The Nuclear Physics Weekly I

Anish

August 6, 2021

Contents

1 Week 1

1.1 Overview

The first lecture started with outlining and discussion of modalities for the program. We then moved on to introduction of various topics in nuclear physics. Topics discussed in the first lecture are listed below:

- Fundamental Forces : Gravitation, Electromagnetic, Strong and Weak Forces
- Standard Model : Quarks and Bosons (specifically gluons)
- QCD - Theoretical framework for strong nuclear force.
- Lattice QCD - A non-perturbative approach to solving a QCD Systems.
- Particle Physics : Feynman diagrams and Resonance.

1.2 Introduction

1.2.1 Strong force

Strong force was then introduced in more detail. Strong force is the force that keeps protons and neutrons together. Quarks inside them interact through this strong force with gluons acting as force carriers. Gluons play a similar role in QCD as electron play in QED but keep in mind that gluons are much more involved than photons because color - analog of charge in QCD - of quarks can change unlike charge and gluons have to interact with differently colored quarks whilst conserving color of the confined system and they also interact with themselves unlike photons.

1.2.2 QCD

Dr. Briceno then expanded a bit more on QCD. Quarks and gluons carry different "color" (*red, green, blue*) and quarks come in 6 different "flavors" : *up, down, strange, charm, top, bottom.* Note that the flavors at odd places are in one family and than on even are in one. Properties of quarks in a family remain same other than the mass which get heavier to the right. Also note that heavier quarks are much more unstable that the lighter ones and hence up, down and strange quark are most abundant.

We never find quarks independently in the wild, they are always found in confined state. These structures are called *Hadrons.* Hadrons are composite particles made up of two or more quarks. They are always color neutral. Hadron made of quark-antiquark pair (even number of quarks) are called *Mesons*. Hadrons composed of Odd number of quarks (at least 3) are called *Baryons*. Keep in mind that these confined structures don't just have the specifies number of quarks, more quark-antiquark pairs pop in and out of existence forming a sea of quarks inside the Hadrons.

While reading up more on QCD, I found that one the reason QCD was proposed as the framework for Strong force was the propoerty of *asymptotic freedom*. Asymptotic freedom is the phenomenon of weak coupling for hard (high energy) gluons and high coupling for soft (low energy) gluons. This is one of the features of QCD due to which it was proposed. It is also the reason why QCD gets really hard to solve as low energies. At low energy an effective attraction happens between particles on opposite side of fermi surface which leads to pairing instability, which then results in big changes even when the perturbation is small, making the problem non-perturbative.

1.2.3 Lattice QCD

As we just discussed above, QCD at low energy is non-perturbative! Approach towards solving QCD system at high energy - which are perturbative - don't work on low energy systems. At low energies the coupling is higher coupling constant is higher and the contribution of more complex interactions is much more than in high energy systems making the calculations much more complex. Lattic QCD gets around this by taking a non-perturbative approach to solving QCD. Quarks are represented as lattice sites and gluons as links connecting them.

There are three main questions Lattice QCD is used to answer at JLab:

1. Nuclear structure : How do nucleons come together to form low-lying nuclei?
2. Hadron Structure : How do quarks come together to form hadrons?
3. Spectroscopy : What are bound states of QCD?

1.2.4 Feynman diagram

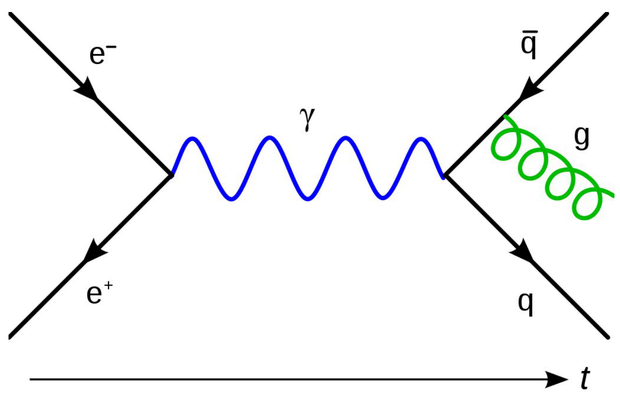

Figure 1: Feynman Diagram of Electron Positron interaction resulting in formation of quark-antiquark pair. The blue line represents photon and the green line represents gluon.

Feynman diagrams, proposed by Richard Feynman, are graphical representation of rigorous mathematical equations that govern particle interactions. One axis of the diagram (x-axis usually, unlike the above figure) represents space and the other axis time. Particles are represented as lines in the diagram and the vertices represent particle interactions.

1.2.5 QCD Spectroscopy

QCD Spectroscopy is basically probing the decaying of Hadrons. A lot of phyiscs can be extracted from this, such as, production mechanisms, prominent decay channels, structure and more! Lattice QCD is able the only tools available to us right now which generates stable states, Resonant amplitudes exactly.

