AA 279A - Space Mechanics
Winter 2017

Lecture 8 Notes
Monday, 6 February 2017
Prof. Andrew Barrows

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Prof. Barrows' Office Hours 2/6–2/27:

Mon 2/6 → 3:00 – 5:00
Wed 2/8 → 3:00 – 5:00
Fri 2/10 → 9:30 – 11:30 ← PSet 4 due 2/13
Mon 2/13 → 3:00 – 5:00
Wed 2/15 → 3:00 – 5:00
Thu 2/16 → 9:30 – 11:30 ← PSet 5 due 2/20
← Midterm 2/22
Mon 2/27 → 10:30 – 12:30
← PSet 6 due 3/1

ROADMAP FOR NEXT FOUR LECTURES

Orbital Transfers

Delta V or "ΔV"

Coplanar maneuvers
- Hohmann transfer
- Bi-elliptic transfer
- One-tangent burn
- Oberth effect

TODAY

Non-coplanar transfers

Combined maneuvers

Continuous thrust and kinematic inefficiency

Launch to Earth orbit

Phasing

Interplanetary Trajectories

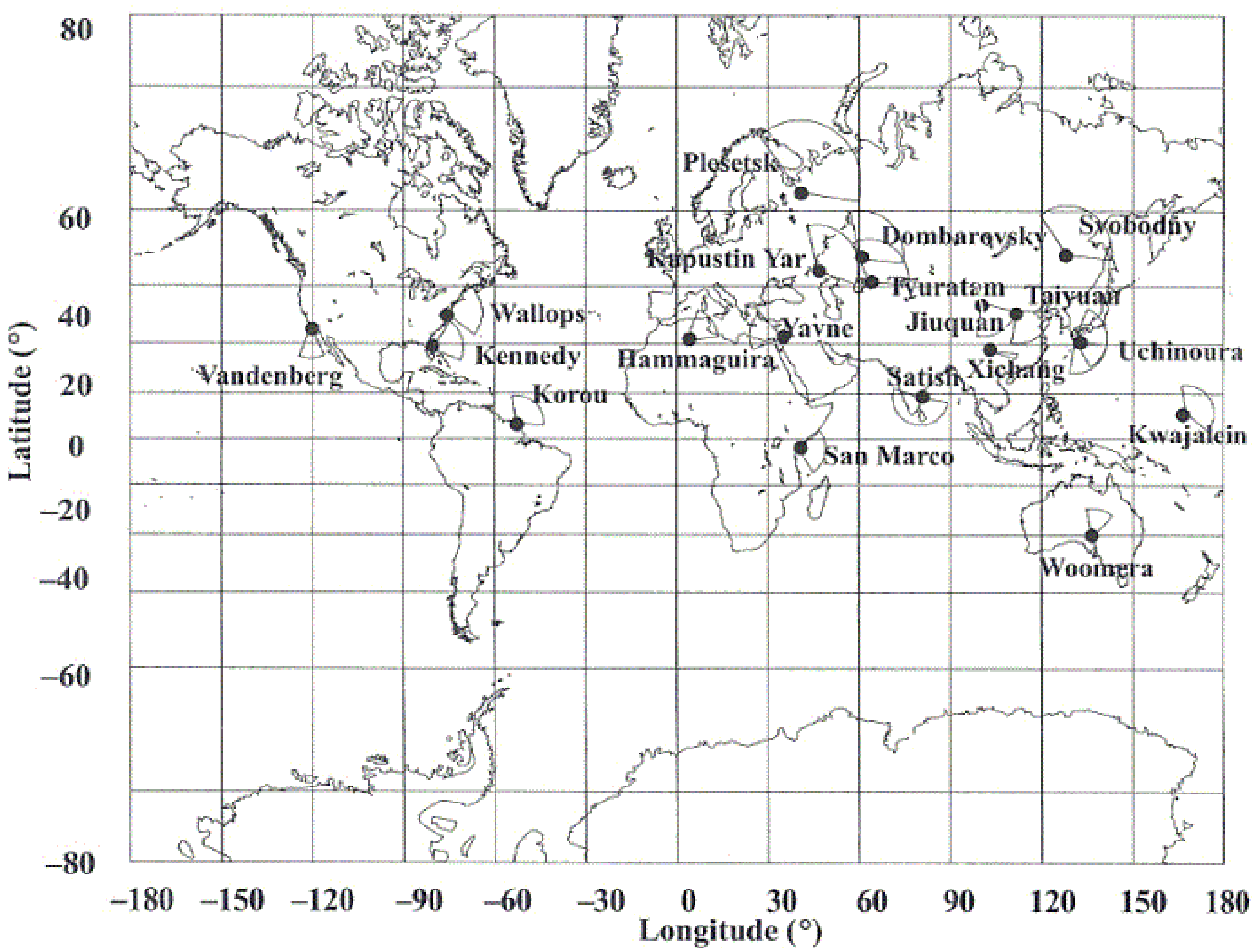
Method of Patched Conics

Hyperbolic departures, arrivals, flybys

Numerical Integration of Orbital EOMs

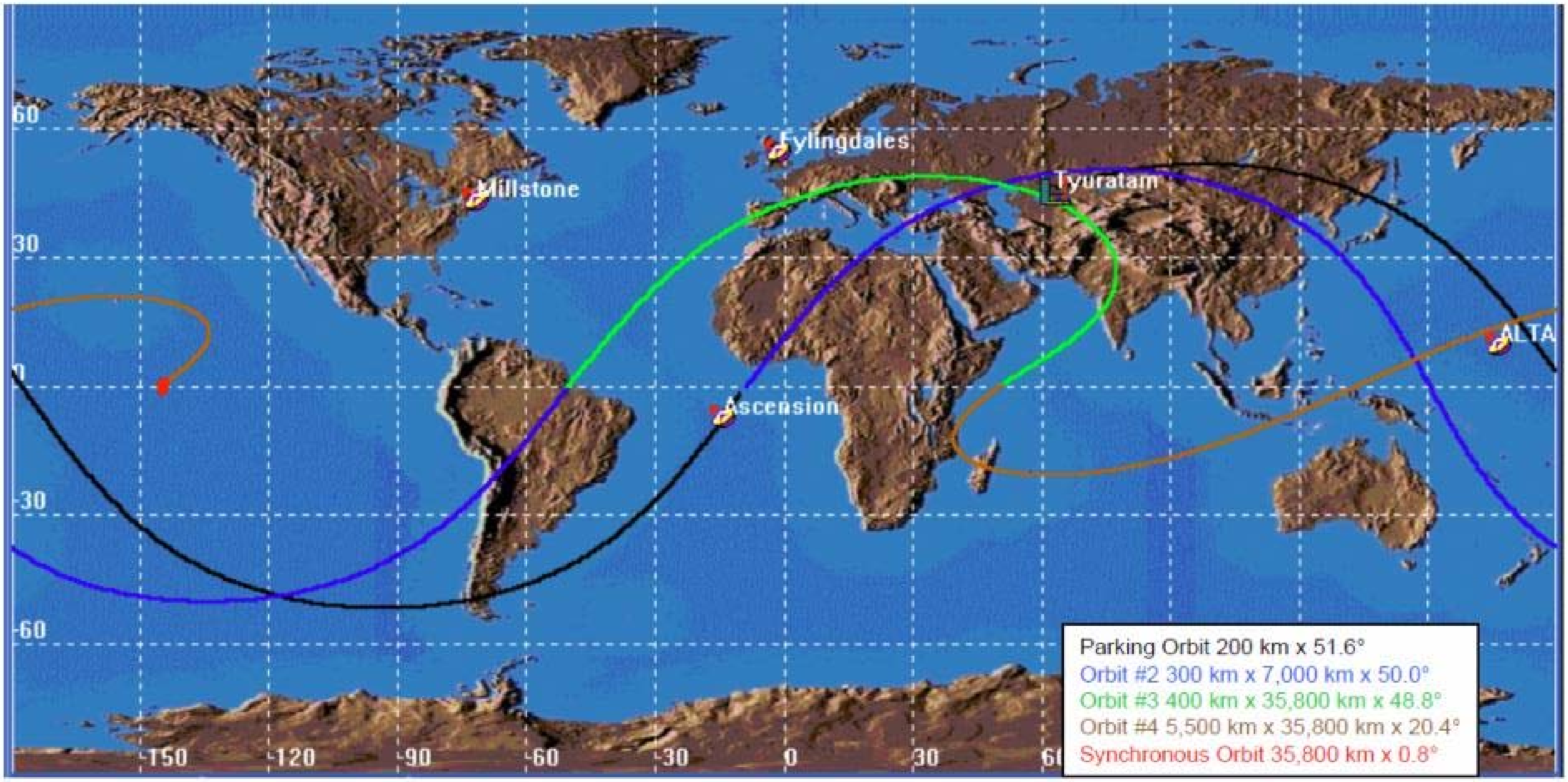
Reading for lectures 8 and 9
(on orbital transfers)
BMW 3.3 - 3.4, 8.3 - 8.3.2, 8.4
Vallado 6.2 - 6.6, 6.7, 6.7.1

Reading for Lecture 10
(on interplanetary trajectories)
BMW 7.4, 8.3.3 - 8.3.5
Vallado 12.1 - 12.2.4, 12.4



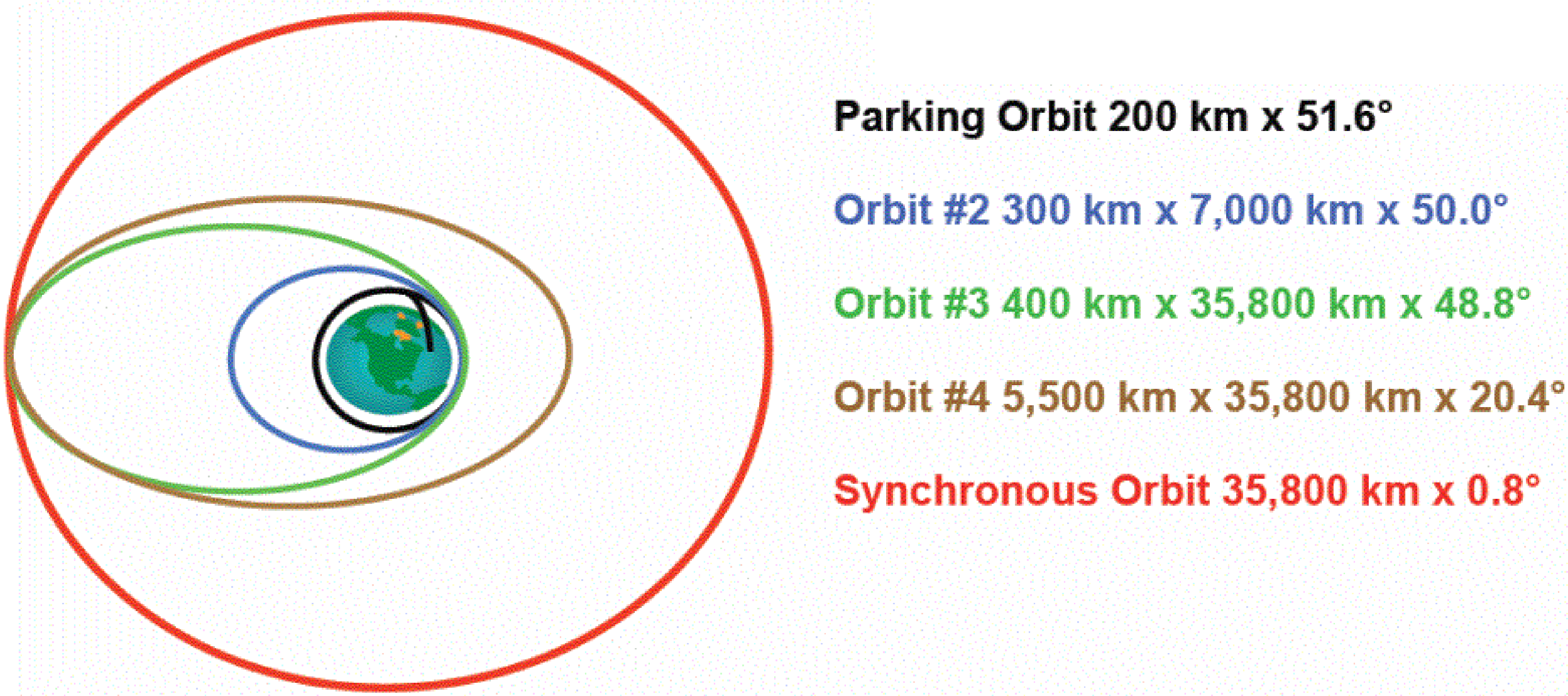
A MORE COMPLICATED EXAMPLE

Proton M/Briz M groundtrack



A MORE COMPLICATED EXAMPLE (CONT'D)

