AA 279 C – SPACECRAFT ADCS: LECTURE 14

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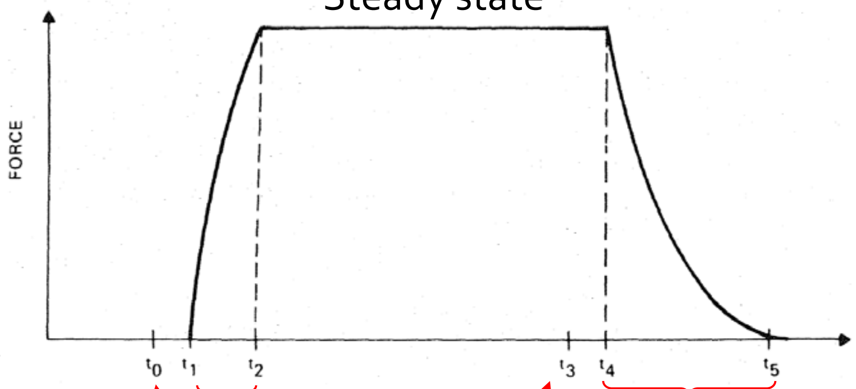
Thrusters hardware

- All jets or thrusters produce thrust by expelling propellant in the opposite direction, in particular collective acceleration of propellant molecules
- Resultant torques or forces are used to control
  - Attitude
  - Spin rate
  - Nutation
  - Speed of momentum wheels
  - Orbit (translational motion)
- The energy comes from
  - Chemical reaction (hot gas) → larger force, smaller specific impulse
  - Thermodynamic expansion (cold gas) → smaller force, larger specific impulse
  - Electric propulsion (ion) → low TRL, largest specific impulse
- Wertz provides hardware description (17.10), mathematical models (17.4), and typical control laws (19.3)

Thrusters profile (general case)

