AA 279 C – SPACECRAFT ADCS: LECTURE 6

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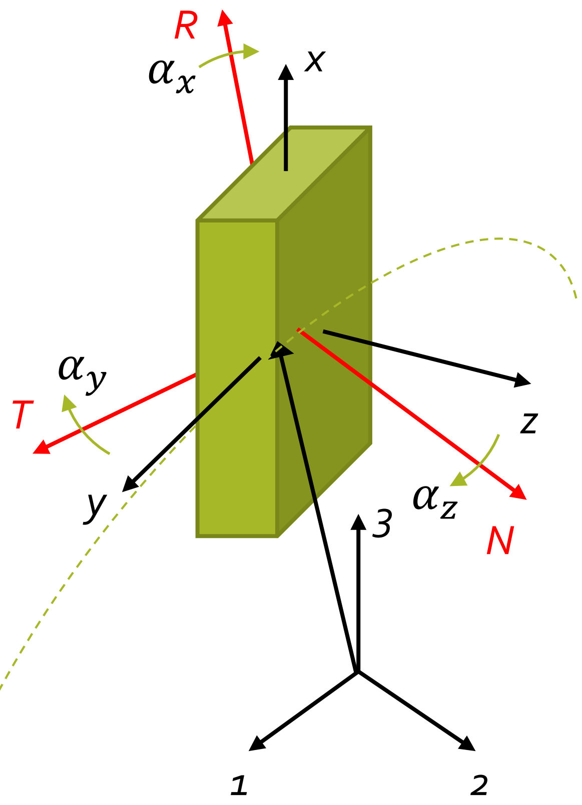
Single-Spin, Angles (1)



- In order to study the orientation of the spacecraft relative to the orbital frame, we select the following equilibrium configuration (step 2 of analysis procedure)

$$\begin{cases} \bar{\omega}_x = \bar{\omega}_R = 0 \\ \bar{\omega}_y = \bar{\omega}_T = 0 \\ \bar{\omega}_z = \bar{\omega}_N \end{cases} ; \ \vec{\omega}_{xyz} = \vec{A}\vec{\omega}_{RTN} = \begin{bmatrix} 1 & \alpha_z & -\alpha_y \\ -\alpha_z & 1 & \alpha_x \\ \alpha_y & -\alpha_x & 1 \end{bmatrix} \vec{\omega}_{RTN}$$

Alignment with RTN frame at equilibrium

Perturbation of equilibrium through small Euler angles which change orientation and speed

$$\vec{\omega}_{RTN} = \begin{Bmatrix} \dot{\alpha}_x \\ \dot{\alpha}_y \\ \dot{\alpha}_z + \bar{\omega}_N \end{Bmatrix} \Rightarrow \vec{\omega}_{xyz} = \begin{Bmatrix} \dot{\alpha}_x + \alpha_z \dot{\alpha}_y - \alpha_y (\dot{\alpha}_z + \bar{\omega}_N) \\ \dot{\alpha}_y - \alpha_z \dot{\alpha}_x + \alpha_x (\dot{\alpha}_z + \bar{\omega}_N) \\ \alpha_y \dot{\alpha}_x - \alpha_x \dot{\alpha}_y + (\dot{\alpha}_z + \bar{\omega}_N) \end{Bmatrix} \Rightarrow \vec{\omega}_{xyz} = \begin{Bmatrix} \dot{\alpha}_x - \alpha_y \bar{\omega}_N \\ \dot{\alpha}_y + \alpha_x \bar{\omega}_N \\ \dot{\alpha}_z + \bar{\omega}_N \end{Bmatrix}$$

Perturbed angular velocity in RTN

II order perturbations

Perturbed angular velocity in body frame

Single-Spin, Angles (2)

- We substitute perturbed angular velocity in Euler equations and neglect second order terms (step 3 of analysis procedure)

$$\begin{cases} I_x\left(\ddot{\alpha}_x - \dot{\alpha}_y\bar{\omega}_N\right) + \bar{\omega}_N\left(I_z - I_y\right)\left(\dot{\alpha}_y + \alpha_x\bar{\omega}_N\right) = 0 \\ I_y\left(\ddot{\alpha}_y + \dot{\alpha}_x\bar{\omega}_N\right) + \bar{\omega}_N\left(I_x - I_z\right)\left(\dot{\alpha}_x - \alpha_y\bar{\omega}_N\right) = 0 \\ I_z\ddot{\alpha}_z = 0 \end{cases}$$

