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AA 279 A – Space Mechanics
Lecture 1: Notes

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- Brief historical review
- Kepler's and Newton's laws
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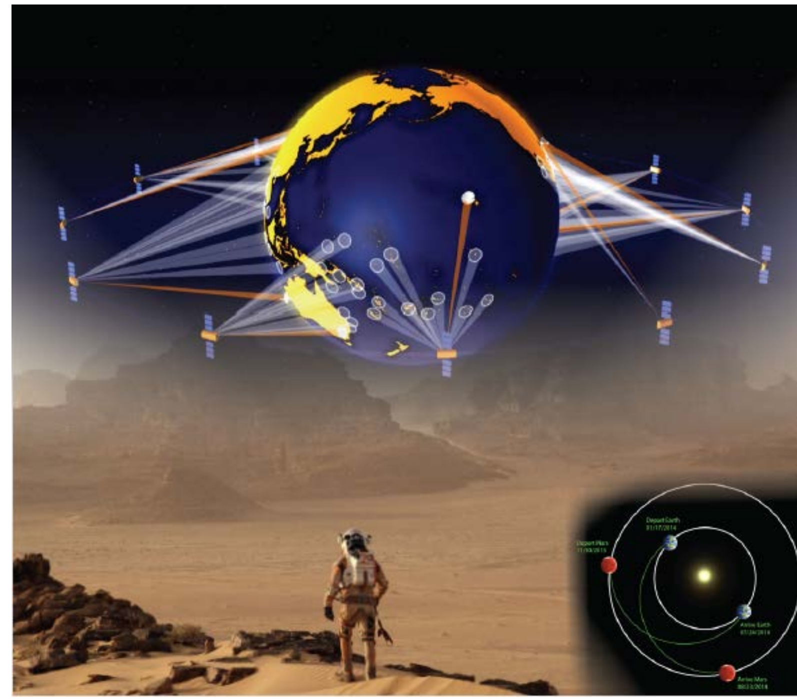

- Reading for this week
  - Bate 1.1-1.6, 1.11
  - Montenbruck 1-2.1.3
  - Vallado 1.1-1.3.4

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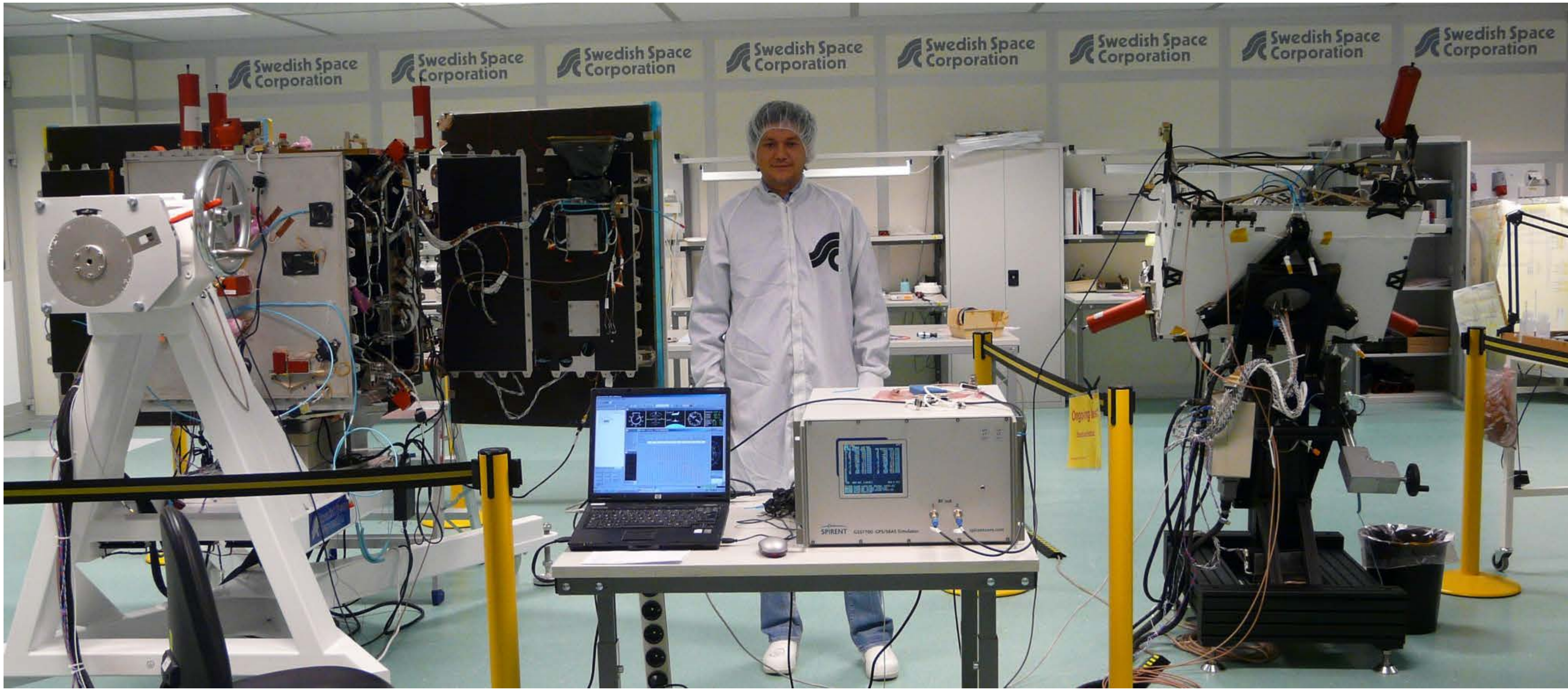
JPL astrodynamicist Rich Purnell has the answer... possibly



