The Nuclear Physics Weekly III

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1 Theory

1.1 Recap

We started with revisiting scattering amplitude from previous lecture. Scattering amplitude characterizes the probability of an interaction to occur.

$$iM \propto \langle final|\, S - 1 \,|initial\rangle$$

1.2 Scattering theory

Model independent features i.e. features present independent of what interaction is occuring.

- Symmetry
  - Spacetime $\implies$ Lorentz Invarience (can be ignored in low energy systems)
  - Internal $\implies$ Flavor, Baryon Numbers
- Unitarity
  - Probability Conservation $\implies$ The S matrix is a unitary operator

    Let Ψ be such that $|\,|\Psi\rangle\,|^2 = 1$, Then U is a unitary operator iff $|U\,|\Psi\rangle\,|^2 = 1 \implies U^\dagger U = 1$
- Analyticity
  - Causality $\implies$ Amplitudes are boundary values of analytic functions in complex energy plane
- Crossing
  - CPT symmetry $\implies$ Relates particle-anti-particle in scattering process.

1.3 Unitarity

Unitarity implies a relationship between imaginary part of M and modulus of M.

$$Im(M) = \rho|M|^2 \text{ for } E^* \geq 2m$$

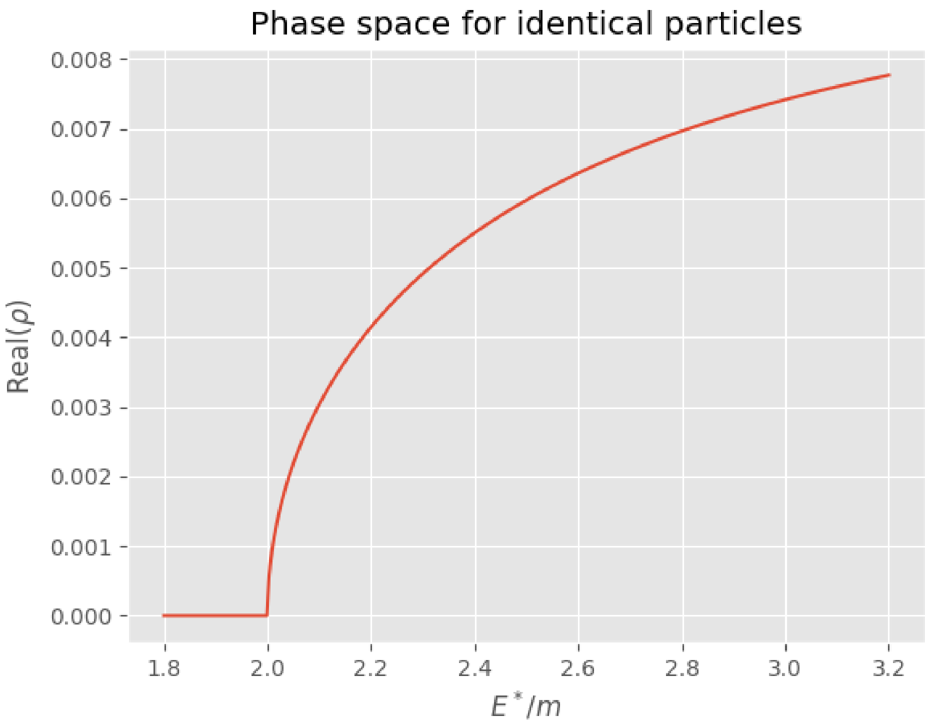
Here, ρ is the *kinematic function* which characterizes <u>onshell</u> scattering of two-particles

$$\rho = \frac{\xi q^*}{8\pi E^*}$$

where q^* is relative momentum between particles in center-of-mass frame and equals to $q^* = \frac{1}{2}\sqrt{E^{*2} - 4m^2}$

2 Assignment Questions

1. Phase space for identical particles in the range $1.8 \leq E^*/m \leq 3.2$



2. We have

$$\mathcal{M} = |\mathcal{M}|e^{i\delta}$$

and,

$$Im(\mathcal{M}) = \rho|\mathcal{M}|^2$$

Now,

$$\mathcal{M} = |\mathcal{M}|e^{i\delta}$$

$$\implies \mathcal{M} = |\mathcal{M}|(\cos\delta + i \cdot \sin\delta)$$

Equating the known relation to imaginary part of the above equation, we get,

$$\rho|\mathcal{M}|^2 = |\mathcal{M}| \cdot \sin\delta$$

$$\implies |\mathcal{M}| = \frac{1}{\rho}\sin\delta$$

Plugging this in the polar form of complex number, we get,

$$\mathcal{M} = \frac{1}{\rho}\sin\delta e^{i\delta}$$

3. From previous result,

$$\mathcal{M} = \frac{1}{\rho}\sin\delta e^{i\delta}$$

$$\implies \mathcal{M}^{-1} = \rho\frac{e^{-i\delta}}{\sin\delta}$$

$$\implies \mathcal{M}^{-1} = \rho\frac{\cos\delta - i\sin\delta}{\sin\delta} = \rho\left(\cot\delta - i\right)$$

$$\implies Im(\mathcal{M}^{-1}) = -\rho$$

4.

$$\mathcal{M} = \mathcal{K}\frac{1}{1 - i\rho\mathcal{K}}$$

$$\implies \mathcal{M}^{-1} = \frac{1}{\mathcal{K}} - i\rho$$

Equating this to $\mathcal{M}^{-1} = \rho(\cot\delta - i)$,we get,

$$\mathcal{K}^{-1} = \rho\cot(\delta)$$