AA 279 C – SPACECRAFT ADCS: LECTURE 11

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State Vector Parameters (Examples)

- Single-spin stabilized satellite subject to small torques (*kin*)

Right ascension and declination of spin axis

Azimuth and latitude of one axis of magnetometer triad in body coordinates

$$\mathbf{x} = (\alpha, \delta, \omega, \theta, \phi_0, f, b)^{\mathrm{T}}$$

Angular velocity about spin axis

Slope of magnetometer calibration curve and residual bias

- Three-axis stabilized earth pointing satellite subject to small torques (*kin*)

Initial pitch, roll, and yaw angles and rates

Residual bias of magnetometer $_j$

$$\mathbf{x} = (p, \dot{p}, r, \dot{r}, y, \dot{y}, \theta_x^1, \theta_y^1, \theta_z^1, \theta_x^2, \theta_y^2, \theta_z^2, \theta_x^3, \theta_y^3, \theta_z^3, b_1, b_2, b_3)^{\mathrm{T}}$$

Angles between magnetometer ^{j} and body axis $_i$

- Three-axis stabilized inertial pointing satellite subject to high torques (*dyn*)

Attitude and angular velocity

Constant environmental torques in body frame

$$\mathbf{x} = (q_1, q_2, q_3, q_4, \omega_x, \omega_y, \omega_z, m_1, m_2, m_3, m_4, s_1, s_2, s_3, s_4, c_x, c_y, c_z, d_x, d_y, d_z)^{\mathrm{T}}$$

Moment of inertia and constant speed bias on each reaction wheel

Spacecraft magnetic dipole in body frame

Real-time State Propagation (Examples)

- In many applications it is possible to avoid the numerical integration of the Euler equations to propagate the attitude within the state estimation
- For a spin-stabilized satellite, one could even assume a constant angular velocity over the period of interest and build the attitude matrix at any time t

$$\vec{A}(t_0+\Delta t) = \begin{bmatrix} \cos(\omega\Delta t) & \sin(\omega\Delta t) & 0 \\ -\sin(\omega\Delta t) & \cos(\omega\Delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \vec{A}(t_0)$$

- For a three-axis stabilized earth pointing satellite, for short time periods and small torques, the Euler angles are small and can be propagated as

$$p(t_0+\Delta t) = p(t_0) + \dot{p}(t_0)\Delta t\,; r(t_0+\Delta t) = r(t_0) + \dot{r}(t_0)\Delta t\,; y(t_0+\Delta t) = y(t_0) + \dot{y}(t_0)\Delta t$$

- More accurately, for a general satellite and short time periods the quaternions can be propagated as

$$\vec{q}(t_0+\Delta t) = e^{\frac{1}{2}\Omega(t_0)\Delta t}\vec{q}(t_0) = \left[\cos\left(\frac{\omega\Delta t}{2}\right)\vec{I} + \frac{1}{\omega}\sin\left(\frac{\omega\Delta t}{2}\right)\Omega(t_0)\right]\vec{q}(t_0)$$

Observation Model (Examples)

- A satellite uses a three-axis induction magnetometer which measures the earth's magnetic field in the sensor frame (here body axes) at time t_{m}

$$\vec{B}_{\mathrm{meas}}(t_{\mathrm{meas}}) = \vec{F}\vec{V}_{\mathrm{meas}} + \vec{V}_0$$

Calibration curve
Output voltage
Calibration bias

- The predicted measurement must be computed from the current state

$$\vec{B}_{\mathrm{pred}} = \vec{A}\vec{B}_I + \vec{b}$$

Estimated calibration bias
Magnetometer frame attitude
Modelled inertial magnetic field

- In general, the predicted measurement requires a short propagation step where the state is propagated to the time of the measurement

$\vec{A}(t_{\mathrm{meas}})$ from $\vec{A}(t_{\mathrm{state}})$ (see previous slide)

- If the measurement is chosen to be the magnetometer voltage, then the modelled observation will be given by

$$\vec{y} = \vec{V}_{\mathrm{meas}}\ ;\ \vec{z} = \vec{V}_{\mathrm{pred}} = \vec{F}^{-1}(\vec{B}_{\mathrm{pred}} - \vec{V}_0)$$

Function of state

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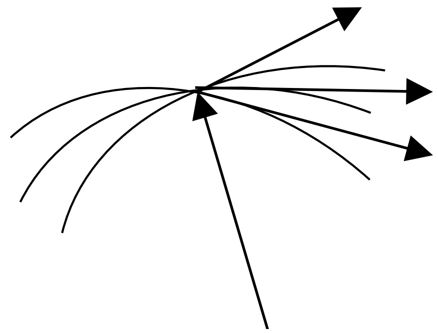
Sensitivity Matrix (Examples)

- Sensitivity represents the partial derivative of measurements w.r.t. state
- Answers the question: how much does measurement change if I change state

$$\boldsymbol{z}\,[\mathrm{nx1}] = \boldsymbol{H}\,[\mathrm{nxm}]\,\boldsymbol{x}\,[\mathrm{mx1}]$$