Simone D'Amico in the clean room ... with the PRISMA satellites

Course Introduction (1)

- Definition of Astrodynamics (from Griffin and French, 1991)

Astrodynamics is the study of the motion of man-made objects in space, subject to both natural and artificially induced forces

- This definition combines features of
  - Celestial mechanics (motion of celestial objects)
  - Orbital dynamics (orbit motion of all objects)
  - Attitude dynamics (orientation of objects in space)

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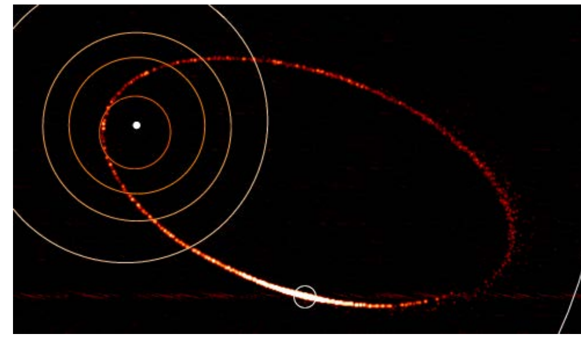
Course Introduction (2)



- Why AA279A?
  - Entry point into “Astro”
  - Language of space mechanics
  - Fundamentals of spacecraft motion
  - Simulation of “real-world” orbits
- After AA279A?
  - AA279B: Advanced Space Mechanics
  - AA279C: S/C Attitude Determination&Control
  - AA279D: Distributed Space Systems

Skybox Imaging + Google

Capella Space

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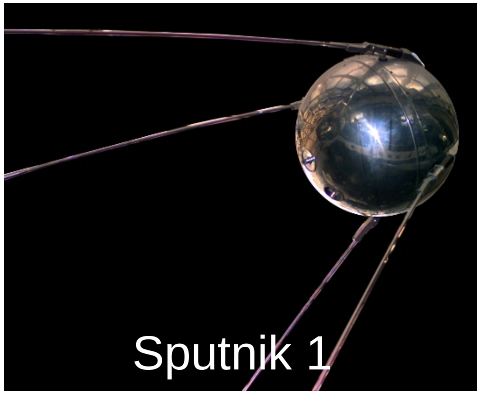
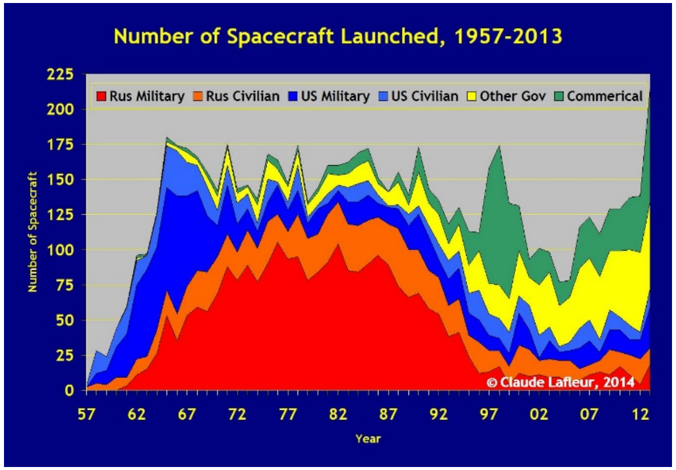
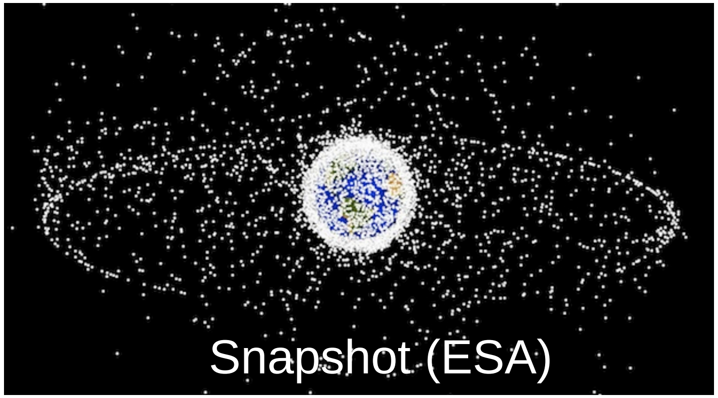
Course Introduction (3)



