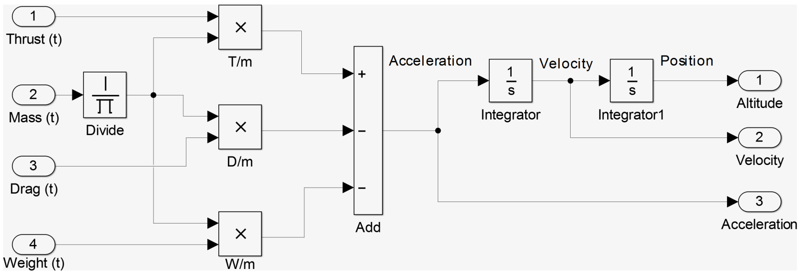
Space Concordia - Rocketry Division

Aurelius CR-2-4G - Structural Team



Engineering Simulation for Rocket Flight Analysis

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List of Abbreviations

Abbreviation	Description	Function of	Units
AOA, α	Angle of Attack		radians
COP	Center of pressure		N/A
COG	Center of gravity	time	N/A
Re	Reynolds Number	$\rho, \mu, \vec{v}, L$	dimensionless
Re_{crit}	Critical Reynolds Number	$\rho, \mu, \vec{v}, L$	dimensionless
I_{zz}	Pitch/Yaw Moment of Inertia	time	m^4
D	Drag Force (combined)		N
W	Weight of the Rocket		N
R	Specific Gas Constant		$Jkg^{-1}K^{-1}$
T	Thrust of the Rocket		N
t_f	Fin thickness	distance	m
L_{cf}	Aerodynamic Chord Length of Fins	distance	m
c	Speed of sound	$\sqrt{\gamma RT}$	
R_a	Surface Finish	*distance*	microns
M	Mach Number	$\vec{v}, c$	dimensionless
D_{pa}, C_{pa}	Parasitic Drag Force, Coefficient		
D_{fb}, C_{fb}	Body Drag Force, Coefficient		
D_{fp}, C_{fp}	Fin Pressure Drag Force, Coefficient		
D_{pr}, C_{pr}	Pressure Drag Force, Coefficient		
D_{in}, C_{in}	Interference Drag Force, Coefficient		
D_{ba}, C_{ba}	Base Drag Force, Coefficient		
D_{sk}, C_{sk}	Skin Friction Drag Force, Coefficient		
D_{aoa}, C_{aoa}	Additional Angle of Attack Drag Force, Coefficient		
C_{MC}	Corrective Moment Coefficient		
C_{FN}	Normal Force Coefficient		
C_{PDM}	Propulsive Damping Moment Coefficient		
C_{ADM}	Aerodynamic Damping Moment Coefficient		
A_{wb}	Area of Wetted Body		m^2
A_{wf}	Area of Wetted Fins		m^2
A_{fr}	Frontal Reference Area		m^2
A_{fp}	Fin Planform Area		m^2
A_{fe}	Exposed Fin Planform Area		m^2
OD,ϕ_{bt}	Outer Diameter		m
L	Total Length of Rocket		m
h_n	Height of the nose cone		m
S_{fc}	Thrust Specific Fuel Consumption		$\frac{g}{s} \cdot \frac{1}{N} = \frac{s}{m}$
$\dot{m}_{fc}$	Mass Flow Rate due to Fuel Consumption		$\frac{g}{s} \cdot \frac{1}{N} = \frac{s}{m}$
T_{avg}	Average Thrust		N
t_{burn}	Burn Time		s
m_{m_t}	Total Motor Mass		g
W_{m_t}	Total Motor Weight		N
F_N	Aerodynamic Normal Force		N
F_A	Aerodynamic Axial Force		N
F_L	Aerodynamic Lift Force		N
S_{lm}	Longitudinal Stability Margin		Calibers
f_B	Fineness Ratio		dimensionless
μ	Dynamic Viscosity		Ns/m^2
ν	Kinematic Viscosity	μ, ρ	m^2/s
λ	Angular Acceleration		rad/s^2
ω	Angular Velocity		rad/s
θ	Angular Position		$radians$

Table 2: List of Abbreviations

Engineering Simulation for Rocket Flight Analysis

Overview

The goal of this project is to create an Engineering Simulation for Rocket Flight Analysis in Matlab to model the rocket flight characteristics of the CR-2_4G Rocket.

Specifically the model should verify the maximum altitude and velocity. Further developments are explored for future enhancement. A modular development pattern will be followed order to support future expansion. Unit testing of simulator logic will be undertaken where reasonable, and further validation will be provided by testing the overall model against 3rd party flight data where available.

The model must take as input the current structural design parameters, thrust information, and simulated ambient conditions of the launch environment. Certain parameters will be dynamic; as the motor expends fuel, the position of the center of gravity and center of pressure will shift with decreasing mass. Additionally the moments of inertia will be altered. These dynamic parameters must be considered to maximize the accuracy of the model.

Beyond the primary goal of validating the rocket flight performance through simulation, this project is an educational tool to enhance the knowledge and learning of the team members and of the *Space Concordia* society as a whole. Furthermore, it will serve as a starting point for future controls and simulations applications to come, many of which are discussed in the **Future Enhancements** section.

Definition of the Problem

An engineering simulation is needed to test the flight performance of a high-powered rocket. A high-powered rocket is defined as having between 160 Ns and 40,960 Ns total impulse [1]. A team from the *Space Concordia* association at *Concordia University* is entering a submission into the *International Rocket Engineering Competition* (IREC) run by the *Experimental Sounding Rocket Association* (ESRA). The competition provides performance targets which must be met in order to be eligible to win. These targets are held as design requirements for performance, and are listed in the **Requirements** section below.

Requirements

The Performance Model must provide the maximum altitude and velocity of the rocket in subsonic flight under a known thrust curve and known dimensional parameters.

- 2a Static stability between 2 and 3 calibers
- 2b Dynamic stability above 0
- 2c Max velocity at launch rail 30.5 m/s
- 2d Vehicle max speed mach 0.9
- 2e Vehicle reaches 10,000 ft altitude (+1000 feet / - 0 feet)
- 2f Vehicle doesn't experience resonant pitching/yawing moment

[SCRD 2016 Specifications and Requirements]

Validation

- Sub-Models will be unit tested with 3rd party data to ensure functional validity where possible.
- Once all Sub-Models are validated individually, the overall model will be validated using 3rd-party data.
  - Primary 3rd-party simulation data source will be OpenRocket simulations
  - Where possible, actual 3rd-party rocket flight data will be used to validate the model

Problem Solving Approach

Kinematics

Kinematics is the study of the motion without consideration of the forces in play. This analysis can be simplified by considering the entire mass of a body at a convenient point, such as the *Center of Gravity* (point-mass). Of particular interest are the position, velocity, and acceleration of the point [2].

Dynamics

Dynamics is the study of motion which considers the forces in play. It is useful to consider bodies as *rigid bodies* in order to simplify the forces that act on them (e.g. ruling out the stiffness of the body) [2]. Forces acting at a distance d from the center of gravity create a rotation about the center of gravity.

Decoupling the Model

If it can be said that the translation of the body at its center of gravity (kinematics) does not impact the rotation of the body about its center of gravity by a force applied at some other point (the *Center of Pressure*), the analysis can be greatly simplified. Such a *mutually independent system of equations* could be solved rather easily, as the translational model could be solved independently of the rotational model, and vice-versa [2]. Coupled systems of equations, on the other hand, would have to be solved simultaneously.

As it happens, the translational model of the rocket is not fully decoupled from the rotational model. As the rocket rotates while travelling through the air at an angle of attack, it presents a larger reference area and thus increases the drag force, causing a negative acceleration in the translation model. Likewise, an increase in the translational velocity of the rocket creates a larger lift force when the rocket is rotating at an angle of attack. This lift force, which is applied at the rocket's center of pressure, creates a rotation about the center of gravity.

However, it is possible to proceed with a decoupled analysis with the assistance of careful assumptions and approximations within acceptable accuracy. If accepted, this would imply a *weak coupling* between the translational model and the rotational model [2].

Assumptions

- subsonic flight
- axis-symmetric rigid body rocket
- single cylindrical body
- Von Karman nose shape
- three or four trapezoidal fins
- passively controlled (no active thrust or stability control)
- constant fuel expenditure rate
- vertical/linear flight within 5 degrees [3]
- the *Ideal Gas Law* applies throughout the flight
- humidity in the air is ignored
- steady-state irrotational flow around the body [4]
- fully aligned thrust [3]
- smooth transition between nose cone and body tube (no shoulder)
- rocket does not have a boattail
- rocket has a single rectangular launch lug

Coordinate System

Position

Rocket Coordinates

World Coordinates

Orientation

Input Parameters

Before beginning the modeling process, it is necessary to understand the inputs of the system.

Many parameters can be considered unchanging during flight, and are from this point referred to as *Static Parameters.* Other inputs change as a function of time or velocity, and must be carefully handled within the system - these parameters are herein referred to as *Dynamic Parameters.*

Atmospheric Model

The *International Standard Atmosphere* model is assumed to describe the pressure, temperature, density and viscosity conditions of the surrounding air during launch.

[5]

Viscosity

Previously, in-house simulations were conducted using the *velocity_model* created by Alex Botros. Of interest to this report, is the method by which the viscosity of the working fluid was calculated. It will include a review of the previous methods use and it will introduce *Sutherland's law.* As well, both methods will be compared and changes to the existing model will be proposed.

Previous Work

In the *velocity_model* the absolute viscosity was never actually calculated. Instead, the kinematic viscosity was calculated (as shown below) as part of the function to solve for the *Reynolds Number* of the vehicle.

$$\nu = (-1E-14)T^3 + (1E-10)T^2 + (3E-8)T - (3E-6) \tag{1}$$

As can be clearly seen, the kinematic viscosity was estimated based on the temperature T. It is assumed that the temperature is in Kelvin. When the function shown here is plotted, it creates the following result.

Static Parameters

The following tabulated parameters are considered to remain constant during flight

The following table lists all input parameters to the system TODO export the parametric spreadsheet

Table 3: Input Parameters

Parameter	Units	Status	Source
area_fin_frontal	m^2	Static	CATIA
area_face_fin	m^2	Static	CATIA
width_fins_at_tube	m	Static	CATIA
fins_count	N/A	Static	CATIA
thickness_fins_at_tube	m	Static	CATIA
area_surface_nose	m^2	Static	CATIA
height_fins	m	Static	CATIA
diameter_outer	m	Static	CATIA

Parameter	Units	Status	Source
surface_roughness	m	Static	CATIA
length_total_rocket	m	Static	CATIA
pressure_ambient_ground	Pa	Static	Manual
temperature_ambient_ground	C	Static	Manual
distance_tip_to_COG	m	Dynamic	CATIA
distance_tip_to_COP	m	Dynamic	Manual
diameter_outer_launch_guide	m	Static	CATIA
diameter_inner_launch_guide	m	Static	CATIA
moment_inertia_rocket	m^4	Dynamic	CATIA

TODO expand

These parameters are written in the parametric spreadsheet shared with the design team. The CAD software populates values related to the structural design, and others are entered manually. The performance model reads them at the beginning of execution to simulate the latest design iteration.

Dynamic Parameters

Parameters listed as *dynamic* in the table above are provided as initial values which are then recalculated by the model throughout the simulated flight.

Force

Force is a change in momentum with time, and is related by Newton's Second Law

$$F = \frac{m\Delta\vec{v}}{\Delta t} = m\vec{a} \tag{2}$$

Impulse

Impulse is the product of force and integration of a differential (infinitesimal period) of time between the time periods in which it was applied.

$$J = \int_{t_1}^{t_2} F dt \tag{3}$$

This is also known as the *Total Impulse*, or the *total change in momentum*, and can be calculated as the average thrust over a given time period.

Thrust

Thrust is the mechanical force that drives the flight of the rocket. It is a vector quantity of magnitude and direction. *Thrust* is a reaction force in the opposite direction of accelerating fluid (exhaust gas) caused by the combustion of fuel, and is assumed to be aligned with the longitudinal axis of the rocket.

Mass Flow Rate

Mass Flow Rate is found by the product of fluid density, velocity, and cross-sectional area.

$$\dot{m} = \rho\vec{v}A \tag{4}$$

NASA - Thrust

Thrust Equation

$$T = \dot{m}\Delta\vec{v} \tag{5}$$

NASA - Thrust Equation

Thrust curves are provided by the manufacturer, and are rated at a constant fuel expenditure rate known as the specific fuel consumption (S_{fc})

Table 6 shows an example of the motor data provided by ThrustCurve.org:

Parameter	Value
Manufacturer	Cesaroni Technology
Entered	May 20, 2009
Last Updated	Jun 26, 2014
Mfr. Designation	6819M1540-P
Common Name	M1540
Motor Type	reload
Delays	P
Diameter	75.0mm
Length	75.7cm
Total Weight	5906g
Prop. Weight	3624g
Cert. Org.	Canadian Association of Rocketry
Cert. Designation	6819-M1540-IM-P
Cert. Date	
Average Thrust	1537.0N
Maximum Thrust	2328.8N
Total impulse	6819.4Ns
Burn Time	4.4s
Case Info	Pro75-5G
Propellant Info	Imax
Availability	regular

Table 6: Sample Motor Data

Source: http://www.thrustcurve.org/motorsearch.jsp?id=673

Thrust Specific Fuel Consumption

Thrust Specific Fuel Consumption is how much fuel is burned for a given time.

$$S_{fg} = \frac{m}{t_{burn}} \cdot \frac{1}{T_{avg}} \tag{6}$$

$$\left[\frac{g}{s} \cdot \frac{1}{N} = \frac{s}{m}\right] \tag{7}$$

Since at the time of writing the S_{fc} was not provided by the manufacturer, the following calculations are used for a first approximation.

Assumptions

- all propellant is spent during the motor burn time
- final S_{fg} determined is constant during burn
- the motor info provided is accurate

It should be noted that a variance in thrust of $\pm 20\%$ is possible. This and other variance factors are taken into account in the *Statistical Analysis* section.

From the table above, the dry propellant weight is given as 3624 grams. The Average Thrust is given as 1537.0 Newtons, and the total burn time is given as 4.4 seconds.

Thus, the *Thrust Specific Fuel Consumption* can be determined as follows:

$$S_{fg} = \frac{3.624\,kg}{4.4\,s} \cdot \frac{1}{1537.0\,N} \approx 0.00053587 \frac{kg}{N \cdot s} = 5.3587 \times 10^{-4} \frac{kg}{N \cdot s} \tag{8}$$

This rate is considered constant.

Weight

As fuel is expended in generating thrust, the weight of the rocket is reduced. One assumption that can be made is that the change of mass with time is "proportional to the impulse of the motor up to that point" [3].

$$\Delta M_i = -\frac{M_f \int_0^i T dt}{\int_0^\infty T dt} \tag{9}$$

Where:

- M_f is the total mass of fuel
- T is the thrust
- ΔM_i is the change in mass of fuel between time i and t

[3]

We can also use the *Thrust Specific Fuel Consumption* to determine the corresponding reduction in weight during burn, if we assume that the mass flow rate of fuel during burn is constant.

First remove the Average Thrust term to isolate the mass flow rate:

$$\dot{m} = S_{fG} \cdot T_{avg} = 5.3587 \times 10^{-4} \frac{kg}{N \cdot s} \cdot 1537.0\,N = 0.8236\,kg/s \tag{10}$$

This equation can be expressed in terms of Weight through Newton's $2^n d$ law: $F = m\vec{a}$

$$\dot{W}_m = \dot{m} \cdot \vec{g} = 0.8236\,kg/s \cdot 9.81\,m/s^2 \approx 8.0799 N/s \tag{11}$$

To develop a relation for the change in weight as a function of S_{f_c}

$$W_f(t) = (m_{f_i} kg - \Delta m(t)) \cdot \vec{g} \tag{12}$$

$$W_f(t) = (3.624\,kg - \Delta m(t)) \cdot 9.81\,m/s^2 \tag{13}$$

$$W_f(t) = W_{f_i} - \Delta W_f(t) \tag{14}$$

$$\Delta W_f(t) = \int \dot{W} dt = \dot{W} \cdot t \tag{15}$$

$$\Delta W_f(t) = \frac{\Delta m(t) \cdot \vec{g}}{t} \cdot t = \Delta m(t) \cdot g \tag{16}$$

$$\Delta m_f(t) = S_{fg} \cdot t \tag{17}$$

Finally, the motor weight as a function of time is

$$W_m(t) = W_{m_t} - \Delta W_f(t) = W_{m_t} - \dot{m}_{fc} \cdot t \tag{18}$$

Matlab Implementation

The following figure shows the output of the dynamic weight calculation in Matlab. The Thrust and Weight curves produce output similar to OpenRocket as expected.

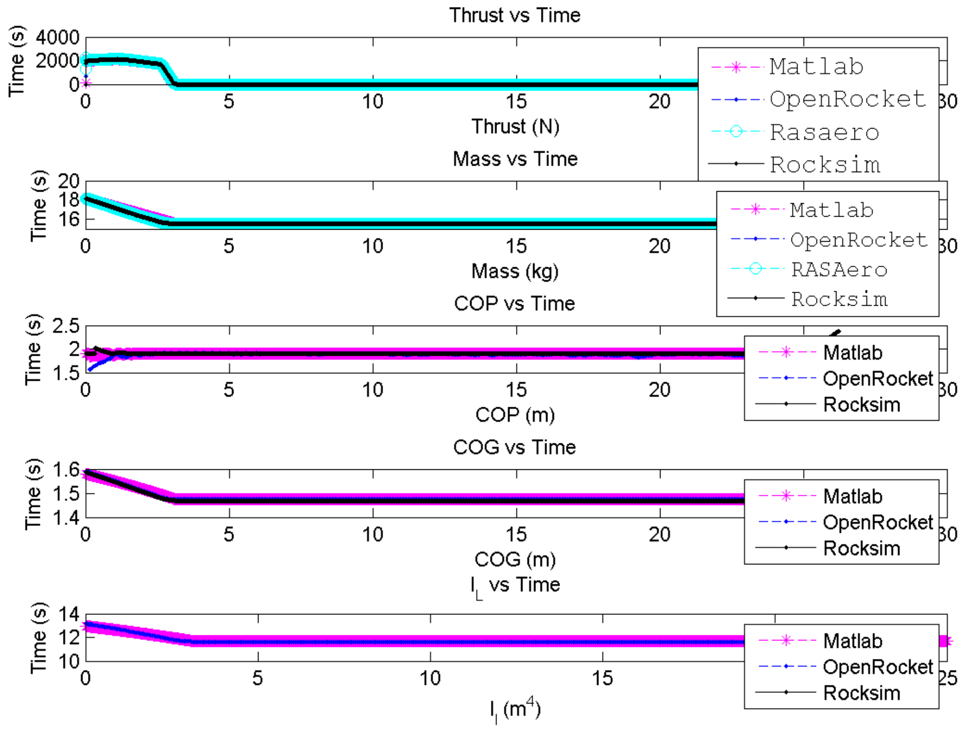

Figure 1: Dynamic Weight Calculation Test Output

Center of Pressure

The *Center of Pressure* (COP) is the location (point) where the aerodynamic forces can be said to be acting, simplifying the complex distribution of forces across the rocket and its features.

