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Question 1: Hohmann Transfer

1. What other information suggests that Hohmann transfers were being used.

Other than the explicit mention of the Hohmann transfer window in the film, there are two evidence that suggest that Hermes used this maneuver to get from Mars to Earth and vice-versa. The first one is the scene for Project Elrond which shows an image of a Hohmann transfer from Earth to Mars. The second is the scene that mentions the time it takes for the crew to come from Mars back to Earth.

The Martian Shooting Script 27.

CONTINUED:

ANNIE
Why don't we talk to Lewis? Let's go to CAPCOM and ask her directly right now.

Vincent shoots Teddy a glance. After a moment, Teddy understands what it means.

TEDDY
No. If Watney is really alive... we don't want the *Ares 3* crew to know.

ANNIE
What? How can you not tell them?

TEDDY
They have another ten months on their trip home. Space travel is dangerous. We need them alert and undistracted.

ANNIE
They already think he's dead.

VINCENT
And they'd be devastated to find out they abandoned him alive.

Figure 1: Trip Duration [1]

Figure 1 above shows Teddy saying that the Ares 3 crew has 10 more months until they reach Earth. 10 months is equivalent to about 295 days. Since Earth and Mars' orbit around the sun are slightly elliptical, there is a range for how long a Hohmann transfer will take. Below shows the results for the minimum and maximum time.

Minimum time calculation assumes that Mars initial location is at its periapsis with respect to the sun and Earth's final location is at its periapsis with respect to the sun. Maximum time assumes that the planet's locations are at their apoapsis.

Table 1: Minimum and maximum value for the transfer ellipse from Mars to Earth.

	Minimum	Maximum
Semimajor Axis, a_{TE}	$176846515\ km$	$200653485\ km$
Eccentricity, e_{TE}	0.1682	0.2420
Period, T_{TE}	$4.0562 \times 10^7\ s$	$4.9022 \times 10^7 s$
Duration, t_{TE}	$234.7339\ days$	$289.6946\ days$

Based on the results, the 10 months' trip duration said the film is relatively close to the maximum possible duration for a Hohmann transfer from Mars back to Earth. The difference is about 5 days. Considering that the dialog in the movie are not exact values, the estimated time frame can be considered valid proven by calculating the time range for a Hohmann transfer from Mars to Earth.

The second evidence is with the document shown in the film for Project Elrond.

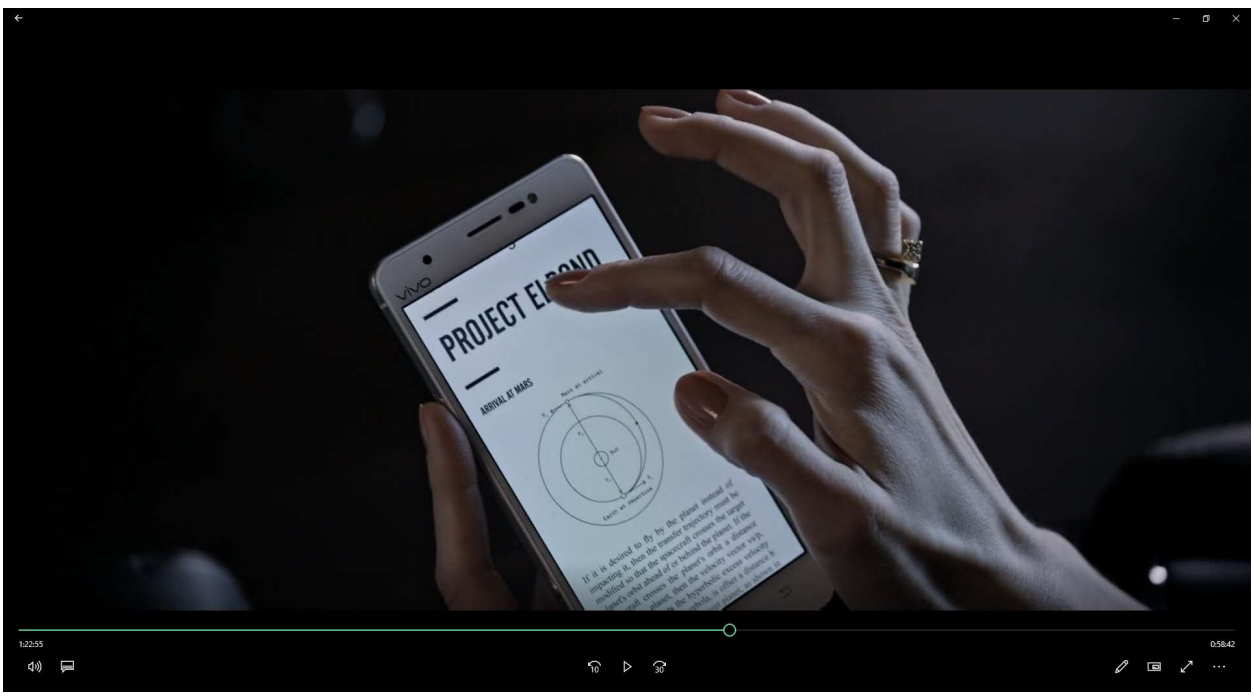

Figure 2: Project Elrond [1].

Figure 2 above shows a diagram of a Hohmann transfer from Earth to Mars. It correctly shows the direction of motion of the spacecraft as it departs Earth and arrive at Mars. As learn in class, a Hohmann transfer only utilizes the first half of the ellipse to get from one orbit to another. This scene also clearly shows the different parameters such as the periapsis radius, apoapsis radius, and apse line for the maneuver.

2. How would you respond to Rich Purnell's presentation? Critique and grade this explanation. What he got right and wrong.

After the presentation, I would immediately ask for the calculations and have it validated by another person who is an expert with Astrodynamics. Then I would ask for what all the risks involved and the possible consequences for going through with the proposed plan. I would ask questions on whether crew's body handle the extra time required to do the mission, and can the Hermes continue with the added distance.

