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Stellar Inertial Attitude Determination For LEO Spacecraft

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Abstract

This paper describes the normal mode attitude determination algorithm for the low earth orbit (LEO) spacecraft using stellar inertial sensors. Included in the algorithm is a 6 state Extended Kalman Filter (EKF) which corrects the spacecraft attitude error as well as gyro bias error using celestial observations from star trackers. The detailed derivation of the EKF using a quaternion formulation is given in the paper. Time-domain simulations are presented to show the predicted filter performance which is a function of star density (or number of stars seen), star catalog error, star tracker noise, and gyro noises. Covariance analysis indicates that precision attitude with accuracy better than 12 arc seconds (3 sigma) can be met with the proposed attitude determination and control system hardware components.

1. Introduction

The low earth orbit (LEO) Earth Observing System (EOS) PM spacecraft is the second in NASA's EOS series which provide scientific data for monitoring changes in the global Earth system. The EOS PM spacecraft accommodates six instruments that focus on the Earth's land processes and radiation data collection. One of the key functions required in the spacecraft's Attitude Determination and Control System (ADCS) is to report accurate three-axis spacecraft attitude and orbit data to the scientific instruments via the Guidance, Navigation, and Control (GN&C) 1553 bus.

This paper describes an ADCS that provides precision attitude determination for the LEO spacecraft satisfying NASA EOS PM mission. The proposed ADCS contains attitude determination sensors : Litton's Spacecraft Inertial Reference Unit (SIRU), Hughes' HD 1003 Star Trackers (STs), and flight software that computes the spacecraft attitude.

Output data from the SIRU is sampled and filtered by the spacecraft computer every 0.05 seconds and sent to the ADCS every 0.5 seconds. The spacecraft attitude expressed as a quaternion between the Earth-Centered-Inertial (ECI) reference frame and the spacecraft body reference frame, is updated in real time using the current estimated quaternion and the SIRU derived rates to determine the changes in attitude over the last 0.5 second cycle. This attitude propagation is simply done by a quaternion multiplication operation :

$$\hat{Q}^{ECI}_{s/c}(t+\Delta t) = \hat{Q}^{ECI}_{s/c}(t) * \hat{\omega}^{s/c}_{ECI\leftarrow s/c} \tag{1}$$

$$\hat{\omega}^{s/c}_{ECI\leftarrow s/c} = \begin{bmatrix} \frac{\Delta\hat{\theta}_x}{2} & \frac{\Delta\hat{\theta}_y}{2} & \frac{\Delta\hat{\theta}_z}{2} & 1 - \frac{\left(\Delta\hat{\theta}\right)^2}{8} \end{bmatrix}^T \tag{2}$$

where

$$\left(\Delta\hat{\theta}\right)^2 = \Delta\hat{\theta}_x^2 + \Delta\hat{\theta}_y^2 + \Delta\hat{\theta}_z^2$$

and $\Delta\hat{\theta}_x$, $\Delta\hat{\theta}_y$, $\Delta\hat{\theta}_z$ are the measured spacecraft's roll, pitch, yaw incremental angles over the 0.5 second period respectively. The errors in the initial attitude estimate, the errors in the uncompensated SIRU drifts, and the error introduced through approximate quaternion propagation need to be corrected or compensated for so that the computed spacecraft attitude meets the allocated attitude knowledge requirements (12 arcsec, 3σ, per axis).

To correct the attitude error, a Stellar Update Filter (SUF) using celestial observations from the STs is implemented in the ADCS. The ST built by Hughes Danbury Optical System (HDOS) is a state-of-the-art solid state star tracker which has a 8^0x8^0 field-of-view and is capable of tracking up to six stars simultaneously with star magnitude 6 or brighter. Each tracker provides accurate two-axis star centroid position and star magnitude data for a particular tracked star. On-orbit catalog stars (approximately 1183 stars with visual magnitude 5 or brighter) are stored in the spacecraft computer. Upon each star sighting, a star identification is carried out. If a star is uniquely identified, the star residuals - the difference between the measured star centriod positions and the estimated star centriod positions are inputted to the SUF.

It is the function of the SUF to process the received stellar residuals to produce an estimate of the errors in the current attitude knowledge and the gyro drift rate biases so that the updated attitude meets the attitude knowledge requirement. The purpose of this paper is (i) to derive the detailed algorithms or equations to be implemented in the SUF to estimate these errors; (ii) to describe the time-domain and covariance analysis simulation models developed for evaluating the filter performance; and (iii) to present computer simulation results used to allocate some of the critical subsystem requirements.

