平成 29 年度 S2 学期定期試験問題

試験科目　　　　　　　　出題教員名

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One can use English and/or Japanese for answering the questions. One can assume the existence of the standard "idealistic" market conditions used in the lecture course; liquidity, completeness, absence of arbitrage, and the existence of a unique risk-neutral measure when it is used. One can use the following functions for the standard normal distribution:

$$\phi(z) := \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right), \qquad N(x) := \int_{-\infty}^{x}\phi(z)dz\ .$$

$\log(\cdot)$ denotes the natural logarithm with base "e". $1bp$:=0.01%.

Problem 1

Suppose that the exchange rate $(X_t, t \geq 0)$ of a foreign currency, i.e. the price of the unit amount of the foreign currency (FC) in terms of the domestic currency (DC), is given by

$$X_s = X_0 + \int_0^s (r_d - r_f)X_t dt + \int_0^s X_t \sigma dW_t^{\mathbb{Q}},\ s \geq 0$$

with $X_0 > 0$. Here, $r_d, r_f > 0$ are domestic as well as foreign risk-free interest rates assumed to be constant. $\sigma > 0$ is a given constant representing the volatility of the exchange rate. $W^{\mathbb{Q}}$ is a one-dimensional Brownian motion under the domestic risk-neutral measure $\mathbb{Q}$. Answer the following questions.

(1.1) Explain the definition of the "forward" exchange rate. Moreover, give the current $(t = 0)$ forward exchange rate F of the above FC for the forward contract with maturity $T > 0$.

(1.2) Apply Itô formula to $(\log X_t, t \geq 0)$. In addition, express X_T at some future time $T > 0$ in terms of $\{X_0, T, r_d, r_f, \sigma, W_T^{\mathbb{Q}}\}$.

(1.3) Calculate the probability $\mathbb{Q}(\{X_T > K\})\ \left(= \mathbb{E}^{\mathbb{Q}}[\mathbf{1}_{\{X_T > K\}}]\right)$.

(1.4) Calculate the expectation $\mathbb{E}^{\mathbb{Q}}\left[X_T \mathbf{1}_{\{X_T \leq K\}}\right]$.

Suppose that there exists a bank offering the following FX-linked structured deposit: *(i) At time 0, the client deposits c (> 0) unit of FC (by paying cX_0 in DC). (ii) At time T, the client receives*

$$\begin{cases} ce^{r_f T} & \textit{in FC} \quad \textit{if} \quad X_T \leq K \\ cX_0 e^{RT} & \textit{in DC} \quad \textit{if} \quad X_T > K \end{cases}$$

with some $R > 0$. Here, $K > 0$ is a given constant.

(1.5) Give the possible maximal loss for the depositor in DC without including opportunity costs.

(1.6) Express the present value (at time 0) of the deposit for the client in terms of DC, and then derive the *fair* deposit rate R_* which is the rate R that makes the present value of the deposit equal to cX_0.

(1.7) Give the first order Taylor expansion of $\log(N(x))$ with respect to x around the origin.

(1.8) Suppose that K is equal to the forward exchange rate F derived in (1.1) and that σ is sufficiently small. Give the approximation of $R_* - r_d$ up to the first order of σ.

(1.9) Suppose $\sigma = 10\%$, $T = \dfrac{1}{2\pi}$ ($\simeq$ 1.9 month). Estimate $R_* - r_d$ using the result of (1.7).

Problem 2

Consider a one-period binomial stock model. The current time is $t_0 = 0$ and the terminal time is t_1 (> 0). The stock is a risky asset whose current price is $S_0 = s$ and the price S_{t_1} at time t_1 is either $s \times u$ with probability p_u or $s \times d$ with probability p_d without any dividend payments. Here, p_u, p_d are the probabilities under the empirical measure $\mathbb{P}$. There also exists a risk-free bond with the current price $B_0 = 1$. The bond price at t_1 is given by $B_{t_1} = e^{rt_1}$. Here, all the parameters are constant and satisfy $r, s, p_u, p_d > 0$, $0 < d < u$, and $p_u + p_d = 1$.

