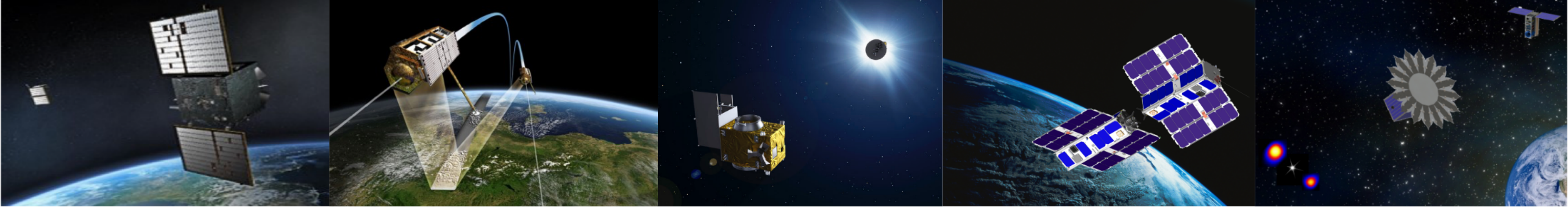


aa.stanford.edu

damicos@stanford.edu
people.stanford.edu/damicos

stanford.edu

# AA 279 A – Space Mechanics
# Lecture 15: Notes

**Simone D’Amico, PhD**
**Assist. Prof., Aeronautics and Astronautics (AA)**
**Director, Space Rendezvous Laboratory (SLAB)**
**Satellite Advisor, Stanford Space Student Initiative (SSI)**

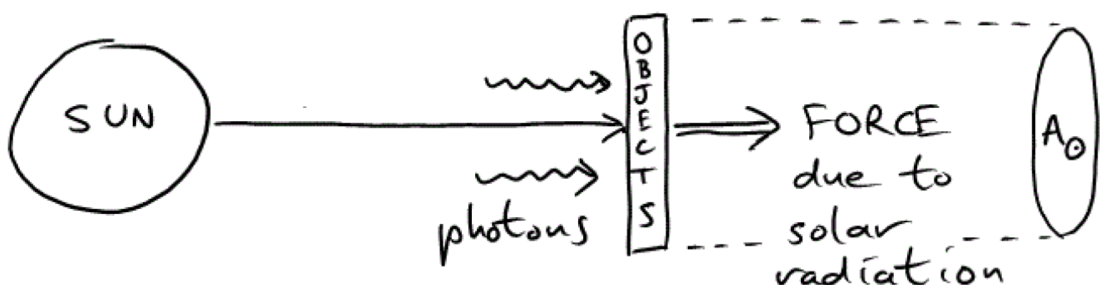
# Table of Contents

# Solar Radiation Pressure (1)

- Sunlight hitting object 'S' causes a force

$$\overrightarrow{f_{SRP}} = p_{SRP} C_{SRP} \frac{A_{SRP}}{M} \frac{\overrightarrow{r_{Sun \to Sat}}}{r_{Sun \to Sat}}$$

  - $p_{SRP}$= Solar radiation pressure = Solar flux / speed of light = $4.57 \cdot 10^{-6}$ [N/m$^2$]
  - $C_{SRP}$= SRP coefficient = Absorption + Reflection (specular and diffused)
  - $A_{SRP}$= Cross-section area (size, attitude)
  - $M$ = Mass of object 'S'



# Solar Radiation Pressure (2)

- Simplified effect of solar radiation pressure on orbit
  - Earth satellite starts in circular orbit


  - SRP force acts to speed up satellite at P and slow it down at A


  - Later, we end up with eccentric orbit lowered at P and raised at A with no change in semi-major axis and specific mechanical energy

damicos@stanford.edu
people.stanford.edu/damicos

# Third-Body Forces

$$\boxed{\vec{f}_3 = \frac{\mu_3}{r_{23}^3}\vec{r}_{23} - \frac{\mu_3}{r_{13}^3}\vec{r}_{13}}$$

