AA 279A - Space Mechanics
Winter 2017

Lecture 10 Notes

Monday, 13 February 2017
Prof. Andrew Barrows

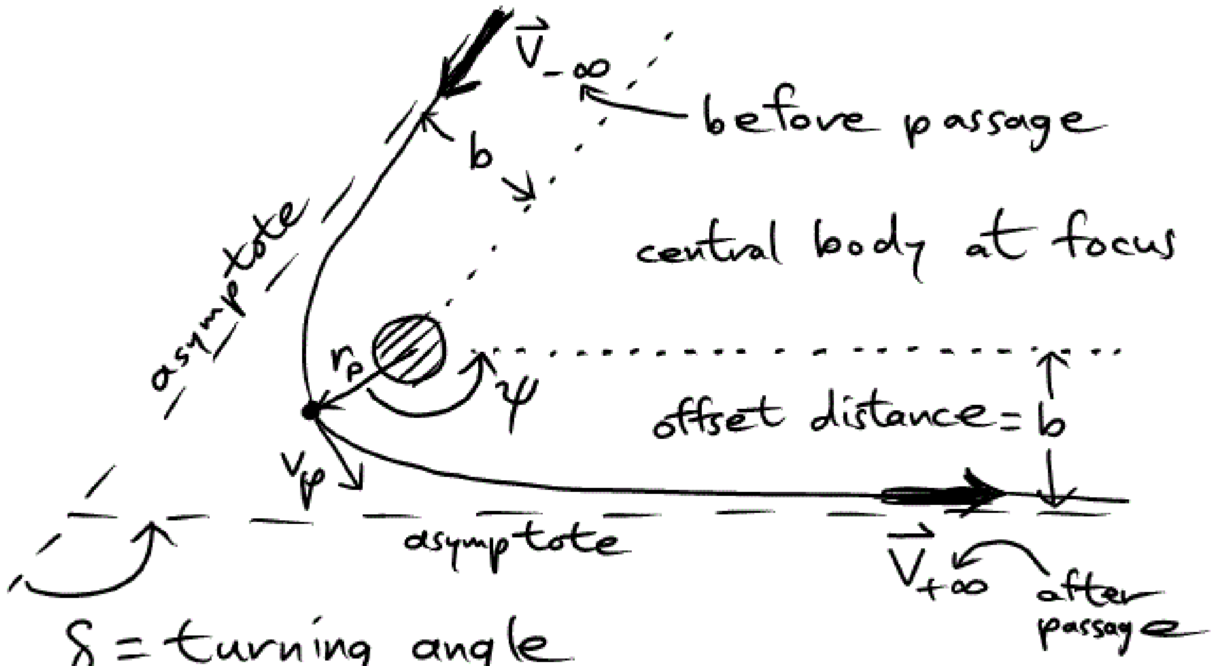
Reading for Lecture 11 (numerical integration)

BMW 9.5, 9.6
Montenbruck 4.1 - 4.1.3
Vallado (4th ed.) 8.4, 8.5

Topics for today

Interplanetary transfers
Method of patched conics
Hyperbolic departures and arrivals
Planetary flybys
Sphere of influence

HYPERBOLAS: A REFRESHER



δ = turning angle

$$V_\infty^2 = (\text{hyperbolic excess speed})^2 = V^2 - V_{esc}^2$$

$$\sin\frac{\delta}{2} = \frac{1}{e} \Rightarrow \delta = 2\sin^{-1}\left(\frac{1}{e}\right) \; ; \; \psi = \cos^{-1}\left(-\frac{1}{e}\right)$$

$$\mathcal{E} = \frac{V_\infty^2}{2} = \frac{V_p^2}{2} - \frac{\mu}{r_p} \qquad (>0)$$

$$h = \text{constant} = |\vec{r}\times\vec{v}|_{\text{at }\infty} = b\,V_\infty = r_p\,V_p$$

$$e = \sqrt{1 + \frac{2\mathcal{E}h^2}{\mu^2}} = \sqrt{1 + \frac{b^2 V_\infty^4}{\mu^2}} = 1 + \frac{r_p V_\infty^2}{\mu}$$