(2.1) Give q_u (the probability of the upward movement) and q_d (the probability of the downward movement) under the risk-neutral measure $\mathbb{Q}$, which is defined as a probability measure satisfying $\mathbb{E}^{\mathbb{Q}}[e^{-rt_1}S_{t_1}] = s$. Furthermore, give the conditions that make $\mathbb{Q}$ well-defined and equivalent to $\mathbb{P}$, i.e., $q_u > 0, q_d > 0$ and $q_u + q_d = 1$.
(2.2) Derive the price (at t_0) of a call option of expiry t_1 with strike K. Here, K is assumed to satisfy $sd < K < su$.

Now, let generalize it to the equally-spaced n-period model: $0 = t_0 < t_1 < \cdots < t_n = T$. Up and down movement factor (u, d), its probability (p_u, p_d), the bond interest rate r are assumed to be the same as the previous model for every time interval: Answer the following question.

(2.3) Consider the cancelable forward contract where the investor receives one share of the stock at time T by paying the cash amount K. K is assumed to satisfy $sd^n < K < su^n$. In this contract, the investor is allowed to cancel (i.e. terminate the contract) at any time $t_1, \cdots, t_{n-1}$ by paying the penalty fee $c > 0$. (The fee is to be paid at the time when the contract is canceled.) Write down the induction algorithm which gives the present value of this exotic contract in such a way that it can be straightforwardly translated into a computer code. [One can use the risk-neutral pricing technique without giving its justification.]

Problem 3

Let a time-partition $0 = T_0 < T_1 < T_2 < T_3 < T_4$ be equally spaced with $T_{i+1} - T_i = 1$(year) for every i. We consider a forward swap starting from T_1 with maturity T_4 with the amortizing notional: The notional amount N_i for the period $[T_{i-1}, T_i]$ is given by $(N_2, N_3, N_4) = (3, 2, 1)$ (billion Yen). At $T_i (i \in \{2, 3, 4\})$, the investor pays $L(T_{i-1}, T_i)$ the risk-free *simple* rate (announced at T_{i-1} in the market for the period $[T_{i-1}, T_i]$) and receives the some fixed rate K. For simplicity, the day-count fraction is assumed to be 1 for both the floating and fixed rate payments.

(3.1) Using the current zero coupon bond prices $P(0, T_i)$ with maturity T_i, $i = \{1, 2, 3, 4\}$, express the forward swap rate F, which is the fixed rate K that makes the present value of the above swap zero.
(3.2) Draw a bar graph of deltas (ie. so-called PV01's) of the above contract from the view point of the investor with respect to the change (+1bp) of each swap rate of the standard (spot-start) swaps with maturities (1yr,2yr,3yr, 4yr). One can use the approximation with single-digit round number assuming that the interest rates are low enough.
(3.3) Suppose that 2yr and 4yr (spot-start) swap rates changed +1bp and −3bp, respectively, on the next day. The other swap rates were unchanged. As the change of the present value of the above amortizing swap from this rate change, select the most appropriate one from the following choices (unit Yen) [Assume interest rates are low enough.]
(a) -4×10^6, (b) -2×10^5, (c) 2×10^5, (d) 1×10^6, (e) 4×10^6

Problem 1 floating

Suppose that the exchange rate $(X_t, t \geq 0)$ of a foreign currency, i.e. the price of the unit amount of the foreign currency (FC) in terms of the domestic currency (DC), is given by

$$X_s = X_0 + \int_0^s (r_d - r_f)X_t dt + \int_0^s X_t \sigma dW_t^{\mathbb{Q}}, \; s \geq 0$$

with $X_0 > 0$. Here, $r_d, r_f > 0$ are domestic as well as foreign risk-free interest rates assumed to be constant. $\sigma > 0$ is a given constant representing the volatility of the exchange rate. $W^{\mathbb{Q}}$ is a one-dimensional Brownian motion under the domestic risk-neutral measure $\mathbb{Q}$. Answer the following questions.

(1.1) Explain the definition of the "forward" exchange rate. Moreover, give the current $(t = 0)$ forward exchange rate F of the above FC for the forward contract with maturity $T > 0$.

