multivariate_t3

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1 Multivariate statistics Test 3

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1.1 Task 1: Probit regression

Data: File *LI*, variables

- *stflife* (satisfaction with life: 0-extremely unsatisfied,…,10-extremely satisfied)
- *stfhlth* (satisfaction with health system: 0-extremely unsatisfied,…,10-extremely satisfied)
- *trstprt* (trust in political parties: 0-no trust at all,…,10-complete trust)
- *pray*: how often pray: 1-every day,…, 7- never
- *state*: 0-Israel, 1-LT

Dependent binary variable *state*, regressors *stflife*, *stfhlth*, *trstprt*, *pray*. Treat all regressors as interval variables.

Task: Perform Probit Analysis.

First of all, let's load the data and take a look.

[1]:
```
import pyreadstat
import pandas as pd
import matplotlib.pyplot as plt
import statsmodels.formula.api as smf
from scipy.stats import chi2
from sklearn.metrics import roc_curve

pd.options.display.float_format = '{:.4f}'.format


df_state, metadata_state = pyreadstat.read_sav("LI.sav")

df_state.describe()
```

[1]:
```
        stflife   stfedu  stfhlth     pray  trstplc  trstprt    state
count 413.0000 413.0000 413.0000 413.0000 397.0000 381.0000 413.0000
mean    5.7942   4.0605   4.7361   4.8886   5.5390   2.4016   0.5569
std     2.5786   2.3137   2.6321   2.0294   2.6057   2.2778   0.4974
```

```
min       0.0000    0.0000    0.0000    1.0000    0.0000    0.0000    0.0000
25%       4.0000    2.0000    3.0000    4.0000    4.0000    0.0000    0.0000
50%       6.0000    4.0000    5.0000    5.0000    6.0000    2.0000    1.0000
75%       8.0000    6.0000    7.0000    7.0000    7.0000    4.0000    1.0000
max      10.0000    9.0000   10.0000    7.0000   10.0000   10.0000    1.0000
```

The dataset has six columns (*stflife*, *stfedu*, *stfhlth*, *pray*, *trstplc*, *trstprt*, *state*) and 413 rows. However, we will not be using *stfedu* and *trstplc* columns in this task because the task description does not mention them.

The *trstprt* column has 32 missing values. We will remove these rows from the dataset.

[2]:

```
df_state = df_state.dropna(subset=["trstprt"]).reset_index(drop=True)

df_state.describe()
```

[2]:

```
         stflife    stfedu   stfhlth      pray   trstplc   trstprt     state
count 381.0000  381.0000  381.0000  381.0000  375.0000  381.0000  381.0000
mean    5.8373    4.0761    4.7585    4.8924    5.4800    2.4016    0.5617
std     2.5782    2.3095    2.6526    2.0265    2.5456    2.2778    0.4968
min     0.0000    0.0000    0.0000    1.0000    0.0000    0.0000    0.0000
25%     4.0000    2.0000    3.0000    4.0000    4.0000    0.0000    0.0000
50%     6.0000    4.0000    5.0000    5.0000    6.0000    2.0000    1.0000
75%     8.0000    6.0000    7.0000    7.0000    7.0000    4.0000    1.0000
max    10.0000    9.0000   10.0000    7.0000   10.0000   10.0000    1.0000
```

This has left us with 381 rows and no missing values in the relevant columns.

1. Present parameter estimates for the initial model with highlighted statistically nonsignificant variables.

We fit the initial Probit regression model using all relevant predictor variables.

[3]:

```
initial_model = smf.probit("state ~ stflife + stfhlth + trstprt + pray",␣
 ↪data=df_state).fit()

initial_model.summary2().tables[1]
```

```
Optimization terminated successfully.
         Current function value: 0.488566
         Iterations 6
```

[3]:

```
             Coef.  Std.Err.       z  P>|z|   [0.025   0.975]
Intercept   1.7953    0.3129  5.7381 0.0000   1.1821   2.4085
stflife    -0.2347    0.0350 -6.6993 0.0000  -0.3034  -0.1660
stfhlth    -0.1720    0.0325 -5.2931 0.0000  -0.2357  -0.1083
trstprt     0.0284    0.0333  0.8539 0.3931  -0.0368   0.0936
pray        0.1139    0.0366  3.1071 0.0019   0.0420   0.1857
```

The parameter estimates for the initial model are presented above in the column `Coef.`. Their statistical significance is in the column `Pr(>|z|)`.

Not all variables are statistically significant. Statistically insignificant variables (with significance level 0.05) are:

- `trstprt` (p-value = 0.393)

We remove the statistically non-significant variable `trstprt` and fit the final Probit regression model.

