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AA 279 A – Space Mechanics
Lecture 7: Notes

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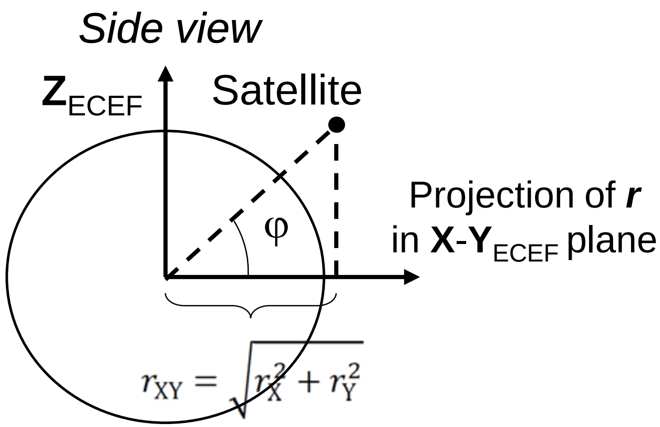
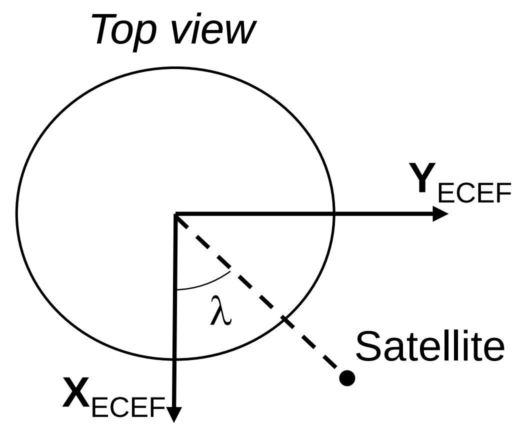
Geocentric Coordinates (Spherical Earth)





$$\varphi = \arcsin\left(\frac{r_Z}{\sqrt{r_X^2 + r_Y^2 + r_Z^2}}\right)$$

Geocentric latitude and longitude **from/to** position in ECEF

$$\lambda = \arctan\left(\frac{r_Y}{r_X}\right)$$

Note: $r = R_E + h$

Height above sphere

$$\vec{r}_{ECEF} = \begin{bmatrix} (R_E + h)\cos\varphi\cos\lambda \\ (R_E + h)\cos\varphi\sin\lambda \\ (R_E + h)\sin\varphi \end{bmatrix}$$

Projection onto XY

Projection onto X and Y

Plotting Groundtracks

- We can put it all together and plot ground-tracks from orbital elements knowing the Earth's orientation at some reference time
- Typical inputs are a (or n), e, i, Ω, ω, M_0 (or ν_0), and t_0 (UTC or GPS time)

1) Do for time $t = t_0 \rightarrow t_{end}$
2) Propagate Mean anomaly (definition)

$$M = M_0 + n(t - t_0)$$

3) Find eccentric anomaly (Kepler's equation)

$$M = E - e\sin E$$

4) Compute perifocal position (and velocity)

$$\vec{r}_{\mathrm{PQW}} = \begin{bmatrix} a(\cos E - e) & a\sqrt{1-e^2}\sin E & 0 \end{bmatrix}^t$$

5) Rotate to ECI (exact!)

$$\vec{r}_{\mathrm{IJK}} = \vec{R}_{\mathrm{z}}(-\Omega)\vec{R}_{\mathrm{x}}(-i)\vec{R}_{\mathrm{z}}(-\omega)\vec{r}_{\mathrm{PQW}}$$