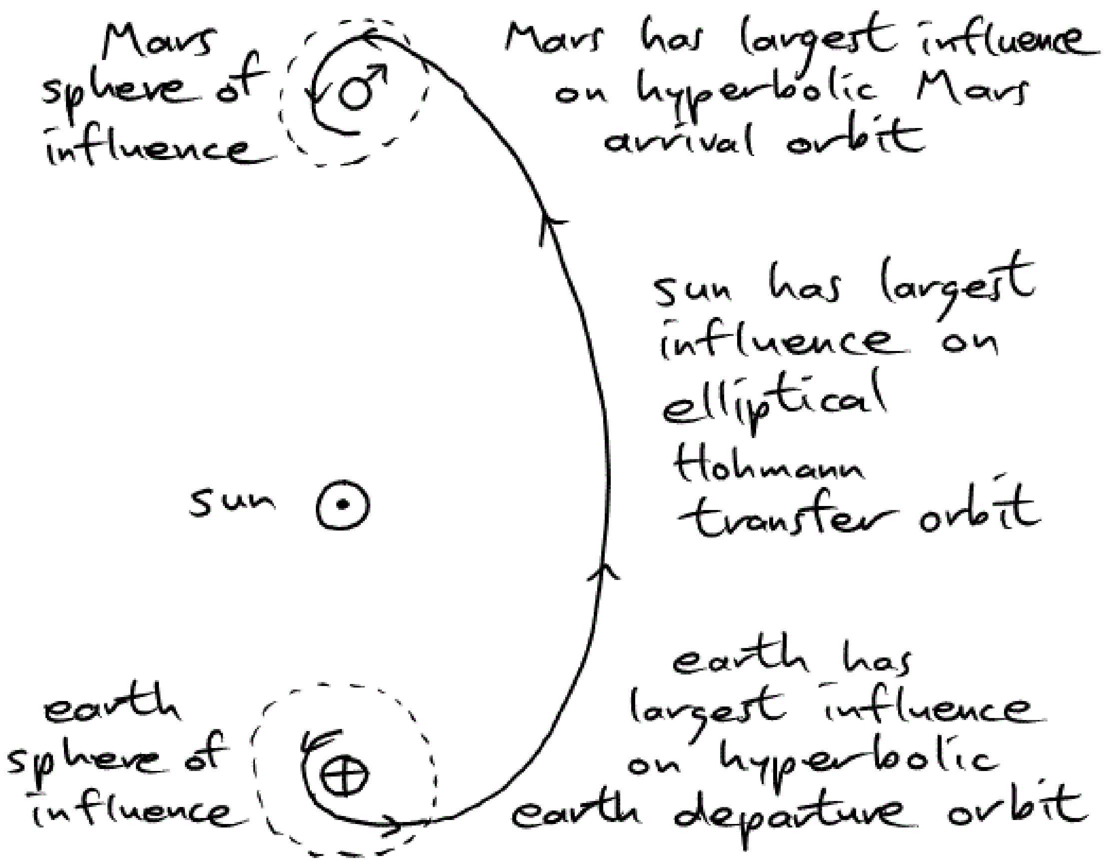
$$b = \frac{\mu}{V_\infty^2}\sqrt{e^2-1} = \text{'offset distance'} \quad \left(\text{BMW calls this 'y'}\right)$$

METHOD OF PATCHED CONICS

... Therefore, we'll approximate with "Patched Conics," i.e. we'll patch together conic section orbits about bodies of largest gravitational influence at a given time. This turns the N-body problem into a series of simpler two-body problems.

EXAMPLE (will break down step-by-step)

Transfer from LEO, to heliocentric Hohmann transfer orbit, to orbit around Mars



STEPS FOR PATCHED CONIC ANALYSIS

1) Determine heliocentric transfer orbit, ignoring gravity due to planets. (i.e. 'turn off' their gravity)

2) Determine planetocentric (typically geocentric) departure orbit, ignoring gravity due to sun and other planets.

3) Determine planetocentric arrival orbit, ignoring gravity due to sun and other planets. And then either:

   a) Inject into orbit about target planet,

   OR

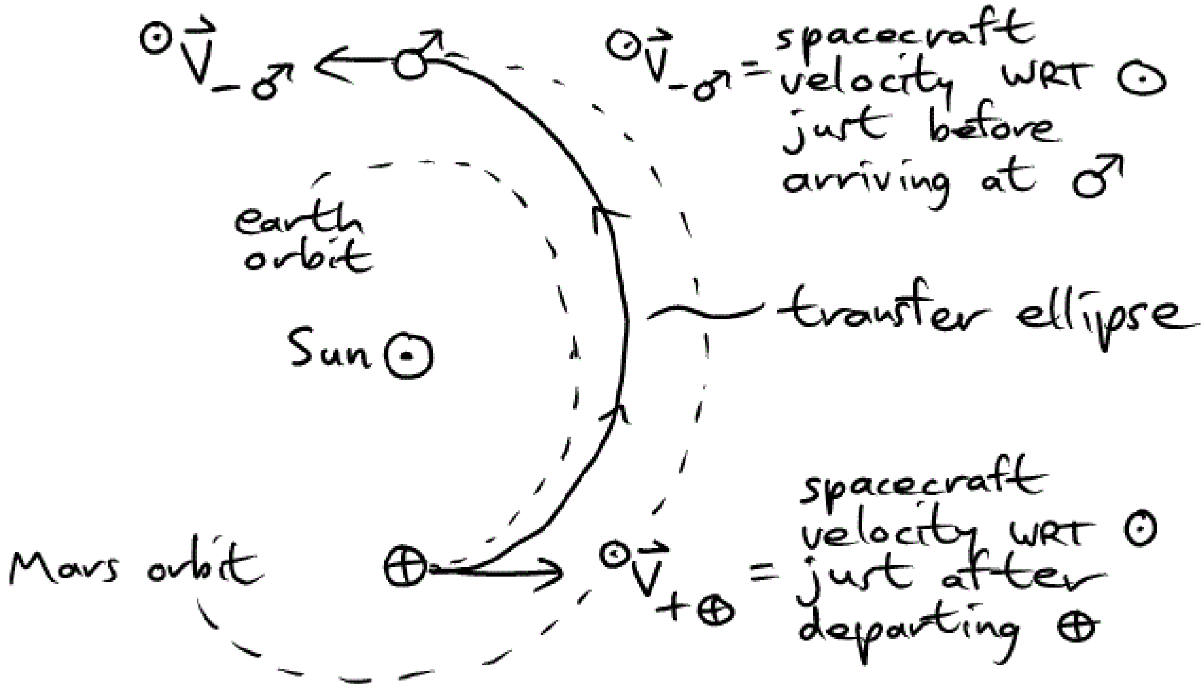
   b) Fly by target planet to change velocity vector.

Method of patched conics works well for preliminary mission design. For detailed planning, greater accuracy, and cases where patched conics don't work well (e.g. lunar return orbits), turn to numerical simulation.

HELIOCENTRIC TRANSFER ORBIT

Can use Hohmann transfer relations developed previously.