- Note: quantities are taken at the same time since propagation is handled in the time update
- Does velocity affect position? **NO**
- Do angles affect position? **YES**
- Example partial derivatives: **Sun vs Euler angles**



$$S_x = (c\phi c\psi + s\phi s\psi s\theta)S_{x_{eci}} + (c\theta s\psi)S_{y_{eci}} + (-c\psi c\phi + c\phi s\psi s\theta)S_{z_{eci}}$$

$$\frac{\partial S_x}{\partial \theta} = (c\theta s\phi s\psi)S_{x_{eci}} - (s\theta s\psi)S_{y_{eci}} + (c\theta c\phi s\psi)S_{z_{eci}}$$

$$\frac{\partial S_x}{\partial \phi} = (-s\phi c\psi + c\phi s\psi s\theta)S_{x_{eci}} + (s\phi c\psi + s\phi s\psi s\theta)S_{z_{eci}}$$

$$\frac{\partial S_x}{\partial \psi} = (-s\psi c\phi + c\psi s\phi s\theta)S_{x_{eci}} + (c\psi c\theta)S_{y_{eci}} + (s\psi c\phi + c\psi c\phi s\theta)S_{z_{eci}}$$

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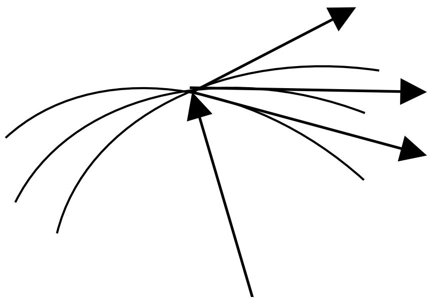
- Example partial derivatives: **Sun vs Quaternion**



$$S_x = (q_1^2 - q_2^2 - q_3^2 + q_4^2)S_{x_{eci}} + 2(q_1 q_2 + q_3 q_4)S_{y_{eci}} + 2(q_1 q_3 - q_2 q_4)S_{z_{eci}}$$

$$\frac{\partial S_x}{\partial q_1} = 2q_1 S_{x_{eci}} + 2q_2 S_{y_{eci}} + 2q_3 S_{z_{eci}}$$

$$\frac{\partial S_x}{\partial q_2} = -2q_2 S_{x_{eci}} + 2q_1 S_{y_{eci}} - 2q_4 S_{z_{eci}}$$

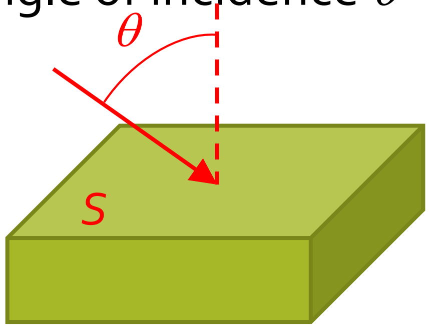
$$\frac{\partial S_x}{\partial q_3} = -2q_3 S_{x_{eci}} + 2q_4 S_{y_{eci}} + 2q_1 S_{z_{eci}}$$

$$\frac{\partial S_x}{\partial q_4} = 2q_4 S_{x_{eci}} + 2q_3 S_{y_{eci}} - 2q_2 S_{z_{eci}}$$

Sun Sensors (1)

- Since the radiation intensity of the Sun is orders of magnitude larger than the stars, these sensors are referred to as Sun sensors
- The source is unique for most satellite missions, and Sun sensors are typically calibrated to work at distances of 1 AU
- We distinguish between measurements of the Sun's position (typically affected by 0.5° error), or of the Sun's contour to decrease this error
- The sensor is hit by radiation and produces an electric signal based on the sensitive surface S, its optical properties α, and the angle of incidence θ

$$I = \underbrace{\alpha S}_{\text{From manufacturer}} \cos\theta$$

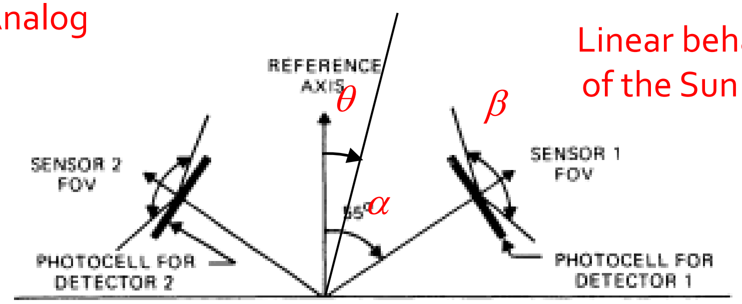


- Measuring the output current provides the direction of the radiation sources relative to the surface
- In practice this relationship is linear only for small θ
- For large angles, the current depends on the angle itself, not only on its cosine