1.2.6 Resonance

Resonance in particle physics is the phenomenon where probability of an interaction occurring is amplified at a particular frequency or energy. Resonance occurs because a bound state, if it can decay, will decay. When the energy of colliding particles in enough to produce rest mass, it forms and then decays quickly by strong force.

1.3 Other work

Beside the lecture, I did some reading on introductory particle physics (see Reading List) to prime myself with some (very) basic particle physics. I learnt about :

- Elementary Particles : Fermions (Leptons and Quarks) and Bosons; Hadrons (mesons and baryons)
- Particle Conservation Laws : Baryon Number, Lepton Number, Strange (not in weak)
- The Standard Model : Combines Electroweak theory and QCD.
- Drawing (basic) Feynman diagram : Complex particle interaction equations represented as diagrams with time as y-axis and position as x-axis.

1.4 Reading List

1. Particle Physics and Cosmology Ch-11 University Physics Volume 3
2. QCD Made Simple - Physics Today 53, 8, 22 (2000)
3. Particle Concepts - HyperPhysics

The Nuclear Physics Weekly II

Anish

August 15, 2021

Contents

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1 Scattering Theory : Complex Numbers review

1.1 Why Complex Numbers

- Complex numbers often come up as shortcuts for calculations in physics.
  Example : Wave equation $\Psi(x,t) = Re[Ae^{i(\omega t - kx)}]$
- Not just as shortcuts, Complex number is *fundamental* to Quantum Physics.

1.2 Scattering

- Scattering Amplitude M, a complex number, establishes the probability of a scattering process.
- M is a function of Center-of-momentum (frame in which total momentum of the system is zero) Energy , denoted as M.
- Probability of an interaction is proportional to $|M|^2$
- In a scattering amplitude-Energy plot, we observe a cusp at the *threshold energy.*
- Threshold energy is the minimum energy at which scattering can occur as below this energy level, probability of scattering is zero.

1.3 Basic Complex Number

- $z = a + i \cdot b$
- Often written in polar format as $z = r \cdot e^{i \cdot \alpha}$. Here, r is magnitude and α is the argument.

1.4 Further Reading

- Townsend, A Modern Approach to Quantum Mechanics
- Sakurai, Modern Quantum Mechanics
- Taylor, Scattering Theory
- International Summer School On Reaction Theory (2017 Lectures - Team 1,2)

2 More on Complex Numbers

2.1 Analytic functions

A complex analytic function, also known as *holomorphic function*, defined in some domain D of argand plane is such that it is single-valued and differentiable everywhere at all points in the domain.

Example: $f(z) = \sum_{n=0}^{N} a_n \cdot z^n$ where $a_n \in \mathbb{C}$

2.2 Visualising Complex functions

Plotting functions that are from $\mathbb{R}$ to $\mathbb{R}$ is a trivial task which can be done with 2D surface. But for complex functions where both domain and range in 2-dimensional, we need a 4D surface and hence impossible. But we can get (somewhat) around this problem by using a technique called *Domain Coloring.*

Domain coloring works by coloring the argand plane, say W, with help of a function that define color at each point in W. We then color each point of another argand plane, say Y, such that color of point $z \in Y$ corresponds to color of point $f(z)$ in W.

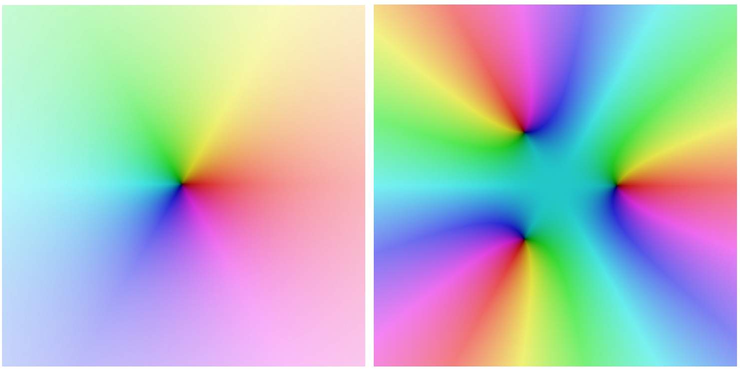


Figure 1: Color function (left) and Domain Coloring of $f(z) = z^3 - 1$ (right).

2.3 Pole singularity

A complex function $f(z)$ has pole singularity of order n at z_0 if $f(z) \cdot (z - z_0)^n$ is an holomorphic function.

Example: $f(z) = \frac{g(z)}{z-z_0}$ where $g(z)$ is holomorphic has pole singularity or order 1 at z_0.

Domain coloring can be helpful in inspecting singularities. $f(z) = \frac{z^2+z-1}{(z-i)(z+1)}$ is plotted in Figure 2 and we can clearly observe a peculiar character of the plot at the poles i.e. i and -1.

