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AA 279 A – Space Mechanics
Lecture 12: Notes

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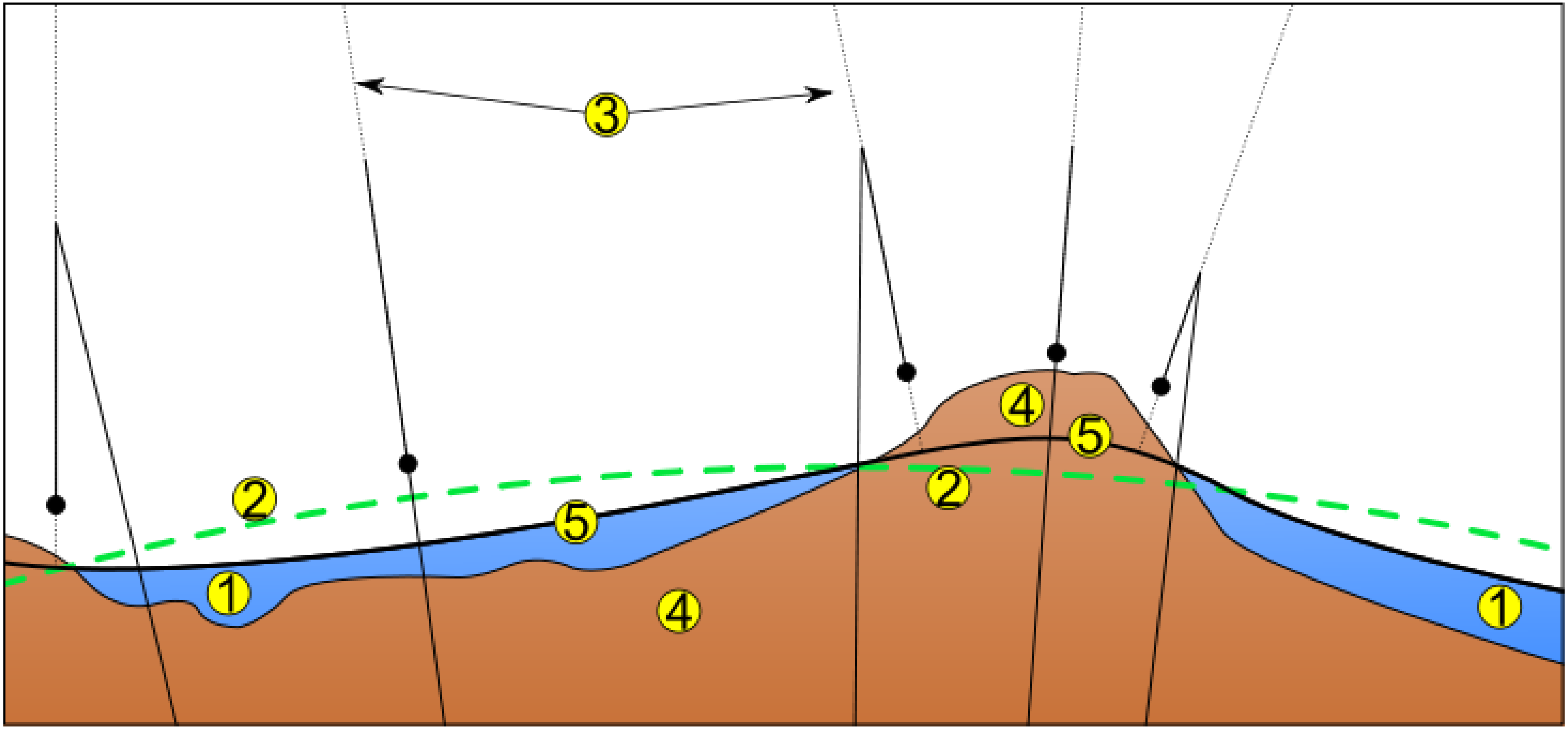
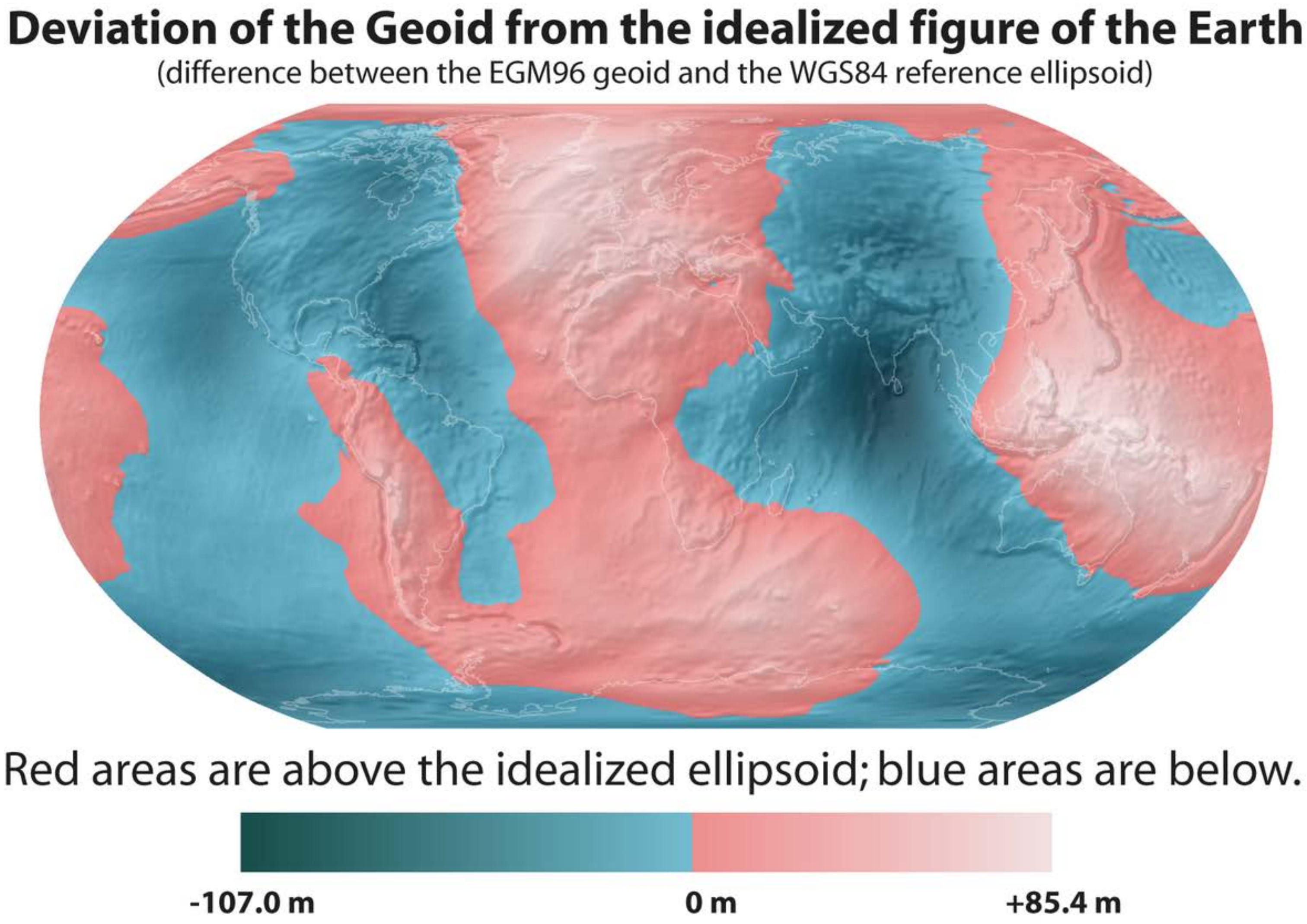
Earth’s Shapes (1)