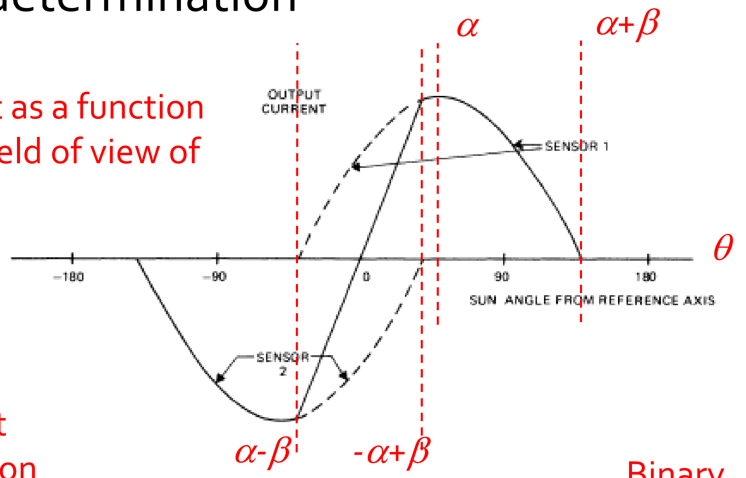
Sun Sensors (2)

- As a consequence, the field of view is limited to avoid non-linearities through a cage or box and multiple narrow field of view elements are combined
- Sun position sensors: large field of view, seeks dependency on $\sin\theta$ rather than $\cos\theta$, analog or digital, for attitude determination

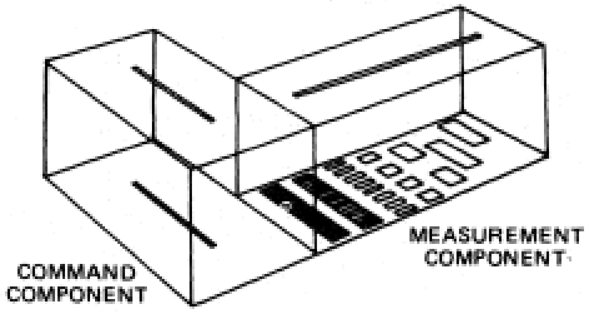
Analog

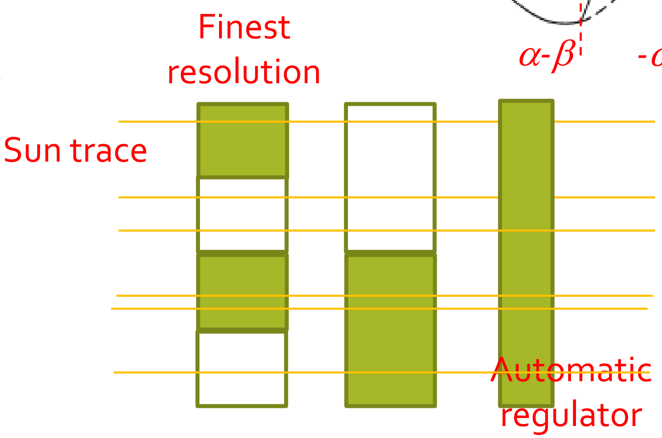


Linear behavior of output as a function of the Sun angle over a field of view of 15 °



Digital

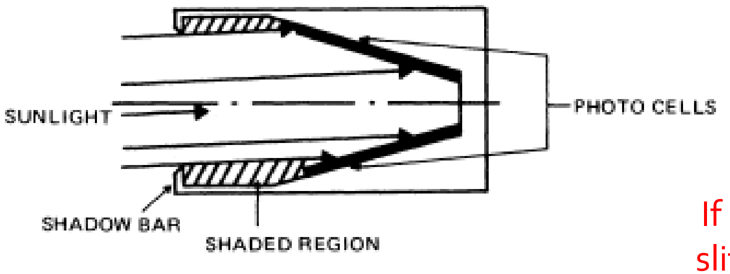




Binary	Dec
10	2
11	3
01	1

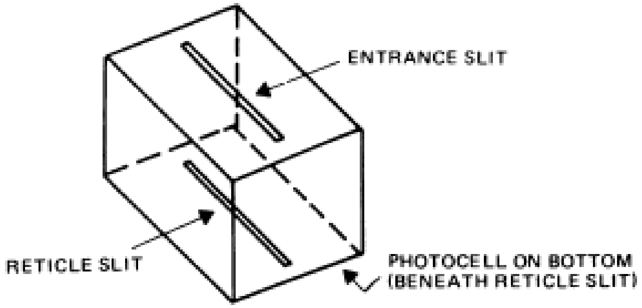
Sun Sensors (3)

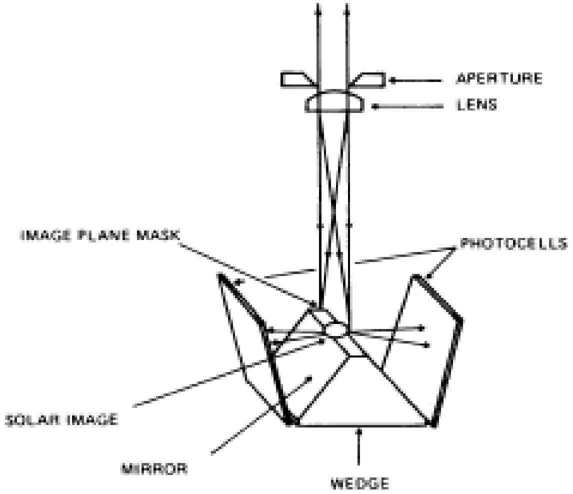
- Sun presence sensors: narrow field of view (0.5°), binary info, for operational constraints (e.g., Sun exclusion)



