parameter vector. Sufficient excitation of the system must occur for the method to provide useful answers. The system is not properly conditioned, or properly excited, if the regressor matrix is singular. The manipulation of the equation of motion into the standard form is introduced in the next section.

Estimation Method

The elements of the inertia matrix as well as the position vector to the center of mass must be extracted to form the parameter vector. The nonlinear cross product involving the inertia matrix causes difficulty in the extraction. Multiplying equation 1 by the transpose of the angular rate vector eliminates the nonlinear term.

$$w^T\left(I\dot{w}+w\times Iw\right)=w^T\left(B+\sum_i \mathbf{r}_i\times F_i\right)\Rightarrow w^T I\dot{w}=w^T\left(B+\sum_i \mathbf{r}_i\times F_i\right) \tag{5}$$

The term on the left hand side of equation 5 is the time rate of change of the rotational kinetic energy.

$$\dot{T}_r=\frac{d}{dt}\left(\frac{1}{2}w^T Iw\right)=\frac{1}{2}\dot{w}^T Iw+\frac{1}{2}w^T I\dot{w}=w^T I\dot{w} \tag{6}$$

Integration of equation 6 eliminates the need to measure $\dot{w}$. In equation 7, T_{r0} is the kinetic energy at t=0.

$$\int_{to}^{t}\left(w^T I\dot{w}\right)dt=\frac{1}{2}w^T Iw\Big|_t-T_{r0} \tag{7}$$

The next step is to form a vector consisting of the elements of the inertia matrix, $\hat{\mathbf{I}}$.

$$\frac{1}{2}w^T Iw\Big|_t=\frac{1}{2}\Omega_I^T(t)\hat{\mathbf{I}} \tag{8}$$

where

$$\Omega_I^T(t)=\left[w_x^2\quad w_y^2\quad w_z^2\quad 2w_xw_y\quad 2w_xw_z\quad 2w_yw_z\right]_t \tag{9}$$

Combining equations 5-9 yields equation 10

$$\frac{1}{2}\Omega_I^T(t)\hat{\mathbf{I}} = \int_{to}^{t} w^T\left(B + \sum_i r_i \times F_i\right)dt + T_{r0} \tag{10}$$

The equation of motion is now in the standard form of the regression model where the elements of the matrices are defined as follows

$$f_k^T = \frac{1}{2}\Omega_I^T(t) \tag{11}$$

$$y_k = \int_{to}^{t} w^T\left(B + \sum_i r_i \times F_i\right)dt + T_{r0} \tag{12}$$

$$q = \hat{\mathbf{I}} \tag{13}$$

Equation 10 is only suitable for estimating the moments of inertia. Further work must be done if the location of the center of mass is also unknown.

The vector from the origin of the body-fixed coordinate system to the center of mass, **R**, must be introduced. The geometric relationship

$$r_i = r_i - \mathbf{R} \tag{14}$$

is used with equation 10 to introduce **R** as an unknown.

$$\frac{1}{2}\Omega_I^T(t)\hat{\mathbf{I}} + \int_{to}^{t} w^T(\mathbf{R} \times \overline{\mathbf{F}})dt = \int_{to}^{t} w^T\left(B + \sum_i r_i \times F_i\right)dt + T_{r0} \tag{15}$$

The integrand on the left hand side of equation 12 can be manipulated to remove **R** from the cross product.

$$w^T(\mathbf{R} \times \overline{\mathbf{F}}) = (\overline{\mathbf{F}} \times w)^T \mathbf{R} \tag{16}$$

Also, **R** can be removed from the integral if it is assumed constant.

$$\frac{1}{2}\Omega_I^T(t)\hat{\mathbf{I}} + \Omega_F^T(t, t_0)\mathbf{R} = \int_{to}^{t} w^T\left(B + \sum_i r_i \times F_i\right)dt + T_{r0} \tag{17}$$

where

$$\Omega_F^T(t,t_0) = \int_{to}^{t} \left(\overline{\mathbf{F}} \times w\right)^T dt \tag{18}$$