1: Ocean, 2: Ellipsoid, 3: Plumb, 4: Continents, 5: Geoid

Earth’s Shapes (2)



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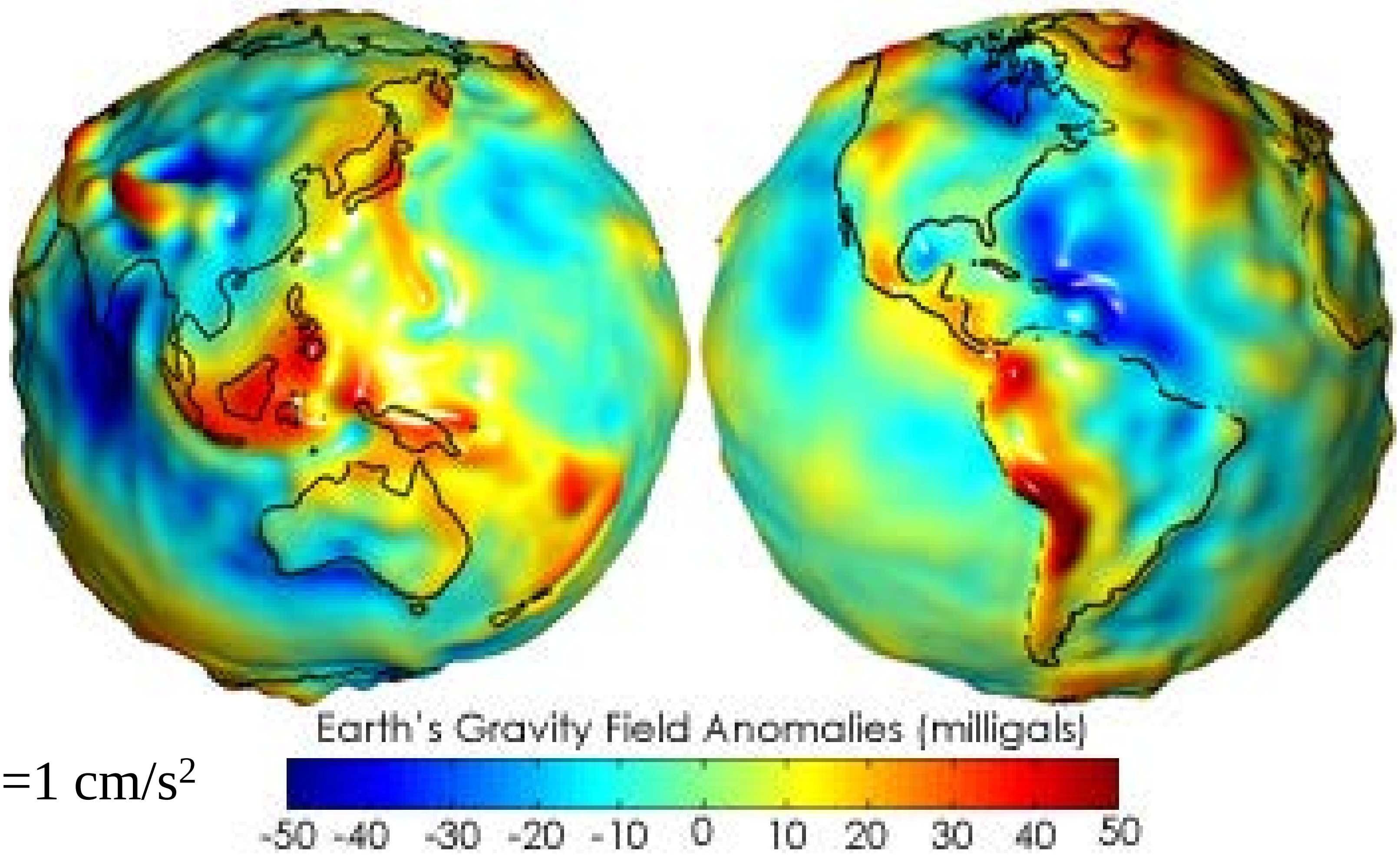
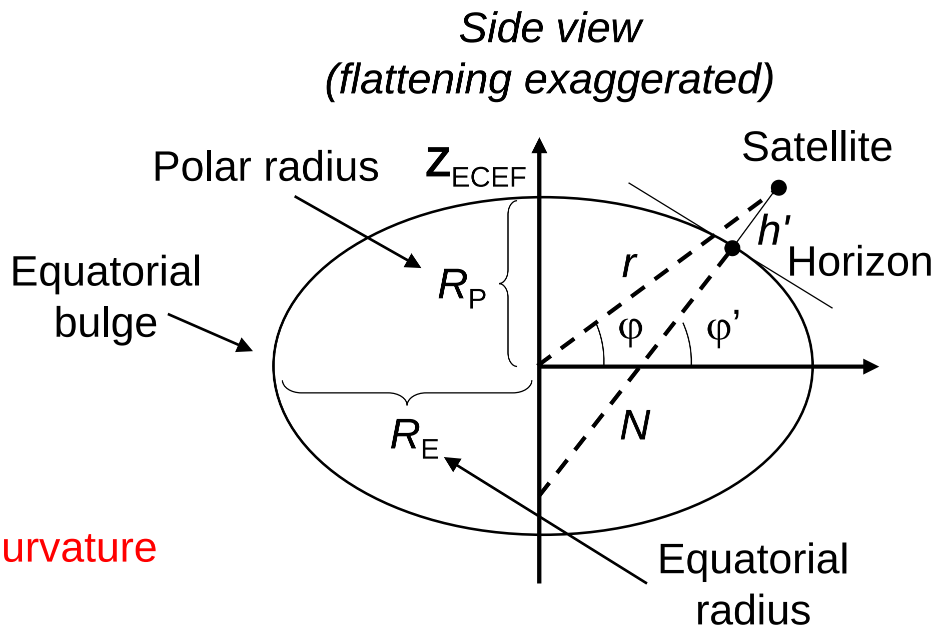
Earth's Shapes (3)