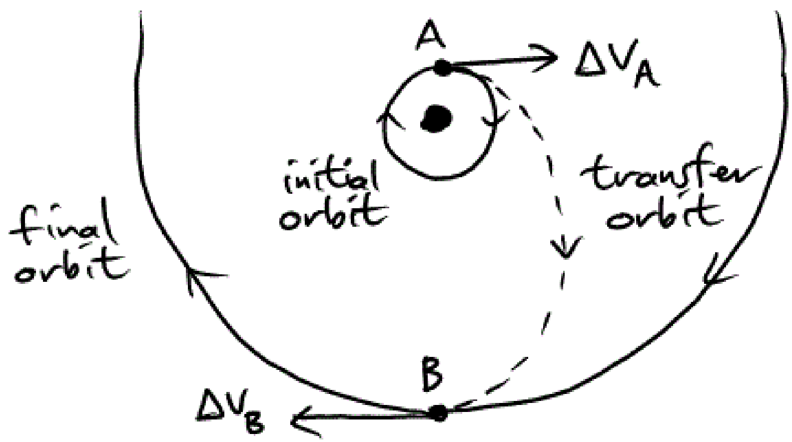
Proton M / Briz M mission profile



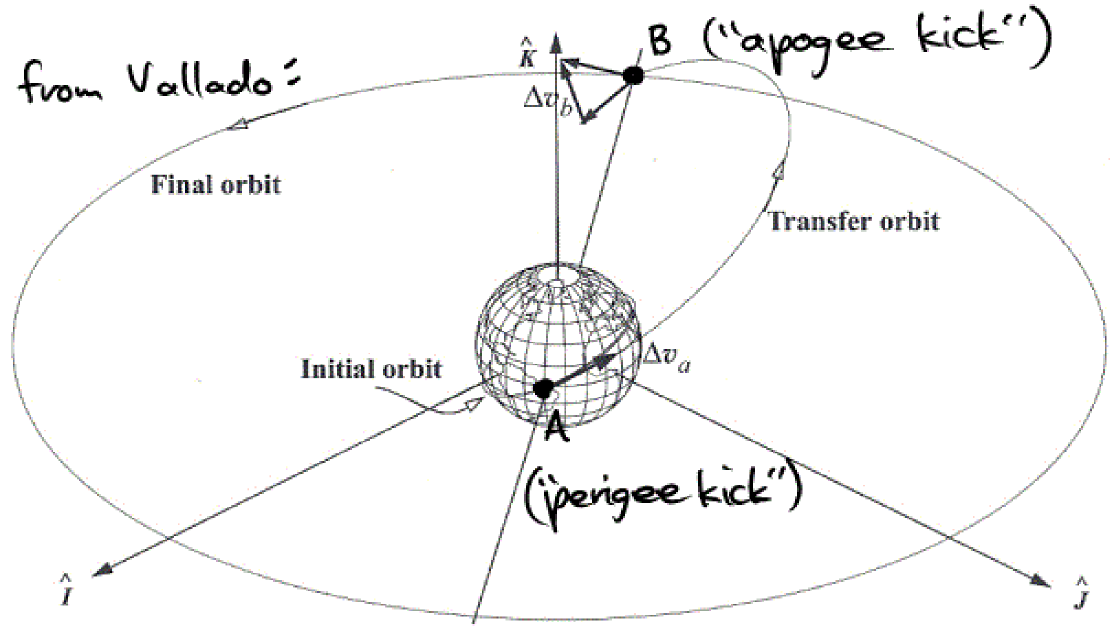
ORBITAL MANEUVERING

Simply put: maneuvering from one orbit to another using thrust (or perhaps solar pressure).