Equation 17 can be used for simultaneous estimation of both the inertia matrix and the location of the center of mass. The elements of the regression model matrices are defined as follows

$$f_k^T = \left[\frac{1}{2}\Omega_I^T(t) \quad \Omega_F^T(t,t_0)\right] \tag{19}$$

$$y_k = \int_{to}^{t} w^T\left(B + \sum_i r_i \times F_i\right)dt + T_{r0} \tag{20}$$

$$q = \left[\hat{\mathbf{I}}^T \quad \mathbf{R}^T\right]^T \tag{21}$$

The assumption that **R** remains constant is valid if there is no moving mass on the body. Moving mass causes the moments and products of inertia to change as well as the location of the center of mass. The appropriate form of the equation of motion is developed for the case of a moving mass as follows. The position vector **R** is separated into two components. The first is the contribution of the body to the location of the center of mass, $\mathbf{R}_M$, which is constant. The second component of **R** is the contribution of the moving mass, $\mathbf{R}_m$, which varies with time. The inertia vector, $\hat{\mathbf{I}}$, must also be split into two components in the same way as **R** was separated. The contribution to the inertia vector from the body is defined as $\mathbf{I}_M$. The variable contribution of the moving mass is defined as $\mathbf{I}_m$.

$$\mathbf{R} = \mathbf{R}_M + \mathbf{R}_m \tag{22}$$

$$\hat{\mathbf{I}} = \mathbf{I}_M + \mathbf{I}_m \tag{23}$$

where

$$\mathbf{R}_M = \frac{Mr_M}{M+m} \tag{24a}$$

$$\mathbf{R}_m = \frac{mr_m}{M+m} \tag{24b}$$

Equation 15 can now be rewritten as

$$\frac{1}{2}\Omega_I^T(t)(\mathbf{I}_M + \mathbf{I}_m(t)) + \int_{to}^{t} (\overline{\mathbf{F}} \times w)^T \mathbf{R}_M \, dt + \int_{to}^{t} (\overline{\mathbf{F}} \times w)^T \mathbf{R}_m(t) \, dt = \int_{to}^{t} w^T \left(B + \sum_i r_i \times F_i \right) dt + T_{r0} \tag{25}$$

The integral containing $\mathbf{R}_M$ on the left hand side of equation 24 can be treated in the same manner as before, since $\mathbf{R}_M$ is constant. The integral containing $\mathbf{R}_m$ must be moved to the right hand side and integrated with time. The term containing $\mathbf{I}_m$ must also be moved to the right hand side and integrated with time. The corresponding definitions of the elements of the regression model matrices are as follows

$$f_k^T = \left[\frac{1}{2}\Omega_I^T(t) \quad \Omega_F^T(t, t_0) \right] \tag{26}$$

where

$$\Omega_F^T(t, t_0) = \frac{M}{M+m} \int_{to}^{t} (\overline{\mathbf{F}} \times \mathbf{w})^T \, dt \tag{27}$$

$$y_k = \int_{to}^{t} \omega^T \left(B + \sum_i r_i \times F_i \right) d\tau - \frac{m}{M+m} \int_{to}^{t} \sum_i (F_i \times \omega)^T r_m \, d\tau + T_{r0} - \frac{1}{2}\Omega_I^T \mathbf{I}_m(t) \tag{28}$$

$$q = \left[\mathbf{I}_M^T \quad r_M{}^T \right]^T \tag{29}$$

Simulation

Simulation of the estimation of mass properties requires integration of the equation of motion. The inertia matrix and location of the center of mass are assumed known and a disturbing torque is applied to the system. Equation 1 is numerically integrated for the duration of the excitation. The solution to the system is the angular rate history. Noise is then added to the solution before the parameter estimation algorithm is applied. A comparison between the input and output mass properties can then be made. The disturbing torque can be produced by any of the actuators available on the spacecraft simulator. However, the equation of motion developed in this paper only applies to the case of thrusters or linear actuators. Momentum wheels would require use of the gyroscopic equation.

Noise

Sensor noise must be addressed in estimation of the mass properties. The regressor matrix of equation 2 is always properly conditioned in the presence of noise. However, the excitation may not be sufficient for the true regressor matrix to be properly conditioned. The noise prevents the algorithm from settling to the true optimal solution.[3] Therefore, the applied external torques must not cause a noise-free singular regressor matrix.

System Model

Actuators available on spacecraft simulators were modeled and simulated. The origin of the body fixed coordinate system was placed at the center of rotation of the simulator. A 1-2-3 Euler angle sequence was used to determine the rotation matrix from the body frame to the inertial frame. The two frames are aligned at t=0 for each case.