2. Derivation of Stellar Update Filter

The derivation of SUF consists of three steps : (1) to derive the so called "measurement equations" which defined the unique relationship between the reported stellar residuals and the errors to be estimated; (2) to linearize the measurement equations and dynamic equations that describe the dynamic behavior of the errors; and (3) to apply the extended Kalman filtering technique to obtain an unbiased estimator .

Measurement Equations - Stellar Residuals

The star residuals, ΔH and ΔV are the differences between the measured star horizontal/vertical positions, H_m and V_m and the estimated star horizontal/vertical positions, $\tilde{H}$ and $\tilde{V}$:

$$\begin{bmatrix} \Delta H \\ \Delta V \end{bmatrix} = \begin{bmatrix} \tilde{H} \\ \tilde{V} \end{bmatrix} - \begin{bmatrix} H_m \\ V_m \end{bmatrix} \tag{3}$$

The measured star horizontal/vertical positions will be corrupted with the measurement noises, ΔH_m and ΔV_m such as star tracker charge coupled device (CCD) noise and centroiding error:

$$\begin{bmatrix} H_m \\ V_m \end{bmatrix} = \begin{bmatrix} H \\ V \end{bmatrix} - \begin{bmatrix} \Delta H_m \\ \Delta V_m \end{bmatrix} \tag{4}$$

where H and V are the true star horizontal/vertical positions.

The estimated star horizontal/vertical positions are obtained as follows :

$$\begin{bmatrix} \tilde{H} \\ \tilde{V} \end{bmatrix} = G_\alpha \begin{bmatrix} \dfrac{\tilde{x}_s}{\tilde{z}_s} & \dfrac{\tilde{y}_s}{\tilde{z}_s} \end{bmatrix}^T \tag{5}$$

where G_α is the star tracker scale factor,

$$\begin{bmatrix} \tilde{x}_s \\ \tilde{y}_s \\ \tilde{z}_s \\ 0 \end{bmatrix} = \hat{Q}^{ECI}_{S/c} * Q^{s/c}_{ST} * \begin{bmatrix} \tilde{r}_{ECI}(1) \\ \tilde{r}_{ECI}(2) \\ \tilde{r}_{ECI}(3) \\ 0 \end{bmatrix} * Q^{ST}_{s/c} * \hat{Q}^{s/c}_{ECI} \tag{6}$$

$$\begin{bmatrix} \tilde{r}_{ECI}(1) \\ \tilde{r}_{ECI}(2) \\ \tilde{r}_{ECI}(3) \end{bmatrix} = \begin{bmatrix} \cos(\tilde{DEC})\cos(\tilde{RA}) \\ \cos(\tilde{DEC})\sin(\tilde{RA}) \\ \sin(\tilde{DEC}) \end{bmatrix} \tag{7}$$

$\tilde{RA}$ and $\tilde{DEC}$ are the identified star right ascension and declination angles, and "*" denotes the quaternion multiplication [1].

The identified star right ascension and declination angles are never known exactly, instead, they are corrupted with the star catalog errors, ΔRA and ΔDEC:

$$\begin{bmatrix} \tilde{DEC} \\ \tilde{RA} \end{bmatrix} = \begin{bmatrix} DEC \\ RA \end{bmatrix} - \begin{bmatrix} \Delta DEC \\ \Delta RA \end{bmatrix} \tag{8}$$

where RA and DEC are the true star right ascension and declination angles. The computed spacecraft attitude given by equations (1) and (2) will contain errors caused by initial attitude estimation error and gyro noises such as bias error, scale factor error, and random walk error.

Definition of State Variables

Before one can derive the filter to estimate the attitude errors, one needs to define the desired state variables to be estimated and the noise parameters to be included in the filter. Once the state variables are defined, one can then obtain the dynamic equations for each of the state variables. It is noted that for a non-linear estimation problem that we are dealing with, the filter performance often depends on the state variables that one selects. On the one hand, one would select minimum number of states to improve the filter's observability, on the other hand, one would select states having simple dynamics in order to ease filter's implementation.