When it comes to Rich Purnell's explanation, I rate it 5/10 because he got the main message across as plainly as he could. But considering he is presenting to NASA employees; he could have gone more in-depth with his explanations. The aspects of the maneuver that he got wrong are the planetary position, timing, lack of sun to rotate about, Hermes' acceleration towards Earth, and noise he made indicative of propulsion. The aspects he got right are that the Hermes' is currently traveling towards the Earth, the slingshot maneuver.

For the actual maneuver, the planets will be rotating about the sun at different rates given the radii of the orbits that they are in. The initial and final positions of the must be predetermined to match the time it takes for the Hohmann transfer. Assuming that Mars and Earth have circular orbits, the initial phase angle between the two planets must be about $75.1°$. This will ensure that the transfer ellipse will meet up Earth at the final position. Rich Purnell could have demonstrated the planetary positions and timings with the empty glasses on the board room table. The sun was not indicated in this presentation. Although it is safe to assume that the NASA employees would know that it exists somewhere in between the two planets, it is more accurate to have another person to represent the sun. The plan to accelerate towards Earth is not accurate. The whole point of a gravity assist maneuver is to preserve fuel while gaining velocity. Accelerating would mean that it would require the engine to burn fuel to produce thrust. Another issue with the acceleration is that gaining speed through propulsion will result in orbit or trajectory to change. This means that accelerating could potentially lead to the Hermes' completely missing the target as it would gain energy from the increased velocity. Lastly, the noise he made as he was doing the fly-by indicates the use of propulsion during the maneuver. This again goes against the whole purpose of a gravity assist which is to gain velocity without expending any or minimal fuel for minor course correction.

What he got right are the general direction of the Hermes and the slingshot maneuver. In the presentation, Hermes was travelling towards the Earth which was the case at the

given setting. The flaw is with it the orbit which is not as eccentric as he made it appear. He also explain the slingshot maneuver correctly for the most part.

The Martian Shooting Script 69.

CONTINUED: (2)

TEDDY
Teddy.
(then)
I'm the Director of NASA.

RICH
Okay, Teddy, you're Earth. And right now the *Hermes* is heading towards you and is about to start its month-long deceleration to intercept. But, instead, I'm proposing...
(demonstrates)
We start accelerating immediately, to preserve velocity and gain even more. We don't intercept Earth at all, but we come close enough to use a gravity assist to adjust course. While we're doing that...

He grabs the pen out of Teddy's pocket.

RICH (CONT'D)
We resupply with the probe --

VINCENT
The Taiyeng Shen.

Figure 3: Slingshot maneuver

Other than the accelerating part, his explanation of a slingshot maneuver is accurate. The idea behind it come close enough to the planet to gain or reduce velocity by changing direction with the help of the planet's gravitational pull. The flaw with his demonstration is that the turn angle appeared to be 180 degrees which is not likely.

3. Would you, as the Director of NASA, move forward with the Gravity assist maneuver based on the demonstration? Explain why or why not.

Based on the demonstration, I will not go through with the plan. In order to make the best decision, I would require more details and proof of the suggested solution. Trusting Rich Purnell's word is not enough. Without showing any of the math, or simulation, it is difficult to assess the validity of his claims. Furthermore, once he provides the calculations, another person or more will have to check it and approve it. NASA has this publish a way to determine the whether a particular mission or project is worth it based on the risk matrix [2].

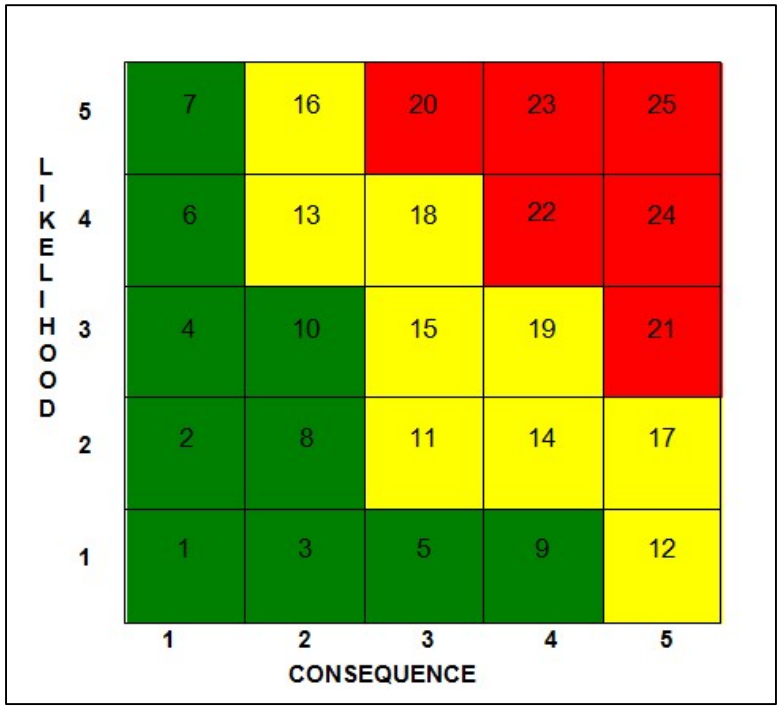

Figure 4: Risk Matrix [2].

Figure 4 above shows an example of risk matrix. The green indicates low risks, yellow medium risks, and red for high risks.

Risk can be quantified based on the likelihood of an event happening and multiplying it by the consequences for a given event. For this case, there are several events that must be considered. The first one is the fly-by maneuver, then the trip back to Mars, then the rescue plan to get Mark Watney, and the trip back home. For all four events, the consequences in case of failure are catastrophic as lives would be lost. But in terms of likelihood, the highest would be the rescue plan, followed by the fly-by maneuver around Earth, and the trip to and from Mars are the least likely to fail. A study regarding the radiation risk for a human mission to Mars indicate that galactic cosmic rays exposure rate increases about three time as spacecraft ventures out of the Earth's orbit into deep space [3]. Overall, the risk is too high to go forward with the mission. In the film, Bruce said, "We either have a high chance of killing one person, or low chance of killing six." [1]. Which is not completely true because both choices have a high risk to end up being catastrophic.

