AA 279 C – SPACECRAFT ADCS: LECTURE 9

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Solar Radiation Pressure Torque (1)

- Radiation incident on a spacecraft's surface produces a force which results on a torque about the center of mass
- The force per unit area or radiation pressure on the surface equals the difference between incident and reflected momentum flux
- Since the solar radiance varies as the inverse square of the distance from the Sun, the solar radiation pressure is essentially altitude dependent
- Radiation torque depends on
  - Intensity and spectral distribution of the incident radiation
  - Geometry of the surface and its optical properties
  - Orientation of the Sun vector relative to the spacecraft
- Major sources of radiation pressure are
  - Solar illumination
  - Planet's albedo (reflected solar radiation)
  - Earth and atmosphere
  - Solar wind

500 km	1000	15000	60000
1358 W/m^2	1358	1358	1358
600	500	30	7
150	117	14	2

Solar constant: F

Solar Radiation Pressure Torque (2)

- Mean momentum flux on surface normal to Sun's radiation

$$P = F/c$$

Solar constant (solar activity)

- Radiation pressure on elemental area that is completely absorbed

$$d\vec{f}_{absorbed} = -PC_a \cos\theta \hat{\vec{S}} dA$$

Absorption coefficient

- Radiation pressure on elemental area that is specularly reflected

$$d\vec{f}_{specular} = -2PC_s \cos^2\theta \hat{\vec{N}} dA$$

Specular reflection coefficient

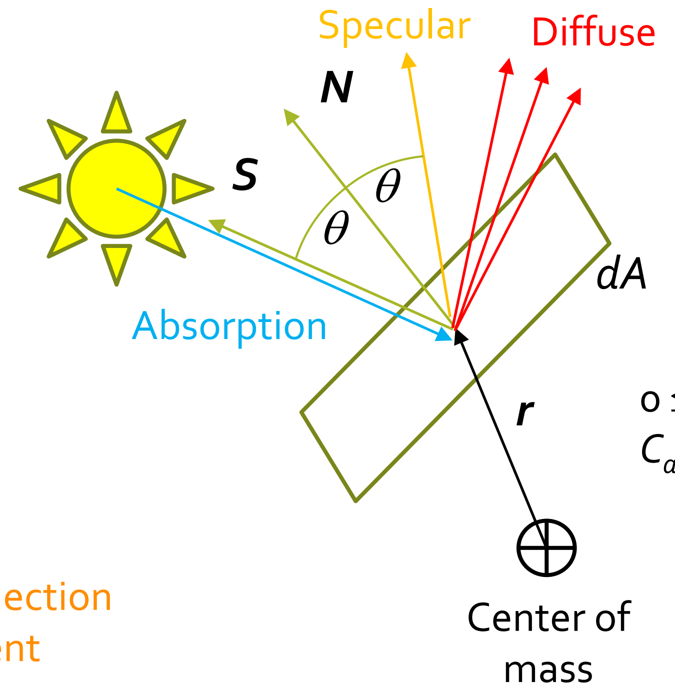
- Radiation pressure on elemental area that is diffusely reflected

$$d\vec{f}_{diffuse} = PC_d \left(-\frac{2}{3}\cos\theta\hat{\vec{N}} - \cos\theta\hat{\vec{S}}\right) dA$$

Diffuse reflection coefficient

- The total radiation force is given by

$$\boxed{d\vec{f}_{total} = -P\left[(1 - C_s)\hat{\vec{S}} + 2\left(C_s\cos\theta + \frac{1}{3}C_d\right)\hat{\vec{N}}\right]\cos\theta dA}$$



$0 \leq \theta \leq 90$ deg

$C_a + C_s + C_d = 1$

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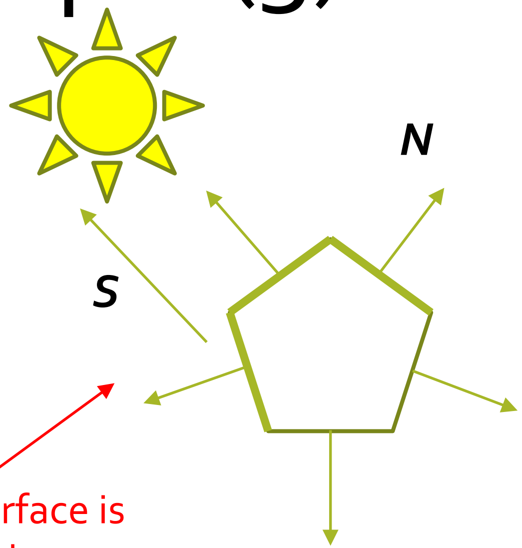
Solar Radiation Pressure Torque (3)



- The solar radiation torque acting on the spacecraft is given by the general expression

$$\vec{M}_s = \int \vec{r} \times d\vec{f}_{total}$$

- For simplicity, the integral is approximated by the sum of individual contributions of surface elements

Center of pressure of *i*th element

$$\vec{M}_s = \sum_{i=1}^{n} \vec{r}_i \times \vec{F}_i = \sum_{i=1}^{n} \vec{r}_i \times e_i \int d\vec{f}_{total_i}$$