3rd equation is still decoupled from other 2, but now unstable

- The first 2 equations are second order and can be re-written as 4 equations of first order in the standard form

$$\begin{cases} \ddot{\alpha}_x = (1 - k_x)\bar{\omega}_N\dot{\alpha}_y - k_x\bar{\omega}_N^2\alpha_x \\ \ddot{\alpha}_y = -(1 + k_y)\bar{\omega}_N\dot{\alpha}_x + k_y\bar{\omega}_N^2\alpha_y \\ \dot{\alpha}_x = \dot{\alpha}_x \\ \dot{\alpha}_y = \dot{\alpha}_y \end{cases} \Rightarrow \begin{Bmatrix} \ddot{\alpha}_x \\ \ddot{\alpha}_y \\ \dot{\alpha}_x \\ \dot{\alpha}_y \end{Bmatrix} = \begin{bmatrix} 0 & (1 - k_x)\bar{\omega}_N & -k_x\bar{\omega}_N^2 & 0 \\ -(1 + k_y)\bar{\omega}_N & 0 & 0 & +k_y\bar{\omega}_N^2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{\alpha}_x \\ \dot{\alpha}_y \\ \alpha_x \\ \alpha_y \end{Bmatrix}$$

- The moments of inertia are

$$\begin{cases} -1 < k_x = \left(I_z - I_y\right)/I_x < 1 \\ -1 < k_y = \left(I_x - I_z\right)/I_y < 1 \end{cases}$$

- Eigenvalues

$$\begin{cases} \lambda_{1,2} = \pm\sqrt{k_x k_y} \\ \lambda_{3,4} = \pm i \end{cases}$$

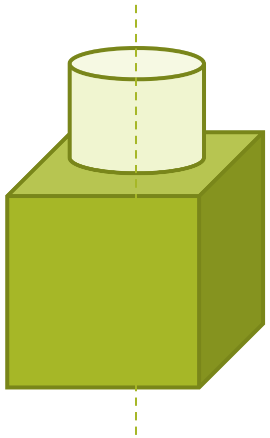
- Stability condition

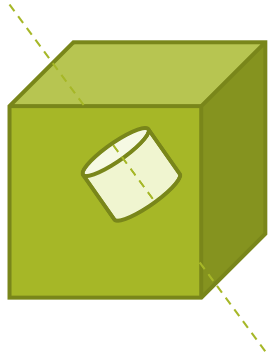
$$\left(I_z - I_y\right)\left(I_x - I_z\right) < 0$$

Same result as for angular velocities

Dual-Spin, Introduction

- Problems of single-spin satellite
  - size and geometry, or volume of the satellite are constrained by the launcher
  - rotation about the axis of maximum inertia is not always desired
  - desired angular velocity might be too small to provide angular momentum vector for stability
- Alternative solution
  - Passive stabilization using a dual-spin satellite
  - Dual-spin means that a momentum wheel or a rotor rotate about an arbitrary axis relative to satellite
- The rotor changes the total angular momentum vector so to facilitate stability of rotations about arbitrary axes





Dual-Spin, No Dissipations (1)

- The total angular momentum vector is given by the sum of the angular momentum vector of the satellite (considering that the rotor is not moving) and the rotor (moving relative to satellite)

$$\begin{cases} \vec{L}_{\mathrm{SAT}} = I_x\omega_x\vec{x} + I_y\omega_y\vec{y} + I_z\omega_z\vec{z} \\ \vec{L}_{\mathrm{ROT}} = I_r\omega_r\vec{r} \end{cases} \Rightarrow \vec{L} = \begin{cases} I_x\omega_x + I_r\omega_r r_x \\ I_y\omega_y + I_r\omega_r r_y \\ I_z\omega_z + I_r\omega_r r_z \end{cases}$$

Angular velocity of rotor relative to satellite

1. The Euler equations can be re-written considering the additional rotor term

$$\begin{cases} I_x\dot{\omega}_x + I_r\dot{\omega}_r r_x + \left(I_z - I_y\right)\omega_y\omega_z + I_r\omega_r\left(\omega_y r_z - \omega_z r_y\right) = M_x \\ I_y\dot{\omega}_y + I_r\dot{\omega}_r r_y + \left(I_x - I_z\right)\omega_z\omega_x + I_r\omega_r\left(\omega_z r_x - \omega_x r_z\right) = M_y \\ I_z\dot{\omega}_z + I_r\dot{\omega}_r r_z + \left(I_y - I_x\right)\omega_x\omega_y + I_r\omega_r\left(\omega_x r_y - \omega_y r_x\right) = M_z \\ I_r\dot{\omega}_r = M_r \end{cases}$$