6) Find Greenwich sidereal time from UTC or GPS time

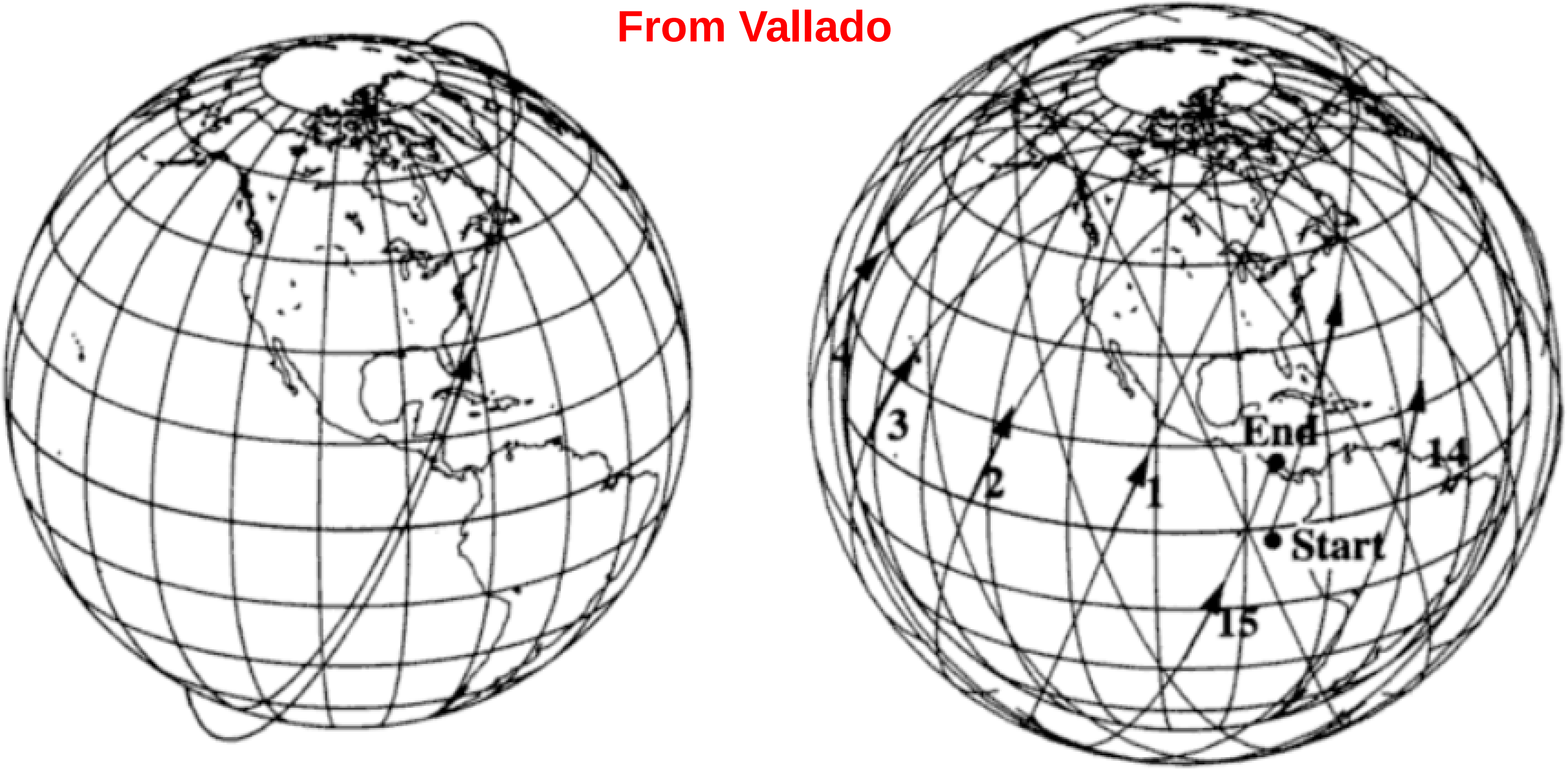
$$t_{UTC} \rightarrow t_{UT1} \rightarrow MJD \rightarrow \Theta$$

7) Rotate to ECEF (approximate!)

$$\vec{r}_{\mathrm{XYZ}} = \vec{R}_{\mathrm{z}}(\Theta)\vec{r}_{\mathrm{IJK}}$$

8) Find geocentric coordinates iteratively λ, φ, h

From Vallado



Fixed Earth

Rotating Earth

Figure 2-20. **Three-D Groundtracks.** With the Earth fixed, the satellite's orbital plane is visible, but this figure doesn't represent the satellite as it travels through time. In reality, the Earth moves under the "fixed" orbital plane. When a groundtrack is displayed over the positions it would occupy over time, the result is a "ball-of-yarn." Although this view *does* show the true positions, it's not particularly useful because it contains so much information. I've numbered each revolution for reference.

