AA 279 C – SPACECRAFT ADCS: LECTURE 8

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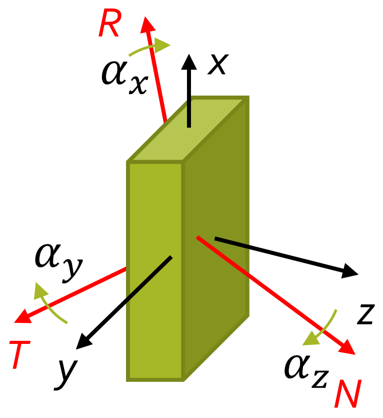
Modes of Oscillation with Gravity Gradient



- We consider a satellite which is nearly axial-symmetric

$$I_x \to 0\,;\, I_y \to I_z$$

- The eigenvalues of the plant matrix provides the frequencies of attitude motion when subject to gravity gradient

$$\begin{cases} s^2 + 3n^2 k_N = 0 \\ s^4 + s^2 n^2 (1 + 3k_T + k_T k_R) + 4n^4 k_T k_R = 0 \end{cases} \Rightarrow \begin{cases} s_{1,2} = \pm i\sqrt{3n^2 k_N} \\ s^4 + 4s^2 n^2 \sim 0 \end{cases}$$

$$\Rightarrow \begin{cases} s_{1,2} \sim \pm\, in\sqrt{3} \\ s_{3,4} = 0\,,\, s_{5,6} = \pm i2n \end{cases} \Rightarrow \begin{cases} f_N \to n\sqrt{3} \\ f_R \to 0 \\ f_T \to 2n \end{cases}$$

1.7 pitch oscillations in 1 orbit } Precession

No yaw oscillations

2 roll oscillations in 1 orbit } Nutation

- Since the frequency of natural perturbations is similar to these modes of oscillations, we should avoid resonance by staying far from this so-called critical point identified by $K_T = 1$, $K_R = 0$

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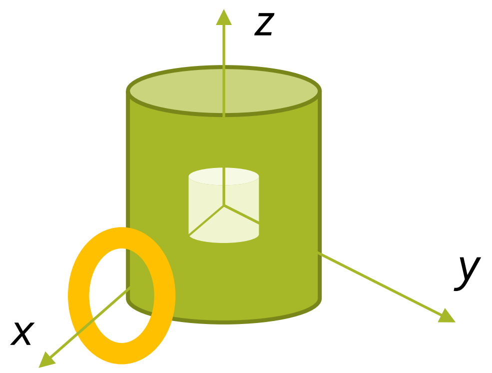
Passive Damping of Modes of Oscillations

- The actual motion of a dual spin satellite subject to gravity gradient (generic satellite) is the sum of multiple harmonic motions at different frequencies
  - **Nutation**: conic motion (e.g. of pitch axis) about the spin axis
    - f_x and f_y (perturbations) are undesired and need to be removed
  - **Precession**: pendulum motion (e.g. of roll and yaw axis) about the spin axis
    - f_z (perturbation) is undesired and need to be removed
- These perturbations can be removed through a damper which brings the satellite back to its nominal configuration
- This is a passive device which makes use of dissipation, thus the target attitude must be a configuration of stable equilibrium
- Dissipation can be created inside the satellite through relative motion caused by the motion of the satellite itself

Damping-Off Nutation (1)



- The perturbation of nutation is removed through zeroing the perturbations ω_x and ω_y
- To this end, a ring with fluid can be placed around the *x* or *y* axes
- Similar to a momentum wheel, the rotation of the fluid changes the total angular momentum vector

$$\vec{L} = \begin{cases} I_x\omega_x + I_f\omega_f \\ I_y\omega_y \\ I_z\omega_z + I_r\omega_r \end{cases}$$