If optical axis of cylinder is not aligned with Sun, photo cells produce a different signal

If Sun is in the place defined by slits, the photo cell produces an impulse (and time for spinning)



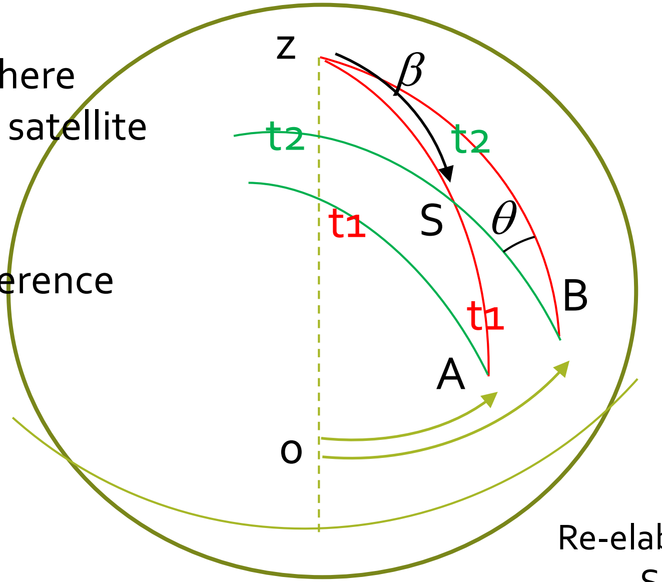


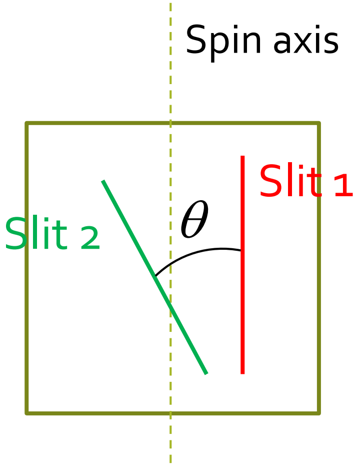
If Sun is aligned with reference axis, no signal. Otherwise mirrors→photo cells produce signal

Sun Sensors (4)

- Simple model of a Sun presence sensor for a spinning satellite built from two slits, where one slit is parallel to the spin axis and one slit is tilted

Celestial sphere
centered at satellite
z: zenith
S: sun
o: initial reference





$$AB = \omega(t_2 - t_1) = \omega\Delta t \underset{\text{SAB}}{\Rightarrow} \overbrace{\tan\beta = \frac{\tan\theta}{\sin(\omega\Delta t)}}^{\text{Re-elaboration provides Sun direction}} \Rightarrow \overbrace{\Delta t = \frac{1}{\omega}\operatorname{asin}(\frac{\tan\theta}{\tan\beta})}^{\text{Functioning principle provides measurement output}}$$

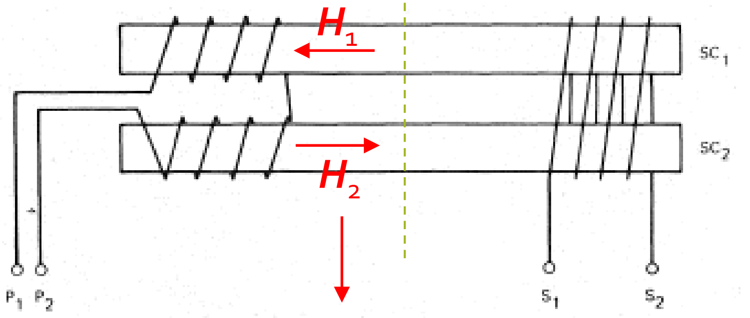
Magnetometers (1)

- Sun and earth sensors provide directions, whereas a magnetometer provides 3 independent measurements: direction and intensity of magnetic field
- The measurement is based on ferromagnetic induction through a coil which surrounds ferromagnetic material
- The coil outputs a voltage proportional to the variation of magnetic field flux
- The variation of magnetic field flux must be generated through
  - Spinning motion, since orbital motion is not sufficient, or
  - Generation of a variable magnetic field (fluxgate magnetometers)