[4]:

```
final_model = smf.probit("state ~ stflife + stfhlth + pray", data=df_state).
 ↪fit()
```

```
Optimization terminated successfully.
         Current function value: 0.489525
         Iterations 6
```

2. For the final model present Chi square likelihood ratio omnibus test.

[5]:

```
# likelihood ratio statistic
lr_stat = -2 * (initial_model.llf - final_model.llf)
# degrees of freedom
df_diff = initial_model.df_model - final_model.df_model
# p-value
p_value = chi2.sf(lr_stat, df_diff)

print(f"Chi-square likelihood ratio test statistic: {lr_stat}")
print(f"Degrees of freedom: {df_diff}")
print(f"P-value: {p_value}")
```

```
Chi-square likelihood ratio test statistic: -0.730486049556248
Degrees of freedom: 1.0
P-value: 1.0
```

The p-value for our test is 1.0, which means that the final model is not statistically significantly different from the initial model. That makes sense because we only removed one non significant variable from the initial model.

3. Estimates for all variables and their statistical significance.

[6]:

```
final_model.summary2().tables[1]
```

[6]:

```
             Coef.  Std.Err.       z  P>|z|   [0.025  0.975]
Intercept   1.8667    0.3013  6.1964 0.0000   1.2763  2.4572
stflife    -0.2337    0.0350 -6.6848 0.0000  -0.3022 -0.1652
stfhlth    -0.1742    0.0324 -5.3747 0.0000  -0.2377 -0.1107
pray        0.1137    0.0366  3.1046 0.0019   0.0419  0.1854
```

The parameter estimates for the final model are presented above in the column `Coef.`. Their statistical significance is in the column `Pr(>|z|)`.

This time all variables are statistically significant.

4. Model equation.

We can present the final model equation using the parameter estimates.

[7]:
```
# extract coefficients
coefficients = final_model.params
intercept = coefficients["Intercept"]
stflife_coef = coefficients["stflife"]
stfhlth_coef = coefficients["stfhlth"]
pray_coef = coefficients["pray"]

# model equation
model_equation = f"state = Φ({intercept:.4f} + {stflife_coef:.4f} * stflife +␣
 ↪{stfhlth_coef:.4f} * stfhlth + {pray_coef:.4f} * pray)"
print(model_equation)
```

```
state = Φ(1.8667 + -0.2337 * stflife + -0.1742 * stfhlth + 0.1137 * pray)
```

5. Classification Table.

We can present the classification table for the final model.

[8]:
```
# predict the probabilities
df_state["predicted_prob"] = final_model.predict(df_state)

# classify based on threshold=0.5
df_state["predicted_state"] = (df_state["predicted_prob"] > 0.5).astype(int)

# create the classification table
classification_table = pd.crosstab(
    df_state["state"].astype(int),
    df_state["predicted_state"],
    rownames=["Actual"],
    colnames=["Predicted"],
)
classification_table
```

[8]:
```
Predicted    0    1
Actual
0          122   45
1           45  169
```

The classification table above shows there are:

- 169 true positives
- 122 true negatives
- 45 false positives
- 45 false negatives

6. Sensitivity and Specificity.

Using the classification table, we can calculate the sensitivity and specificity for the final model.

[9]:

```
tn, fn = classification_table.iloc[0]
fp, tp = classification_table.iloc[1]

sensitivity = tp / (tp + fn)
specificity = tn / (tn + fp)

print(f"Sensitivity: {sensitivity:.4f}")
print(f"Specificity: {specificity:.4f}")
```

```
Sensitivity: 0.7897
Specificity: 0.7305
```

