multivariate_t6

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1 Multivariate statistics Test 6: EFA + CA

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1.1 Exploratory Factor Analysis

Data: File `Beer.sav`, variables

- `cost`
- `size`
- `alcohol`
- `color`
- `aroma`
- `taste`

Task: perform factor analysis.

First of all, let's load the data and take a look.

[ ]:
```python
import numpy as np
import pandas as pd
import pyreadstat
from factor_analyzer import FactorAnalyzer, calculate_kmo
from matplotlib import pyplot as plt

pd.options.display.float_format = '{:.4f}'.format

df_beer, metadata_beer = pyreadstat.read_sav("data/Beer.sav")

df_beer.describe()
```

[ ]:
```
           COST     SIZE  ALCOHOL  REPUTAT    COLOR    AROMA    TASTE      SES  \
count 231.0000 231.0000 231.0000 231.0000 231.0000 220.0000 231.0000 231.0000
mean   48.8095  45.7143  49.0476  48.3333  49.5238  44.7500  65.9524   3.3333
std    34.1574  34.3800  33.4755  23.6214  27.0465  25.8209  24.2353   2.5378
min     0.0000   0.0000  10.0000   0.0000   0.0000   0.0000  25.0000   0.0000
25%    15.0000  15.0000  20.0000  30.0000  30.0000  27.5000  50.0000   2.0000
50%    50.0000  35.0000  35.0000  40.0000  50.0000  45.0000  65.0000   3.0000
75%    80.0000  80.0000  70.0000  65.0000  75.0000  66.2500  90.0000   5.0000
```

```
max   100.0000 100.0000 100.0000 100.0000  95.0000  90.0000 100.0000   8.0000

          GROUP
count 231.0000
mean    1.4286
std     0.4959
min     1.0000
25%     1.0000
50%     1.0000
75%     2.0000
max     2.0000
```

There are 11 missing values in the `aroma` column. We are going to remove them.

The dataset has three additional columns not described in the task: `reputat`, `ses` and `group`. We are not going to remove them to avoid using them accidentally.

[49]:
```python
df_beer = df_beer.drop(["REPUTAT", "SES", "GROUP"], axis=1)
df_beer = df_beer.dropna()
df_beer.columns = df_beer.columns.str.lower()

df_beer
```

[49]:
```
        cost    size  alcohol   color   aroma    taste
0    90.0000 80.0000  70.0000 50.0000 70.0000  60.0000
1    75.0000 95.0000 100.0000 55.0000 40.0000  65.0000
2    10.0000 15.0000  20.0000 40.0000 30.0000  50.0000
3   100.0000 70.0000  50.0000 75.0000 60.0000  80.0000
4    20.0000 10.0000  25.0000 30.0000 35.0000  45.0000
..       …       …        …       …       …        …
225  20.0000  5.0000  10.0000 60.0000 50.0000  95.0000
226  70.0000 60.0000  70.0000 10.0000 15.0000  25.0000
227  50.0000 15.0000  20.0000 10.0000  5.0000  50.0000
228  75.0000 50.0000  95.0000  0.0000  0.0000  40.0000
229  15.0000 10.0000  25.0000 95.0000 80.0000 100.0000

[220 rows x 6 columns]
```

1.1.1 KMO

We are going to calculate the KMO measure of adequacy to check if the data is suitable for factor analysis (i.e. KMO > 0.5).

[57]:
```python
kmo_all, kmo_model = calculate_kmo(df_beer)
print(f"KMO Model: {kmo_model:.3f}")
```

```
KMO Model: 0.628
```

The KMO of 0.628 is acceptable.

1.1.2 Variance explained by factors

Variance explained by each factor is shown in the table below.

[163]:
```
ev, v = np.linalg.eig(df_beer.corr())
ev = ev / ev.sum()
ev = np.sort(ev)[::-1]
ev
```

[163]:
```
array([0.4781151 , 0.43414613, 0.04000927, 0.02392752, 0.01761521,
       0.00618677])
```

[164]:
```
df_var_explained = pd.DataFrame(
    {
        "factor": range(1, 7),
        "eigenvalues": ev,
        "proportion": ev / ev.sum(),
        "cumulative": np.cumsum(ev / ev.sum()),
    }
)
```