(1.2) Apply Itô formula to $(\log X_t, t \geq 0)$. In addition, express X_T at some future time $T > 0$ in terms of $\{X_0, T, r_d, r_f, \sigma, W_T^{\mathbb{Q}}\}$.

(1.3) Calculate the probability $\mathbb{Q}(\{X_T > K\})$ $\left(= \mathbb{E}^{\mathbb{Q}}[\mathbf{1}_{\{X_T > K\}}]\right)$.

(1.4) Calculate the expectation $\mathbb{E}^{\mathbb{Q}}\left[X_T \mathbf{1}_{\{X_T \leq K\}}\right]$.

$$\frac{dX_t}{X_t} = (r_d - r_f)dt + \sigma dW_t^{Q}$$

(1.1) "forward" exchange rate F is fixed rate K that makes P.V. of FRA zero.
(=a binding contract)

$$\cancel{F = \frac{1}{\delta}\left(\frac{P(0,?)}{P(0,T)} - 1\right)}$$

よくわからん...

$$\underline{F = e^{(r_d - r_f)T} X_0} = e^{\Delta r T} X_0 = \frac{X_0}{P_\Delta(0,T)}$$

(1.2) $f(x) = \log x$ $\quad \frac{\partial f}{\partial t} = 0, \quad \frac{\partial f}{\partial x} = \frac{1}{x}, \quad \frac{\partial^2 f}{\partial x^2} = -\frac{1}{x^2}$

$$d(\log X_t) = \frac{dX_t}{X_t} + \frac{1}{2}\left(-\frac{dX_t}{X_t}\right)^2 = (r_d - r_f)dt + \sigma dW_t^{Q} - \frac{1}{2}\sigma^2 dt$$

$$= \underline{\left(r_d - r_f - \frac{\sigma^2}{2}\right)dt + \sigma dW_t^{Q}}$$

[0,T]で積分

$$\log X_T = \log X_0 + \left(r_d - r_f - \frac{\sigma^2}{2}\right)T + \sigma W_T^{Q}$$

$$\underline{X_T = X_0 \exp\left\{\left(r_d - r_f - \frac{\sigma^2}{2}\right)T + \sigma W_T^{Q}\right\}}$$

$$X_0 \exp\left\{\left(\Delta r - \frac{\sigma^2}{2}\right)T + \sigma W_T^{Q}\right\}$$

$$(1.3)\quad \mathbb{Q}(\{X_T>K\})=\mathbb{Q}\left(\exp\left\{\left(r_d-r_f-\frac{\sigma^2}{2}\right)T+\sigma\sqrt{T}Z\right\}>\frac{K}{X_0}\right)$$

$$=\mathbb{Q}\left(Z>-\boxed{\overset{d_-}{\overset{\shortparallel}{\frac{\ln X_0-\ln K+\left(r_d-r_f-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}}}}\right)=\mathbb{Q}(Z>-d_2)$$

$$=N(d_2)$$

$$\Delta r=r_d-r_f\qquad F=e^{\Delta r T}X_0$$

$$(1.4)\quad \mathbb{Q}(\{X_T\leq K\})=\mathbb{Q}(\{Z\leq -d_2\})$$

※ここで $\mathbb{Q}(\{Z\geq d_2\})$ とやってはいけない！

$\mathbb{E}^{\mathbb{Q}}[X_T\mathbb{1}\{X_T\leq K\}]$ ← put の一部

$$=X_0\exp\{(r_d-r_f)T\}\int_{-\infty}^{-d_-}e^{-\frac{\sigma^2}{2}T+\sigma\sqrt{T}Z}\phi(Z)dZ$$

$$=\int_{-\infty}^{-d_2}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{Z^2}{2}+\sigma\sqrt{T}Z-\frac{\sigma^2}{2}T\right)dZ=\int_{-\infty}^{-d_2}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{(Z-\sigma\sqrt{T})^2}{2}\right)dZ$$