GRACE Gravity Field (GGM01S)



Gal=1 cm/s^2

Geodetic Coordinates (Ellipsoidal Earth)



- Earth's actual shape is closer to an oblate ellipsoid with eccentricity (from side) $e_E \approx 0.0818$ and R_E - $R_P \approx 21$ km

$$R_P = R_E\sqrt{1 - e_E^2}$$ Recall scale factor

$$N = \frac{R_E}{\sqrt{1 - e_E^2 sin^2\varphi'}}$$ Modified radius of curvature

- The relationship between Cartesian and polar coordinates is developed similar to the Kepler's equation, using the "auxiliary circle" concept

$$\vec{r}_{ECEF} = \begin{bmatrix} (N + h')\cos\varphi'\cos\lambda \\ (N + h')\cos\varphi'\sin\lambda \\ (N(1 - e_E) + h')\sin\varphi' \end{bmatrix}$$

Geodetic latitude and longitude to position in ECEF

Updated Ground-Tracks with Geodetic Coordinates

- This can be done iteratively or by direct methods which involve the solution of a quartic equation

1) Note that geodetic and geocentric longitudes are identical

$$\lambda = \lambda'$$

2) Use geocentric latitude as first guess for geodetic latitude

$$\varphi'_{i=0} = \varphi = \arcsin\left(r_{\mathrm{Z}} / \sqrt{r_{\mathrm{X}}^2 + r_{\mathrm{Y}}^2 + r_{\mathrm{Z}}^2}\right)$$

3) Compute modified radius of curvature

$$N_i = \frac{R_{\mathrm{E}}}{\sqrt{1 - e_{\mathrm{E}}^2 sin^2 \varphi'_i}}$$

4) Update geodetic latitude

$$\varphi'_{i+1} = \arctan\left[\left(r_{\mathrm{Z}} + N_i e_{\mathrm{E}}^2 sin\varphi'_i\right) / r_{\mathrm{XY}}\right]$$

5) If tolerance is violated go back to 3), otherwise go to 6)

$$\left|\varphi'_{i+1} - \varphi'_i\right| > \mathrm{Tolerance}$$

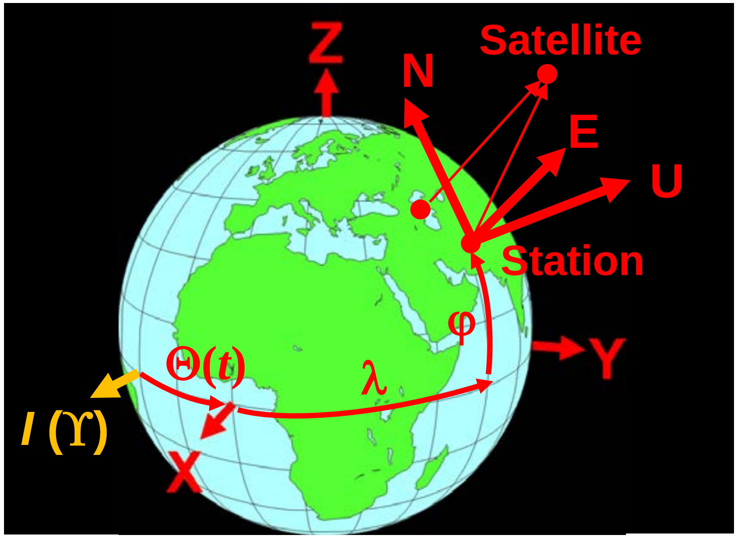
6) Compute height above ellipsoid

$$h' = \frac{r_{\mathrm{XY}}}{\cos\varphi'} - N$$

Topocentric Coordinate System

- Natural coordinate system for satellite's motion description from ground station
- This is a local tangent (ENU or ENZ) reference system with axis
  - ***E*** aligned with meridian passing trough ground station
  - ***N*** aligned with parallel passing through ground station
  - ***U*** normal up to horizontal plane
- The satellite's local cartesian coordinates are