Primary coil generates variable magnetic field

$V_p \rightarrow$ $H_1 \rightarrow B_1$

$H_2 \rightarrow B_2$

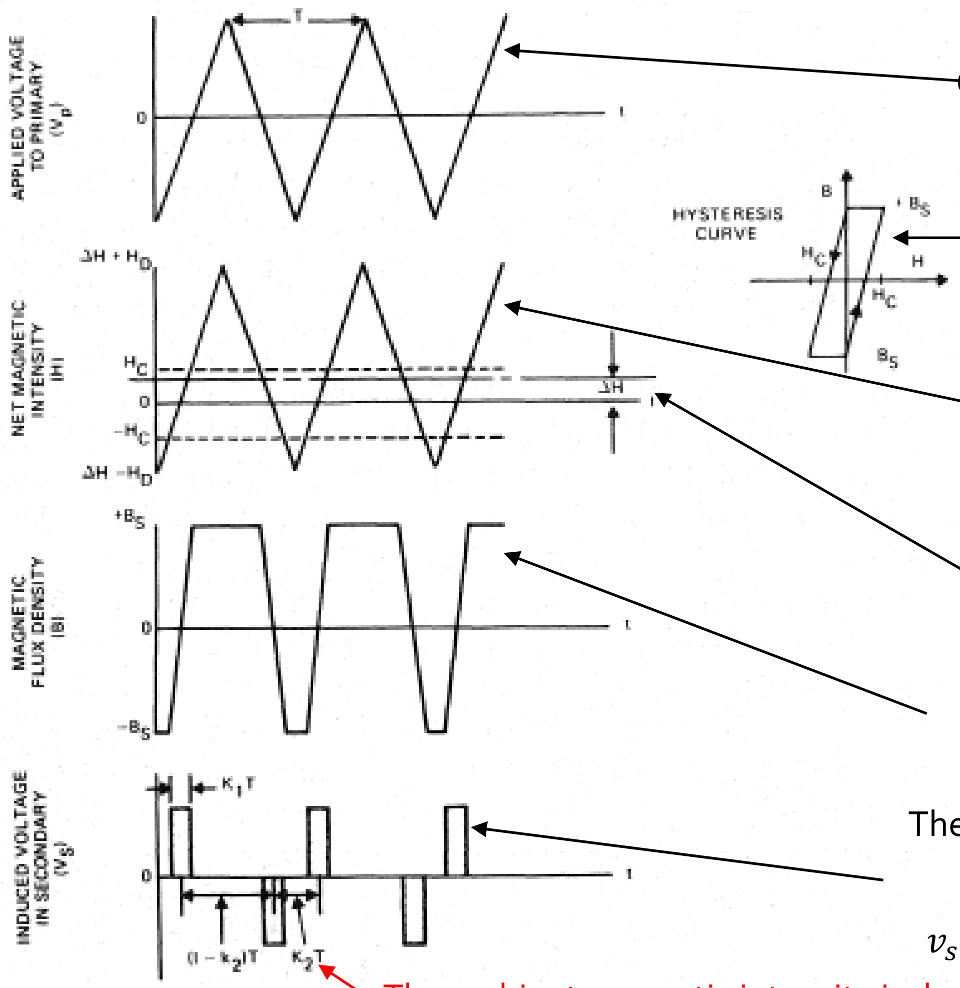


Secondary coil measures induced current

$B_1+B_2+B_E \rightarrow V_s$

The two cores are saturated in opposite directions such that the ambient magnetic field can be sensed by the secondary coil

Magnetometers (2)



Command on primary with current period T

General relationship between induction and magnetic field (with hysteresis)

Induction on the two cores is proportional to the input current with opposite signs

Ambient magnetic induction causes an offset

Magnetic field is generated on each core according to induction law

The derivative of the algebraic sum of the magnetic fields produces a voltage according to

$$v_S = -\frac{d(B_1 + B_2 + B_E)}{dt} \Rightarrow \Delta H = (1 - 2K_2)H_D$$

The ambient magnetic intensity is derived from the pulse spacing read at the secondary coil

Magnetometers (3)

- Usually vector magnetometer systems are employed which consists of three mutually orthogonal single-axis fluxgate magnetometers mounted
  - as a single unit within the spacecraft
  - disperse within the spacecraft
  - to a boom

Calibrated by measuring magnetometer response when spacecraft is placed in Helmholtz coil

- The output of a vector magnetometer system is

$$\vec{V} = \vec{F}\vec{B} + \vec{V}_0$$

Output of the three units

Magnetometer bias voltage

3x3 matrix including scale factors and alignment data

- The analog output is passed through an analog-to-digital converter to provide the digitized output through the c_i conversion factors

$$\vec{N}_V = \begin{bmatrix} \mathrm{Int}(c_1 V_1 + 0.5) \\ \mathrm{Int}(c_2 V_2 + 0.5) \\ \mathrm{Int}(c_3 V_3 + 0.5) \end{bmatrix} \Rightarrow \vec{B} = \vec{F}^{-1}(\vec{V} - \vec{V}_0) = [F_{ij}/d_i] \begin{bmatrix} N_{v1}/(c_1 d_1) - V_{01}/d_1 \\ N_{v2}/(c_2 d_2) - V_{02}/d_2 \\ N_{v3}/(c_3 d_3) - V_{03}/d_3 \end{bmatrix}; d_i = \left[\sum_{j=1}^{3} F_{ij}^2\right]^{0.5}$$