2.4 Cauchy's theorem

Cauchy's theorem is one of the most important theorem in complex analysis. It states that if $f(z)$ is analytic everywhere within a simply-connected region then for every simple closed path C lying in the region, we have,

$$\oint_C f(z) \cdot dz = 0$$

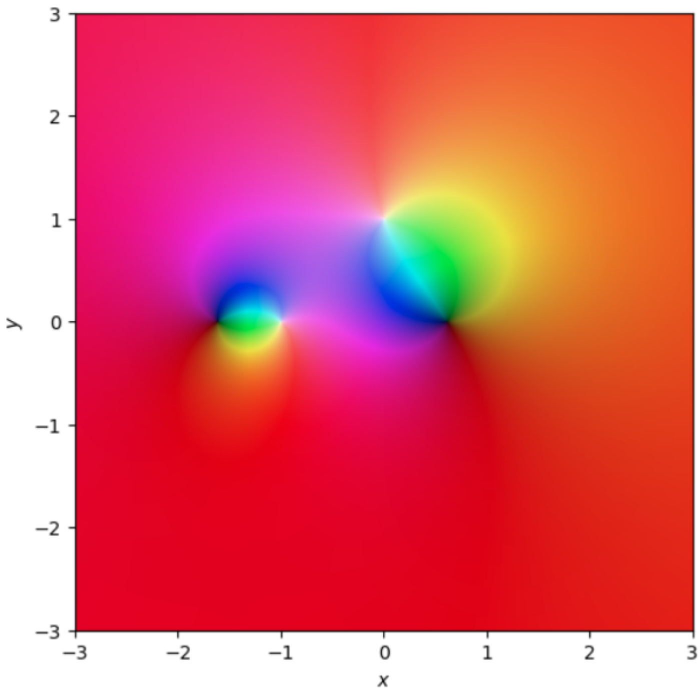


Figure 2: $f(z) = \frac{z^2+z-1}{(z-i)(z+1)}$

Example: z^2 is analytic everywhere. Hence for any simple contour C $\oint_C z^2 \cdot dz = 0$

2.5 Cauchy's Integral Formula

If $f(z)$ is analytic everywhere inside and on the boundary C fo a simply-connected region then for any point z_0 inside C, we have,

$$\oint_C \frac{f(z)}{z - z_0} \cdot dz = 2\pi i f(z_0)$$

Example: Evaluate $\oint_C \frac{z^2+z-1}{(z-i)(z+1)} \cdot dz$ where C is circular region of radius $r < \sqrt{2}$ around i.

The only pole of the integrand in C is at i. Rewrite the function as $\frac{\frac{z^2+z-1}{(z+1)}}{z-i}$. Thus, by Cauchy's integral formula the integral equates to $2 \cdot \pi \cdot \frac{i-2}{i+1}$.

2.6 Branch Cut

A branch cut in the curve along which an analytic multivalued function is discontinuous.

Example: Real and Imaginary parts of $\sqrt{z}$ are plotted separately in Figure 3 which shows a branch cut along the negative x-axis. The discontinuity is apparent in Figure 4.

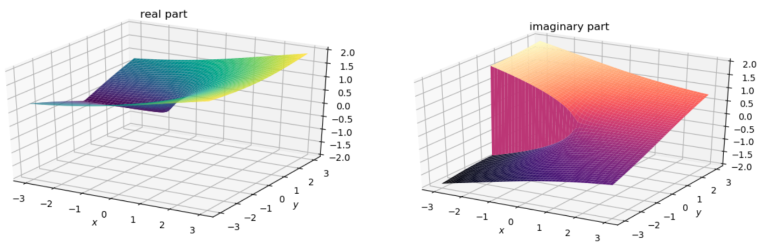


Figure 3: Real (left) and Imaginary(right) component of $f(z) = \sqrt{z}$.

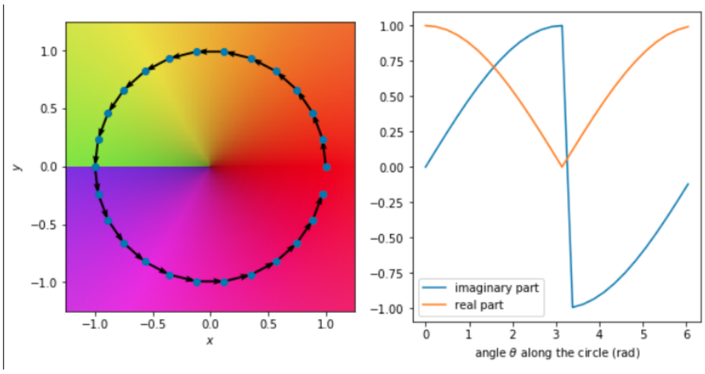


Figure 4: Discontinuity in $\sqrt{z}$

3 References

- Visualizing complex analytic functions using domain coloring - Hans Lundmark 2004
- Cauchy's Theorem - HELM 2008
- Winding Number and domain coloring - Grant Sanderson, 3blue1brown
- Weisstein, Eric W. "Branch Cut." From MathWorld–A Wolfram Web Resource

The Nuclear Physics Weekly III

Anish

September 1, 2021

Contents

1 Theory

1.1 Recap

We started with revisiting scattering amplitude from previous lecture. Scattering amplitude characterizes the probability of an interaction to occur.

$$iM \propto \langle final| S - 1 |initial\rangle$$

1.2 Scattering theory

Model independent features i.e. features present independent of what interaction is occuring.

