AA 279 C – SPACECRAFT ADCS: LECTURE 13

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Attitude Control through MW and RW (1)

- The Euler equations represent the attitude dynamics about three axes
- For a single- or dual-spin satellite, two equations are coupled, thus
  - 1 MW and 1 RW are sufficient to obtain active stability
  - 2 RW are sufficient to obtain active stability
  - 2 MW are sufficient to obtain active stability
- For MW and RW, the actual command is given through an electric motor by

$$\vec{M}_c = \dot{\vec{L}}_w = \vec{I}_w \dot{\vec{\omega}}_w$$

- But this is only 1 of the three terms which are caused by the actuator (see previous slide). Theoretically one could use the pseudo-inverse to solve for

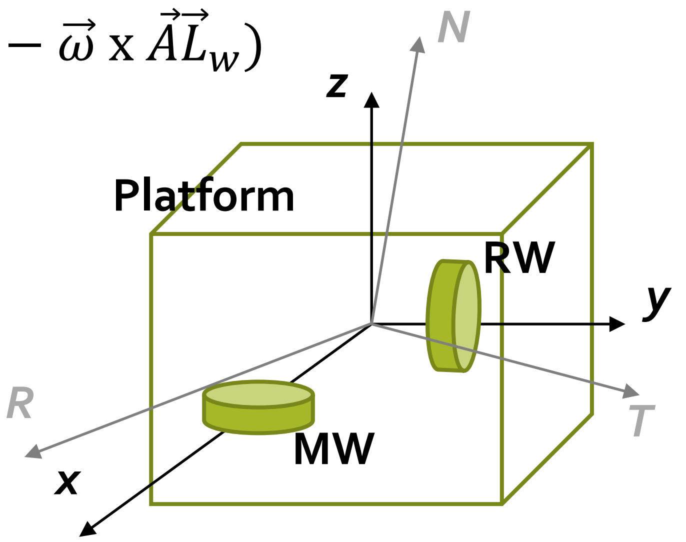
$$\dot{\vec{L}}_w = \vec{A}^* (\vec{I}\dot{\vec{\omega}} + \vec{\omega} \text{ x } \vec{I}\vec{\omega} + \vec{\omega} \text{ x } \vec{A}\vec{L}_w - \vec{M})$$

- But this equation is unnecessarily complicated, highly non-linear, and requires knowledge of the satellite rotational acceleration and disturbance torques

Attitude Control through MW and RW (2)

- In practice we go back to the Euler equations, use a control law from linear control theory, and solve for our command from

$$\vec{M}_c = -\vec{\vec{A}}\dot{\vec{L}}_w - \dot{\vec{\vec{A}}}\vec{L}_w - \vec{\omega} \text{ x } \vec{\vec{A}}\vec{L}_w \Rightarrow \dot{\vec{L}}_w = \vec{A}^*(-\vec{M}_c - \vec{\omega} \text{ x } \vec{\vec{A}}\vec{L}_w)$$



- Example: Earth pointing with 1 MW and 1 RW

$$\begin{cases} I_x\dot{\omega}_x + (I_z - I_y)\omega_y\omega_z + I_{wz}\omega_{wz}\omega_y - I_{wy}\omega_{wy}\omega_z = 3n^2(I_z - I_y)c_y c_z \\ I_y\dot{\omega}_y + I_{wy}\dot{\omega}_{wy} + (I_x - I_z)\omega_z\omega_x - I_{wz}\omega_{wz}\omega_x = 3n^2(I_x - I_z)c_z c_x \\ I_z\dot{\omega}_z + I_{wz}\dot{\omega}_{wz} + (I_y - I_x)\omega_x\omega_y + I_{wy}\omega_{wy}\omega_x = 3n^2(I_y - I_x)c_x c_y \\ I_{wy}\dot{\omega}_{wy} = M_{cy}\,;\, I_{wz}\dot{\omega}_{wz} = M_{cz} \end{cases}$$