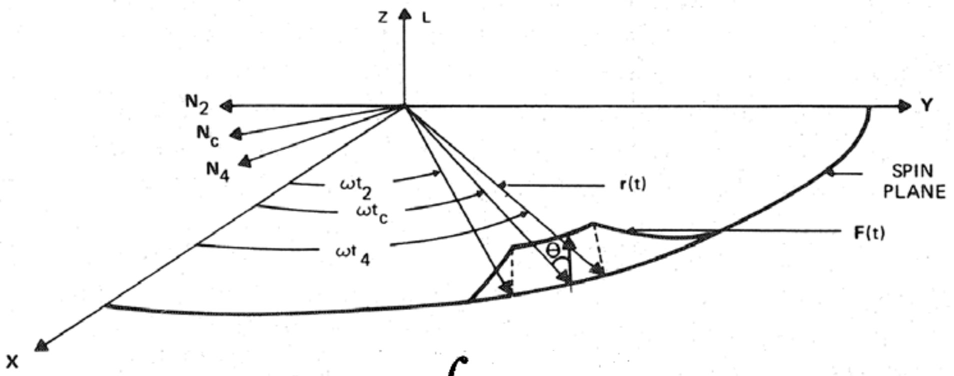


Rise time

Fall time

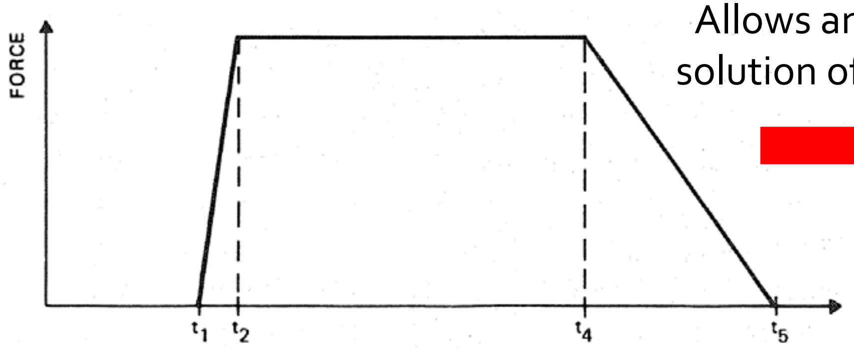
Commanded start time

Commanded stop time



$$I_c \equiv \int N_{\|}(t)\,\mathrm{d}t$$

Average torque over burn



Allows analytical solution of integral

$$\tan \omega t_c = b/a$$

$$\frac{\omega I_c}{r \sin\theta} = a \cos \omega t_c + b \sin \omega t_c$$

$$a = \frac{\cos \omega t_2 - \cos \omega t_1}{\omega(t_2 - t_1)} - \frac{\cos \omega t_5 - \cos \omega t_4}{\omega(t_5 - t_4)}$$

$$b = \frac{\sin \omega t_2 - \sin \omega t_1}{\omega(t_2 - t_1)} - \frac{\sin \omega t_5 - \sin \omega t_4}{\omega(t_5 - t_4)}$$

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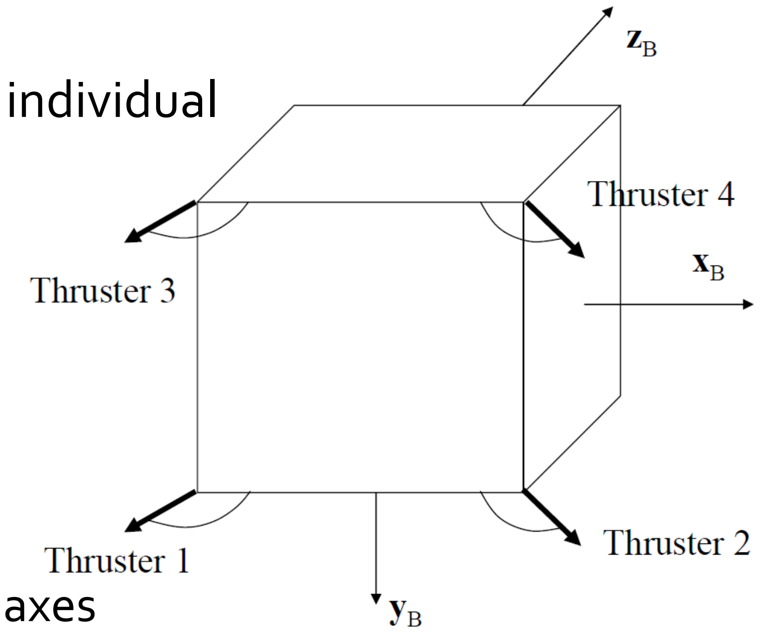
Thrusters distribution (1)



- The distribution of forces and torques from spacecraft axes to individual thrusters depends on the thruster configuration
  - Mounting position ***r***
  - Direction ***e***
- Often 4 thrusters are used in a tilted configuration as shown in the figure
  - Symmetrically mounted w.r.t. ***z*** (force direction)
  - Canted in the ***x-z*** plane
  - Each thruster produces a torque of equal magnitude around all axes
- Example: thruster 1 produces torque *a*>0 about ***x***, *b*>0 about ***y***, c<0 around ***z***
- In general, we make use of a torque matrix ***A*** to compute the control torque ***M*** from the thrust vector ***t***

$$\vec{A} = [[\vec{r}_1 \mathrm{x}]\vec{e}_1 \quad [\vec{r}_2 \mathrm{x}]\vec{e}_2 \quad [\vec{r}_3 \mathrm{x}]\vec{e}_3 \quad [\vec{r}_4 \mathrm{x}]\vec{e}_4] = \begin{bmatrix} a & a & -a & -a \\ b & -b & b & -b \\ -c & c & c & -c \end{bmatrix} \Rightarrow \vec{M} = \vec{A}\vec{t}$$

Thrusters distribution (2)

- Note that if all thrusters are actuated with the same value, or equivalently ***t*** = [1 1 1 1]^t, then the resulting torque is zero and the resulting thrust vector

$$\frac{\vec{f}}{f} = \sum_{i=1}^{4} \vec{e}_i = \begin{pmatrix} 0 \\ 0 \\ 4e_{iz} \end{pmatrix}^t$$