For each case the initial angular velocity was set equal to zero, leading to zero initial rotational kinetic energy. Trapezoidal approximations of the integrals on the right hand side of the regression matrix element definition equations were made. The inevitable misalignment of the center of mass with the center of rotation of the spacecraft simulator results in a torque about the center of mass. A force through the center of rotation, which is equal and opposite to the weight of the simulator, was modeled for each case.

Excitations for Mass Property Estimation

Case 1

The first actuators used for excitation are linear actuators. Linear actuators have masses which can traverse along one dimension. The movement of the mass affects the mass properties of the system. The moment of inertia and location of the center of mass vary as the mass moves along the traverse. The variations are proportional to the ratio of the moving mass to the body mass. The torque supplied by these actuators is the cross product between the position vector and the weight of the mass. A limitation of linear actuators is that the force is always in the inertial z direction. The moving mass was modeled as a point mass. The mass of the body was assumed to be known.

Case 2

The next actuators modeled are thrusters. Thrusters are able to provide body-fixed forces in any direction. The torque supplied is equal to the cross product between the position vector and the direction of the force. Mass properties variations can be neglected while using thrusters if the duration of the excitation is short.

Estimation of the Mass Properties of an Air-Bearing Spacecraft Simulator

The mass properties being estimated are the inertia matrix and the location of the center of mass. Both of these properties depend on the configuration of the spacecraft simulator. Different configurations are often used on simulators depending on what simulation is being conducted. Alignment of the center of mass with the center of rotation is a desirable condition due to the simplified dynamics of the system. The linear actuator masses may be moved to produce the alignment after the location of the center of mass is determined.

Linear Actuators

The linear actuator model was tested. A schematic of the location of the linear actuator is given in figure 1. The mass on the linear actuator, m, was taken to be 1kg while the body mass, M, was set equal to 100 kg. The mass started at one end of the linear actuator and traversed to the other end at a constant speed in a period of 10 sec. The length of the traverse was 1m. The distance in the x-direction to the linear actuator, from the center of mass, was 0.5 m. The inertia matrix was chosen to show the ability of the algorithm to estimate asymmetric bodies. All of the inputs listed above were chosen to properly excite the system, however, they are all generic and do not necessarily represent a real simulator. The input and output parameter values along with the associated error are given in table 1. The convergence of the estimated values of the moments of inertia can be seen in figure 2.

Input:	$I = \begin{bmatrix} 6 & 0.5 & 1 \\ 0.5 & 7 & 0.2 \\ 1 & 0.2 & 8 \end{bmatrix} kg \cdot m^2$ $\mathbf{R} = [0 \quad 0.08 \quad 0]^T m$	
Output:	$I = \begin{bmatrix} 5.91 & 0.51 & 1.11 \\ 0.51 & 6.86 & 0.23 \\ 1.11 & 0.23 & 8.14 \end{bmatrix} kg \cdot m^2$ $\mathbf{R} = [-0.00 \quad 0.079 \quad 0.00]^T m$	$I = \begin{bmatrix} 1.48 & 1.96 & 11.2 \\ 1.96 & 2.07 & 16.67 \\ 11.2 & 16.67 & 1.72 \end{bmatrix} \%$ $\mathbf{R} = [0.00 \quad 1.28 \quad 0.00]^T \%$
Output (.1% noise):	$I = \begin{bmatrix} 6.05 & 0.80 & 1.70 \\ 0.80 & 7.45 & 0.46 \\ 1.70 & 0.46 & 9.41 \end{bmatrix} kg \cdot m^2$ $\mathbf{R} = [-0.004 \quad 0.078 \quad 0.00]^T m$	$I = \begin{bmatrix} 0.89 & 60.4 & 70.1 \\ 60.4 & 6.41 & 129 \\ 70.1 & 129 & 17.62 \end{bmatrix} \%$ $\mathbf{R} = [0.00 \quad 2.9 \quad 0.00]^T \%$

Table 1 Input and output mass properties using linear actuators

The linear actuator excitation was able to provide useful results for the moments of inertia and the position of the center of mass. The products of inertia were not estimated very well, especially when noise was added to the system. The limitation of the direction of the force could have led to the inability to accurately predict the products of inertia.