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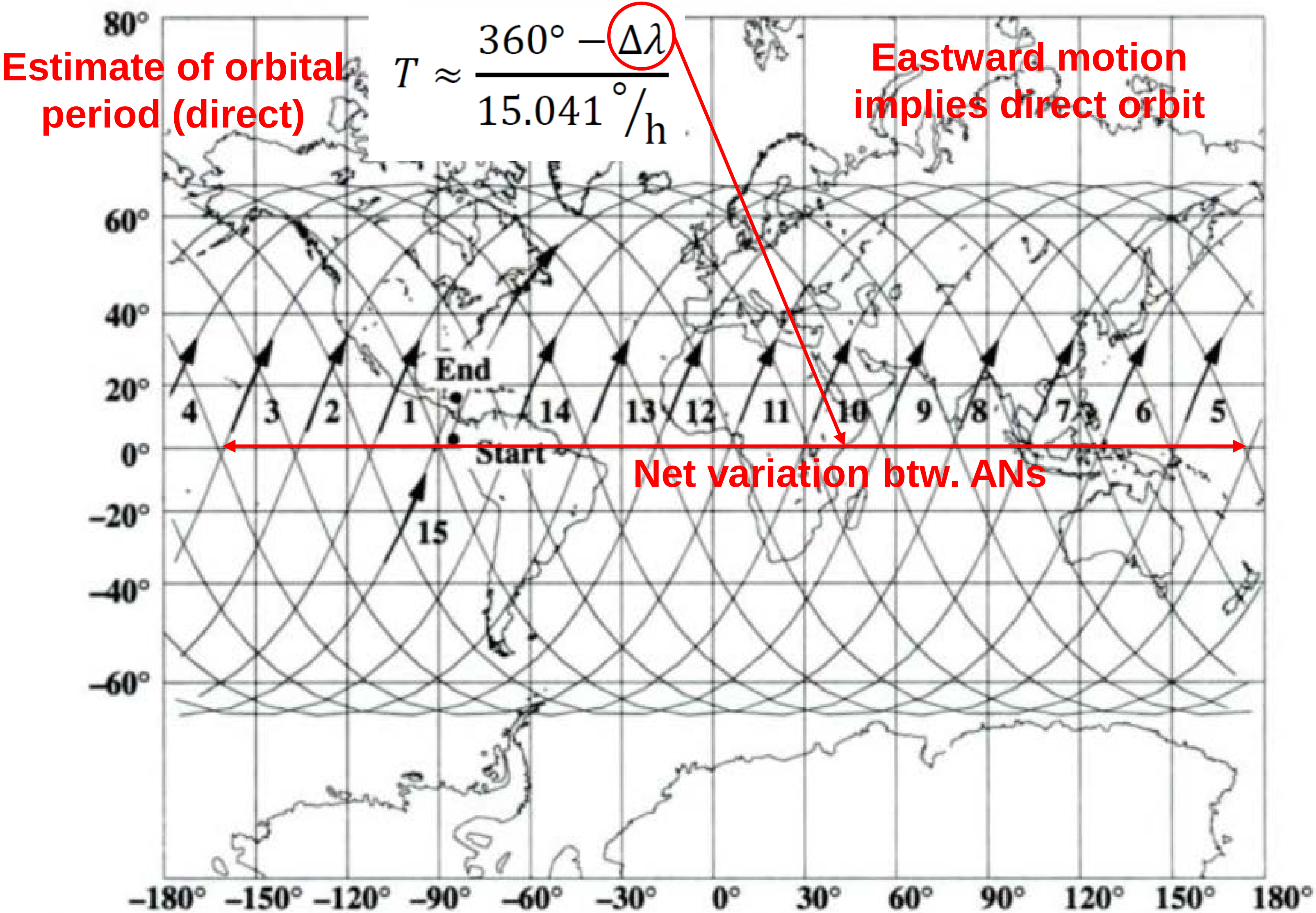