$$(Z'=Z-\sigma\sqrt{T})=\int_{-\infty}^{-d_2-\sigma\sqrt{T}}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{Z'^2}{2}\right)dZ'=\int_{-\infty}^{-(d_2+\sigma\sqrt{T})}\phi(Z)dZ=N(-d_1)$$

$$\mathbb{E}^{\mathbb{Q}}[X_T\mathbb{1}\{X_T\leq K\}]=X_0e^{(r_d-r_f)T}N(-d_1)=FN(-d_1)$$

$$※\ d_\pm=\frac{\ln X_0-\ln K+\left(r_d-r_f\pm\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$=\frac{\ln\left(\frac{F}{K}\right)\pm\frac{\sigma^2}{2}T}{\sigma\sqrt{T}}\qquad F=X_0e^{(r_d-r_f)T}$$

Suppose that there exists a bank offering the following FX-linked structured deposit: *(i) At time* 0*, the client deposits* c (>0) *unit of FC (by paying* cX_0 *in DC). (ii) At time* T*, the client receives*

$$\begin{cases} ce^{r_f T} & \text{in } FC \quad \text{if} \quad X_T \le K \\ cX_0 e^{RT} & \text{in } DC \quad \text{if} \quad X_T > K \end{cases}$$

with some $R > 0$*. Here,* $K > 0$ *is a given constant.*

(1.5) Give the possible maximal loss for the depositor in DC without including opportunity costs. ?

(1.6) Express the present value (at time 0) of the deposit for the client in terms of DC, and then derive the *fair* deposit rate R_* which is the rate R that makes the present value of the deposit equal to cX_0.

(1.7) Give the first order Taylor expansion of $\log(N(x))$ with respect to x around the origin.

(1.8) Suppose that K is equal to the forward exchange rate F derived in (1.1) and that σ is sufficiently small. Give the approximation of $R_* - r_d$ up to the first order of σ.

(1.9) Suppose $\sigma = 10\%$, $T = \frac{1}{2\pi}$ ($\simeq$ 1.9 month). Estimate $R_* - r_d$ using the result of (1.7).

(in DC)

(1.5) depositor ??

$0 \quad cX_0 \quad T$

$$\begin{cases} cX_T e^{r_f T} & X_T \le K \quad \text{円高} \quad 80\text{円/ドル} \\ cX_0 e^{RT} & X_T > K \quad \text{円安} \quad 120\text{円/ドル} \end{cases}$$

$$X_T = X_0 \exp\left\{\left(r_d - r_f - \frac{\sigma^2}{2}\right)T + \sigma W_T^Q\right\}$$

$$cf(T) = +cX_T e^{r_f T} = cX_0 e^{\left(r_d - \frac{\sigma^2}{2}\right)T} e^{\sigma\sqrt{T}z}$$

minimal

$$0 = X_0 \exp(-\infty) < X_T \le K$$

$$\text{at } t=0 \ -cX_0 \longrightarrow \text{at } t=T \ +0$$

$$0 - cX_0 = -cX_0 \quad \text{maximal loss} = cX_0$$

why? $\Longrightarrow$ in terms of D.C.だから

$$(1.6) \quad e^{-r_d T} \mathbb{E}^Q\left[ce^{r_f T} X_T \mathbb{1}_{\{X_T \le K\}} + cX_0 e^{RT} \mathbb{1}_{\{X_T > K\}}\right]$$

$$= e^{-r_d T}\left[ce^{\cancel{r_f}} X_0 e^{(r_d - \cancel{r_f})T} N(-d_1) + cX_0 e^{RT} N(d_2)\right]$$

$$= e^{-r_d T} cX_0 \left\{e^{r_d T} N(-d_1) + e^{RT} N(d_2)\right\}$$

$$= cX_0\left(N(-d_1) + e^{(R - r_d)T} N(d_2)\right)$$

//

(1.6) (つづき)

$t=0$で預けた分

R_* is R that makes the P.V. of deposit (at T) cX_0.