- The objective of any thruster actuation algorithm is to invert the actuation equation and, at the same time, take into account the constraints on the range of the individual thrust vectors

Thruster limits

$$\vec{t} = \vec{A}^{*-1}\vec{M} + k\vec{1}\ ;\ t_{min} \le t_i \le t_{max}$$

Pseudoinverse   Control law   Constant chosen to obtain positive overall thrust

- Possible choices of *k* are

$$k = t_{min} - \min_i\left(\vec{A}^{*-1}\vec{M}\right)_i\ ;\ \text{if}\ \left(\vec{A}^{*-1}\vec{M}\right)_i < t_{min}$$

$$k = t_{max} - \max_i\left(\vec{A}^{*-1}\vec{M}\right)_i$$

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Thrusters modulation (1)

- Normally thrusters are not used in a proportional fashion but through pulse width modulation, i.e. the thrusters provide either zero or full thrust
- The pulse width is lower bounded to a minimum impulse bit
- Each actuation command t_i provided by the distribution algorithm must be converted into an on-off sequence by a modulator
- A pseudorate modulator use linear dynamic feedback so that at steady state the desired thrust is obtained in average

$$\bar{t} \sim t_i = \frac{p}{P}$$

Pulse width

Repetition period

- A simplified digital implementation of a pseudorate modulator is as follows

Remainder (0 at start)

Number of pulses

$$\frac{t_i(k)}{t_{max}} := \frac{t_i(k)}{t_{max}} + r(k-1) \qquad p_i(k) = floor\left(p_{max} \cdot \frac{t_i(k)}{t_{max}}\right) \qquad r(k) = \frac{t_i(k)}{t_{max}} - \frac{p_i(k)}{p_{max}}$$

Next time interval

$$k := k + 1$$

Thrusters modulation (2)

- More sophisticated pulse-width-modulators can be used for fine control
- The attitude control accuracy requirement for the GRACE mission is $\pm$0.5mrad ($\pm$0.03deg) and $\pm$10mrad in pitch/yaw and roll respectively
- To this end
  - cold gas thrusters are operated in on-off pulse width during 1Hz sample
  - when the attitude error exceeds the threshold level (deadband), the impulse duration (thruster on-time with 1msec resolution) is calculated
  - the minimum on-time is 30ms, while the maximum is 1s (100% duty cycle)
  - the pulse size increases monotonically between these two values
  - during transient the on-time value is based on a polynomial function of the control tracking error
  - at steady state an adaptive feature is used upon ground command where the pulse width is increased or decreased depending on the excursion of the error inside the deadband

"Polynomial" part description:

The "Polynomial" part of the design computes the On-Time (pulse width) as a polynomial-function of the error.

Algorithm:
The error is first converted into a number of deadbands (DB): X value

```
X = abs(error)/DB;
if X >= 1,
  Then the On-Time  is given by:
        ON-TIME = A*X^2 + B*X + C
  with A,B,C parameters.
  ON-TIME is saturated between 0.03 sec and 1 sec.
  ON-TIME = ON-TIME * sign(error);
Else,
  ON-TIME = 0;
End;
```

Parameters Description:
The parameter A ,B ,C , DB are defined for each channel (roll, pitch and yaw)
Depending on the mode of the ADCS, the coefficients A,B,C are set to different values.
When the mode is CICPM, these coefficients are determined so that the On-Time vs Error (in number of deadband) has the following straight line shape:

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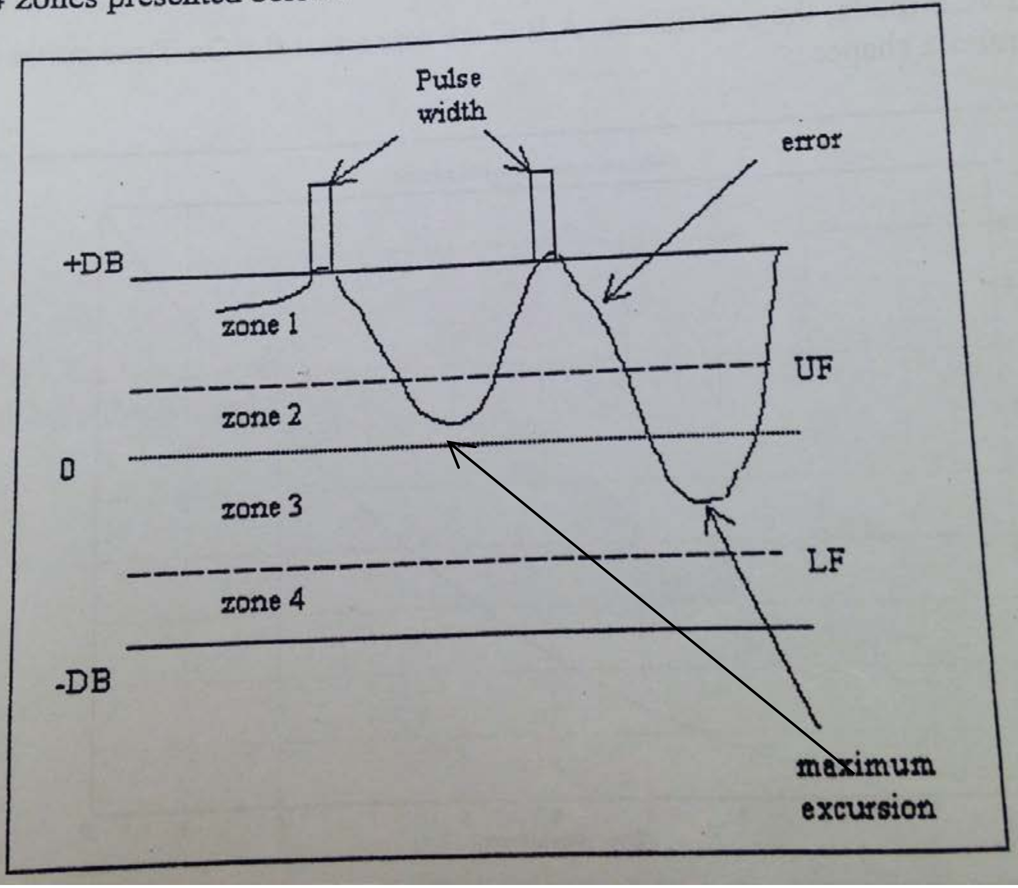
"Adaptative" part description:

When the Spacecraft enters a phase where the error is kept inside the deadband (Steady State part), the modulator can be switched by ground command to the adaptative algorithm (function Adapt_pw_modulator). Therefore, the **on_off_Adapt** boolean is set to 1.

This algorithm is based on the excursion of the error inside the deadband, and increases or decreases the pulse width depending on this excursion.

To do so, it defines 4 zones presented below:



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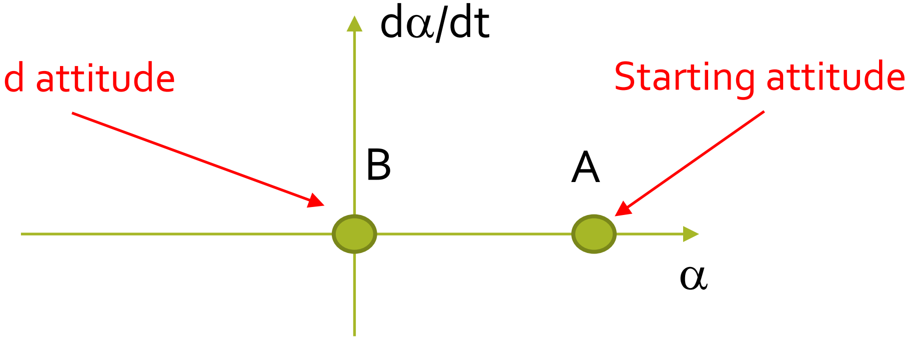
Attitude Maneuvers (1)