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Magnetometers (4)

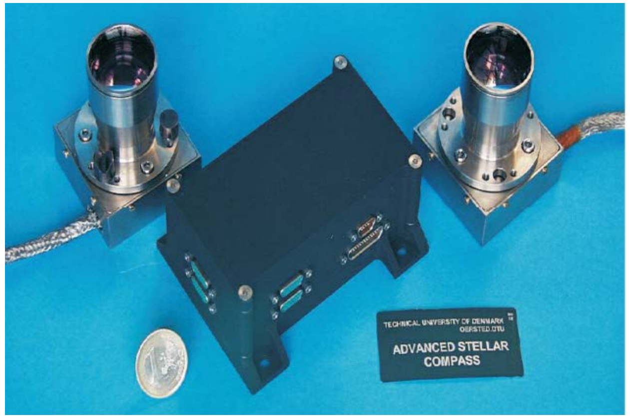
- The attitude rotation matrix is determined by the comparison of the measured magnetic field in body or principle axes and the model of the magnetic field available on-board

$$\vec{B}_{\mathrm{model}} = \vec{A}\vec{B}_{\mathrm{meas}}$$

- A good model of the magnetic field is necessary for this purpose. The lower the altitude, the higher the necessary order and degree
- Unfortunately both measurements and model are affected by errors which provide different vector magnitudes, but the rotation matrix does not change magnitudes
- A normalization is necessary to preserve accuracy. Typically we trust our model and change the magnitude of the measurement to match the model
- As a consequence, the vector magnetometer is not able to provide a three-axis attitude determination because only 1 direction (unit vector) is provided

Star Sensors (1)

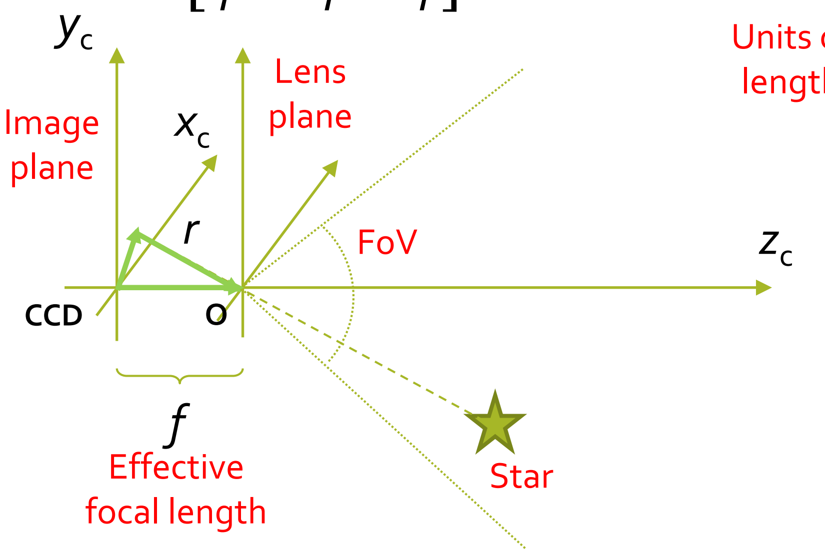
- More complex than sun and earth sensors because it seeks objects with smaller luminous intensities
- It is an optical camera which provides a matrix of pixels corresponding to the image of the starry sky
- The typical camera model used for a star sensor is the pinhole model



Camera head units (CCD) and digital processing unit of star sensor used on PRISMA

$$\hat{\vec{r}} = \begin{bmatrix} \frac{x_c}{r} & \frac{y_c}{r} & \frac{f}{r} \end{bmatrix} ; r = \sqrt{x_c^2 + y_c^2 + f^2}$$

Units of length



Star Sensors (2)

- Star sensors must be calibrated to remove the following typical errors
  - Origin of lens plane and image plane are not aligned

$$\begin{cases} x_c'' = x_{CCD} - x_o \\ y_c'' = y_{CCD} - y_o \end{cases}$$

Components of intersection of optical axis with the CCD plane

  - Pixels have different width and height

$$\begin{cases} x_c' = x_c'' \\ y_c' = y_c'' \dfrac{dy}{dx} \\ d' = \sqrt{x_c'^2 + y_c'^2} \end{cases}$$

CCD pixel width and height

Intrinsic parameters

  - The optic chain causes distortions (function of distance from center)

$$\begin{cases} x_c = (1 + kd'^2)x_c' \\ y_c = (1 + kd'^2)y_c' \end{cases}$$

Lens distortion coefficient

- The undistorted coordinates are used in the pinhole model (previous slide)

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Star Sensors (3)

- It is necessary to recognize which stars are being observed though a map of the sky or a star catalogue which is an integral part of the star sensor
- Star catalogues are distinguished by the
  - Maximum number of catalogued stars
  - Minimum luminous intensity of catalogued stars
- The sensitivity of a star sensor limits its capability to detect stars below a certain luminous intensity or above a certain magnitude (inversely proportional to the luminous intensity)
- In addition to position and magnitude, star catalogues can provide emission spectra and very small yearly movements of the stars
- To improve resolution, the field of view of a star sensor is limited to a few degrees. The smaller the field of view, the higher the resolution or the number of pixels per degree
- It is necessary to protect the star sensor from other sources of radiations, typically the sun, earth, and moon. This is done though baffles.