East-North-Up Triad



$$\vec{r}_{\mathrm{ENU}} = \vec{R}_{\mathrm{XYZ}\rightarrow\mathrm{ENU}}\left(\vec{r}_{\mathrm{XYZ}}^{\,\mathrm{Satellite}} - \vec{r}_{\mathrm{XYZ}}^{\,\mathrm{Station}}\right)$$

Here λ, φ are the station's geocentric longitude and latitude (geodetic can be used)

Montenbruck (Page 37)

$$\boxed{\vec{R}_{\mathrm{XYZ}\rightarrow\mathrm{ENU}} = (\vec{E} \quad \vec{N} \quad \vec{U})^t}$$

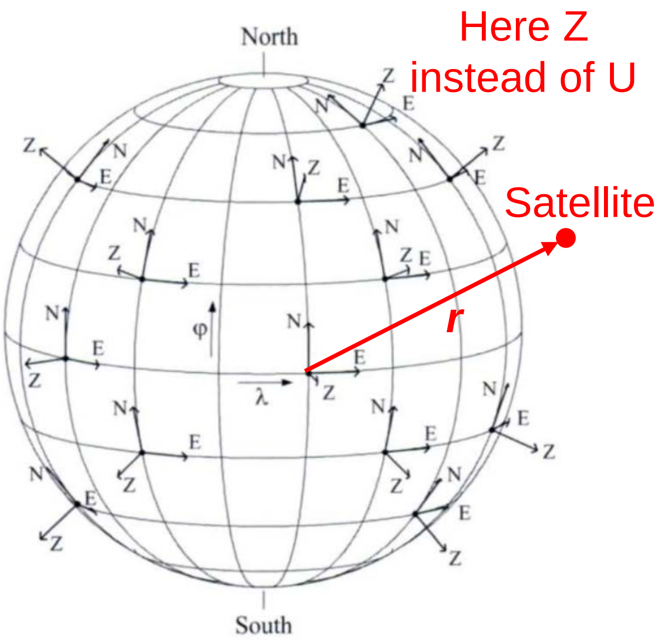
$$\boxed{\vec{r}_{\mathrm{XYZ}}^{\,\mathrm{Satellite}} = \vec{R}_{\mathrm{z}}(\Theta)\vec{r}_{\mathrm{IJK}}}$$

$$\boxed{\vec{r}_{\mathrm{XYZ}}^{\,\mathrm{Station}} = R_{\mathrm{E}}(\cos\varphi\cos\lambda \quad \cos\varphi\sin\lambda \quad \sin\varphi)^t}$$

3D Relative Motion w.r.t. Ground Station (1)

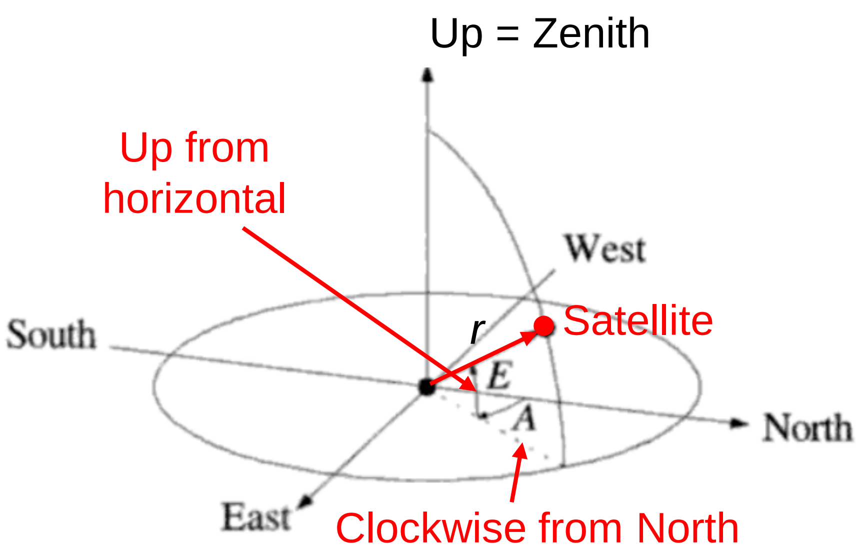
Topocentric triad for geodetic coordinates (λ, φ, h)

$$\vec{E}_{\mathrm{XYZ}} = \begin{pmatrix} -\sin\lambda \\ +\cos\lambda \\ 0 \end{pmatrix} ; \; \vec{N}_{\mathrm{XYZ}} = \begin{pmatrix} -\sin\varphi\cos\lambda \\ -\sin\varphi\sin\lambda \\ \cos\varphi \end{pmatrix} ; \; \vec{U}_{\mathrm{XYZ}} = \begin{pmatrix} \cos\varphi\cos\lambda \\ \cos\varphi\sin\lambda \\ \sin\varphi \end{pmatrix}$$



From Cartesian to spherical coordinates

$$A = \arctan(r_{\mathrm{E}}/r_{\mathrm{N}}) \; ; \; E = \arctan\left(r_{\mathrm{U}}\Big/\sqrt{r_{\mathrm{E}}^2 + r_{\mathrm{N}}^2}\right)$$