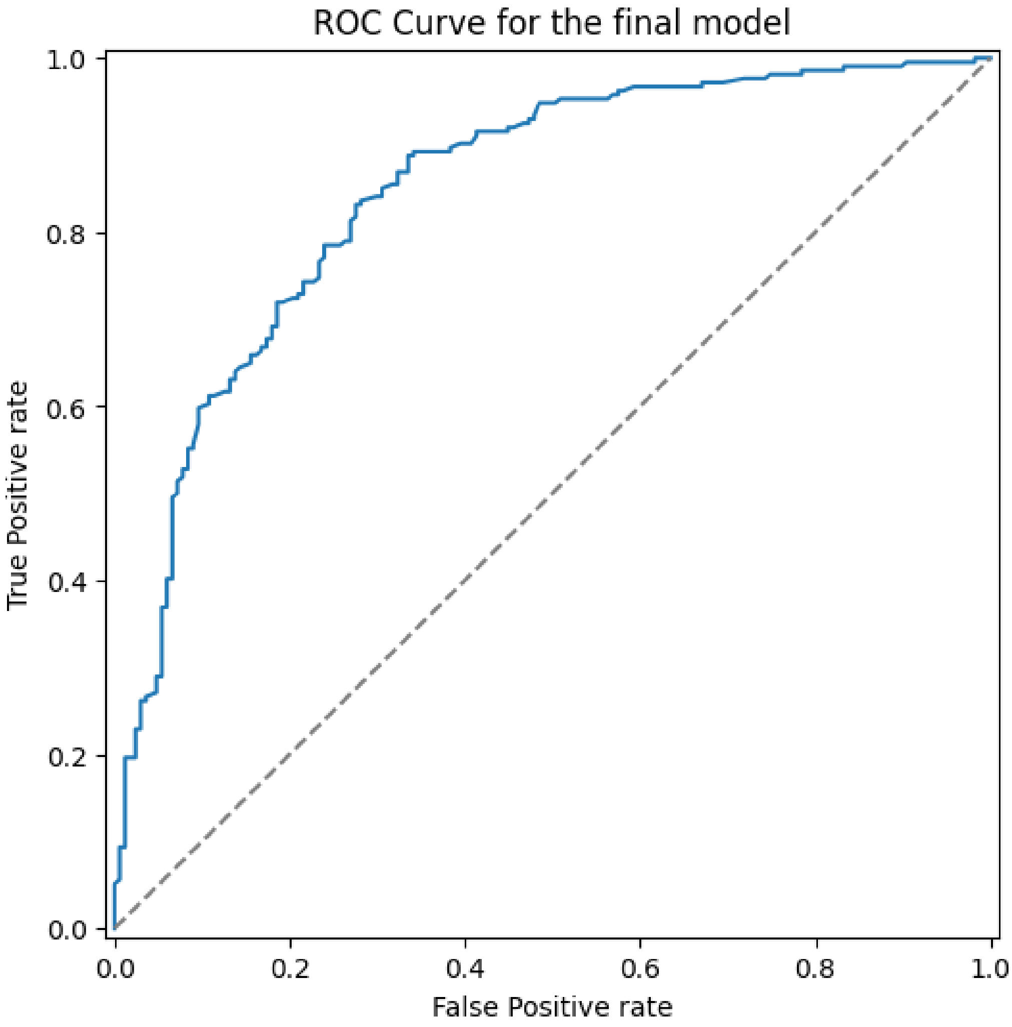
7. ROC curve.

We can also use the predicted probabilities to plot the ROC curve for the final model.

[10]:

```
# calculate the ROC curve
fpr, tpr, thresholds = roc_curve(
    df_state["state"],
    df_state["predicted_prob"],
)

# plot the ROC curve
plt.figure(figsize=(6, 6))
plt.plot(fpr, tpr, label="ROC curve")
plt.plot([0, 1], [0, 1], color="gray", linestyle="--")
plt.xlim([-0.01, 1.01])
plt.ylim([-0.01, 1.01])
plt.xlabel("False Positive rate")
plt.ylabel("True Positive rate")
plt.title("ROC Curve for the final model")
plt.show()
```



8. Forecast for stflife=5, stfhlth=7, trstprt = 6, pray =6 (use only variables from the final model).

Let’s use the final model to forecast the state for stflife = 5, stfhlth = 7, trstprt = 6, pray = 6.

[11]:
```
stflife = 5
stfhlth = 7
trstprt = 6  # trstprt is not in the final model!
pray = 6

# calculate the probability of state=1
predicted_prob = final_model.predict({"stflife": stflife, "stfhlth": stfhlth,␣
  ↪"pray": pray})[0]
print(f"predicted_prob: {predicted_prob:.4f}")
```

```
predicted_prob: 0.5638
```

1.2 Task 2: Poisson regression

Data: File *partners*, variables

- *age*: respondents age.
- *partners*: number of sexual partners last year.
- *sexeduc*: Sex education in public schools (1-Favor, 2-Oppose)
- *polviews*: 1-extremely liberal, … 7-extremely conservative.

Task: Perform Poisson regression modeling partners by remaining variables. Results must include:

First of all, let's load the data and take a look.

[12]:
```
df_partners, metadata_partners = pyreadstat.read_sav("partners.sav")

df_partners.describe()
```

[12]:
```
            age  polviews  sexeduc  tvhours  partners  married
count 188.0000  188.0000 188.0000 188.0000  188.0000 188.0000
mean   37.6755    3.9894   1.1223   2.8830    1.3830   1.5957
std    14.5336    1.3641   0.3286   2.2604    1.3252   0.4921
min    19.0000    1.0000   1.0000   0.0000    0.0000   1.0000
25%    27.0000    3.0000   1.0000   2.0000    1.0000   1.0000
50%    34.0000    4.0000   1.0000   2.0000    1.0000   2.0000
75%    44.0000    5.0000   1.0000   3.2500    1.0000   2.0000
max    87.0000    7.0000   2.0000  20.0000    7.0000   2.0000
```

The dataset has six columns (*age*, *polviews*, *sexeduc*, *tvhours*, *partners*, *married*) and 188 rows. However, we will not be using *tvhours* and *married* columns in this task because the task description does not mention them.