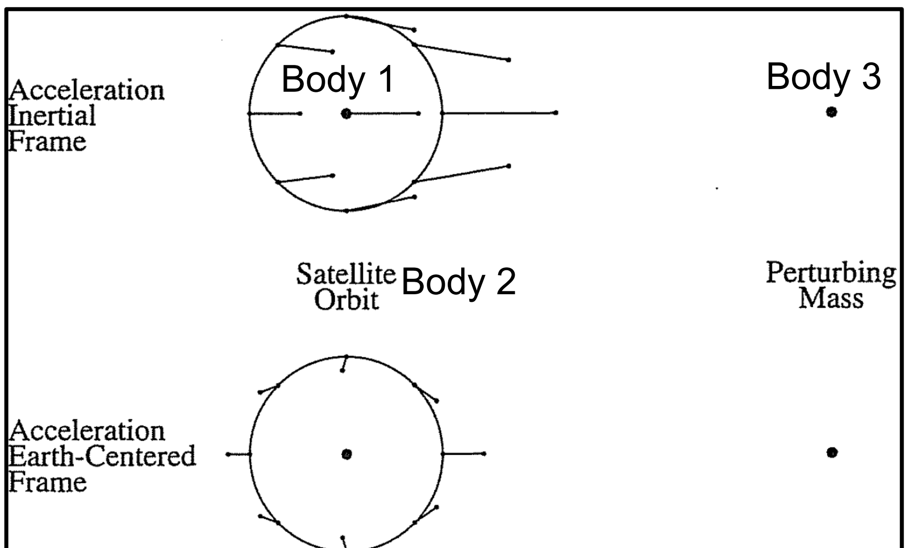
- Depend on relative acceleration
- Dominant above GEO
- Approximate formulas for small eccentricity $e_3$ of the 3rd body

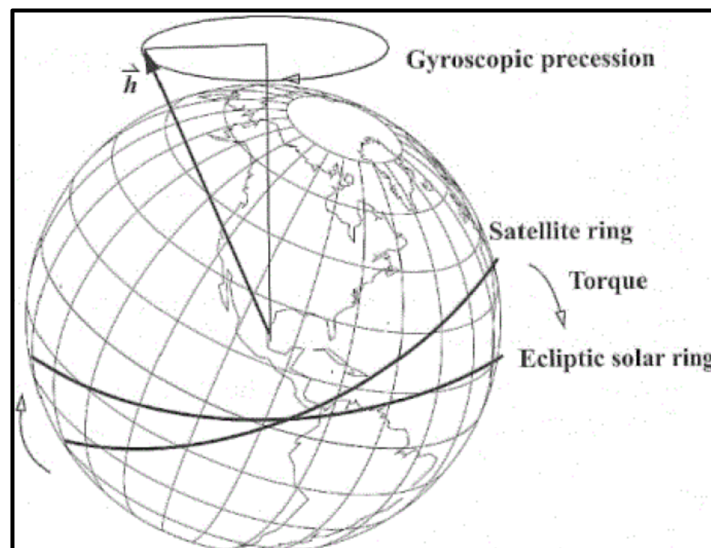
$$\left(\frac{d\Omega}{dt}\right)_{\mathrm{AVG}} = -\frac{3\mu_3(2+3e^2)[2-3\sin^2(i_3)]}{16 r_{23}^3 n\sqrt{1-e^2}}\cos i$$

$$\left(\frac{d\omega}{dt}\right)_{\mathrm{AVG}} = \frac{3\mu_3[2-3\sin^2(i_3)]}{16 r_{23}^3 n\sqrt{1-e^2}}[e^2+4-5\sin^2 i]$$

View from above perifocal plane (+ or – radial depending on alignment)

3D view (regression about normal to 3rd body orbital plane)