$$\cancel{cX_0}\left(N(-d_1)+e^{(R-r_d)T}N(d_2)\right)=\cancel{cX_0}$$

$$e^{(R-r_d)T}=\frac{1-N(-d_1)}{N(d_2)}=\frac{N(d_1)}{N(d_2)}$$

$$\underline{R_*=r_d+\frac{1}{T}\ln\left(\frac{N(d_1)}{N(d_2)}\right)}\ /\!/$$

(1.7)

$$f(x)=\ln(N(x))=\ln\left(\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}e^{-\frac{z^2}{2}}dz\right),\quad f(0)=\ln\left(\frac{1}{2}\right)=-\ln 2$$

$$f'(x)=\frac{\phi(x)}{N(x)}=\cancel{e^{-\frac{x^2}{2}}\Big/\int_{-\infty}^{x}e^{-\frac{z^2}{2}}dz}$$

$$f'(0)=\frac{\phi(0)}{N(0)}=\frac{\frac{1}{\sqrt{2\pi}}e^{-0}}{\frac{1}{2}}=\sqrt{\frac{2}{\pi}}$$

$$\underline{\ln(N(x))\approx-\ln 2+\sqrt{\frac{2}{\pi}}\,x+o(|x|)}\ /\!/$$

(1.8) $K=F$のとき, $d_\pm=\dfrac{\ln\left(\frac{F}{K}\right)\pm\frac{\sigma^2}{2}T}{\sigma\sqrt{T}}=\pm\dfrac{1}{2}\sigma\sqrt{T}$

$$R_*-r_d=\frac{1}{T}\ln\left(\frac{N(d_1)}{N(d_2)}\right)=\frac{1}{T}\left\{\ln(N(d_1))-\ln(N(d_2))\right\}$$

$$\approx\frac{1}{T}\left\{\left(-\cancel{\ln 2}+\sqrt{\frac{2}{\pi}}d_1\right)-\left(-\cancel{\ln 2}+\sqrt{\frac{2}{\pi}}d_2\right)\right\}=\frac{1}{T}\sqrt{\frac{2}{\pi}}(d_1-d_2)$$

$$=\frac{1}{T}\sqrt{\frac{2}{\pi}}\,\sigma\sqrt{T}=\underline{\sqrt{\frac{2}{\pi T}}\,\sigma+o(\sigma)}\ /\!/$$

dとσは同じオーダーなので…

$$\left((1.8)\ R_f - r_d = \sqrt{\frac{2}{\pi T}}\,\sigma \right)$$

(1.9) $\sigma = 10\%$, $T = \frac{1}{2\pi}$ year $= 1.9$ month

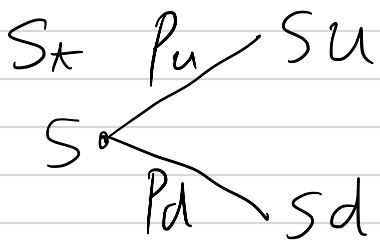
$$R_f - r_d = \sqrt{\frac{2}{\pi T}}\,\sigma = \sqrt{\frac{2}{\cancel{\pi}} \times 2\cancel{\pi}} \times 0.1 = 0.2$$

$$\underline{20\%}$$

Problem 2

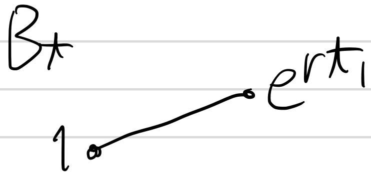
Consider a one-period binomial stock model. The current time is $t_0 = 0$ and the terminal time is t_1 (> 0). The stock is a risky asset whose current price is $S_0 = s$ and the price S_{t_1} at time t_1 is either $s \times u$ with probability p_u or $s \times d$ with probability p_d without any dividend payments. Here, p_u, p_d are the probabilities under the empirical measure $\mathbb{P}$. There also exists a risk-free bond with the current price $B_0 = 1$. The bond price at t_1 is given by $B_{t_1} = e^{rt_1}$. Here, all the parameters are constant and satisfy $r, s, p_u, p_d > 0$, $0 < d < u$, and $p_u + p_d = 1$.