Listed below are the 6 states that were selected for our baseline SUF design - Δq_1, Δq_2, Δq_3 for spacecraft attitude corrections; Δb_x, Δb_y, Δb_z for SIRU gyro bias corrections :

$$Q_{ECI}^{S/C} = \hat{Q}_{ECI}^{S/C} * \Delta Q_{ECI}^{S/C} \tag{9}$$

$$\Delta Q_{ECI}^{S/C} = \begin{bmatrix} \Delta q_1 & \Delta q_2 & \Delta q_3 & 1 \end{bmatrix}^T ;$$

$$\left|\Delta q_i\right| << 1 ; \text{and}$$

$$b_x = \hat{b}_x + \Delta b_x \tag{10}$$

$$b_y = \hat{b}_y + \Delta b_y \tag{11}$$

$$b_z = \hat{b}_z + \Delta b_z \tag{12}$$

where $\Delta Q_{ECI}^{S/C}$ is the spacecraft attitude error quaternion. The inclusion of three gyro bias corrections as state variables is necessary since when there are occasional large no star gaps in the star field (with no ability to correct the gyro propagated attitude error), the knowledge of the gyro bias becomes important. It is noted that the misalignments between the ST coordinate frame and the spacecraft body reference frame are not included here, since they are treated as separated error terms in the overall attitude error budget.

<u>Dynamic Equations</u>

The SIRU gyro bias errors can be modeled as unknown but constant parameters. Hence their dynamics are simply given by :

$$\Delta \dot{b}_x = \eta_x \tag{13}$$

$$\Delta \dot{b}_y = \eta_y \tag{14}$$

$$\Delta \dot{b}_z = \eta_z \tag{15}$$

where η_x, η_y, η_z are gyro rate random walk noises. The dynamics of error quaternion $\Delta Q_{ECI}^{S/C}$ can be obtained in following two steps : First, differentiating both sides of equation (9) which gives :

$$\frac{d}{dt}\left(\Delta Q_{ECI}^{S/C}\right) = \hat{Q}_{S/C}^{ECI} * \frac{d}{dt}\left(Q_{ECI}^{S/C}\right) - \hat{Q}_{S/C}^{ECI} * \frac{d}{dt}\left(\hat{Q}_{ECI}^{S/C}\right) * \Delta Q_{ECI}^{S/C} \tag{16}$$

where $Q_{ECI}^{S/C}$ and $\hat{Q}_{ECI}^{S/C}$ satisfy two differential equations :

$$\frac{d}{dt}\left(Q_{ECI}^{S/C}\right) = -\,\omega_{ECI\leftarrow S/C}^{S/C} * Q_{ECI}^{S/C} \tag{17}$$

$$\frac{d}{dt}\left(\hat{Q}_{ECI}^{S/C}\right) = -\,\hat{\omega}_{ECI\leftarrow S/C}^{S/C} * \hat{Q}_{ECI}^{S/C} \tag{18}$$

with

$$\omega_{ECI\leftarrow S/C}^{S/C} = \begin{bmatrix} \frac{1}{2}\omega_x \\ \frac{1}{2}\omega_y \\ \frac{1}{2}\omega_z \\ 0 \end{bmatrix} ; \quad \hat{\omega}_{ECI\leftarrow S/C}^{S/C} = \begin{bmatrix} \frac{1}{2}\hat{\omega}_x \\ \frac{1}{2}\hat{\omega}_y \\ \frac{1}{2}\hat{\omega}_z \\ 0 \end{bmatrix}$$

and ω_x, ω_y, ω_z are the true spacecraft angular rates; $\hat{\omega}_x$, $\hat{\omega}_y$, $\hat{\omega}_z$ are the measured spacecraft angular rates; Second, substituting equations (17), (18) into equation (16) which results in :

$$\frac{d}{dt}\left(\Delta Q_{ECI}^{S/C}\right) = \hat{Q}_{S/C}^{ECI} * \Delta\omega * \hat{Q}_{ECI}^{S/C} * \Delta Q_{ECI}^{S/C} \tag{19}$$

$$\text{with } \Delta\omega = -\,\omega_{ECI\leftarrow S/C}^{S/C} + \hat{\omega}_{ECI\leftarrow S/C}^{S/C}$$

<u>Linear Dynamic Equations:</u>

Our next step is to linearize both the non-linear dynamic equations - equation (19) and non-linear measurement equations - equation (3).