- Symmetry
  - Spacetime $\implies$ Lorentz Invarience (can be ignored in low energy systems)
  - Internal $\implies$ Flavor, Baryon Numbers
- Unitarity
  - Probability Conservation $\implies$ The S matrix is a unitary operator

    Let Ψ be such that $| |\Psi\rangle |^2 = 1$, Then U is a unitary operator iff $|U |\Psi\rangle |^2 = 1 \implies U^\dagger U = 1$
- Analyticity
  - Causality $\implies$ Amplitudes are boundary values of analytic functions in complex energy plane
- Crossing
  - CPT symmetry $\implies$ Relates particle-anti-particle in scattering process.

1.3 Unitarity

Unitarity implies a relationship between imaginary part of M and modulus of M.

$$Im(M) = \rho|M|^2 \text{ for } E^* \geq 2m$$

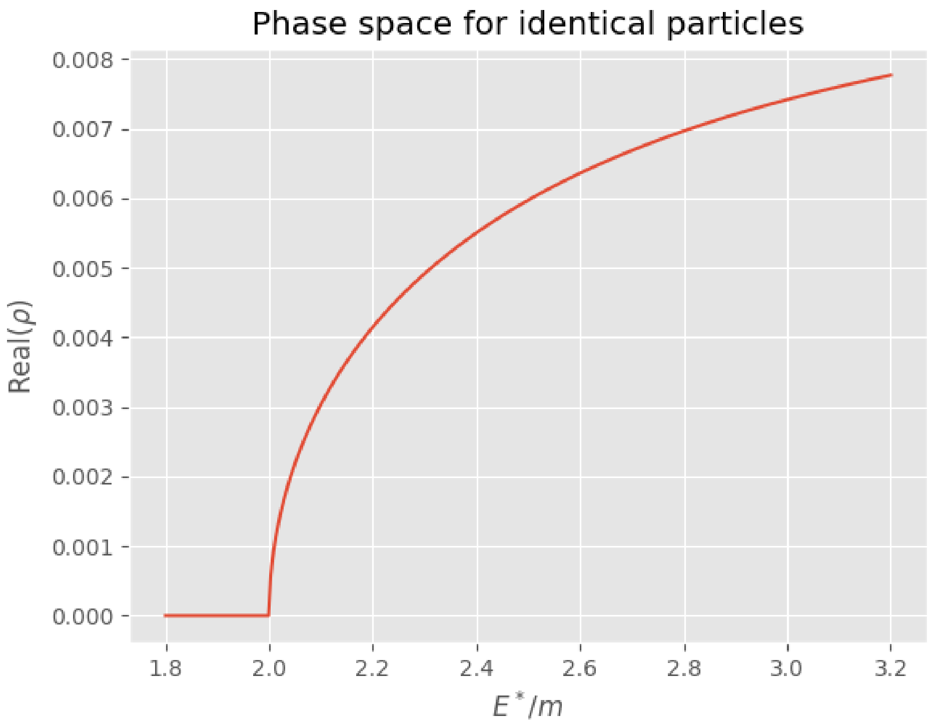
Here, ρ is the *kinematic function* which characterizes onshell scattering of two-particles

$$\rho = \frac{\xi q^*}{8\pi E^*}$$

where q^* is relative momentum between particles in center-of-mass frame and equals to $q^* = \frac{1}{2}\sqrt{E^{*2} - 4m^2}$

2 Assignment Questions

1. Phase space for identical particles in the range $1.8 \le E^*/m \le 3.2$



2. We have

$$\mathcal{M} = |\mathcal{M}|e^{i\delta}$$

and,

$$Im(\mathcal{M}) = \rho|\mathcal{M}|^2$$

Now,

$$\mathcal{M} = |\mathcal{M}|e^{i\delta}$$
$$\implies \mathcal{M} = |\mathcal{M}|(\cos\delta + i \cdot \sin\delta)$$

Equating the known relation to imaginary part of the above equation, we get,

$$\rho|\mathcal{M}|^2 = |\mathcal{M}| \cdot \sin\delta$$
$$\implies |\mathcal{M}| = \frac{1}{\rho}\sin\delta$$