Question 2: MAV Intercepts Hermes

How much impulse velocity must the main engine produce for the MAV to intercept the Hermes?

Based on the conversation between the JPL, NASA director, and advisors what they want to happened is for the MAV to escape Mars' sphere of influence and meet up with Hermes there.

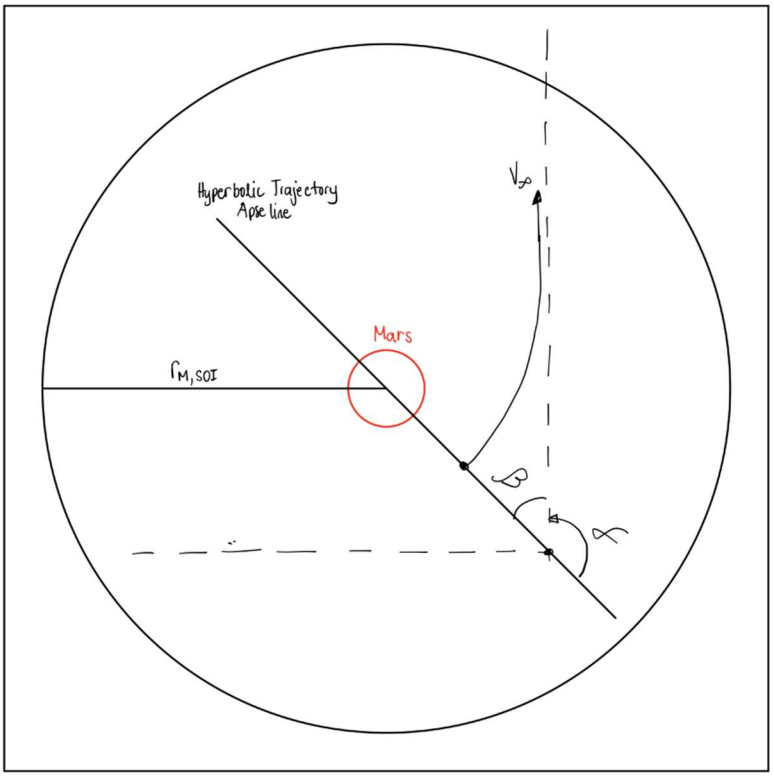

Figure 5: Planetary Departure

Based on the discussion among the astronauts aboard Hermes, the relative velocity at interception must be 5 m/s. The $V_D^{(v)}$ is the planet's heliocentric velocity. V_1 is the MAV's velocity at the sphere of influence. V_1 relative to the Hermes must be 5 m/s:

$$V_1 - V_{Hermes} = 5\ m/s$$

Mars' Heliocentric Velocity, $V_D^{(v)}$

$$V_D^{(v)} = \sqrt{\frac{\mu_{Sun}}{R_M}}$$

$$V_D^{(v)} = \sqrt{\frac{(1.32712 \times 10^{11}\ km^3/s^2)}{227.9 \times 10^6\ km}}$$

$$V_D^{(v)} = 24.1314 \frac{km}{s}$$

Calculate Hermes' velocity based on a Hohmann transfer:

$$V_{Hermes} = \sqrt{2\left(\frac{\mu_{Sun}}{R_M} - \frac{\mu_{Su}}{2a_{te}}\right)}$$

$$V_{Hermes} = \sqrt{2\left(\frac{1.32712 \times 10^{11}\ km^3/s^2}{227.9 \times 10^6\ km} - \frac{1.32712 \times 10^{11}\ km^3/s^2}{2(188750000)}\right)}$$

$$V_{Hermes} = 21.4835 \frac{km}{s}$$

Calculate V1

$$V_1 = V_{Hermes} + 0.005 \frac{km}{s}$$

$$V_1 = 21.4835 \frac{km}{s} + 0.005 \frac{km}{s}$$

$$V_1 = 21.4885 \frac{km}{s}$$

Calculate excess velocity, v_∞

$$v_\infty = \Delta V_D = V_D^{(v)} - V_1$$

$$v_\infty = 24.1314 \frac{km}{s} - 21.4885 \frac{km}{s}$$

$$v_\infty = 2.6429 \frac{km}{s}$$

Based on the actual scene of trying to rescue Mark, the MAV will be 68 km away from the Hermes travelling at 11 m/s relatively, with remaining 39minutes and 12 seconds before time to intercept. Its current velocity is $850\ m/s$ at altitude $1843\ km$.

$$\Delta v = v_p - v_c = (v_c)\left(\sqrt{2 + \left(\frac{v_\infty}{v_c}\right)^2} - 1\right)$$

$$\Delta v = v_p - v_c = \left(0.85\frac{km}{s}\right)\left(\sqrt{2 + \left(\frac{2.6429\frac{km}{s}}{0.85\frac{km}{s}}\right)^2} - 1\right)$$

$$\Delta v = 2.0534\frac{km}{s}$$

Thus, the delta V required from the main engine in order to intercept with the Hermes is about 2.0534 km/s

Question 3: Slingshot Maneuver

1. Perform a Hohmann Transfer calculation of the Hermes that includes the resulting gravity assist around the Earth. Assume hyperbolic periapsis of 500 km.

