AA 279A - Space Mechanics
Winter 2017

Lecture 9 Notes

Wednesday, 8 February 2017
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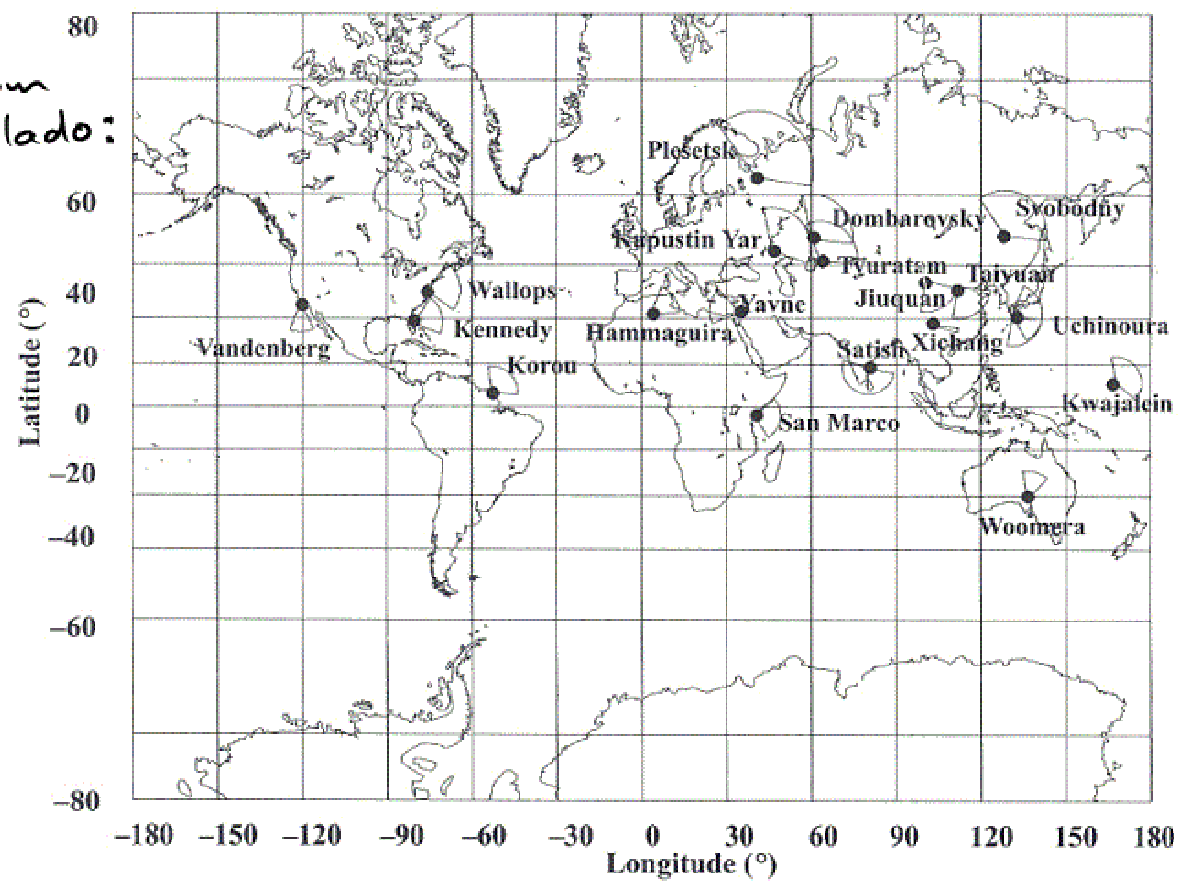
Topics for today
- Non-coplanar transfers
- Combined orbital maneuvers
- Kinematic inefficiency
- Continuous-thrust transfers
- Launching into earth orbit
  - ΔV required
  - Launch geometry

NON-COPLANAR MANEUVERS

We'll soon see that orbital plane changes (via out-of-plane burns) can get very 'expensive' in terms of propellant (ΔV). So why not simply launch into an orbit that has the final orientation we want?

from Vallado:

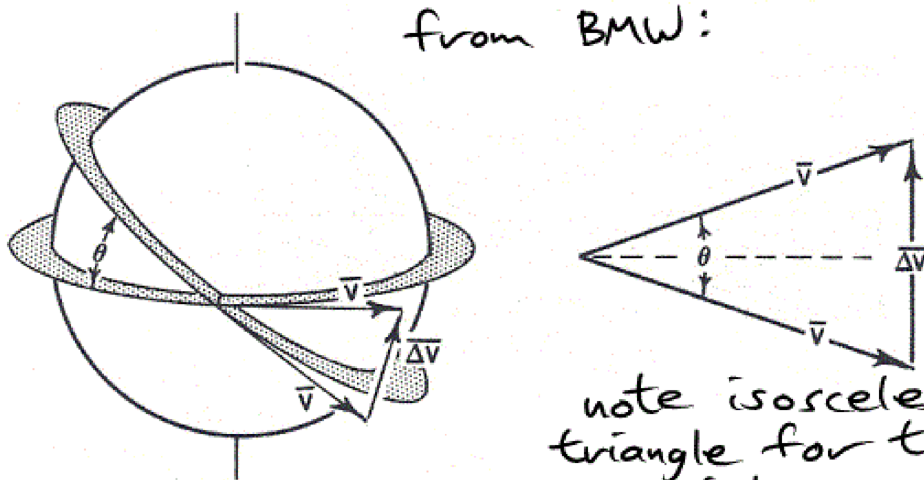


Answer: Launch site geometry may prevent it!

(Recall that if our launch site is at latitude ϕ, we can't launch directly to prograde orbits with inclination less than $|\phi|$.)