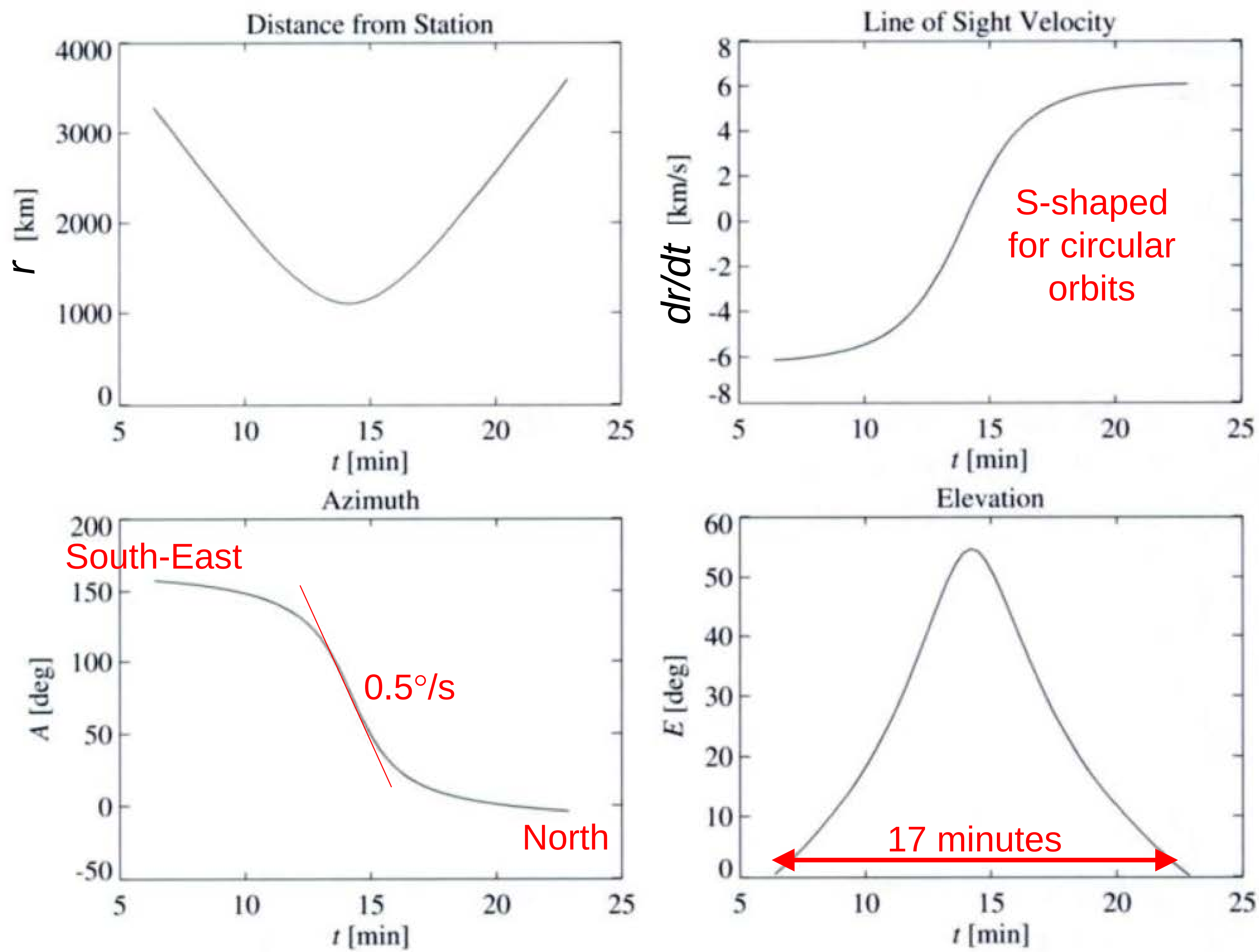
$$\vec{r}_{\mathrm{ENU}} = r\begin{pmatrix} \sin A\cos E \\ \cos A\cos E \\ \sin E \end{pmatrix}$$

From spherical to Cartesian coordinates

$$\vec{v}_{\mathrm{ENU}} = \dot{r}\begin{pmatrix} \sin A\cos E \\ \cos A\cos E \\ \sin E \end{pmatrix} + r\dot{E}\begin{pmatrix} -\sin A\sin E \\ -\cos A\sin E \\ \cos E \end{pmatrix} + r\dot{A}\begin{pmatrix} \cos A\cos E \\ -\sin A\cos E \\ 0 \end{pmatrix}$$

Velocity as time derivative of position
(taken in rotating frame!)

3D Relative Motion w.r.t. Ground Station (2)