Hohmann Transfer from Mars to Earth

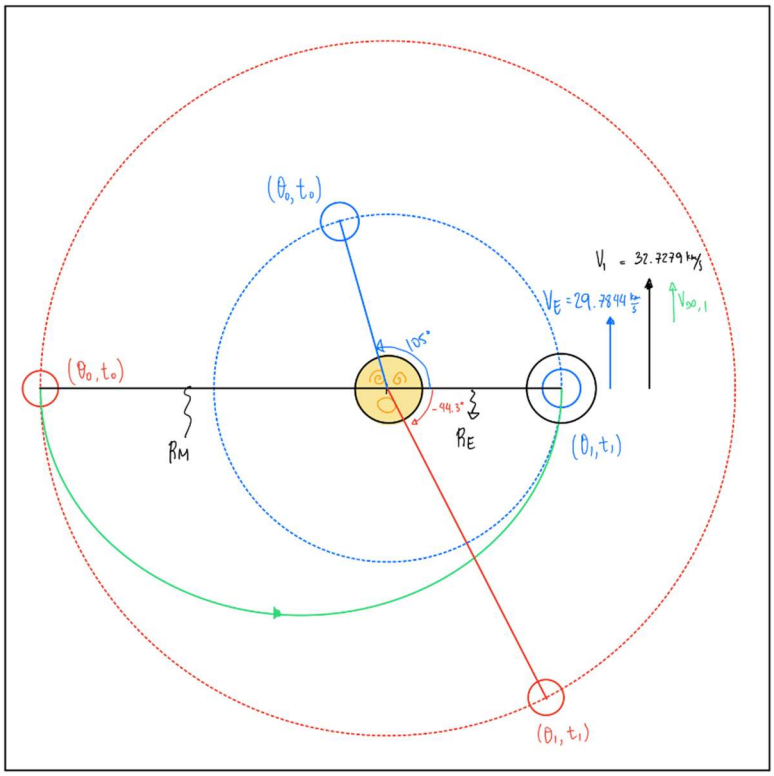

Figure 6: Hohmann Transfer from Mars to Earth

Table 2: Hohmann Transfer ellipse parameters

Transfer Ellipse	**Calculated Values**
Semi-Major Axis, a_{TE}	$188750000\ km$
Eccentricity, e_{TE}	0.2074
Period, T_{TE}	$4.4726 \times 10^7\ s$
Travel Duration, t_{TE}	$258.83\ days$
Velocity at Perihelion, V_1	$32.7279\ km/s$
Earth's Heliocentric Velocity, V_E	$29.7844\ km/s$

Sample Calculations:

```
% Hohmann Transfer from Mars to Earth
a_te = (R_E+R_M)/2 %[km]
e_te = ((R_M-R_E)/(R_E+R_M))
T_te = (2*pi*a_te^(3/2))/sqrt(mu_S) %[s]
t_days = (T_te/2/24/60/60)
% inbound velocity at the T.E's periapse
V1 = sqrt(2*((mu_S/R_E)-(mu_S/(2*a_te)))) %[km/s]
V_E = sqrt(mu_S./R_E) %[km/s] Velocity of Earth around the Sun
```

Gravity Assist around the Earth:

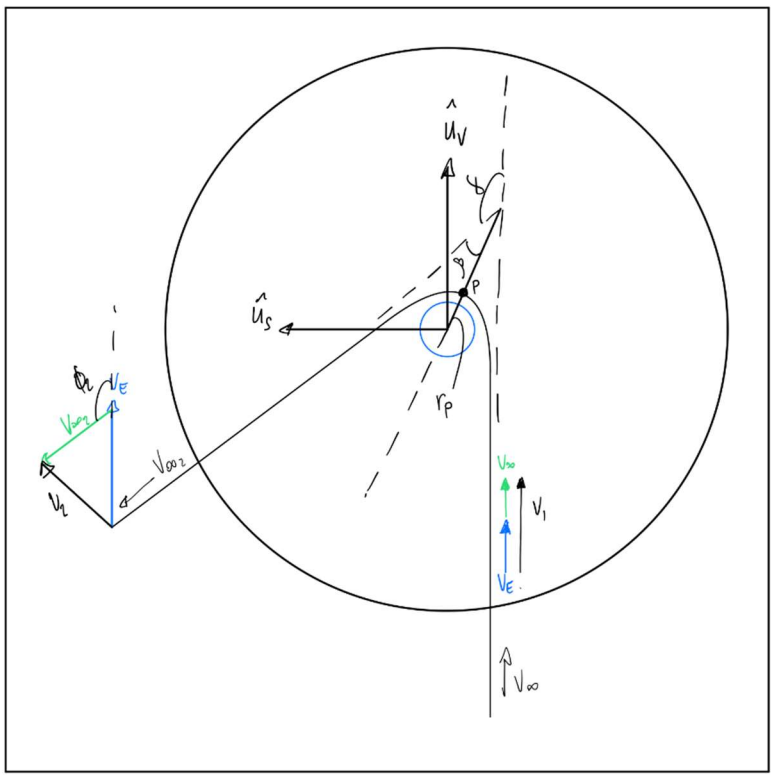

Figure 7: Hyperbolic trajectory in Earth centered frame

Boundary Condition inbound:

Since the Earth is at the periapsis, the direction of V_1 with respect to V_E is $0°$. Thus, the inbound excess velocity $v_{\infty,1} = V_1 - V_E$.

Hyperbolic Trajectory	**Calculated Values**
Excess Velocity, $v_{\infty,1}$	$2.9435\ km/s$
Periapsis Radius, $r_{p,HT}$	$6878\ km$
Semi-major Axis, a_{HT}	$-4.6007 \times 10^4 km$
Eccentricity, e_{HT}	1.1495
Turn Angle, δ_{HT}	$120.9042\ °$
Aiming Radius, Δ_{HT}	2.6080×10^4

Boundary Condition outbound:

At the outbound, the excess velocity is turned $\phi_2 = \phi_1 + \delta = 120.9042°$. The heliocentric velocity at the exit is $\overrightarrow{V_2} = (28.2727\ \hat{u}_v + 2.5256\ \hat{u}_s)\ km/s$