Thrusters

The thruster model was tested. The thrusters were taken to be located at the coordinate (.5,.5,0). A force in the negative z direction equal to 10 N was supplied by the thrusters. The same inertia matrix was chosen as was used in the linear actuator case. Again, the inputs listed above were chosen to properly excite the system, however, they are all generic and do not necessarily represent a real simulator. The output parameter values along with the associated error are given in table 2.

Output:

$$I = \begin{bmatrix} 5.92 & 0.49 & 1.06 \\ 0.49 & 6.84 & 0.22 \\ 1.06 & 0.22 & 8.04 \end{bmatrix} kg \cdot m^2 \qquad I = \begin{bmatrix} 1.32 & 2.02 & 5.59 \\ 2.02 & 2.25 & 7.92 \\ 5.59 & 7.92 & 0.49 \end{bmatrix} \%$$

$$\mathbf{R} = \begin{bmatrix} -0.00 & 0.080 & 0.00 \end{bmatrix}^T m \qquad \mathbf{R} = \begin{bmatrix} 0.00 & 0.40 & 0.00 \end{bmatrix}^T \%$$

Output (.1% noise)

$$I = \begin{bmatrix} 6.07 & 0.60 & 1.44 \\ 0.60 & 7.12 & 0.27 \\ 1.44 & 0.27 & 8.94 \end{bmatrix} kg \cdot m^2 \qquad I = \begin{bmatrix} 1.15 & 19.8 & 43.9 \\ 19.8 & 1.77 & 35.65 \\ 43.9 & 35.65 & 11.73 \end{bmatrix} \%$$

$$\mathbf{R} = \begin{bmatrix} -0.00 & 0.079 & 0.00 \end{bmatrix}^T m \qquad \mathbf{R} = \begin{bmatrix} 0.00 & 0.72 & 0.00 \end{bmatrix}^T \%$$

Table 2 Output mass properties using thrusters

The thruster excitation was also able to provide useful results for the moments of inertia and the position of the center of mass. The estimated products of inertia were more accurate then in the linear actuator system. The simplicity of the model, no time-variant mass properties, could have added to the better estimation. Having the thrusters allowed for a greater range of excitation. The forces could be applied in more than one direction, resulting in more accurate estimations. The different inputs resulted in similar results as those shown above. The error of the estimation grew by a factor of about 10 for the moments of inertia about the unexcited axes when noise was added. The estimation for the moments of inertia about the excited axes remained just as accurate when noise was present. The force directed in the negative *z* direction was chosen to show a direct comparison with the linear actuator case.

Conclusions

The algorithm developed to estimate the mass properties of a spacecraft simulator produces useable results when no noise is present, using either thrusters or linear actuators. The moment of inertia about the excitation axes could still be accurately estimated in the presence of noise. The accuracy of the estimation for all other parameters is drastically reduced when noise is added to the system. The thruster system is able to provide slightly better results than the linear actuator system for a number of reasons. The thruster model is a simpler, time-invariant mass property system. All of the axes can be excited using a series of tests in order to obtain information about all three moments of inertia. The linear actuators are limited to sufficient excitation about only two axes. Further contribution to the error includes the numerical integration of the angular rates to obtain the attitude. Implementation of attitude sensors could mitigate this source of error. Absolute attitude knowledge was not modeled since attitude sensors are not currently available on the Virginia Tech spacecraft simulators. Error!