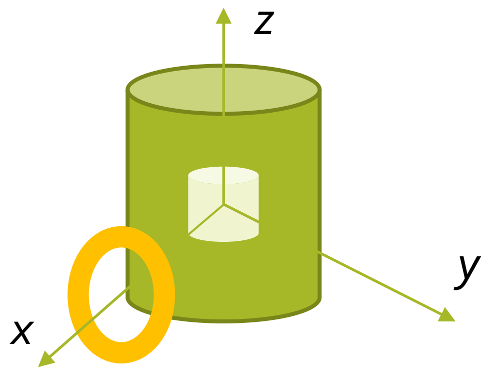
Angular velocities relative to satellite

Contain inertia of still wheel and fluid

Similar to equations of dual-spin satellite with second wheel

$$\begin{cases} I_x\dot{\omega}_x + I_f\dot{\omega}_f + \left(I_z - I_y\right)\omega_y\omega_z + I_r\omega_r\omega_y = 0 \\ I_y\dot{\omega}_y + (I_x - I_z)\omega_z\omega_x - I_r\omega_r\omega_x + I_f\omega_f\omega_z = 0 \\ I_z\dot{\omega}_z + I_r\dot{\omega}_r + \left(I_y - I_x\right)\omega_x\omega_y - I_f\omega_f\omega_y = 0 \\ I_r\dot{\omega}_r = 0\,; I_f\dot{\omega}_f = -c\omega_f \end{cases}$$

Damping-Off Nutation (2)



- The new equilibrium is given by

$$\begin{cases} \omega_x = 0 \\ \omega_y = 0 \\ \omega_z = \overline{\omega_z} \end{cases} \qquad \begin{matrix} \omega_f = 0 \\ \omega_r = \overline{\omega_r} \end{matrix}$$

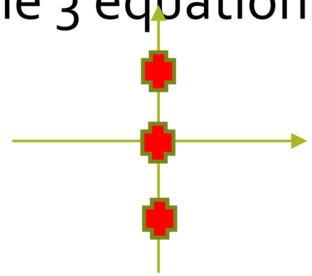
- Perturbation and linearization provide

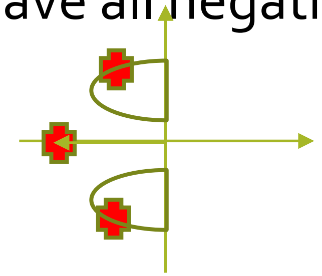
$$\begin{cases} I_x \dot{\omega}_x + I_f \dot{\omega}_f + \left(I_z - I_y\right)\omega_y \overline{\omega_z} + I_r \overline{\omega_r} \omega_y = 0 \\ I_y \dot{\omega}_y + (I_x - I_z)\overline{\omega_z}\omega_x - I_r \overline{\omega_r}\omega_x + I_f \omega_f \overline{\omega_z} = 0 \\ I_z \dot{\omega}_z + I_r \dot{\omega}_r = 0 \\ I_r \dot{\omega}_r = 0\,;\, I_f \dot{\omega}_f = -c\omega_f \end{cases}$$

As usual the equations in z are decoupled from x and y, thus the damper does not alter the dynamics

- We can tune *c* such that the poles of the linear system given by the 3 equations in x and y have all negative real part

c = 0 (periodic stability)





c ↑ (asymptotic stability)

But for too large *c* (viscosity) relative motion disappears

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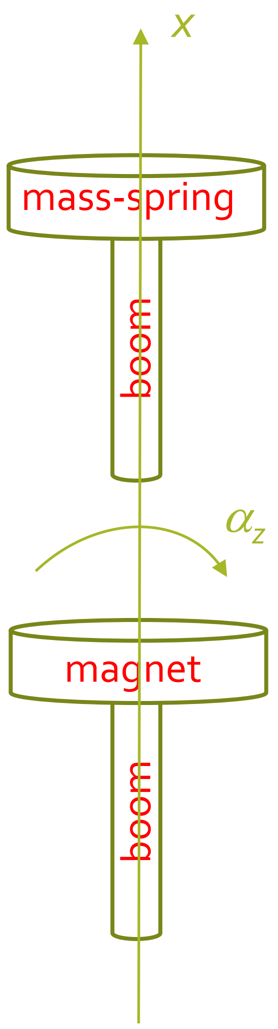
Damping-Off Precession