Figure 2-21. Groundtrack of a Satellite in Circular Orbit on a Rotating Earth. This form of the two-dimensional groundtrack is very useful because it shows the actual locations over a rotating Earth. Also notice the mismatch in the starting and ending points after 15 revolutions.

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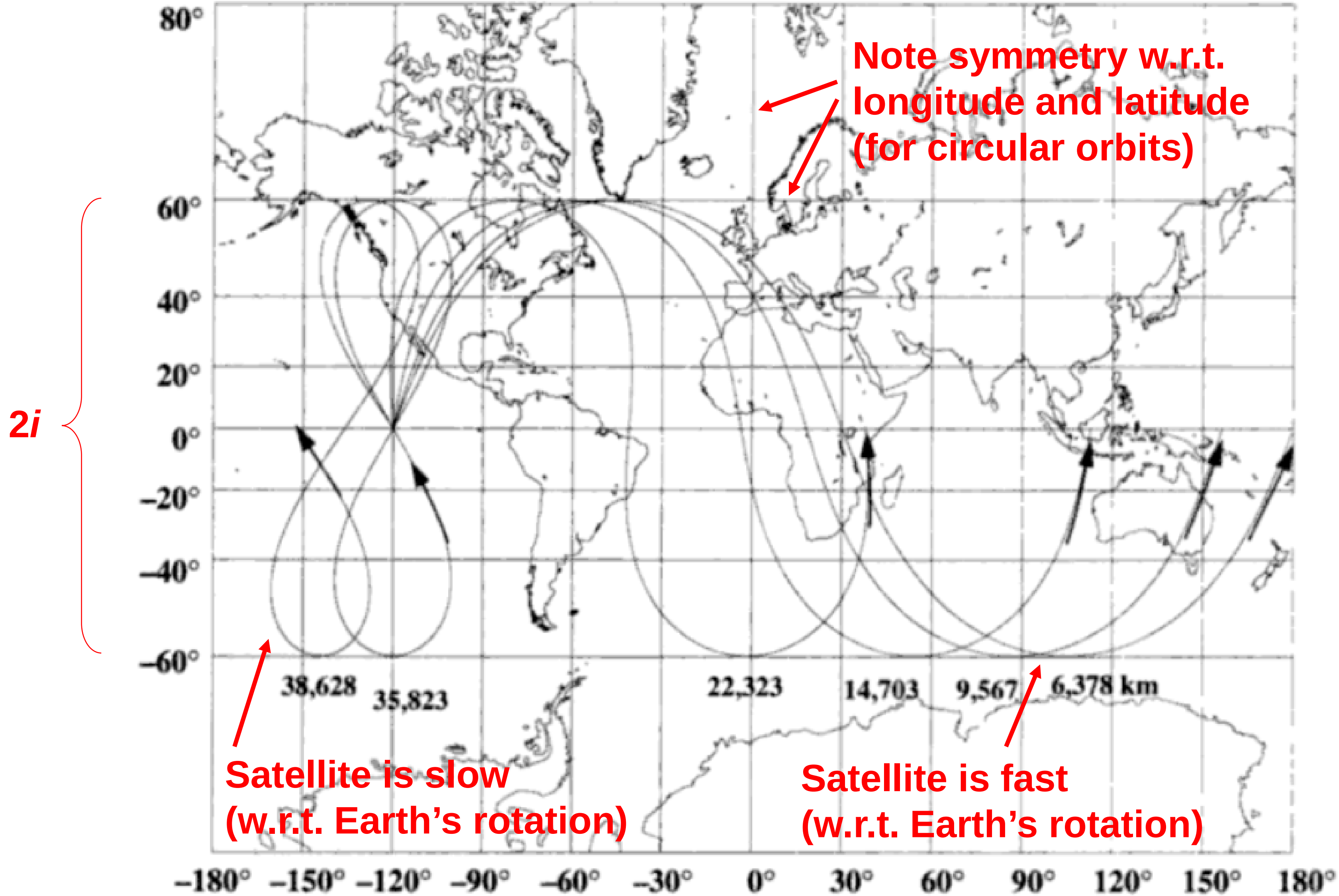