- So far we have learned how to keep a certain attitude through a linear control law under nonlinear dynamics
- We have neglected the time variable which is important during attitude maneuvers
- Our goal is now to shift from a start-attitude to an end-attitude in the minimum possible time
- This problem is nonlinear and does not have an exact solution

Single axis
for now

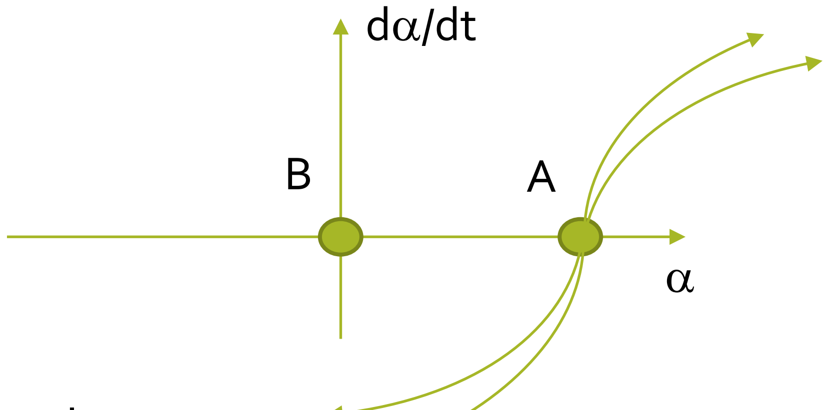


Normalized
control variable

$$I\ddot{\alpha} = M \Rightarrow \ddot{\alpha} = u \Rightarrow \alpha = \dot{\alpha}_0 t + \frac{1}{2}ut^2 + \alpha_0 = \frac{\dot{\alpha}_0}{u}(\dot{\alpha} - \dot{\alpha}_0) + \frac{1}{2u}(\dot{\alpha} - \dot{\alpha}_0)^2 + \alpha_0 \Rightarrow \boxed{(\alpha - \alpha_0) = \frac{1}{2u}(\dot{\alpha}^2 - \dot{\alpha}_0^2)}$$

Attitude Maneuvers (2)

- The attitude motion is parabolic in the phase space with
  - Right or left parabola branches depending on the sign of u
  - Larger or smaller parabolas depending on applied torque



- In order to design the minimum-time maneuver

Maneuver time

Maneuver amplitude

$$\dot{\alpha}^2 = f(\alpha, \text{const}) \Rightarrow \dot{\alpha} = \frac{1}{\dot{\alpha}} f(\alpha, \text{const}) \Rightarrow dt = \frac{\dot{\alpha}}{f(\alpha,\text{const})} d\alpha \Rightarrow \int_{t0}^{tf} dt = \int_{\alpha 0}^{\alpha f} \frac{\dot{\alpha}}{f(\alpha,\text{const})} d\alpha$$

$$\Rightarrow \int_{t0}^{tf} dt = \int_{\alpha 0}^{\alpha f} \frac{1}{\dot{\alpha}} d\alpha$$