Equations of motion for dual-spin satellite (4 unknowns)

Dual-Spin, No Dissipations (2)

2. Equilibrium

Simplest possible equilibrium

Satellite rotation about principal axis is equilibrium if rotor is along that axis

$$\begin{cases} (I_z - I_y)\bar{\omega}_y\bar{\omega}_z + I_r\bar{\omega}_r(\bar{\omega}_y r_z - \bar{\omega}_z r_y) = 0 \\ (I_x - I_z)\bar{\omega}_z\bar{\omega}_x + I_r\bar{\omega}_r(\bar{\omega}_z r_x - \bar{\omega}_x r_z) = 0 \\ (I_y - I_x)\bar{\omega}_x\bar{\omega}_y + I_r\bar{\omega}_r(\bar{\omega}_x r_y - \bar{\omega}_y r_x) = 0 \\ \omega_r = \bar{\omega}_r = \text{const} \end{cases} \Rightarrow \begin{cases} \bar{\omega}_y = 0 \\ \bar{\omega}_x = 0 \\ \bar{\omega}_z \neq 0 \\ \omega_r = \bar{\omega}_r = \text{const}, r_x = r_y = 0 \end{cases}$$

3. Perturbation

Re-notation

$$\begin{cases} \omega_y = \omega_y \\ \omega_x = \omega_x \\ \omega_z = \bar{\omega}_z + \omega_z \\ \omega_r = \bar{\omega}_r + \omega_r \end{cases} \Rightarrow$$

4. Substitution in equations of motion

Neglecting II order perturbations

$$\begin{cases} I_x\dot{\omega}_x + (I_z - I_y)\omega_y\bar{\omega}_z + I_r\bar{\omega}_r\omega_y = 0 \\ I_y\dot{\omega}_y + (I_x - I_z)\bar{\omega}_z\omega_x - I_r\bar{\omega}_r\omega_x = 0 \\ I_z\dot{\omega}_z + I_r\dot{\omega}_r = 0 \\ I_r\dot{\omega}_r = 0 \end{cases}$$

3rd and 4th equations are decoupled from other 2 (and stable)

Dual-Spin, No Dissipations (3)

5. Linear system

$$\begin{cases} I_x\dot{\omega}_x + \left[\left(I_z - I_y\right)\overline{\omega}_z + I_r\overline{\omega}_r\right]\omega_y = 0 \\ I_y\dot{\omega}_y + \left[\left(I_x - I_z\right)\overline{\omega}_z - I_r\overline{\omega}_r\right]\omega_x = 0 \end{cases} \Rightarrow \begin{cases} I_x\dot{\omega}_x + a\omega_y = 0 \\ I_y\dot{\omega}_y + b\omega_x = 0 \end{cases} \Rightarrow \dot{\vec{x}} = \vec{A}\vec{x}\,;\, \vec{A} = \begin{bmatrix} 0 & -\frac{a}{I_x} \\ -\frac{b}{I_y} & 0 \end{bmatrix}$$

$$\det\left(\lambda\vec{I} - \vec{A}\right) = 0 \Rightarrow \lambda^2 - \frac{ab}{I_xI_y} = 0 \Rightarrow \lambda_{1,2} = \pm\sqrt{\frac{ab}{I_xI_y}}$$

6. Stability (mathematical, not yet physical)

$$\mathrm{Re}(\lambda_i) < 0\,, \forall i \Rightarrow ab < 0 \Rightarrow \begin{cases} \left[\left(I_z - I_y\right)\overline{\omega}_z + I_r\overline{\omega}_r\right] > 0 \\ \left[\left(I_z - I_x\right)\overline{\omega}_z + I_r\overline{\omega}_r\right] > 0 \end{cases} \text{OR} \begin{cases} \left[\left(I_z - I_y\right)\overline{\omega}_z + I_r\overline{\omega}_r\right] < 0 \\ \left[\left(I_z - I_x\right)\overline{\omega}_z + I_r\overline{\omega}_r\right] < 0 \end{cases} \Rightarrow$$

