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AA 279 A – Space Mechanics
Lecture 6: Notes

Simone D'Amico, PhD
Assist. Prof., Aeronautics and Astronautics (AA)
Director, Space Rendezvous Laboratory (SLAB)
Satellite Advisor, Stanford Space Student Initiative (SSI)

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- Reading for next week (lectures 7 and 8)
  - Bate n/a
  - Montenbruck 6.2.2, 9.1.1
  - Vallado 6.8

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Earth-Centered Earth-Fixed Coordinate System

- Solar Day (d) – Familiar w.r.t. Sun

  Time between two subsequent meridian transits of sun is traditionally measured in solar days of **86400 [solar sec]**

- Sidereal Day – w.r.t. inertial direction

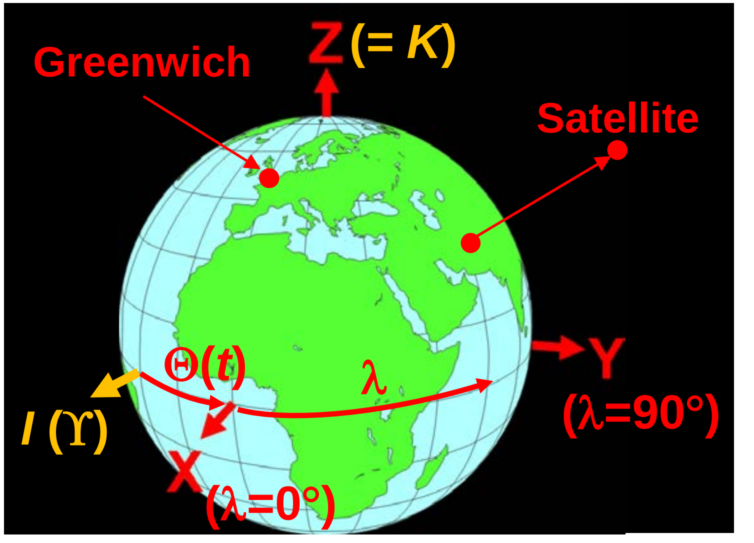
  Time taken for earth to rotate once w.r.t. inertial space, i.e. for Θ to span 360°, takes about **86164 [solar sec]**, i.e. about 4 solar minutes shorter than solar day

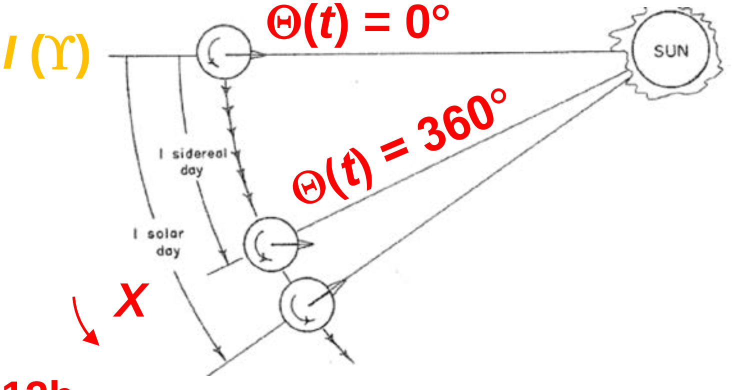
- Neglecting small secular changes, the *Greenwich mean sidereal time* is given by

$$\Theta(d) = 280.4606° + 360.9856473° \cdot d$$

Solar time in days since January 1, 2000 at 12h

Ideal configuration ECI/ECEF





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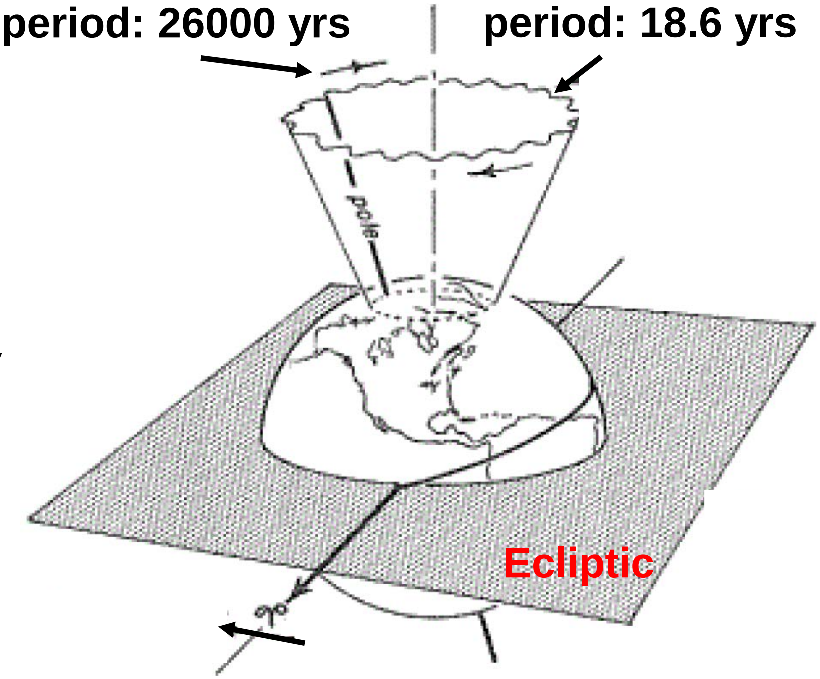
Time and Reference Systems

