Procedure for Satellite Crossing and Visibility Analysis

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1 Coordinate Reference Frame

All computations are performed in the **True Equator Mean Equinox (TEME)** inertial frame to ensure consistency between:

- Target propagation using SGP4 from TLE,
- Tracker propagation using Keplerian elements transformed to TEME.

The state vectors are defined as:

$$\mathbf{r} = [x, y, z]^T, \quad \mathbf{v} = [\dot{x}, \dot{y}, \dot{z}]^T \tag{1}$$

2 Relative Geometry and FOV Crossing

2.1 Relative Position Vector

The relative position vector from Tracker to Target is:

$$\mathbf{d}(t) = \mathbf{r}_{tgt}(t) - \mathbf{r}_{trk}(t) \tag{2}$$

The relative range is:

$$\rho(t) = \|\mathbf{d}(t)\| \tag{3}$$

2.2 Field-of-View (FOV) Condition

The camera boresight is aligned with the Tracker velocity vector $\mathbf{v}_{trk}$.

The angular separation between the boresight and the target direction is:

$$\cos\theta = \frac{\mathbf{d} \cdot \mathbf{v}_{trk}}{\|\mathbf{d}\| \, \|\mathbf{v}_{trk}\|} \tag{4}$$

A **crossing event** is detected when:

$$\theta \leq \frac{FOV}{2} = 15^\circ \tag{5}$$

3 Visibility Conditions

A target is considered **visible/detectable** only if all physical constraints are satisfied simultaneously.

3.1 Range Constraint

$$\rho(t) < R_{max} \tag{6}$$

where:

$$R_{max} = 1000 \text{ km} \tag{7}$$

3.2 Illumination (Sunlight) Condition

Let:

- $\mathbf{r}_{tgt}$ = Earth → Target
- $\mathbf{r}_{sun}$ = Earth → Sun
- R_E = Earth radius

Define:

$$f_{e,1} := (\mathbf{r}_{tgt} \cdot \mathbf{r}_{sun} > 0) \tag{8}$$

$$f_{e,2} := \left[\frac{(\mathbf{r}_{tgt} \cdot \mathbf{r}_{sun})^2}{\|\mathbf{r}_{sun}\|^2} - \|\mathbf{r}_{tgt}\|^2 + R_E^2 < 0 \right] \tag{9}$$

The target is sunlit if:

$$f_e = f_{e,1} \wedge f_{e,2} \tag{10}$$

This represents a cylindrical Earth shadow model.

3.3 Above-Horizon (Line-of-Sight) Condition

To ensure Earth does not block the line-of-sight between Tracker and Target:
Let:

$$\mathbf{d} = \mathbf{r}_{tgt} - \mathbf{r}_{trk} \tag{11}$$

$$r_{o,e} = \sqrt{\|\mathbf{r}_{trk}\|^2 - R_E^2} \tag{12}$$

The above-horizon condition is:

$$f_o := (\|\mathbf{d}\| \, r_{o,e} + \mathbf{d} \cdot \mathbf{r}_{trk} > 0) \tag{13}$$

This ensures that the Target is not geometrically occluded by Earth.

4 Final Detection Condition

A target is considered **detectable** if:

$$f_{vis} = (\theta \leq 15^\circ) \wedge (\rho < 1000) \wedge f_e \wedge f_o \tag{14}$$

5 Time Propagation and Event Grouping

The simulation is performed over:

$$T = 86400 \text{ s} \tag{15}$$

with discrete time steps:

$$t_{n+1} = t_n + \Delta t \tag{16}$$

where $\Delta t = 5$ s.

Crossing timestamps are recorded whenever the FOV condition is satisfied. Consecutive timestamps separated by exactly Δt are grouped into continuous intervals:

$$[t_{start}, t_{end}] \tag{17}$$

representing one geometric crossing event.