The *Center of Pressure* changes with the normal force distribution on the rocket, which is driven by *Angle of Attack* [6].

$$COP = COP(\alpha)$$

A wind tunnel is the best way to approximate this point, but an analytic method is available.

Barrowman's Equations

Barrowman's Equations are used to determine the *Center of Pressure*.

$$\bar{X} = \frac{(C_{N\alpha})_n \,\bar{x}_n + (C_{N\alpha})_{cb} \,\bar{x}_{cb} + (C_{N\alpha})_{fb} \,\bar{x}_{fb}}{C_{N\alpha}} \tag{19}$$

Where:

- $C_{N\alpha}$ is the *Stability Derivative*
- subscript $_n$ refers to the nose cone
- subscript $_{cb}$ refers to the cylindrical body
- subscript $_{fb}$ refers to the fin set in the presence of the body
- $\bar{x}$ refers to the component centroid

[6]

Nose Cone COP

LV-Haack Nose Cone COP

$$\bar{X}_n = 0.437 h_n \tag{20}$$

[7]

Von Karman Nose Cone COP

$$\bar{X}_n = 0.500 h_n \tag{21}$$

[7]

Fin Set COP

The force acting on the fins can be calculated using the following equation (applies only for identically shaped fins, in sets of 3, 4, or 6).

$$(C_{n\alpha})_f = C_{in} \frac{4n\left(\frac{s}{d}\right)^2}{1+\sqrt{1+\left(\frac{2l}{a+b}\right)^2}} \tag{22}$$

Where:

- a is the fin tip chord length
- b is the fin root chord length

- s is the fin height
- l is the distance between the root center and the tip center
- C_{in} is a coefficient for the interference effects of the air flow near the fin-body interface

$$C_{in} = 1 + \frac{OD/2}{OD/2 + s} \tag{23}$$

[6]

The location of the *Center of Pressure* for the fin set is as follows.

$$\bar{X}_{fb} = X_f + \frac{m(b+2a)}{3(b+a)} + \frac{1}{6}\left[b + a - \frac{ba}{b+a}\right] \tag{24}$$

[3]

Where:

- X_f is the distance from the tip of the nose cone to the point where the leading edge of the fin meets the body tube [3]
- a is the fin tip chord length
- b is the fin root chord length
- s is the fin height
- m is the fin sweep length

Also, see this Center of Pressure Calculator online.

Cylindrical Body COP

The centroid of a cylindrical body will be half its length

$$\bar{x}_{ct} = \frac{1}{2} l_{cb} \tag{25}$$

Wind tunnel tests performed in 1918 and 1919 demonstrated that the normal force generated by a cylindrical body at an angle of attack of less than 10 degrees is negligible

[6].

Stability Derivative

The *Stability Derivative* $C_{N\alpha}$ is a dimensionless parameter, used to calculate the force normal to the longitudinal axis, and is dependent on the shape of the component. It is the slope of the *Normal Force Coefficient* plotted against the angle-of-attack. For low angles of attack, it is nearly constant.

TODO show figure

???

The total *Stability Derivative* is the sum of all i rocket component stability derivatives

$$C_{N\alpha} = \sum C_{N\alpha(i)} \tag{26}$$

[3]

Nose Cone

$$C_{N\alpha(n)} = 2 \tag{27}$$

Rocket Body

$$C_{N\alpha(bt)} = 0 \tag{28}$$

Fins

$$C_{N\alpha(fs)} = ... \tag{29}$$

Rocket Body Lift Correction

Barrowman's Method neglects the lift generated by the rocket body. Galejs [8] suggests the following adjustment to provide a compensated *Coefficient of Normal Force due to Body Lift*

$$C_{N(L)} = K\frac{A_p}{A_ref}\alpha^2 \tag{30}$$

Where:

- $K = 1$
- A_p is the rocket planform area excluding the fins
- A_{ref} is the reference area of the rocket

[3]

Equation 30 is divided by α to be added to $C_{N\alpha}$ calculated and used in Equation 19.

TODO All COP components must be modified by the lift coefficient

$$C_{N\alpha^2} = K\frac{A_p}{A_ref}\alpha \tag{31}$$

This correction is applied at the centroid of the planform area.

[3]

Transonic Considerations

Barrowman's Equations are based on assumptions that are only valid in subsonic flight. In the transonic and supersonic regions, what new effects are introduced that would affect the location of the *Center of Pressure*?

Center of Gravity

$$y_{cg} = \frac{m_1y_1 + m_2y_2 + ... + m_ny_n}{\sum_{j=1}^{n} m_j} \tag{32}$$

$$COG(t) = \frac{m_1y_1 + (m_2 - \Delta m)y_2}{m_1 + m_2 - \Delta m(t)} \tag{33}$$

Where COG(t) is the Center of Gravity as a function of time, m_1 is the static mass (combination of nose cone, body tube, and fins), m_2 is the initial mass of the motor, and $\Delta m(t)$ is the change of mass as a function of time due to fuel expenditure.

We consider the motor as a point mass centered at the geometric center of the motor casing. This simplifies the calculation of the center of gravity of the rocket as fuel is expended, as only the mass of the motor is changing, and not the location of its particular center of mass.

Moments of Inertia

The instantaneous moment of inertia is determined by relating the moment of inertias of the static structure and the dynamic structure through the parallel axis theorem evaluated at the total center of gravity (COG).

The sum of moment of inertias evaluated through the *parallel axis theorem* nets the total rocket moment of inertia.

$$I_n = I_{cm(n)} + M_P d^2 \tag{34}$$

$$I_T(t) = \sum I_n \tag{35}$$

Where:

- $I_T(t)$ is the total moment of inertia of the rocket as a function of time
- I_n is the component vector (either static or dynamic moment of inertia)

[3]

Longitudinal Moment of Inertia

To the *Moment of Inertia* related to the pitch/yaw of the rocket is the *Longitudinal Moment of Inertia*.

$$I = \frac{mL^2}{12} \tag{36}$$

[TODO source dynamics textbook]

In keeping with the assumption of the motor as a point mass in the volumetric center of the motor casing, the dynamic *Longitudinal Moment of Inertia* is calculated as follows.

$$I_s + m_s r_{0\to1}^2 \tag{37}$$

Where $r_{0\to1}$ is the distance between the static center of gravity (the COG of the nose cone, body tube, and fins) and the instantaneous center of gravity of the rocket. I_s is provided by CATIA.

$$I_m = \frac{m_m L_m}{12} + m_m r_{0\to2}^2 \tag{38}$$

Where L_{motor} is the length of the motor casing, and $r_{0\to2}$ is the distance between the motor center of gravity and the rocket center of gravity.

Then, the rocket *Longitudinal Moment of Inertia* is the sum, shown as follows

$$I_r = I_m + I_s \tag{39}$$

Rocket Orientation

Building on the material explored in the Angular Stability model, this section explains the orientation of the rocket during flight.

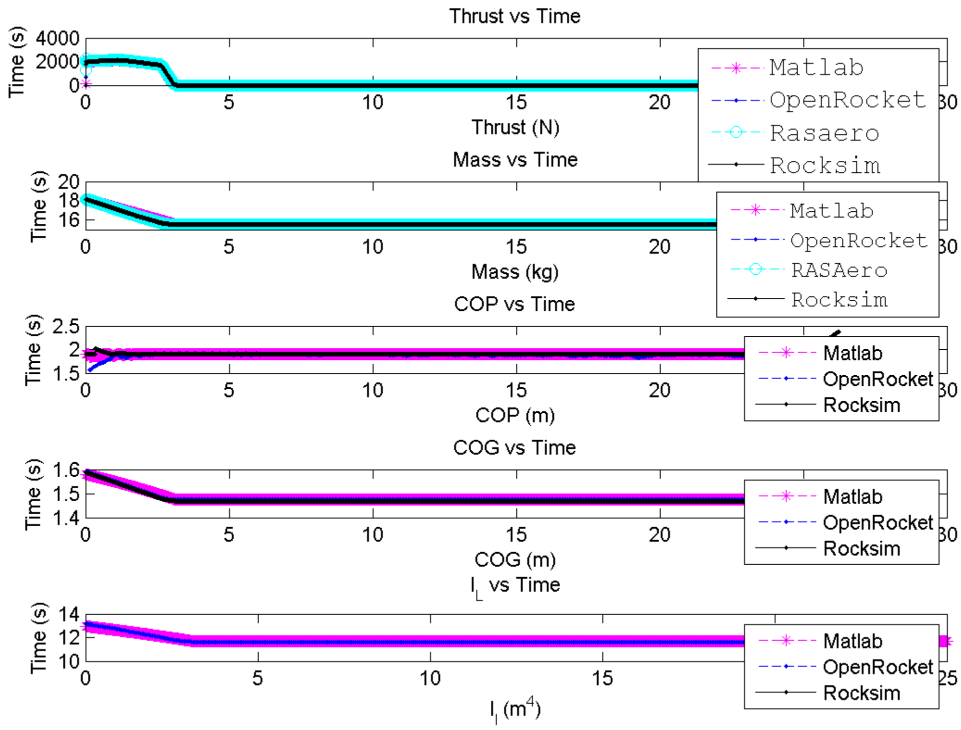

Figure 2: Forces and Moments Experienced by rocket in flight

Rotations

Spherical Coordinates and *Euler Angles* are commonly used to describe the orientation of an object, however both systems encounter singularities where the orientation is ambiguous. Special cases are required to handle these singularities, which complicate the analysis and programming.

Quaternions

Quaternions are commonly used to describe spatial rotation, avoiding singularities.

An initial position is taken as reference to describe subsequent changes in orientation. Vectors can be transformed from rocket coordinates to world coordinates, and the reverse.

Leonhard Euler proved that

$$e^{i\phi} = \cos\phi + i\sin\phi \tag{40}$$

Thus, $e^{i\phi}$ lies on the unit circle in the complex plane, and has a unit length.

Multiplication

$$(a+bi)(c+di) = re^{i\phi}se^{i\phi} = rse^{i(\phi+\theta)}$$

$$i^2 = j^2 = k^2 = ijk = -1$$

Quaternions

$$H = a + bi + cj + dk : a, b, c, d \forall Rj$$

i, j, k are all square roots of -1

$$ij = k = -ji$$
$$jk = i = -kj$$
$$ki = j = -ik$$

If we consider three-dimensional space to be purely imaginary quaternions:

$$R^3 = xi + yj + zk : x, y, z \forall R$$

Rotations are done using unit quaternions

$$\cos\phi + i\sin\phi$$
$$\cos\phi + j\sin\phi$$
$$\cos\phi + k\sin\phi$$

Which can be rewritten as

$$e^{i\phi}, e^{i\phi}, e^{k\phi}$$

For example, taking any arbitrary unit quaternion $(vector)$ **u**

$$u = u_1 i + u_2 j + u_3 k = e^{u\phi}$$

We can rotate another arbitrary vector **v** about the axis in the **u** direction

$$e^{u\phi} v e^{-u\phi}$$

http://math.ucr.edu/~huerta/introquaternions.pdf

Complex Plane

Parameters needed for quaternion analysis

- rocket mass
- **reference length**
- angle-of-attack
- reference area
- longitudinal moment of inertia
- radial moment of inertia
- thrust force
- drag force
- weight force
- pitching moment
- pitch damping moment
- yawing moment
- yaw damping moment
- ~~roll moment~~
- ~~roll forcing moment~~
- ~~roll damping moment~~
- normal force
- side force
- pitch rate
- raw rate
- roll rate
- wind velocity

Coordinate System

Position

Rocket Coordinates

World Coordinates

Rocket Moments

A *Pitch Moment* and *Pitch Damping Moment* are defined, which are different than the *Corrective Moment Coefficient* and the *Damping Moment Coefficient*. Note: a complementary *Yaw Moment* and *Yaw Damping Moment* are implied, with exactly the same considerations for motion along the uncoupled complementary yaw-axis.

Pitch Moment

The *Pitch Moment* is taken from the tip of the nose cone, it must be moved to the COG to mirror *Corrective Moment Coefficient*.

$$M_{pm}(x) = -\frac{1}{2}\rho v^2 C_m A_{ref} * L_{ref} \tag{41}$$

TODO why minus?

Where:

- ρ is the local atmospheric density
- v is the rocket velocity relative to the surrounding air
- $A_r ef$ is the reference area (frontal or side?)
- L_{ref} is ???

Pitch Moment Coefficient

$$C_m = C_m - F_s \cdot COG \cdot L_{ref} \tag{42}$$

Where:

- F_s is the side force

Fin-set Pitch Moment Coefficient

$$M_{pm} = \frac{C_N \times COP}{L_{ref}} \tag{43}$$

Body Tube and Fin Set Interference Pitch Moment Coefficient

$$M_{pm} = \frac{C_N \times COP_x}{L_{ref}} \tag{44}$$

Barrowman Calculation

$$total.getCM + forces.getCM \tag{45}$$

Final Solver Pitch Moment Coefficient

$$M_{pm} = M_{pm} + (\text{randomness}) \tag{46}$$

Some randomness is added to avoid an "over-perfect" solution

Pitch Damping Moment

- significant only near apogee

Vertical (AOA $< 5^\circ$) Flight Model

Particular Assumptions

The rocket is to be launched on a guide that may have a $\pm$ 5° angle. Considering the small angle approximation, the sine of 5 degrees or less is approximately equal to the angle in radians, or zero.

For a 1% error:

$$sin(\theta \leq 15^\circ) \approx 0 \tag{47}$$

[9]

This assumption greatly simplifies the simulation analysis. We consider that the rocket flies perfectly vertical (experiencing no significant drift) into still (quiescent) air for which density is described by the International Standard Atmosphere (ISA) model.

Simplified Model

The dynamics of the rocket flights can be simplified to a sum of forces.

Simplifying the rocket flight as ideally one-dimensional, with the positive z-direction being upwards from the launch pad, the impulse is equal to the thrust of the rocket minus the weight of the rocket and the drag forces of the rocket interacting with the surrounding air.

$$m(t)\ddot{z}(t) = T(t) - D(\dot{z}) - W(t) \tag{48}$$

Mass is a function of time, which is explained in the *Dynamic Parameters* section. Drag is a function of velocity, which is explained in *Drag Model* section. Acceleration can be expressed as the first derivative of velocity and also the second derivative of position, each with respect to time.

$$\vec{a} = \dot{v} = \ddot{z} \tag{49}$$

Each force component can be rearranged and expressed as follows:

$$\vec{a}_T = \frac{T(t)}{m(t)}, \vec{a}_W = \frac{W(t)}{m(t)}, \vec{a}_D = \frac{D(v)}{m(t)} \tag{50}$$

The net upward acceleration is: $\vec{a}_T - \vec{a}_W - \vec{a}_D$

The sum of forces can be rearranged and acceleration can be solved for:

$$\vec{a} = \ddot{z} = \frac{1}{m(t)}(T(t) - D(\dot{z}) - W(t)) \tag{51}$$

Acceleration can be integrated to find position and velocity.

$$\vec{v} = \int \vec{a} dz \tag{52}$$

$$z = \iint \vec{a} dz \tag{53}$$

Integration of equation (51) in the model is represented by the $\frac{1}{s}$ block. The model is pictured in Figure 3.

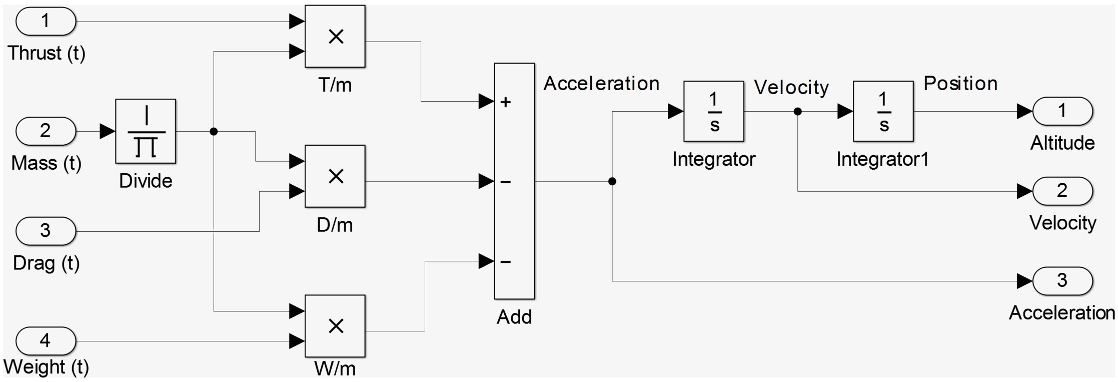

Figure 3: Vertical Flight Model - Simplified

Weathercocking

Weathercocking is a phenomenon when the rocket tends to alter its trajectory and fly into the wind. If the rocket is stable, and has a sufficiently high damping ratio, the rocket eventually reaches a near-zero angle-of-attack parallel to the velocity vector of the wind, only in the opposite direction.

We can modify Equation 48 to account for weathercocking and provide the actual altitude reached, as well as the amount of drift experienced.

Altitude accounting for weathercocking

$$m(t)\ddot{z}(t) = T(t)\cos\theta - D(\dot{z})\cos\theta - W(t) \tag{54}$$

Where:

- z is the upward direction (normal from the ground)
- θ is the angle between the current rocket trajectory and the z-axis

Drift accounting for weathercocking

$$m(t)\ddot{z}(t) = T(t)\sin\theta - D(\dot{z})\sin\theta \tag{55}$$

Alternatively

The altitude lost to weathercocking can be calculated based on the

Aerodynamic Geometry

Overview

Related to aerodynamic geometry of the rocket, the specific parameters of interest are the following:

- Outer Diameter of Rocket (OD)
- Total Length of Rocket (L)
- Height of Nose Cone (h_n)
- Thickness of Fins
- Number of Fins
- Width of Fins
- Surface Area of Nose

Surface Roughness

Surface Roughness is the deviation in the normal direction from a surface of its features. It contributes to *Skin Friction Drag*

Fineness Ratio

The *Fineness Ratio* is the ratio of the length to the outer diameter

$$f_B = \frac{L}{OD} \tag{56}$$

Fins

Aerodynamic Chord Length of Fins

Since there is no airfoil on the fin design, the *Aerodynamic Chord Length of the Fins* (L_{cf}) is equal to the height of the fins.