Linearization of equation (19) can be obtained by carrying out quaternion multiplication and truncating the higher order terms. The result gives the following linear dynamic equations :

$$\frac{d}{dt}\left(\Delta Q_{ECI}^{S/C}\right) = \hat{Q}_{S/C}^{ECI} * \{\delta\eta + \delta b\} * \hat{Q}_{ECI}^{S/C} \tag{20}$$

$$\text{where } \delta\eta = \frac{1}{2}\begin{bmatrix} \delta\eta_x \\ \delta\eta_y \\ \delta\eta_z \\ 0 \end{bmatrix} ; \quad \delta b = \frac{1}{2}\begin{bmatrix} \delta b_x \\ \delta b_y \\ \delta b_z \\ 0 \end{bmatrix}$$

and $\delta\eta_x$, $\delta\eta_y$, $\delta\eta_z$ are gyro angle random walk noises. In state-space representation, equations (13) - (15) and equation (20) can be combined and expressed as follows:

$$\dot{x} = A x + B \omega_n \tag{21}$$

where x = 6x1 state vector =

$$\begin{bmatrix} \Delta q_1 & \Delta q_2 & \Delta q_3 & \frac{1}{2}\Delta b_x & \frac{1}{2}\Delta b_y & \frac{1}{2}\Delta b_z \end{bmatrix}^T$$

$$A = \begin{bmatrix} 0_{3x3} & \hat{C}^{S/C}_{ECI} \\ 0_{3x3} & 0_{3x3} \end{bmatrix}; \quad B = \begin{bmatrix} \hat{C}^{S/C}_{ECI} & 0_{3x3} \\ 0_{3x3} & I_{3x3} \end{bmatrix}$$

$$\omega_n = \frac{1}{2}\begin{bmatrix} \delta\eta_x & \delta\eta_y & \delta\eta_z & \eta_x & \eta_y & \eta_z \end{bmatrix}^T \text{ with}$$

$E\left\{\omega_n(t)\omega_n^T(t)\right\} = Q_n$, and $\hat{C}^{S/C}_{ECI}$ is the 3x3 direction cosine matrix representing the estimated spacecraft attitude with respect to ECI frame, and Q_n is the 6x6 process noise matrix.

<u>Linear Measurement Equations</u>

Linearization of measurement equations is accomplished by taking partial derivatives of equation (3) with respect to the defined state variables and noise parameters. The procedure is straightforward, however, it is very tedious. We skip the detailed derivations and summarize the following results :

$$y = \begin{bmatrix} \Delta H \\ \Delta V \end{bmatrix} = \begin{bmatrix} H_1 & H_2 \end{bmatrix} x + \begin{bmatrix} G_1 & G_2 \end{bmatrix} n \tag{22}$$

where n = 4x1 noise vector = $\begin{bmatrix} \Delta H_m & \Delta V_m & \Delta DEC & \Delta RA \end{bmatrix}^T$, $H_2 = 0_{2x3}$;

$$H = \begin{bmatrix} H_1 & H_2 \end{bmatrix}; \; G = \begin{bmatrix} G_1 & G_2 \end{bmatrix}; \; G_1 = I_{2x2};$$

$$H_1 = -2[D]\left[C^{S/C}_{ST}\right]\left[\hat{C}^{ECI}_{S/C}\right] \text{skew\_rECI}$$

$$G_2 = -[D]\left[C^{S/C}_{ST}\right]\left[\hat{C}^{ECI}_{S/C}\right] G_{20}$$

$$D = G_\alpha \begin{bmatrix} \frac{1}{z_{sm}} & 0 & -\frac{x_{sm}}{z^2_{sm}} \\ 0 & \frac{1}{z_{sm}} & -\frac{y_{sm}}{z^2_{sm}} \end{bmatrix}$$

$$x_{sm} = \frac{H_m z_{sm}}{G_\alpha}; \; y_{sm} = \frac{H_m z_{sm}}{G_\alpha}; \; z_{sm} = \frac{1}{\sqrt{1 + \left(\frac{H_m}{G_\alpha}\right)^2 + \left(\frac{V_m}{G_\alpha}\right)^2}}$$

$$G_{20} = \begin{bmatrix} -\sin\tilde{DEC}\cos\tilde{RA} & -\cos\tilde{DEC}\sin\tilde{RA} \\ -\sin\tilde{DEC}\sin\tilde{RA} & \cos\tilde{DEC}\cos\tilde{RA} \\ \cos\tilde{DEC} & 0 \end{bmatrix}$$

$$\text{skew\_rECI} = \begin{bmatrix} 0 & -\tilde{r}_{ECI}(3) & \tilde{r}_{ECI}(2) \\ \tilde{r}_{ECI}(3) & 0 & -\tilde{r}_{ECI}(1) \\ -\tilde{r}_{ECI}(2) & \tilde{r}_{ECI}(1) & 0 \end{bmatrix}$$

with $E\{n(t)n^T(t)\} = R$ = 4x4 measurement noise matrix.

