

Relation $W = \{A, B, C, D, E, F, G, I, J, K, L, M, N, P, Q, R, S, T\}$

$F = \{$
 $AB \rightarrow C$
 $A \rightarrow DEFG$
 $E \rightarrow G$
 $F \rightarrow DJ$
 $G \rightarrow DI$
 $D \rightarrow KL$
 $D \rightarrow M$
 $DM \rightarrow NP$
 $L \rightarrow D$
 $PQR \rightarrow ST$
 $PR \rightarrow S$
 $\}$

1. Determine a candidate key

Step 1: determine a super key

Selecting all attributes that appear on the left hand side but not on the right hand side, we have $\{ABQR\}$

Calculate $\{ABQR\}^+ = \{ABQR CDEF GHIK LMNP ST\}$

If the closure contains all attributes, it is a super key.

Step 2: check if the super key is minimal

Remove one attribute a time and then recalculate the closure

$\{BQR\}^+$ doesn't contain A

$\{AQR\}^+$ doesn't contain B

$\{ABR\}^+$ doesn't contain Q

$\{ABQ\}^+$ doesn't contain R

If no attribute can be removed from the super key, it is minimal and is a candidate key.

2. Calculating the minimal cover

Step 1: Every right hand side has a single attribute

$F = \{$
 $A \rightarrow D \quad A \rightarrow E \quad A \rightarrow F \quad A \rightarrow G$
 $E \rightarrow G$
 $F \rightarrow D \quad F \rightarrow J$
 $G \rightarrow D \quad G \rightarrow I$
 $D \rightarrow K \quad D \rightarrow L$
 $D \rightarrow M$
 $DM \rightarrow N \quad DM \rightarrow P$
 $L \rightarrow D$
 $PQR \rightarrow S \quad PQR \rightarrow T$
 $PR \rightarrow S$
 $\}$

Step 2: Every left hand side is irreducible

Test $AB \rightarrow C$:

To test whether B can be removed for the Left-Hand-Side(LHS), calculate $\{A\}^+$. If B is in $\{A\}^+$, it is redundant; otherwise it is not.
 $\{A\}^+ = \{A, D, E, F, G, J, K, L, M, N, P, S, T\}$. Since B is not in $\{A\}^+$, B is not redundant.
 To test whether A can be removed for the LHS, calculate $\{B\}^+$. If A is in $\{B\}^+$, it is redundant; otherwise it is not.
 Calculate $\{B\}^+ = \{B\}$. Since A is not in $\{B\}^+$, A is not redundant.

In the same way, you can test other FDs where the left hand side contains more than one attribute.

After Step 2, we have

```
F={ AB->C
  A->D  A->E  A->F  A->G
  E->G
  F->D  F->J
  G->D  G->I
  D->K  D->L
  D->M
  D->N  D->P
  L->D
  PQR->S  PQR->T
  PR->S }
```

Step 3: remove redundant FDs

Test whether A->D is redundant, calculate $\{A\}^+$ without A->D, if D is in $\{A\}^+$, it means A->D is redundant. In the same way you can test the other FDs.

```
F={ AB->C
  A->D  A->E  A->F  A->G
  E->G
  F->D  F->J
  G->D  G->I
  D->K  D->L
  D->M
  D->N  D->P
  L->D
  PQR->S  PQR->T
  PR->S }
```

The minimal cover is:

```
F={ AB->C
  A->E  A->F
  E->G
  F->D  F->J
  G->D  G->I
```

$D \rightarrow K$ $D \rightarrow L$
 $D \rightarrow M$
 $D \rightarrow N$ $D \rightarrow P$
 $L \rightarrow D$
 $PQR \rightarrow T$
 $PR \rightarrow S$ }

3. Normalising to 3NF

Step 1: Create a relation based on each FD

R1(ABC) {AB→C}
 R2(AE) {A→E}
 R3(AF) {A→F}
 R4(EG) {E→G}
 R5(FD) {F→D}
 R6(FJ) {F→J}
 R7(GD) {G→D}
 R8(GI) {G→I}
 R9(DK) {D→K}
 R10(DL) {D→L}
 R11(DM) {D→M}
 R12(DN) {D→N}
 R13(DP) {D→P}
 R14(LD) {L→D}
 R15(PQRT) {PQR→T}
 R16(PRS) {PR→S}

Step 2: Merge FDs with the same left hand side

R1(ABC) {AB→C}
 R2(AEF) {A→E, A→F}
 R3(EG) {E→G}
 R4(FDJ) {F→D, F→J}
 R5(GDI) {G→D, G→I}
 R6(DKLMNP) {D→K, D→L, D→M, D→N, D→P}
 R7(LD) {L→D}
 R8(PQRT) {PQR→T}
 R9(PRS) {PR→S}

Step 3: Merge tables with equivalent keys

find tables that are subsets of other groups and try to merge them

R7(LD) is a subset of R6(DKLMNP). R7 can be merged with R6. We have

R1(ABC) {AB→C} candidate key is AB
 R2(AEF) {A→E, A→F}. candidate key is A
 R3(EG) {E→G} candidate key is E
 R4(FDJ) {F→D, F→J} candidate key is F
 R5(GDI) {G→D, G→I} candidate key is G
 R6(DKLMNP) {D→K, D→L, D→M, D→N, D→P, L→D}, either D or L is a candidate key

R7(PQRT) {PQR->T}
R8(PRS) {PR->S}

candidate key is PQR
candidate key is PR