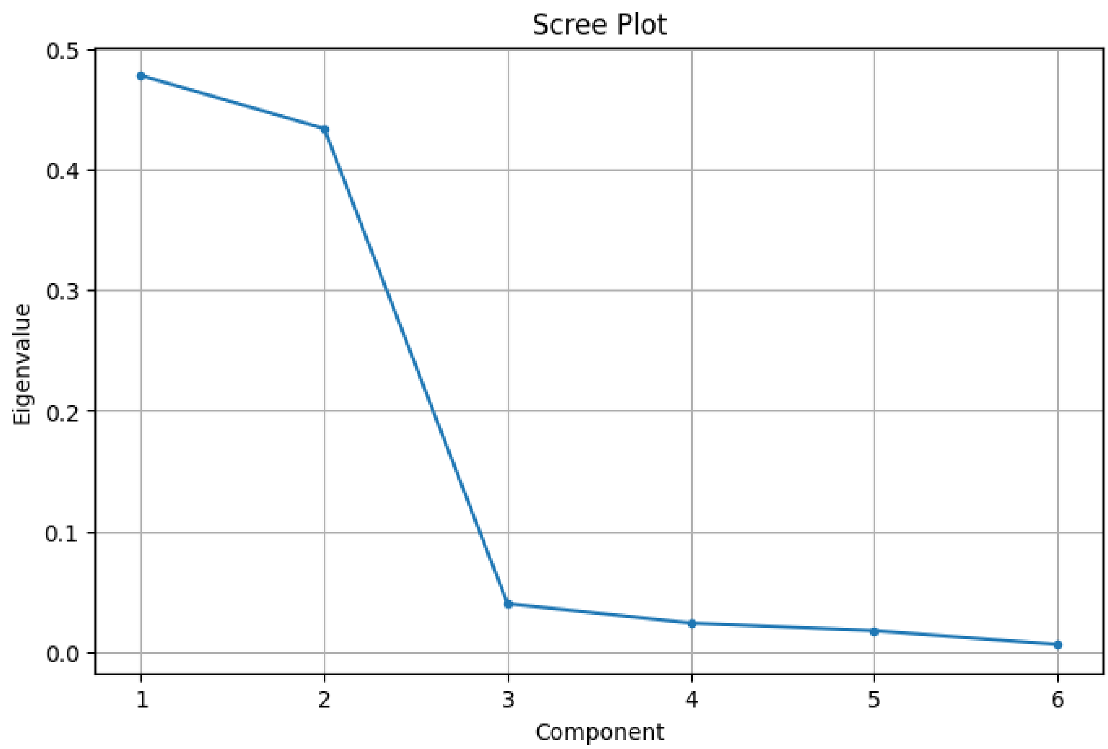
[165]:
```
df_var_explained
```

[165]:

	factor	eigenvalues	proportion	cumulative
0	1	0.4781	0.4781	0.4781
1	2	0.4341	0.4341	0.9123
2	3	0.0400	0.0400	0.9523
3	4	0.0239	0.0239	0.9762
4	5	0.0176	0.0176	0.9938
5	6	0.0062	0.0062	1.0000

1.1.3 Scree plot

[166]:
```
plt.figure(figsize=(8, 5))
plt.plot(
    df_var_explained["factor"],
    df_var_explained["eigenvalues"],
    marker=".",
    label="Eigenvalues",
)
plt.title("Scree Plot")
plt.xlabel("Component")
plt.ylabel("Eigenvalue")
plt.grid()
plt.show()
```



Based on the above table and plot we're going to choose 2 factors with eigenvalues > 1. Cumulatively they explain 91.2% of the variance (see table above).

[167]:
```
n_factors = 2
```

1.1.4 Communalities

The table below shows the communalities of the variables. All of them are above 0.2, which is acceptable.

[191]:
```
fa = FactorAnalyzer(rotation="varimax", method="principal", n_factors=n_factors)
fa.fit(df_beer)

communalities = fa.get_communalities()
pd.DataFrame(communalities, index=df_beer.columns, columns=["Communalities"])
```

[191]:

	Communalities
cost	0.8458
size	0.9430
alcohol	0.8911
color	0.9499
aroma	0.9236
taste	0.9201

1.1.5 Total variation explained by extracted factors

We will confirm that the total variation explained by the two extracted factors is still 91.2%.

[192]:
```
df_var_explained = pd.DataFrame(
    np.transpose(fa.get_factor_variance()),
    columns=["eigenvalues", "proportion", "cumulative"],
)
df_var_explained.insert(0, "factor", range(1, 3))
df_var_explained
```

[192]:

	factor	eigenvalues	proportion	cumulative
0	1	2.7960	0.4660	0.4660
1	2	2.6776	0.4463	0.9123

1.1.6 Rotated factor solution

The rotated factor loadings are shown in the table below.