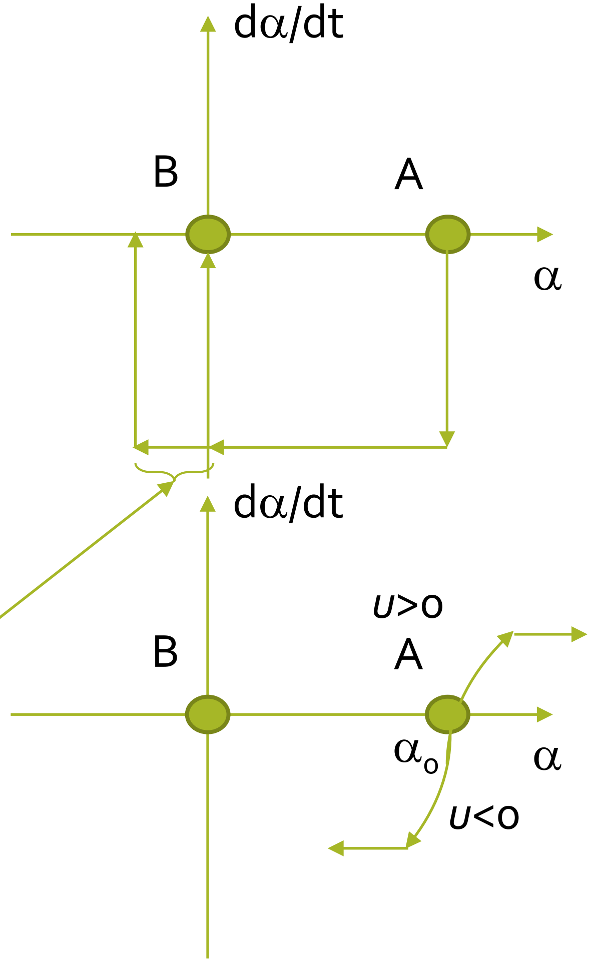
To minimize time, we have to maximize the angular velocity with the constraint that the angular velocity is zero at the end of the maneuver (nonlinear, large torque)

Attitude Maneuvers (3)

- The attitude motion in the phase space is given by a parabola translated by a constant term given by the initial conditions

$$\alpha = \frac{1}{2u}\dot{\alpha}^2 + \text{const}$$

- An ideal maneuver applies
  - infinite torque (impulse) in an infinitesimal time interval
  - a coasting phase after switching off the actuator with constant high angular velocity (natural motion)
  - when zero attitude error is achieved, a second impulse with opposite sign is applied to stop the attitude motion
- Such an impulsive maneuver requires very large torques (thrusters) and is affected by errors due to small timing errors in the second impulse
- In the real case, since the torque is finite, parabola arcs are followed and not straight segments



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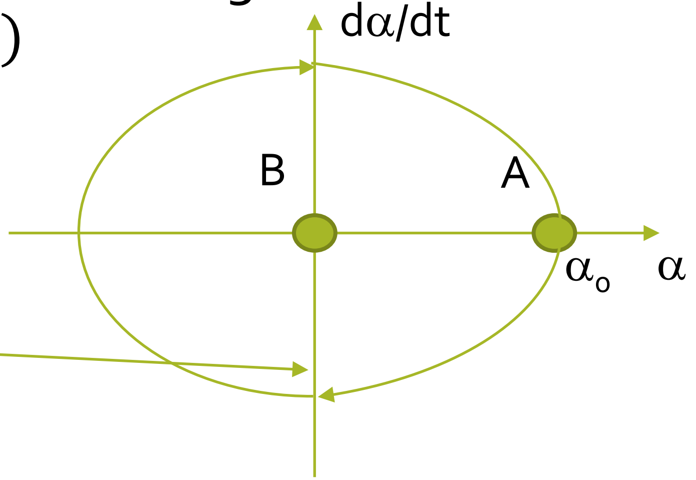
Attitude Maneuvers (4)

- The sign of *u* depends on the sign of the attitude angle α

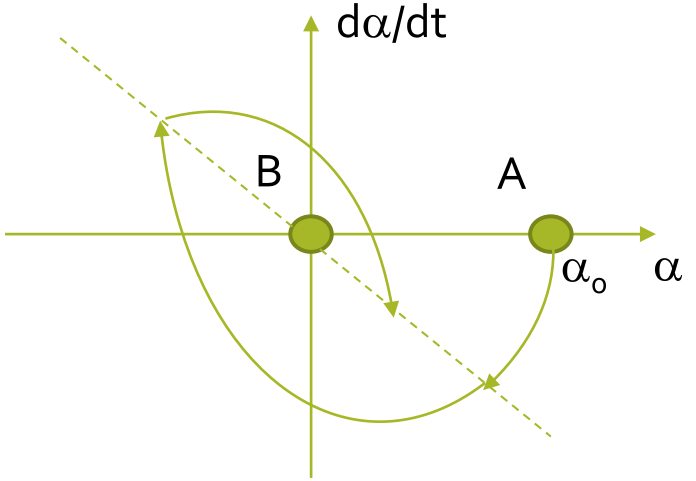
$$u = -u_{\max}\text{sign}(\alpha)$$



- Such a control law produces an harmonic motion which never converges to the target attitude following new arcs of parabolas each time α changes sign
- To damp this behavior, it is necessary to switch the control sign before reaching the target