damicos@stanford.edu
people.stanford.edu/damicos

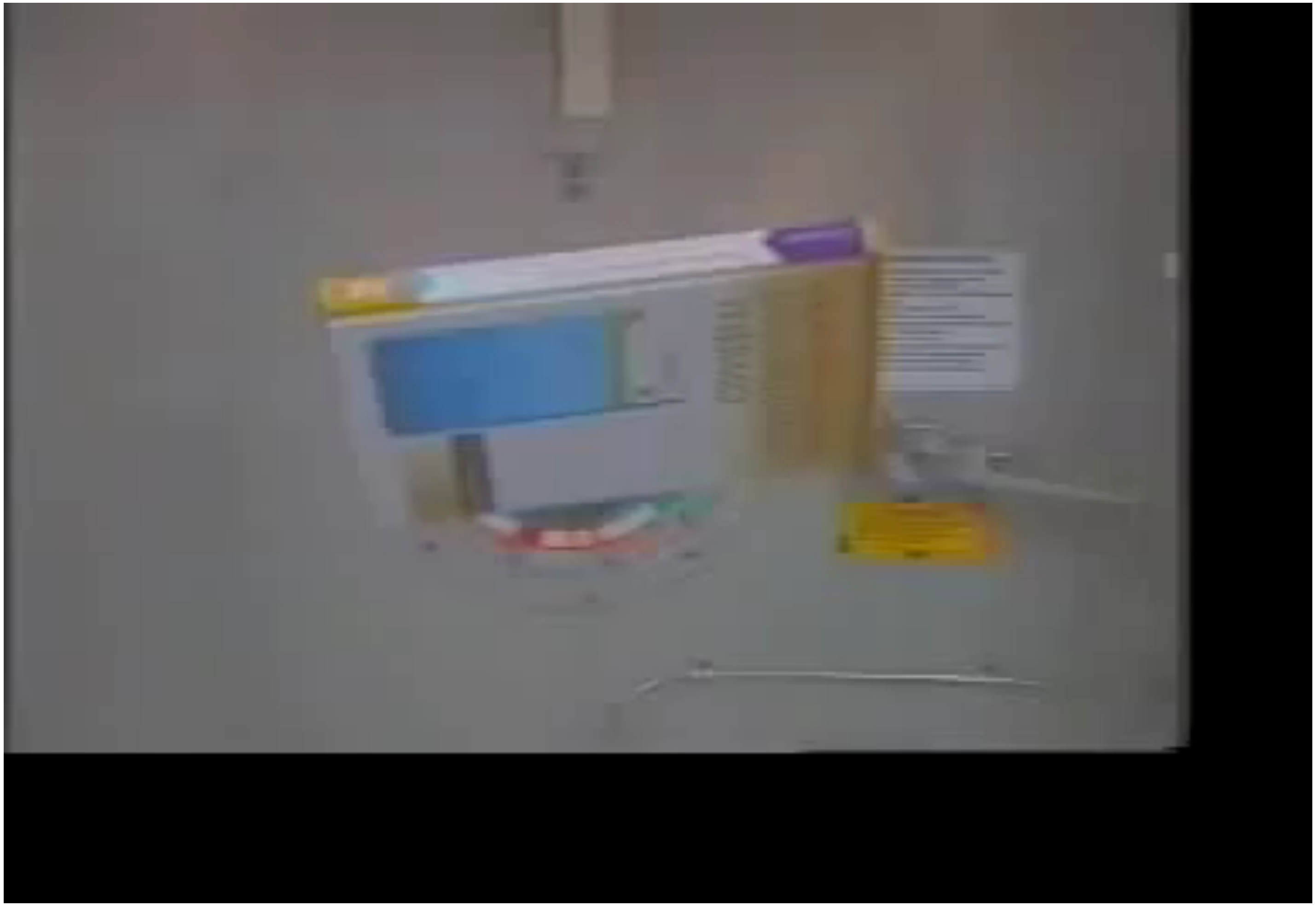
# After AA279A?

- AA279C: Spacecraft Attitude Determination and Control (Spring 2017)
  - Rotational motion in space
  - Project based (no exams): ADCS