[193]:
```
df_loadings = pd.DataFrame(fa.loadings_, index=df_beer.columns)
df_loadings.columns = [f"Factor {i}" for i in range(1, n_factors + 1)]
df_loadings
```

[193]:

	Factor 1	Factor 2
cost	-0.0648	0.9174
size	0.0787	0.9679
alcohol	0.0189	0.9438
color	0.9708	0.0856
aroma	0.9608	0.0230
taste	0.9590	-0.0228

1.1.7 Variables related to each factor and Factor names

Based on the loadings provided above, we can see the following:

The first factor is strongly related to `color`, `aroma`, and `taste` with loadings > 0.9, all of them positively correlated. Based on that, we will interpret the first factor as the "taste" factor.

The second factor is strongly related to `cost`, `size`, and `alcohol` with loadings > 0.9, all of them positively correlated. Based on that, we will interpret the second factor as the "size" factor.

Thus, we have two factors: * "taste" factor, positively related to `color`, `aroma`, and `taste` * "size" factor, positively related to `cost`, `size`, and `alcohol`

1.2 Correspondence Analysis

Data: Dataframe below

- `lib`: liberal
- `slib`: slightly liberal
- `mod`: moderate

- scon: slightly conservative
- con: conservative

Task: perform correspondence analysis.

[289]:
```
import numpy as np
import pandas as pd
from matplotlib import pyplot as plt
from scipy.stats import chi2_contingency
```

[202]:
```
df_politics = pd.DataFrame(
    {
        "Protestant": [98, 116, 324, 163, 214],
        "Catholic": [33, 40, 139, 61, 52],
        "Jewish": [9, 6, 7, 5, 3],
        "None": [41, 28, 43, 16, 7],
    },
    index=["lib", "slib", "mod", "scon", "con"],
)

df_politics
```

[202]:

	Protestant	Catholic	Jewish	None
lib	98	33	9	41
slib	116	40	6	28
mod	324	139	7	43
scon	163	61	5	16
con	214	52	3	7

1.2.1 Chi-square test

[222]:
```
chi2_result = chi2_contingency(df_politics)
print(f"Chi-Square Statistic: {chi2_result.statistic:.3f}")
print(f"p-value: {chi2_result.pvalue}")
print(f"Degrees of Freedom: {chi2_result.dof}")
print("Expected Frequencies:")
pd.DataFrame(chi2_result.expected_freq, columns=df_politics.columns,␣
 ↪index=df_politics.index)
```

```
Chi-Square Statistic: 84.434
p-value: 5.8455426657118e-13
Degrees of Freedom: 12
Expected Frequencies:
```

[222]:

	Protestant	Catholic	Jewish	None
lib	117.8754	41.8683	3.8648	17.3915
slib	123.7367	43.9502	4.0569	18.2562
mod	334.0890	118.6655	10.9537	49.2918

```
scon     159.5552   56.6726  5.2313 23.5409
con      179.7438   63.8434  5.8932 26.5196
```

The Chi-square test is significant ($p < 0.05$), so we can proceed with correspondence analysis.

1.2.2 Eigenvalues and cumulative variance explained by 2 dimensions

We are going to perform SVD on the contingency table and calculate the eigenvalues and the cumulative variance explained by 2 dimensions.

[300]:
```
residuals = (df_politics - chi2_result.expected_freq) / np.sqrt(chi2_result.
 ↪expected_freq)
total = df_politics.values.sum()

svd_result = np.linalg.svd(residuals, full_matrices=False)

ev = svd_result.S**2 / (total * np.min(df_politics.shape))
cumulative_var = np.cumsum(ev / ev.sum())

pd.DataFrame(
    {
        "Dimension": range(1, 5),
        "Eigenvalues": ev,
        "Cumulative Variance": cumulative_var,
    }
).iloc[:2]
```

[300]:

	Dimension	Eigenvalues	Cumulative Variance
0	1	0.0131	0.8706
1	2	0.0019	0.9944

The first two dimensions explain 99.44% of the variance.

