Enhanced Differential Evolution for Job Shop Scheduling Optimization

***Abstract -*** *Job shop scheduling is well-known as one of the hardest combinatorial optimization problems and has been demonstrated to be NP-hard problem. In the past decades, several researchers have devoted their effort to develop evolutionary algorithms such as Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) for job shop scheduling problem. Differential Evolution (DE) algorithm is a more recent evolutionary algorithm which has been widely applied and shown its strength in many application areas. However, the applications of DE on scheduling problems are still limited. This paper proposes a one-stage differential evolution algorithm (1ST-DE) for job shop scheduling problem. The proposed algorithm employs random key representation and permutation of m-job repetition to generate active schedules. The performance of proposed method is evaluated on a set of benchmark problems and compared with results from an existing PSO algorithm. The numerical results demonstrated that the proposed algorithm is able to provide good solutions especially for the large size problems with relatively fast computing time.*

*Keywords: Job Shop Scheduling; Differential Evolution; Local Search; Combinatorial Optimization;*

*Evolutionary Algorithms; DEAP Framework; Iterative Optimization; Scheduling Heuristics; Metaheuristic;*

*Gantt Chart.*

# INTRODUCTION

The Job Shop Scheduling Problem (JSP) stands as one of the most extensively studied combinatorial problems over several decades. The classical JSP involves scheduling a set of jobs on a defined set of machines, aiming to optimize a specific criterion. This optimization is subject to constraints, ensuring that each job undergoes processing operations on all machines, which are predetermined and fixed. Recognized as NP-hard (Garey et al., 1976), exact algorithms such as integer programming, mixed-integer programming, and the branch-and-bound method face inefficiency, particularly with increasing problem sizes. Recent research endeavors have pragmatically shifted towards developing evolutionary algorithms to attain high-quality or near-optimal solutions within reasonable timeframes. Various approaches to solving JSP include, but are not limited to, the Genetic Algorithm (GA) (Wang et al., 2004; Concalves et al., 2005; Yamada and Nakano, 1995), Ant Colony Optimization (ACO) (Huang et al., 2008; Udomsakdigool and Kachitvichyanukul, 2006, 2008), and Particle Swarm Optimization (PSO) (Pratchayaborirak and Kachivichyanukul, 2007; Pongchairerks and Kachivichyanukul, 2009).

Differential evolution (DE) is one of the latest evolutionary algorithms developed by Storn and Price in Differential Evolution (DE), introduced by Storn and Price in 1995, stands out as one of the latest evolutionary algorithms. Researchers have increasingly focused on DE due to its advantages, including a minimal number of control variables coupled with strong performance in search ability and convergence. Its widespread application has demonstrated notable strengths across various problem domains, spanning scientific, engineering, and financial applications (Price et al., 2005; Chakraborty, 2008).

Despite its success, the utilization of DE for combinatorial optimization remains relatively limited. Recent attempts have emerged to apply DE to address scheduling problems. Godfrey et al. (2006) employed DE in tackling the flow shop scheduling problem. Quan et al. (2007) introduced a discrete differential evolution algorithm (DDE) tailored for the permutation flow shop with makespan criteria. Wang et al. (2008) proposed a self-adaptive DE (SDE) to enhance global convergence properties and mitigate premature convergence issues associated with conventional DE. Liu et al. (2009) extended the application of DDE to the job shop scheduling problem, incorporating special mutation and crossover operators to handle discrete variables in JSP.

Notably, the results indicated that DDE inherits the advantages of the original DE, showcasing rapid convergence rates and high solution quality for small-sized problems. However, for larger problems, DDE exhibited a loss of diversity, leading to solutions becoming easily trapped at local optima.

# JOB SHOP SCHEDULING PROBLEM

The classical Job Shop Scheduling Problem (JSP) involves the allocation of n distinct jobs to be processed across m different machines, governed by two primary sets of constraints: precedence constraints and conflict constraints. Each job comprises a series of sequential operations, and each operation must be executed on a predetermined machine with a known and fixed processing time. The machines operate independently of each other, with negligible setup time and transfer time between operations. Factors such as machine breakdown and pre-emption are not considered in this problem.