- Using our standard procedure and linearization

$$\begin{cases} I_x(\ddot{\alpha}_x - n\dot{\alpha}_y) + (I_z - I_y)(n\dot{\alpha}_y + n^2\alpha_x) + I_{wz}\overline{\omega}_{wz}(\dot{\alpha}_y + n\alpha_x) - I_{wy}\omega_{wy}n = 0 \\ I_x(\ddot{\alpha}_y - n\dot{\alpha}_x) + (I_z - I_y)(n\dot{\alpha}_x - n^2\alpha_y) - I_{wz}\overline{\omega}_{wz}(\dot{\alpha}_x - n\alpha_y) + I_{wy}\dot{\omega}_{wy} - 3n^2(I_x - I_z)\alpha_y = 0 \\ I_z\ddot{\alpha}_z + I_{wz}\dot{\omega}_{wz} + 3n^2(I_y - I_x)\alpha_z = 0 \\ I_{wy}\dot{\omega}_{wy} = M_{cy}\,;\, I_{wz}\dot{\omega}_{wz} = M_{cz} \end{cases}$$

2 pitch eqs. decoupled from other 3 yaw and roll eqs.

Attitude Control through MW and RW (3)

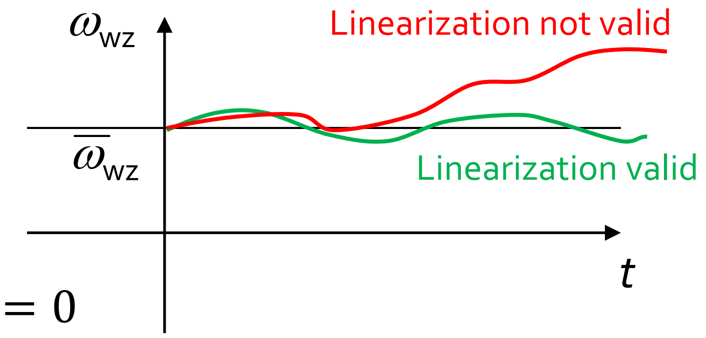
- The problem is decoupled and can be split in two independent parts as long as the linearization assumption holds. For pitch

$$\begin{cases} I_z\ddot{\alpha}_z + 3n^2\left(I_y - I_x\right)\alpha_z = -I_{wz}\dot{\omega}_{wz} = -M_{cz} \\ I_{wz}\dot{\omega}_{wz} = M_{cz} \end{cases}$$

Control part moved to the right hand side

Actuator command directly known from control law

- Given the control law for pitch, the time evolution of ω_{wz} is also given. Large control actions can cause variations of ω_{wz} which bring the system far from the equilibrium with time



- One possibility to compute the control law is

$$M_{cz} = +k_p\alpha_z + k_d\dot{\alpha}_z$$

- which provides an harmonic oscillator

Oscillator frequency

$$\ddot{\alpha}_z + \frac{k_d}{I_z}\dot{\alpha}_z + \frac{\left[3n^2\left(I_y - I_x\right) + k_p\right]}{I_z}\alpha_z = \ddot{\alpha}_z + 2\xi f\dot{\alpha}_z + 2f^2\alpha_z = 0$$

Damping

- The roots of the characteristic equation can be used to obtain critical dumping

$$k_d = 2\sqrt{I_z\left[3n^2\left(I_y - I_x\right) + k_p\right]}\,;\; k_p = f^2/I_z$$

Desired frequency of response

Integration with Magnetic Actuation (1)

- Independent from the actuator, we can always write the Euler equations excluding the perturbation torques

$$\vec{I}\dot{\vec{\omega}} + \vec{\omega} \times \vec{I}\vec{\omega} = \vec{M}_c$$

- From the principles of control theory, after linearization, we can compute the control law

$$\vec{M}_c = \vec{f}(\vec{\alpha}, \dot{\vec{\alpha}})$$

- The desired control torque needs to be realized through the actuator functioning principle, e.g.
  - RW

$$\vec{M}_c = \dot{\vec{L}}_w = \vec{g}(\vec{\alpha}, \dot{\vec{\alpha}})$$

Can be solved for the actuator command, provided that angular velocity is < saturation

  - Magnetorquer

$$\vec{M}_c = \vec{m} \times \vec{B} = \begin{bmatrix} 0 & B_z & -B_y \\ -B_z & 0 & B_x \\ B_y & -B_x & 0 \end{bmatrix} \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix}$$

Cannot be solved for the actuator command, because the matrix is singular

Integration with Magnetic Actuation (2)

- We can combine RW and magnetorquers to prevent saturation of the wheel and at the same time enable three-axis magnetic control. In particular we can substitute the magnetorquer along *z* with a RW

$$\vec{M}_c = \vec{m} \times \vec{B} + \vec{A}\dot{\vec{L}}_w = \begin{bmatrix} 0 & B_z & 0 \\ -B_z & 0 & 0 \\ B_y & -B_x & 1 \end{bmatrix} \begin{pmatrix} m_x \\ m_y \\ \dot{L}_w \end{pmatrix}$$