1.2.3 Vizualization (biplot)

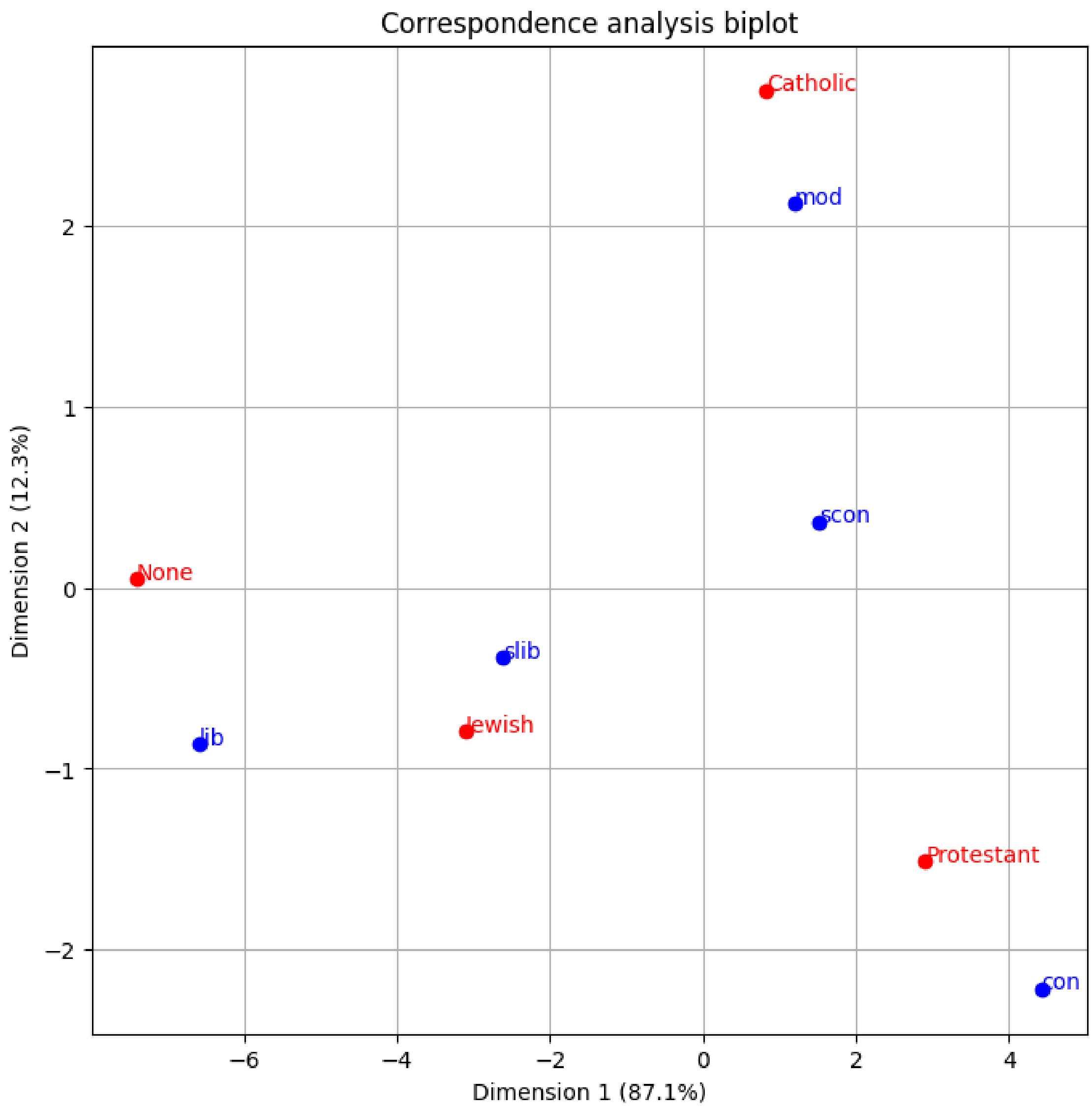
[328]:
```
row_coordinates = svd_result.U[:, :2] @ np.diag(svd_result.S[:2])
col_coordinates = svd_result.Vh[:2].T @ np.diag(svd_result.S[:2])

plt.figure(figsize=(8, 8))

for i, (x, y) in enumerate(row_coordinates):
    plt.scatter(x, y, color="blue")
    plt.text(x, y, df_politics.index[i], color="blue")

for i, (x, y) in enumerate(col_coordinates):
    plt.scatter(x, y, color="red")
    plt.text(x, y, df_politics.columns[i], color="red")
```

```
plt.title("Correspondence analysis biplot")
plt.xlabel("Dimension 1 (87.1%)")
plt.ylabel("Dimension 2 (12.3%)")
plt.grid()
plt.show()
```



Comments:

- Political views and religions close together are strongly associated.
- Catholics appear to have a tendency to be moderate, Protestants tend to be conservative, Jewish people tend to be slightly liberal, and atheists tend to be liberal.