Therefore, sometimes need to do plane change maneuvers. Example of pure plane change with no change to a or e:

from BMW:



note isosceles triangle for this special case of $|\vec{V}_{after}| = |\vec{V}_{before}|$

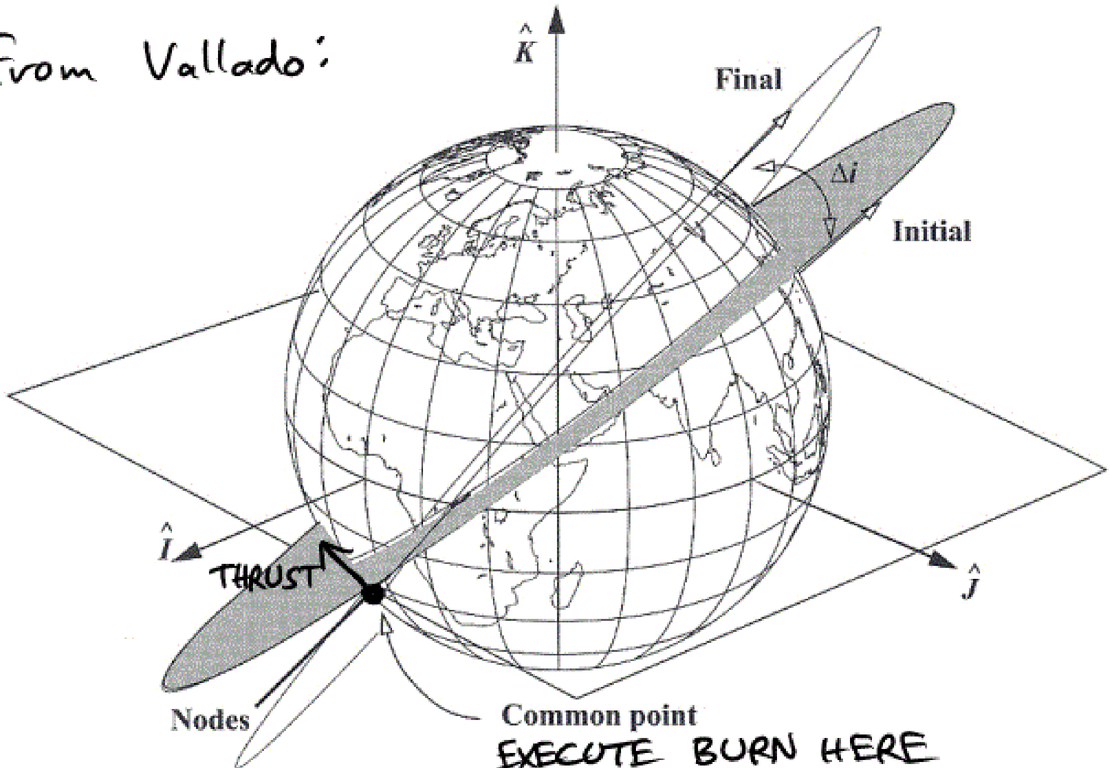
Plane change maneuvers will affect Ω and i.

CHANGES TO INCLINATION ONLY

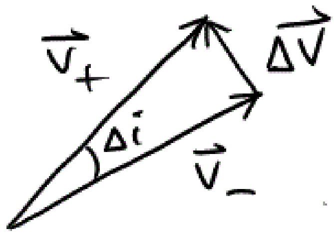
Inspection of diagrams above and below suggests that if we want to change i without affecting Ω, we should apply ΔV (mostly perpendicular to orbital plane) as object crosses equator (i.e. at a NODE).

Example of inclination change only:

from Vallado:



For pure inclination change, length of $\vec{v}$ doesn't change:



for this isosceles triangle

$$\Delta v = 2 v \sin \frac{\Delta i}{2}$$

NOTE: For $\Delta i = 60^\circ$, $\Delta v = v$
That's a lot of extra propellant mass to carry!

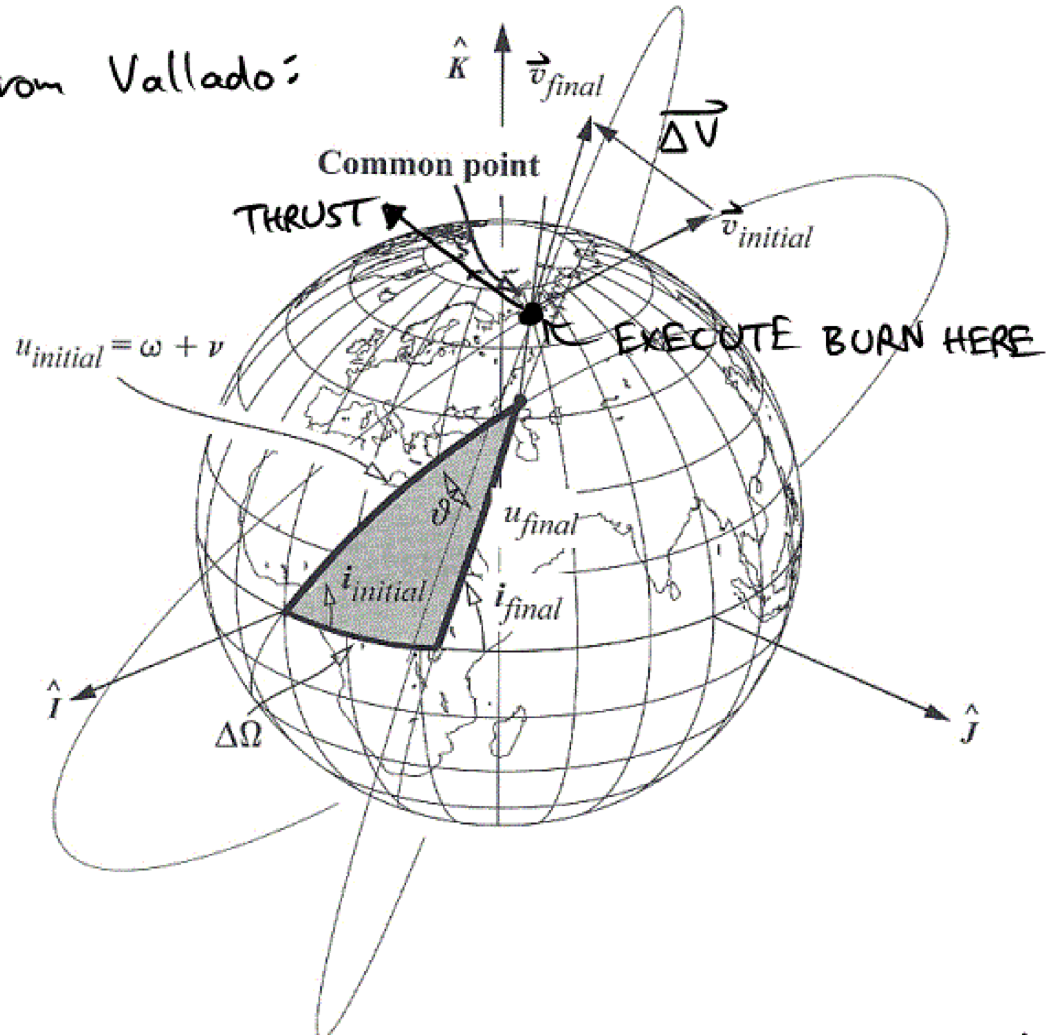
Burn takes place at intersection of initial and final planes. $\vec{h}$ vector rotates, as if an impulsive torque had been applied to entire orbit.