Areas

Reference areas are required to calculate the drag force.

Wetted Body Area

The *Wetted Body Area* is the combined area of all surfaces in contact with moving air.

Frontal Reference Area

The *Frontal Reference Area* is the projected area of the rocket perpendicular to the direction of air flow. For perfectly vertical flight and quiescent air conditions, this is the precise projection of the tip face of the rocket.

[TODO show figure]

Frontal Reference Area at Angle of Attack

When the rocket pitches into the free stream at an angle of attack, a greater portion of the rocket comprises the frontal reference area. We do not account for this, as it would be extremely complex to evaluate. We consider the frontal reference area does not change with angle-of-attack.

Planform Area

Nose Profile

Von Karman (Haack)

A *Von Karman* nose profile has been selected by the design team, other profiles will not be supported in the initial version of the model. The *Von Karman* nose profile is a *Haack Series* geometry, designed to minimize theoretical pressure drag [niskanen2013]. This profile excels in subsonic flow conditions, and performs well in transonic flow conditions [nassaNoseCone] - as such is it well suited for the current mission.

The equation for the *Haack Series* is

$$r(x) = \frac{R}{\sqrt{\pi}}\sqrt{\theta - \frac{1}{2}sin(2\theta) + \kappa \sin^3\theta} \tag{57}$$

Where

$$\theta = \cos^{-1}\left(1 - \frac{2x}{L}\right) \tag{58}$$

Nose Profile Design

Nose Profile Design

Aerodynamic Center

The *Aerodynamic Center* is the point where the *Pitching Moment* does not change with angle-of-attack

Sources:

- http://ocw.mit.edu/courses/aeronautics-and-astronautics/16-100-aerodynamics-fall-2005/lecture-notes/16100lectre10_cg.pdf
- http://www.digplanet.com/wiki/Aerodynamic_center

Drag Model

Rockets in flight experience multiple sources of drag. The total drag effect is the sum of all specific drag effects.

Figure 4 depicts the types of drag forces to be expected in subsonic flight at *zero-angle of attack*.

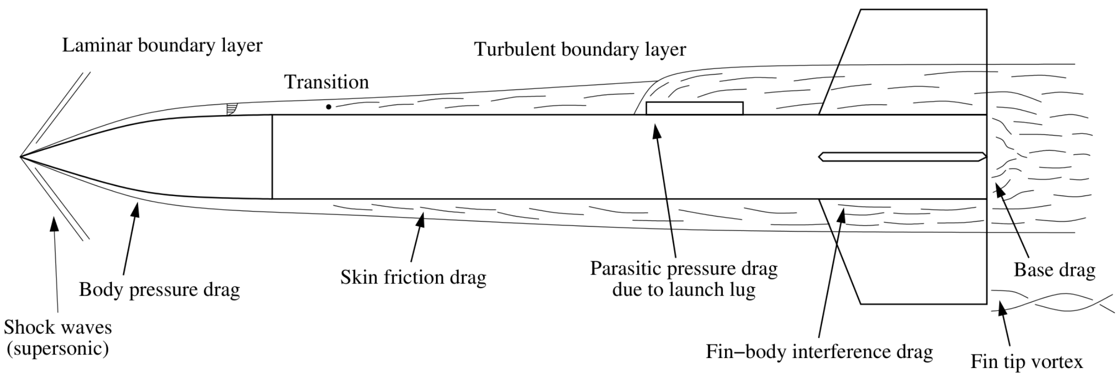

Figure 4: Rocket Drag Sources - Subsonic Flight

[4]

The two main contributing factors to *Drag Force* are *Skin Friction* and pressure distribution effects. Pressure distribution effect are broken down into body pressure and parasitic drag effects, among others [4]. These and other drag forces are detailed in this section.

The drag model must take the parametric design parameters and applicable dynamics parameters (see *Data Model*) to output the Drag Force and combined drag coefficient.

Mach Number

Mach Number (M) is the ratio of the airspeed to the speed of sound for air at a given temperature

The speed of sound (c) is calculated as follows

$$c = \sqrt{\gamma R T} \tag{59}$$

The *Ideal Gas Law* states that

$$P = \rho R T \tag{60}$$

We assume that the *Ideal Gas Law* applies, and use it to solve for RT using pressure and density.

$$RT = \frac{P}{\rho}$$

Thus we can calculate the speed of sound as follows

$$c = \sqrt{\gamma \frac{P}{\rho}} \tag{61}$$

Where p is the local pressure, ρ is the local density, and γ is the *adiabatic index*, known as the *isentropic explansion factor* - it is the ratio of the specific heats of a gas at constant pressure and constant volume.

[10]

The *Mach Number* is then the ratio of the air velocity to the sound speed of the local air

$$M = \frac{\vec{v}}{c} \tag{62}$$

Mach Regions

Velocity regions are defined, in which aerodynamic effects are known to vary considerably. The following velocity regions are established for further discussion.

Mach Region (M)	Classification
0.3 < 0.8	Subsonic
0.8 < M < 1	Transonic
1 < M < ~5	Supersonic
M > ~5	Hypersonic

Table 8: Mach Regions

[4]

As the rocket is constrained not to exceed Mach 0.9, much of the flight will be in the subsonic region, greatly simplifying much of the analysis. However, transonic effects cannot be ignored when at a Mach Number greater than 0.8.

Incompressible Flow

For Mach < 0.3,

> In the incompressible flow regime the forces can be divided into pressure force and viscous force

Pressure Force is due to fluid stagnation on areas of the rocket, as well as due to the low pressure region created beyond the rocket at is passes quickly through the air.

Viscous Force is due to boundary layer effects and interactions of moving air with surfaces. These forces are highly dependent on Reynolds number. [3]

Compressible Flow Correction

Special considerations apply when compressibility effects are in play. These effects occur above Mach 0.3 [3], which will be easily exceeded by the transonic upper limit of Mach 0.9 mandated by the competition.

> At low speeds (incompressible flow), the aerodynamic coefficients are functions of the angle of attack (α) and Reynolds number (Re).

$$C_i(M < 0.3) = C_i(\alpha, Re) \tag{63}$$

> At higher speeds (compressible, Ma $\geq$ 0.4) they are also a function of Mach number.

$$C_i(M \geq 0.3) = C_i(\alpha, Re, M) \tag{64}$$

Particular correction factors are recommended for ranges of Mach number

Mach Number	Correction Factor
$M < 0.3$	N/A
$0.3 < M < 0.8$	$C_i^{'} = \frac{C_i}{\sqrt{1-M^2}}$
$0.8 < M < 1.1$	$C_i^{'} = \frac{C_i}{\sqrt{1-(0.8)^2}}$
$M > 1.1$	$C_i^{'} = \frac{C_i}{\sqrt{M^2-1}}$

Table 10: Prandtl-Glauert Compressible Flow Correction Factors

Where C_i is the incompressible drag coefficient and $C_i^{'}$ is the compressibility corrected drag coefficient [3].

Turbulent Effects

> A turbulent boundary layer induces a notably larger skin friction drag than a laminar boundary layer

[4]

Stagnation Pressure

Stagnation Pressure is the pressure on the normal surfaces to airflow.

For a cylindrical rocket, it can be approximated as follows [4]

$$\frac{q_{stag}}{q} = \begin{cases} 1 + \frac{M^2}{4} + \frac{M^4}{40} & M < 1 \\ 1.84 - \frac{0.76}{M^2} + \frac{0.166}{M^4} + \frac{0.035}{M^6} & M > 1 \end{cases} \tag{65}$$

Where q_{stag} is and q is

Then, the *Pressure Drag Coefficient* can be expressed as a function of *Mach Number*

$$C_{pr} = 0.85\frac{q_{stag}}{q} \tag{66}$$

Reynolds Number

The *Reynolds Number* is a dimensionless number which describes the ratio of the kinematic effects of a fluid to viscous effects.

$$Re = \frac{\rho\vec{v}d}{\mu} \tag{67}$$

[10]

Critical Reynolds Number

The *Critical Reynolds Number* (Re_{crit}) is the value of *Reynolds Number* where the flow changes from laminar to turbulent. This is greatly dependent on the surface roughness [munson2013].

[4] gives the *Critical Reynolds Number* as

$$R_{crit} = \frac{\vec{v}x}{\nu} \tag{68}$$

Where:

- $\vec{v}$ is the free stream air velocty
- x is the distance along the body from the nose cone tip where turbulent flow begins
- ν is the kinematic viscosity of air

For $Re_{crit} = 5 \times 10^5$

- $\nu = 1.5 \times 10^-5m^2/s$
- $v_0 = 100m/s$
- $x = 7cm$ from the nose tip, where turbulent flow begins

[4]

Trinh, Khanh Tuoc See Fluids Text book

Surface roughness has a considerable influence on *Critical Reynolds Number*. It can be determined as follows.

$$R_{crit} = 51\left(\frac{R_s}{L}\right)^{-1.039} \tag{69}$$

[4]

Actual Reynolds Number

The *Actual Reynolds Number* can be expressed in the following form:

$$Re = \frac{\vec{v}L}{\nu} \tag{70}$$

Where:

- $\vec{v}$ is the free stream velocity
- L is the length of the rocket
- ν is the kinematic viscosity of the air in free stream

Drag Force and Coefficients

The total drag force is a function of air velocity (relative to the rocket body) drag coefficient, reference area, and air density.

$$D_f = D_f(\vec{v}, C_d, A_{ref}, \rho) \tag{71}$$

The drag coefficient C_d is the sum of all component drag coefficients

$$C_d = \sum C_i = C_{pa} + C_{fo} + C_{pr} + C_{in} + C_{ba} + C_{sk} + C_{fp} + C_{wa} + C_{bt} + C_{aoa} \tag{72}$$

From Fluid Mechanics [source?]

$$D_f = \frac{1}{2} C_d A_{ref} \rho \vec{v}^2 \tag{73}$$

Viscous Drag Effects

Skin Friction Drag

Skin Friction Drag is due to viscous effects during flight, and is significantly influenced by surface roughness.

$$D_{sk} = \frac{1}{2} \rho \vec{v}^2 A_{wet} C_{sk} \tag{74}$$

[10]

Where

$$C_{sk}, (A_{wet}, M, \frac{\epsilon}{l}) \tag{75}$$

$\frac{\epsilon}{l}$ is the relative roughness of the surface

With the critical and actual Reynolds Numbers determined, the *Uncorrected Skin Friction Drag Coefficient* can now be conditionally determined

$$C_{sk_{uncorrected}} = \begin{cases} 0.0148 & Re < 10^4 \\ \dfrac{1}{(1.5 \ln Re - 5.6)^2} & 10^4 < Re < Re_{crit} \\ 0.032 \left(\dfrac{R_a}{L}\right)^{0.2} & Re > Re_{crit} \end{cases} \tag{76}$$

[4]

Two other sources describe the cases for Skin Friction Drag Coefficient differently.

$$C_{sk_{uncorrected}} = \begin{cases} \frac{1.328}{\sqrt{Re}} & Re \leq Re_{crit} \\ \frac{0.074}{Re^{1/5}} & 10^4 < Re < Re_{crit} \end{cases} \tag{77}$$

[3] and [2] agree on the above.

The *Skin Drag Coefficient Corrected for Compressibility* is:

Conversely, Niskanen evaluates the corrected skin drag coefficient as follows

$$C_{sk_{corrected}} = C_{sk_{uncorrected}} \times \begin{cases} (1 - 0.1M^2) & \text{Subsonic} \\ \left[(1 + 0.15M^2)^{0.58}\right]^{-1} & \text{Supersonic} \\ (1 + 0.18M^2)^{-1} & \text{Roughness Limited} \end{cases} \tag{78}$$

Finally, the *Normalized and Corrected Skin Friction Drag Coefficient* is:

$$C_{sk} = \frac{C_{sk,c}\left[\left(1 + \frac{1}{2f_B}\right) \cdot A_{wb} + \left(1 + \frac{2t_f}{L_{cf}} \cdot\right) A_{wf}\right]}{A_{ref}} \tag{79}$$

Where f_b is the *Fineness Ratio*, the ratio of the length of the rocket divided by the outer diameter. L_{cf} is the aerodynamic chord length of the fins, and t_f is the thickness of the fins

[4]

$$Re_{crit} = 51 \left(\frac{R_a}{L}\right)^{-1.039} \tag{80}$$

Pressure (Form/Profile) Drag

This is the drag caused by the pressure exerted on the surface of an object as it moves through a free stream [10].

$$C_{pr}, D_{pr}(A_{ref}, M) \tag{81}$$

Body Drag

Body Drag is the drag on the rocket forebody (pressure drag?)

$$C_{fb} = \left[1 + \frac{60}{(l_{TR}/d_b)^3} + 0.0025\frac{l_b}{d_b}\right]\left[2.7\frac{l_n}{d_b} + 4\frac{l_b}{d_b} 2\left(1 - \frac{d_d}{d_b}\right)\frac{l_c}{d_b}\right] \cdot C_{f(fb)} \tag{82}$$

Where l_{TR} is the total length of the rocket body, l_c is the length of the boat tail, d_b is the maximum body diameter and d_d is the diameter of the rocket base. C f(fb) is the coefficient of viscous friction on the rocket forebody (defined later in (45))

Fin Pressure Drag

The *Fin Pressure Drag* depends on the fin profile. The current rocket will use a square (rectangular) profile, and can be determined as follows.

$$C_{fp}, D_{fp}(A_{ref}, M) \tag{83}$$

Leading Edge pressure drag

$$C_{D,LE} = C_{D,stag} = 0.85\frac{q_{stag}}{q} \tag{84}$$

The *Body Base Drag Coefficient* is

$$C_{base} = \begin{cases} 0.12 + 0.13M^2 & M < 1 \\ \dfrac{0.25}{M} & M > 1 \end{cases} \tag{85}$$

For perpendicular orientation of the fin edges to air flow, the stagnation pressure defined in Equation 65 is used.

$$\frac{q_{stag}}{q} = \begin{cases} 1 + \dfrac{M^2}{4} + \dfrac{M^4}{40} & M < 1 \\ 1.84 - \dfrac{0.76}{M^2} + \dfrac{0.166}{M^4} + \dfrac{0.035}{M^6} & M > 1 \end{cases}$$

[4]

Von Karman Nose Pressure Drag

Most nose cone shapes can be approximated to produce zero pressure drag at subsonic velocities, however complications arise for transonic and supersonic velocities. A semi-empirical method can be employed in the latter conditions.

> The curves of the pressure drag coefficient as a function of the nose fineness ratio f_N can be closely fitted with a function of the form

$$C_{d_pressure} = \frac{a}{(f_N + 1)^b} \tag{86}$$

Where a and b are calculated from two data points corresponding to fineness ratios 0 and 3

???

In subsonic and transonic regions, pressure drag of nose cones is calculated as follows:

$$\begin{cases} 0.8 \cdot \sin^2\phi & M \approx 0 \\ a \cdot M^b + 0.8 \cdot \sin^2\phi & M \approx 0.8 \end{cases} \tag{87}$$

Where a and b are computed by interpolation to fit the drag coefficient and the derivative of the drag coefficient at the lower bound of the transonic region.

The cause of this drag is slight flow separation, and as such cannot be corrected due to compressibility effects.

[4]

Base Drag

Base drag is caused by a low pressure region generated behind the base of the rocket as it moves quickly through the atmosphere [4]. Specifically, it is due to boundary separation between the flow past the rocket and the surrounding air [3]. The flowing air attempts to make a sharp turn around the sudden geometry change at the base end of the rocket, however, viscous effects resist this change in direction. As a result, pressure cannot be equalized in the space directly behind the rocket and a low-pressure (vacuum) region forms [11]. This low-pressure region has an effect analogous to *pulling* the rocket against its direction of flight.

$$C_{ba}, D_{ba}((A_{ref}, M)) \tag{88}$$

$$C_{ba} = \begin{cases} 0.12 + 0.13M^2 & M < 1 \\ \dfrac{0.25}{M} & M > 1 \end{cases} \tag{89}$$

???, pg.50

In reality, this low pressure region is disturbed by the thrust envelope from the motor. Thus, we would expect base drag to be different during the motor burn time than during the free flight after all fuel was exhausted. Considering the thrust envelope is at this moment beyond the scope of the project. Instead, an accepted approximation is to subtract the area of the motor from the area of the base when calculating drag force [4].

$$D_{ba} = \frac{1}{2} C_{ba} \rho (A_{tube,base} - A_{motor,base}) \vec{v}^2 \tag{90}$$

We can normalize the base drag coefficient to take this into account.

$$C_{ba,normalized} = C_{ba} * A_{tube,base} / A_{motor,base} \tag{91}$$

Shoulder Pressure Drag

The drag coefficient of the shoulder interfacing the body tube is assumed to be equal to that of the body tube itself, and also assumes a smooth interface. This is likely to be sufficient for subsonic velocites [4], and for the scope of this project it is neglected entirely.

Parasitic Drag

Parasitic drag is the drag due to body features not explicitly designed and/or imperfections not easily approximated. Examples include launch guides, ventilation holes, surface roughness, and any damage during flight.

PARASITIC DRAG IS CURRENTLY NEGLECTED IN THE MODEL

$$C_{pa}, D_{pa}(A_{ref}, M) \tag{92}$$

Where C_{stag} is the *Stagnation Drag Coefficient* [see equation from fin pressure drag section]

We consider the most significant source of *Parasitic Drag* to be the launch lug. If there is no significant airflow through the launch lug, we can approximate it as a cylinder next to the rocket body. *Niskanen* states that a launch lug with a length at least two times its width has a drag coefficient of 0.74, with its reference area being the frontal area. Stagnation pressure proportionally influences the drag coefficient [4].

The following equation relates the launch lug diameter ϕ_{lug} to the launch lug tube length l_{lug}.

$$C_{pa} = \left(1.3 - 0.3 \frac{l_{lug}}{\phi_{lug}}, 1\right)_{max} \cdot C_{stagnation} \tag{93}$$

Where L is the rocket length, h_n is the height of the nose cone, OD is the outer diameter of the rocket, and $C_{stagnation}$ is the stagnation coefficient [4].