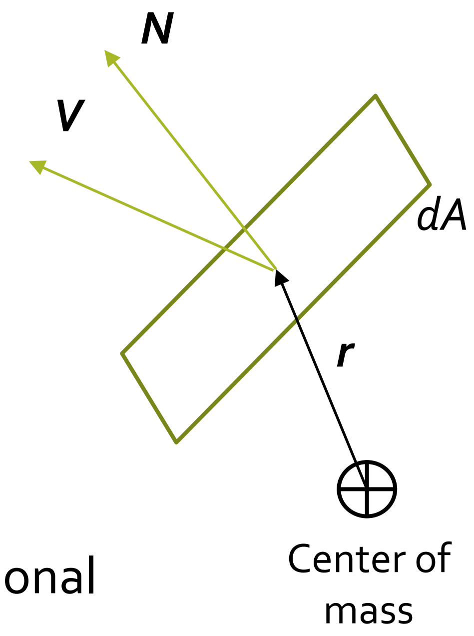
1 or 0 if surface is illuminated or not

GEOMETRIC FIGURE	FORCE
PLANE WITH SURFACE AREA A AND NORMAL $\hat{N}$: $\theta = \cos^{-1}(\hat{S} \cdot \hat{N})$	$-PA\cos\theta \left[(1 - C_s)\hat{S} + 2\left(C_s \cos\theta + \frac{1}{3} C_d\right)\hat{N}\right]$
SPHERE OF RADIUS r	$-P(4\pi r^2)\left(\frac{1}{4} + \frac{1}{9} C_d\right)\hat{S}$
RIGHT CIRCULAR CYLINDER OF RADIUS r, SYMMETRY AXIS $\hat{Z}$, AND HEIGHT h; ψ SUN ANGLE MEASURED FROM SYMMETRY AXIS, $A_1 = 2rh$, $A_2 = \pi r^2$	$-P\left(\left\{\left[\sin\psi\left(1 + \frac{1}{3} C_s\right) + \frac{\pi}{6} C_d\right] A_1 + \left(1 - C_s\right)\cos\psi A_2\right\}\hat{S} + \left[\left(-\frac{4}{3} C_s \sin\psi - \frac{\pi}{6} C_d\right)\cos\psi A_1 + 2\left(C_s \cos\psi + \frac{1}{3} C_d\right)\cos\psi A_2\right]\hat{Z}\right)$

Aerodynamic Torque

- The impact of atmospheric molecules on a spacecraft's surface produces a force which results on a torque about the center of mass
- This force can be modelled as an elastic impact without reflection, i.e. the incident particles loose their entire energy on collision

$$d\vec{f}_{aero} = -\frac{1}{2} C_D \rho V^2 \left(\vec{\hat{V}} \cdot \vec{\hat{N}}\right) \vec{\hat{V}} dA$$



- where $\vec{\hat{V}}$ is the unit vector in the direction of the translational velocity $\vec{V}$ of the surface element relative to the incident stream

$$\vec{V} = \vec{V}_{CM} + \vec{\omega} \times \vec{r} \, (\sim \vec{V}_{CM})$$

Translation of satellite CM relative to atmosphere

Rotation of satellite relative to atmosphere

- The total torque is evaluated from individual contributions of surface elements similar to the solar radiation pressure

Center of Pressure and Shadowing

- The position of the center of pressure, $\boldsymbol{r}_{cp}$ is required to compute the torque contribution of each surface element. The center of pressure is at the intersection of
  - the line of action of the single force which replaces the resultant (radiation or drag) force and
  - the plane passing through the center of mass of the spacecraft perpendicular to the line of action
- In mathematical terms, the center of pressure of the *i*th surface is such that

$$\vec{r}_{cp_i} \; x \; \vec{F}_i = \int \vec{r} \times d\vec{f}_{total_i}$$

- Solar radiation and drag torques are reduced by the shadows cast by one part of the spacecraft on another. Shadowing reduces the total force and also shifts the center of pressure
- The extent of shadowing is a function of the geometrical design of the spacecraft, the incident Sun angle (for radiation) or orientation relative to the velocity vector (for drag)

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Internal Disturbances

- Internal torques are defined as undesired torques exerted on the main body of a spacecraft by internal moving parts
- Internal torques can alter the system's rotational kinetic energy and exchange angular momentum
- Not being real torques, they are better modelled as changes of the mass distribution