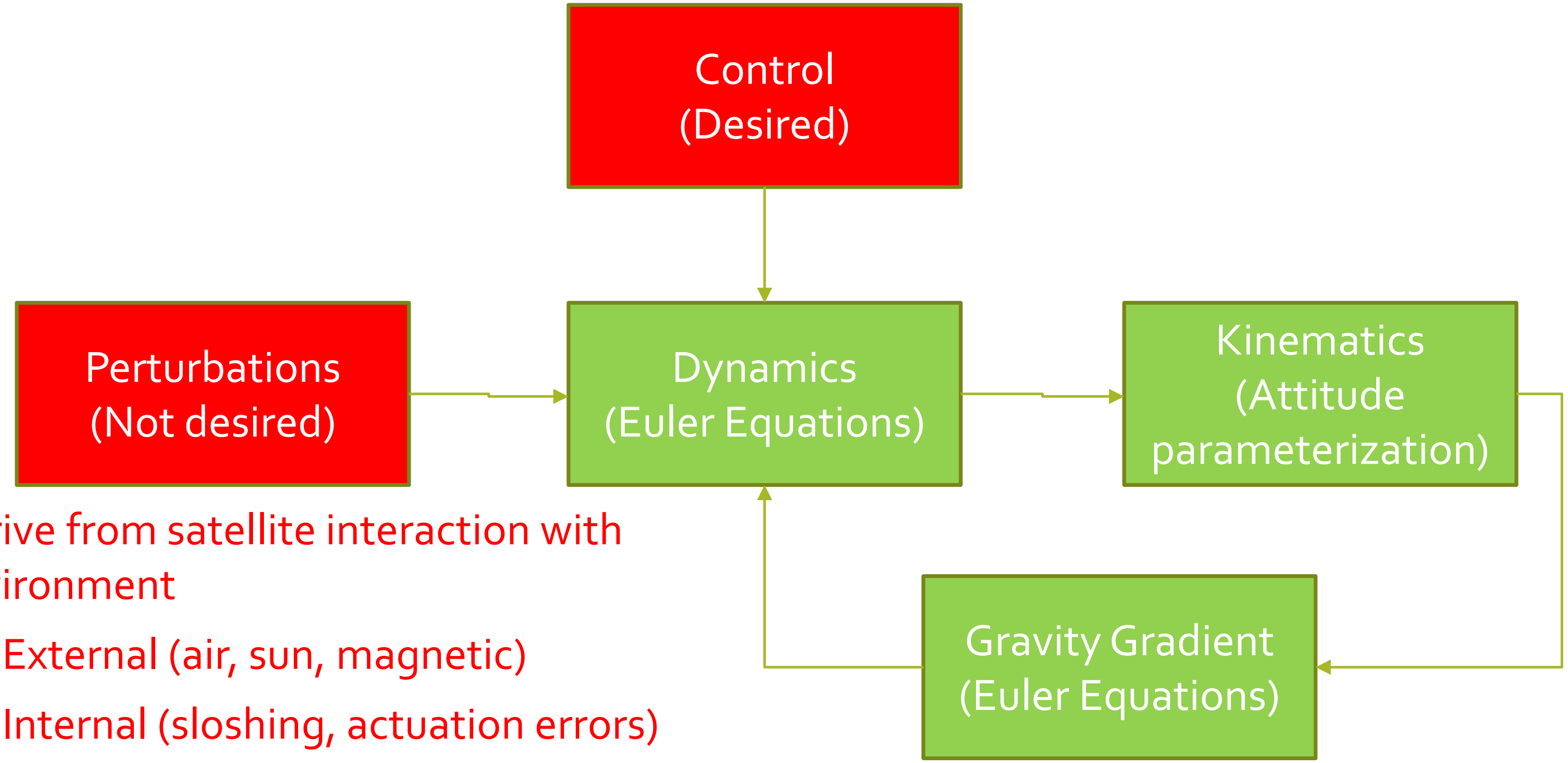
- The perturbation of precession is described by an harmonic oscillator about the z-axis (slow)