Plugging this in the polar form of complex number, we get,

$$\mathcal{M} = \frac{1}{\rho}\sin\delta e^{i\delta}$$

3. From previous result,

$$\mathcal{M} = \frac{1}{\rho}\sin\delta e^{i\delta}$$
$$\implies \mathcal{M}^{-1} = \rho\frac{e^{-i\delta}}{\sin\delta}$$
$$\implies \mathcal{M}^{-1} = \rho\frac{\cos\delta - i\sin\delta}{\sin\delta} = \rho\left(\cot\delta - i\right)$$
$$\implies Im(\mathcal{M}^{-1}) = -\rho$$

4.

$$\mathcal{M} = \mathcal{K}\frac{1}{1 - i\rho\mathcal{K}}$$

$$\implies \mathcal{M}^{-1} = \frac{1}{\mathcal{K}} - i\rho$$

Equating this to $\mathcal{M}^{-1} = \rho(\cot\delta - i)$,we get,

$$\mathcal{K}^{-1} = \rho\cot(\delta)$$

The Nuclear Physics Weekly IV

Anish

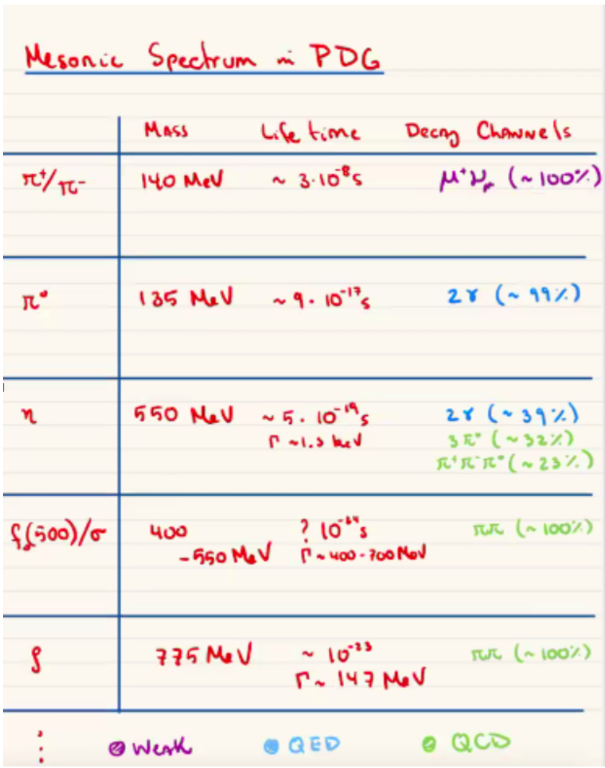
September 2, 2021

Contents

1 Overview

We start with revisiting our motivation for the current research which is to understand how different particles decay and why do they do so. The challenging part of this process is finding all the stages of decay process which may involve intermediate particles with ultra-low lifetime. QCD spectroscopy deals with just this and Lattice QCD is the tool that is used to accomplish this.

We then moved on to Particle Data Group's website where different particles and their decay products with (a lot) more details is maintained. A particle can decay in multiple pathways - which obey all the conservation rules - with different probabilities, some higher than others. For example $\pi^{\pm}$ decays into $\mu^{\pm}$ and ν_{mu} 99% of the time. Other decay pathways are there but they have low probability. The lifetime also depends on the force through which the particle is decaying. Stronger the force, faster will be the decay. Hence, particles decaying through QCD will have much lower lifetime compared to other decay mechanisms.



Mesonic Spectrum in PDG

	Mass	Life time	Decay Channels
π^+/π^-	140 MeV	$\sim 3\cdot 10^{-8}$ s	$\mu^+\nu_\mu$ ($\sim$100%)
π^0	135 MeV	$\sim 9\cdot 10^{-17}$ s	2γ ($\sim$99%)
η	550 MeV	$\sim 5\cdot 10^{-19}$ s $\Gamma \sim 1.3$ keV	2γ ($\sim$39%) $3\pi^0$ ($\sim$32%) $\pi^+\pi^-\pi^0$ ($\sim$23%)
$f_0(500)/\sigma$	400 – 550 MeV	? 10^{-24} s $\Gamma \sim 400-700$ MeV	$\pi\pi$ ($\sim$100%)
ρ	775 MeV	$\sim 10^{-23}$ $\Gamma \sim 147$ MeV	$\pi\pi$ ($\sim$100%)
⋮			

Weak QED QCD

Figure 1: Some common particles and their decay products. Green → QCD, Blue → QED, Violet → Weak

QCD is very strongly interacting. This make it mathematically much more complex than other forces. So for simplifying things, we consider a simpler standard model where forces other than QCD are not present. As a consequence of this, particles that majorly decay through forces other than QCD are essentially stable (lifetime $\to \infty$). For example, $\pi^{\pm}$ which decays generally through weak force is now stable. This is not a catastrophic simplification because most mesonic and baryonic particles decay through QCD.