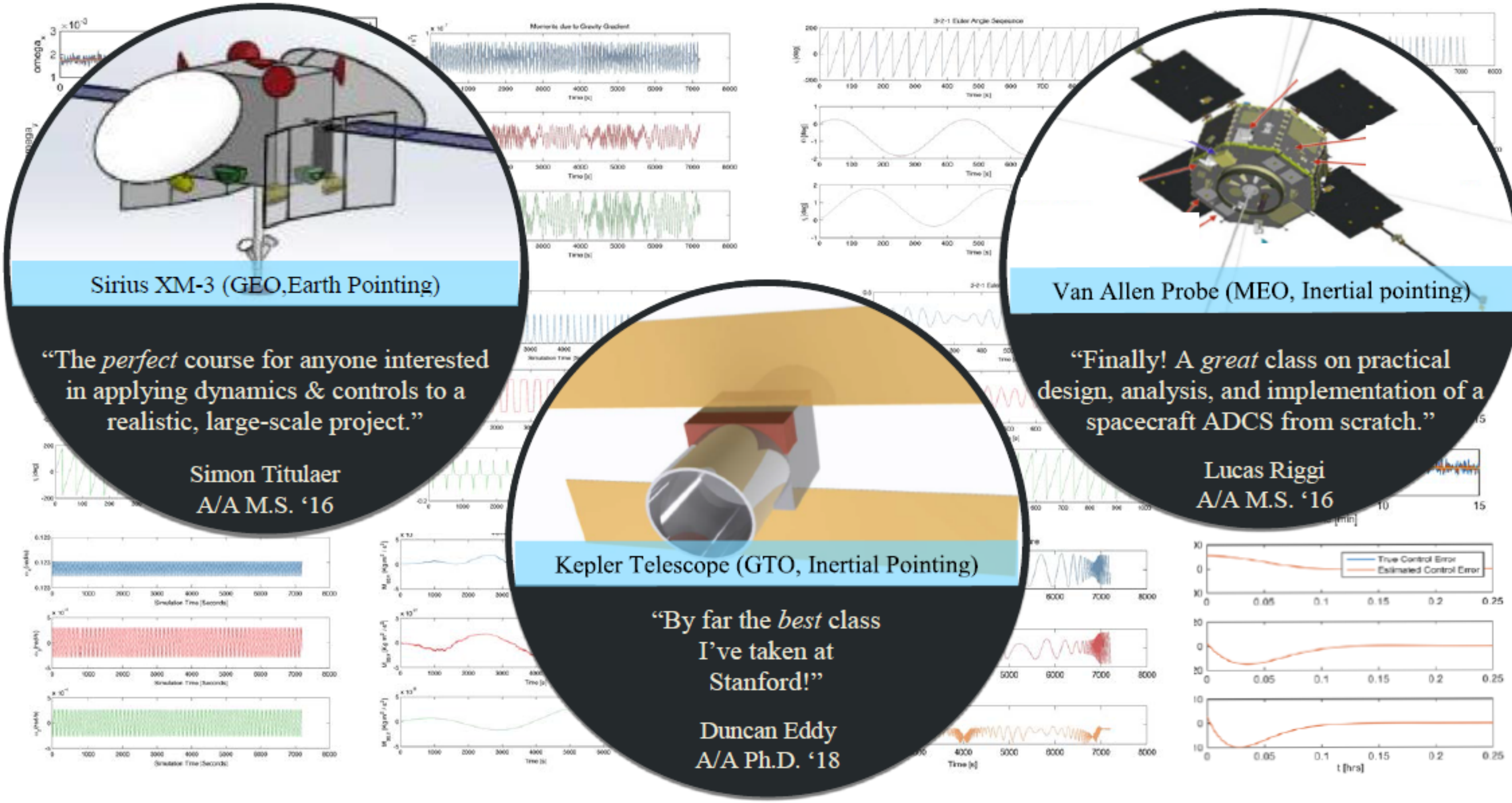
- AA279D: Spacecraft Formation-Flying and Rendezvous (Spring 2018)
  - Relative motion in space
  - Project based (no exams): GN&C

- Topics include
  - State representations and dynamics modelling
  - State determination and estimation
  - Guidance and control
  - Sensors and actuators
  - Numerical simulation