The reference area of the launch lug is given as follows

$$\pi \cdot (r_{ext,lug}^2 - r_{int,lug}^2) \cdot \left[1 - \left(\frac{l_{lug}}{\phi_{lug}}\right)\right]_{+ve} \tag{94}$$

The *Parasitic Drag Coefficient* can be normalized to the reference area of the launch lug.

$$C_{pa_{norm}} = C_{pa} \cdot \left(\pi \cdot (r_{ext}^2 - r_{int}^2) \cdot \left[1 - \left(\frac{L-h}{OD}\right)\right]_{+ve}\right) \tag{95}$$

[4]

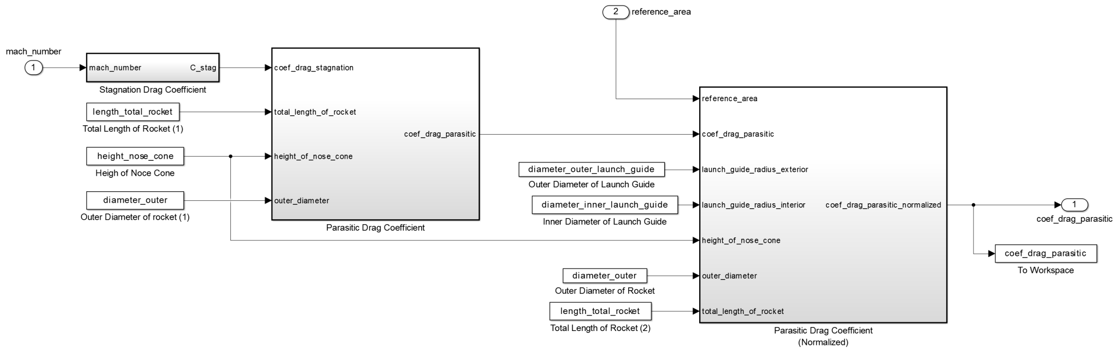

Figure 5: Matlab Implementation of Parasitic Drag Coefficient

Interference Drag

Interference Drag is caused due to effects of air flow at the interfaces of the fins and the body.

$$C_{in}, D_{in}(A_{ref}, M) \tag{96}$$

$$C_{in} = 2C_{sk,fins}\left(1 + 2\frac{T_f}{l_m}\right)\frac{4n(A_{f_p} - A_{f_e})}{\pi d_f^2} \tag{97}$$

Where:

- $C_{sk,fins}$ is the coefficient of skin friction (due to viscous effects) on the fins
- n is the number of fins
- A_{f_p} is the fin planform area

$$A_{f_p} = A_{f_e} + \frac{1}{2}d_f l_r \tag{98}$$

- A_{f_e} is the exposed planform area of the fin

$$A_{f_e} = \frac{1}{2}(l_r + l_t)l_s \tag{99}$$

[3]

Interference Drag effects are small in comparison to other drag effects [4], and are thus ignored at this stage of the project.

Wave Drag

Wave drag is drag associated with shock waves (independent of viscous effects).

> At transonic speed, shock waves form at the nose tip and at the leading edge of the fins ... Momentum is transferred from the rocket to the surrounding air via these shockwaves

Boat-Tail Drag

A *boat-tail* is a reduction in diameter of the body tube towards the base of the rocket. Our rocket does not have a boat-tail, thus *Boat-Tail Drag* considerations are ignored.

Additional Drag at Angle of Attack

When the rocket flies at a non-zero angle of attack, additional drag considerations must be made. The reference area the rocket becomes larger as the rocket is pitched into the free stream, exposing more of the rocket body to pressure and stagnation effects.

In Figure 6, velocity $\vec{v}$ is the apparent velocity of the center of pressure relative to the surrounding air.

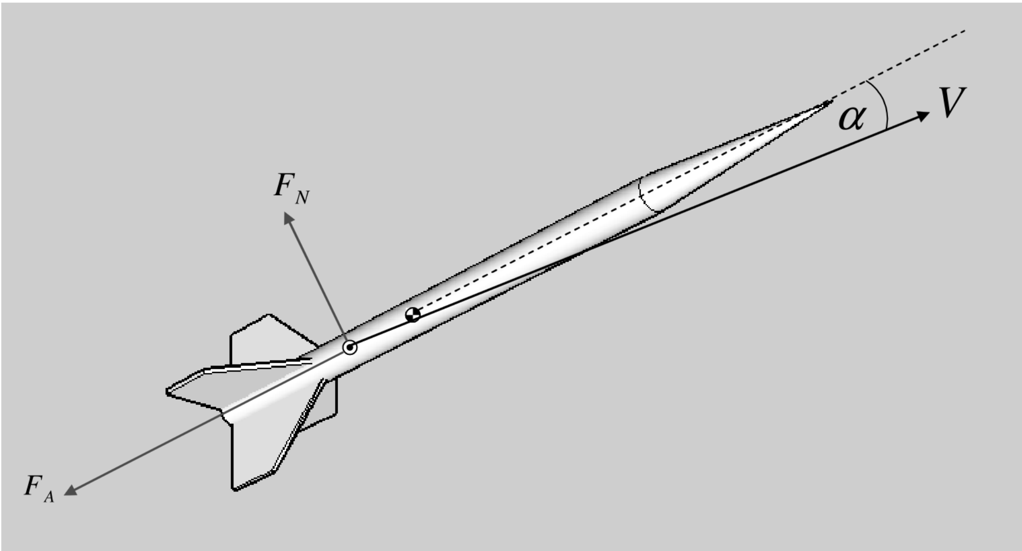

Figure 6: Rocket Drag Forces - Axial vs. Normal Caption

[3]

In the following analysis, additional rocket drag coefficients are determined to be added to the *zero angle of attack* drag coefficient. This analysis is derived with the aid of additional coefficients determined experimentally in wind tunnel tests on rocket models [3] [2].

$$C_{aoa} = C_{Db(\alpha)} + C_{Df(\alpha)} \tag{100}$$

Rocket Body Drag at Angle of Attack

$$C_{Db(\alpha)} = 2\delta\alpha^2 + \frac{3.6\eta(1.36L - 0.55h_n)}{\pi \cdot OD}\alpha^3 \tag{101}$$

Where:

- α is the angle of attack
- L is the total rocket length
- OD is the outer diameter of the rocket
- h_n is the height of the nose cone
- δ and ν are experimentally determined coefficients
- OD is the outer diameter of the rocket

[3]

Rocket Fin Drag at Angle of Attack

$$C_{Df(\alpha)} = \alpha^2 \left[1.2\frac{A_{fp}4}{\pi OD_f^2} + 3.12(k_{fb} + k_{bf} - 1)\left(\frac{A_{fe}4}{\pi OD_f^2}\right)\right] \tag{102}$$

Where:

- k_{fb} is the fin-body coefficient

$$k_{fb} = 0.8065R_s^2 + 1.1553R_s \tag{103}$$

- k_{bf} is the body-fin coefficient

$$k_{bf} = 0.1935R_s^2 + 0.8174R_s + 1 \tag{104}$$

- R_s is the fin section ratio

$$R_s = \frac{l_{TS}}{d_f} \tag{105}$$

- l_{TS} is the total span of the fins
- OD_f is the diameter of the body tube at the base of the fin mount

[3]

Alternatively

[2] shares a function determined for *Total drag coefficient due to angle-of-attack*

$$C_d(\alpha) = 16.83\alpha^2 + 8.9\alpha^3 \tag{106}$$

Matlab Implementation

Figure 7 below shows the *Simulink* implementation of the calculation of the drag model

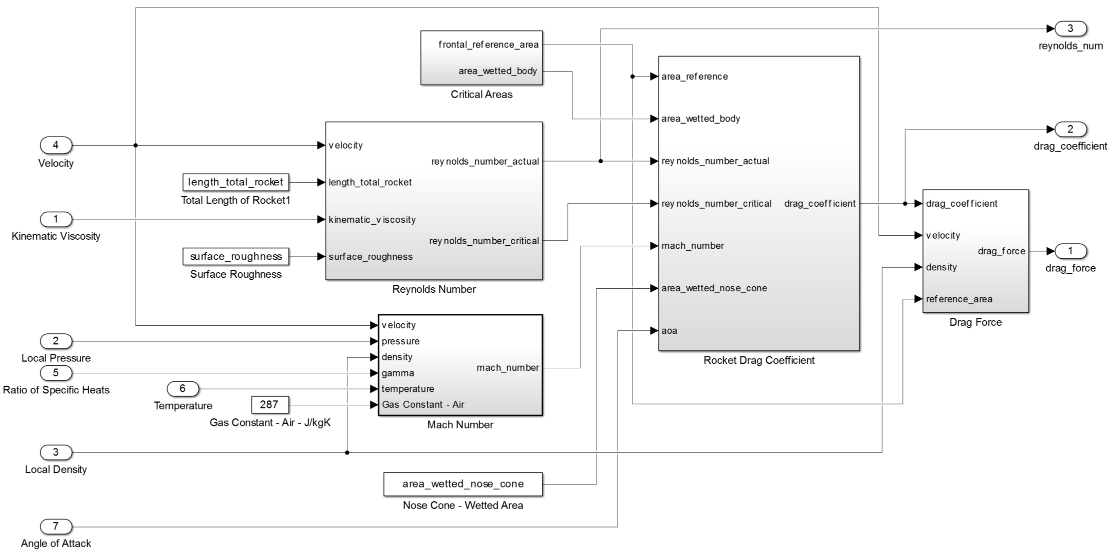

Figure 7: Rocket Drag Model

Figure 8 below shows the *Simulink* implementation of the calculation of drag coefficient

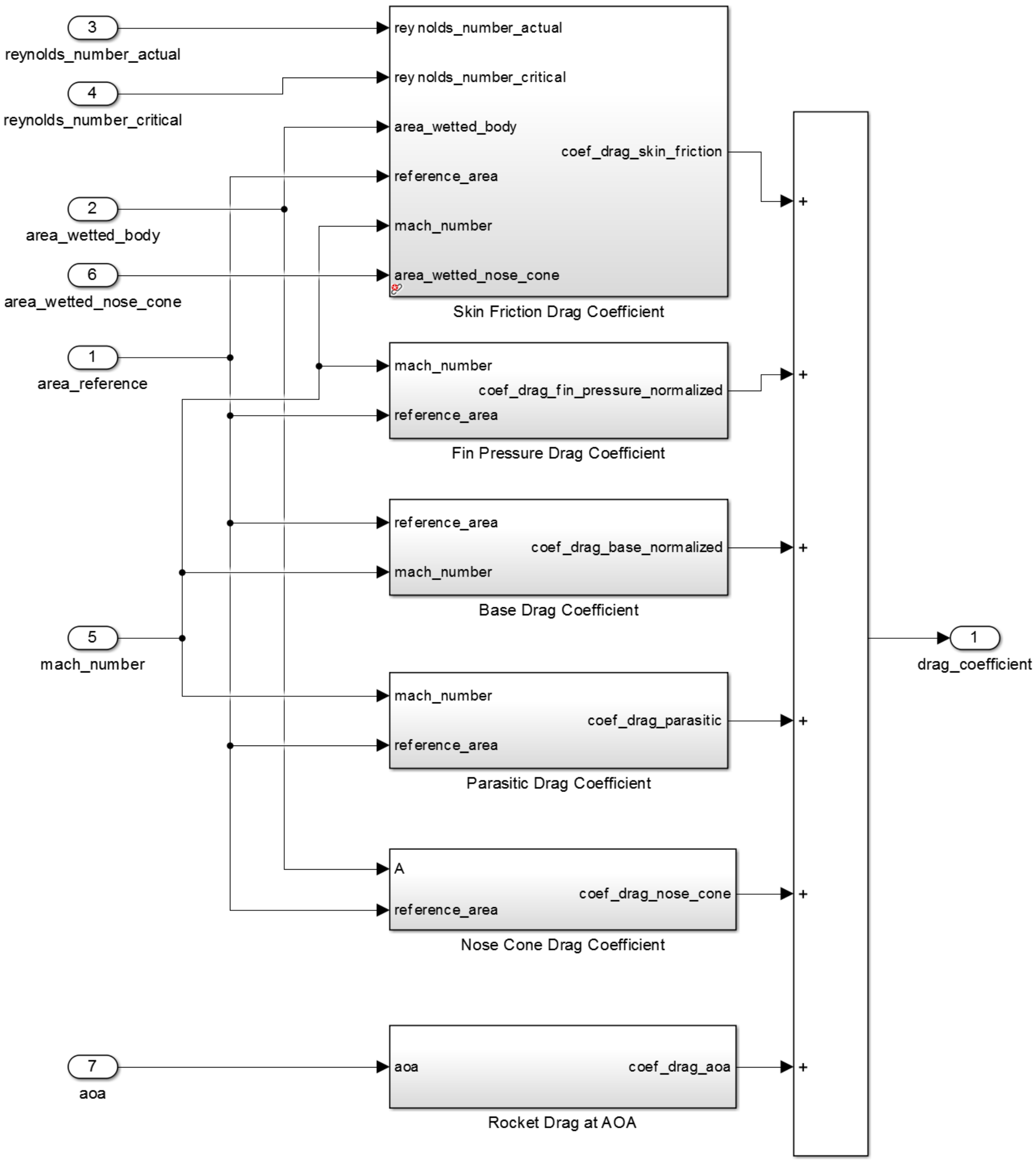

Figure 8: Rocket Drag Coefficient Model

Angular Flight Stability (Pitch, Yaw) Analysis

Overview

Due to disturbances such as wind, and imperfections and imbalances in the construction, the rocket will tend to fly at an *Angle of Attack* into the free stream, wherein the velocity vector (taken from the *Center of Gravity*) is not parallel with the longitudinal axis. This will cause non-linear changes to the magnitude of the aerodynamic forces, which, as a further simplification, can be said to be acting about the *Center of Pressure*. In order for the aerodynamic forces to straighten the rocket in its forward motion, and to stabilize the oscillatory rotation about the COG, the COP must be located behind the COG.

> "One of the first principles any rocket designer must learn is that a rocket will fly only if the center of gravity is ahead of the center of pressure far enough to allow the air currents to cause a stabilizing effect."

http://www.nar.org/NARTS/TR13.html

Dynamic Stability Analysis

The rocket must exhibit dynamic stability, wherein oscillations are reduced by the damping moment.

The rocket should experience a resonant motion in response to aerodynamic forces such that it doesn't sustain at the natural frequencies that can cause structural or component damage. This value must be verified regularly with the design team to ensure compliance.

Characteristic Equations

$$C_1\ddot{\alpha} + C_2\dot{\alpha} + C_3\alpha = 0 \quad (107)$$

Vibrations

$$m\ddot{x} + c\dot{x} + kx = 0 \quad (108)$$

Controls

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \quad (109)$$

- ζ is the damping coefficient, and is tied to the frequency of the system

Harmonic Response

Particular Response

Homogeneous Response

Undamped Response

Complete Response

Coordinate System, Equations of Motion

The moment arm about the COG is the distance of the COP from the tip of the nose cone, minus the distance of the COG from the tip of the nose cone. Then, the sum of forces at the COP is the *Restoring Force* (F_R) minus the *Damping Force* (F_D), and the sum of the Moments about the COG is expressed as follows.

The *Moment* of a rigid body about its COG can be expressed as the product of the *Moment of Inertia* of the rigid body and the *Angular acceleration* of the body.

$$M = I\lambda \tag{110}$$

- λ is the *angular acceleration* of the rigid body, which is the second time derivative of the angular displacement

$$\lambda = \ddot{\alpha}$$
$$\omega = \dot{\alpha}$$

- ω is the *angular velocity*, which is the first time derivative of the angular displacement
- α is the *angle of attack*

Longitudinal Static Stability Margin

The *Longitudinal Static Stability Margin* (S_{lm}) is the distance between the *Center of Gravity* and the *Center of Pressure* divided by the outer diameter of the body tube when the rocket is positioned at an angle-of-attack (α) of zero **???**.

$$S_{lm} = \frac{COP - COG}{OD}$$

When traveling under a non-zero angle of attack, the Stability Margin is adjusted using the body lift correction factor Equation **??**.

The result is dimensionless, however the ratio determined is measured in the number of *calibers*.

2a - The static stability margin falls above 2 (but less than 3) calibers at launch

Static Stability Margin

Out-of-scope

- range (drift)
- roll

Requirement

- 2a - The static stability margin falls above 2 (but less than 3) calibers at launch
- 2b - The dynamic stability is greater than 0 even in winds up to 8.33 m/s
- 2f - The vehicle does not experience resonant pitching/yawing motion in flight

Assumptions

- small angle of attack (less than 10°)
- incompressible flow
- neglect viscous forces
- neglect compressibility effects [3]
- neglect lift force on the body tube [3]
- neglect the effect of roll due to having 3 fins vs 4

Derivation of the Harmonic Motion Equation

Suppose a high-powered rocket is launched in quiescent air vertically, and flies straight without wobbling. Then, suppose a small and momentary disturbance (e.g. a short gust of wind) is experienced on the side of the rocket causing an angular deflection, . If the rocket is *stable*, a restoring force causes a *corrective moment* which will act in the opposite direction of the deflection. This *corrective moment* can be considered a function of angular displacement [2].

$$M_{corrective} = F(\alpha) \tag{111}$$

As the rocket gains velocity in the direction opposite the disturbance, a *damping moment* is generated as a result of the relative speed of the air, in the direction orthogonal to the longitudinal axis. As this *damping moment* opposes the angular velocity caused by the *corrective moment*, its sign is opposite to the angular velocity. The *damping moment* is also a function of angular velocity [2].

$$M_{damping} = G(\omega) \tag{112}$$

Then, taking a sum of Moments, the rotation of the rocket can be described as follows [2]

$$I\lambda = -F(\alpha) - G(\omega)$$
$$I\left(\frac{d^2\alpha}{dt^2}\right) = -F(\alpha) - G\left(\frac{d\alpha}{dt}\right)$$

$$I\left(\frac{d^2\alpha}{dt^2}\right) + F(\alpha) + G\left(\frac{d\alpha}{dt}\right) = 0 \tag{113}$$

This nonlinear, homogenous, differential equation can not be solved exactly [2].