1. Estimates for variance and mean of partners.

[13]:
```
mean_partners = df_partners["partners"].mean()
variance_partners = df_partners["partners"].var()

print(f"Mean of partners: {mean_partners:.4f}")
print(f"Variance of partners: {variance_partners:.4f}")
```

```
Mean of partners: 1.3830
Variance of partners: 1.7563
```

2. Parameter estimates for the initial model with highlighted statistically non-significant variables.

We fit the initial Poisson regression model using all relevant predictor variables.

[14]:
```
initial_model = smf.poisson("partners ~ age + sexeduc + polviews",␣
 ↪data=df_partners).fit()

initial_model.summary2().tables[1]
```

```
Optimization terminated successfully.
         Current function value: 1.438090
         Iterations 6
```

[14]:

```
             Coef.  Std.Err.       z  P>|z|  [0.025  0.975]
Intercept  2.0128    0.3482  5.7805 0.0000  1.3303  2.6952
age       -0.0207    0.0053 -3.8748 0.0001 -0.0311 -0.0102
sexeduc   -0.5512    0.2545 -2.1653 0.0304 -1.0501 -0.0523
polviews  -0.0903    0.0485 -1.8625 0.0625 -0.1854  0.0047
```

The parameter estimates for the initial model are presented above in the column `Coef.`. Their statistical significance is in the column `Pr(>|z|)`.

Not all variables are statistically significant. Statistically insignificant variables (with significance level 0.05) are:

- `polviews` (p-value = 0.063)

We remove the statistically non-significant variable `polviews` and fit the final Poisson regression model.

[15]:

```python
final_model = smf.poisson("partners ~ age + polviews", data=df_partners).fit()
```

```
Optimization terminated successfully.
         Current function value: 1.452596
         Iterations 6
```

3. For the final model present Chi square likelihood ratio omnibus test.

[16]:

```python
# likelihood ratio statistic
lr_stat = -2 * (initial_model.llf - final_model.llf)
# degrees of freedom
df_diff = initial_model.df_model - final_model.df_model
# p-value
p_value = chi2.sf(lr_stat, df_diff)

print(f"Chi-square likelihood ratio test statistic: {lr_stat}")
print(f"Degrees of freedom: {df_diff}")
print(f"P-value: {p_value}")
```

```
Chi-square likelihood ratio test statistic: -5.454461517427603
Degrees of freedom: 1.0
P-value: 1.0
```

The p-value for our test is 1.0, which means that the final model is not statistically significantly different from the initial model. That makes sense because we only removed one non significant variable from the initial model.

4. Estimates for all variables and their statistical significance.

[17]:

```python
final_model.summary2().tables[1]
```

[17]:

```
              Coef.  Std.Err.       z  P>|z|  [0.025  0.975]
Intercept  1.5307    0.2645  5.7872 0.0000  1.0123  2.0491
age       -0.0219    0.0053 -4.0891 0.0000 -0.0323 -0.0114
polviews  -0.1101    0.0475 -2.3208 0.0203 -0.2031 -0.0171
```

The parameter estimates for the final model are presented above in the column `Coef.`. Their statistical significance is in the column `Pr(>|z|)`.

This time all variables are statistically significant.

5. Model equation.

We can present the final model equation using the parameter estimates.

[18]:

```
# extract coefficients
coefficients = final_model.params
intercept = coefficients["Intercept"]
age_coef = coefficients["age"]
polviews_coef = coefficients["polviews"]

# model equation
model_equation = f"partners = exp({intercept:.4f} + {age_coef:.4f} * age +␣
  ↪{polviews_coef:.4f} * polviews)"
print(model_equation)
```

```
partners = exp(1.5307 + -0.0219 * age + -0.1101 * polviews)
```

6. Forecast for age = 40, sexeduc= 1, polviews=1 (use only variables from the final model).

Let’s use the final model to forecast the number of sexual partners for age = 40, sexeduc = 1, polviews = 1.

[19]:

```
age = 40
sexeduc = 1   # sexeduc is not in the final model!
polviews = 1

# calculate the expected number of partners
expected_partners = final_model.predict(pd.DataFrame({"age": [age], "polviews":␣
  ↪[polviews]}))
print(f"Forecasted number of partners: {expected_partners[0]:.4f}")
```

```
Forecasted number of partners: 1.7267
```