CHANGES TO Ω ONLY FOR POLAR ORBIT

Apply ΔV at south or north pole.

CHANGES TO BOTH i AND Ω FOR CIRCULAR ORBIT

from Vallado:



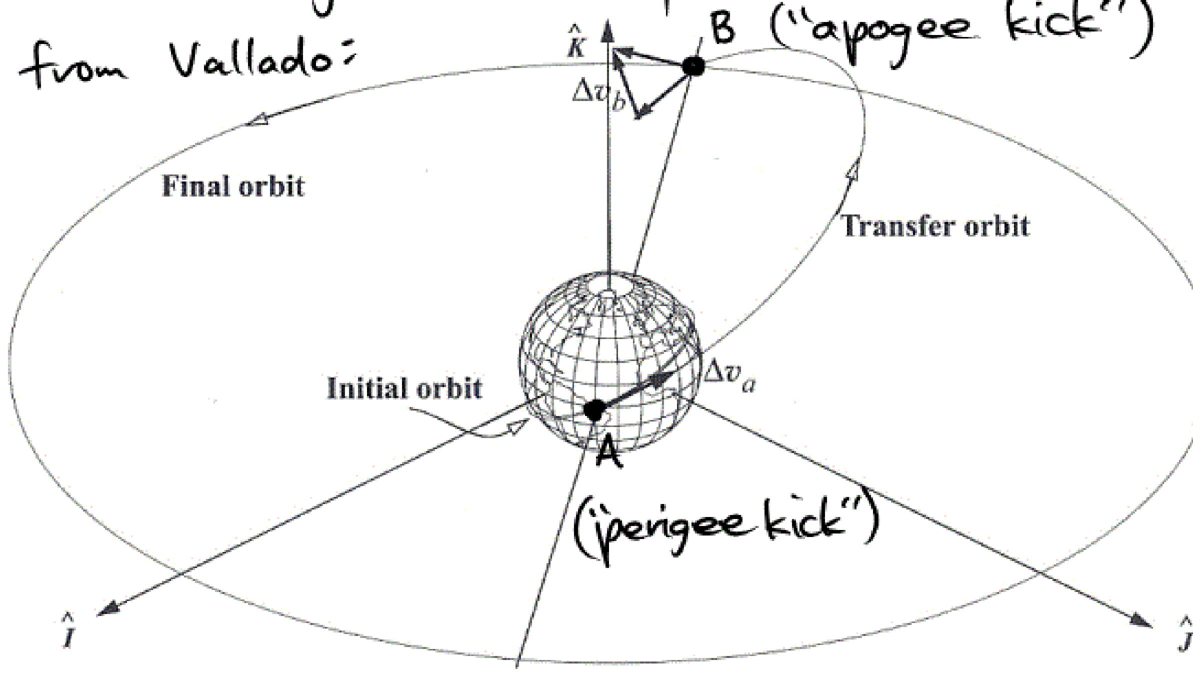
ΔV triangle is still isosceles. Given $\Delta\Omega$, i_{final} and $i_{initial}$, can use spherical trigonometry to find angles $u_{initial}$, u_{final}, and ϑ. Then can find ΔV.

COMBINED MANEUVERS

A burn that has in-plane and out-of-plane components can change a, e, Ω, i, etc. all at once. Multiple burns of this type can accomplish complete changes of orbit, phasing, intercept, rendezvous, etc.

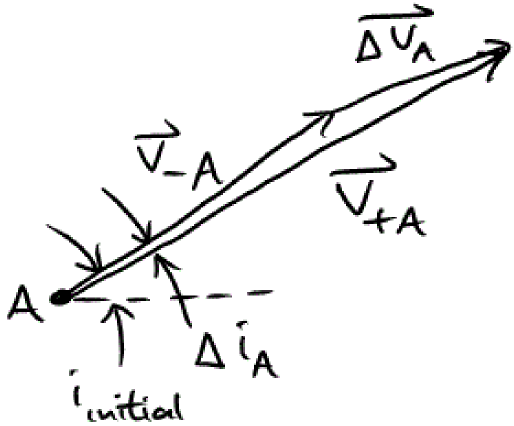
COMMON EXAMPLE • Transfer from circular, inclined LEO "parking orbit" to geostationary orbit:

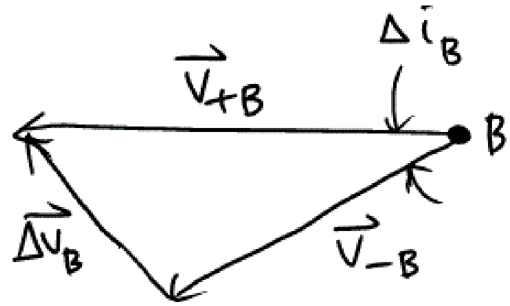
from Vallado:



at A: decrease inclination by Δi_A and establish V_{tA} according to Hohmann transfer

at B: arrive with V_{tB} and burn to circularize and decrease inclination to zero at A (perigee kick) at B (apogee kick)





$(\Delta i_B = i_{initial} - \Delta i_A)$

$$\Delta V_A = \sqrt{V_{-A}^2 + V_{+A}^2 - 2V_{-A}V_{+A}\cos\Delta i_A}$$ ← Law of Cosines

$$\Delta V_B = \sqrt{V_{-B}^2 + V_{+B}^2 - 2V_{-B}V_{+B}\cos\Delta i_B}$$

There will be an optimum choice for Δi_A (and therefore $\Delta i_B = i_{initial} - \Delta i_A$) that minimizes ΔV_{total}.