Linearization Approximation

Linear Approximations of 113 are made considering small values of α and ω, known as the *small-perturbation theory* [2]. This linearization process provides ~~constant~~ coefficients, which we will denote C_1 for the *Corrective Moment Coefficient* and C_2 for the *Damping Moment Coefficient*.

$$F(\alpha) \approx C_1 \cdot \alpha$$
$$G\left(\frac{d\alpha}{dt}\right) \approx C_2 \cdot \frac{d\alpha}{dt}$$

$$I\left(\frac{d^2\alpha}{dt^2}\right) + C_1(\alpha) + C_2\left(\frac{d\alpha}{dt}\right) = 0 \tag{114}$$

Preliminary Considerations

Rocket Normal Force

The *Rocket Normal Force* is the resultant force applied at the *Center of Pressure* perpendicular to the longitudinal axis of the rocket, when the rocket flies at an angle-of-attack.

$$F_N = \frac{1}{2}\rho\vec{v}^2 A_c C_N \tag{115}$$

[3]

Where A_c is the cross-sectional area of the body tube, and C_N is the *Normal Force Coefficient*, and is a function of angle-of-attack (α). The small angle approximation is applied, wherein small angles can be approximated as a linear function of the angle.

$$C_N = C_{N\alpha} \cdot \alpha \tag{116}$$

[3]

Where $C_{N\alpha}$ is the *Stability Derivative*

Corrective Moment Coefficient

The *Corrective Moment Coefficient* describes the reaction of the rocket against a disturbance about its longitudinal axis.

$$C_{MC} = \frac{1}{2}\rho\vec{v}^2 A_{ref} C_{N\alpha}(COP - COG) \tag{117}$$

Where:

- ρ is the local density of air
- $\vec{v}$ is the velocity of the rocket
- A_{ref} is the reference area of the rocket flying into the free stream
- $C_{N\alpha}$ is the *Stability Derivative* ~~*Normal Force Coefficient*~~
- $(COP - COG)$ is the distance between the *Center of Pressure* and *Center of Gravity*

Note: a rocket with a high *Corrective Moment Coefficient* is going to weathercock faster at lower velocities.

Corrective Moment Coefficient

Dimensional Analysis

$$\frac{kg}{m^3}\left[\frac{m}{s}\right]^2 m^2 m = \frac{kg \cdot m}{s^2} \cdot m \tag{118}$$

Damping Moment Coefficient

As the rocket responds to a disturbance, the *Corrective Moment* reactions forces act in an oscillating manner - weathercocking into the wind, then turning back towards the vertical direction. In order to reach dynamic stability, this oscillation must decay and settle to a reasonable response. The *Damping Moment Coefficient* represents how fast the response settles towards zero.

There are two *Damping Moment Coefficients* to consider, the *Aerodynamic Damping Moment Coefficient* and the *Propulsive Damping Moment Coefficient*.

Then the *Damping Moment Coefficient* is the sum of the two moment components coefficients.

$$C_{DM} = C_{ADM} + C_{PDM} \tag{119}$$

Aerodynamic Damping Moment Coefficient

Each rocket component contributes to the *Aerodynamic Damping Moment Coefficient*

$$C_{ADM} = \frac{1}{2}\rho\vec{v}A_{ref}\sum\left(C_{N\alpha,x}\cdot\left[COP_x - COG\right]^2\right) \tag{120}$$

NOTE: Why isn't $\vec{v}$ SQUARED? It might have something to with the fact that the ADM is a function of angular displacement, and DM is a function of angular velocity??

Where:

- ρ is the local density of air
- $\vec{v}$ is the velocity of the rocket
- A_{ref} is the reference area of the rocket flying into the free stream
- $\cancel{C_{NF,x}}$ $C_{N\alpha}$ is the ~~*Normal Force Coefficient*~~ *Stability Derivative*
- COP_x is the distance of *Center of Pressure* of the rocket component to the nose cone tip
- COG is the distance between the rocket *Center of Gravity* to the nose cone tip

Dimensional Analysis

$$\frac{kg}{m^3}\frac{m}{s}m^2m^2 = \frac{kg\cdot m}{s}\cdot m \tag{121}$$

Propulsive Damping Moment Coefficient

Also known as *Jet Damping*, as propulsion creates forward momentum, it resists rotation of the rocket.

$$C_{PDM} = \dot{m}\left(d_{tip,nozzle} - COG\right)^2 \tag{122}$$

Jet Damping - Dimensional Analysis

$$\dot{m}\left(d_{tip,nozzle} - COG\right)^2 : \left[\frac{kg}{s}\cdot m^2\right]$$

$$M = fd : \left[\frac{kg\cdot m^2}{s^2}\right]$$

Note: why is the *Jet Damping Moment* missing a 1/t?

Damping Moment Coefficient - Source

Pitch Damping Moment

The *Pitch Damping Moment* is a moment opposing the *Rocket Restoring Moment* and dampens the oscillation.

$$0.55\frac{l^4 r_t}{A_{ref}d}\frac{\omega^2}{v_0^2} \tag{123}$$

According to [4], the *Pitch Damping Moment* is essentially insignificant until near apogee. This is because it is proportional to $\frac{\omega^2}{v_0^2}$ (as seen in 123), which will be near zero until apogee due to very small angular velocities made smaller by squaring the ω term.

The *Pitch Damping Moment* of each rocket component must be calculated individually. For instance, the *Pitch Damping Moment* of a fin is as follows.

$$C_{damp} = 0.6\frac{NA_{fin}d_{COP}^3}{A_{ref}d}\frac{\omega^2}{v_0^2} \tag{124}$$

General Homogeneous Response

The characteristic, linearized, homogeneous yaw/pitch response is given as:

$$I_L \frac{d^2\alpha_x}{dt^2} + C_2 \frac{d\alpha_x}{dt} + C_1 \alpha_x = 0 \tag{125}$$

[2]

A solution over a known range of acceptable values of the coefficients above is:

$$\alpha_x = Ae^{-Dt} \sin(\omega t + \phi) \tag{126}$$

Where:

- t is the time passed since the "observation of the dynamic response has begun, not the time elapsed since the rocket was launched" [2]
- ω is the *frequency of oscillation* (not literally the angular velocity of the rocket)

$$\omega = \sqrt{\frac{C_1}{I_L} - \frac{C_2^2}{4I_L^2}} \tag{127}$$

- ϕ is the *phase angle* in radians

$$\phi = \arctan\left(\frac{\alpha_{xo}\omega}{D\alpha_{xo} + \Omega_{xo}}\right) \tag{128}$$

- D is the *inverse time constant*

$$D = \frac{C_2}{2I_L} \tag{129}$$

- A is the *initial amplitude*

$$A = \frac{\alpha_{xo}}{sin\phi} \tag{130}$$

- α_{xo} is the value of α_x at $t = 0$

[2]

The initial conditions for this solution are

- a_{xo} = some non-zero angle-of-attack
- $\Omega_{xo} = 0$

This equation is represented in the model as follows

$$I_L(D^2 - \omega^2) - C_2 D + C_1 = 0$$
$$-2I_L D + C_2 = 0$$

[2]

We can consider the rocket to be *restored* when the angle of attack stabilizes below 5% of A.

[2]

The *Natural Frequency* of the rocket at the current air speed for the homogeneous solution is

$$\omega_n = \sqrt{\frac{C_1}{I_L}} \tag{131}$$

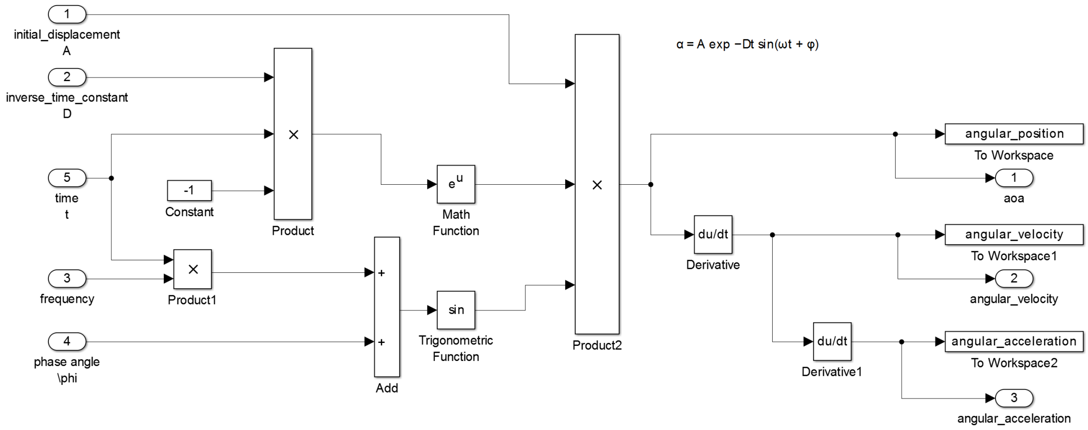

Figure 9: Angular Flight Model - Simplified

Note: it would appear that this response only reflects the physical system for non-decreasing values of C_2, which would cause the exponential term to increase with time and cause the amplitude to grow. Although the damping coefficient remains relatively constant, the inverse time-constant is only a function of $\frac{C_2}{2I_L}$. As velocity decreases in the rocket coasting phase, C_2 drops proportional to the square of the velocity and thus the inverse-time constant decreases enough with respecting time, that Dt is decreasing and thus e^{-Dt} will begin to increase.

Attach image **???**

Complete Response to Step Input

Complete Response to Impulse Input

A valid solution to the impulse response within the finite horizon is a mathematical guarantee of stability.

???

Delta-Dirac Function

$$u(t) = \int_{-\infty}^{\infty} \delta(u - \tau)d\tau \tag{132}$$

???

Convolution Theorem

The integral of a linear operator is the linear operator of the integral

Steady State Response to Sinusoidal Forcing

Rocket Damping Ratio

The *Rocket Damping Ratio* is calculated as follows.

$$\zeta = \frac{C_2}{2 \cdot \sqrt{C_1 I_L}} \tag{133}$$

Where:

- C_1 is the *Corrective Moment Coefficient*
- C_2 is the *Damping Moment Coefficient*
- I_L is the *Longitudinal Moment of Inertia*

[2]

Underdamped Case

$$0 < \frac{C_2^2}{4I_L^2} < \frac{C_1}{I_l} \tag{134}$$

The fastest response is when $\zeta = \dfrac{\sqrt{2}}{2}$

Overdamped Case

$$\frac{C_2^2}{4I_L^2} > \frac{C_1}{I_l} \tag{135}$$

Critically Damped Case

$$\frac{C_1}{I_l} = \frac{C_2^2}{4I_L^2} \tag{136}$$

[2]

Rocket Natural Frequency

$$\omega_n = \sqrt{\frac{C_1}{I_L}} \tag{137}$$

Where:

- C_1 is the *Corrective Moment Coefficient*
- I_L is the *Longitudinal Moment of Inertia*

[2]

Time Constants of the Response

TODO

Complete response to step input

Complete response to impulse input

[mandell1973, pg.123]

AOA as a function of velocity

In order to plot the real system behavior, it may be possible to solve Equation 113 where α_x is a function of velocity, and solve for α_x by twice integrating $\ddot{\alpha_x}$.

$$I\left(\frac{d^2\alpha}{dt^2}\right) + F(\alpha) + G\left(\frac{d\alpha}{dt}\right) = 0$$

$$\frac{\delta^2\alpha_x}{\delta t^2} = \dot{v} = \ddot{x}$$

$$\frac{\delta\alpha_x}{\delta t} = v = \dot{x}$$

$$\frac{\delta\alpha_x}{\delta t} = v = \dot{x}$$

Somehow get to:

$$\frac{d^2}{dv^2}\cdots$$

Corrections

Compressibility Correction

Barrowman's Method neglects compressibility effects, however these effects cannot be neglected above Mach 0.3.

Wind Disturbance

We are interested in the damping ratio of the rocket as it stabilizes towards *zero angle of attack* in reaction to angular disturbances.

As we consider the rocket to have ideal dimensional accuracy, the main source of flight disturbance is wind.

Impulse Disturbance

We can test the ability of the rocket to stabilize due to an initial angular disturbance, by applying an initial angle of attack. This simulates a small gust of wind hitting the rocket just as it takes off.

Constant Disturbance

We can test the ability of the rocket to stabilize due to a constant disturbance force, as well as applying an initial *angle of attack*. This simulates a constant wind force coming from a single direction. As the density of air goes down with increases altitude, this assumes that the wind speed picks up at higher altitudes to maintain the constant wind force.

Alternatively, we could model a constant wind speed of $8.33m/s$, and apply the ISA Model for the density as a function of altitude to determine the changing wind force as the rocket climbs.

Future Enhancement

To account for wind turbulence in future models, two commonly used Wind Models are explored.

Kaimal Wind Model

$$\frac{S_u(f)}{\sigma_u^2} = \frac{4L_{1u}/U}{(1 + 6fL_{1u}/U)^{5/3}} \tag{138}$$

Von Karman Wind Model

$$\frac{S_u(f)}{\sigma_u^2} = \frac{4L_{2u}/U}{(1 + 70.8(fL_{2u}/U)^2)^{5/3}} \tag{139}$$

Where

- $\frac{S_u(f)}{\sigma_u^2}$ is the *Spectral Density Function* of turbulence velocity
- f is the turbulence frequency
- σ_u is the standard deviation fo the turbulence velocity
- L_{1u} and L_{2u} are length parameters
- U is the average wind speed

[4]

More Reading

There are further resources on rocket flight stability here: - http://www.apogeerockets.com/Tech/Rocket_Stability

Solver Algorithm

The numerical algorithm chosen for performing the simulation can impact the accuracy of the results. The following discussion introduces available algorithms and their tradeoffs.

Algorithms

Interpolation

Matlab Cubic Spline

Runge-Kutta-Fehlberg

Integration method order: Integration step size: Convergence Tolerance

Validation

Integration Testing

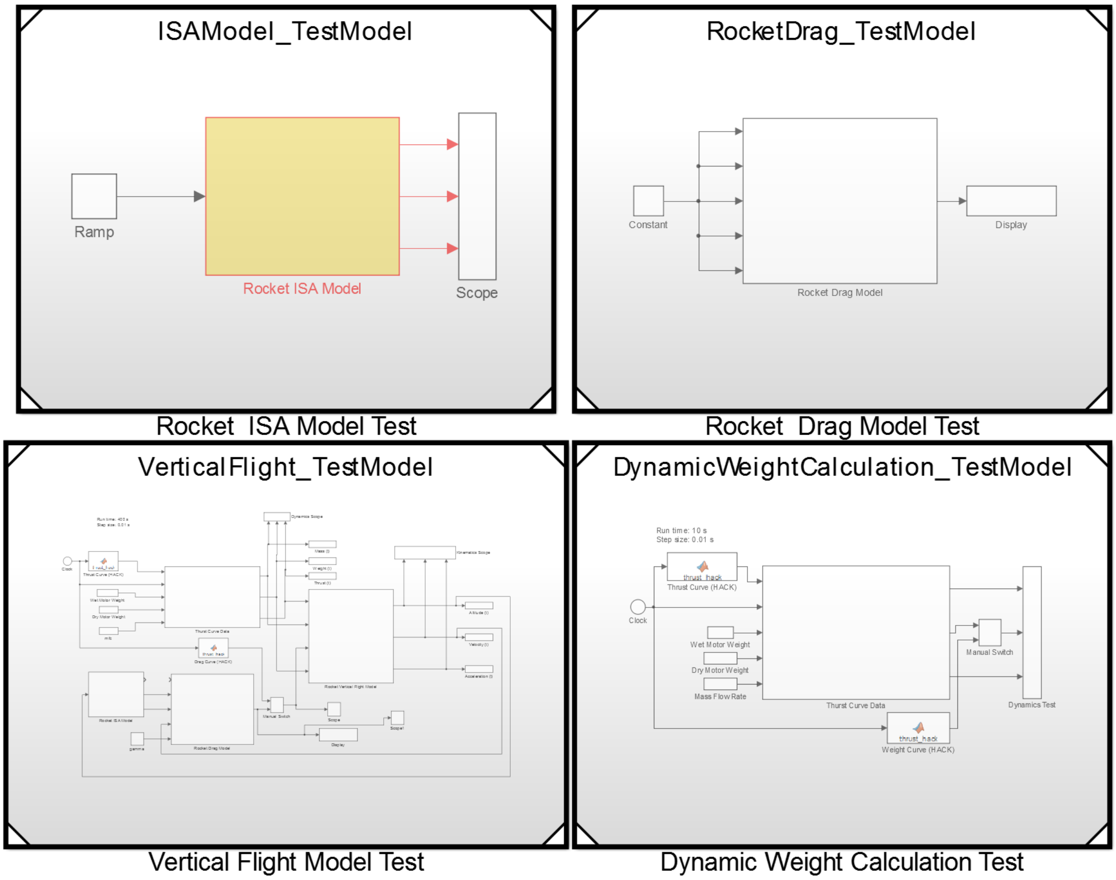

Figure 10: All Integration Tests

Comparison with Experimental Data

The University of Louisville was kind enough to share experimental rocket flight data, as well as their rocket dimensions so that our simulation could be compared to real-world experimental data.

