



Q&A of GrabCut

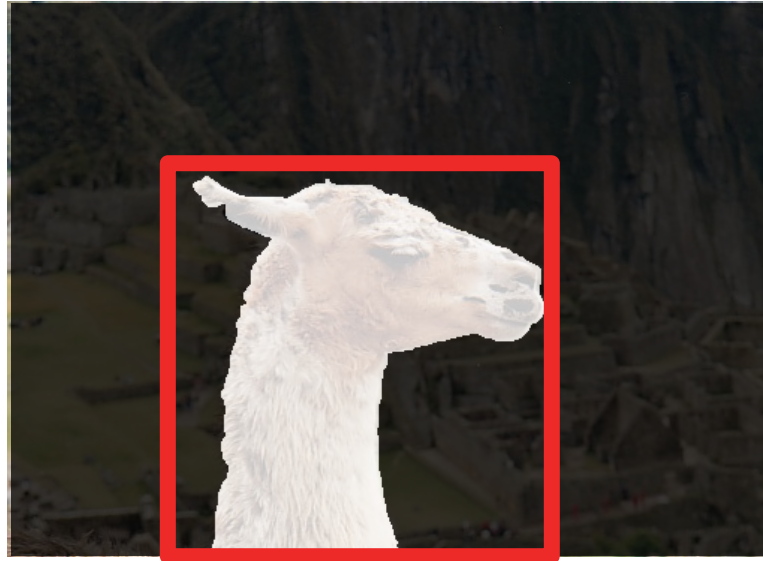
Philipp Krähenbühl

Goal



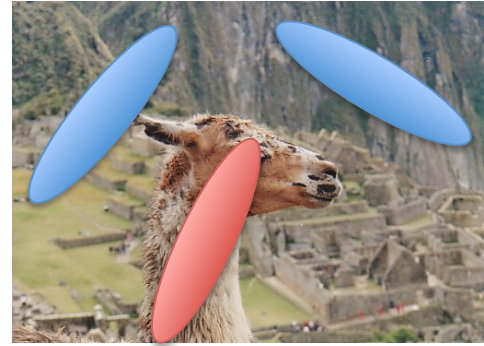
- Bounding Box
 - provided

Goal



- Segmentation of object within bounding box

Overview



- Boykov & Jolly segmentation model

$$E(\alpha, z) = \sum_n D(\alpha_n, z_n) + \gamma \sum_{n,m} w_{n,m} [\alpha_n \neq \alpha_m]$$

- Segmentation using Graph Cuts

- GMM foreground and background model

$$D(\alpha_n, \theta, z_n)$$

- Iterative optimization

- Graph Cuts
- GMM estimation

Initialisation

- User initialises trimap T by supplying only T_B . The foreground is set to $T_F = \emptyset$; $T_U = \overline{T_B}$, complement of the background.
- Initialise $\alpha_n = 0$ for $n \in T_B$ and $\alpha_n = 1$ for $n \in T_U$.
- Background and foreground GMMs initialised from sets $\alpha_n = 0$ and $\alpha_n = 1$ respectively.

Iterative minimisation

1. *Assign GMM components to pixels:* for each n in T_U ,

$$k_n := \arg \min_{k_n} D_n(\alpha_n, k_n, \theta, z_n).$$

2. *Learn GMM parameters from data \mathbf{z} :*

$$\underline{\theta} := \arg \min_{\underline{\theta}} U(\underline{\alpha}, \mathbf{k}, \underline{\theta}, \mathbf{z})$$

3. *Estimate segmentation:* use min cut to solve:

$$\min_{\{\alpha_n: n \in T_U\}} \min_{\mathbf{k}} \mathbf{E}(\underline{\alpha}, \mathbf{k}, \underline{\theta}, \mathbf{z}).$$

4. Repeat from step 1, until convergence.
5. Apply border matting (section 4).

User editing

- *Edit:* fix some pixels either to $\alpha_n = 0$ (background brush) or $\alpha_n = 1$ (foreground brush); update trimap T accordingly. Perform step 3 above, just once.
- *Refine operation:* [optional] perform entire iterative minimisation algorithm.