Actual Earth's rotation dynamics

- So far we have tacitly assumed the availability of a unique time and reference system for the equation of motion (inertial)
- Traditionally, time and reference systems have been tied to the Earth's rotation and its revolution around the Sun
- In practice the Earth's orbit is slightly eccentric (e = 0.0167). In addition we don't live in a 2-body system and the rotational dynamics of the Earth is complex (perturbations are covered later)
- The Moon, Sun, and other planets act on a non-spherical Earth and produce torques on top of the purely radial attractive force
- This causes secular, periodic and short-term variations of the vernal equinox, equator, and Earth's rotation axis

Precession's period: 26000 yrs

Nutation's period: 18.6 yrs



Secular drift westward of vernal equinox

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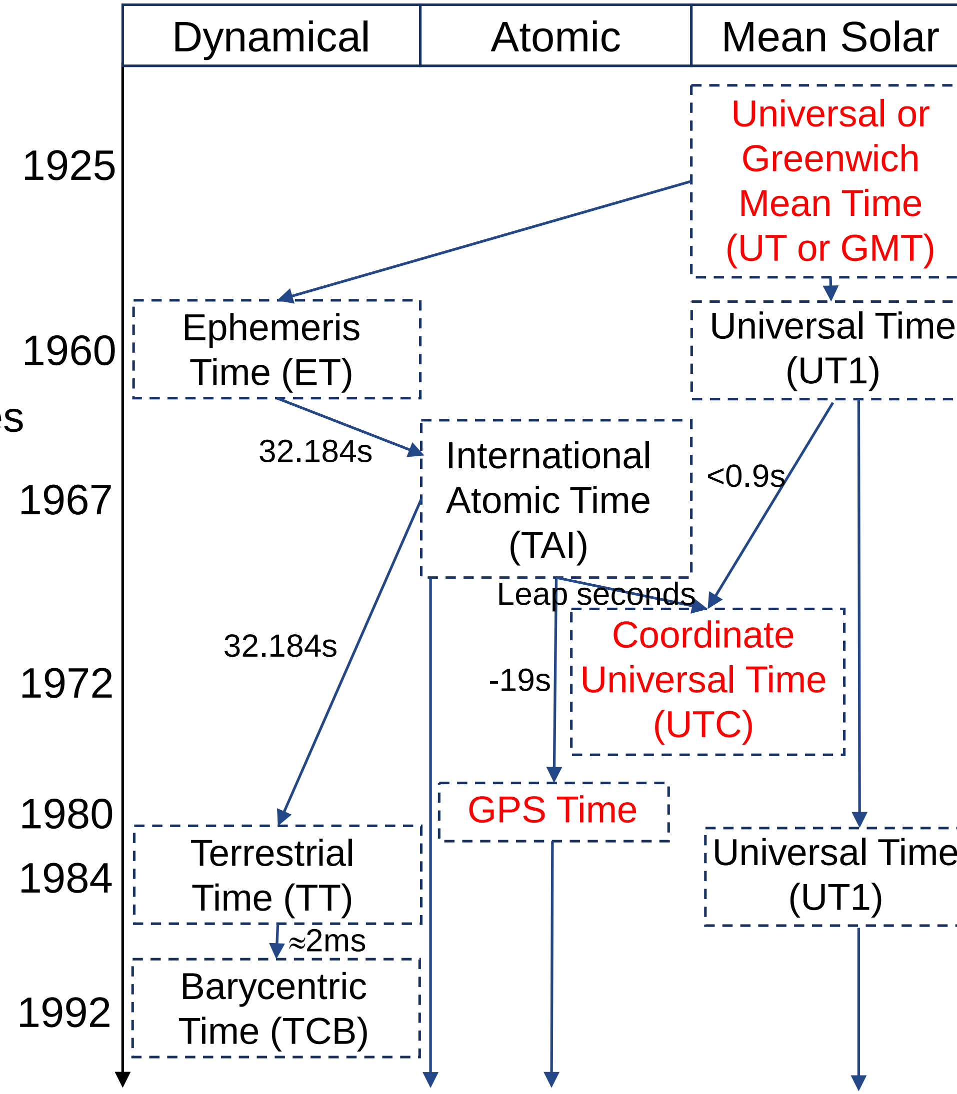
Evolution of Time Scales