The precedence constraints dictate that each operation of a particular job must be processed sequentially, while the conflict constraints ensure that each machine can handle only one operation at any given time. The objective is to strategically sequence operations on machines, determining the start and end times for each operation to optimize specific criteria, all within the confines of the established constraints.

# DIFFERENTIAL EVOLUTION

Differential Evolution (DE) is one of the Evolutionary Algorithms (EAs) for global optimization over continuous search space (Storn and Price 1995). Its theoretical framework is simple and requires inexpensive computation in term of CPU time (Bin *et al*. 2008). Due to its advantage of relatively few control variables but performing well in convergence, DE has been widely applied and shown its strengths in many application areas (Godfrey and Donald 2006; Quan *et al*. 2007; Qian *et al*. 2008).

As a population-based search method, DE starts with randomly generate initial population of size *N* of *D*-dimensional vectors. A solution in DE algorithm is represented by *D*-dimensional position of a vector. Each variable’s value in the dimensional space is represented as the real number. The key idea behind DE is a new mechanism for generating trial vectors. DE generates trial vectors by mutation and crossover operation. Then, the replacement of an individual, so called selection operation, occurs only if the trial vector outperforms its corresponding vector. These processes are repeated until some stopping criteria are met. Thus, DE population evolve through repeated cycles of three main DE operators; mutation, crossover and selection.

# The Classic DE

Currently, there are several DE variants. Generally, the mutation is the main operation which makes each variant distinct from one another. The classic version of DE is the simplest and most popular scheme used in literatures. The mutation operator in the classic DE generates a mutant vector by adding a weighted difference between two randomly selected vectors to the third randomly selected vector. This scheme has proven to be effective in solving many multimodal optimization problems due to its good exploration capability (Price *et al*. 2005; Chakraborty 2008). The procedures in the classic DE (Price *et al*. 2005) can be described as follows.

# Population Initialization

DE starts with initializing the population of size *N* of *D*-dimensional vectors. The lower bound, *bL*, and upper bound, *bU*, for the value in each dimension *jth* (*j*=0,1,…,*D*-1) must be specified. At initialization step (g =0), the *jth* value of the *ith* vector is randomly generated as follows:

Where 𝑢j is a uniform random number in the range [0, 1].

# Mutation

Once initialized, DE mutates and combines current target vectors to produce mutant vectors. For each target vector, at generation *g*, the mutant vector, *Vi,g*, is generated according to the following equation:

It is noted that X𝑟1, X𝑟2, and X𝑟3 are randomly selected vectors from the population. They are mutually exclusive and different from the *i*th target vector, *Xi,g*. *F* is a scale factor which controls the scale of the difference vector between X𝑟2, and X𝑟3*,* added to the base vector, X𝑟1 *.*

# Crossover

DE applies crossover operator on *Xi,g* and *Vi,g* to generate the trial vector *Zi,g*. In the classic DE, the uniform crossover is employed and the trial vector is generated by the following equation.

Where 𝐶𝑟 controls the probability of selecting the value in each dimension for a trial vector from a mutant vector.

# Selection

The selection operation is performed on each target vector, *Xi,g* ,and its corresponding trial vector, *Zi,g*, to determine the survival vector for the next generation. The vector, *Xi,g+1*, is selected according to the greedy criteria

It is noted that this selection scheme is applicable for minimization problem. Once each individual in the current population is updated, the population continue to evolve

through mutation, crossover, and selection operation until some stopping criteria are met.

This classic DE is commonly denoted as DE/rand/1/bin, where DE stands for DE, rand is the type of base vector selected to be perturbed, 1 is the number of difference vector for permutation, and bin stands for binomial distribution of the number of inherited dimension values of mutant vectors.

1. **APPLICATION OF DE ON JSP**

Since DE is first designed for continuous domain, in order to apply DE to combinatorial problem i.e. scheduling problem, solution vectors in DE need to be transformed into a schedule. The procedures of solution mapping are illustrated in the following example.

# Solution Representation

A solution of the problem can be represented using a vector with dimensions equal to the total number of operations processed on all machines. Consider an example of two jobs and three machines in JSP in table 1.

According to this example, the total number of dimension, equal to the total number of operation, is 6. Figure 1 illustrates random key representation encoding scheme (Bean 1994) where each value in a vector dimension is initially generated with a uniform random number in range [0, 1].