Table 3: Outbound Condition and the resulting orbit

Outbound Orbit	**Calculated Values**
Phase angle between v_∞ and V_E ,ϕ_2	$120.9042°$
Outbound Heliocentric velocity, $\lvert\overrightarrow{V_2}\rvert$	$28.3852\ km/s$
Angular Momentum, h_2	$4.2296 \times 10^9\ km^2/s$
True Anomaly, θ_2	$220.13°$
Eccentricity, e_2	0.1275
Aphelion, $R_{a,2}$	$154.51 \times 10^6\ km$
Perihelion, $R_{p,2}$	$119.55 \times 10^6\ km$

2. Obtain the location where the Hermes will intersect Mars' orbit.

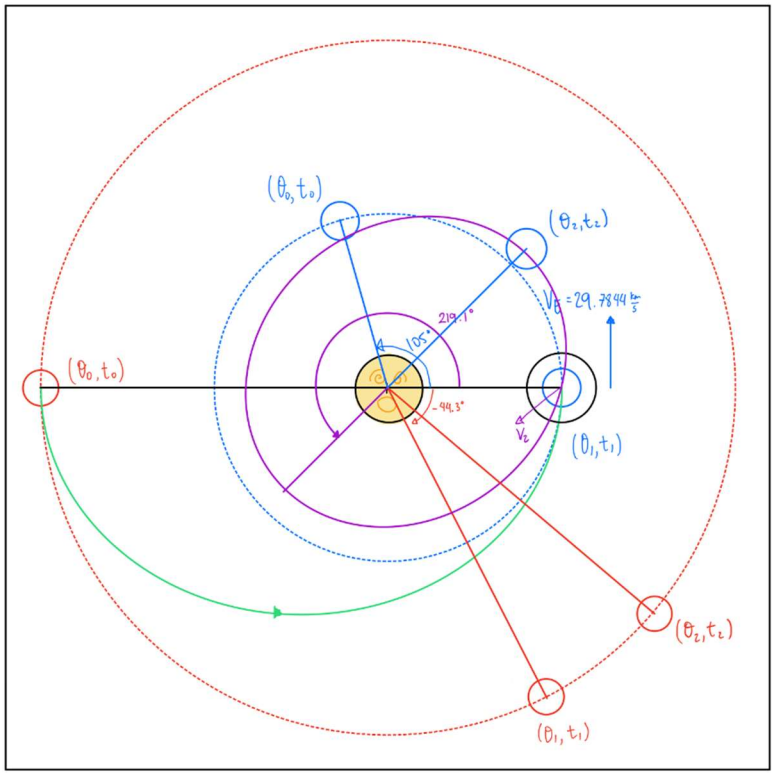

Figure 8: Post Fly-by orbit

Based on the results, the outbound orbit will not intersect Mars' orbit. The maximum distance from the sun that the post fly by orbit will achieve is $154.51 \times 10^6\ km$. In order to intersect with Mars, parameters such as aiming radius, and eccentricity must change. The angle of approach will also affect the trajectory post fly by.

```
% Post Fly by
% Eqtn 8.91
Adj = ((h2^2)/(mu_S*R_E))-1 % eCos(theta)
% Eqtn 8.92
Opp = (-V2_vector(2)*h2)/(mu_S) %eSin(theta)
% True Anomaly
theta2_d = atan(Opp/Adj)+pi;%eCos(theta)
theta2_deg = rad2deg(theta2_d)
% eccentricity
e2 = Opp/(sin(theta2_d))
% ra2
ra2= h2^2/(mu_S*(1-e2))
% rp2
rp2 = h2^2/(mu_S*(1+e2))
r_close = R_M- ra2
```

```
% Period
a2 = (ra2+rp2)/2; %km
T2 = (2*pi*a2^(3/2))/sqrt(mu_S) %[s]
%timing for the closest locaiton
theta3_d = theta2_d-pi
% Eccentric Anomaly
E2= 2*atan(sqrt((1-e2)/(1+e2))*tan(theta3_d/2))
% Mean Anomaly
M2 = E2-e2*sin(E2)
% time elapsed
t2_d = (M2/(2*pi))*T2; %s
% Mars'position 2
thm2 = 2*pi*(t2_d/T_M)
thm2_deg = rad2deg(thm2)
```

3. How close will Mars actually be to this point when the Hermes intersects the Martian orbit.

Given the post fly-by orbit did not intersect Mars' orbit, the closest distance between the two orbit is at aphelion. This closest distance to Mars' orbit is about $73.395 \times 10^6 km$

$$r_{close} = R_M - R_{a,2}$$

$$r_{close} = 227.9 \times 10^6\ km - 154.51 \times 10^6\ km$$

$$r_{close} = 73.395 \times 10^6 km$$

But the distance between Hermes and Mars at this time is $187.77 \times 10^6 km$

$$r_{closest} = R_{herm} \quad - R_M$$

$$r_{closest} = \left(R_{a,2}\cos(\theta_h)\, i + R_{a,2}\sin(\theta_h) j\right) - (R_M \cos(\theta_m) i - R_M \sin(\theta_m) j)$$

$$r_{closest} = \left(R_{a,2}\cos(39.1°)\, i + R_{a,2}\sin(39.1°) j\right) - (R_M \cos(14.25°) i - R_M \sin(14.25°) j)$$

$$r_{closest} = (119.9 \times 10^6 i + 97.4 \times 10^6 j) - (220.9 \times 10^6 i - 56.1 \times 10^6 i)$$

$$r_{closest} = (-100.98 \times 10^6 i + 153.54 \times 10^6) km$$

$$|r_{closest}| = 187.77 \times 10^6 km$$

4. Assume hyperbolic periapsis of 500 km. Why do you need to specify this distance in your analysis?

The hyperbolic periapsis distance is required to be specified because it greatly affects the behavior of the hyperbolic trajectory. The following equations depend on the $r_{p,HT}$:

$$e_{HT} = 1 + \frac{r_{p,HT} v_{\infty,1}^2}{\mu_E}$$

$$\Delta_{HT} = r_{p,HT} \sqrt{\frac{e_{HT} + 1}{e_{HT} - 1}}$$

Eccentricity is a parameter that dictates many other parameters such as turn angle, true anomaly of asymptote, and aiming radius. By increasing $r_{p,HT}$, it will decrease turn angle which will increase how the heliocentric velocity at the outbound for inner planet slingshot maneuver.