Figure 2-22. Effects of Increasing Altitude for Groundtracks on a Rotating Earth. Notice how the span of the groundtrack shrinks as the altitude increases. All groundtracks shown are for circular orbits. Altitudes are given in kilometers above the Earth's surface.

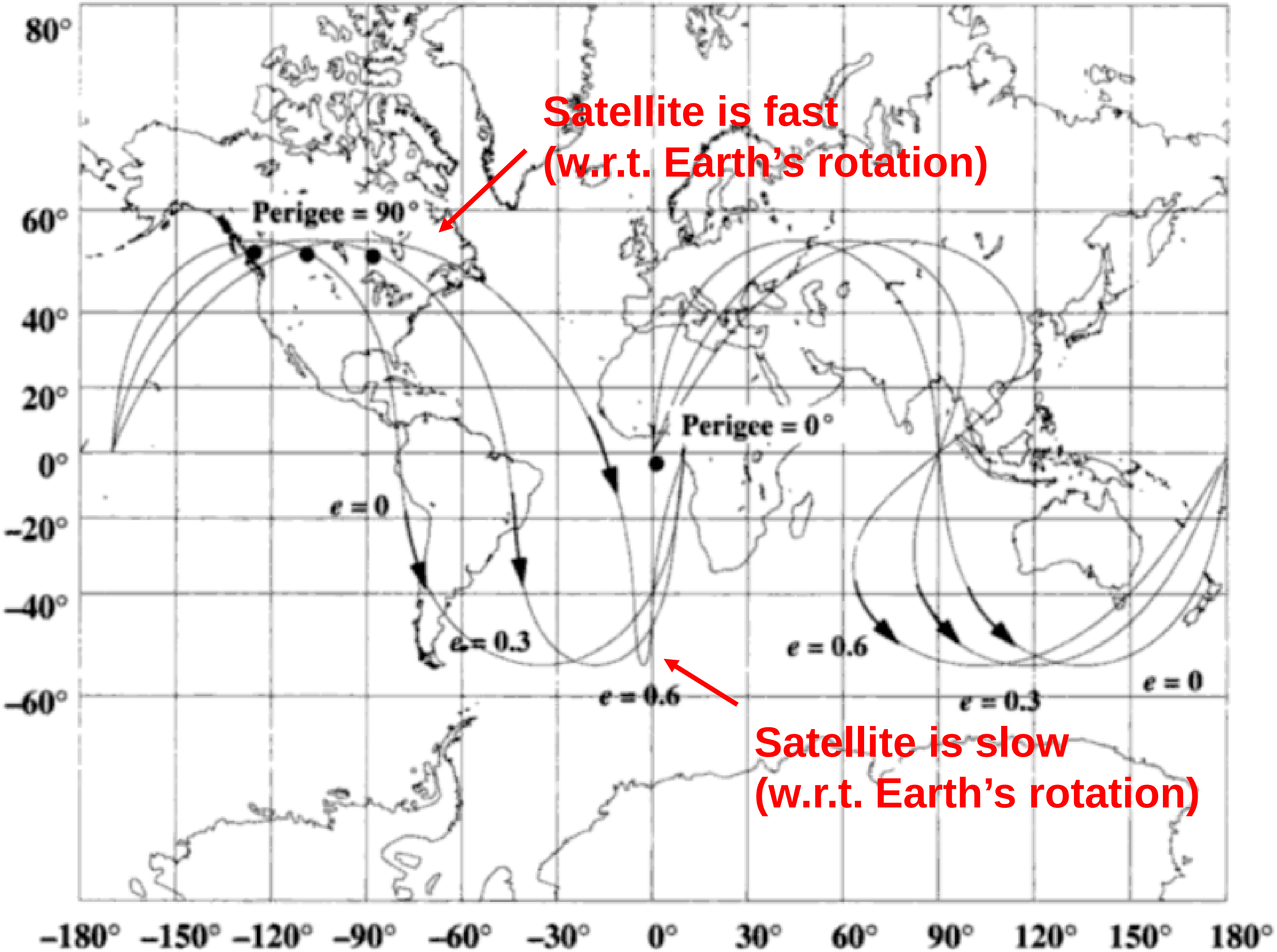
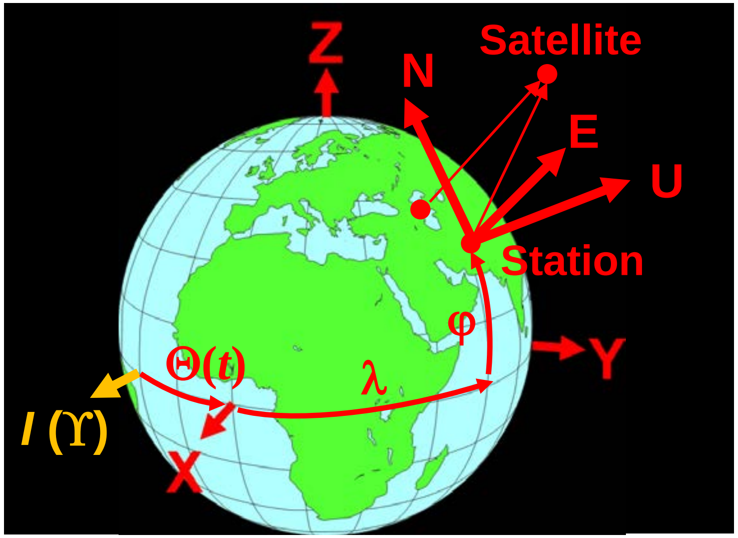