<u>Discretization of Linear Dynamic Equations</u>

There is one more step before one can apply the Kalman filtering technique to the above formulated estimation problem. That is : to discretize the linear dynamic equations, or in other words, to convert the continuous system into a discrete system :

$$x(t + \Delta t) = \phi(\Delta t)\, x(t) + \Gamma(\Delta t)\, \omega_n(t) \tag{23}$$

where Δt is the filter sampling period, $\phi(\Delta t)$ is the state transition matrix which is given by :

$$\phi(\Delta t) = e^{A\Delta t} \tag{24}$$

(if A is a constant matrix), and $\Gamma(\Delta t)$ is the process noise sensitivity matrix which is given by :

$$\Gamma(\Delta t) = \int_0^{\Delta t} e^{A\tau} B(\tau)\, d\tau \tag{25}$$

<u>Kalman Filter</u>

Finally, we are ready for deriving the SUF. Based on equations (22) and (23), the stellar update filter can be implemented as follows:

(a) Propagation: Between the star sighting, the filter propagates the covariance matrix P and the estimated state vector $\hat{x}$:

$$P^-(t+\Delta t) = \phi(\Delta t)\,P^+(t)\,\phi^T(\Delta t) + \Gamma(\Delta t)Q_n\Gamma^T(\Delta t) \quad (26)$$

$$\hat{x}^-(t+\Delta t) = \phi(\Delta t)\,\hat{x}^+(t) \quad (27)$$

where the covariance matrix P is defined as :

$$P^+(t) = E\left\{\left(x(t) - \hat{x}^+(t)\right)\left(x(t) - \hat{x}^+(t)\right)^T\right\} \quad (28)$$

(b) Correction: Upon the star sighting at time $t+\Delta t$, first, the filter computes the measurement matrix, H and the measurement noise sensitivity matrix, G as defined in equation (22); second, it computes the filter gain matrix, $K_c(t+\Delta t)$:

$$K_c(t+\Delta t) = P^-(t+\Delta t)H\left(HP^-(t+\Delta t)H^T + GRG^T\right)^{-1}$$

third, it updates the covariance matrix, P and the estimated state vector $\hat{x}$:

$$P^+(t+\Delta t) = \left(I_{6x6} - K_c(t+\Delta t)H\right)P^-(t+\Delta t)$$

$$\left(I_{6x6} - K_c(t+\Delta t)H\right)^T + K_c(t+\Delta t)GRG^TK_c^T(t+\Delta t)$$

$$\hat{x}^+(t+\Delta t) = \hat{x}^-(t+\Delta t) + K_c(t+\Delta t)\Delta y(t+\Delta t)$$

$$\Delta y(t+\Delta t) = y(t+\Delta t) - H\hat{x}^-(t+\Delta t)$$

and finally, the filter updates the quaternion and gyro bias error estimates :

$$\hat{Q}^{S/C}_{ECI}(t+\Delta t)^+ = \hat{Q}^{S/C}_{ECI}(t+\Delta t)^- * \Delta Q_1$$

$$\Delta\hat{b}_x(t+\Delta t)^+ = \Delta\hat{b}_x(t+\Delta t)^- - 2\,d\hat{x}(4)$$

$$\Delta\hat{b}_y(t+\Delta t)^+ = \Delta\hat{b}_y(t+\Delta t)^- - 2\,d\hat{x}(5)$$

$$\Delta\hat{b}_z(t+\Delta t)^+ = \Delta\hat{b}_z(t+\Delta t)^- - 2\,d\hat{x}(6)$$

where $d\hat{x} = K_c(t+\Delta t)\Delta y(t+\Delta t)$ and

$$\Delta Q_1 = \left[d\hat{x}(1) \quad d\hat{x}(2) \quad d\hat{x}(3) \quad 1\right]^T$$

Figure 1 shows a block diagram and signal flow of the baseline attitude determination algorithms implemented with the 6 state Extended Kalman Filter (EKF).