5. What is actually missing in your analysis and therefore approximated by the need to specify this value?

For this analysis, the aiming radius was not specified which will help determine the how and where the spacecraft will approach the sphere if influence. The angular momentum of the hyperbolic trajectory can be calculated as:

$$h_{HT} = v_\infty \Delta$$

By choosing the larger aiming radius, Δ, the angular momentum increases. By combining this equation with the latus rectum, p, aiming radius can be related to eccentricity

$$\Delta = \sqrt{\frac{a(1 - e^2)\mu}{v_\infty^2}}$$

Assuming that semi-major axis, a, gravitational parameter, μ, and excess velocity, v_∞ are all fixed value aiming radius is directly related to eccentricity. Increasing eccentricity will translate to higher velocity gains from the slingshot maneuver. Periapsis radius will be calculated after choosing the aiming radius.

Question 4: Rescue Plan

What moral, ethical, and technical reasons why the crew would or would not be told this information. What would you have done? When would you have told the crew.

Looking at it from the moral and ethical perspective, the crew must be of told of Mark Watney's survival eventually but not right away. The reason is because although the truth will ultimately come out. Judging the rightness and wrongness of an action is determined by the motives of the person [4]. The director claims to want to withhold the truth to ensure that the crew remains focus throughout the remaining of their mission. This is a valid claim given that telling the crew right away will only raise more questions than answer. If I was in the director's I would try to come up with a plan and once there is one that has a high probability of success with minimal risks, then I would let the crew know of the situation. Prior to the crew knowing Watney's survival, they were all devastated especially Commander Lewis. Telling them of the news before any rescue plan is formed will only add to the guilt she already feels.

From the various technical reasons, it is important to tell the crew the of his survival as soon as possible. The reason is because one of the possible rescue plans might involve the Hermes performing a maneuver at a specific time and location. For example, a Hohmann transfer from Earth to Mars requires a planetary alignment in order to execute. Missing this specific window will cost 2.13 years of wait time. Also, the astronauts aboard the Hermes are a group smart people and may help with finding a possible solution to the problem. That is what they are trained to do.

$$T_{syn} = \frac{T_E \cdot T_M}{|T_E - T_M|} = \frac{365.26 \cdot 687.99}{|365.26 - 687.99|} = 777.9\ days = 2.13\ years$$

Rescue Plan Calculations

The plan is to rescue Mark Watney within the wait time for the next Hohmann Transfer window back to Earth.

Table 4: Earth and Mars position and timing.

Position and Time	**Earth**	**Mars**
0	$(0°, 0\ day)$	$(44.33°, 0\ day)$
1	$(255.10°, 258.83\ days)$	$(180°, 255.83\ day)$
2	$(703.25°, 713.53\ days)$	$(418.34°, 713.53\ days)$
3	$(958.34°, 972.36\ days)$	$(554.01°, 972.36\ days)$

```
% Hohmann Transfer
a_te = (R_E+R_M)/2; %[km] Semi-major axis
e_te = ((R_M-R_E)/(R_E+R_M)) %eccentricity
T_te = (2*pi*a_te^(3/2))/sqrt(mu_S); %[s] Period
% Earth
T_E = (2*pi*R_E^(3/2))/sqrt(mu_S); %[s]
V_E = sqrt(mu_S./R_E) %[km/s] Velocity of Earth around the Sun
n_E = (2*pi)/(T_E) % [rad/s]
% Mars
T_M = (2*pi*R_M^(3/2))/sqrt(mu_S); %[s]
V_M = sqrt(mu_S./R_M) %[km/s] Velocity of Mars around the Sun
n_M = (2*pi)/(T_M) %[rad/s]


% Time and Position 0 Departure from Earth
t0 = 0; %seconds
th0e = 0; %rad
th0m = pi-2*pi*(T_te/(2*T_M));
phi0 = th0m - th0e;
t0_days = 0 %days
th0e_deg = 0 %deg
th0m_deg = rad2deg(pi-2*pi*(T_te/(2*T_M)))
phi0_deg = rad2deg(th0m - th0e)


% Time and Position 1 Arrival at Mars
t1 = T_te/2; %seconds
th1e = th0e + n_E*t1;
th1m = th0m + n_M*t1;
%phi1 = phi0 + (n_E-n_M)*t1;
phi1 = th1m-th1e;
t1_days = (T_te/2)/(24*60*60) %days
th1e_deg = rad2deg(th1e) %deg
th1m_deg = rad2deg(th1m) %deg
```

```
phi1_deg = rad2deg(phi1)


% Time and Position 2 Wait until next phi aligns
t2 = (-2*phi1-2*pi*1)/(n_M-n_E); %s
t2_days = t2/24/60/60 % days
th2e = th1e + n_E*t2;
th2m = th1m + n_M*t2;
th2e_deg = rad2deg(th2e) %deg
th2m_deg = rad2deg(th2m) %deg


% Hohmann Transfer from Mars to Earth
t3 = t1; %seconds
th3e = th2e + n_E*t3; %rad
th3m = th2m + n_M*t3; %rad
t3_days = t3/24/60/60 %seconds
th3e_deg = rad2deg(th3e) %deg
th3m_deg = rad2deg(th3m) %deg
```

References

[1] R. Scott, Director, *The Martian.* [Film]. United States : 20th Century Fox, 2015.

[2] NASA, "S3001: Guidelines for Risk Management," NASA, 2017.

[3] F. A. Cucinotta, M.-H. Y. Kim, L. J. Chappell and J. L. Huff, "How Safe is Safe Enough? Radiation risk for Human Mission to Mars," *PLoS One,* vol. 8, no. 10, 2013.

[4] J. D. Rummel, "Some Ethical Consideration in Space Exploration," SETI Institute, 2017.