# AA279C: Start from basic principles



# AA279C: End with full ADCS



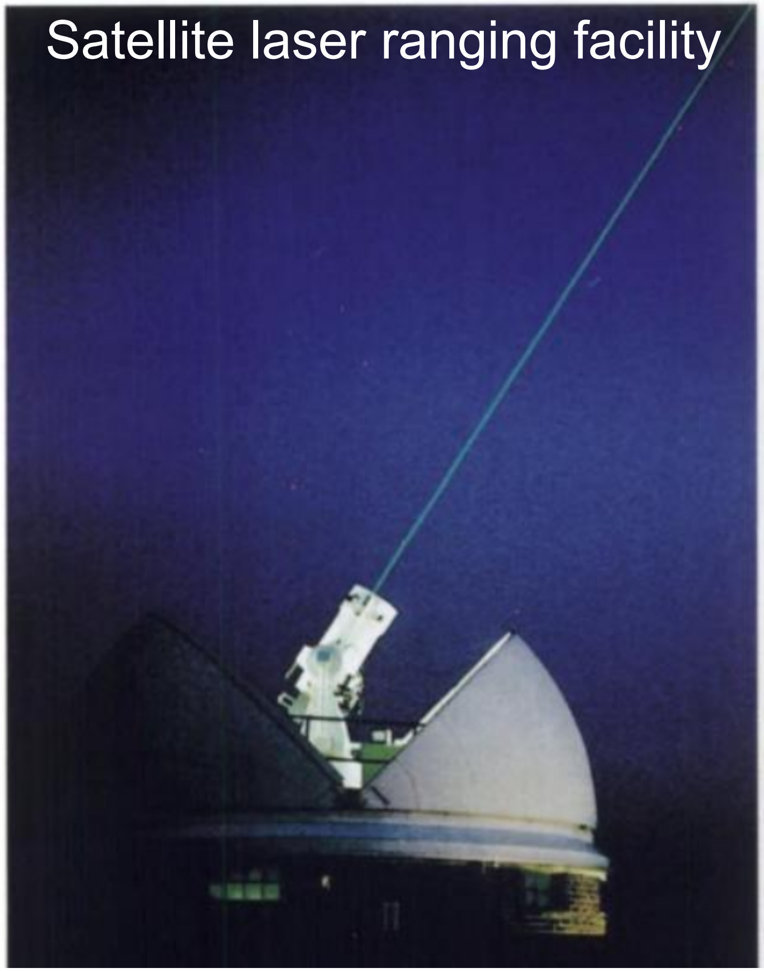
# AA279D: GN&C for multiple spacecraft

- Video of DEOS


damicos@stanford.edu
people.stanford.edu/damicos

# Navigating in Space (1)

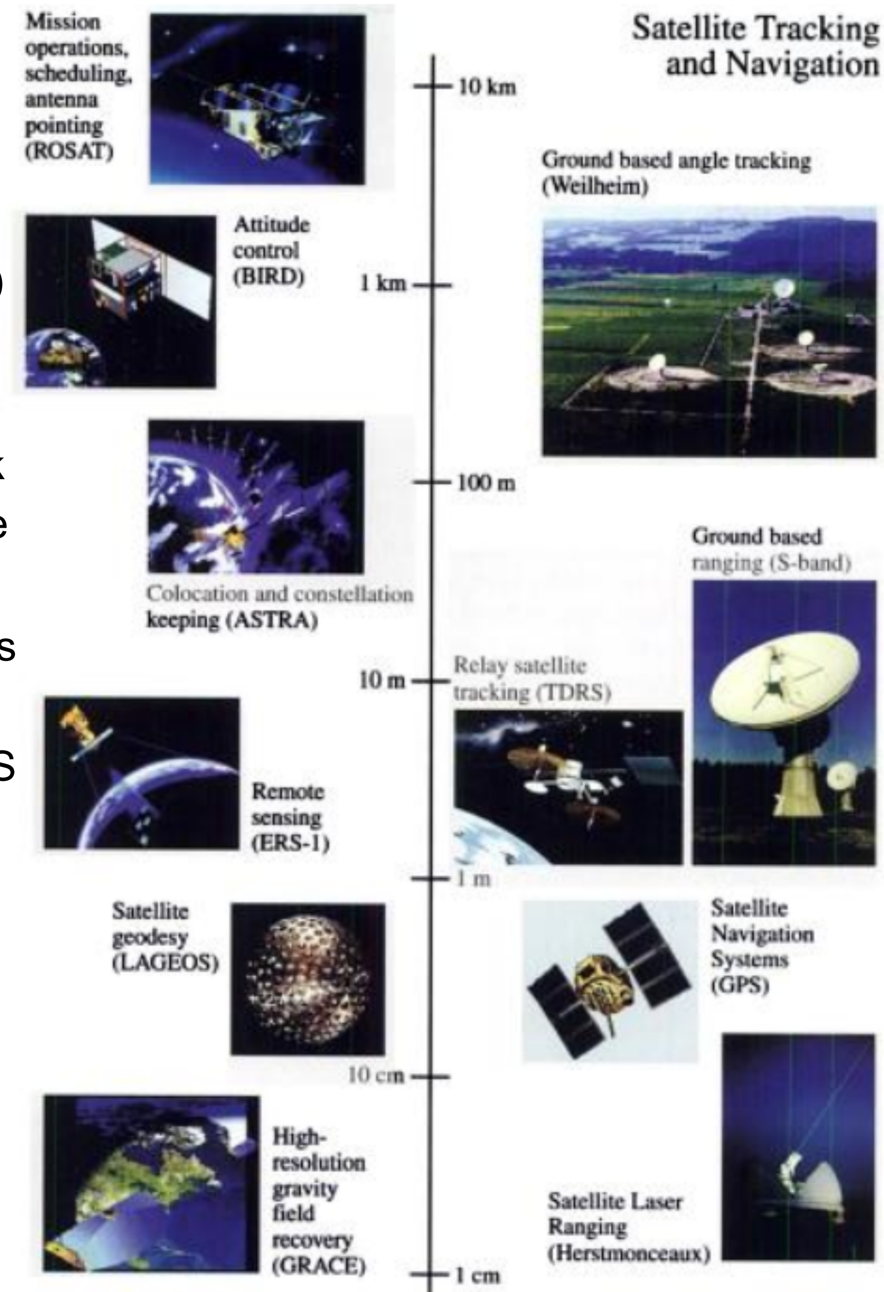
- Navigation is an essential part of spacecraft operations, it includes
  - planning
  - determination
  - prediction
- Measurements related to the position of a satellite or its rate of change are based on
  - radio signals (to/from ground antenna)
    - angles (azimuth, elevation)
    - slant range (round-trip light time)
    - range rate (Doppler shift)
  - optical signals (telescopes, laser)
    - imaging (plane-of-sky position)
    - range (round-trip light time)
- Limitation: station contact required!