- According to our definition, Astrodynamics' birthdate is October 4, 1957



- Today, several thousands man-made satellites orbit the Earth…



- …together with countless pieces of space debris, but **how do they move**?

Most Popular Orbits

Sun-synchronous orbit

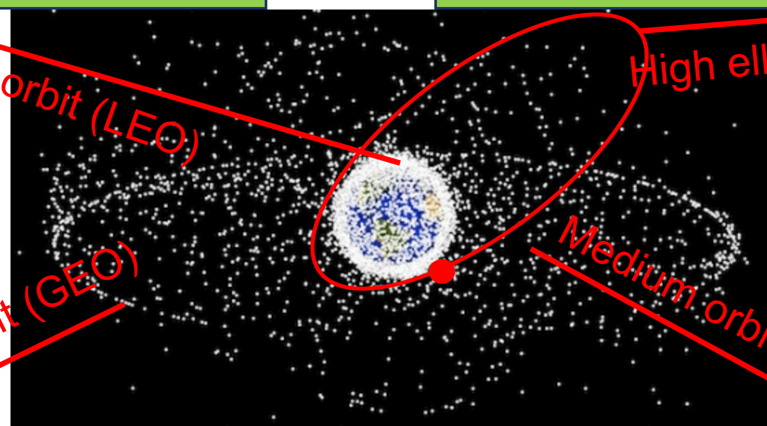
500-1500 km

Sweet spot over the pole that lets the satellite stay in one time…

Molniya orbit (semi-synch)

1000x40000 km

Combines high inclination and eccentricity to maximize viewing time over high latitudes…



Geostationary orbit (geo-synch)

35800 km

Sweet spot over the equator that lets the satellite stay over one place…

GNSS constellations (semi-synch)

20000 km

Multiple suitably shifted orbits that repeat twice every day and let a minimum number of satellites always visible from any point…

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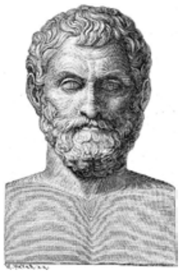
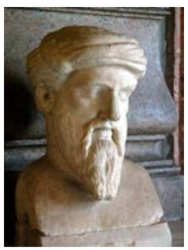
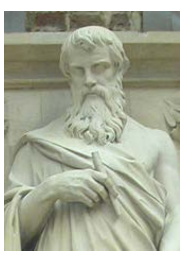
Brief historical review (Ancient Greece)



Thales (Turkey, 624-546 BC)
- Astronomy
- Length of the year
- Predicted eclipses
- Earth is a sphere



Pythagorous (Greece, 570-495 BC)
- Geometry
- Earth and comets around Sun
- Earth rotates about its own axis
- Each planet emits musical note



Euclid (Egypt, 330-275 BC)
- Geometry (writings were lost)
- Conic sections
- Apollonius: Excentric and epicycle
- Aristarchus: Earth around Sun

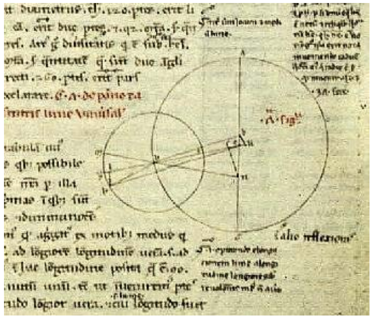
Eratosthenes (Libya, 276-194 BC)
- Estimate Earth's radius (1%)
- Based on Sun's light rays and simple geometry