EXAMPLE Hohmann transfer from earth orbit to Mars orbit



$$a_{transfer} = \frac{^{\odot}r_{⊕} + {}^{\odot}r_{♂}}{2} \qquad \mathcal{E}_t = -\frac{\mu_{\odot}}{2a_t} = \frac{^{\odot}v^2}{2} - \frac{\mu_{\odot}}{^{\odot}r}$$

$$^{\odot}V_{+⊕} = \sqrt{2\mu_{\odot}\left(\frac{1}{^{\odot}r_{⊕}} - \frac{1}{^{\odot}r_{⊕} + {}^{\odot}r_{♂}}\right)}$$

$$= \sqrt{2\left(1\,\frac{AU^3}{TU_{\odot}^2}\right)\left(\frac{1}{1\,AU} - \frac{1}{1\,AU + 1.52\,AU}\right)}$$

$$^{\odot}V_{+⊕} = 1.098\left[\frac{AU}{TU_{\odot}}\right] = 32.71\left[\frac{km}{sec}\right]$$

$$
{}^{\odot}V_{-\text{♂}} = \sqrt{2\mu_{\odot}\left(\frac{1}{r_{\text{♂}}} - \frac{1}{r_{\oplus} + r_{\text{♂}}}\right)}
$$

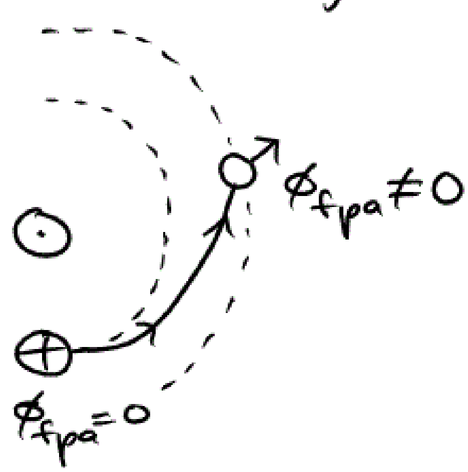
$$
= \sqrt{2(1)\left(\frac{1}{1.52} - \frac{1}{1+1.52}\right)}
$$

$$
{}^{\odot}V_{-\text{♂}} = 0.723 \left[\frac{AU}{TU_{\odot}}\right] = 21.52 \left[\frac{km}{sec}\right]
$$

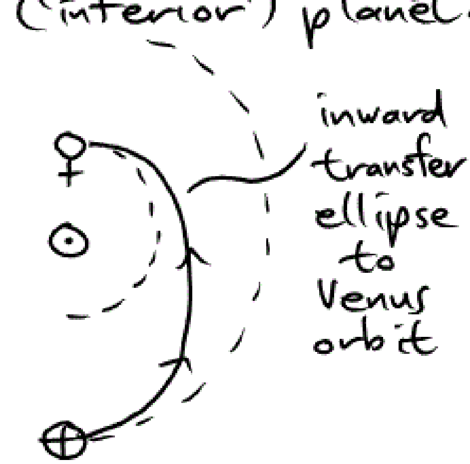
Notes:

① We assumed that timing/phasing of earth and Mars within their orbits was such that Mars is in the right place when spacecraft gets to Mars orbit. (Requires 'synodic period' and 'phasing angle' from next lecture.)

② Not all transfers are Hohmann transfers. Can use one-tangent or no-tangent burns:

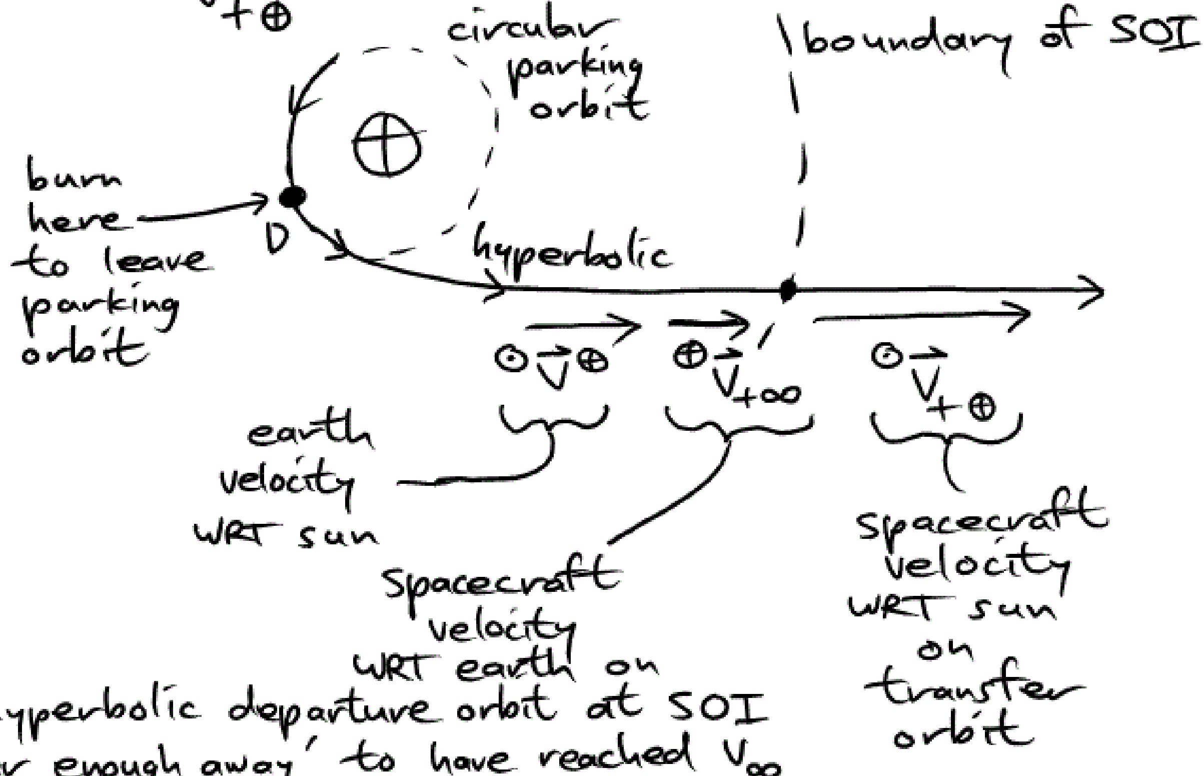


③ Mars is farther from sun than earth. Can also go to inner ('inferior') planet:



EARTH DEPARTURE

Now that heliocentric transfer orbit (and ${}^{\odot}\vec{r}$ and ${}^{\odot}\vec{v}$ at its endpoints) are defined, need to find size and location of a ΔV to allow departure from earth's 'sphere of influence' (SOI) with ${}^{\odot}\vec{V}_{+\oplus}$:



Matching velocities on either side of the boundary gives the 'patch conditions':

$${}^{\odot}\vec{r}_{+\oplus} = {}^{\odot}\vec{r}^{\oplus} + {}^{\oplus}\vec{r}_{+\infty} \quad \text{(we won't use this one)}$$

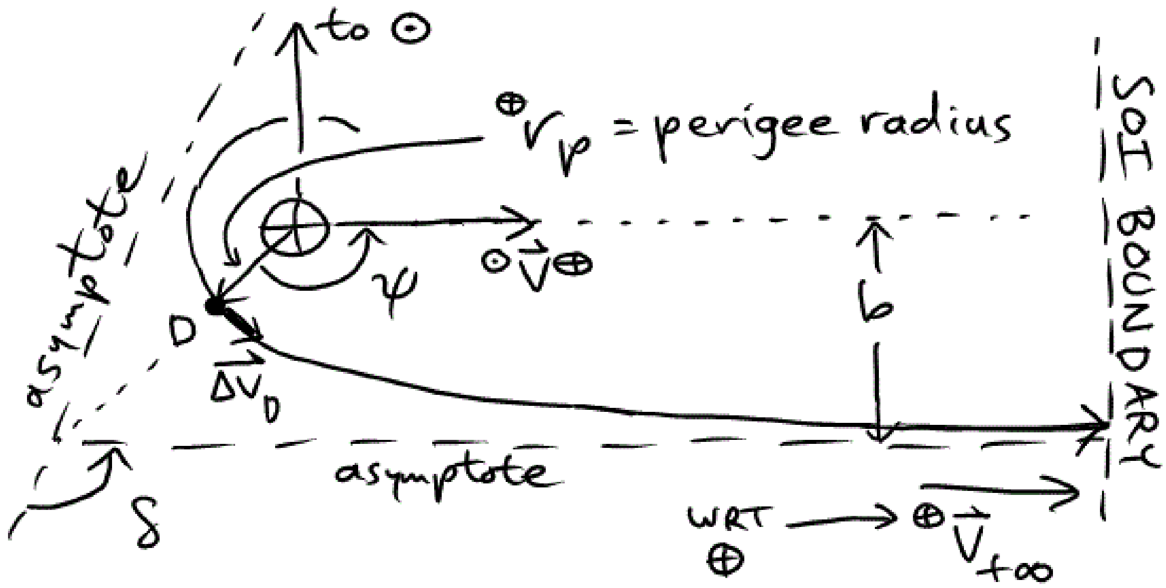
$${}^{\odot}\vec{V}_{+\oplus} = {}^{\odot}\vec{V}^{\oplus} + {}^{\oplus}\vec{V}_{+\infty} \quad \left(+ \underbrace{\cancel{{}^{\odot}\vec{\omega}^{\oplus}}}_{=0} \times {}^{\oplus}\vec{r}_{+\infty} \right)$$

These give ← from heliocentric xfer

$${}^{\oplus}\vec{V}_{+\infty} = {}^{\odot}\vec{V}_{+\oplus} - {}^{\odot}\vec{V}^{\oplus}$$ ← just $V_{circular}$ for $\oplus$

In order to produce the ${}^{\oplus}\vec{V}_{+\infty}$ that allows us to depart earth's SOI (a ΔV of sorts), we'll use an impulsive thrust at D. (This will be the actual ΔV_D that expends propellant.)

EARTH-RELATIVE HYPERBOLA FOR DEPARTURE TO SUPERIOR (OUTER) PLANET



Our job is:

Given: ${}^{\oplus}\vec{V}_{+\infty}$ and ${}^{\oplus}r_p$

Find: location (ψ) and size (ΔV_D) of rocket burn at D, and also b

Note: b is a <u>result</u> we'll simply live with. (It is small compared to heliocentric transfer orbit.)