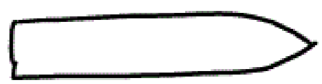
WHERE TO APPLY OUT-OF-PLANE ΔV TO MAXIMIZE PLANE CHANGE

From simple Law-of-Cosines geometry, to maximize plane change, choose <u>direction</u> of $\vec{\Delta V}$ roughly $\perp$ to orbital plane, and choose <u>timing</u> so $\vec{V}$ is small when thrust is applied. PLANE CHANGES ARE EXPENSIVE in ΔV. Usually want to do at low speed/large distance.

KINEMATIC INEFFICIENCY

Reconsider the Rocket Equation, but this time include gravity pulling backward:

Thrust

$m_{sc}g \longleftarrow$  $\longrightarrow v_{ex}\dot{m}_p = -v_{ex}\dot{m}_{sc}$

v_{ex} = propellant exhaust velocity $= g_0 I_{sp}$

$\dot{m}_p$ = propellant mass flow rate (>0)

$\dot{m}_{sc} = -\dot{m}_p$ = rate of change of spacecraft mass

Over Δt, rocket burns Δm_p (>0) of propellant mass as V changes by ΔV and spacecraft mass decreases from m_{sc} to $m_{sc} - \Delta m_p$:

$$\sum F_{\substack{\text{on}\\\text{spacecraft}}} = m_{sc}\,a$$

$$-v_{ex}\dot{m}_{sc} - m_{sc}g = m_{sc}\,a$$

$$\int\left(\frac{-v_{ex}\dot{m}_{sc}}{m_{sc}} - g\right)dt = \int a\,dt$$

$$-v_{ex}\int \frac{1}{m_{sc}}\frac{dm_{sc}}{dt}\,dt - g\int dt = \int \frac{dv}{dt}\,dt$$

$$-V_{ex}\int_{m_{sc}}^{m_{sc}-\Delta m_p} \frac{1}{m_{sc}}\, dm_{sc} - g\int_{t}^{t+\Delta t} dt = \int_{V}^{V+\Delta V} dv$$

$$\Rightarrow \boxed{\Delta V = V_{ex} \ln\left(\frac{m_{sc}}{m_{sc}-\Delta m_p}\right) - g\,\Delta t}$$

If thrust is non-impulsive (i.e. $\Delta t \neq 0$ and $\dot{m}_p \neq \infty$) the amount of ΔV created by burning Δm_p is reduced by $(g\Delta t)$. This quantity is called "gravity drag" and results from having to fight gravity during a non-zero time period.

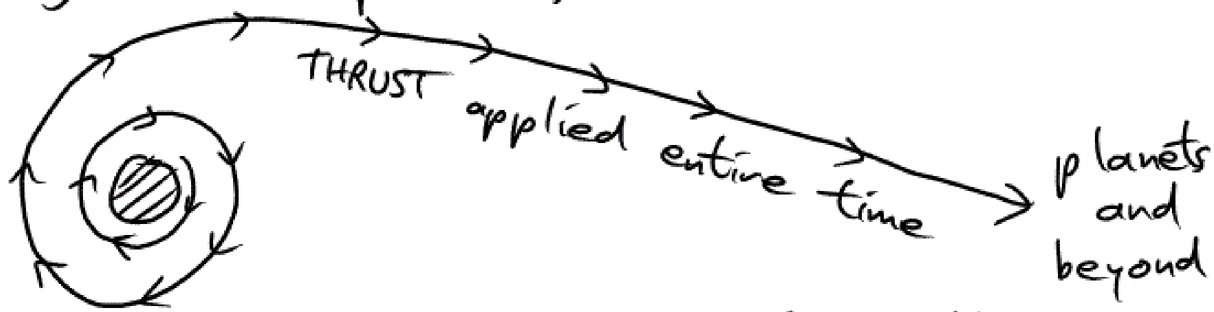
This loss is called <u>KINEMATIC INEFFICIENCY</u>.

It's of concern for low/continuous-thrust propulsion where $\dot{m}_p$ is small and Δt is long.

HOWEVER, if low-thrust transfer is done with engine with significantly higher I_{sp} than Hohmann/chemical rocket combination, <u>we can potentially use far less</u> propellant mass despite needing more ΔV.

CONTINUOUS-THRUST TRANSFERS

These use rockets with high Specific Impulse (I_{sp} translates to propulsive mass efficiency) but low thrust levels. These thrusters are operated continuously or over long time periods, with low thrust:



Ion engines, Hall-effect thrusters, others

(Specific Impulse $I_{sp} = \frac{V_{exhaust}}{g_0}$ 1,500 [sec] & higher)

EXAMPLE

ESA SMART-1 Satellite

- Launched 2003, Crashed into moon 2006
- Hall effect thruster with Xenon ion propellant I_{sp}=1,640 [sec], powered by 1,200 W solar panels
- Thrust: 68 mN (~0.25 ounces of force)
  → acceleration: $0.7 \left[\frac{m/sec}{hr}\right] = 2 \times 10^{-5}$ [g's]
- Crashed (intentionally) into moon by thrusting 1/3 - 1/2 of each orbit on perigee side. Raising orbit took > 1 [year]

CONTINUOUS-THRUST SPIRAL CLIMB

To model "climb" away from low earth orbit, assume spacecraft spirals outward so slowly that it behaves like a slowly-expanding circle:

$$\mathcal{E} = -\frac{\mu}{2a} = KE + PE = KE - \frac{\mu}{r} \simeq KE - \frac{\mu}{a}$$

$$\rightarrow \quad KE \simeq \frac{\mu}{2a} = -\mathcal{E}, \quad PE \simeq -\frac{\mu}{a} = 2\mathcal{E}$$

$$\rightarrow \quad \frac{\delta PE}{\delta \mathcal{E}} = +2 \quad , \quad \frac{\delta KE}{\delta \mathcal{E}} = -1$$

As $\mathcal{E}$ increases by each $\Delta\mathcal{E}$ added due to thrusting, KE (speed) decreases by $\Delta\mathcal{E}$ while PE (orbital radius) increases by $2\Delta\mathcal{E}$. Paradoxical? (Later, you'll learn about the 'drag paradox' - the reverse of this.)