- We can now invert the matrix to obtain

Command to the actuator

$$\begin{pmatrix} m_x \\ m_y \\ \dot{L}_w \end{pmatrix} = \frac{1}{B_z^2} \begin{bmatrix} 0 & -B_z & 0 \\ B_z & 0 & 0 \\ B_z B_x & B_z B_y & B_z^2 \end{bmatrix} \begin{pmatrix} M_{cx} \\ M_{cy} \\ M_{cz} \end{pmatrix}$$

Torques desired from control law

- An extremely simple control law is the B-dot law which despins the spacecraft relative to the earth's magnetic field vector and stabilizes a MW spin axis (typically to the orbit normal)

$$\vec{M}_c = -k_p \dot{\vec{B}} \text{ or } \vec{M}_c = -k_p \left(\dot{\vec{B}} - \dot{\vec{B}}_{\mathrm{des}} \right) \text{ or } \vec{M}_c = -\vec{M}_{\max} \mathrm{sign}(\dot{\vec{B}})$$

Positive constant

Measured in body frame

Modelled

Max dipole from magnetorquer

Control Law for Large Tracking Errors (1)

- General Euler equations for a rigid satellite subject to gyroscopic actuators, gravity gradient, un-modelled disturbances, and other actuators

$$\vec{I}\dot{\vec{\omega}} + \vec{\omega} \times \vec{I}\vec{\omega} + \vec{A}\dot{\vec{L}}_w + \dot{\vec{A}}\vec{L}_w + \vec{\omega} \times \vec{A}\vec{L}_w = \vec{M}_g + \vec{M}_d + \vec{M}_c$$

- The control law is designed using an auxiliary equation where all colored terms are grouped in a single control term on the right side

$$\vec{I}\dot{\vec{\omega}} + \vec{\omega} \times \vec{I}\vec{\omega} = \vec{M}_{cc}$$

- This equation might prove difficult to linearize for large control tracking errors, thus we often use brute force by using instead

$$\vec{I}\dot{\vec{\omega}} = \vec{M}_{ccc}$$

- This equation is first order and can be linearized to a standard harmonic oscillator using a PD controller

$$\vec{I}\ddot{\vec{\alpha}} = \vec{M}_{ccc} = -k_p\vec{\alpha} - k_d\dot{\vec{\alpha}} \Rightarrow \vec{I}\ddot{\vec{\alpha}} + k_d\dot{\vec{\alpha}} + k_p\vec{\alpha} = 0$$

Tuned to obtain damping at ~0.7 and frequency ~10n

- This control law does not work for large angles, since the attitude motion is coupled across all axes, and needs to be modified

Control Law for Large Tracking Errors (2)

- The modification of the control law is done by considering the definition of the attitude control error matrix

$$\vec{A}_E = \vec{A}_S \vec{A}_T^t$$

Attitude tracking error

Satellite desired attitude from mission objective

Satellite attitude from sensors

- Expanding the attitude error matrix

Unit vector of satellite triad

Unit vector of target triad

$$\vec{A}_E = \begin{bmatrix} \vec{x}_S \cdot \vec{x}_T & \vec{x}_S \cdot \vec{y}_T & \vec{x}_S \cdot \vec{z}_T \\ \vec{y}_S \cdot \vec{x}_T & \vec{y}_S \cdot \vec{y}_T & \vec{y}_S \cdot \vec{z}_T \\ \vec{z}_S \cdot \vec{x}_T & \vec{z}_S \cdot \vec{y}_T & \vec{z}_S \cdot \vec{z}_T \end{bmatrix} \sim \begin{bmatrix} 1 & \alpha_z & -\alpha_y \\ -\alpha_z & 1 & \alpha_x \\ \alpha_y & -\alpha_x & 1 \end{bmatrix}$$

Only valid for small angles

- The controller aims at zeroing the off-diagonal terms, or equivalently at making the corresponding unit vectors as perpendicular as possible. The nonlinear version of the control tracking errors is given by $\vec{x}_S \cdot \vec{y}_T$, $\vec{x}_S \cdot \vec{z}_T$, and $\vec{y}_S \cdot \vec{z}_T$