Figure 2-23. Effects of Eccentricity and Perigee Location for Groundtracks on a Rotating Earth. This figure shows two sets of orbits with different perigees. Each set contains three orbits consisting of eccentricity values of 0.0, 0.3, and 0.6. Notice how distorted each groundtrack becomes with increased eccentricity. Perigee locations at ±180° result in equal but opposite images.

Topocentric Coordinate System

- Natural coordinate system for satellite's motion description from ground station
- This is a local tangent (ENU) reference system with axis
  - ***E*** aligned with meridian passing trough ground station
  - ***N*** aligned with parallel passing through ground station
  - ***U*** normal up to horizontal plane
- The satellite's local tangent coordinates are given by

East-North-Up Triad



$$\vec{r}_{\mathrm{ENU}} = \vec{R}_{\mathrm{XYZ}\rightarrow\mathrm{ENU}}\left(\vec{r}_{\mathrm{XYZ}}^{\,\mathrm{Satellite}} - \vec{r}_{\mathrm{XYZ}}^{\,\mathrm{Station}}\right)$$

Here λ, φ are the station's geocentric longitude and latitude respectively

Montenbruck (Page 37)

$$\boxed{\vec{R}_{\mathrm{XYZ}\rightarrow\mathrm{ENU}} = (\vec{E} \quad \vec{N} \quad \vec{U})^t}$$

$$\boxed{\vec{r}_{\mathrm{XYZ}}^{\,\mathrm{Satellite}} = \vec{R}_z(\Theta)\vec{r}_{\mathrm{IJK}}}$$

$$\boxed{\vec{r}_{\mathrm{XYZ}}^{\,\mathrm{Station}} = R_{\mathrm{E}}(\cos\varphi\cos\lambda \quad \cos\varphi\sin\lambda \quad \sin\varphi)^t}$$

Backup