damicos@stanford.edu
people.stanford.edu/damicos

# Navigating in Space (2)

- In order to overcome visibility restrictions, satellite constellations are employed
- Tracking and Data Relay Satellites (>1983)
  - 6 geosynchronous satellites
  - Two ground terminals (New Mexico)
  - Goddard Space Flight Center network
  - Relay to/from users < 3000km altitude
- Global Positioning System (>1973)
  - 32 satellites in 6 evenly spaced planes
  - 20200 km, 12 sidereal hours
  - 5 monitor stations (equipped with GPS receiver and atomic clock)
  - 1 master control station (collect tracking data and compute satellite orbits/clocks)
  - 3 ground control stations (for daily command upload)



# Preliminary Orbit Determination (1)

- Ground-based satellite observations depend directly on the satellite's motion with respect to the center of the Earth
- They can be used to deduce the orbital elements
- We first consider situations in which a satellite orbit must be determined from a small set of available measurements
- This is likely to occur during tracking of foreign spacecraft, unforeseen launcher injection errors, detection of pieces of the space debris, near-Earth asteroids, etc.
- At least 6 independent measurements are required to uniquely determine an orbit if no further assumptions are made (6 DoF)
- Deriving orbital elements from this minimum set of observations is commonly referred to as preliminary orbit determination (OD)
- Techniques like the least-squares method can later be employed to further refine the OD as more measurements become available (next courses)

Why?

How?

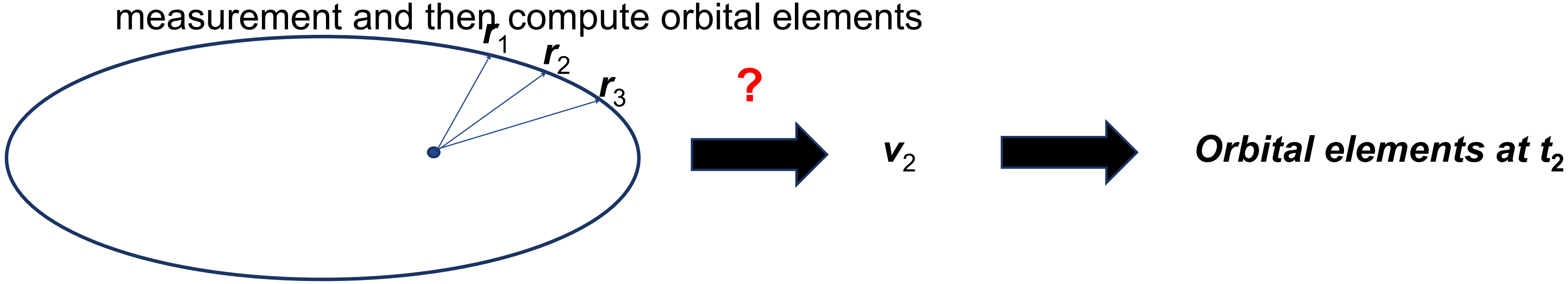
# Preliminary Orbit Determination (2)

- Based on the unperturbed two-body problem, a variety of different analytical OD methods has been developed
- They are generally divided into *Laplacian* and *Gaussian* type methods, referring to the two scientists that devised the prototypes in the last 18$^{th}$ and early 19$^{th}$ centuries
- *Laplacian* methods
  - derive inertial position and velocity in the middle of the observation interval
  - work for various combinations of measurements
  - not well suited for long tracking arcs if velocity info is obtained from interpolation of position measures
- *Gaussian* methods
  - derive orbital elements
  - from widely spaced direction measurements
  - not well suited for short tracking arcs
- We address 1 simple Gaussian OD method

Methods?