Flight Details

March 29, 2015

Date	Elevation	Ground Pressure	Ground Temperature	Wind Speed	Humidity	Lau
2015/03/29	405 ft (123,48 m)	?? Pa	?? K (61.00 °F)	14 km/h (3.89 m/s)	~39 %	3.05
2015/03/29	432 ft (131.71 m)	?? Pa	?? K (58.21 °F)	14 km/h (3.89 m/s)	~39 %	3.05
2015/03/29	410 ft (125.00 m)	?? Pa	?? K (70.79 °F)	14 km/h (3.89 m/s)	~39 %	3.05
2015/03/29	417 ft (127.13 m)	?? Pa	?? K (61.32 °F)	14 km/h (3.89 m/s)	~39 %	3.05
2015/03/29	446 ft (135.97 m)	?? Pa	?? K (61.50 °F)	14 km/h (3.89 m/s)	~39 %	3.05

Motor Details

Cesaroni L935

Parameter	Value
Manufacturer:	Cesaroni Technology
Entered:	Oct 6, 2009
Last Updated:	Jun 26, 2014
Mfr. Designation:	3147L935-P
Common Name:	L935
Motor Type:	reload
Diameter:	54.0mm
Length:	64.9cm
Total Weight:	2542g
Prop. Weight:	1567g
Cert. Org.:	Canadian Association of Rocketry
Cert. Date:	Aug 27, 2009
Average Thrust:	933.8N
Maximum Thrust:	1585.6N
Total impulse:	3146.8Ns
Burn Time:	3.4s
Isp:	205s
Case Info:	Pro54-6GXL
Propellant Info:	Imax
Data Sheet:	link
Availability:	regularCesaroni L935

Plots

Simulation Execution

Simulation Configuration

Historical Weather Data for Green River, Utah

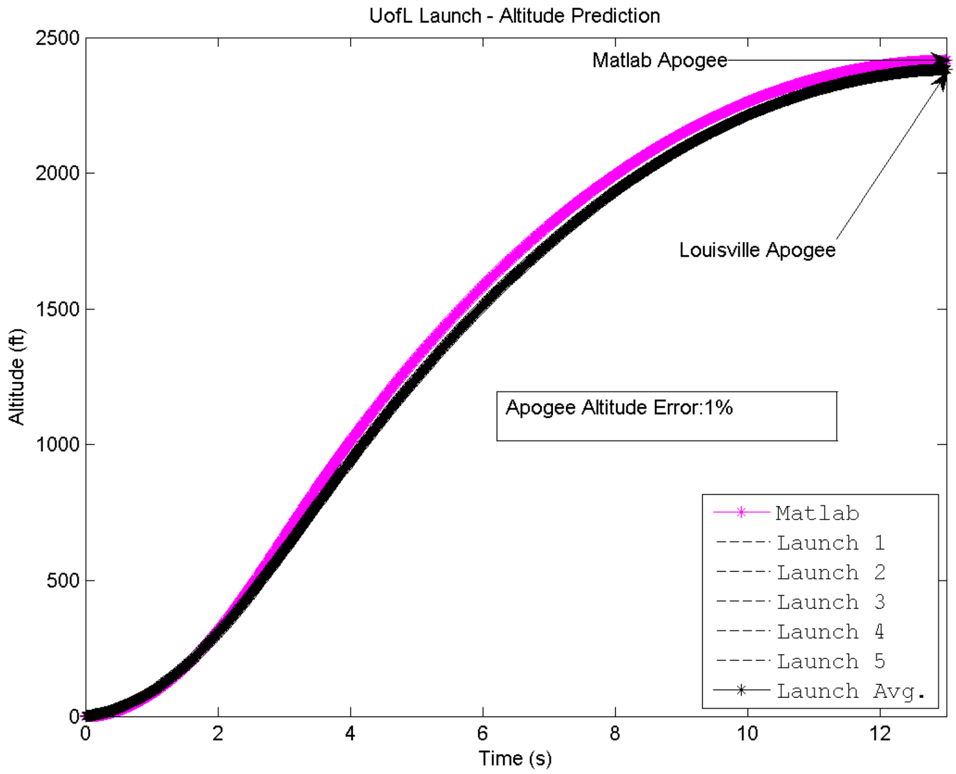

Figure 11: Altitude Plot of Simulation Data vs Louisville Rocket Launches

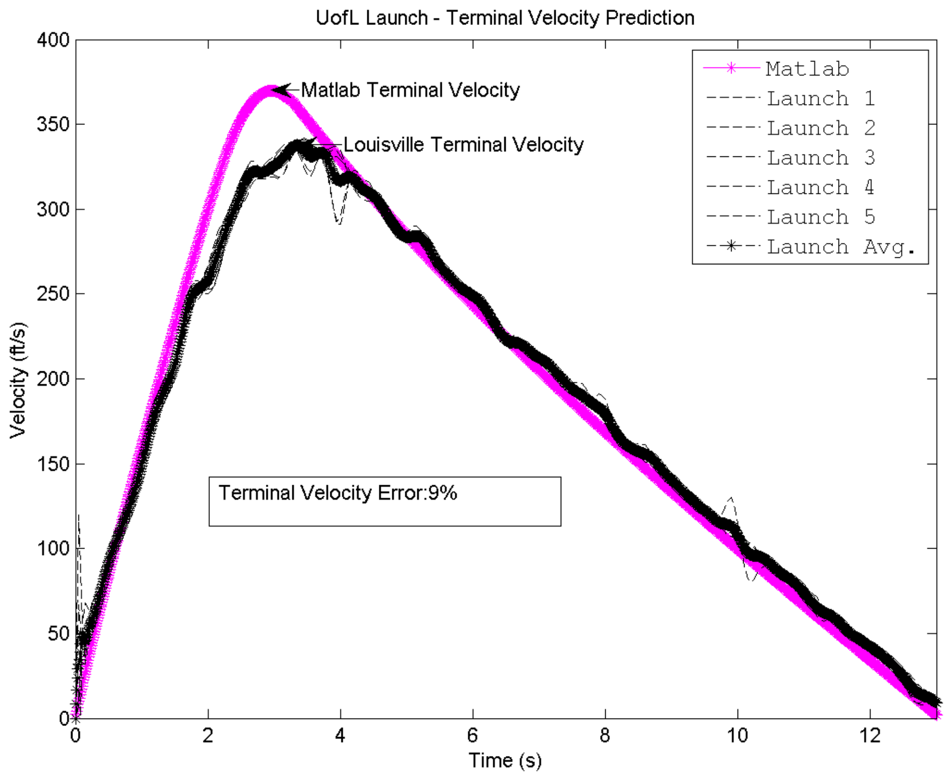

Figure 12: Velocity Plot of Simulation Data vs Louisville Rocket Launches

Date	Elevation	Ground Pressure	Ground Temperature	Wind Speed	Humidity
2015/06/15	4,078 ft (1,243 m)	101300 Pa	298.15 K (25 °C)	6 km/h (1.6 m/s)	33 %
2014/06/15	4,078 ft (1,243 m)	100700 Pa	296.15 K (23 °C)	20 km/h (5.56 m/s)	8 %
2013/06/15	4,078 ft (1,243 m)	101000 Pa	299.15 K (26 °C)	10 km/h (2.78 m/s)	19 %

Table 14: Historical Weather Conditions, Green River, Utah

General Conditions

For all tests, the following conditions were used, based on historical data for June 25th, 2015 at 12:00PM (noon) near Green River, Utah [12].

Elevation	Humidity	Launch Guide Length
4,078 ft (1,243 m) **???**	33 %	5.4864 (18 ft)

Table 16: General Simulation Conditions

Best Case

The best case scenario is with no wind, and a 0° launch angle, and the lowest air pressure.

Wind Speed	Ground Pressure	Ground Temperature	Launch Guide Angle
0 m/s	101000 Pa	298.15 K (25 °C)	0°

Table 18: Best Case Simulation Conditions

Worst Case

The worst case scenario, is the maximum wind condition permitted for launch by the competition, and a launch guide angle, and the highest pressure.

Wind Speed	Ground Pressure	Ground Temperature	Launch Guide Angle
8.33 m/s	101325 kPa	288.15 K (15 °C)	10°

Table 20: Worst Case Simulation Conditions

Simulation Software

The following models were executed.

OpenRocket

RockSim

RASAero

Matlab

Matlab Models

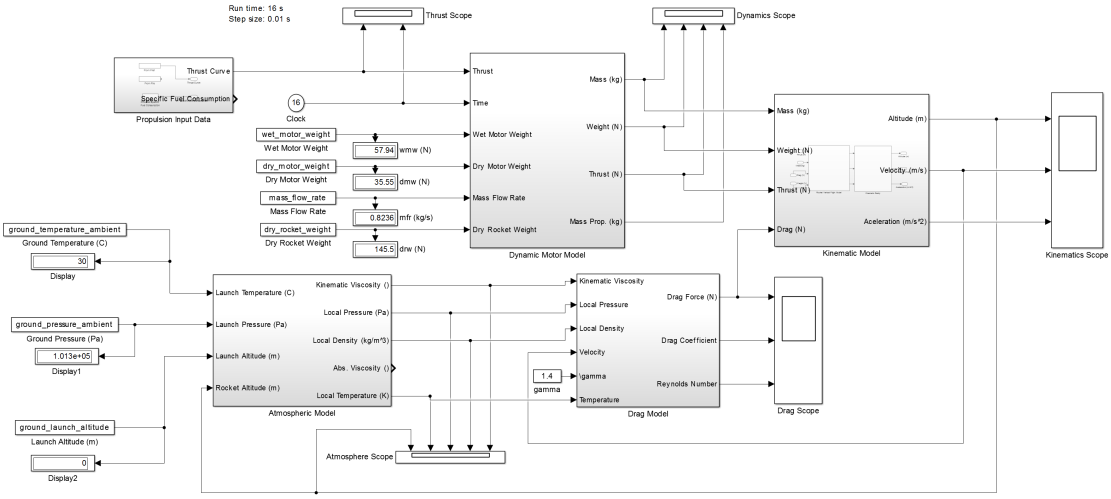

Figure 13: Vertical Model in Simulink, angle of attack less than 5 degrees

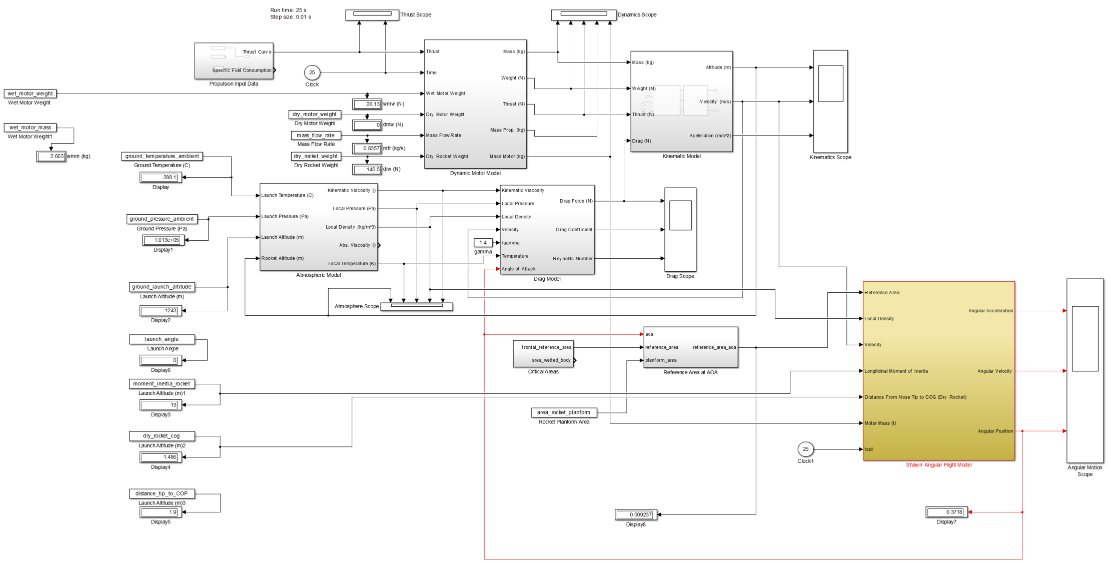

Figure 14: Full Model in Simulink, angle of attack less than 15 degrees

Summary

Best Case

Software	Case	Velocity leaving launch rail	Max Acc.	Max Vel.	Max. Alt.	Time to apogee

Worst Case

Software	Case	Velocity leaving launch rail	Max Acc.	Max Vel.	Max. Alt.	Time to apogee

Observations

Primary Plots

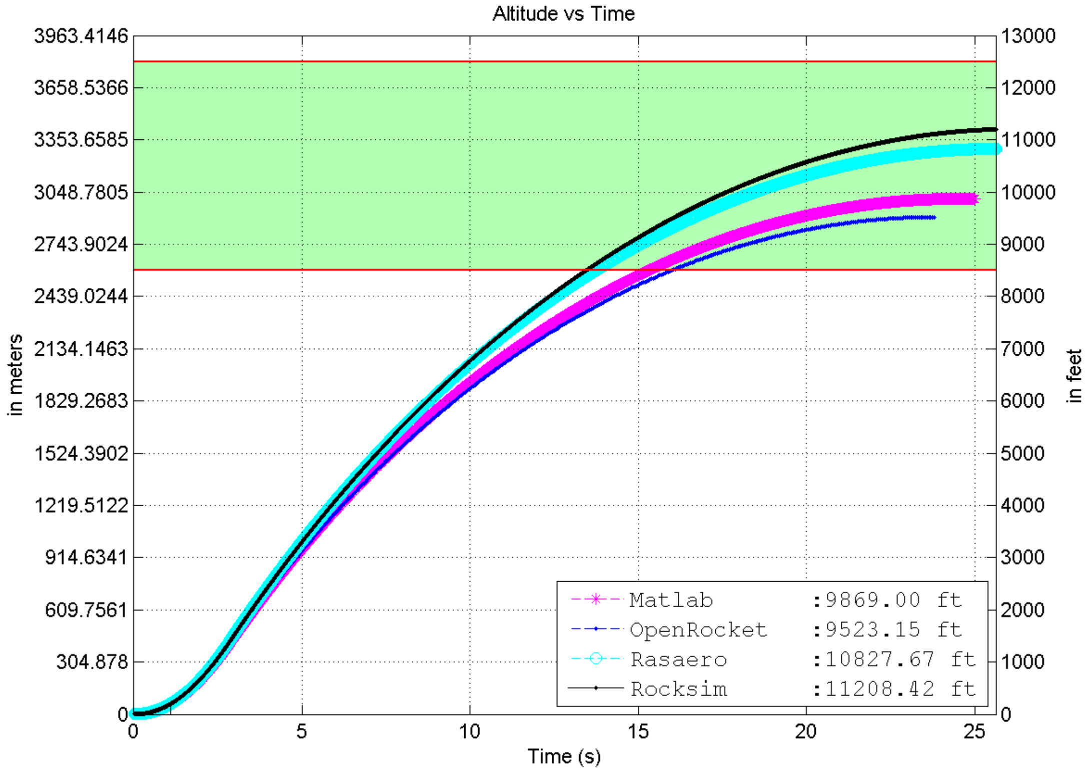

Figure 15: Altitude as a Function of Time

Homogeneous Response to Initial Conditions

$$\alpha_{xo} = 4\text{deg}$$
$$\omega_{xo} = 0\text{deg/s}$$

A rocket can be considered restored from a disturbance if the angle of attack decays to 5% of the initial amplitude [2].

The ideal damping ratio is between 0.05 and 0.30 **???**

The natural frequency of the rocket to pitch/yaw is **???**. This does not come into resonance with the oscillatory response of the rocket to disturbance.