We then looked at how higher the mass a particle has, more will be the possible decay states. This is simply due to the fact that a particle with high mass will have higher energy which can then in turn be used to produce more particles. Higher the energy, more will be the particles of which the energy clears production threshold of and hence more possible decay states. This is illustrated in the figure below.

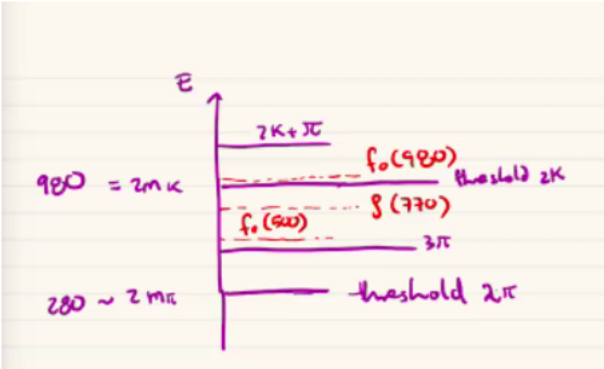


Figure 2: Energy thresholds for different particles.

We then discussed how these hardonic resonances are produced in different facilities. Three different ways to prodcuing a ρ resonance are show below in figure below. Note that although these resonances are created through different mechanisms, the ρ thus produced have same properties i.e., resonances are universal.

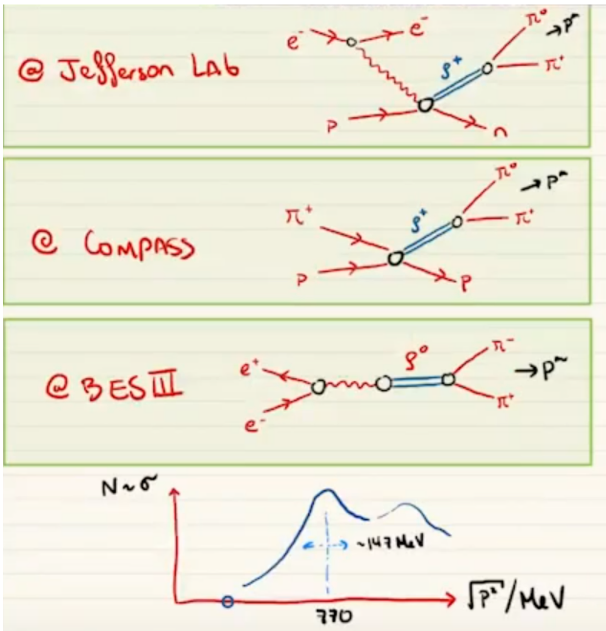


Figure 3: ρ resonance production at Jlab, COMPASS and BES III.

The Nuclear Physics Weekly V

Anish

October 5, 2021

Contents

1 Introduction

This week we covered Feynman diagrams in more detail and looked at the rules governing it and how we convert those diagrams to the underlying mathematical equations.

2 Feynman diagram

In Feynman diagrams, a line represents a freely propagating particle and a circle represents an interaction. There are certain rules for Feynman diagram in φ^4 theory for writing down a mathematical equation from the given diagrams. The rules being:

1. Every interaction point leads to a factor of $i\lambda$ where λ some real number.
2. Every external line leads to a factor of $+1$
3. Every internal line leads to a factor of $\frac{i}{p^2-m^2+i\epsilon}$. These internal lines are also knows as Feynman propagators.
4. Every Feynman propagator having unknown momentum leads to a factor of $\int \frac{dk_i}{(2\pi)^4}$ i.e., integrating over all possible momentum.
5. Factor of $\frac{1}{s}$ where s is the symmetric factor. Symmetric factor is the number of permutation of propagators that leaves the diagram unchanged.

3 Examples

These are couple of examples that Dr. Briceno's left to us as homework.

1. Calculating symmetry factor

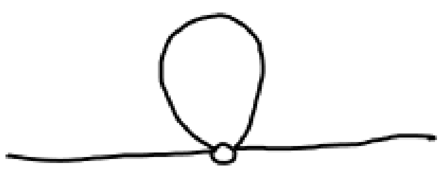


Figure 1: (a)

(a) There is only one internal line in this diagram which leads to only one possible permutation without changing the diagram. Hence, **symmetry factor** $= \mathbf{1}$.