h = 960 km
i = 97°
e = 0

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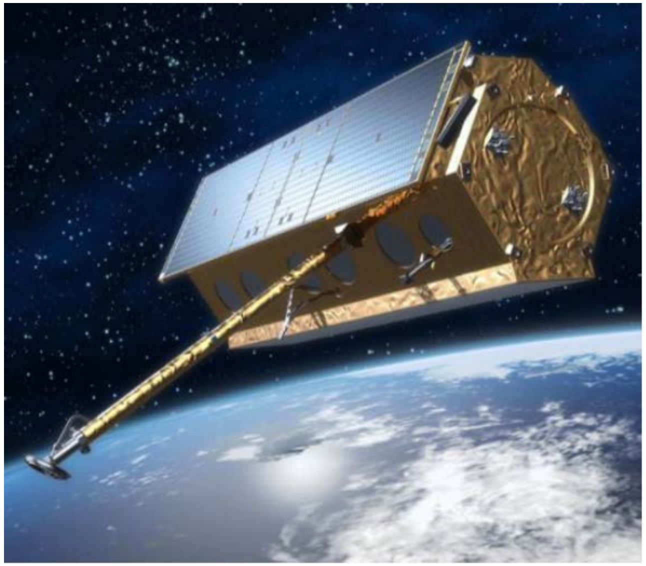
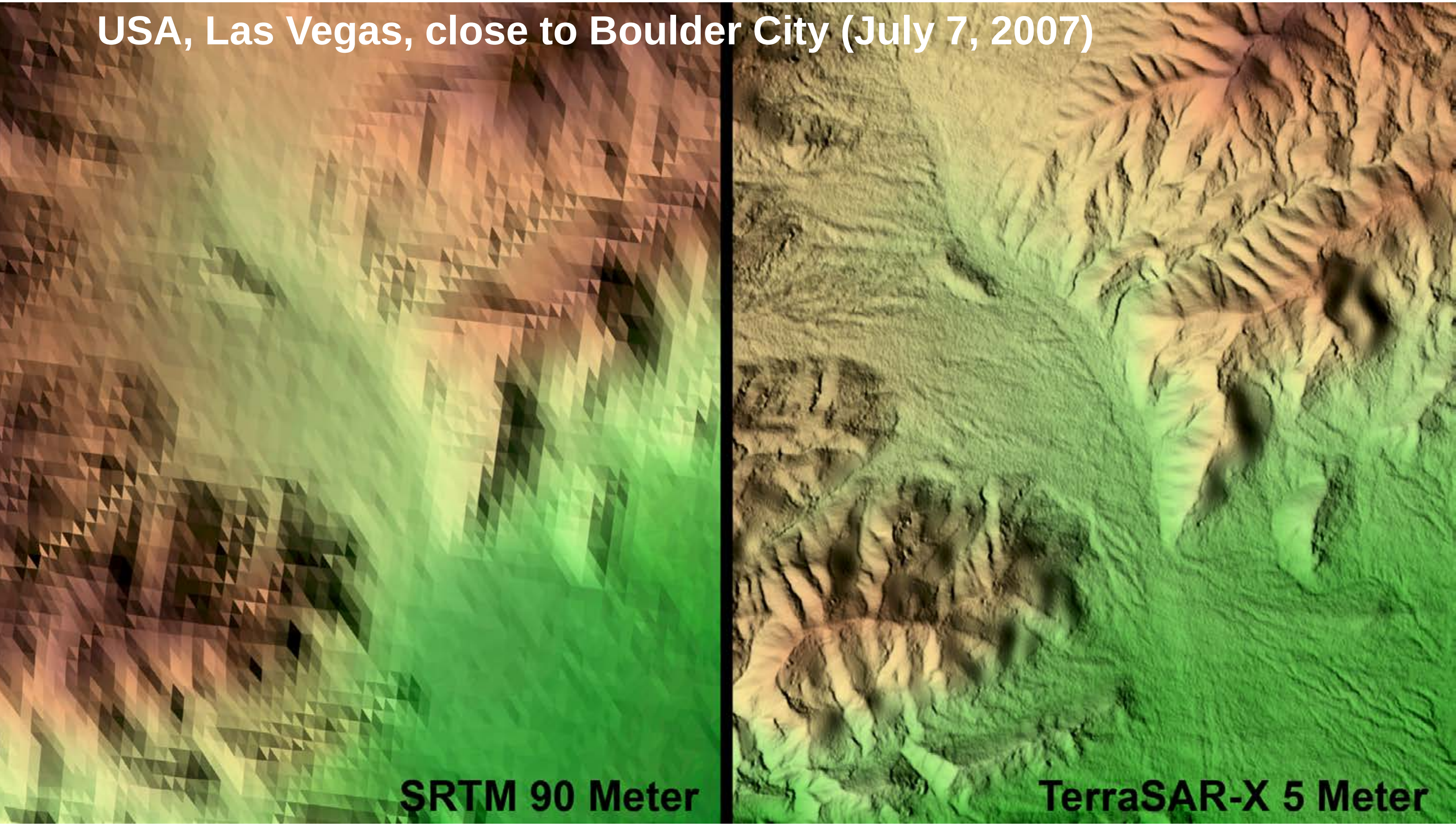
TerraSAR-X Mission Overview

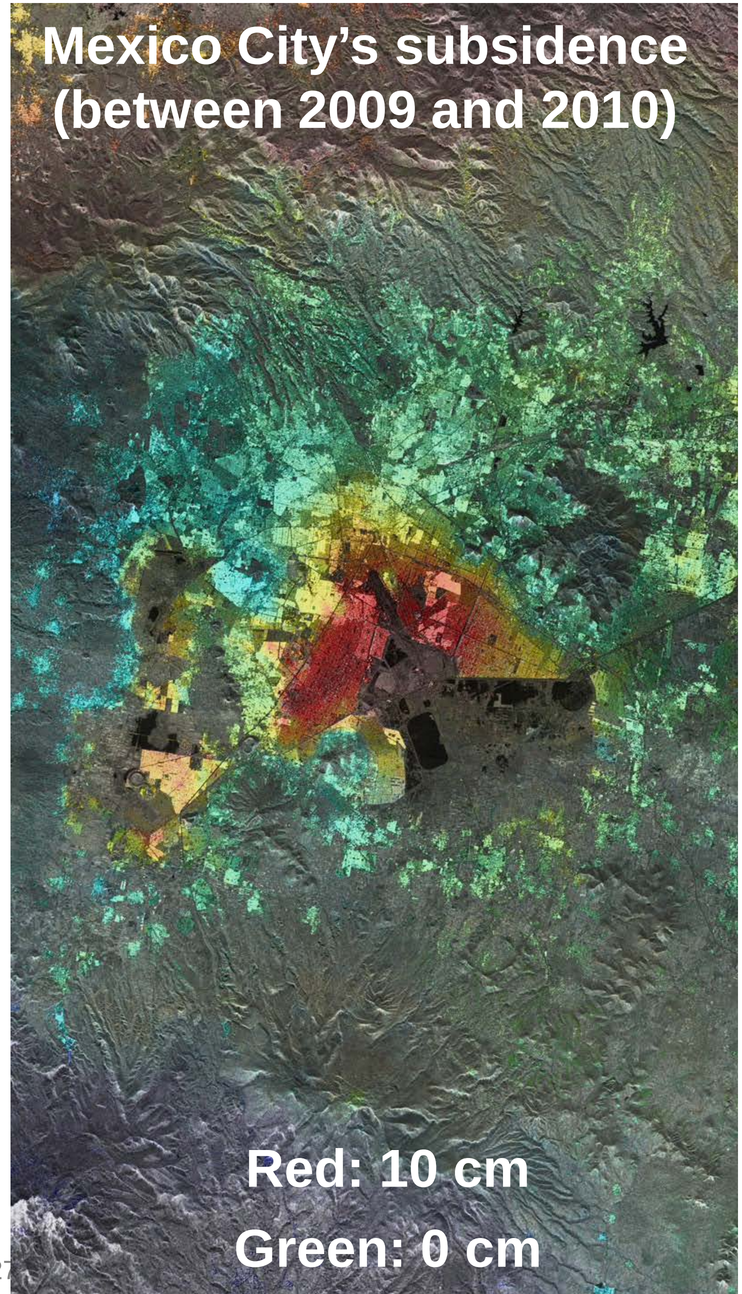
- Spacecraft
  - Size: 5 x 2.4 m
  - Mass: 1340 kg
  - Payload: X-band Synthetic Aperture Radar
- Launch
  - Dnepr-1 from Baikonour
  - June 15, 2007
- Operations
  - Weilheim S-band ground station, Germany
  - Mission lifetime: 5.5 years
- Primary mission goal
  - Repeat Pass Interferometry