SIMPLE COPLANAR MANEUVER

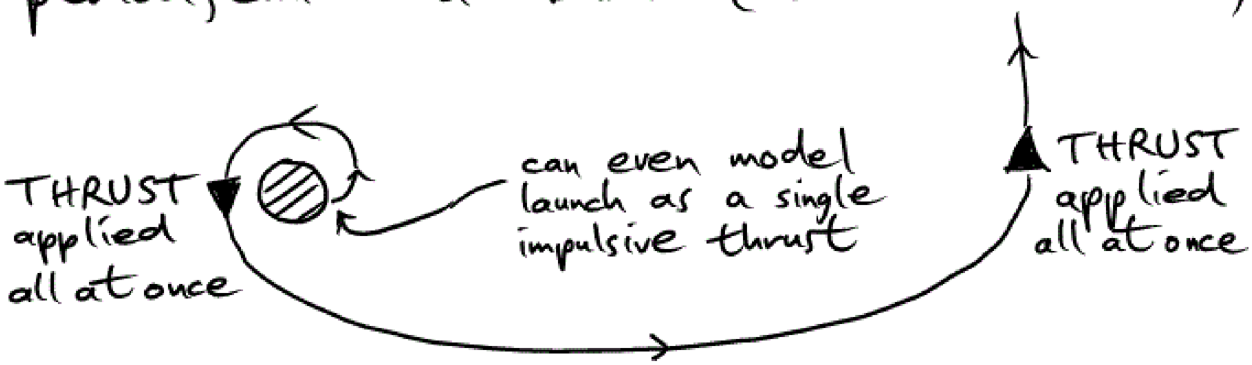


COMPLEX COMBINED MANEUVER



IMPULSIVE MANEUVERS

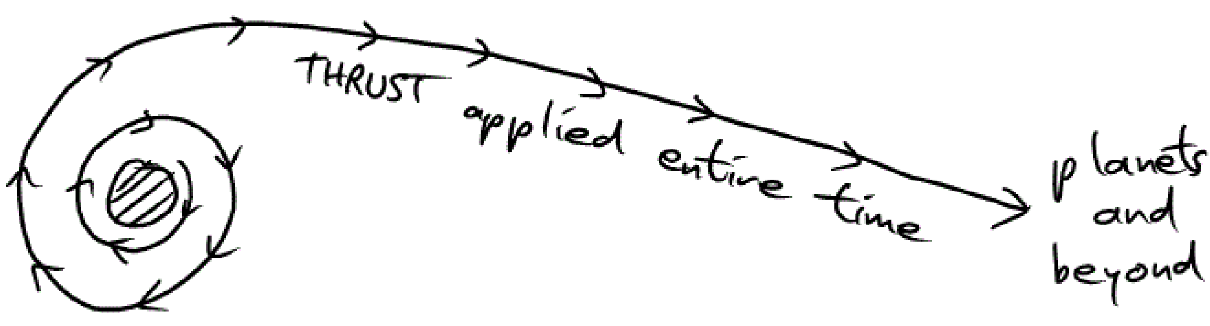
(Typically) larger engines applying larger amounts of thrust over short time period, called a "burn." (few minutes or less)



Chemical rockets
(I_{sp} up to 500 [sec] for liquids, less for solids)

CONTINUOUS-THRUST MANEUVERS

Small engines applying small amounts of thrust over long periods of time.



Ion engines, Hall-effect thrusters, ...
(Specific Impulse $I_{sp} = \frac{V_{exhaust}}{g_0}$ 1,500 [sec] & higher)

"DELTA V" OR "ΔV"

Magnitude of velocity vector change during orbital maneuvering. Typically used as measure of the 'magnitude' of an impulsive maneuver. Related to the mass of propellant used (of big concern to mission designers and spacecraft operators). Consider a spacecraft thrusting with <u>no</u> <u>gravity</u> <u>or</u> <u>other</u> <u>external</u> <u>forces</u>:

$$F_{\text{on spacecraft}} = m_{\text{spacecraft}}\, a$$

$$\int \frac{F}{m_{sc}}\, dt = \int a\, dt$$

Magnitude of F is $V_{\text{exhaust}}\,\underbrace{\dot{m}}_{>0}{}_{\text{propellant}}$.

Since $\dot{m}_{\text{propellant}} = -\dot{m}_{sc}$

$$\int \frac{-V_{ex}\,\dot{m}_{sc}}{m_{sc}}\, dt = \int a\, dt$$

$$-V_{ex} \int \frac{1}{m_{sc}} \frac{dm_{sc}}{dt}\, dt = \int \frac{dv}{dt}\, dt$$

end

$$-V_{ex}\int_{m_{sc}}^{m_{sc}-\Delta m_p}\frac{1}{m_{sc}}\,dm_{sc}=\int_{V}^{V+\Delta V}dV \qquad (\Delta m_p>0)$$

beginning

$$\Rightarrow \boxed{\Delta V = V_{ex}\,\ln\left(\frac{m_{sc}}{m_{sc}-\Delta m_p}\right)}$$

TSIOLKOVSKY'S ROCKET EQUATION

>0 $\quad>0\quad$ >0 since $\Delta m_p>0$

NOTE: This is independent of $\dot{m}_p$ and Δt.
$\Delta m_p=\dot{m}_p\,\Delta t$ is what matters here.

For 'small' burns, $\Delta m_p \ll m_{sc}$ and using $\ln\frac{1}{1-x}\simeq x$ for small x gives

$$\Delta V \approx \frac{V_{ex}}{m_{sc}}\,\Delta m_p$$

for 'small' burns only

$>0 \qquad\qquad >0$

I.e., achieving large ΔV requires using lots of (valuable) propellant mass.

We'll get used to the 'language' of ΔV...

OPTIMAL TRANSFERS

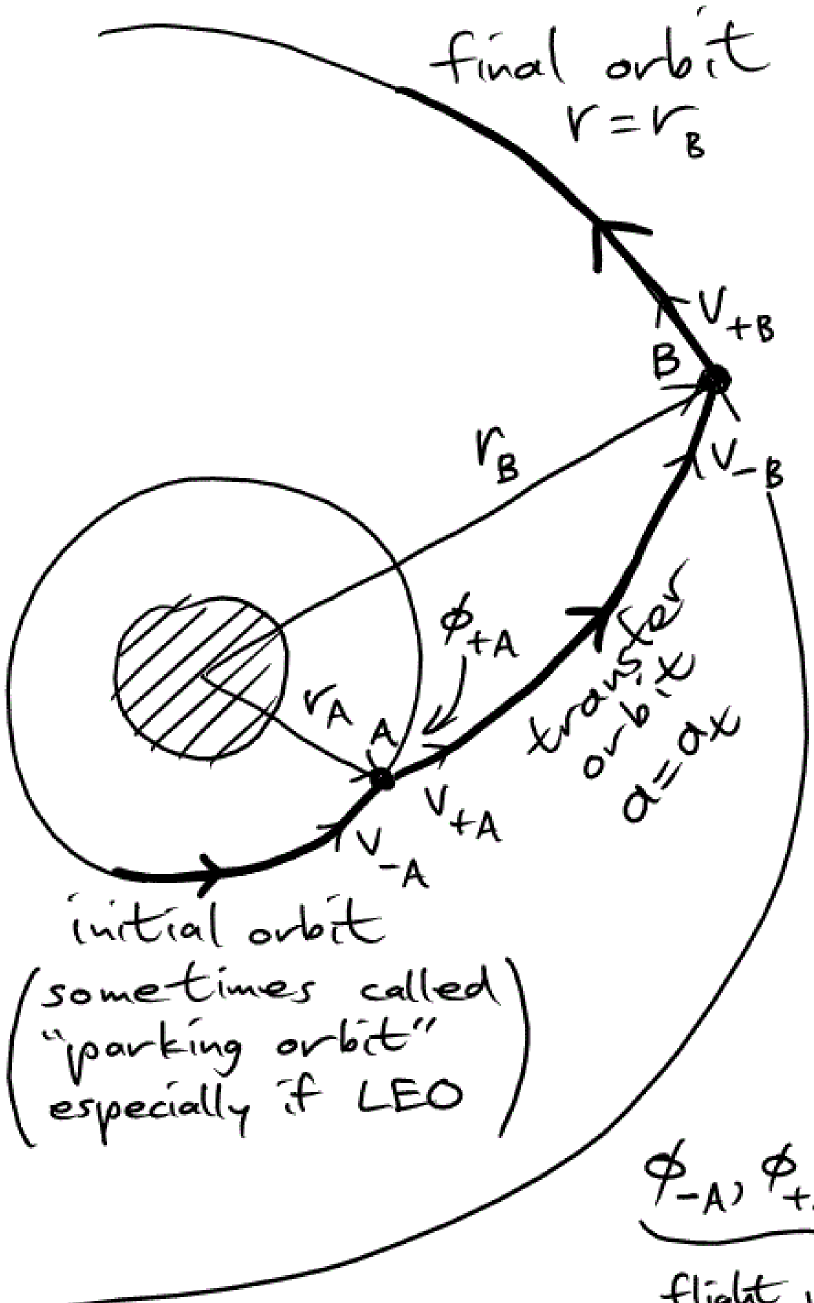
We typically want to optimize transfer orbit and burns for minimum ΔV, minimum time, minimum Δmass (in cases where this is different from min ΔV), or combinations thereof. Also have constraints on time, phasing, launch parameters, etc.