[5] L. Burke, "An Examination of "The Martian" Trajectory," NASA/Langley Research Center, Hampton , 2015.

MATLAB

```
%{

AER 721 Orbital Dynamics

Project 3: The Martian

Jann Cristobal

500815181

%}

mu_S = 132712000000; %[km^3/s^2]
mu_E = 398600; %[km^3/s^2]

mu_M = 42828; %[km^3/s^2]

R_E = 149.6e6;%[km]

R_M = 227.9e6;%[km]

re = 6378; %[km]

rm = 3396; %[km]

m_S = 1.989e30; %[kg]

m_E = 5.974e24; %[kg]

m_M = 641.9e21; %[kg]

e_E = 0.0167;

e_M = 0.0935;
```

Question 1

```
% What suggest that Hermes used Hohmann Transfer
% Max Hohmann Transfer Duration from Mars to Earth

R_Emax = R_E*((1-e_E^2)/(1+e_E*cos(pi)))
```

```
R_Emax = 152098320
```

```
R_Mmax = R_M*((1-e_M^2)/(1+e_M*cos(pi)))
```

```
R_Mmax = 2.4921e+08
```

```
a_temax = (R_Emax+R_Mmax)/2 %[km]
```

```
a_temax = 200653485
```

```
e_temax = ((R_Mmax-R_Emax)/(R_Emax+R_Mmax))
```

```
e_temax = 0.2420
```

```
T_temax = (2*pi*a_temax^(3/2))/sqrt(mu_S) %[s]
```

```
T_temax = 4.9022e+07
```

```
t_maxdays = (T_temax/2/24/60/60)
```

```
t_maxdays = 283.6946
```

```
% Min Hohmann Transfer Duration from Mars to Earth
R_Emin = R_E*((1-e_E^2)/(1+e_E*cos(0)))
```

```
R_Emin = 147101680
```

```
R_Mmin = R_M*((1-e_M^2)/(1+e_M*cos(0)))
```

```
R_Mmin = 2.0659e+08
```

```
a_temin = (R_Emin+R_Mmin)/2 %[km]
a_temin = 176846515
```

```
e_temin = ((R_Mmin-R_Emin)/(R_Emin+R_Mmin))
```

```
e_temin = 0.1682
```

```
T_temin = (2*pi*a_temin^(3/2))/sqrt(mu_S) %[s]
```

```
T_temin = 4.0562e+07
```

```
t_mindays = (T_temin/2/24/60/60)
```

```
t_mindays = 234.7339
```

Question 2

```
v_mav = 0.85; %[km/s]
z_mav = 1843; %[km]

r_mav = rm+z_mav; %[km]

a_mav = -(mu_M)/(2*((v_mav^2/2)-(mu_M/r_mav))); %[km]
```

```
V_M = sqrt(mu_S/R_M) % km/s Mars' Heliocentric Velocity
```

```
V_M = 24.1314
```

```
a_te = (R_E+R_M)/2 %[km]

V_hermes = sqrt(2*((mu_S/R_M)-(mu_S/(2*a_te))))
```

```
V_hermes = 21.4835
```

```
% km/s Hermes' Heliocentric Velocity

v_infinity = V_M - (V_hermes+0.005) % km/s Excess velocity
```

```
v_infinity = 2.6429
```

```
Delta_v = v_mav*(sqrt(2+(v_infinity/v_mav)^2)-1)
```

```
Delta_v = 2.0534
```

```
% km/s Delta V required for MAV to interset with the Hermes.
```