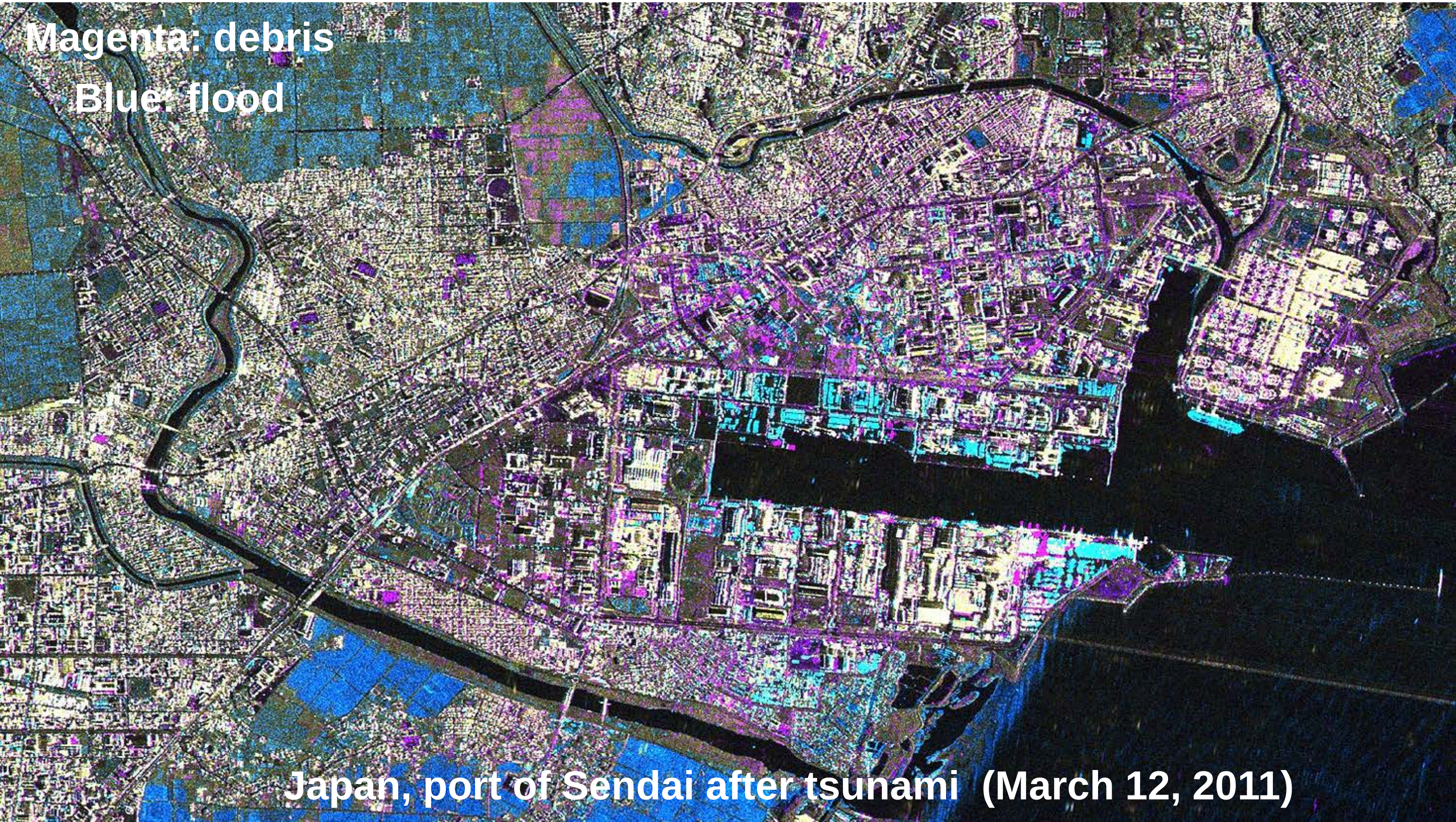
First TerraSAR-X Digital Elevation Model