# OD from 3 position vectors (1)

- In favorable cases a satellite may allow simultaneous distance and angle measurements yielding directly the 3D position relative to the ground station (topocentric coordinates)
- Knowing the ground station location, these measurements can be converted to the position with respect to the center of mass of the Earth
- *Gibbs* (American scientist) developed a geometric method to compute orbital elements from 3 such widely spaced (>1°) position vectors
- The constraint is that $\boldsymbol{r}_1$, $\boldsymbol{r}_2$, and $\boldsymbol{r}_3$ must lie on a conic section (coplanar) and be sequential. In practice 2-3° error out-of-plane ($i$,$\Omega$) is tolerable.
- The idea is to find $\boldsymbol{v}_2$ corresponding to the intermediate position measurement and then compute orbital elements




damicos@stanford.edu
people.stanford.edu/damicos

# OD from 3 position vectors (2)

- The overall procedure is to find a constant (middle velocity vector) which is common between each of the given vectors. *Rationale*: if the 3 vectors are coplanar, one vector must be a linear combination of the other two.
- Vallado derives the following algorithm (Section 7.5.1)

1. Compute the cross products between the input position vectors

$$\vec{Z}_{12} = \vec{r}_1 \text{x} \vec{r}_2 \; ; \; \vec{Z}_{23} = \vec{r}_2 \text{x} \vec{r}_3 \; ; \; \vec{Z}_{31} = \vec{r}_3 \text{x} \vec{r}_1$$

2. Check coplanar and spacing constraints

$$\alpha_{\text{cop}} = \sin^{-1}\left(\frac{\vec{Z}_{23} \cdot \vec{r}_1}{|\vec{Z}_{23}||\vec{r}_1|}\right) < \varepsilon_{\text{cop}}$$

$$\left.\begin{aligned} \alpha_{12} &= \cos^{-1}\left(\frac{\vec{r}_1 \cdot \vec{r}_2}{|\vec{r}_1||\vec{r}_2|}\right) \\ \alpha_{23} &= \cos^{-1}\left(\frac{\vec{r}_2 \cdot \vec{r}_3}{|\vec{r}_2||\vec{r}_3|}\right) \end{aligned}\right\} > \varepsilon_{\text{spa}}$$

3. Compute several intermediate vectors

$$\vec{N} = r_1 \vec{Z}_{23} + r_2 \vec{Z}_{31} + r_3 \vec{Z}_{12}$$

$$\vec{D} = \vec{Z}_{23} + \vec{Z}_{31} + \vec{Z}_{12}$$

$$\vec{S} = (r_2 - r_3)\vec{r}_1 + (r_3 - r_1)\vec{r}_2 + (r_1 - r_2)\vec{r}_3$$

4. Solve for the second velocity

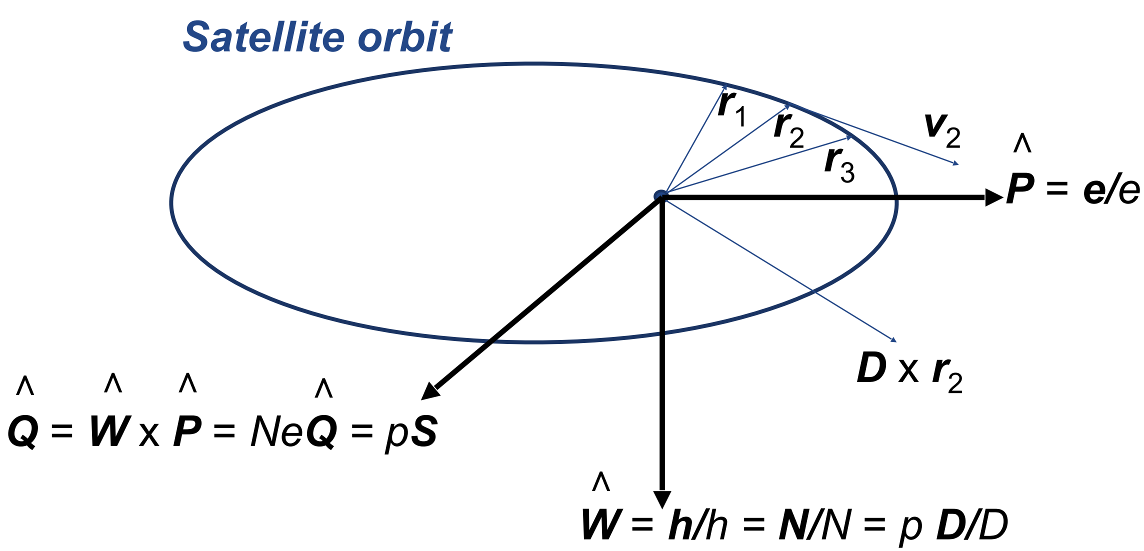
$$\vec{B} = \vec{D} \text{ x } \vec{r}_2$$