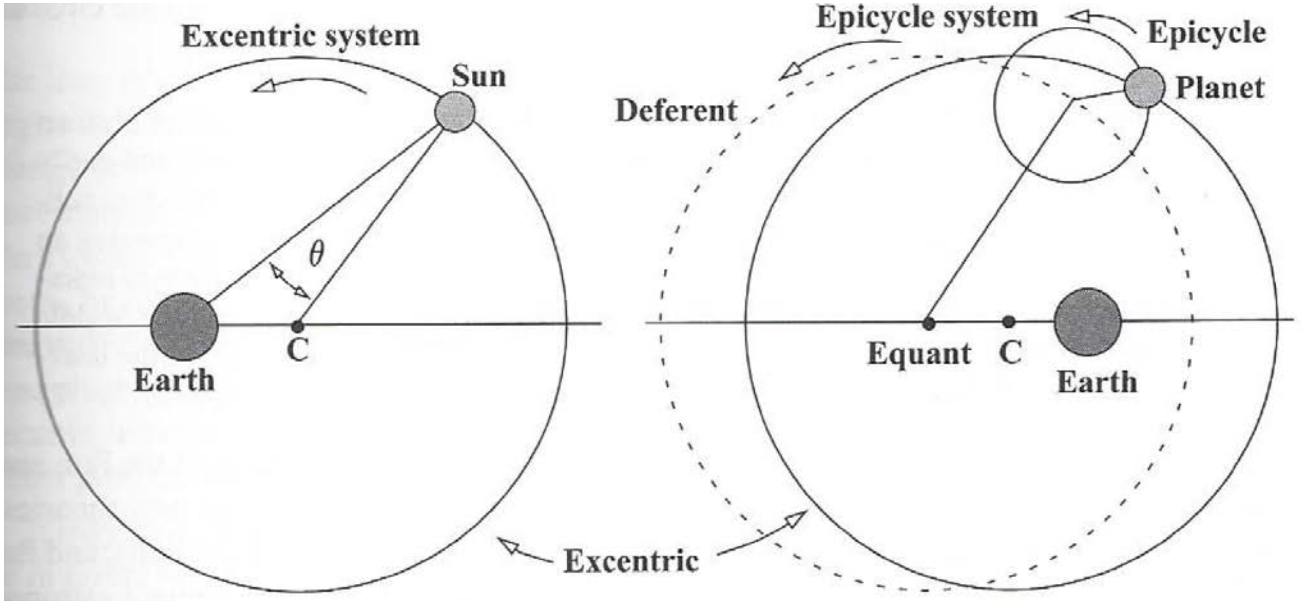
Hipparcus (Turkey, 190-120 BC)
- Spherical trigonometry
- Earth is center of universe
- Cataloged 1000 stars by brightness
- Excentric and epicycle

Caesar (Italy, 100-44 BC)
- Julian calendar in 46 BC
- 365.25 days
- Leap day every four years
- Error of 11 minutes/year

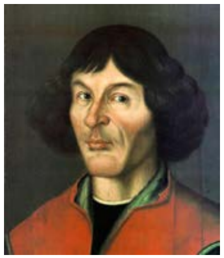
Brief historical review (Roman Empire)

Ptolemy (Egypt, 90-168 AD)
- 13-volume work
- "The Mathematical Collection" or the "Almagest"
- Earth-centered solar system
- Final refinements of excentric and epicycle
- Extremely complex theory which "served" the purpose





Brief historical review (Revolution)

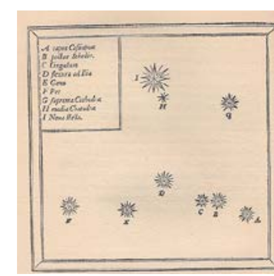


Copernicus (Poland, 1473-1543 AD)
- Astronomer
- Sun-centered heliocentric theory
- New numbers and data, details on planetary motion
- Small epicycles still necessary to match observations


NICOLAI
COPERNICI TO
RINENSIS DE REVOLVTIONI-
bus orbium cœlestium,
Libri VI.

Galileo Galilei (Italy, 1564-1642 AD)
- Astronomer, Mathematician, Physicist
- Scientific method through empirical observations (telescope)
- Observed Jupiter’s moons and planets
- Defended heliocentric view (and circular motion)

Tycho Brahe (Denmark, 1546-1601 AD)
- Last major “naked eye” Astronomer
- Placed supernovae and comets outside atmosphere
- Copious accurate observations…
- …left to his assistant…



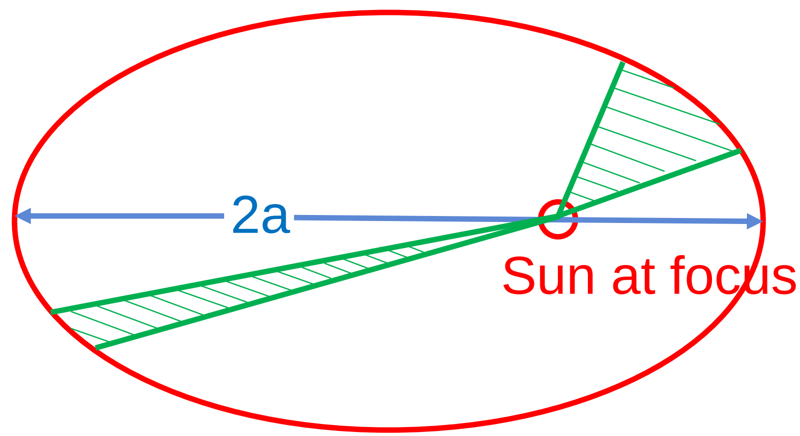
Kepler’s Laws (What?)