- Solar time
  - Tied to motion of Earth/Sun
  - Based on *mean* vernal equinox
  - Non-uniform
- Dynamical time
  - Tied to motion of solar system bodies
  - Based on dynamics model/theory
  - Uniform (theoretical)
- Atomic time
  - Tied to periodic properties of atoms
  - Based on laboratory measurements
  - Uniform (practical)
- Conversions between time scales are needed depending on adopted Earth's rotation model



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Simplified Calendrical Calculations

- Earth's rotation is typically expressed through Greenwich mean sidereal time Θ
- Spacecraft time is typically expressed through more familiar t_{UTC} (or t_{GPS}) time
- Intermediate steps are required to convert between these time scales
- Since the civilian calendar is not well suited to handle time calculations, a Modified Julian Date (MJD) is used
- The algorithm to compute MJD from a calendar date is given in "Montenbruck" on page 321 (A.1.1)

1) Obtain necessary time differences $t_{\mathrm{UT1}} - t_{\mathrm{UTC}}$ (and $t_{\mathrm{UTC}} - t_{\mathrm{GPS}}$) from IERS bulletin
2) Compute universal time t_{UT1}
3) Compute Modified Julian Date (MJD) corresponding to t_{UT1}
4) Compute approximate Greenwich mean sidereal time in [deg]

$$\Theta(d) = 280.4606° + 360.9856473° \cdot d$$

$$d = MJD - 51544.5$$

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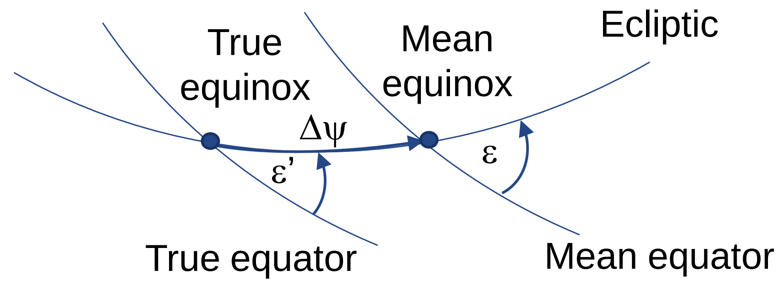
Conventional Reference Systems

- **Celestial Reference Frame (CRF)**
  - Quasi-inertial
  - Origin at Earth's center of mass
  - I-axis aligned with mean equinox at J2000 epoch
  - K-axis normal to mean equator's plane at J2000 epoch
- Determined from a set of precise coordinates of extragalactic radio sources
- Mean equator and equinox of J2000 were defined by the International Astronomical Union (IAU)
- *Earth Mean Equator and Equinox of J2000* (EME2000) is a practical implementation based on the FK5 star catalogue

- **Terrestrial Reference Frame (TRF)**
  - Co-rotating with Earth but slightly moving w.r.t. Earth's crust
  - Origin at Earth's center of mass
  - Z-axis aligned with International Earth Rotation Service (IERS) pole
  - X-axis defined by intersection of plane normal to Z and Greenwich mean meridian
- Determined from a set of precise coordinates of ground stations and updated every year
- *World Geodetic System 1984* (WGS84) is another popular implementation based on the Global Positioning System (GPS)

?

Coordinate Transformation



- The transformation between CRF and TRF is accomplished by models for
  - Precession

    Secular change in orientation of Earth's rotation axis and equinox
  - Nutation

    Periodic and short-term variation of equator and equinox
  - Sidereal Time (w.r.t. UT1)

    Earth's rotation about its axis
- These models are supplemented by the IERS Earth Observation Parameters (EOP), comprising
  - Observations of UT1-TAI difference
  - Measured coordinates of rotation axis relative to IERS pole

$$\vec{r}_{\mathrm{MeanOfDate}} = \vec{R}_{\mathrm{Pre}}(t)\vec{r}_{\mathrm{CRS}}$$

≈ 50''/year (1.54km/year)

$$\vec{r}_{\mathrm{TrueOfDate}} = \vec{R}_{\mathrm{Nut}}(t)\vec{r}_{\mathrm{MeanOfDate}}$$

≈ 20'' (618m)

$$\vec{r}_{\mathrm{TRS'}} = \vec{R}_{\mathrm{Rot}}(t)\vec{r}_{\mathrm{TrueOfDate}} = \vec{R}_{z}(\Theta + \Delta\psi\cos\varepsilon)\vec{r}_{\mathrm{TrueOfDate}}$$

Equation of Equinoxes: ≈ 20'' (618m)

UT1-UTC: ≈ 15'' (464m)

$$\vec{r}_{\mathrm{TRS}} = \vec{R}_{\mathrm{Pol}}(t)\vec{r}_{\mathrm{TRS'}}$$

≈ 0.3'' (10m)

$$\vec{r}_{\mathrm{TRS}} = \vec{R}_{\mathrm{Pol}}(t)\vec{R}_{\mathrm{Rot}}(t)\vec{R}_{\mathrm{Nut}}(t)\vec{R}_{\mathrm{Pre}}(t)\vec{r}_{\mathrm{CRS}}$$

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