Next, the permutation of 3-repetition of 3 jobs (Bierwirth 1995) is applied with a sorting list rule to decode an individual vector into a sequence of operations as shown in figure 2. The advantage of this approach is that any permutation of this representation always provides a feasible schedule. Then, the operation-based approach (Cheng *et al*. 1996) is employed to generate a schedule. The decoded individual is transformed into a schedule by taking the first operation from the list, then the second operation, and so on. During the process of generating a schedule, each operation is allocated to a required machine in the best available position without delaying other scheduled operations. This procedure results in an active schedule as shown in figure 3.

# A Single Stage DE (1ST- DE)

This paper proposes a one-stage differential evolution (1ST-DE) with a single population. In 1ST-DE, initial population is randomly generated according to equation (6).To obtain trial vectors, the mutation operation is carried out in the same way as the classic DE by equation (7), however, this study uses the exponential crossover operation. Starting at the randomly picked dimension index *š*, each dimension value of trial vector are inherited from its mutant vector, *Vi,g* , as long as *uj* ≤ 𝐶𝑟. The first time when *uj* >𝐶𝑟, all the remaining dimension value are taken from the target vector, *Xi,g*. An integer, *L* indicates the number of consecutive dimension indexes on which crossover is performed. The exponential crossover scheme can be expressed in equation (10).

**COMPUTATIONAL EXPERIMENT**

* 1. **Parameter Setting**

In this study, the crossover rate 𝐶𝑟 is set to 0.2. Some experiments were carried out to determine the choice of scale factors *F* in the range [0, 2]; 0.5, 1, 1.5, and 2. The results showed that the suitable value of *F* for JSP in this study is 1.5. Particularly compared with PSO, DE population size and number of iterations are set equally to those in previous PSO studies as 100 and 500 respectively (Pratchayaborirak and Kachitvichyanukul 2007).

# Experimental Results

The experiments are implemented using the C# language of the Microsoft Visual Studio 8.0. The program runs on the platform of Intel®CoreTM 2 Dou CPU 1.67GHz with 3062 MRAM.

The performance of 1ST-DE is evaluated using several benchmark JSP; FA20, LA01, LA21, LA29, LA35 and LA37 representing different problem sizes. The results are compared to those obtained from 1ST-PSO (a one-stage PSO) having the same algorithm as in previous PSO studies (Pratchayaborirak and Kachitvichyanukul 2007) under the same conditions such as encoding and decoding scheme, population size and number of iterations. Table 2 shows the best, average, standard deviations of makespan, and computing time from 10 runs of each algorithm for each problem. The t-test was performed to statistically compare the results of two algorithms with 95% confident interval. The average convergence graph are also observed and illustrated in figure 4.

It is noted that since this experiment focuses on the performance of DE and PSO algorithms, the 1ST-DE and 1ST-PSO used here are the simple versions without any additional improvements such as re-initialization and local search strategies.

According to the results from table 2 and figure 4, it is observed that all of solutions obtained from the proposed DE are equal to or better than those obtained from 1ST-PSO with relatively faster computing time. The statistical t-test indicates that 1ST-DE outperforms 1ST-PSO in all cases except for small problem size LA01 where both algorithms can easily find the optimal solution. The average convergence graph clearly demonstrates faster convergence behavior of 1ST-DE compared to 1ST-PSO.

# CONCLUSION

This paper proposes a one-stage differential algorithm (1ST-DE) for solving JSP. The proposed algorithm employs the classic DE mutation scheme with exponential crossover operation. The random key representation and permutation of m-job repetition is applied to generate an active schedule. The performance of proposed method is evaluated on a set of benchmark problems and compared with results from an

The experimental results indicate that under the same experimental conditions: encoding and decoding scheme, population size and number of iterations, the 1ST-DE is superior to 1ST-PSO in term of solution quality and computing time especially for the large size problems. In addition, the proposed DE demonstrates faster convergence behavior than that in 1ST-PSO.

In conclusion, it can be claimed that DE has mechanisms to effectively search for better solutions than PSO. The ongoing researches are under investigation to improve DE performance for solving a wide range on scheduling problems.

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