INTUITIION

We'll expect that in order to change the 'size' (a) or 'shape' (e) of an orbit, we'll need to apply ΔV ($\rightarrow$ thrust) <u>in</u> the orbital plane.

We'll expect that in order to change the orbital plane or direction of $\vec{h}$ (i.e. change Ω or i), we'll need to apply ΔV ($\rightarrow$ thrust) <u>perpendicular to</u> the orbital plane.

NOTATION AND TERMINOLOGY



A = point of first impulsive burn with

$$\Delta V_A = \left| \underset{\text{just after A}}{\vec{V}_{+A}} - \underset{\text{just before A}}{\vec{V}_{-A}} \right|$$

B = point of second impulsive burn with

$$\Delta V_B = \left| \underset{\text{just after B}}{\vec{V}_{+B}} - \underset{\text{just before B}}{\vec{V}_{-B}} \right|$$

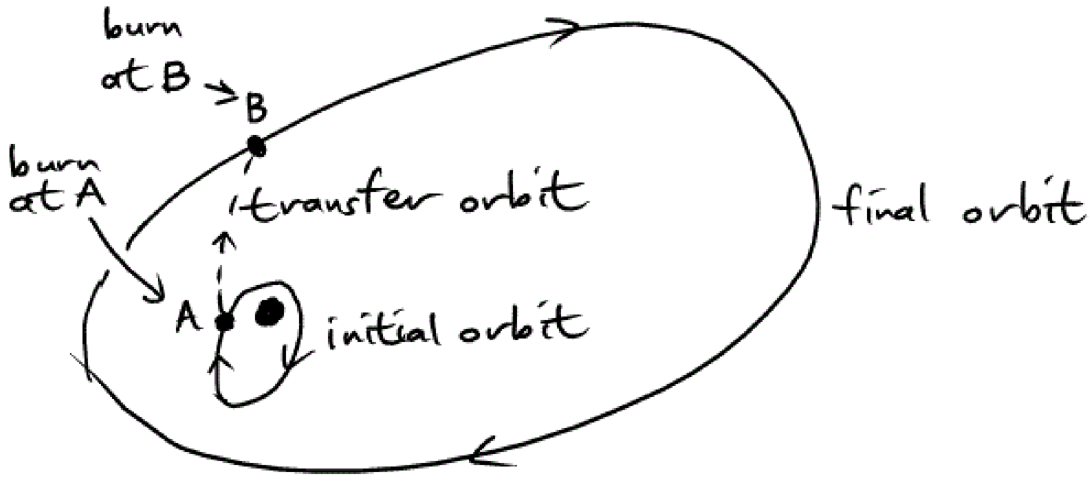
$\underbrace{\phi_{-A}, \phi_{+A}, \phi_{-B}, \phi_{+B}}_{\text{flight path angles}}$ all exist

If orbital plane changing, also have $\Delta i_A, \Delta i_B$, etc.

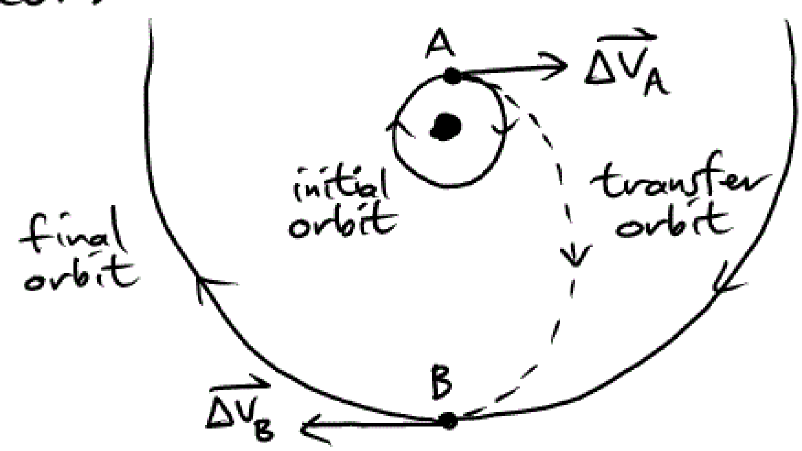
Transfer orbits can be circular, elliptical, parabolic, or hyperbolic.