(2.1) Give q_u (the probability of the upward movement) and q_d (the probability of the downward movement) under the risk-neutral measure $\mathbb{Q}$, which is defined as a probability measure satisfying $\mathbb{E}^{\mathbb{Q}}[e^{-rt_1} S_{t_1}] = s$. Furthermore, give the conditions that make $\mathbb{Q}$ well-defined and equivalent to $\mathbb{P}$, i.e., $q_u > 0, q_d > 0$ and $q_u + q_d = 1$.
(2.2) Derive the price (at t_0) of a call option of expiry t_1 with strike K. Here, K is assumed to satisfy $sd < K < su$.



($\mathbb{P}$: emprical)

$0 < d < u$



(2.1)
$$\begin{cases} s = e^{-rt_1}\mathbb{E}^{\mathbb{Q}}[S_{t_1}] = e^{-rt_1}(su\,q_u + sd\,q_d) \\ q_u + q_d = 1 \end{cases}$$

$$\Rightarrow e^{rt_1} = u q_u + d(1 - q_u) \quad \therefore \begin{cases} q_u = \dfrac{e^{rt_1} - d}{u - d} \\ q_d = \dfrac{u - e^{rt_1}}{u - d} \end{cases}$$

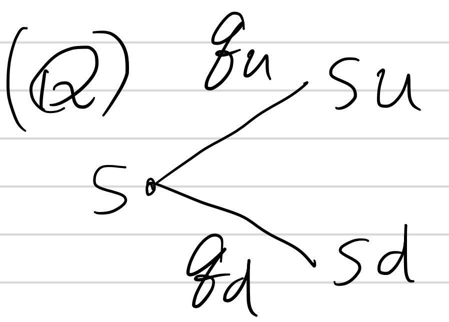
$q_u > 0, q_u > 0$ より、

condition: $d < e^{rt_1} < u$ を得る.

(2.3) Consider the cancelable forward contract where the investor receives one share of the stock at time T by paying the cash amount K. K is assumed to satisfy $sd^n < K < su^n$. In this contract, the investor is allowed to cancel (i.e. terminate the contract) at any time $t_1, \cdots, t_{n-1}$ by paying the penalty fee $c > 0$. (The fee is to be paid at the time when the contract is canceled.) Write down the induction algorithm which gives the present value of this exotic contract in such a way that it can be straightforwardly translated into a computer code. [One can use the risk-neutral pricing technique without giving its justification.]

p.v.

(2.2) assume $sd < K < su$



Call option

$(su-K)_+ = su-K$

$(sd-K)_+ = 0$

$$C_0 = e^{-rt_1} \mathbb{E}^{Q}[C_{t_1}] = e^{-rt_1}(q_u(su-K) + q_d \cdot 0)$$

$$= e^{-rt_1} \frac{e^{rt_1}-d}{u-d}(su-K)$$

(2.3) $sd^n < K < su^n$

index: $S(i,k)$ (time: t_i, k up) $= su^k d^{i-k}$

$\tilde{V}$ not cancellable fwd contract

V cancellable fwd contract

at $t_n (=T)$ $V(n,k) = S(n,k) - K$ $(0 \le k \le n)$

at t_{n-1} $\tilde{V}(n-1,k) = e^{-r\Delta t}(q_u V(n,k+1) + q_d V(n,k))$

$(-c)$: cancellation fee

$$V(n-1,k) = \max\{\tilde{V}(n-1,k), -c\}$$

⋮ (Repeat)

$$V(i,k) = \max\{e^{-r\Delta t}(q_u V(i+1,k+1) + q_d V(i+1,k)), -c\}$$

$\begin{cases} 1 \le i \le n-1 \\ 0 \le k \le i \end{cases}$

$$V(0,0) = e^{-r\Delta t}(q_u V(1,1) + q_d V(1,0))$$

Problem 3

Let a time-partition $0 = T_0 < T_1 < T_2 < T_3 < T_4$ be equally spaced with $T_{i+1} - T_i = 1$(year) for every i. We consider a forward swap starting from T_1 with maturity T_4 with the amortizing notional: The notional amount N_i for the period $[T_{i-1}, T_i]$ is given by $(N_2, N_3, N_4) = (3, 2, 1)$ (billion Yen). At $T_i (i \in \{2,3,4\})$, the investor pays $L(T_{i-1}, T_i)$ the risk-free *simple* rate (announced at T_{i-1} in the market for the period $[T_{i-1}, T_i]$) and receives the some fixed rate K. For simplicity, the day-count fraction is assumed to be 1 for both the floating and fixed rate payments.