Johannes Kepler (Germany, 1571-1630 AD)
- Mathematician, Astronomer
- Used Tycho’s observations to devise a **kinematics** theory of planetary orbits which also applies to satellites
- Three laws of planetary motion

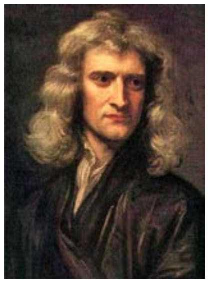
I) Planetary orbits are ellipses with Sun at one focus. [1609]

II) Radius vector to each planet sweeps out equal areas in equal times. [1609]

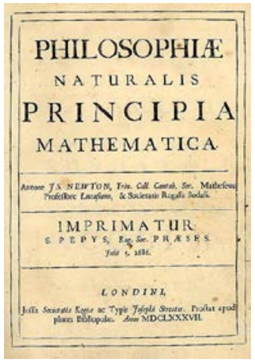
III) $T^2 \propto a^3$, being
T = time to orbit around Sun
a = mean distance to Sun
[1619]



Newton's Laws (Why?)



Isaac Newton (England, 1642-1727 AD)
- Physicist, Mathematician
- Began developing theories while "taking a break from college" (University of Cambridge)
- Father of infinitesimal calculus (with Leibniz)
- Three laws of **dynamics** motion and gravitation [1687]



I) Bodies in uniform motion stay in uniform motion unless acted on by external force.

II) Leibniz notation: $\vec{F} = \dfrac{d}{dt}(m\vec{v})$

III) To every action there is always equal and opposite reaction.

IV) Point mass *k* attracts point mass *j* by applying force F_{jk} in direction from *j* to *k*

$$F_{jk} = \frac{G m_k m_j}{r_{kj}^2}$$

being
$G = 6.67\underline{3} \cdot 10^{-20}$ [$km^3/(kg \cdot s^2)$]
r_{jk} = Distance from j to k

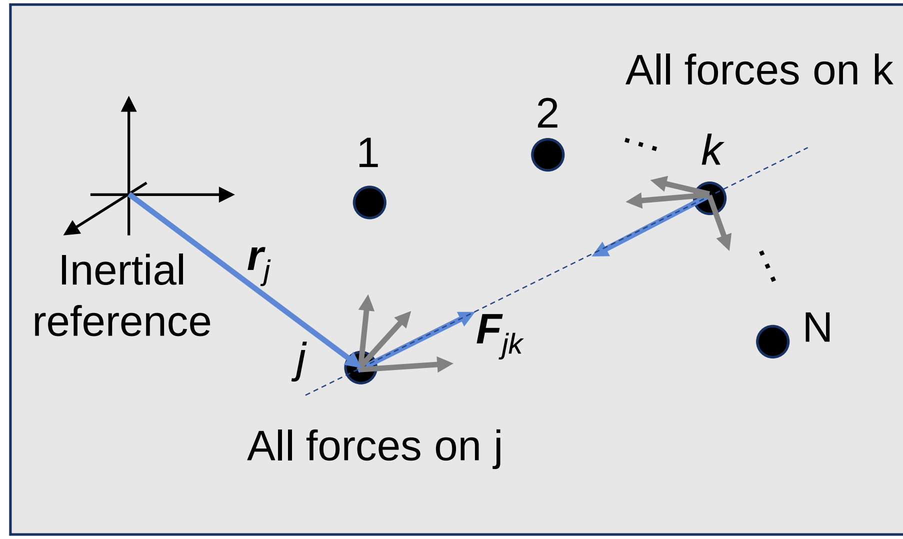
N-Body Problem (from Newton's laws)

- Defining vector from *k* to *j*

$$\vec{r}_{kj} = \vec{r}_j - \vec{r}_k$$

- Applying universal law of gravitation (**IV.**) between *k* and *j*

$$\vec{F}_{jk} = -\frac{G m_k m_j}{r_{kj}^2}\hat{\vec{r}}_{kj} = -G m_k m_j \frac{\vec{r}_{kj}}{r_{kj}^3}$$