COPLANAR MANEUVERS

Transfer between two orbits that share same orbital plane. In general, orbits can be ellipses and need not share axes. Burns can be non-tangential:



However, many important cases involve circular initial and final orbits, and propulsive forces tangent to velocity vector:

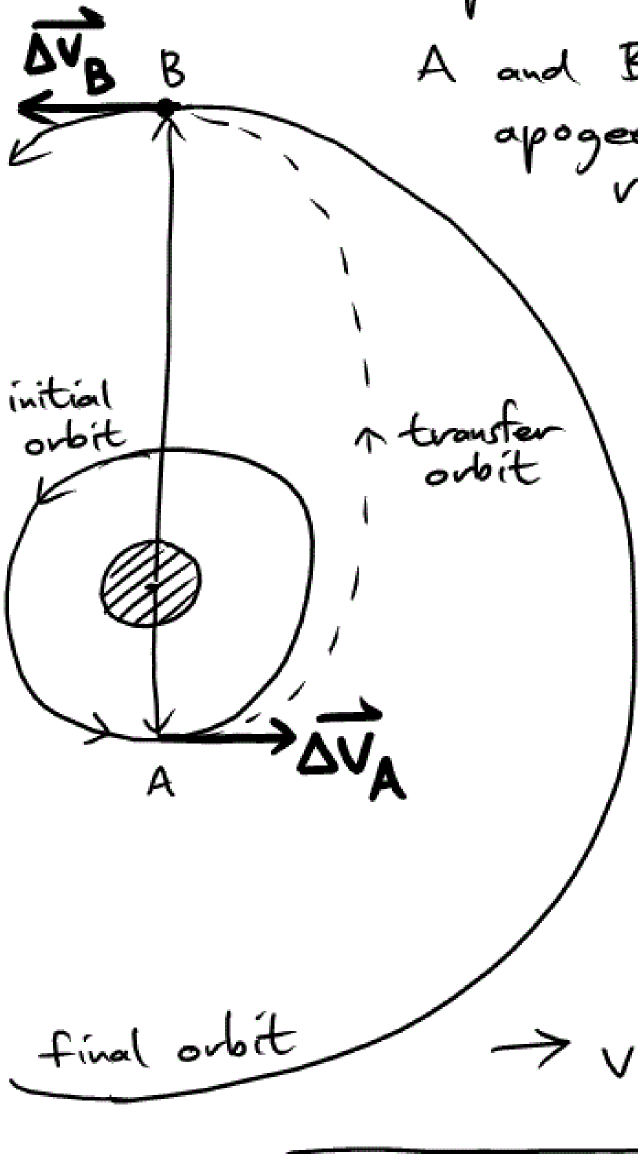


HOHMANN TRANSFER

Walter Hohmann (1880–1945)

Elliptical transfer between two circular coplanar orbits using two tangential burns.

Good news: we already know all the circle and ellipse relations we'll need!



A and B are perigee and apogee of transfer orbit, respectively.

$$\boxed{a_t = \frac{r_A + r_B}{2}} \rightarrow \text{HOHMANN}$$

$$V_{-A} = \sqrt{\frac{\mu}{r_A}} \quad \text{(circular)}$$

$$V_{+B} = \sqrt{\frac{\mu}{r_B}} \quad \text{(circular)}$$

Want V_{+A} and V_{-B} to find ΔV's:

$$\mathcal{E}_t = -\frac{\mu}{2a_t} = \frac{V^2}{2} - \frac{\mu}{r}$$

$$\rightarrow V = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a_t}}$$

$$\rightarrow V = \sqrt{2\mu\left(\frac{1}{r} - \frac{1}{r_A + r_B}\right)}$$ anywhere on transfer ellipse

So at points A and B on transfer ellipse:

$$V_{+A} = \sqrt{2\mu\left(\frac{1}{r_A} - \frac{1}{r_A + r_B}\right)}$$

$$V_{-B} = \sqrt{2\mu\left(\frac{1}{r_B} - \frac{1}{r_A + r_B}\right)}$$

$$\Rightarrow \Delta V_A = \left|\vec{V}_{+A} - \vec{V}_{-A}\right| \qquad \longrightarrow - \rightarrow = \rightarrow$$

$\vdash\!\dashv$ length

$$\boxed{\Delta V_A = \sqrt{2\mu\left(\frac{1}{r_A} - \frac{1}{r_A + r_B}\right)} - \sqrt{\frac{\mu}{r_A}}}$$

→ HOHMANN

$$\Rightarrow \Delta V_B = \left|\vec{V}_{+B} - \vec{V}_{-B}\right| \qquad \longleftarrow - \leftarrow = \leftarrow$$

$\vdash\!\dashv$ length

$$\boxed{\Delta V_B = \sqrt{\frac{\mu}{r_B}} - \sqrt{2\mu\left(\frac{1}{r_B} - \frac{1}{r_A + r_B}\right)}}$$

→ HOHMANN

$$\boxed{\Delta V_{total} = \Delta V_A + \Delta V_B} \qquad \vdash\!\dashv + \vdash\!\dashv = \vdash\!\!\!-\!\!\!-\!\!\!\dashv$$

→ HOHMANN

<u>ΔV's are always positive scalar quantities</u>

The transfer time is one half the ellipse period

$$t_{trans} = \frac{1}{2} 2\pi \sqrt{\frac{a_t^3}{\mu}} \Rightarrow \boxed{t_{trans} = \pi \sqrt{\frac{(r_A + r_B)^3}{8\mu}}}$$

HOHMANN

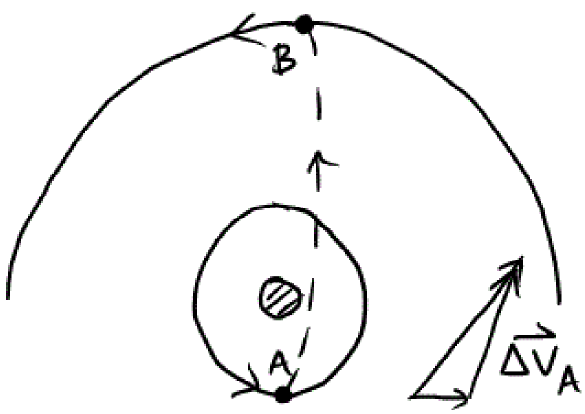
Remember that for the Hohmann tranfer:

$$\phi_{-A} = \phi_{+A} = \phi_{-B} = \phi_{+B} = 0 \rightarrow \text{HOHMANN}$$

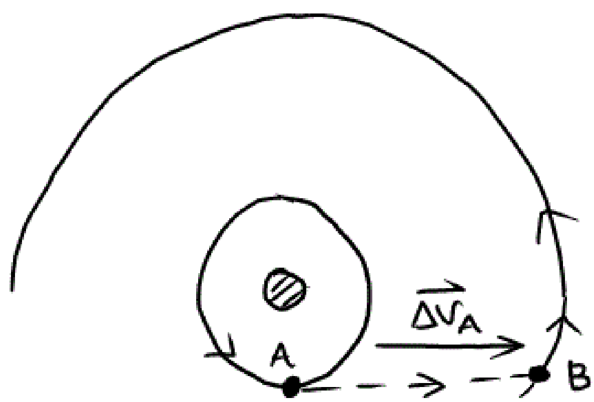
(burns are tangential)