Using hyperbola relations:

$${}^{\oplus}V_{+D} = \sqrt{{}^{\oplus}V_{+\infty}^{2} + \frac{2\mu_{\oplus}}{{}^{\oplus}r_p}} = \text{velocity (on hyperbola) just after burn at D}$$

$${}^{\oplus}V_{-D} = \sqrt{\frac{\mu_{\oplus}}{{}^{\oplus}r_p}} = \text{velocity just before burn at D}$$

$$\Delta V_D = \left| {}^{\oplus}\vec{V}_{+D} - {}^{\oplus}\vec{V}_{-D} \right| = \left| {}^{\oplus}V_{+D} - {}^{\oplus}V_{-D} \right| \quad \left(\text{since they're parallel and point in same direction}\right)$$

$$e = 1 + \frac{{}^{\oplus}r_p \, {}^{\oplus}V_{+\infty}^{2}}{\mu_{\oplus}}$$

$$\psi = \cos^{-1}\left(-\frac{1}{e}\right)$$

$$b = \frac{\mu_{\oplus}}{{}^{\oplus}V_{+\infty}^{2}} \sqrt{e^2 - 1}$$

EXAMPLE Departure from 200km LEO parking orbit to Hohman transfer to Mars orbit

$${}^{\oplus}V_{+\infty} = {}^{\odot}V_{+\oplus} - {}^{\odot}V^{\oplus} \quad \text{(all parallel)}$$

${}^{\odot}V^{\oplus}$ ← simply V_{circ} of earth

$$= 32.71 - 29.79 = 2.92 \left[\frac{km}{sec}\right]$$

$${}^{\oplus}V_{+D} = \sqrt{{}^{\oplus}V_{+\infty}^{2} + \frac{2\mu_{\oplus}}{{}^{\oplus}r_p}} = \sqrt{(2.92)^2 + \frac{2(398600.4418)}{6378 + 200}}$$

$${}^{\oplus}V_{+D} = 11.39 \left[\frac{km}{sec}\right]$$

familiar

$${}^{\oplus}V_{-D} = \sqrt{\frac{\mu_\oplus}{{}^{\oplus}r_p}} = \sqrt{\frac{398600.4418}{6378+200}} = 7.78 \left[\frac{km}{sec}\right]$$

$$\Delta V_D = \left| {}^{\oplus}V_{+D} - {}^{\oplus}V_{-D} \right| = \left| 11.39 - 7.78 \right| = 3.61 \left[\frac{km}{sec}\right]$$

$$\Delta V_D = 3.61 \left[\frac{km}{sec}\right]$$

$$e = 1 + \frac{{}^{\oplus}r_p \, {}^{\oplus}V_{+\infty}^2}{\mu_\oplus} = 1 + \frac{(6378+200)(2.92)^2}{398600.4418} = 1.1407$$

$$\psi = \cos^{-1}\left(-\frac{1}{e}\right) = \cos^{-1}\left(-\frac{1}{1.1407}\right) = 151.2^\circ$$

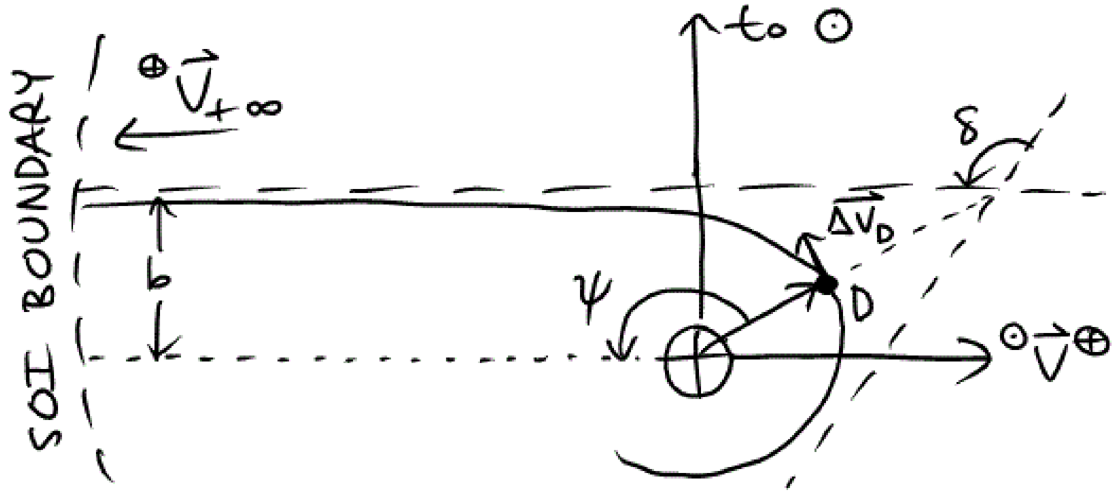
$$\psi = 151.2^\circ$$

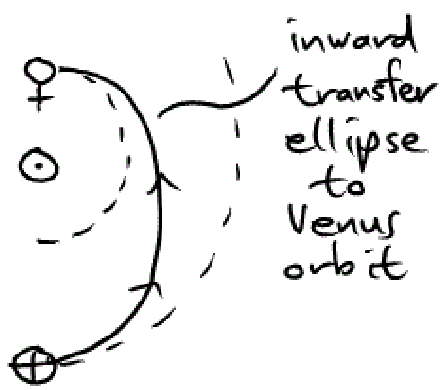
$$b = \frac{\mu_\oplus}{{}^{\oplus}V_{+\infty}^2}\sqrt{e^2-1} = \frac{398600.4418}{(2.92)^2}\sqrt{1.1407^2-1} = 25,656 \,[km]$$

$$b = 25,656 \,[km]$$

If we are going to Venus or Mercury, we need to <u>lose</u> velocity WRT sun so that spacecraft 'falls inward' toward orbit of inferior planet. Same hyperbola relations still hold.

EARTH-RELATIVE HYPERBOLA FOR DEPARTURE TO INFERIOR (INNER) PLANET





Patch condition becomes

$${}^{\odot}\vec{V}_{+\oplus} = {}^{\odot}\vec{V}^{\oplus} + {}^{\oplus}\vec{V}_{+\infty}$$

$$\longrightarrow = \longrightarrow + \longleftarrow$$

Burn at D sends departing spacecraft 'backwards' WRT earth for a slower velocity WRT sun.