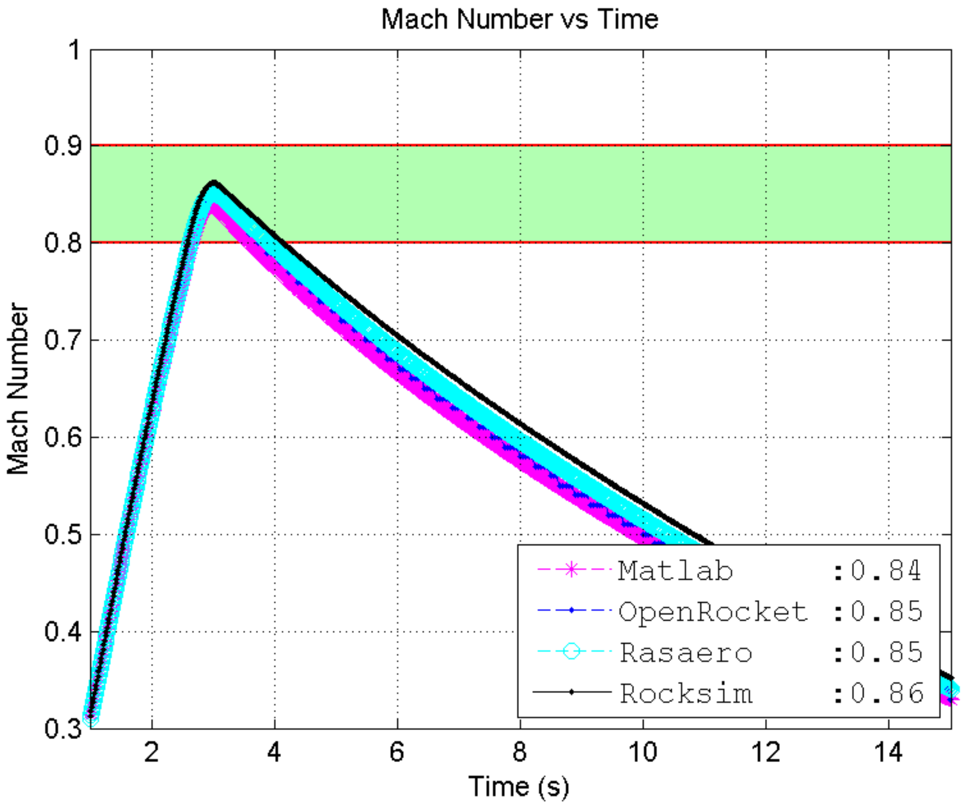

Figure 16: Mach Number as a Function of Time

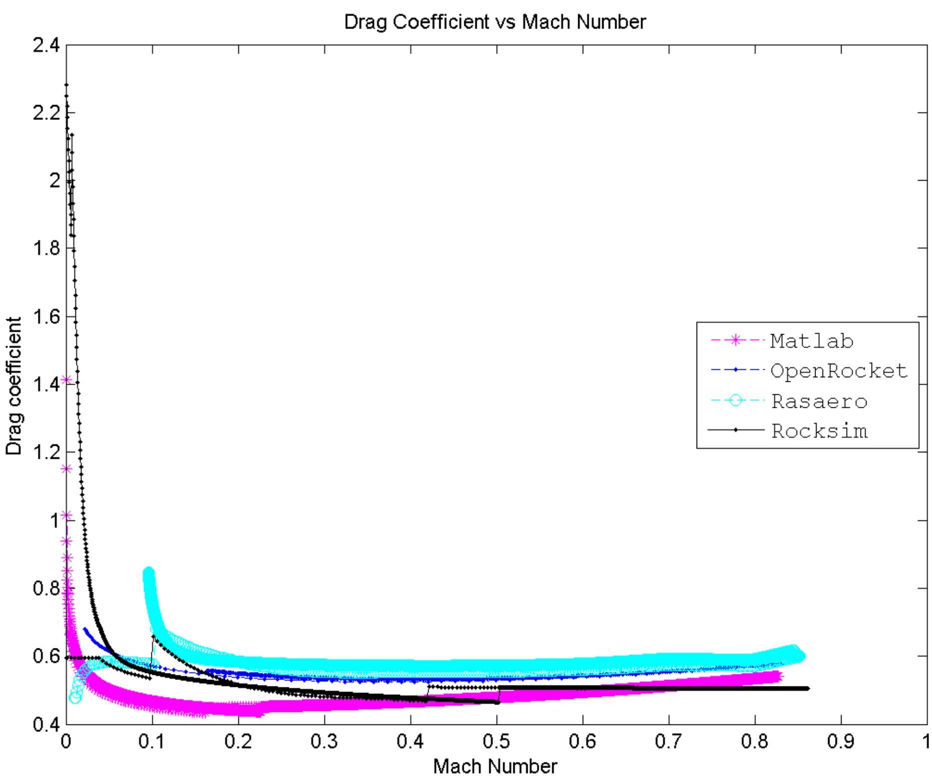

Figure 17: Drag Coefficient as a Function of Mach Number

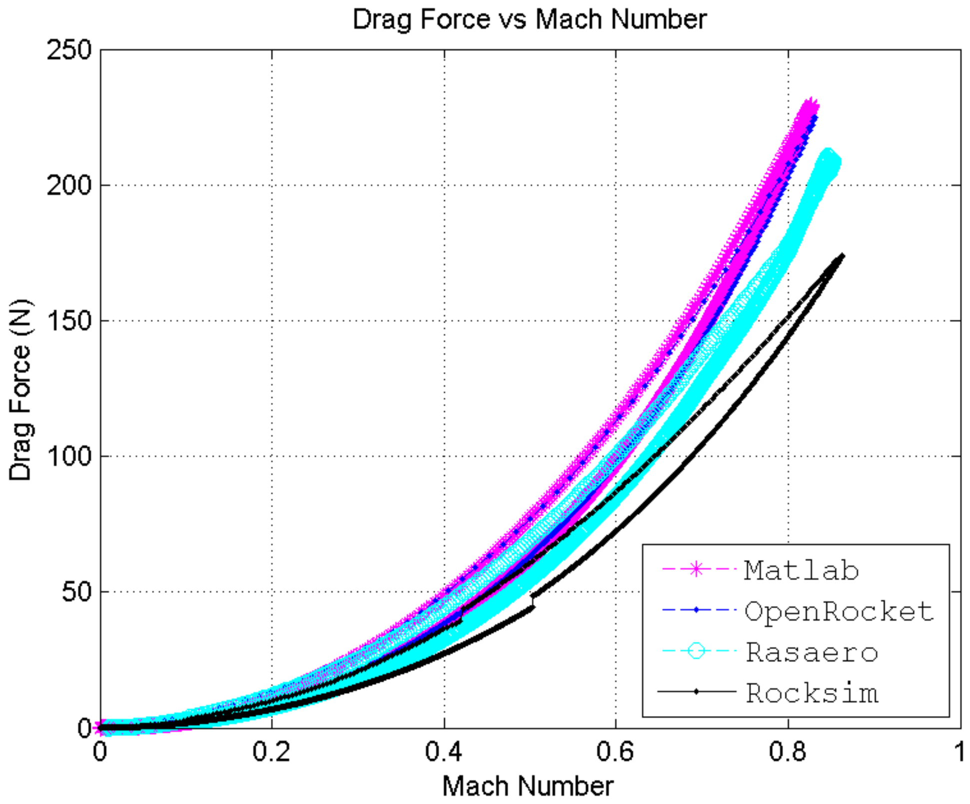

Figure 18: Drag Force as a Function of Mach Number

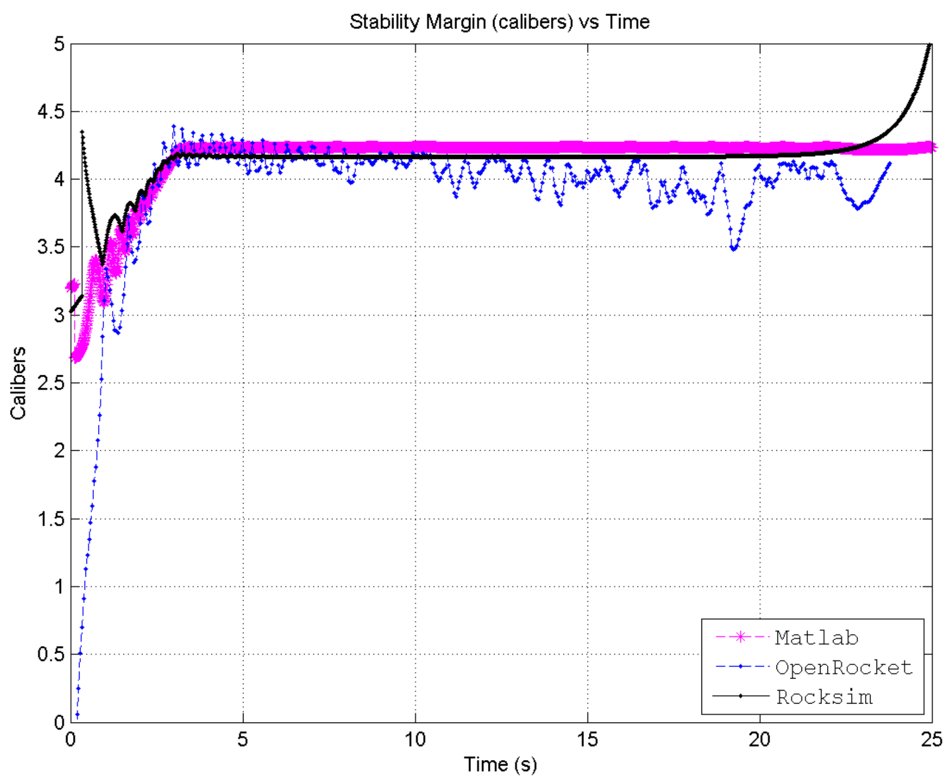

Figure 19: Stability (Calibers) as a Function of Time

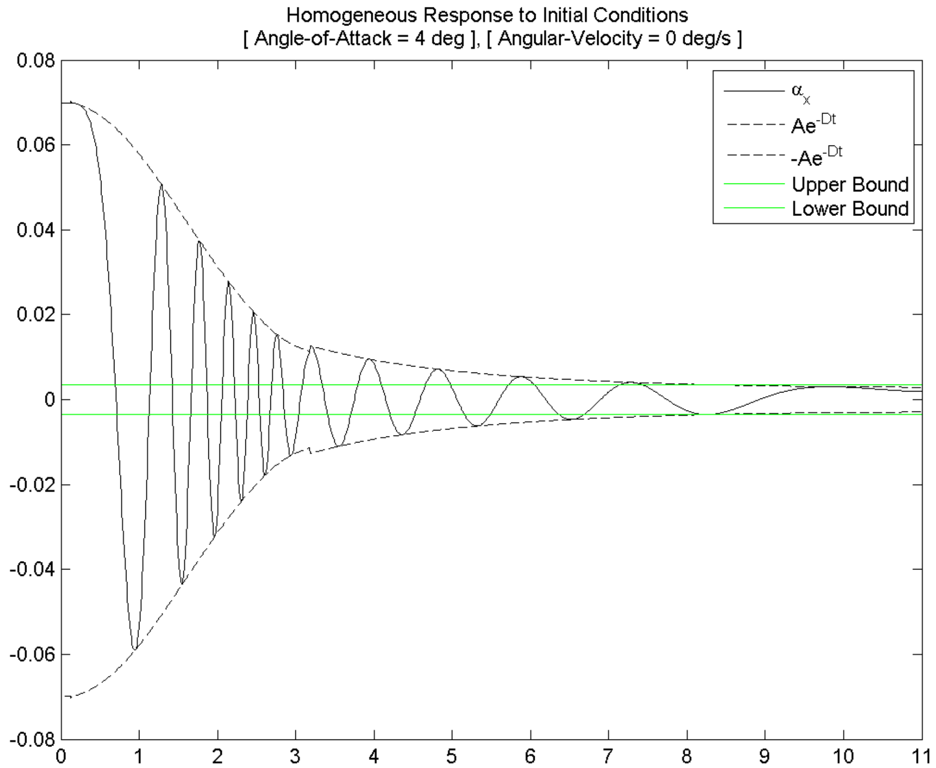

Figure 20: Homogeneous Response

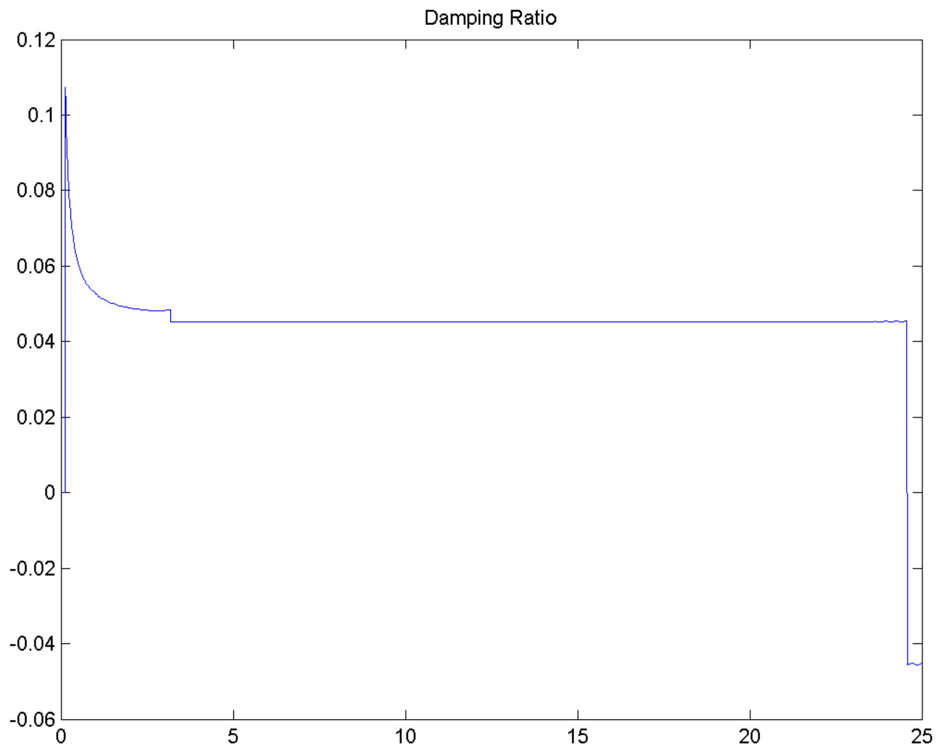

Figure 21: Damping Ratio

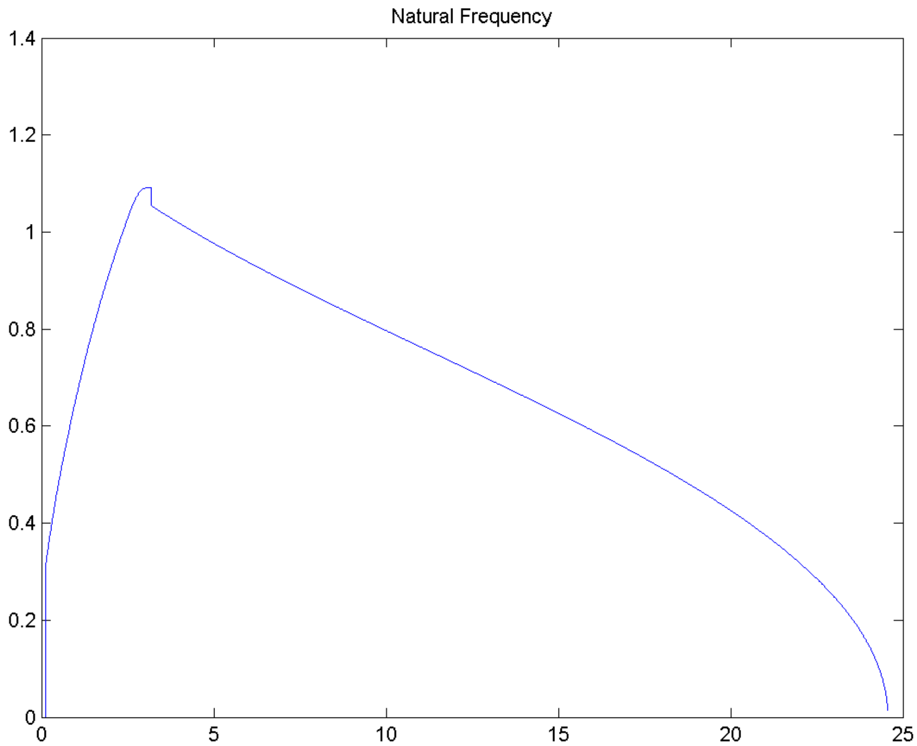

Figure 22: Natural Frequency

Additional Plots

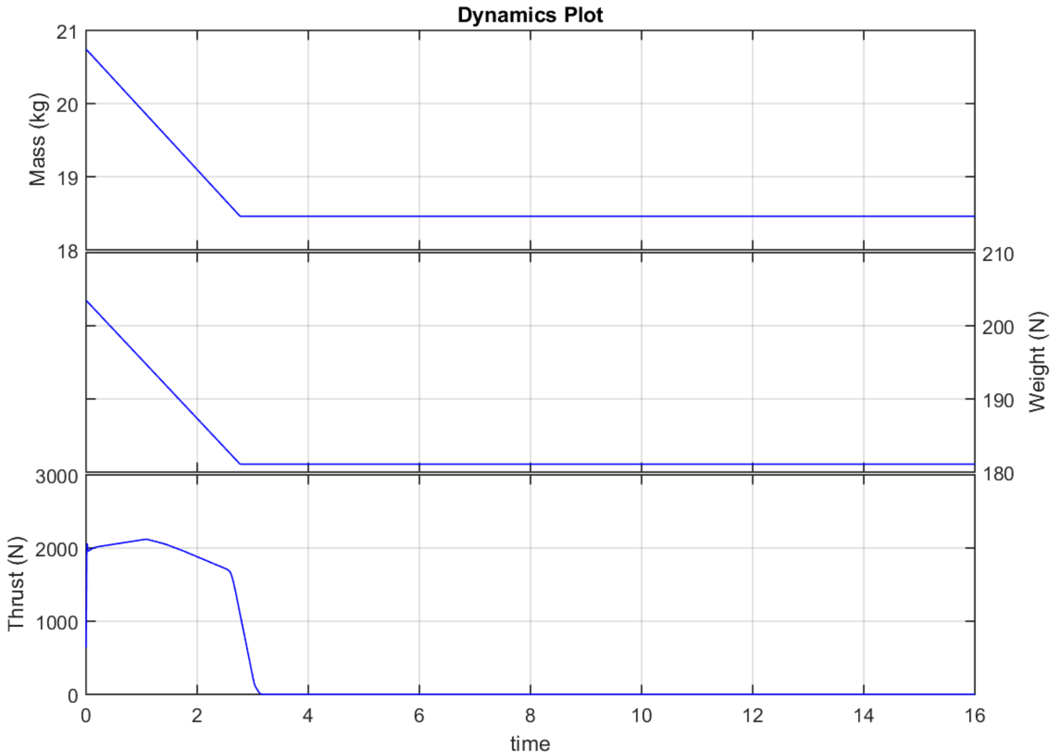

Figure 23: Mass, Weight, and Thrust as a Function of Time

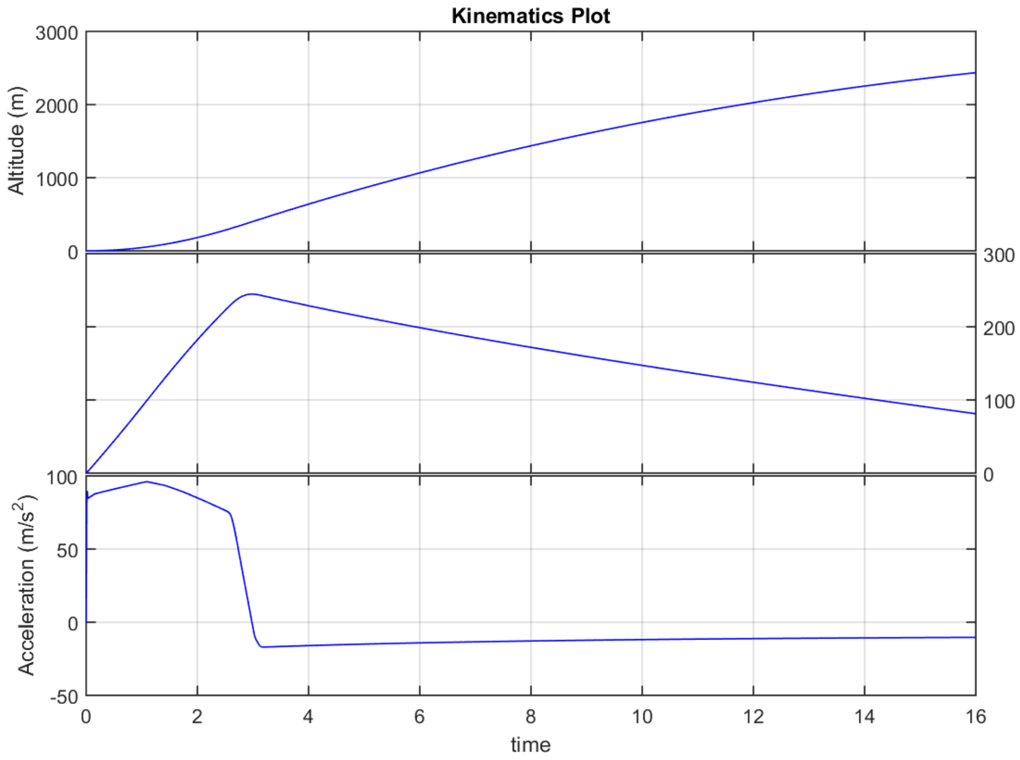

Figure 24: Altitude, Velocity, and Acceleration as a Function of Time

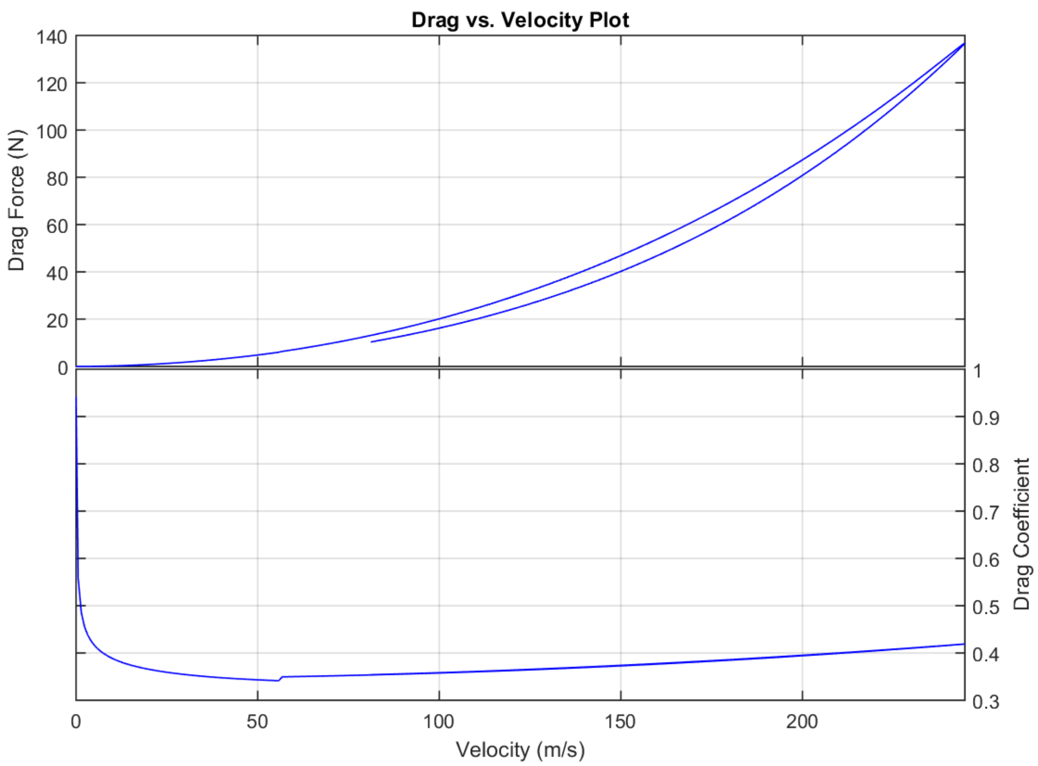

Figure 25: Drag Force, Drag Coefficient, and Reynolds Number as a Function of Velocity