There are plenty of other transfer orbits we could have used to get from the initial to the final orbit. These include some very fast parabolic and hyperbolic transfer orbits involving non-tangential burns:

EXAMPLE 1



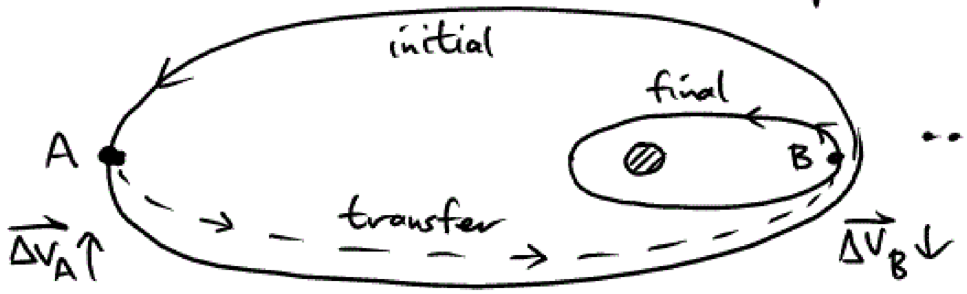
EXAMPLE 2



(note: if 'before' and 'after' velocities aren't parallel, need Law of Cosines to find ΔV)

HOWEVER, Hohmann transfer optimizes for minimum ΔV_{total} for an important set of cases. Turns out that for $r_{final}/r_{initial} < 11.94$, Hohmann minimizes ΔV_{total}.

Hohmann transfer analysis can be easily extended to coaxial ellipses:



... and need not go outward!

$\boxed{\text{EXAMPLE}}$ Hohmann transfer from 200km circular equatorial parking orbit to GEO.

$$r_A = r_\oplus + 200 \text{ km} = 6{,}378 + 200 = 6{,}578 \text{ [km]}$$

$$r_B = \left(\frac{\mu_\oplus}{\omega_\oplus^2}\right)^{1/3} = \left(\frac{398600.4418}{(7.292115\times10^{-5})^2}\right)^{1/3} = 42{,}164 \text{ [km]}$$

$$v_{-A} = \sqrt{\frac{\mu_\oplus}{r_A}} = 7.78 \left[\frac{\text{km}}{\text{sec}}\right]$$ LEO circular speed

$$v_{+B} = \sqrt{\frac{\mu_\oplus}{r_B}} = 3.07 \left[\frac{\text{km}}{\text{sec}}\right]$$ GEO circular speed

$$\Delta V_A = 2.45 \left[\frac{\text{km}}{\text{sec}}\right] \qquad \Delta V_B = 1.48 \left[\frac{\text{km}}{\text{sec}}\right]$$

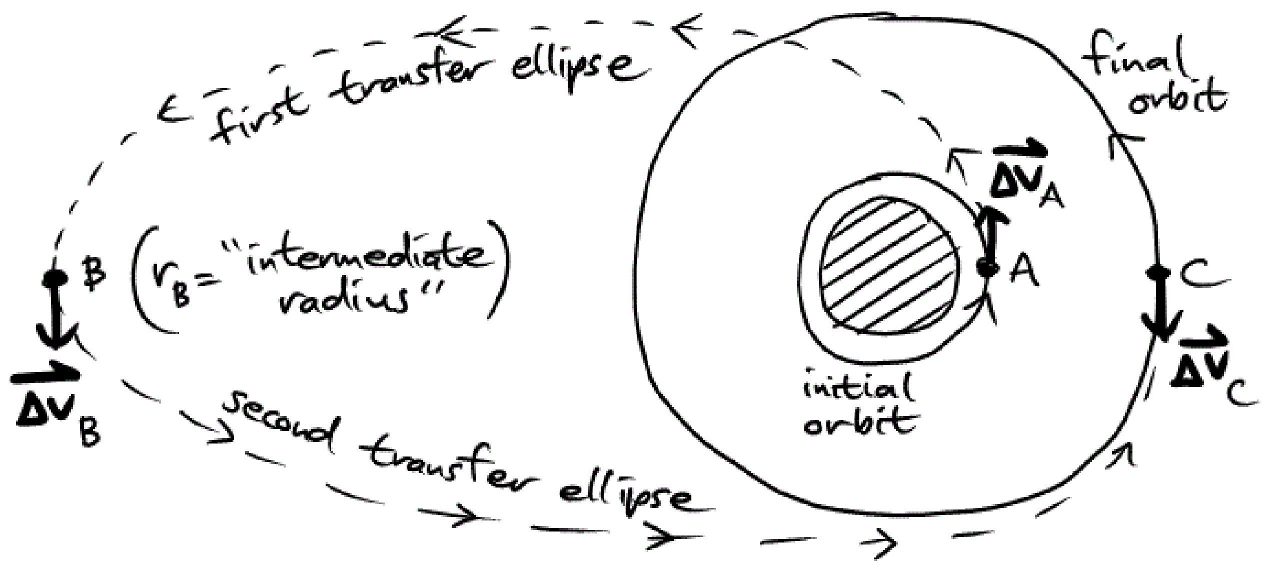
$$\Delta V_{total} = 3.93 \left[\frac{\text{km}}{\text{sec}}\right] \qquad t_{transfer} = 5.26 \text{ [hours]}$$

FYI: FYI: $V_{\substack{\text{equatorial}\\ \text{launch site}}} = 0.47 \left[\frac{\text{km}}{\text{sec}}\right]$

FYI: FYI: $V_{\substack{\text{esc}\\ \text{from}\\ \text{200km}}} = 11.00 \left[\frac{\text{km}}{\text{sec}}\right]$

BI-ELLIPTIC TRANSFER

Combines two Hohmann transfers and does <u>three</u> burns, with the middle burn at a radius larger than $r_{initial}$ and r_{final}:



$\vec{\Delta V}_A \uparrow$ and $\vec{\Delta V}_B \downarrow$ are <u>with</u> motion, $\vec{\Delta V}_C \downarrow$ <u>against</u> motion.