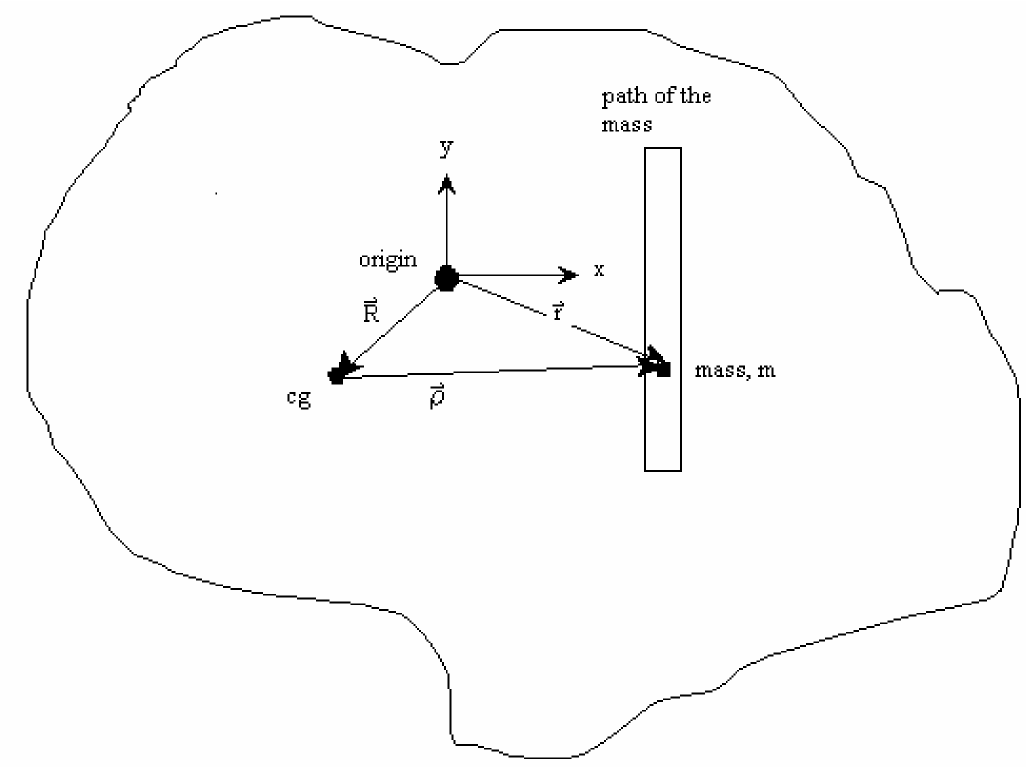

Figure 1 Schematic of the linear actuator system

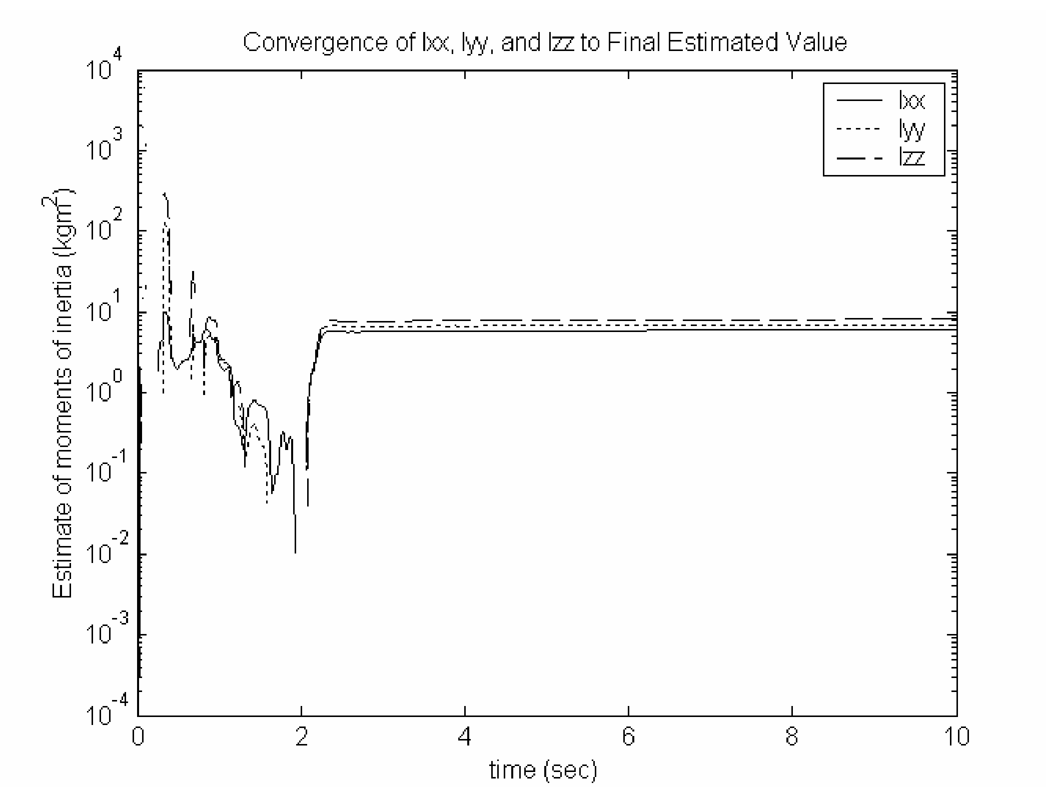

Figure 2 Convergence for the linear actuator system

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