ΔV FOR SPIRAL CLIMB

Let's look at ΔV for a low-thrust transfer from circular orbit with $r = a_1$ to higher circular orbit with $r = a_2$.

$$\frac{d\mathcal{E}}{dt} = \frac{\vec{F}}{m} \cdot \vec{v} = \frac{F}{m_{sc}} \sqrt{\frac{\mu}{a}}$$

thrust

$$\frac{d}{dt}\left(-\frac{\mu}{2a}\right) = -\frac{V_{ex}\,\dot{m}_{sc}}{m_{sc}}\sqrt{\frac{\mu}{a}}$$

$$\sqrt{\frac{a}{\mu}}\frac{\mu}{2a^2}\frac{da}{dt} = -V_{ex}\frac{1}{m_{sc}}\frac{dm_{sc}}{dt}$$

$$\frac{\sqrt{\mu}}{2}\int_{a_1}^{a_2} a^{-3/2}\,da = -V_{ex}\int_{m_{sc}}^{m_{sc}-\Delta m_p}\frac{1}{m_{sc}}\,dm_{sc}$$

$$\frac{\sqrt{\mu}}{2}\left[-\frac{2}{\sqrt{a}}\right]\Bigg|_{a_1}^{a_2} = -V_{ex}\left[\ln m_{sc}\right]\Bigg|_{m_{sc}}^{m_{sc}-\Delta m_p}$$

$$\sqrt{\frac{\mu}{a_1}} - \sqrt{\frac{\mu}{a_2}} = V_{ex}\ln\left(\frac{m_{sc}}{m_{sc}-\Delta m_p}\right)$$

Notice that right side is same as ΔV if there had been no kinematic inefficiency. Call this $\Delta V_{effective}$, which can be thought of as a measure of Δm_p, the propellant mass required.

$$\Delta V_{eff} = \sqrt{\frac{\mu}{a_1}} - \sqrt{\frac{\mu}{a_2}} \quad (>0)$$

(Notice this is independent of thrust.)

Also, $t_{transfer} \approx \frac{\Delta V_{total}}{F/m_{sc}}$ if $m_{sc} \approx$ constant

(Notice this does depend on thrust, F)

EXAMPLE Comparison for transfer from 200 [km] LEO parking orbit to GEO

$$\Delta V_{total_{\substack{impulsive \\ Hohmann}}} = 3.93 \left[\frac{km}{sec}\right]$$

$$t_{transfer_{\substack{impulsive \\ Hohmann}}} = 5.3 \text{ [hours]}$$

$$\Delta V_{eff_{\substack{low-thrust \\ spiral}}} = 4.71 \left[\frac{km}{sec}\right]$$

$$t_{transfer_{\substack{low-thrust \\ spiral}}} = 130.8 \text{ [hours]}$$

* low-thrust thruster sized to provide acceleration $= 0.01 \left[\frac{m}{sec^2}\right] \approx 0.001 \, g_0$

This increase in ΔV required over the (optimal, impulsive) Hohmann transfer is called the 'KINEMATIC INEFFICIENCY' of the continuous low-thrust transfer.

HOWEVER, if low-thrust transfer is done with engine with significantly higher I_{sp} & V_{ex} than Hohmann/chemical rocket combination, <u>we can potentially use far less</u> propellant mass.

LAUNCH INTO EARTH ORBIT

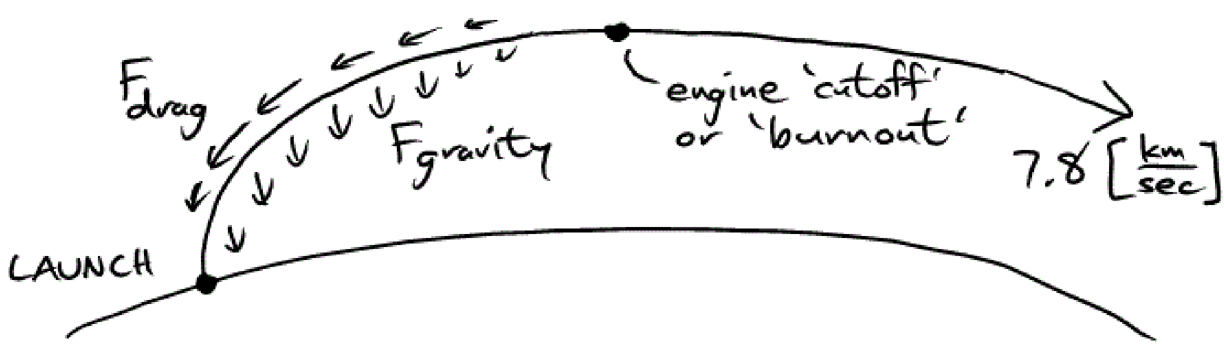
ΔV REQUIRED

Recall that circular velocity for LEO is $\sim 7.8 \left[\frac{km}{sec}\right]$. Amount of ΔV (~propellant) required is significantly more than this due to (in part):

1) CLIMB TO ORBITAL ALTITUDE
   (some thrust used to fight gravity component opposite direction of travel, even with impulsive thrusting)

2) ATMOSPHERIC DRAG
   (some thrust used to fight drag while in earth's atmosphere)

3) KINEMATIC INEFFICIENCY
   (thrust not instantaneous/impulsive, thrust spread out over time)