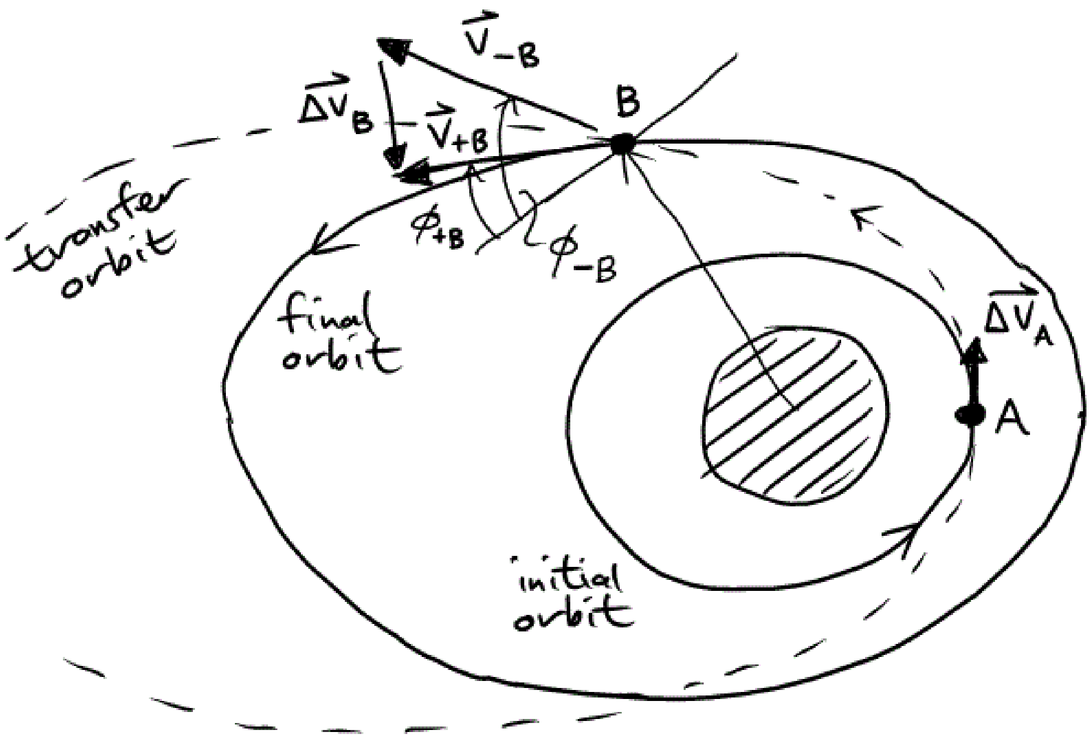
Can use ellipse relationships to find ΔV_A, ΔV_B, ΔV_C, ΔV_{total}, and total transfer time in the same way we did with Hohmann transfer. Don't forget that

$$t_{transfer} = \begin{matrix}\text{half of period}\\ \text{of transfer}\\ \text{ellipse \#1}\end{matrix} + \begin{matrix}\text{half of period}\\ \text{of transfer}\\ \text{ellipse \#2}\end{matrix}$$

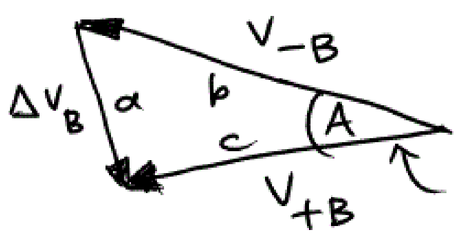
For $r_{intermediate}$ <u>chosen</u> <u>large</u> <u>enough</u>, bi-elliptic transfer minimizes ΔV_{total} for $r_{final}/r_{initial} > 11.94$.

ONE-TANGENT BURN

Includes one burn that is tangential (similar to above) and one that is <u>non-tangential</u>.



Since velocity vectors no longer parallel, need Law of Cosines to find ΔV_B:



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$A = \left|\phi_{+B} - \phi_{-B}\right|$$

$$\Rightarrow \Delta V_B = \sqrt{V_{-B}^2 + V_{+B}^2 - 2V_{-B}V_{+B}\cos(\phi_{-B} - \phi_{+B})}$$

OBERTH EFFECT

Consider we're moving at $\vec{v}$ and want to apply a given $\Delta V = |\overrightarrow{\Delta V}|$ to maximize $\Delta \mathcal{E}$. When and in what direction should we do ΔV?

$$\Delta \mathcal{E} = \mathcal{E}_{\text{after burn}} - \mathcal{E}_{\text{before burn}}$$

$$= \left(\frac{(\vec{v} + \overrightarrow{\Delta V}) \cdot (\vec{v} + \overrightarrow{\Delta V})}{2} - \frac{\mu}{r} \right) - \left(\frac{\vec{v} \cdot \vec{v}}{2} - \frac{\mu}{r} \right)$$

equal

$$\Delta \mathcal{E} = \vec{v} \cdot \overrightarrow{\Delta V} + \underbrace{\frac{\overrightarrow{\Delta V} \cdot \overrightarrow{\Delta V}}{2}}_{\text{small}}$$

So, to maximize $\Delta \mathcal{E}$, choose <u>direction</u> of $\overrightarrow{\Delta V}$ parallel to $\vec{v}$, and choose <u>timing</u> so that $\vec{v}$ is large when thrust is applied. Rocket thrust generates more useful energy at high speed than low speed.

This is called the "Oberth Effect" after Hermann Oberth (1894–1989).

Optimal time to apply ΔV on an interplanetary mission is at closest approach of a flyby.