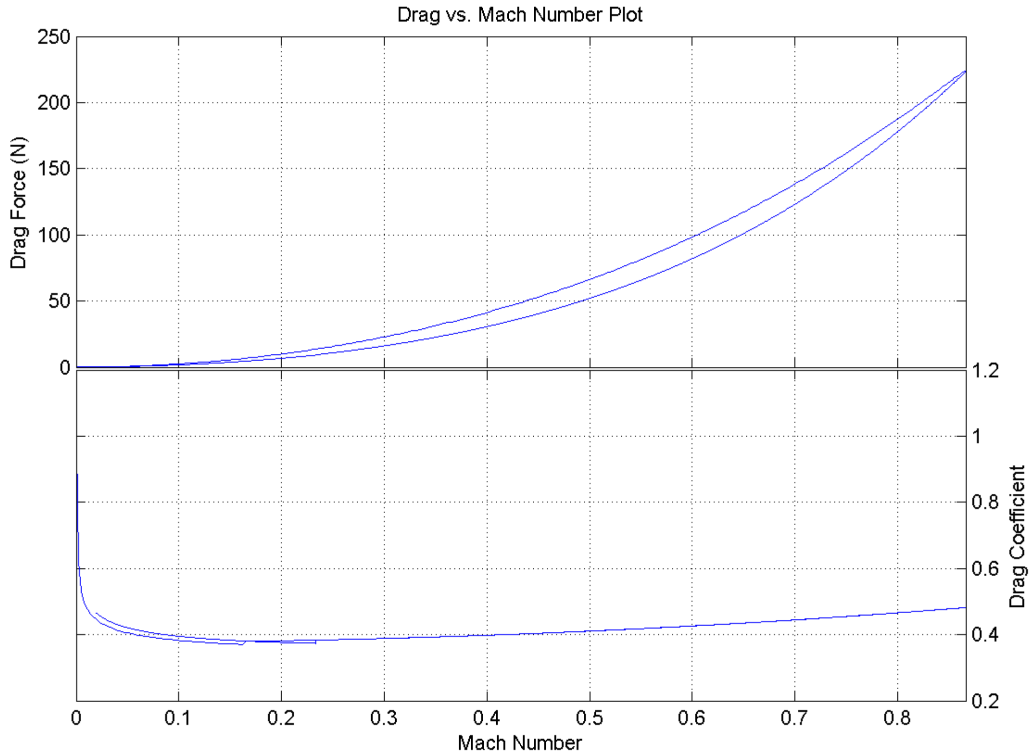

Figure 26: Drag Force, Drag Coefficient, and Reynolds Number as a Function of Mach Number

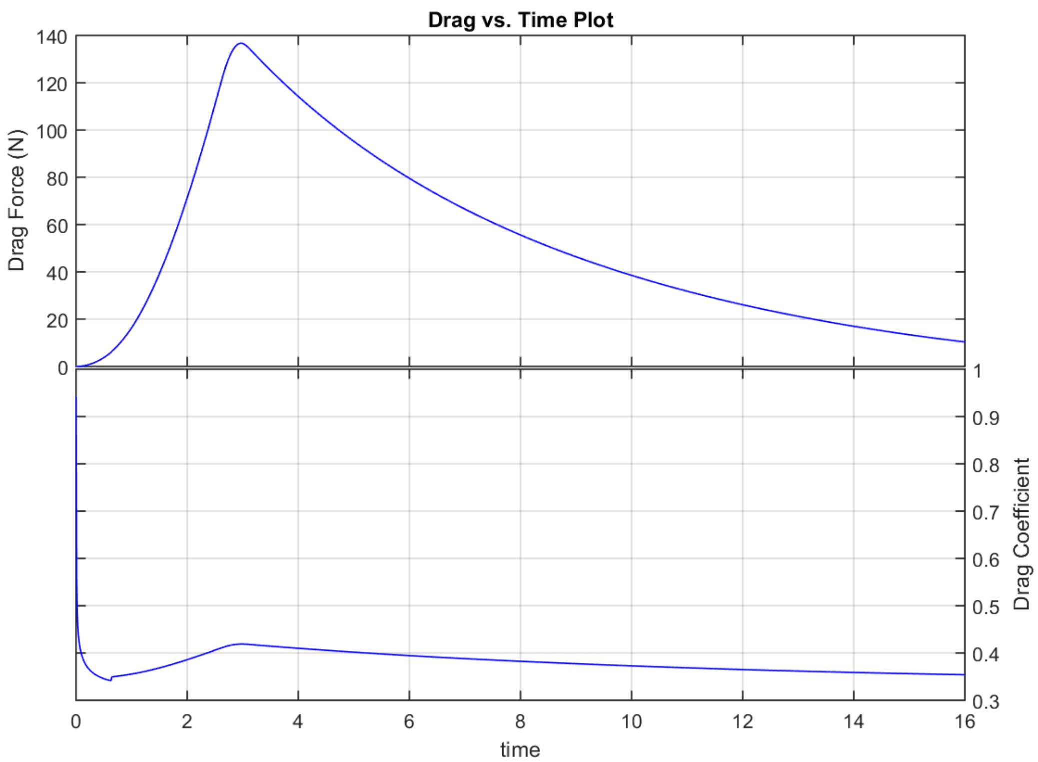

Figure 27: Altitude, Velocity, and Acceleration as a Function of Time

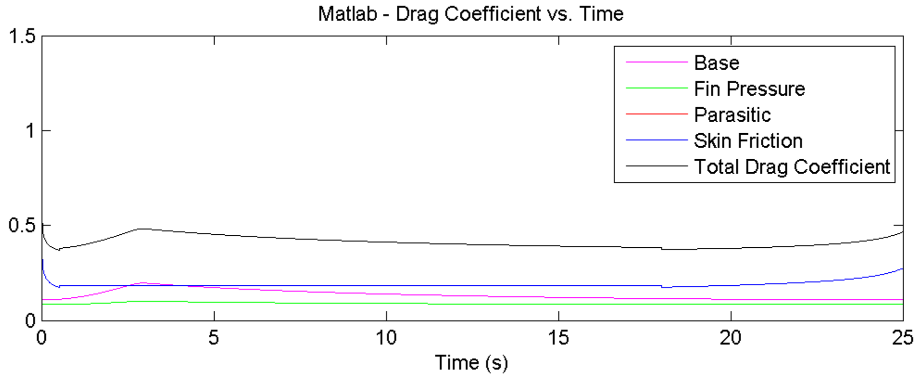

Figure 28: Specific Drag Coefficients as a Function of Time

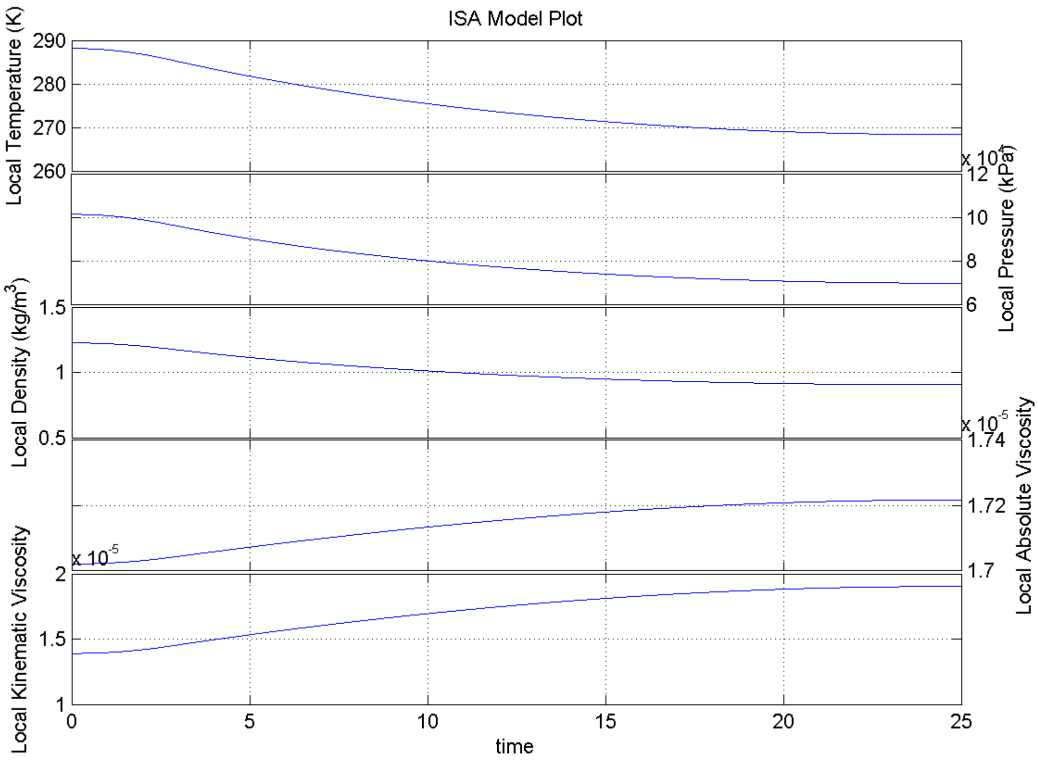

Figure 29: Altitude, Velocity, and Acceleration as a Function of Time

Monte Carlo Method

The *Monte Carlo Method* is

Stochastic Simulations

Conventions

Data Model

Data Store

Data Stores

Data Store

When to Use a Data Store

Data Store Examples

Data Stores and Software Verification > see RTCA DO-331, "Model-Based Development and Verification Supplement to DO-178C and DO-278A," Section MB.6.3.3.b.

The *Data Model* provides static and dynamics parameters as needed by other models in the simulation.

Static Parameters

Many parameters are constant throughout the simulation, notably the structural dimensions. All structural dimensions are generated in the CATIA Design and output to a spreadsheet, which the simulation will load and place in a Matlab *Map Container* instance.

> http://www.mathworks.com/help/matlab/map-containers.html

This instance can be accessed by multiple models to clearly and effectively provide parameter access.

Spreadsheet Import and Mapping

```
function parametric_data_object = data_parametric_model()
%---------------------------------------------------------------------------
%
% data_parametric_model.m
%
% Data Access of Parametric Model
%
% Design Parameters for the Rocket are stored in a spreadsheet.
% These parameters will be accessed and passed along into the simulation to
% determine other parameters and ultimately to evaluate the performance of
% those design values
%
%---------------------------------------------------------------------------
% References
% http://www.mathworks.com/help/simulink/ug/how-to-import-data-from-an-excel-spreadsheet.html
% http://www.mathworks.com/help/matlab/matlab_external/example-reading-excel-spreadsheet-data.htm
%---------------------------------------------------------------------------

% Import the data from Excel
```

```
[pnum,ptxt,praw] = xlsread('Parametric_Data.xlsm','Matlab_Data');

% Extract parameters from the imported data
keyset = ptxt(2:end,1);
valset = praw(2:end,2);

% initialize map object to store parametric data
parametric_data_object = containers.Map(keyset,valset);

end
```

Dynamic Parameters

As discussed in the *Dynamic Parameters* section, many parameters are changing due to flight conditions. An additional *Map Container* instance is created to handle and deliver these changes to the models that need them.

Matlab Conventions

Matlab/Simulink Libraries

Overview

The goal is to create a robust Simulink model that references Matlab code from files that can be tracked by versioning software (Git). The Matlab source should be editable and effect changes in the Simulink model.

By a combination of both, full versioning control can be achieved in the project.

Creating a Library

1. Open Matlab
2. Open Simulink
3. Click File –> New –> Library

Add to path

Permanently add your workspace to the Matlab path. At the command prompt:

```
>> pathtool
```

Alternatively, try this howto

Add to Library Browser

Add to Library Browser

Simulink GOTCHAs

- Atomic Units must be carefully employed to allow the simulation to run

Importing Data

Tabulated input data is relied upon to drive the simulation (see *Dynamic Parameters*). The following configuration is investigated to support this smoothly

Recommended Methods for Importing Data

Load Signal Data for Simulation

Importing Signal Data in Simulink

Import Data Structures

From File

The *From File* block in Simulink allows incremental loading of data

> The From File block reads data from a MAT-file and outputs the data as a signal. The data is a sequence of samples. Each sample consists of a time stamp and an associated data value.

From File

Mat-File Versions

Data is read incrementally from Mat-File versions 7.3 and above Mat-file Versions

nD Lookup Tables

Specifying Time Data

Specifying Time Data in Simulink

Versioning for Matlab Files

Background

- Older versions allowed providing external '.m' file for the *Matlab Function* block in Simulink
- Newer versions are shifting towards the embedded model, where Matlab code is complied for execution on test hardware

Naming conventions in Simulink for Matlab files changed after 2011A

Interpreted Matlab Function

Interpreted Matlab Function blocks are used to reference Matlab files so they can be versioned in Git. *Interpreted Matlab Function* blocks only accept one input and one output, therefore we must pass an array as input and an array as output. The contents of the array will contain our variables of interest. *(De))Mux* and *Bus* blocks may be useful to streamline the model.

1. Write your Matlab function
2. Create a Simulink Model
3. Add the *Interpreted Matlab Function* block
4. Double-click the added block, and enter the name of your function as directed. Select 'OK'
5. Right-click the block, expand the 'Mask', and select 'Create Mask'
6. Add the following in the 'Icon Drawing Commands' box disp('function_name')
7. Add a *Mux* block to combine your inports into a single input array, and a *Demux* port to unpack your output array into outports

Versioning for Libraries

- Saving files as libraries and following the existing use cases in the documentation will allow robust versioning and collaboration workflow

Unit Testing

Simulink Unit Testing

Model Referencing

Model Referencing shall be used to test all libraries for expected behavior.

1. Create a Test Model in which you drag the Library
2. Provide all test inputs and output assertion
3. Create another model to contain all the test cases created in *2*
4. From the *Simulink Library*, drag a *Model Reference* block
5. Edit the *Model Reference* block, providing the name of the Test Model created in *2*
6. Run the model created in *3* to verify the model referencing was successful

More information:

- http://www.mathworks.com/videos/getting-started-with-model-referencing-68918.html
- http://www.mathworks.com/help/simulink/ug/creating-a-model-reference.html
- http://www.mathworks.com/help/simulink/slref/model.html

Exporting Images

High quality figures brings a great deal of value to a report. Simulink Models and Matlab figures can be exported to scalable vector graphics and PDF formats at high quality.

GhostScript

GhostScript is needed to handle the EPS format. It can be downloaded here.

GhostScript and GIMP

GIMP has problems opening EPS files with the default configuration. Follow the instructions here to fix GhostScript in GIMP

Exporting Figures

export-fig is a Matlab library which provides functions to output figures to various formats

Exporting Simulink Models

export Simulink models to publication-quality figures

publication quality graphics in Matlab

LaPrint

Howto

File Organization

- data
- documentation
- functions
- libraries
- models
- referencing
- scripts
- testing

data

data → csv

documentation

- documentation
    - images
    - template

The *documentation* folder contains all markdown files with project documentation. It also contains

documentation → images

Contains all images used in the documentation

documentation → template

Contains LaTeX/Pandoc/Markdown template and styling files

functions

libraries

models

referencing

scripts

testing

Naming Conventions

Variables

All variables must be lowercase, separated by underscores

e.g.

```
wet_motor_weight
```

Functions

All *Matlab Functions* must be CamelCase

e.g.

```
DynamicWeightCalculation
```

Models

All *Matlab Model* names must be CamelCase, and end in the word 'Model'

e.g.

```
DynamicWeightCalculationModel
```

Libraries

All *Matlab Library* names must be CamelCase, and end in the word 'Library'

e.g.

```
DynamicWeightCalculationLibrary
```

Documentation Conventions

Markdown

Markdown is a markup language that is meant to be easy to read and easy to write, as well as easy to convert to HTML, LaTeX, PDF, and other output types.

Python

Python is used to enable additional filters which handle features currently not supported out-of-the-box by Pandoc

Download Python for Windows here

Pandoc

Pandoc is a document converter that in our case is useful in converting the Markdown (.md) files into PDF and HTML

The User Guide is very helpful

Haskell

Haskell is useful in this environment to do some custom scripting

LaTeX

LaTeX is a powerful typesetting language useful for academic writing. Its mathematical expressions are particularly useful for this report.

Citations

Excellent citation discussion

Haskell and Bibtex in Pandoc

IEEE CSL File Another IEEE CSL File

Equations

Wrap functions as follows to enable automatic numbering:

```
\begin{equation}
\label{my_equation}
f(x) = \int \cdot e^{xy}
\end{equation}
```

You can refer to the equation by the label you assigned to it

```
This comment refers to equation \ref{my_equation}
```

LaTeX equations in Markdown+Pandoc

Figures

To automatically number figures, use the following syntax to insert an image:

```
[rocket_drag_model_overview]: images/rocket_drag_model_overview.png "Rocket Drag Model Overview"
![Rocket Drag Model Overview \label{rocket_drag_model_overview_label}][rocket_drag_model_overview
```

Then, in your pandoc command, add the lof variable:

```
pandoc -s ... -V lof=lof
```

You can refer to the figure by the label you assigned to it

```
This comment refers to Figure \ref{rocket_drag_model_overview_label}
```

Tables

To automatically number tables and add captions, add the *capt-of* package to your preamble

```
\usepackage{capt-of}
```

Then, in your pandoc command, add the lot variable:

```
pandoc -s ... -V lot=lot
```

Future Enhancements

Embedded Execution

6DOF System with Quaternions

Porting to Python / OpenModelica

Robust Wind Model

ThrustCurve.org API Integration

ThrustCurve.org has an API we could use to dynamically pull motor data for integration into the simulation

Logging

- it would be nice to enable logging - a master log file that output useful progress information during the simulation

Other

- it would be nice to enable a countdown and mission status to show what stages are encountered

References

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