The Nuclear Physics Weekly V

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1 Introduction

This week we covered Feynman diagrams in more detail and looked at the rules governing it and how we convert those diagrams to the underlying mathematical equations.

2 Feynman diagram

In Feynman diagrams, a line represents a freely propagating particle and a circle represents an interaction. There are certain rules for Feynman diagram in φ^4 theory for writing down a mathematical equation from the given diagrams. The rules being:

1. Every interaction point leads to a factor of $i\lambda$ where λ some real number.
2. Every external line leads to a factor of $+1$
3. Every internal line leads to a factor of $\frac{i}{p^2-m^2+i\epsilon}$. These internal lines are also knows as Feynman propagators.
4. Every Feynman propagator having unknown momentum leads to a factor of $\int \frac{dk_i}{(2\pi)^4}$ i.e., integrating over all possible momentum.
5. Factor of $\frac{1}{s}$ where s is the symmetric factor. Symmetric factor is the number of permutation of propagators that leaves the diagram unchanged.

3 Examples

These are couple of examples that Dr. Briceno's left to us as homework.

1. Calculating symmetry factor

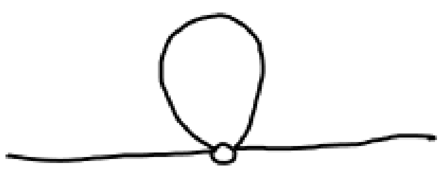

Figure 1: (a)

(a) There is only one internal line in this diagram which leads to only one possible permutation without changing the diagram. Hence, **symmetry factor** $= \mathbf{1}$.

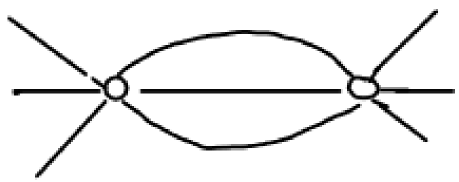

Figure 2: (b)

(b) There are three internal lines which can be rearranged between themselves. Hence the **symmetric factor is 3!** $= \mathbf{6}$

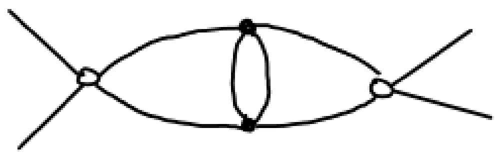

Figure 3: (c)

(c) There are two internal line and in addition to these, we can also observe two lines of exchange in the middle of propagation. The pair can be independently permuted between themselves. So the **symmetric factor is 2!** $\times$ **2!** $= \mathbf{4}$

2. Deriving mathematical expression for Feynman diagrams

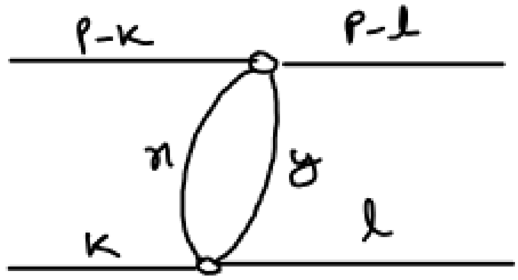

Figure 4: (a)

(a) We know that the total momentum is p. Let momentum of one of the internal line is x, then the other line has momentum has $p - x$.
Conserving momentum at both interaction points, we have,

$$k = x + y + l$$

$$y = x + l - k$$

- By Rule 1, we have two $-i\lambda$ factors in the final equation for two intercation points.

$$f = (-i\lambda)^2 = -\lambda^2$$

- By Rule 2, 4 external lines contribute 1^4 factor, leaving the equation unchanged.

- By Rule 3, The two internal lines contribute a factor of $\frac{i}{x^2-m^2+i\epsilon}$ and $\frac{i}{(x+l-k)^2-m^2+i\epsilon}$ where x and $x+l-k$ are the respective momentums.

$$f = -\lambda^2 \cdot \frac{i}{x^2 - m^2 + i\epsilon} \cdot \frac{i}{(x+l-k)^2 - m^2 + i\epsilon}$$

- By Rule 4, we integrating over all possible momentum for unknown momentas.

$$f = \lambda^2 \int \frac{dx}{(2\pi)^4} \frac{1}{(x^2 - m^2 + i\epsilon) \cdot ((x+l-k)^2 - m^2 + i\epsilon)}$$

- By Rule 5, we divide by a symmetric factor of 2 due to two internal lines that can be permuted. So the final equation is

$$f = \frac{\lambda^2}{2} \int \frac{dx}{(2\pi)^4} \frac{1}{(x^2 - m^2 + i\epsilon) \cdot ((x+l-k)^2 - m^2 + i\epsilon)}$$

(b)

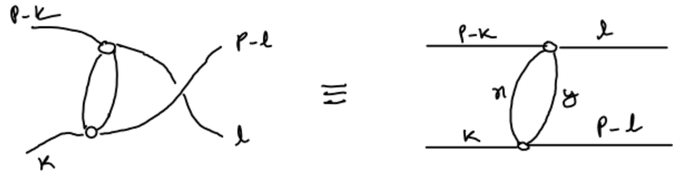

Figure 5: (b)

Observe that this diagram is equivalent to the first problem just that the momentas of the outgoing particles are reversed. But, since the final equation is independent of this and simple redefinition of l to $p-l$ makes the two diagram equivalent, the final equation of this diagram is same as that for problem 2.a, i.e.,

$$f = \frac{\lambda^2}{2} \int \frac{dx}{(2\pi)^4} \frac{1}{(x^2 - m^2 + i\epsilon) \cdot ((x+l-k)^2 - m^2 + i\epsilon)}$$