Typical ΔV_{eff} required to LEO is $9-10 \left[\frac{km}{sec}\right]$

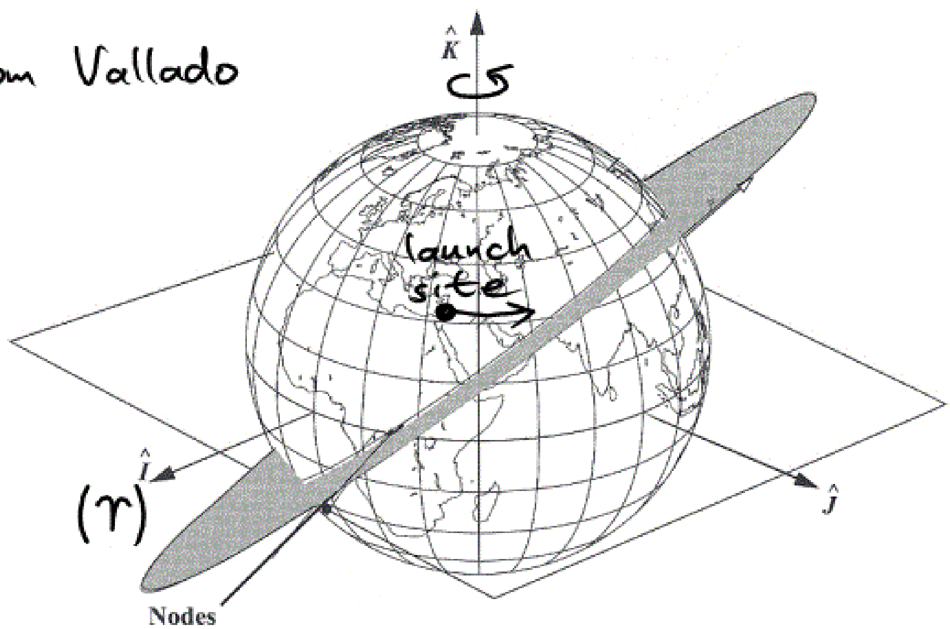
LAUNCH GEOMETRY

Often want to launch into a specific orbital plane (defined by i and Ω). Reasons include:

- rendezvous with something already in orbit (e.g. ISS with $i = 51.6°$ and defined Ω)
- achieve some desired science or remote sensing orbit

(We don't want to change orbital plane after launch, since that's expensive.)

from Vallado

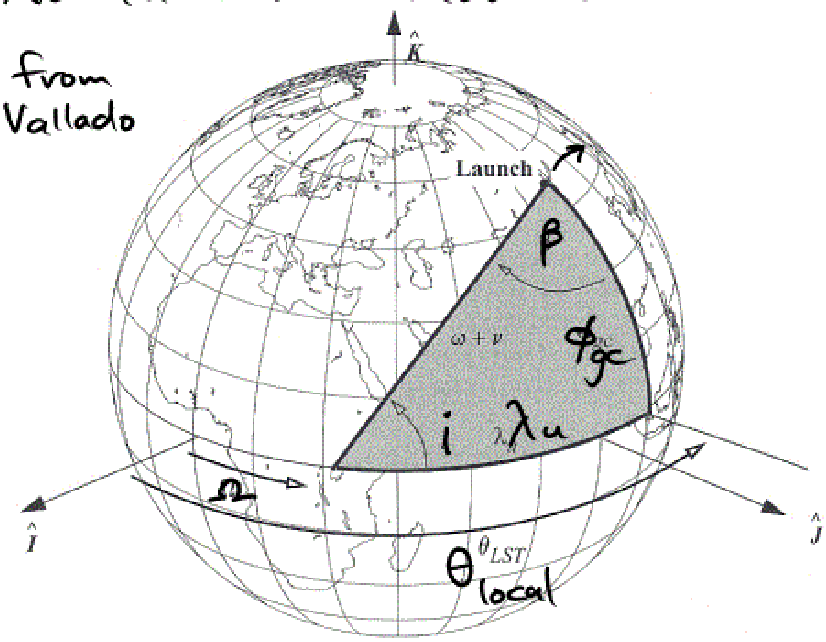


Therefore, need to wait until launch site slides underneath desired orbital plane.

Assuming we can launch directly into orbit on any launch azimuth we choose, we'll generally have two chances per earth revolution (day) to pass under target orbit. These are called "launch windows." (Can be as short as a few seconds.)

At launch window we have:

from Vallado



launch site is at ϕ_{gc}, λ

β is measured clockwise from True North (drawn here to be inside spherical triangle)

Window is defined by β (launch azimuth) and $\theta_{g_{launch}}$ (launch time). Steps in finding these will be:

1) Use spherical trig to find β
2) Use spherical trig to find auxilliary angle λ_u
3) Find $\theta_{g_{launch}}$ and t_{launch}

Performing these steps:

1) Law of Cosines for spherical triangle (Napier):

$$\cos i = \cos \phi_{gc} \sin \beta$$ of launch site

$$\Rightarrow \sin \beta = \frac{\cos i}{\cos \phi_{gc}}$$

(Only has solution for $|\cos i| < \cos \phi_{gc}$ or equivalently $|\phi_{gc}| < i < 180° - |\phi_{gc}|$)

This confirms observation that you can't launch to an inclination lower than the launch site latitude.)

Will normally have two solutions for β, one $<90°$ and one $>90°$, corresponding to 'ascending' and 'descending' parts of orbit.