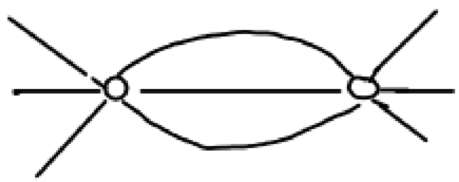


Figure 2: (b)

(b) There are three internal lines which can be rearranged between themselves. Hence the **symmetric factor is 3!** $=$ **6**

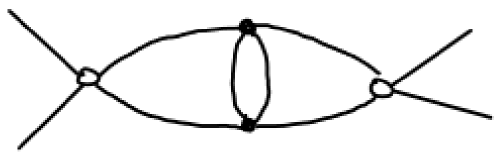


Figure 3: (c)

(c) There are two internal line and in addition to these, we can also observe two lines of exchange in the middle of propagation. The pair can be independently permuted between themselves. So the **symmetric factor is 2!** $\times$ **2!** $=$ **4**

2. Deriving mathematical expression for Feynman diagrams

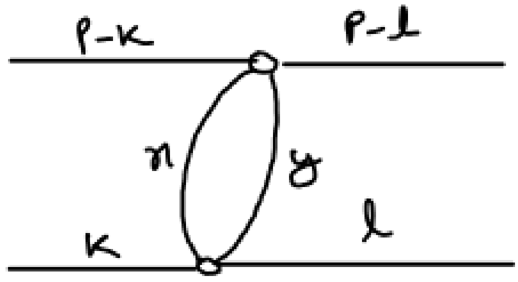


Figure 4: (a)

(a) We know that the total momentum is p. Let momentum of one of the internal line is x, then the other line has momentum has $p - x$.
Conserving momentum at both interaction points, we have,

$$k = x + y + l$$

$$y = x + l - k$$

- By Rule 1, we have two $-i\lambda$ factors in the final equation for two intercation points.

$$f = (-i\lambda)^2 = -\lambda^2$$

- By Rule 2, 4 external lines contribute 1^4 factor, leaving the equation unchanged.

- By Rule 3, The two internal lines contribute a factor of $\frac{i}{x^2-m^2+i\epsilon}$ and $\frac{i}{(x+l-k)^2-m^2+i\epsilon}$ where x and $x+l-k$ are the respective momentums.

$$f = -\lambda^2 \cdot \frac{i}{x^2 - m^2 + i\epsilon} \cdot \frac{i}{(x+l-k)^2 - m^2 + i\epsilon}$$

- By Rule 4, we integrating over all possible momentum for unknown momentas.

$$f = \lambda^2 \int \frac{dx}{(2\pi)^4} \frac{1}{(x^2 - m^2 + i\epsilon) \cdot ((x+l-k)^2 - m^2 + i\epsilon)}$$

- By Rule 5, we divide by a symmetric factor of 2 due to two internal lines that can be permuted. So the final equation is

$$f = \frac{\lambda^2}{2} \int \frac{dx}{(2\pi)^4} \frac{1}{(x^2 - m^2 + i\epsilon) \cdot ((x+l-k)^2 - m^2 + i\epsilon)}$$

(b)

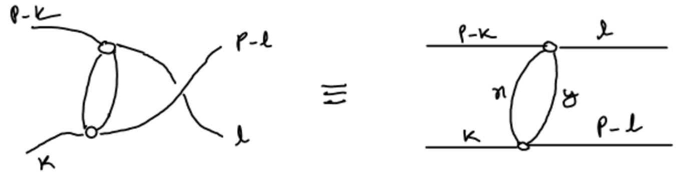


Figure 5: (b)

Observe that this diagram is equivalent to the first problem just that the momentas of the outgoing particles are reversed. But, since the final equation is independent of this and simple redefinition of l to $p-l$ makes the two diagram equivalent, the final equation of this diagram is same as that for problem 2.a, i.e.,

$$f = \frac{\lambda^2}{2} \int \frac{dx}{(2\pi)^4} \frac{1}{(x^2 - m^2 + i\epsilon) \cdot ((x+l-k)^2 - m^2 + i\epsilon)}$$

Final Report | REYES Mentorship program

Anish

October 23, 2021

Contents

This is the final report summarizing what I learnt throughout the REYES mentorship program. First I would like to thank Dr. Raul Briceno, Dr. Andrew Jackura and rest of the REYES staff for successfully conducting such an amazing program.

In the first week, I was introduced to fundamental aspects of nuclear physics including but not limited to Elementary Particle, Standard model, Feynman diagram and Lattice QCD. There are 17 elementary particles in the standard model, divided into fermions and bosons. The fermions or "matter particles" can be further divided into quarks and leptons. Bosons are force carrier particles. Gluon boson in particular facilitates strong force. The theoretical framework for studying strong force is QCD. One of the reasons why QCD is framework used is its propery of *asymptotic freedom*. This property also makes QCD much more computationally expensive at low energies due to high coupling for soft gluons. This is where Lattice QCD comes in, which is a non-perturbative approach to solving QCD. QCD is complex, involving really complex mathematics. This makes it really hard to visualize interactions in QCD. Feynman diagrams then come to rescue us by making it much easier to visualize particle interaction, by interpreting complex mathematical equation as diagrams that can be easily understood.