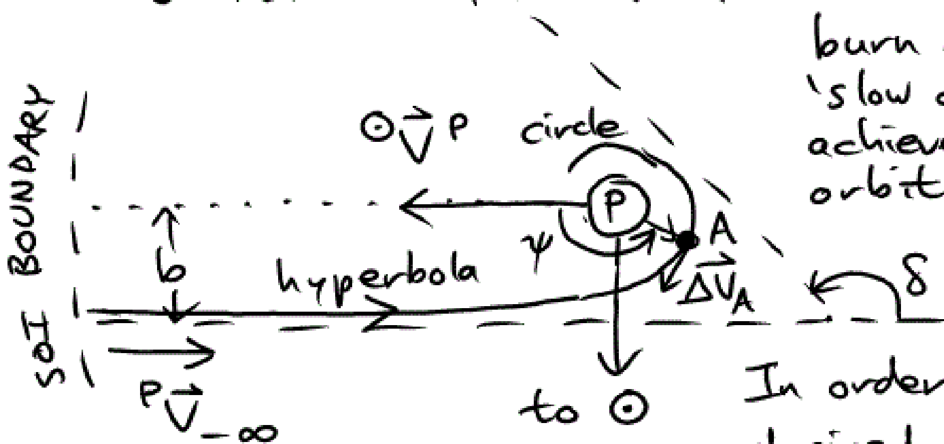
In general, patch condition is a <u>vector</u> relation as we'll soon see.

PLANETARY ARRIVAL WITH ORBIT INJECTION

Ok, we've departed earth and transfered to another planet P. What do we do now? For this case, assume we want to enter circular orbit of radius ${}^{P}r_p$. Same properties of hyperbolas apply here.

For arrival at superior planet from Hohmann transfer, planet will 'catch up' to slower spacecraft from behind:

PLANET-RELATIVE HYPERBOLA FOR ARRIVAL AT SUPERIOR PLANET P



burn at A to 'slow down' and achieve circular orbit with ${}^{P}r_p$

In order to achieve desired ${}^{P}r_p$, we'll need to compute b and make mid-course correction(s) to achieve it.

Patch condition is

$${}^{\odot}\vec{V}_{-P} = {}^{\odot}\vec{V}^{P} + {}^{P}\vec{V}_{-\infty}$$

$\longleftarrow$ from Hohmann; $\longleftarrow$ known; $+ \longrightarrow$ solve for ${}^{P}\vec{V}_{-\infty} = {}^{\odot}\vec{V}_{-P} - {}^{\odot}\vec{V}^{P}$

Our job is:

Given: ${}^{P}\vec{V}_{-\infty}$ and ${}^{P}r_p$

Find: location (ψ) and size (ΔV_A) of rocket burn at A, and b

Note: b is a factor we need to 'build in' to our final heliocentric trajectory using mid-course correction(s) in order to achieve desired periapsis radius

EXAMPLE

Arrival at Mars and injection into circular orbit with 600 km altitude

$${}^{♂}V_{-\infty} = {}^{\odot}V_{-♂} - {}^{\odot}V^{♂}$$ ← simply V_{circ} for Mars (all parallel)

$$= 21.52 - 24.13 = -2.61 \left[\frac{km}{sec}\right]$$

− sign indicates vector direction. Knowing this, we'll use ${}^{♂}V_{-\infty} = 2.61 \left[\frac{km}{sec}\right]$ so we can use hyperbola relations

$${}^{♂}V_{-A} = \sqrt{{}^{♂}V_{-\infty}^2 + \frac{2\mu_{♂}}{{}^{♂}r_p}} = \sqrt{(2.61)^2 + \frac{2(43050)}{3397+600}}$$

$${}^{♂}V_{-A} = 5.32 \left[\frac{km}{sec}\right]$$

$$^{\male}V_{+A} = \sqrt{\frac{\mu_{\male}}{^{\male}r_p}} = \sqrt{\frac{43050}{3397+600}} = 3.28 \left[\frac{km}{sec}\right]$$

$$\Delta V_A = \left| ^{\male}V_{+A} - ^{\male}V_{-A} \right| = \left| 3.28 - 5.32 \right| = 2.04 \left[\frac{km}{sec}\right]$$

$$\Delta V_A = 2.04 \left[\frac{km}{sec}\right]$$

$$e = 1 + \frac{^{\male}r_p \, ^{\male}V_{-\infty}^2}{\mu_{\male}} = 1 + \frac{(3397+600)(2.61)^2}{43050} = 1.6325$$

$$\psi = \cos^{-1}\left(-\frac{1}{e}\right) = \cos^{-1}\left(-\frac{1}{1.6325}\right) = 127.8^\circ$$