$$L_g = \sqrt{\frac{\mu}{ND}}$$

$$\vec{v}_2 = \frac{L_g}{r_2}\vec{B} + L_g \vec{S}$$

# OD from 3 position vectors (3)

- Geometry of vectors for the Gibbs method and fundamental connections
- The original Gibbs' method ended at the computation of $e$ and $p$ from step 3)
- $i$ and $\Omega$ are computed from $\boldsymbol{W}$, $u$ from $\boldsymbol{r}_2$ and $\boldsymbol{W}$, $a$ from $e$ and $p$ as usual




$p = N/D$
$e = S/D$

damicos@stanford.edu
people.stanford.edu/damicos

# OD from 3 position vectors (Example)

- GIVEN:

$\boldsymbol{r}_1$ = 6378.137 $\boldsymbol{K}$ km, $\boldsymbol{r}_2$ = -4464.696 $\boldsymbol{J}$ - 5102.509 $\boldsymbol{K}$ km, $\boldsymbol{r}_3$ = 5740.323 $\boldsymbol{J}$ + 3189.068 $\boldsymbol{K}$ km

- FIND: $\boldsymbol{v}_2$

1. Compute the cross products between the input position vectors

$$\boldsymbol{Z}_{23} = 15{,}051{,}830.62\ \boldsymbol{I}\ \text{km}^2$$

2. Check coplanar and spacing constraints

$$\alpha_{COP} = 0°$$
$$\alpha_{12} = 138.81°$$
$$\alpha_{23} = 160.24°$$

3. Compute several intermediate vectors

$\boldsymbol{N} = 5.3123\cdot10^{11}\ \boldsymbol{I}$ km$^3$

$\boldsymbol{D} = 8.0141\cdot10^{8}\ \boldsymbol{I}$ km$^2$

$\boldsymbol{S} = -3.1490\cdot10^{6}\ \boldsymbol{J} - 8.8301\cdot10^{5}\ \boldsymbol{K}$ km$^2$

4. Solve for the second velocity

$\boldsymbol{B} = 4.0892\cdot10^{11}\ \boldsymbol{J} - 3.5780\cdot10^{11}\ \boldsymbol{K}$ km$^3$

$L_g = 9.6761\cdot10^{8}$ 1/km·s

$\boldsymbol{v}_2 = 5.531148\ \boldsymbol{J} - 5.191806\ \boldsymbol{K}$ km/s

DONE!

# Orbit Determination

- Preliminary orbit determination is used for the direct computation of 6 orbital elements from 6 observations (no a-priori knowledge)
  - Solar system bodies right after their detection
  - Identification of uncatalogued spacecraft
  - Launcher injection errors

---

- Orbit estimation is used for the improvement of a priori orbital elements from an arbitrary set of tracking data
  - Mission planning and operations
  - Scientific orbit determination
  - On-board absolute and relative navigation
  - Interplanetary navigation

6 measurements

$\downarrow$

**Gauss and Laplace methods**

$\downarrow$

6 preliminary orbital elements
+
measurements

$\downarrow$

**Batch and Sequential methods**

$\downarrow$

6 final orbital elements

damicos@stanford.edu
people.stanford.edu/damicos

# Differential Correction

- Strategy common to all estimation methods:
  1. Start with estimated orbital elements (a priori or current)
  2. Take some measurements ("actual" observations)
  3. Use elements to predict what you would have measured ("predicted" or "modelled" observations)
  4. Form "residuals" = "actual" – "modelled" (*pre-fit*)
  5. Use residuals to update the elements, so that the values predicted by the updated elements is a "best fit" for the actual measurements (i.e., minimize *post-fit* residuals)

- Credit to Gauss (from Theoria Motus [1809]):

"*…the most probable value of the unknown quantities will be that in which the sum of the squares of the differences between the actually observed and computed values multiplied by numbers that measure the degree of precision is a minimum.*"

residuals

best fit criteria = least squares

damicos@stanford.edu
people.stanford.edu/damicos