Disaster Monitoring

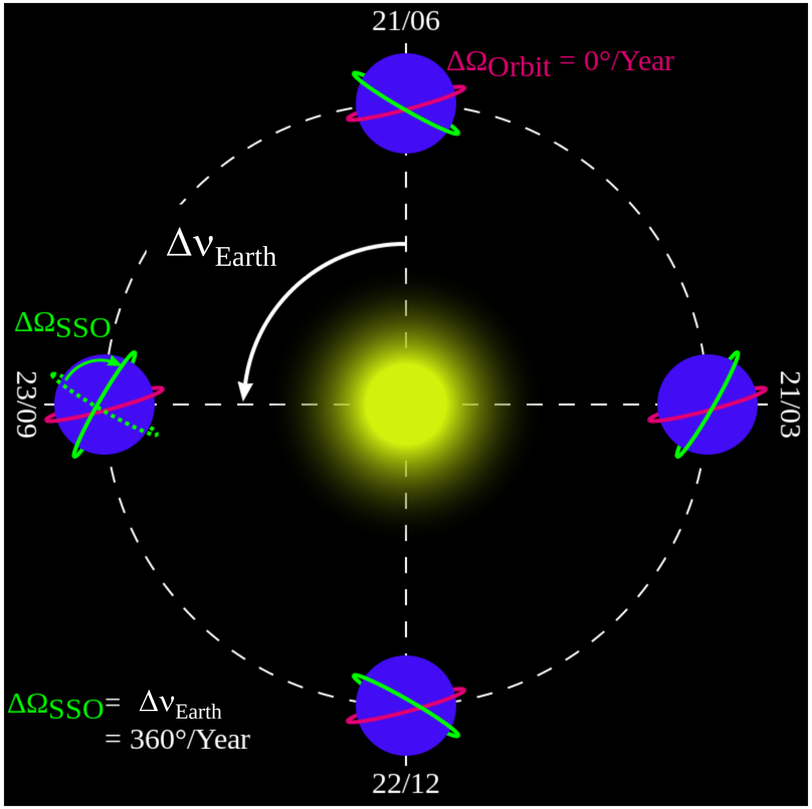


Disaster Monitoring



TerraSAR-X Key Orbit Control Requirements

- Reference orbit (target trajectory)
  - Sun-synchronous, frozen orbit at 510 km
  - Mean local time of ascending node: 18:00
  - Exact repetition after 11 days (167 orbits)
- Baseline (perpendicular to target in ECEF)
  - Nominal: 250 m
  - High accuracy: 30 m
- Allowed thruster pulse budget
  - 4x1N hydrazine thrusters qualified for 1145 cold pulses
  - 567 pulses are allocated for e.g. first acquisition, safety
  - Only **578 pulses** can be used for orbit keeping



Concept of Sun-Synchronous Orbit

Design Orbits: Commensurability

- Two non-zero real numbers T_1 and T_2 are commensurable if and only if their ratio is a rational number, or
- There exists a real number g, and integers *m* and *n*, such that $T_1 = mg$ and $T_2 = ng$

Definition of Commensurability

$$\frac{T_1}{T_2} = \frac{m}{n}$$

Integer numbers

Repeat orbits

$$\frac{2\pi}{1\ [\text{sidereal day}]}\sqrt{\frac{a^3}{\mu}} = \frac{m}{n}$$

Periodic relative motion (formation-flying)

$$\left(\frac{a_1}{a_2}\right)^{3/2} = \left(\frac{\mathcal{E}_2}{\mathcal{E}_1}\right)^{3/2} = \frac{m}{n}$$

- The sum or difference between commensurable variables is periodic
- *Example 1*: T_1 and T_2 are periods of Earth rotation w.r.t. inertial space and satellite orbit period
- *Example 2*: T_1 and T_2 are periods of two satellite orbit periods

Backup

Reference ellipsoid name	Equatorial radius (m)	Polar radius (m)	Inverse flattening	Where used
Maupertuis (1738)	6,397,300	6,363,806.283	191	France
Plessis (1817)	6,376,523.0	6,355,862.9333	308.64	France
Everest (1830)	6,377,299.365	6,356,098.359	300.80172554	India
Everest 1830 Modified (1967)	6,377,304.063	6,356,103.0390	300.8017	West Malaysia & Singapore
Everest 1830 (1967 Definition)	6,377,298.556	6,356,097.550	300.8017	Brunei & East Malaysia
Airy (1830)	6,377,563.396	6,356,256.909	299.3249646	Britain
Bessel (1841)	6,377,397.155	6,356,078.963	299.1528128	Europe, Japan
Clarke (1866)	6,378,206.4	6,356,583.8	294.9786982	North America
Clarke (1878)	6,378,190	6,356,456	293.4659980	North America
Clarke (1880)	6,378,249.145	6,356,514.870	293.465	France, Africa
Helmert (1906)	6,378,200	6,356,818.17	298.3	
Hayford (1910)	6,378,388	6,356,911.946	297	USA
International (1924)	6,378,388	6,356,911.946	297	Europe
Krassovsky (1940)	6,378,245	6,356,863.019	298.3	USSR, Russia, Romania
WGS66 (1966)	6,378,145	6,356,759.769	298.25	USA/DoD
Australian National (1966)	6,378,160	6,356,774.719	298.25	Australia
New International (1967)	6,378,157.5	6,356,772.2	298.24961539	
GRS-67 (1967)	6,378,160	6,356,774.516	298.247167427	
South American (1969)	6,378,160	6,356,774.719	298.25	South America
WGS-72 (1972)	6,378,135	6,356,750.52	298.26	USA/DoD
GRS-80 (1979)	6,378,137	6,356,752.3141	298.257222101	Global ITRS[3]
WGS-84 (1984)	6,378,137	6,356,752.3142	298.257223563	Global GPS
IERS (1989)	6,378,136	6,356,751.302	298.257	
IERS (2003)[4]	6,378,136.6	6,356,751.9	298.25642	[3]