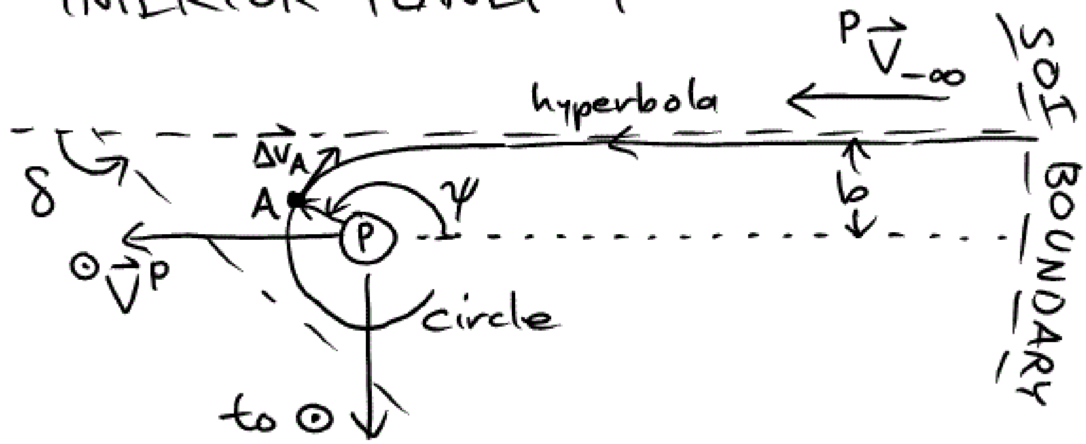
$$\psi = 127.8^\circ$$

$$b = \frac{\mu_{\male}}{^{\male}V_{-\infty}^2} \sqrt{e^2 - 1} = \frac{43050}{(2.61)^2} \sqrt{1.6325^2 - 1} = 8{,}155 \; [km]$$

$$b = 8{,}155 \; [km]$$

If we're instead going to an inferior planet (Venus or Mercury) via heliocentric Hohmann transfer, spacecraft will be 'catching up' to slower planet from behind:

PLANET-RELATIVE HYPERBOLA FOR ARRIVAL AT INFERIOR PLANET P



Patch condition is

$$
{}^{\odot}\vec{V}_{-P} = {}^{\odot}\vec{V}^{P} + {}^{P}\vec{V}_{-\infty}
$$

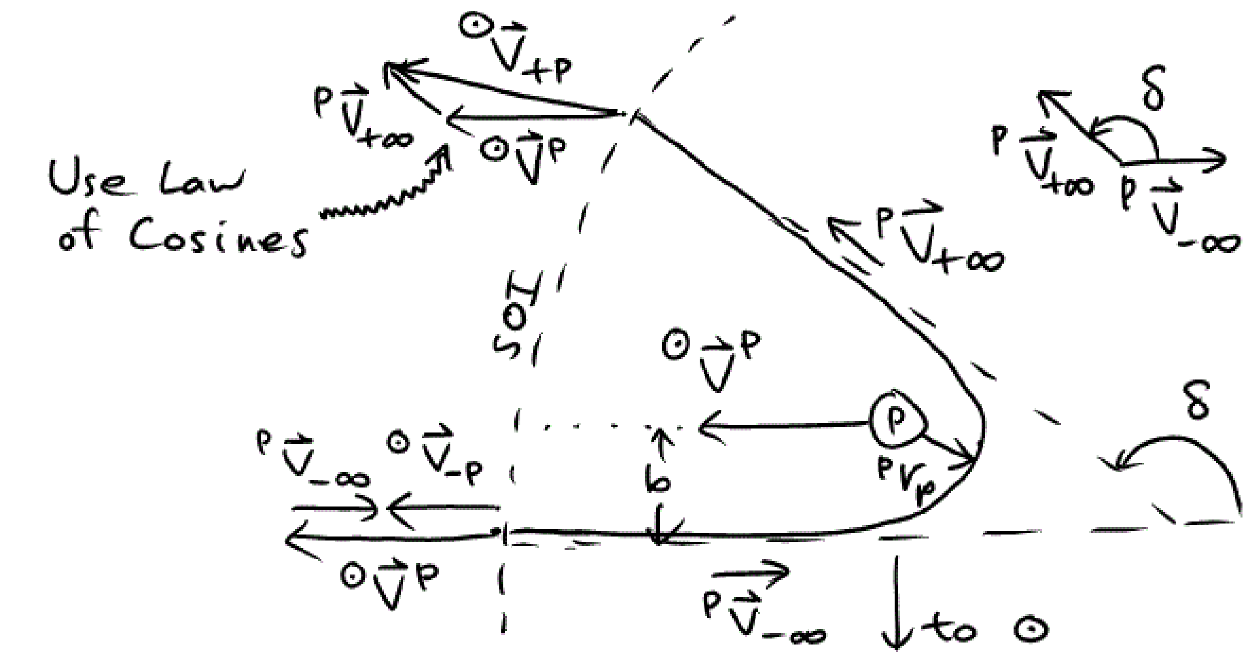
$$
\longleftarrow \qquad \longleftarrow \qquad \leftarrow
$$

Same hyperbola relations still hold.

PLANETARY ARRIVAL WITH FLYBY

Now consider cases where we don't want to inject into planetary orbit at A, but instead 'fly by' the planet by following the second half of the hyperbolic arrival trajectory, thus departing the planet's SOI.

TRAILING SIDE FLYBY



During flyby, heliocentric velocity changed from $^{\odot}\vec{V}_{-p} = \leftarrow$ to $^{\odot}\vec{V}_{+p} =$ ↖

Spacecraft received a large "ΔV" from the planet with no rocket burn!

This is called a "gravity assist" and can be used to save propellant and enable many interplanetary missions that would otherwise use too much propellant.