2) Also from spherical Law of Cosines:

$$\cos \lambda_u = \frac{\cos \beta}{\sin i}$$ (will be used to finding launch time)

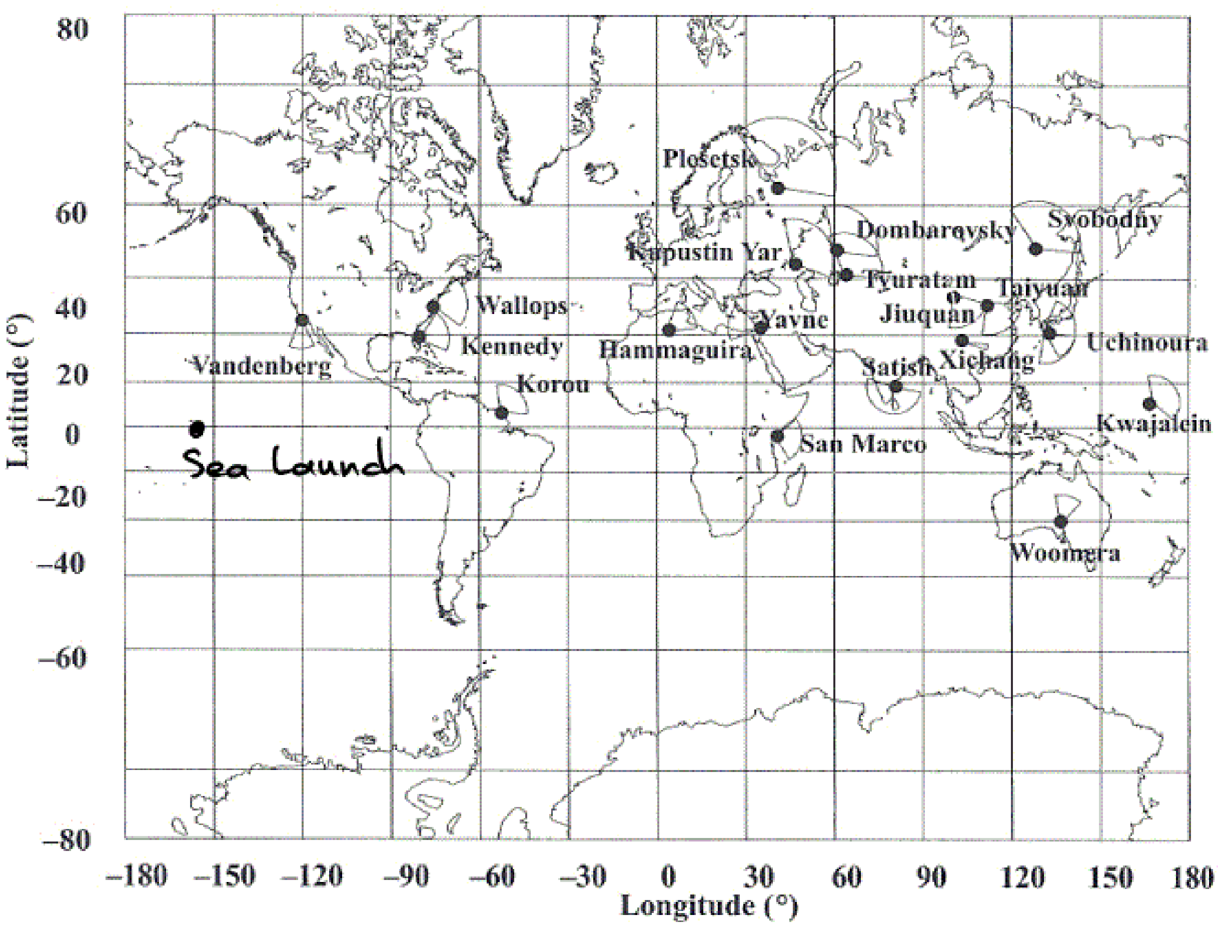
3) At launch time (by inspection)

$$\theta_{local} = \theta_g + \lambda = \Omega + \lambda_u \Rightarrow \theta_{g_{launch}} = \Omega + \lambda_u - \lambda$$

$$\Rightarrow t_{launch} = t_0 + \frac{\theta_{g_{launch}} - \theta_{g_0}}{\omega_\oplus}$$