$$u = -u_{\max}\text{sign}(\alpha + a\dot{\alpha})$$

- Now the control sign changes on the dashed line (commutation line) identified by the argument of the control law
- The tendency to approach the origin is now asymptotic (slow) because the breaking is done before reaching the target attitude

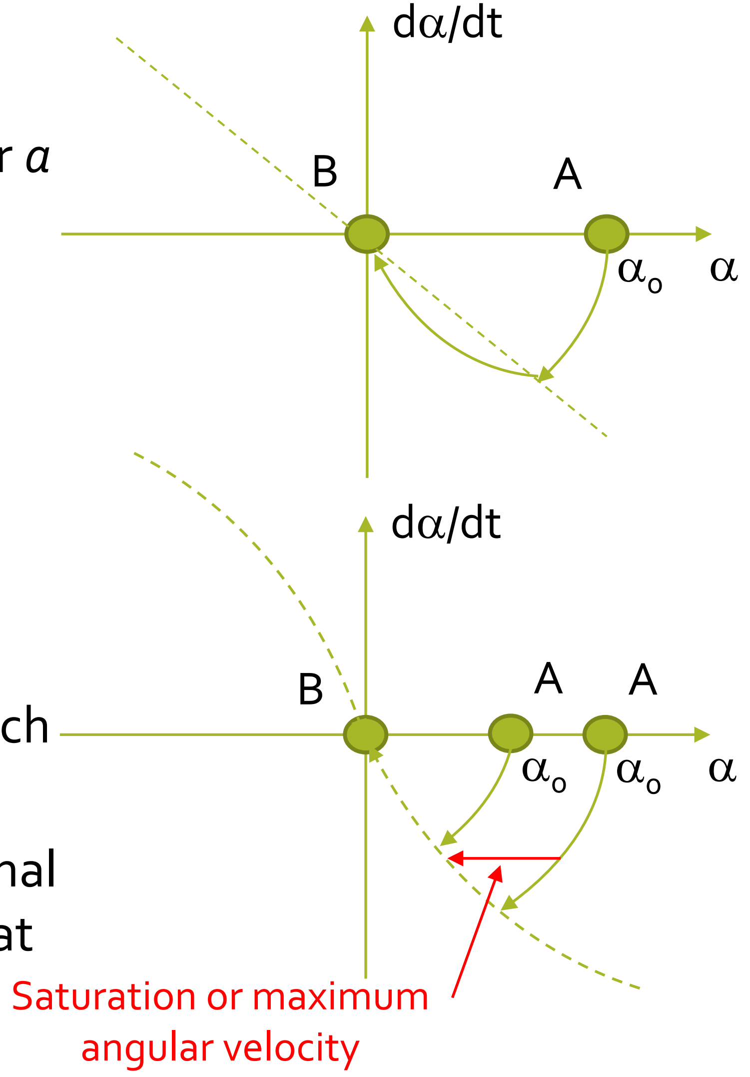


Attitude Maneuvers (5)

- The maneuver time is now very large (ideally infinite), but we can try to tune the parameter a of the commutation line such that only 1 change of sign is necessary (or 2 arcs)
- Unfortunately a depends on the initial condition, but we can choose a commutation line which corresponds to a parabola passing through the origin

$$u = -u_{\max}\operatorname{sign}(\alpha + \frac{1}{2}\dot{\alpha}|\dot{\alpha}|)$$

- Irrespective of the initial condition we will reach the target attitude in minimum time
- In practice actuators limitations and operational constraints force us to set the torque to zero at a maximum angular velocity



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Backup

Backup