- Totaling all forces on *j* due to N bodies

$$\sum \vec{F}_j = -G m_j \underbrace{\sum_{\substack{k=1 \\ k\neq j}}^{N} m_k \frac{\vec{r}_{kj}}{r_{kj}^3}}_{\text{Gravity only}}$$

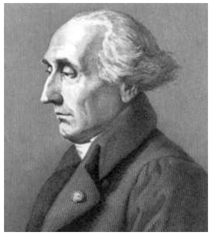
- Applying Netwons's (**II.**) law with constant mass for body j

$$\sum \vec{F}_j = \frac{d}{dt}(m_j \vec{v}_j) = m_j \underbrace{\frac{d^2}{dt^2}(\vec{r}_j)}_{\text{Acceleration}}$$

N-Body Problem (Equations of motion)

- Resulting equation of motion is 2nd order non-linear differential equation for body *j* (N equations for N bodies).

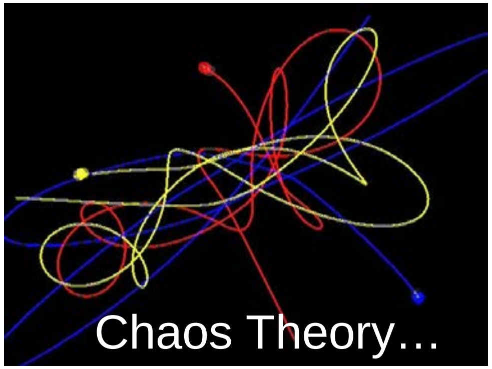
$$\frac{d^2}{dt^2}(\vec{r}_j) = -G \sum_{\substack{k=1 \\ k \neq j}}^{N} m_k \frac{\vec{r}_{kj}}{r_{kj}^3}$$



Lagrangia, Italy (1736-1813 AD)

Euler, Switzerland (1707-1783 AD)

- Analytic solutions available only in rare cases (N = 1, 2, 3). Solutions even rarer when non-gravitational effects considered. For N ≥ 3 we rely on numerical integration.



- In AA 279 A, we will consider only special cases which are of practical relevance
  - Restricted Three-Body Problem (N = 3)
  - Two-Body Problem (N = 2)

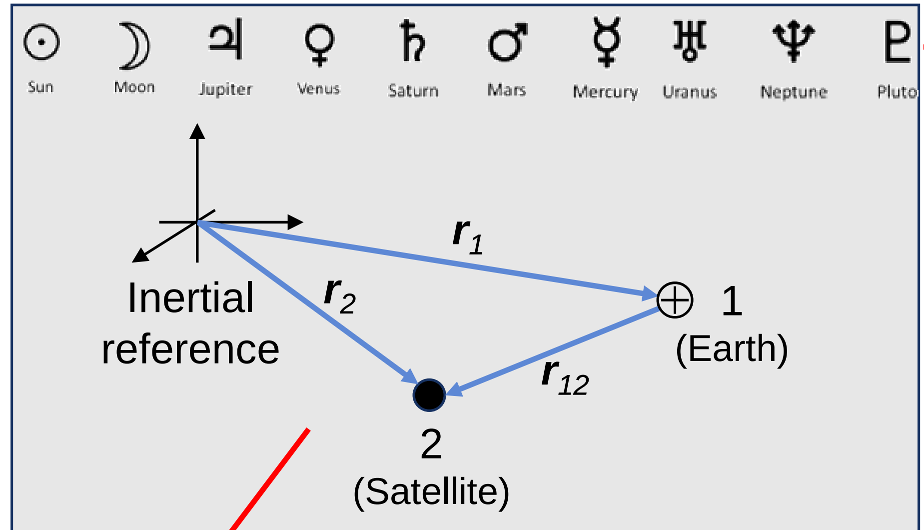
2-Body Problem (1)

- For body 1 (Earth)

$$\frac{d^2}{dt^2}(\vec{r}_1) = -G \sum_{\substack{k=1 \\ k \neq 1}}^{N} m_k \frac{\vec{r}_{k1}}{r_{k1}^3}$$

- For body 2 (Satellite)

$$\frac{d^2}{dt^2}(\vec{r}_2) = -G \sum_{\substack{k=1 \\ k \neq 2}}^{N} m_k \frac{\vec{r}_{k2}}{r_{k2}^3}$$