Control Law for Large Tracking Errors (3)

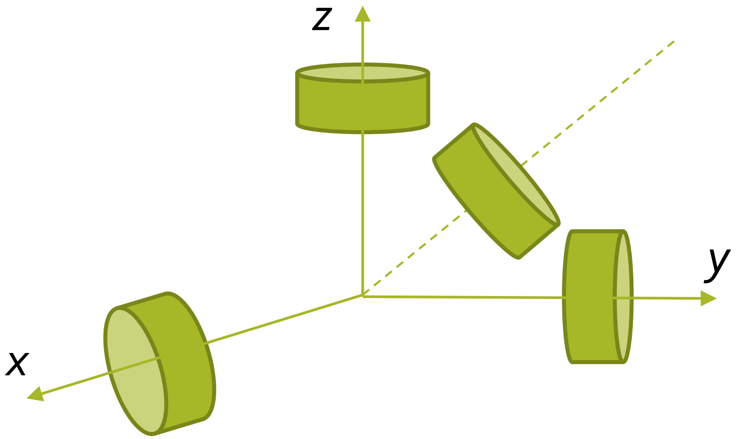
- The control law in the case of large angles (non linearizable) becomes

$$\vec{M}_c = -\overrightarrow{k_p}\vec{\alpha} - \overrightarrow{k_d}\dot{\vec{\alpha}} \rightarrow M_{ci} = -k_p \frac{A_{Ejk} - A_{Ekj}}{2} - k_d \omega_i$$

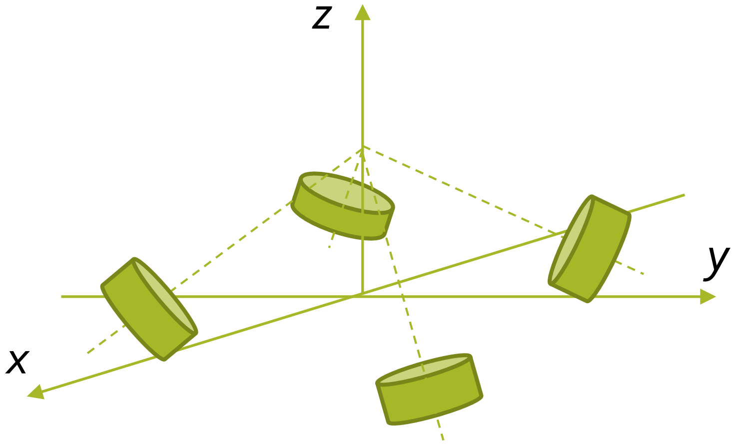
- The non-linear control law matches the linear version for small angles
- Since the difference between the off-diagonal rotation matrix terms is linked to the direction of the Euler axis, this control law is faster in average
- This is because the control law tries to rotate the spacecraft about the Euler axis which gives the minimum consume of energy
- The PD gains are identical for the linear and non-linear control law

Examples of Multiple Gyroscopic Actuators

- 3 RW aligned with x, y, z
- 1 RW aligned with trisectrix



- 4 MW aligned at corners of pyramid with square basis



$$\vec{M}_c = \vec{A}\dot{\vec{L}}_w$$

$$\dot{\vec{L}}_w = \vec{A}^*\vec{M}_c$$

$$\vec{A}^* = \begin{bmatrix} 5/6 & -1/6 & -1/6 \\ -1/6 & 5/6 & -1/6 \\ -1/6 & -1/6 & 5/6 \\ \sqrt{3}/2 & \sqrt{3}/2 & \sqrt{3}/2 \end{bmatrix}$$

$$\vec{A} = \begin{bmatrix} 1 & 0 & 0 & 1/\sqrt{3} \\ 0 & 1 & 0 & 1/\sqrt{3} \\ 0 & 0 & 1 & 1/\sqrt{3} \end{bmatrix}$$

The desired torque can be optimally distributed to all RW

4 MW can create a system equivalent to 3 RW + 1 MW

$$\vec{A}^* = \frac{3}{4\sqrt{3}} \begin{bmatrix} -1 & -1 & 1 \\ +1 & -1 & +1 \\ +1 & +1 & +1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\vec{A} = \frac{1}{\sqrt{3}} \begin{bmatrix} -1 & +1 & +1 & -1 \\ -1 & -1 & +1 & +1 \\ +1 & +1 & +1 & +1 \end{bmatrix}$$

Backup