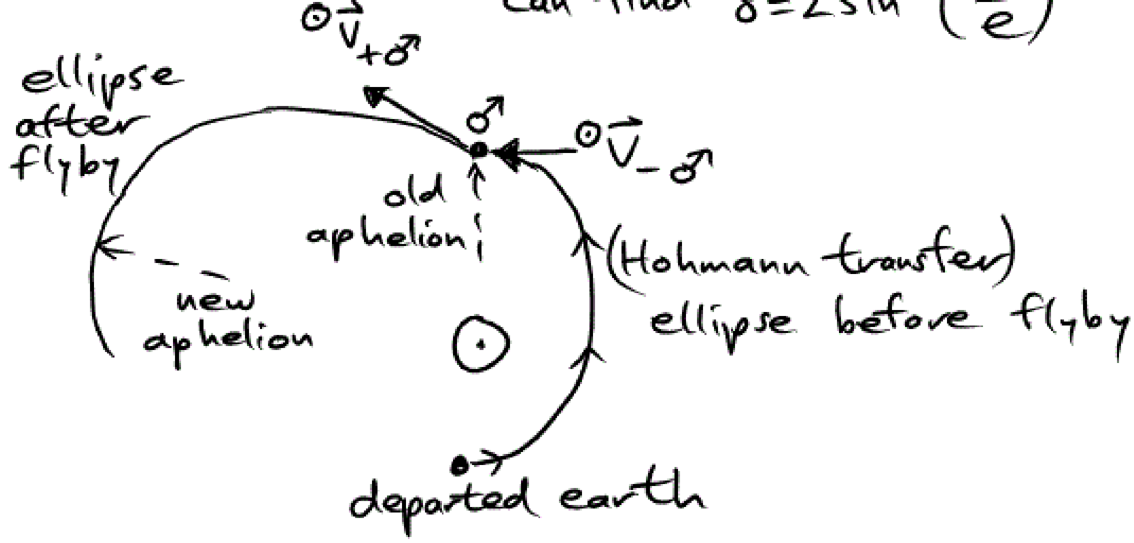
Question: Where does this energy added to the spacecraft come from?

To execute flyby we can choose b and ${}^{P}V_{-\infty}$ to achieve a desired ${}^{P}r_p$, δ, ${}^{P}V_{+\infty}$ or combination thereof.

Note that a flyby changes the heliocentric orbital elements of the spacecraft:

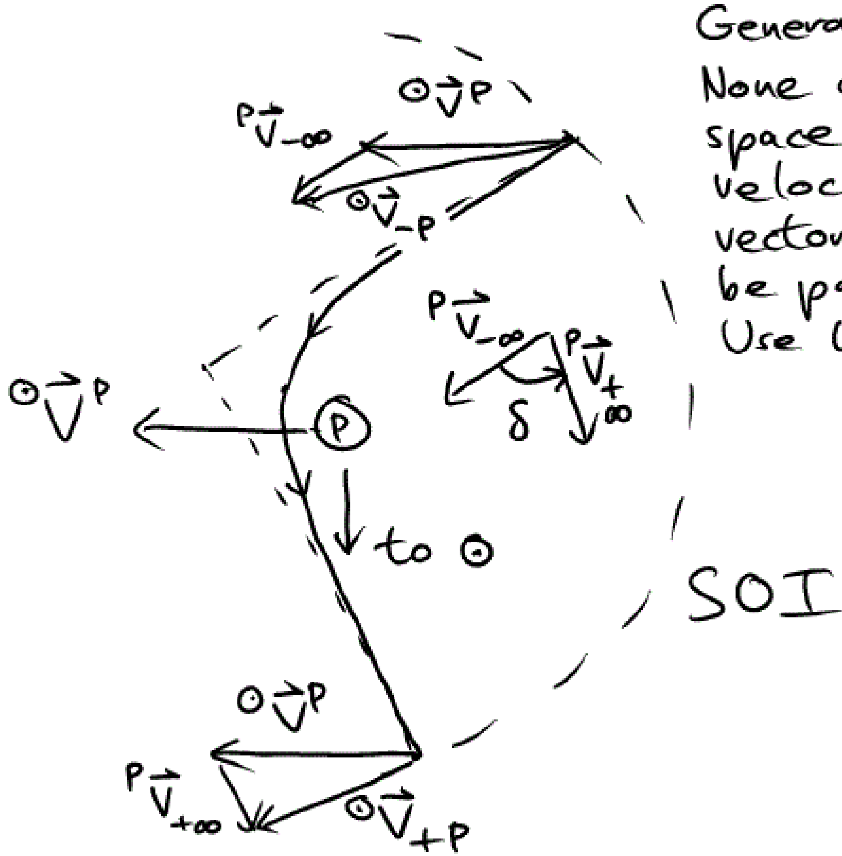
EXAMPLE Counter-clockwise trailing side flyby of Mars

can find $\delta = 2\sin^{-1}\left(\frac{1}{e}\right)$



Whereas a trailing side flyby increases velocity, a leading side flyby can be used to decrease velocity (as might be needed in mission to inferior planet).

LEADING SIDE FLYBY



General Case: None of these spacecraft velocity vectors need to be parallel. Use Law of Cosines

During flyby, heliocentric velocity changed from ${}^{\odot}\vec{V}_{-P} =$ to ${}^{\odot}\vec{V}_{+P} =$

Spacecraft received a "ΔV" from the planet that slowed it down.

You may also see the terms 'sunny side flyby' and 'dark side flyby'. Since there can be ambiguity about what arrival orbit is being described, always use as much detail (e.g. clockwise, counter-clockwise, trailing-, leading-, etc.) and a sketch if possible.

SPHERE OF INFLUENCE

Notice how we haven't needed or used an exact value for this? It can be used to improve accuracy at the patch points, but we're still stuck with the limitations of the two-body assumption. Typically we'll move to numerical simulation rather than using detailed model of SOI and patch points.

That said, one analysis due to Laplace for the SOI of planet P yields

$$\boxed{r_{\substack{SOI \\ \text{of } P}} = \begin{pmatrix}\text{distance} \\ \text{from } \odot \text{ to } P\end{pmatrix}\left(\frac{m_P}{m_\odot}\right)^{2/5}}$$

Within the SOI, planet P 'outcompetes' the sun.