- Motion of body 2 w.r.t. to body 1

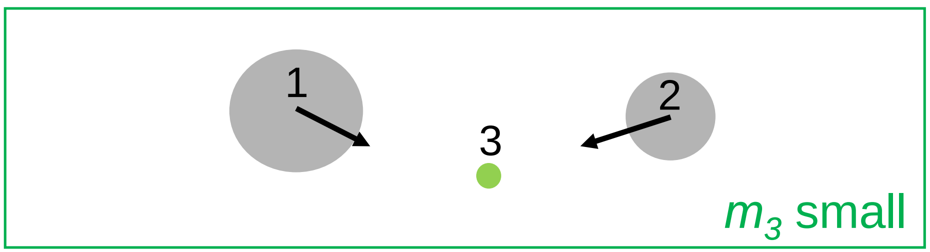
$$\frac{d^2}{dt^2}(\vec{r}_{12}) = \frac{d^2}{dt^2}(\vec{r}_2) - \frac{d^2}{dt^2}(\vec{r}_1) = \overbrace{-G(m_1 + m_2)\frac{\vec{r}_{12}}{r_{12}^3}}^{\text{N = 2 (Dominant)}} - \overbrace{\sum_{k=3}^{N} G m_k \left(\frac{\vec{r}_{k2}}{r_{k2}^3} - \frac{\vec{r}_{k1}}{r_{k1}^3}\right)}^{\text{N} \geq \text{3 (Negligible?)}}$$

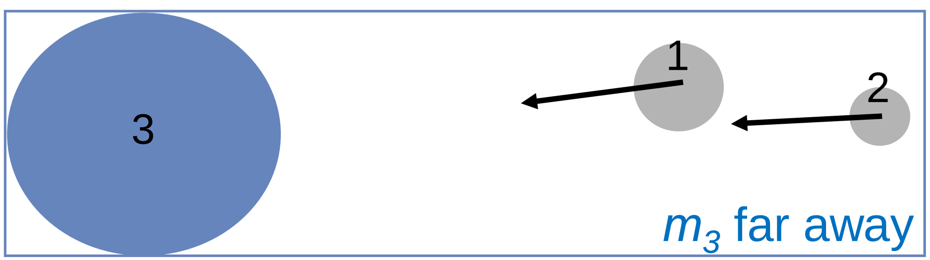
2-Body Problem (2)

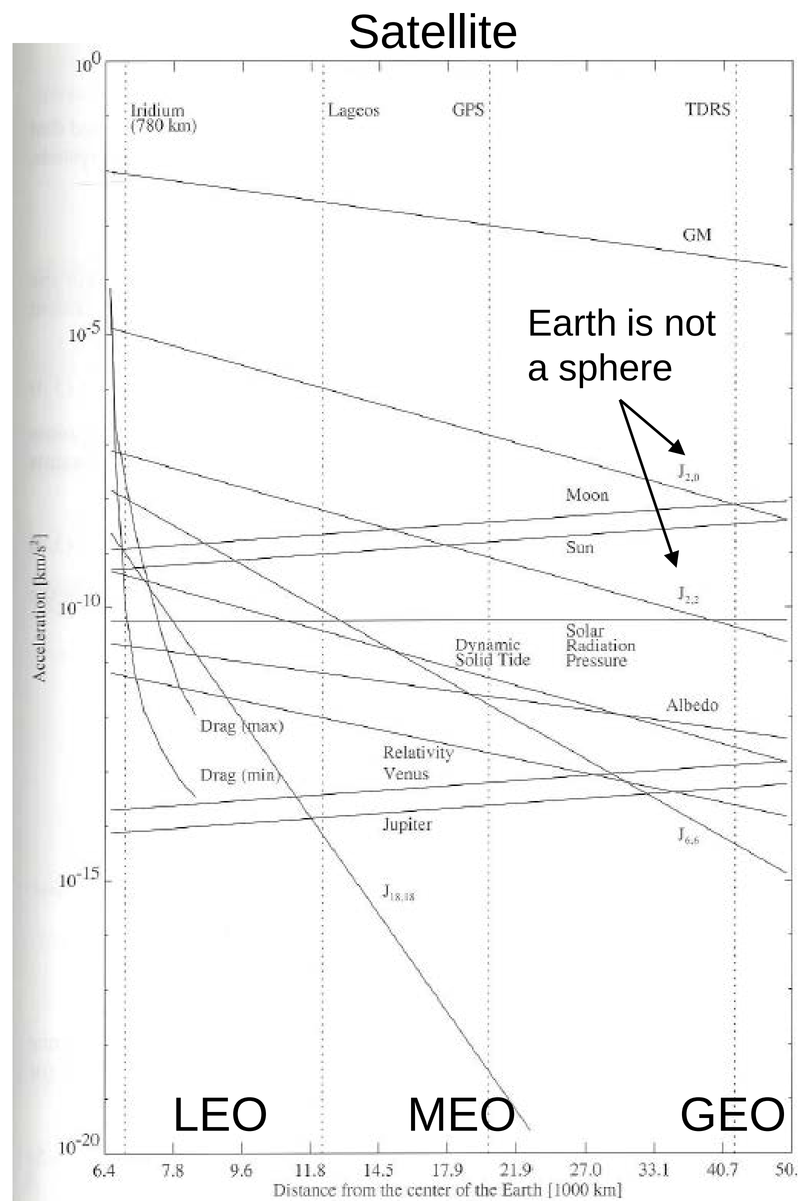
- When can we neglect contributions from $k \geq 3$ bodies?