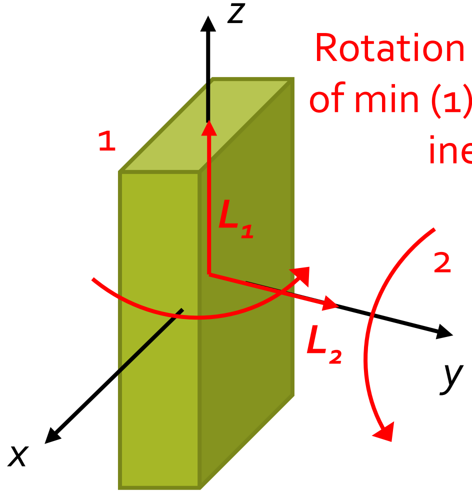
$[I_r\overline{\omega}_r > \left(I_y - I_z\right)\overline{\omega}_z$ AND $I_r\overline{\omega}_r > (I_x - I_z)\overline{\omega}_z]$ OR

$[I_r\overline{\omega}_r < \left(I_y - I_z\right)\overline{\omega}_z$ AND $I_r\overline{\omega}_r < (I_x - I_z)\overline{\omega}_z]$

Irrespective of inertia properties we can stabilize the motion through a proper selection of the rotor's angular velocity, it is even not necessary to spin the satellite ($\overline{\omega}_z = 0$)

Single-Spin, Dissipations (1)

- According to math, two configurations are stable, which correspond to rotation about max. and min. inertia
- In practice, rotational kinetic energy is not conserved (due to dissipations), and system tends to reach equilibrium characterized by minimum energy (among all possible)
- We consider the two identified equilibrium points and assign same magnitude of angular momentum vector (no torques)



Rotation about axis of min (1) or max (2) inertia

$$L_{z1} = I_z\omega_{z1} = L_{y2} = I_y\omega_{y2} \Rightarrow \omega_{y2} < \omega_{z1}$$

$$\begin{cases} 2T_{z1} = I_z\omega_{z1}^2 = L_{z1}\omega_{z1} \\ 2T_{y2} = I_y\omega_{y2}^2 = L_{y2}\omega_{y2} \end{cases} \Rightarrow \boxed{T_{y2} < T_{z1}}$$

In a real physical system, if 1 is the initial condition, the body will reach configuration 2 through an overturning

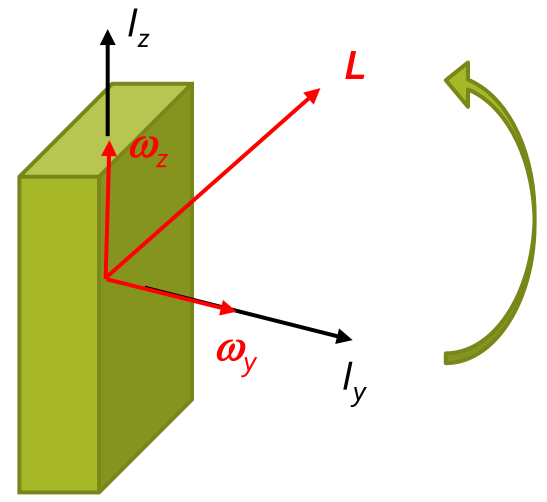
Single-Spin, Dissipations (2)

- General physical effect of dissipation

$$\begin{cases} L^2 = I_z^2\omega_z^2 + I_y^2\omega_y^2 \\ 2T = I_z\omega_z^2 + I_y\omega_y^2 \end{cases} \Rightarrow \begin{cases} I_y\omega_y\dot{\omega}_y = -\dfrac{I_z^2}{I_y}\omega_z\dot{\omega}_z \\ \dot{T} = I_z\omega_z\dot{\omega}_z\left(1 - \dfrac{I_z}{I_y}\right) \end{cases} \Rightarrow$$

$$\begin{cases} \omega_y\dot{\omega}_y = -\dot{T}\,\dfrac{I_z}{I_y(I_y-I_z)} \\ \omega_z\dot{\omega}_z = \dot{T}\,\dfrac{I_y}{I_z(I_y-I_z)} \end{cases} \Rightarrow \quad \boxed{\omega_z \downarrow, \omega_y \uparrow}$$

Rotation about axis of min (z) and max (y) inertia



In a physical system, the axis of max inertia tends to align with the angular momentum vector

- Knowing $\dot{T}$(<0), and the initial ω_y(>0) and ω_z(>0), we can determine the evolution of motion

Backup