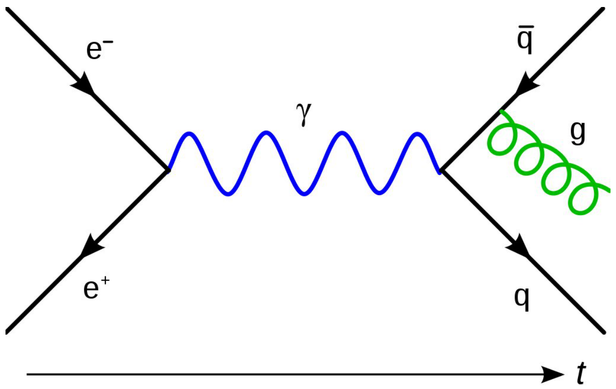


Figure 1: Feynman Diagram of Electron Positron interaction resulting in formation of quark-antiquark pair. The blue line represents photon and the green line represents gluon.

In the second week, we explored complex numbers, their properties and way to visualize them. Scattering amplitude is a complex number that establishes the probability of a scattering process and is a function of the energy in center-of-momentum frame. We then learnt about Analytic function, i.e., functions that are single-valued and differentiable everywhere at all points in some domain D over argand plane. To visualize such complex functions, say $f(z)$, we use Domain coloring, which works by coloring an argand plane W and then the color of each point of another argand plane Y is such that the color of point $z \in Y$ is the color of point $f(z) \in W$.

We looked at more of scattering theory and derived some important relations in week 3. There are some model independent features of scattering theory, such as,

- Spacetime and Internal Symmetry
- Unitarity
- Analyticity

- Crossing

  Unitarity implies a relationship between imaginary part of scattering amplitude and its modulous.

$$Im(M) = \rho|M|^2$$

Although keep in mind that this relationship is only valid when energy is $\geq 2m$ and $\leq$ first inelastic threshold. We derived polar form of scattering amplitude with help of the above relation.

$$\mathcal{M} = \frac{1}{\rho}\sin\delta e^{i\delta}$$

$\mathcal{K}$, which characterizes short range interaction between particles, was then equated to be

$$\mathcal{K}^{-1} = \rho\cot(\delta)$$

Fourth week went on with looking at particle data group's database of particle and their multiple decay modes. A particle can decay in multiple ways, given that the different conservation laws are satisfied. We also found out that lifetime of particles depend on the mechanism through which they are decaying, for example, a particle that decays via strong force will have much less lifetime compared to a particle that decays via weak force. Higher mass of a particle opens up more possible decay mechanisms due to the possibility of creation of more heavier particles. We then discussed how these hardonic resonances are produced in different facilities. Three different ways to prodcuing a resonance are show below in figure below. Note that although these resonances are created through different mechanisms, the thus produced have same properties i.e., resonances are universal.

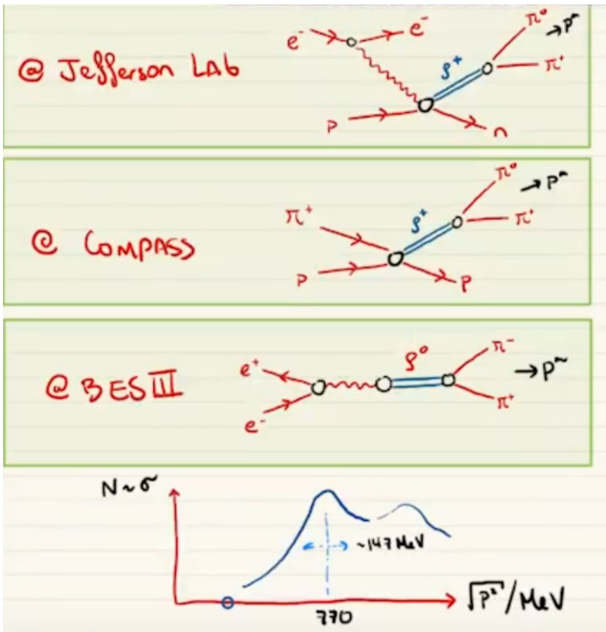


Figure 2: ρ resonance production at Jlab, COMPASS and BES III.

In last couple of weeks, we learnt how to convert feynman diagrams to mathematical equations and details about nucleon-nucleon scattering. There are certain rules for Feynman diagram in φ^4 theory for writing down a mathematical equation from the given diagrams. The rules being:

1. Every interaction point leads to a factor of $i\lambda$ where λ some real number.
2. Every external line leads to a factor of $+1$
3. Every internal line leads to a factor of $\frac{i}{p^2-m^2+i\epsilon}$. These internal lines are also knows as Feynman propagators.
4. Every Feynman propagator having unknown momentum leads to a factor of $\int \frac{dk_i}{(2\pi)^4}$ i.e., integrating over all possible momentum.
5. Factor of $\frac{1}{s}$ where s is the symmetric factor. Symmetric factor is the number of permutation of propagators that leaves the diagram unchanged.