Question 3

```
% Hohmann Transfer from Mars to Earth
a_te = (R_E+R_M)/2 %[km]
```

```
a_te = 188750000
```

```
e_te = ((R_M-R_E)/(R_E+R_M))
```

```
e_te = 0.2074
```

```
T_te = (2*pi*a_te^(3/2))/sqrt(mu_S) %[s]
```

```
T_te = 4.4726e+07
```

```
t_days = (T_te/2/24/60/60)
```

```
t_days = 258.8281
```

```
% inbound velocity at the T.E's periapse

V1 = sqrt(2*((mu_S/R_E)-(mu_S/(2*a_te)))) %[km/s]
```

```
V1 = 32.7279
```

```
h_te = R_E*V1; %[km^2/s] Angular Momentum

V_E = sqrt(mu_S./R_E) %[km/s] Velocity of Earth around the Sun
```

```
V_E = 29.7844
```

```
% Sphere of Influence
re_soi = R_E*(m_E/m_S)^(2/5) %[km] Earth's Sphere of Influence
```

```
re_soi = 9.2466e+05
```

```
rm_soi = R_M*(m_M/m_S)^(2/5); %[km] Mars' Sphere of Influence

A1 = atan(re_soi/R_E);



% Inbound conditions
r_in = a_te*((1-e_te^2)/(1+e_te*cos(-A1)));

v_in = sqrt(mu_S*((2/r_in)-(1/a_te)));

gamma_in = atan((e_te*sin(-A1))/(1+e_te*cos(-A1)));

alpha1 = deg2rad(0);

%alpha1 = acos(V_E/V1)



V1_vector = [V1*cos(alpha1) V1*sin(alpha1)]
```

```
V1_vector = 1×2

   32.7279         0
```

```
V_E_vector = [V_E 0]
```

```
V_E_vector = 1×2

   29.7844         0
```

```
v_inf1_vector = [V1_vector-V_E_vector]
```

```
v_inf1_vector = 1×2

    2.9435         0
```

```
v_inf1 = norm(v_inf1_vector)
```

```
v_inf1 = 2.9435
```

```
rp_HT = re + 500; %[km] this is the rp for the Hyperbolic Trajectory

%e_HT = 1+((rp_HT*v_inf1^2)/(mu_E)) % eccentricity for Hyperbolic Traj.



v_rp = sqrt((2*mu_E/rp_HT)+v_inf1^2)
```

```
v_rp = 11.1611
```

```
%Hyperbola semi-major and eccentricity
a_HT = -mu_E/(v_inf1^2)
```

```
a_HT = -4.6007e+04
```

```
e_HT = 1-rp_HT/a_HT
```

```
e_HT = 1.1495
```

```
%Turn Angle
delta = 2*asin(1/e_HT);

delta_deg = rad2deg(delta)
```

```
delta_deg = 120.9042
```

```
%Beta = (pi-delta)/2;

Beta = acos(1/e_HT)
```

```
Beta = 0.5157
```

```
%True Anomaly of the Asysmptote

theta_inf = acos(-1/e_HT);

%Aiming Radius

Delta = rp_HT*sqrt((e_HT+1)/(e_HT-1))
```

```
Delta = 2.6080e+04
```

```
phi1_HT = atan(v_inf1_vector(2)/v_inf1_vector(1)); %[rad]
phi1_deg = rad2deg(phi1_HT)
```

```
phi1_deg = 0
```

```
% Dark side Approach

phi2 = phi1_HT+delta; %[rad]

% V2_v = v_E-v_inf1*sssin(phi2-(pi/2));

% V2_s = v_inf1*cos(phi2-(pi/2));

V2_vector = V_E_vector + [v_inf1*cos(phi2) v_inf1*sin(phi2)]
```

```
V2_vector = 1×2

   28.2727    2.5256
```

```
V2 = norm(V2_vector)
```

```
V2 = 28.3852
```

```
Delta_V_d = 2*v_inf1*sin(delta/2)
```

```
Delta_V_d = 5.1213
```

```
% Angular Momentum
h2 = R_E*V2_vector(1) %[km^2/s]
```

```
h2 = 4.2296e+09
```

```
% Post Fly by
% Eqtn 8.91

Adj = ((h2^2)/(mu_S*R_E))-1 % eCos(theta)
```

```
Adj = -0.0989
```

```
% Eqtn 8.92
Opp = (-V2_vector(2)*h2)/(mu_S) %eSin(theta)
```

```
Opp = -0.0805
```

```
% True Anomaly
theta2_d = atan(Opp/Adj)+pi;%eCos(theta)

theta2_deg = rad2deg(theta2_d)
```

```
theta2_deg = 219.1301
```

```
% eccentricity
e2 = Opp/(sin(theta2_d))
```

```
e2 = 0.1275
```

```
% ra2
ra2= h2^2/(mu_S*(1-e2))
```

```
ra2 = 1.5451e+08
```

```
% rp2

rp2 = h2^2/(mu_S*(1+e2))
```

```
rp2 = 1.1955e+08
```

```
r_close = R_M- ra2
```

```
r_close = 7.3395e+07
```

Question 4

```
% Hohmann Transfer
a_te = (R_E+R_M)/2; %[km] Semi-major axis

e_te = ((R_M-R_E)/(R_E+R_M)) %eccentricity
```

```
e_te = 0.2074
```

```
T_te = (2*pi*a_te^(3/2))/sqrt(mu_S); %[s] Period

% Earth

T_E = (2*pi*R_E^(3/2))/sqrt(mu_S); %[s]

V_E = sqrt(mu_S./R_E) %[km/s] Velocity of Earth around the Sun
```

```
V_E = 29.7844
```

```
n_E = (2*pi)/(T_E) % [rad/s]
```

```
n_E = 1.9909e-07
```

```
% Mars

T_M = (2*pi*R_M^(3/2))/sqrt(mu_S); %[s]

V_M = sqrt(mu_S./R_M) %[km/s] Velocity of Mars around the Sun
```

```
V_M = 24.1314
```

```
n_M = (2*pi)/(T_M) %[rad/s]
```

```
n_M = 1.0589e-07
```

```
% Time and Position 0 Departure from Earth
t0 = 0; %seconds

th0e = 0; %rad

th0m = pi-2*pi*(T_te/(2*T_M));

phi0 = th0m - th0e;

t0_days = 0 %days
```

```
t0_days = 0
```

```
th0e_deg = 0 %deg
```

```
th0e_deg = 0
```

```
th0m_deg = rad2deg(pi-2*pi*(T_te/(2*T_M)))
```

```
th0m_deg = 44.3292
```

```
phi0_deg = rad2deg(th0m - th0e)
```

```
phi0_deg = 44.3292
```

```
% Time and Position 1 Arrival at Mars
t1 = T_te/2; %seconds

th1e = th0e + n_E*t1;

th1m = th0m + n_M*t1;

%phi1 = phi0 + (n_E-n_M)*t1;

phi1 = th1m-th1e;

t1_days = (T_te/2)/(24*60*60) %days
```

```
t1_days = 258.8281
```

```
th1e_deg = rad2deg(th1e) %deg
```

```
th1e_deg = 255.0971
```

```
th1m_deg = rad2deg(th1m) %deg
```

```
th1m_deg = 180.0000
```

```
phi1_deg = rad2deg(phi1)
```

```
phi1_deg = -75.0971
```

```
% Time and Position 2 Wait until next phi aligns
t2 = (-2*phi1-2*pi*1)/(n_M-n_E); %s

t2_days = t2/24/60/60 % days
```

```
t2_days = 454.7041
```

```
th2e = th1e + n_E*t2;

th2m = th1m + n_M*t2;

th2e_deg = rad2deg(th2e) %deg
```

```
th2e_deg = 703.2467
```

```
th2m_deg = rad2deg(th2m) %deg
```

```
th2m_deg = 418.3438
```

```
% Hohmann Transfer from Mars to Earth
t3 = t1; %seconds

th3e = th2e + n_E*t3; %rad

th3m = th2m + n_M*t3; %rad

t3_days = t3/24/60/60 %seconds
```

```
t3_days = 258.8281
```

```
th3e_deg = rad2deg(th3e) %deg
```

```
th3e_deg = 958.3438
```

```
th3m_deg = rad2deg(th3m) %deg
```

```
th3m_deg = 554.0146
```