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Star Identification (1)

- The sensible element is a matrix of pixels which provides an image that can be processed in different ways
  - Star Tracker: punctual analysis, track a single point (star)
  - Star Mapper: sequential analysis, complete region is processed (sky)
- Irrespective of the approach, for attitude determination, we need to associate the observed stars to the catalogued stars whose inertial direction is known
- Typical star identification techniques are classified as direct match, angular separation match, phase match, and discrete attitude variation
- Often these techniques rely on an initial attitude guess from coarse sensors
- In general, if $\boldsymbol{O}_i$ is the unit vector to the *i*th observed star computed in the camera frame, $\boldsymbol{A}$ is the guess attitude, and $\boldsymbol{S}_i$ is the corresponding catalogued star in the inertial frame, then we seek

$$\underbrace{Max}_{\vec{A}}\left\{\sum_{i=1}^{n} \vec{O}_i^t \vec{A}^t \vec{S}_i\right\} \leq n$$

Possible test to check for wrong correspondences once some of them are given

Star Identification (2)

- *Direct match technique*: observations ***O*** are matched to stars ***S*** if they lie sufficiently close, where ***O*** is obtained from coarse attitude knowledge

$$d(\vec{O},\vec{S}) < \varepsilon$$

Error window radius

- After an observation is checked against all possible catalogue stars
  - No identification (no catalogued star close enough)
  - Ambiguous identification (more catalogued stars close enough)
  - Unique identification (1 catalogues star close enough)
- The score of a direct match technique can be either the number of stars identified or the percentage identified. The score must be sufficiently high.
- This technique may be statistically analyzed using a Poisson distribution. Knowing the level of completeness of the catalogue at the limit magnitude of the sensor (e.g., 98%), and the density of stars (e.g, 1 star/deg^2), the probability that an observation lies inside the window can be computed from ε and the sensor mean expected error

Star Identification (3)

- *Angular separation match technique*: observations $\boldsymbol{O}_1$ and $\boldsymbol{O}_2$ are matched to stars $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ if

Reduced distortion required

$$d(\vec{O}_1, \vec{S}_1) < \varepsilon \,; d(\vec{O}_2, \vec{S}_2) < \varepsilon \,; \left| d(\vec{O}_1, \vec{O}_2) - d(\vec{S}_1, \vec{S}_2) \right| < \mu$$

- If this condition is met by more than one pair of catalog stars, the match is ambiguous
- This problem can be alleviated by polygon methods where a vector of $m = N(N-1)/2$ angular distances is compared with a corresponding vector of catalogued star distances ($N > 2$ is the number of selected observations)
- These methods are computationally expensive and the computation time will increase as N^2
- This techniques are difficult to analyze statistically and the choice of ε and μ are made based on data accuracy and previous experience. Too large values cause ambiguous identifications, while too small values cause no identifications

Star Identification (4)

- *Discrete attitude variation technique*: only used when no a-priori knowledge of the attitude is available, such as in lost-in-space scenarios.
- It requires an array of trial attitudes such as no possible attitude is more than ε angular distance from one of the discrete trial attitudes in the array
- For each trial attitude, apply any of the other identification techniques
- The attitude that gives the highest score is taken to be correct
- Because the number of possible attitudes may be very large (e.g., 40,000 attitudes for ε = 1 deg), refinements to the technique which cut down the number of guesses are critical, e.g.
  - Considering that one matched direction provides a two axis attitude
  - Considering brightness information and limit match to brightest stars

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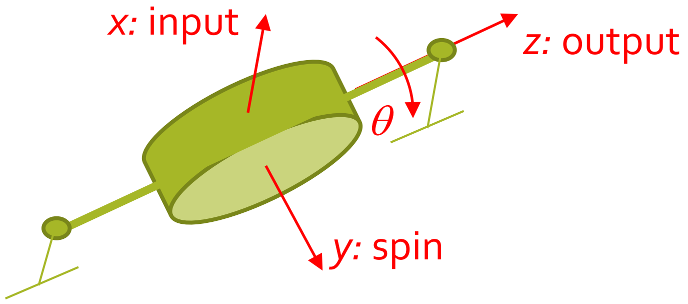
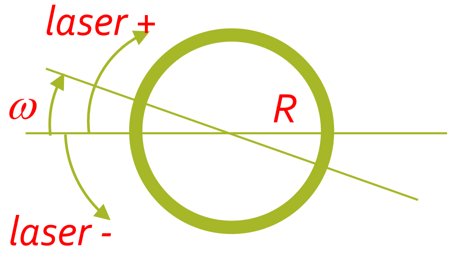
Angular velocity sensors