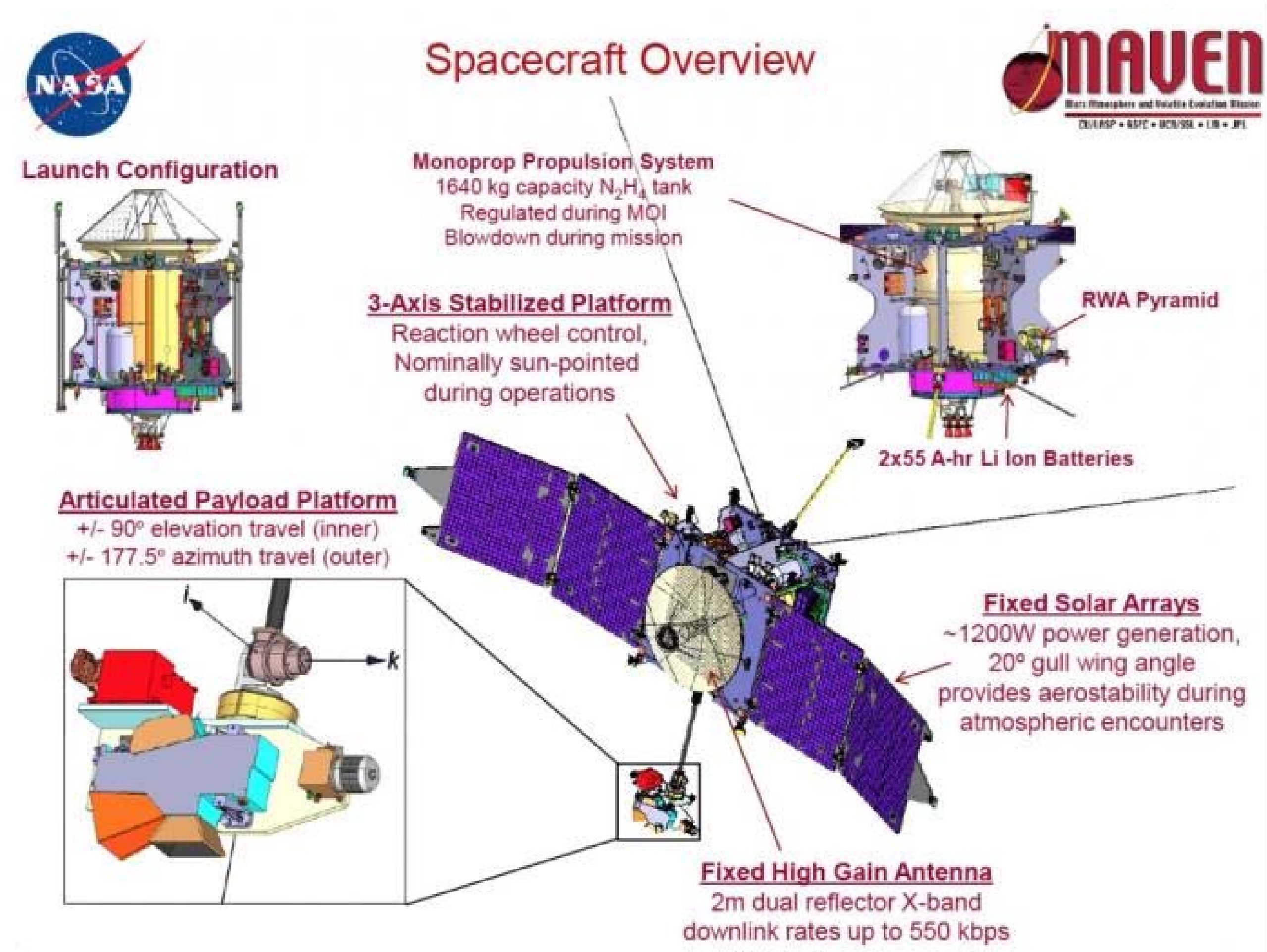
LAUNCH SITE CONSTRAINTS

Not all values of launch azimuth β are available at all launch sites.
Not all times are available either.
Can be limited by:
- range safety, populations/land downrange
- emergency abort procedures
- available range resources
- politics
- other mission requirements (e.g. daytime launch)



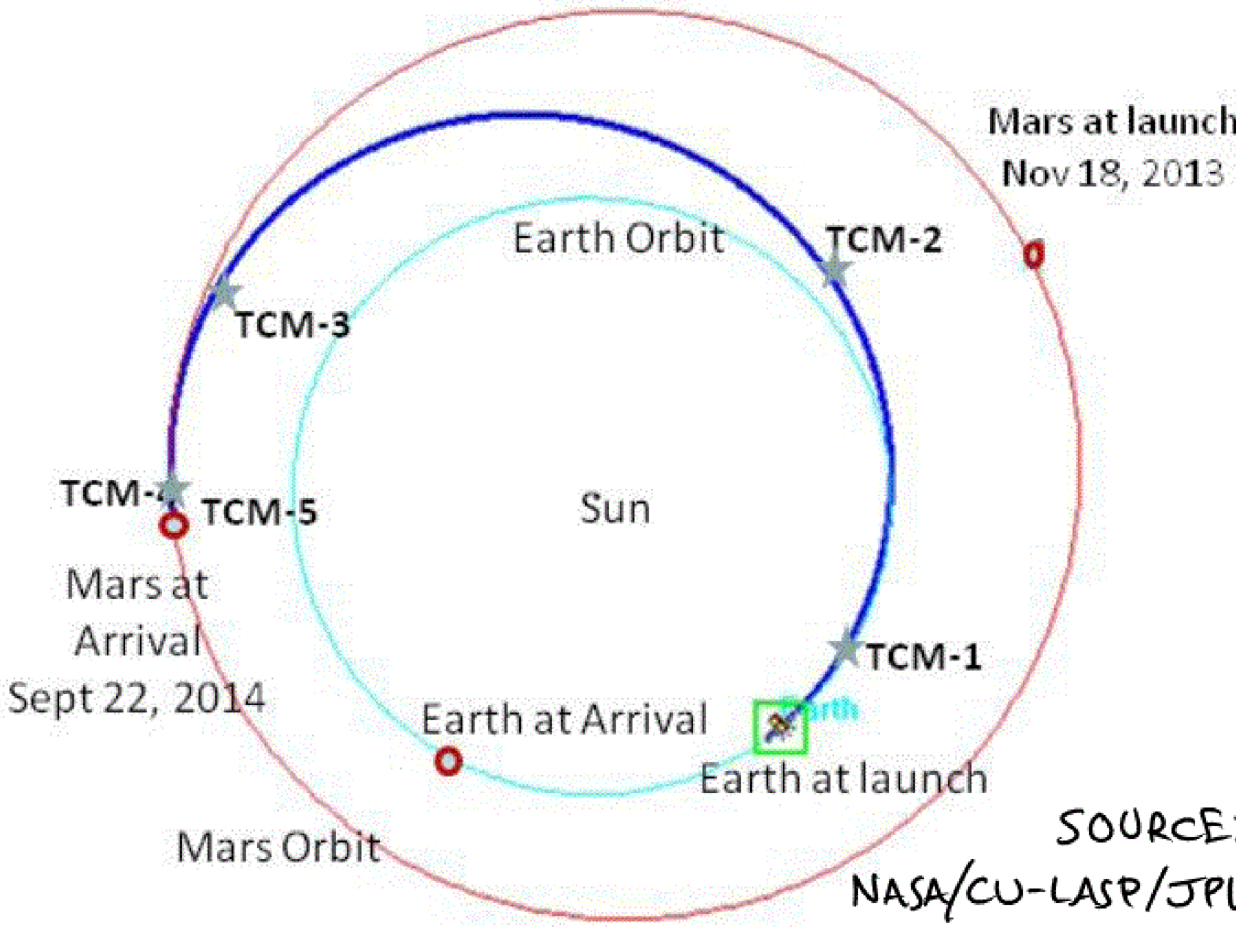
NASA MAVEN TO MARS

- Launched 18 November 2013
- Atlas V 401 Centaur rocket
- 6x 200N rockets on spacecraft
- NASA Scout mission, ~$670M
- Study Martian atmosphere & history



2,550 [kg] - as heavy as an SUV
~2.3 [m] cube
solar panels span 37.5 [m] - long as a schoolbus
had a 2-hour launch window on launch day

MAVEN TRAJECTORY



SOURCE: NASA/CU-LASP/JPL

(Powerful Centaur upper stage enabled straightforward earth departure trajectory.)