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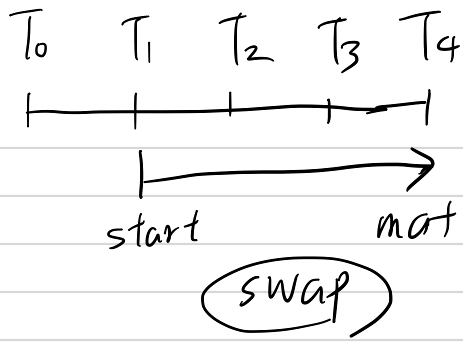
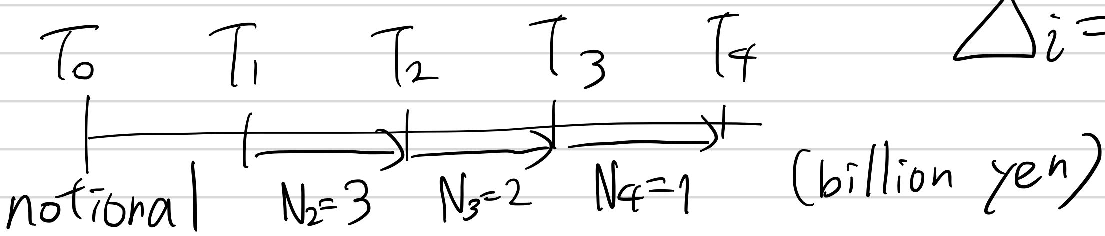


day-count
$\Delta_i = \delta_i = 1$



pays simple rate $L(1,2)$ $L(2,3)$ $L(3,4)$

receives fixed rate K K K

(3.1) $$PV_{fix} = K\sum_{i=2}^{4} N_i P(0,T_i) = K(3P(0,T_2) + 2P(0,T_3) + P(0,T_4))$$

fwd rate

$$PV_{float} = \sum_{i=2}^{4} N_i Fwd(0, T_{i-1}, T_i) P(0,T_i)$$

fwd rate $\neq$ (fwd swap rate)

$$= \sum N_i \frac{1}{\cancel{\delta_i}}\left(\frac{P(0,T_{i-1})}{P(0,T_i)} - 1\right) P(0,T_i)$$

$$= \sum_{i=2}^{4} N_i (P(0,T_{i-1}) - P(0,T_i))$$

$$= 3(P(0,T_1) - P(0,T_2)) + 2(P(0,T_2) - P(0,T_3)) + (P(0,T_3 - P(0,T_4))$$

$$= 3P(0,T_1) - P(0,T_2) - P(0,T_3) - P(0,T_4)$$

fwd swap rate

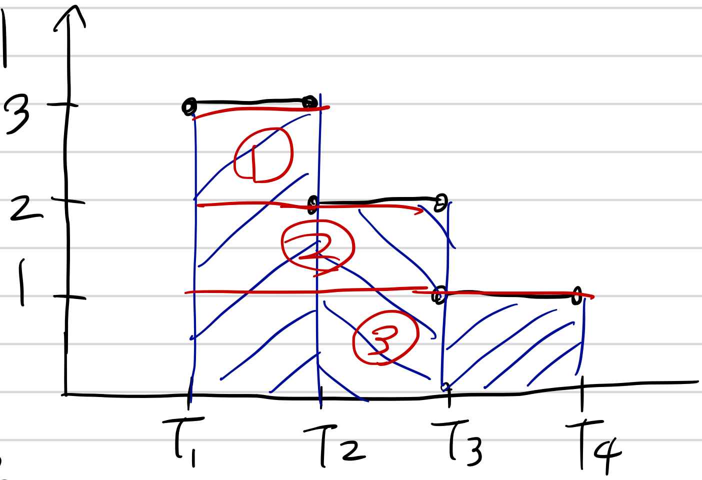
$$\boxed{\therefore F_S = \frac{3P(0,T_1) - P(0,T_2) - P(0,T_3) - P(0,T_4)}{3P(0,T_2) + 2P(0,T_3) + P(0,T_4)}}$$