- Provide direct measurements of angular velocity which must be integrated if the attitude is required
- Typologies used in space
  - Mechanical
    - Oldest, affected by large errors, rate- or rate-integrating-gyros, small



  - Laser
    - New, smaller errors, based on fiber-optics, bigger than mechanical



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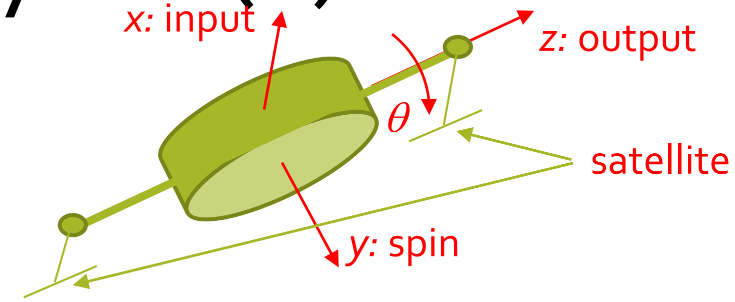
Mechanical Gyros (1)



- A rotor is mounted on a support structure through bearings which allows one degree of rotational motion, here θ about *z*
- The angular velocity of the gyroscope, here ω_r about *y*, provides the following angular momentum

$$\vec{L}_R = I_R \omega_R \vec{\hat{y}} + J_z \dot{\theta} \vec{\hat{z}}$$

Moment of inertia of rotor and structure along z

- The Euler equations can be written for the z axis as follows

$$\frac{d\vec{L}_R}{dt} + \vec{\omega} \times \vec{L}_R \xrightarrow{z} J_z \ddot{\theta} + \omega_x I_R \omega_R = M_z$$

Constraint reaction torque about z

- The angle θ is linked to the angular velocity of the satellite ω_x
  - Rate-gyros (RG) and rate-integrating-gyros (RIG) allow rotation and measure ω_x from steady state θ (RG) or $\dot{\theta}$ (RIG) (inertial platform)
  - Strapdown-gyros (SG) block rotation by applying a torque and measure ω_x from the torque current

Mechanical Gyros (2)

- RG allows rotation and introduces an elastic+viscous contrast of the type spring+damper (passive)

$$J_z\ddot{\theta} + \omega_x I_R \omega_R = M_z = -P\theta - D\dot{\theta} \Rightarrow \text{steady state} \Rightarrow \boxed{\omega_x = -\frac{P\bar{\theta}}{I_R\omega_R}}$$

  The frequency of θ must be much larger than the characteristic frequency of ω_x such that the RG gets to steady state as soon as possible (source of error)

- RIG alleviates this problem by eliminating the elastic (spring) term

$$J_z\ddot{\theta} + \omega_x I_R \omega_R = M_z = -D\dot{\theta} \Rightarrow \text{steady state} \Rightarrow \boxed{\omega_x = -\frac{D\dot{\bar{\theta}}}{I_R\omega_R}}$$

  The satellite body axes rotate w.r.t. to the input axis, thus the gyro measures a combination of the x and y components of the angular velocity

- SG alleviates this problem by keeping the rotation close to zero through an active Proportional-Integral-Derivative (PID) controller

$$J_z\ddot{\theta} + \omega_x I_R \omega_R = M_z = -PID(\theta) \Rightarrow \text{steady state} \Rightarrow \boxed{\omega_x = -\frac{PID(\bar{\theta})}{I_R\omega_R}}$$

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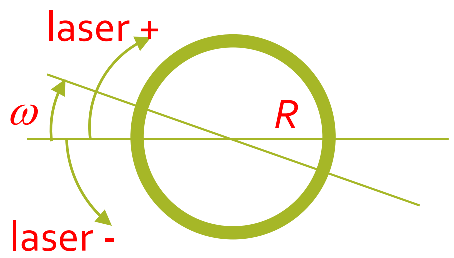
Laser Gyros



- The gyro consists of an optic cavity of radius *R* in rotation at the satellite's angular velocity ω
- Two laser beams start from the same point and follow the circular path in opposite directions, + and –
- The path length of the two beams is given by

Time required to complete travel considering that the starting point has rotated

$$l_+ = 2\pi R + \omega R t_+ = c t_+ \, ; \, l_- = 2\pi R - \omega R t_- = c t_-$$

- The time between the reception of the two laser beams is given by

$$c\Delta t = \omega R (t_+ + t_-)$$

- Substituting t_+ and t_- from the former equations in the latter we obtain

$$\Delta t = \frac{2\pi R \cdot 2}{c^2 - \omega^2 R^2} \omega R \cong \frac{2\pi R \cdot 2}{c^2} \omega R \Rightarrow \omega = \boxed{\frac{\Delta t c^2}{4\pi R^2} = \frac{\Delta t c^2}{4A}}$$

Valid for arbitrary geometry (typically square or rectangle)

- Using fiber-optics instead of a cavity we can increase the path length and thus the area *A*. As a consequence the measurement of Δt is more precise.

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