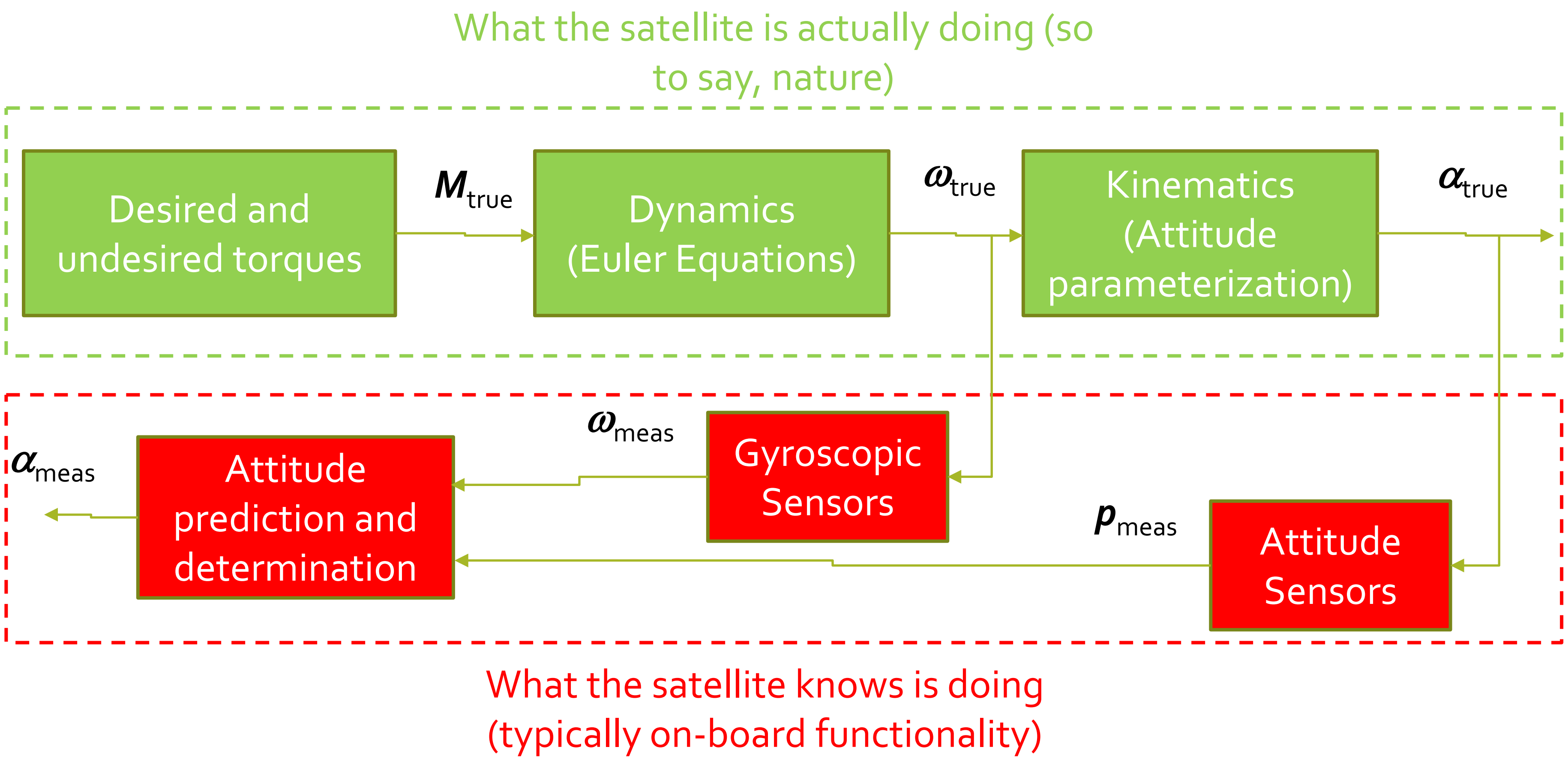
$$\frac{d\vec{L}}{dt} = \vec{I}\dot{\vec{\omega}} + \vec{\omega}\times\vec{I}\vec{\omega} + \dot{\vec{I}}\vec{\omega} = M \Leftrightarrow \frac{d\vec{L}}{dt} = \vec{I}\dot{\vec{\omega}} + \vec{\omega}\times\vec{I}\vec{\omega} = M \overbrace{-\,\dot{\vec{I}}\vec{\omega}}^{\substack{\text{Change of mass distribution} \\ \text{behaves as an external torque}}}$$

- Main internal disturbances
  - Mass expulsion torques (leakage, thrust misalignment, plume impingement)
  - Propellant slosh loads (oscillations of fluid in partially filled tank)
  - Crew motion ($\propto$ amplitude and ratio human's mass/satellite inertia moment)
  - Moving hardware (typically balanced by counter-moving hardware)

Sizing of Disturbance Torques (Max)

Disturbance name	Torque type depending on pointing requirement	Influencing Parameters	Reference Equation $M_{max} =$
Gravity Gradient	Constant for Earth pointing Periodic for inertial pointing	Moments of inertia Altitude	$\frac{3}{2}\frac{\mu}{R^3}\left\|I_{max} - I_{min}\right\|$
Solar radiation pressure	Periodic for Earth pointing Constant for inertial or Sun pointing	Geometry Center of mass Surface properties	distance of center of pressure from center of mass $PA(1+q)\left\|r_p - r_m\right\|$ $q = C_s + C_d$ reflectance
Aerodynamic drag	Constant for Earth pointing Variable for inertial pointing	Geometry Center of mass Altitude	$\frac{1}{2}\rho v^2 A C_d \left\|r_p - r_m\right\|$
Magnetic field	Periodic	Altitude Inclination Satellite's magnetism	$mB_{max} = 2m\frac{RE^3 B_0}{R^3}$

Attitude Determination Architecture



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