(3.2) Draw a bar graph of deltas (ie. so-called PV01's) of the above contract from the view point of the investor with respect to the change (+1bp) of each swap rate of the standard (spot-start) swaps with maturities (1yr,2yr,3yr, 4yr). One can use the approximation with single-digit round number assuming that the interest rates are low enough.

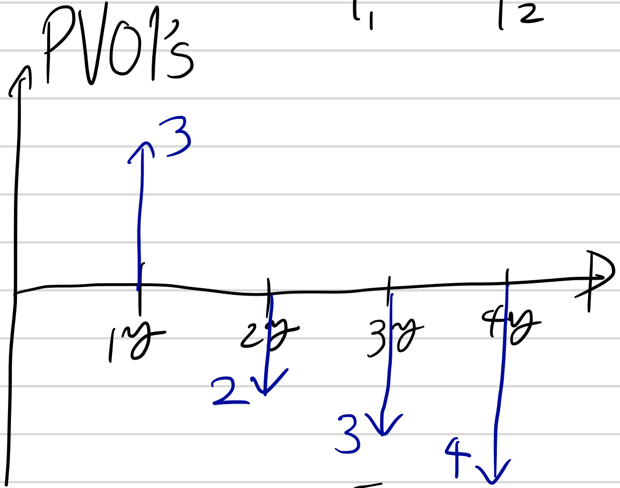
(3.3) Suppose that 2yr and 4yr (spot-start) swap rates changed +1bp and −3bp, respectively, on the next day. The other swap rates were unchanged. As the change of the present value of the above amortizing swap from this rate change, select the most appropriate one from the following choices (unit Yen) [Assume interest rates are low enough.]

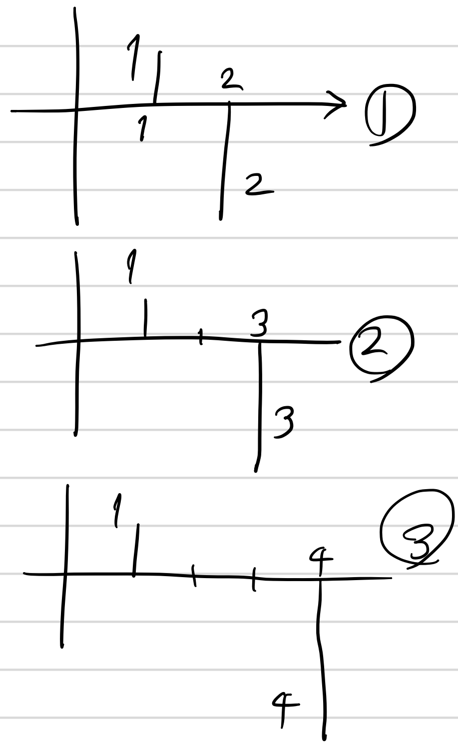
(a) -4×10^6, (b) -2×10^5, (c) 2×10^5, (d) 1×10^6, (e) 4×10^6 (yen)

(3-1) notional



(3.2)





from view point of investor

(3.3) 2yr swap rate +1bp
4yr swap rate −3bp

$$10^9 \times 10^{-4} \times (-2+12) = 10^6$$

d //

$$\frac{\partial F}{\partial r} = \frac{1}{\left(3e^{-2r}+2e^{-3r}+e^{-4r}\right)^2} \times$$

$$\left\{ \left(-3e^{-r}+2e^{-2r}+3e^{-3r}+4e^{-4r}\right)\left(3e^{-2r}+2e^{-3r}+e^{-4r}\right) - \left(3e^{-r}-e^{-2r}-e^{-3r}-e^{-4r}\right)\left(-6e^{-2r}-6e^{-3r}-4e^{-4r}\right) \right\}$$