$$\sum_{k=3}^{N} Gm_k \left(\frac{\vec{r}_{k2}}{r_{k2}^3} - \frac{\vec{r}_{k1}}{r_{k1}^3} \right) \longrightarrow 0$$

small or far away



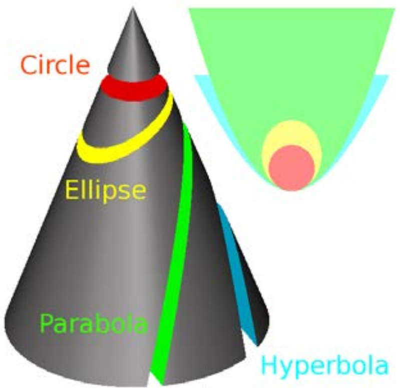




2-Body Problem (3)

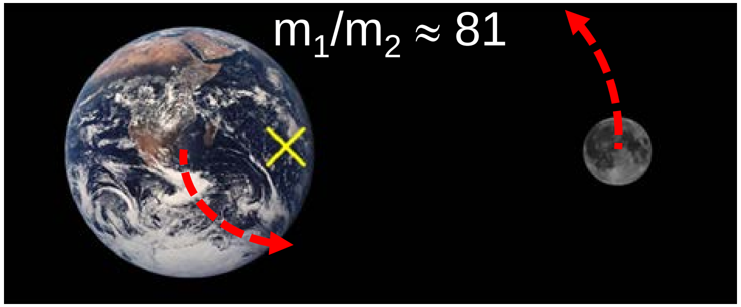
- For N = 2, we are left with a 2nd order nonlinear ordinary differential equation which has closed-form solutions for $\boldsymbol{r}_{12}$.

$$\frac{d^2}{dt^2}(\vec{r}_{12}) = -G(m_1 + m_2)\frac{\vec{r}_{12}}{r_{12}^3}$$



- Solutions are conic sections (circles, ellipses, hyperbolas, parabolas) about the center of mass of the 1-2 system (barycenter)

- Example: The Earth and the Moon orbit each other in nearly circular ellipses with foci at the barycenter (27 days period)



2-Body Problem (4)

- We are usually interested in man-made objects where $m_{\text{Satellite}} \ll m_{\text{Central body}}$

$$G(m_1 + m_2)$$

$$\downarrow$$

$$Gm_1 = Gm_{\substack{\text{Central}\\\text{body}}} = \mu_{\substack{\text{Central}\\\text{body}}}$$

$$\frac{d^2}{dt^2}\vec{r} + \mu\frac{\vec{r}}{r^3} = 0$$

Fundamental Orbital Differential Equation

- In this case we drop the $\boldsymbol{r}_{12}$ notation and assume $\boldsymbol{r}$ being the position of the satellite w.r.t. the center of mass of the spherically symmetric central body

- The **gravitational parameter** μ is specific to a central body and can be measured more accurately than G or $m_{\text{Central body}}$ through laser distance measurements of artificial satellites

$\mu_{\text{Earth}} = 3.986\ 10^5$

$\mu_{\text{Sun}} = 1.327\ 10^{11}$ $\quad \text{km}^3/\text{s}^2$

$\mu_{\text{Moon}} = 4.902\ 10^3$

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2-Body Problem (Solution)

- Assumptions: Spherically symmetric bodies (point masses), only gravitational forces, inertial coordinate system, $m_{\text{Satellite}} << m_{\text{Central body}}$

$$\frac{d^2}{dt^2}\vec{r} + \mu\frac{\vec{r}}{r^3} = 0$$

Fundamental Orbital Differential Equation

$$\begin{cases} \vec{r}(t_0) \\ \vec{v}(t_0) \end{cases}$$

Inertial Position and Velocity

- Orbital motion is governed by 2nd order nonlinear ordinary differential equation and is completely determined by initial conditions at particular time t_0 (**6 Degrees of Freedom**)

- In polar coordinates (r, ν) the fundamental orbital differential equation is solved by the general equation of a conic section (this solution gives a shape, not the time evolution)

$$r(\nu) = \frac{p}{1 + e\cos\nu}$$

Conic Section in Polar Coordinates

Backup