Project 2

Not required

Optional

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Notation

- Trimaps
 - Sets of pixels
 - T_B : background
 - T_F : foreground
 - T_U : undecided
- Segmentation
 - α_i for each pixel i

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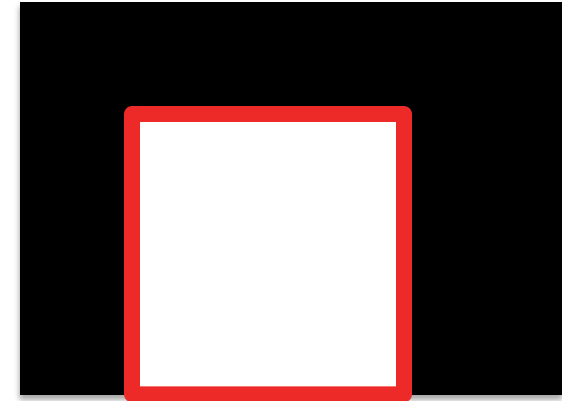
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Initialization



- Outside
 - Background (**fixed**)
 - $\alpha_n = 0$
- Inside
 - Initial foreground
 - $\alpha_n = 1$
 - **updated**

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GMM Initialization

- Find parameters θ
- Standard method
 - random + EM
- Re-estimate θ in step 2
 - Initialization only needed for step 1
 - Trick: Initialize \mathbf{k} instead

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GMM Initialization

- Trick: Initialize k
 - k-means clustering
 - skip step 1 in first iteration

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GMM assignment

- Gaussian log prob.

$$\begin{aligned} D_n(\alpha_n, k_n, \theta, z_n) = & \\ & -\log \pi(\alpha_n, k_n) + \frac{1}{2} \log \det \Sigma(\alpha_n, k_n) \\ & + \frac{1}{2} [z_n - \mu(\alpha_n, k_n)]^T \Sigma(\alpha_n, k_n)^{-1} [z_n - \mu(\alpha_n, k_n)] \end{aligned}$$

- Enumerate all k_n
- Each pixel already assigned to FG or BG (α_n fixed)

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GMM leaning

- Mixture param.

$$\pi(\alpha = 1, k) = \frac{|F(k)|}{\sum_k |F(k)|}$$

$$\mu(\alpha = 1, k) = \text{mean}_{n \in F(k)}(z_n)$$

$$\Sigma(\alpha = 1, k) = \text{cov}_{n \in F(k)}(z_n)$$

- $F(k)$ set of FG pixels assigned to comp. k

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Segmentation

- Find segmentation that minimizes

$$\min_k E(\alpha, k, \theta, z) = \sum_n \underbrace{\min_{k_n} D(\alpha_n, k_n, \theta, z_n)}_{D(\alpha_n, \theta, z_n)} + \gamma \sum_{n,m} w_{n,m} [\alpha_n \neq \alpha_m]$$

- Reduces to Boykov & Jolly
- Solved using GraphCut

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Use **energy.h** and **maxflow.cpp**

```
// initialization
std::vector<Energy::Var> vars(N);
Energy e;

// add a node
vars[i] = e.add_variable();

// add the unary term for a node
e.add_term1(vars[i], u0, u1);
// add the pairwise term for an edge
e.add_term2(vars[i], vars[j], p00,
p01, p10, p11);

// perform energy minimization
Energy::TotalValue mnE = e.minimize();

// get new labels
if (e.get_var(vars[i])) label[i] = 1;
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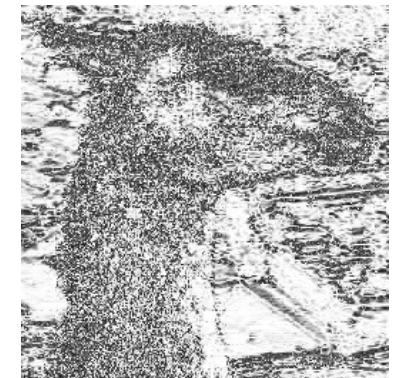
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$$v_{ij} = \gamma \exp(-\beta \|z_i - z_j\|^2)$$



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Optimizations

- Use MEX files for Graph Cut
 - Call C/C++ code from matlab
 - compile: “mex a.cpp b.cpp c.cpp ...”
- Vectorize
 - f : $N \times 3$ matrix of RGB color values
 - a : N -dimensional binary vector (segmentation)
 - $f(a==1,:)$: foreground features
 - $f(a==0,:)$: background features

Implementation Questions?

Extensions

- User interaction or border matting
- Play with GMMs
 - Vary number of components
 - Different initialization
 - Different color space (Lab)
- Different Color model
 - Histogram based model

Extensions

- Different Neighborhood System
 - 4 connected
 - 8 connected
 - fully connected (DenseCRF)
 - Efficient Inference in Fully Connected CRFs with Gaussian Edge Potentials [Krähenbühl and Koltun 2011]
- Different “affinity”
 - Lab color difference
 - Contour detector gPb
 - Contour Detection and Hierarchical Image Segmentation [Arbelaez et al 2010]

Questions?