$$I_z\ddot{\alpha}_z + 3n^2\left(I_y - I_x\right)\alpha_z = 0$$

- Possible dampers
  1. Use boom which generates favorable gravity gradient through change of inertia properties
  2. Use boom with magnet which tends to orient along magnetic field. If precession occurs, magnet moves w.r.t. viscous fluid and dissipates energy
  3. Instead of magnet, we can use a mass-spring mechanism. If precession occurs, rotation causes centrifugal force which is opposed by elastic force. Mass oscillates and dissipates energy



Perturbations Acting on Satellite



Derive from satellite interaction with environment

1. External (air, sun, magnetic)
2. Internal (sloshing, actuation errors)

Even if the equilibrium is stable, these perturbations might change configuration and cause instability

Magnetic Torque (1)

- The magnetic torque is the easiest to model for a spacecraft
- It is caused by the interaction between the planet's magnetic field ***B*** and the satellite's magnetic dipole moment ***m***
- Formally, we express it as the vector product between the two vectors

$$\overrightarrow{M_m} = \vec{m} \times \vec{B}$$

[Nm/T] [T]

- The satellite's magnetic moment
  - Is not known with precision
  - Satellite is like a nucleus surrounded by a coil (current loop)
  - Can be estimated in laboratory
- The planet's magnetic field
  - Depends on satellite's position
  - Modelled as gradient of potential
  - Expressed as a spherical harmonic expansion similar to gravity

$$\vec{B} = -\vec{\nabla} V$$

Geocentric distance, longitude, and colatitude

Gaussian coefficients

Legendre functions

$$V(R, \lambda, \theta) = R_E \sum_{n=1}^{k} \left(\frac{R_E}{R}\right)^{n+1} \sum_{m=1}^{n} (g_n^m \cos m\lambda + h_n^m \sin m\lambda) P_n^m(\theta)$$

Equatorial radius

Magnetic Torque (2)

- The Gaussian coefficients constitute a magnetic field model and are derived empirically by a least-squares fit to field measurements
- Appendix H of Wertz provides the standard IGRF coefficients up to order and degree $n = m = 8$
- The first order provides the magnetic dipole which is already very close to the actual field for our purposes. All coefficients are rarely used, and the higher the altitude, the more accurate the dipole model, e.g.
  - LEO: n=4
  - MEO: n=2
  - GEO: n=1

Dipole strength

Earth dipole direction

Position unit vector

$$\vec{B}(\vec{R}) = -\frac{R_E^3 B_0}{R^3}\left[3\left(\hat{\vec{m}} \cdot \hat{\vec{R}}\right)\hat{\vec{R}} - \hat{\vec{m}}\right]$$

Can be evaluated in any coordinate system

- The axis of the magnetic field does NOT coincide with the rotation axis (~10 deg rotation) and the magnetic poles are routinely monitored
- A rotation is necessary to compute ***B*** from ECEF to ECI coordinates (H-14-23)

Magnetic Torque (3)

- The intensity of the magnetic field goes as $1/R^3$ and is double at the poles as compared with equator
- It is subject to intrinsic perturbations (see the South Atlantic anomaly)
- It is more important to accurately model the magnetic field when used to control the attitude than for perturbation analysis
- In the absence of experimental or reference data, a coarse model for the maximum satellite residual magnetism is given by

$$m_{max} = \mu n S I \sim 4\pi 10^{-7}[H/m] \cdot 1 \cdot S_{\mathrm{SAT}}[m^2] \cdot 0.1[A]$$

Permeability in vacuum

Number of coils

Surface contained by coil

Residual current

- Random values of m could be then generated to be smaller than m_{max}

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Backup