3. SUF Simulation Model

This section describes the computer simulation model that was developed to evaluate the SUF performance. The developed model shown in Figure 2 simulates the spacecraft motion, the characteristics of attitude determination sensors, the data processing that generates the star residuals, the EKF computation, and the spacecraft attitude determination and corrections. The model was generated using MATLAB software tools residing in a Macintosh computer.

Model Description

(1) Spacecraft Motion

A simple orbital motion with a constant spacecraft pitch rate (~1.06×10^{-3} rad/sec.) and zero spacecraft roll and yaw rates is simulated in the model. The spacecraft jitter motion and slew are not considered at the present simulation.

(2) Attitude Determination Sensors - Error Characteristics

Included in the SIRU HRG errors are : gyro bias error, scale factor error, angle random walk, and noise equivalent angle (sampling at 20 Hz). The star centroiding error is dependent on the observed star magnitude. Figure 3 shows the overall star tracker accuracy (including boresight accuracy and noise equivalent angle) as a function of tracked star magnitude in 3 σ, per star, per sample, and per axis.

(3) Catalog Stars

On-orbit catalog stars chosen from the Skymap master catalog were developed using following selection criteria : (1) select stars within Visual Magnitude range from 2.0 to 5.0; (2) remove stars with motion greater than 0.5 arcsec/year; (3) remove stars with position error greater than 1.0 arcsec; (4) remove stars that have another star within 1^0 which has a visual magnitude difference less than 1.0; and (5) remove stars that have another star within 0.1^0 which has a visual magnitude brighter than 2.0 mv dimmer than the candidate. The results are the 1183 catalog stars as shown in Figure 4

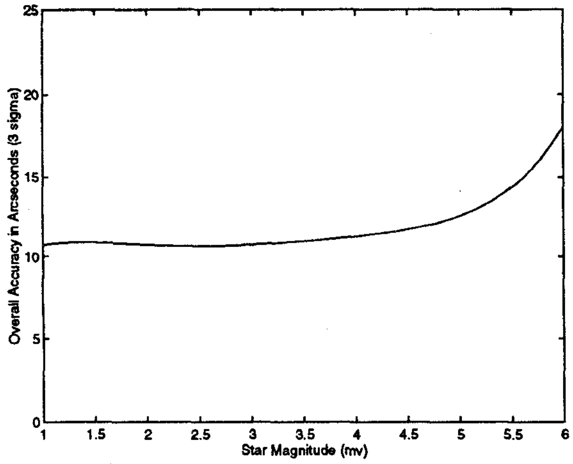

Figure 3. Overall HD-1003 accuracy per star, per sample, per axis at 10 Hz update rate

(4) Set-up parameters and initial conditions:

The following parameters and initial conditions were defined at the beginning of the program:

(a) Gyro parameters : (1 σ, per axis)

o Random walk: 0.0007 deg/sqrt(hr)

o Bias error : 0.006 deg/hr

o Scale factor error : 300 ppm

o Noise equivalent angle: 0.2166 micro rad/sqrt(Hz)

(b) Star tracker error (per axis) : star magnitude dependent (see Figure 3)

(c) Star catalog error : 1 arcsec. (1 σ, per axis)

(d) Initial s/c to ECI attitude error (radian) : $[-4.0\text{ e-}03/2 \;\; 1.0\text{ e-}03/2 \;\; 1.5\text{ e-}03/2 \;\; 1]^T$

(e) Star tracker #1 attitude wrt s/c (degrees) : roll = 15.55; pitch = 0; yaw = -35

(f) Star tracker #2 attitude wrt s/c (degrees) : roll = 15.55; pitch = 0; yaw = 35

(5) Main loop

There are four major do-loop (from outer most loop to innermost loop) in the main program:

j=1:frame	6000 seconds step
i=1: 4	8 second step
k = 1: 4	2 second step
k1= 1:10	0.5 second step

Within the inner most loop (0.05 second step), the spacecraft's roll, pitch, yaw rates are generated and used to determine the true spacecraft attitude. At every 0.5 second step, the SIRU gyros provide measured spacecraft's roll, pitch, yaw rates which are used to propagate the current estimate of spacecraft attitude. At the same time, the filter propagates the covariance matrix, P and the current estimated state vector, $\hat{x}$. The 2 seconds step is set up for recording the data including one sigma covariance values representing spacecraft's roll, pitch, yaw errors, residual gyro bias errors, and true spacecraft attitude errors.

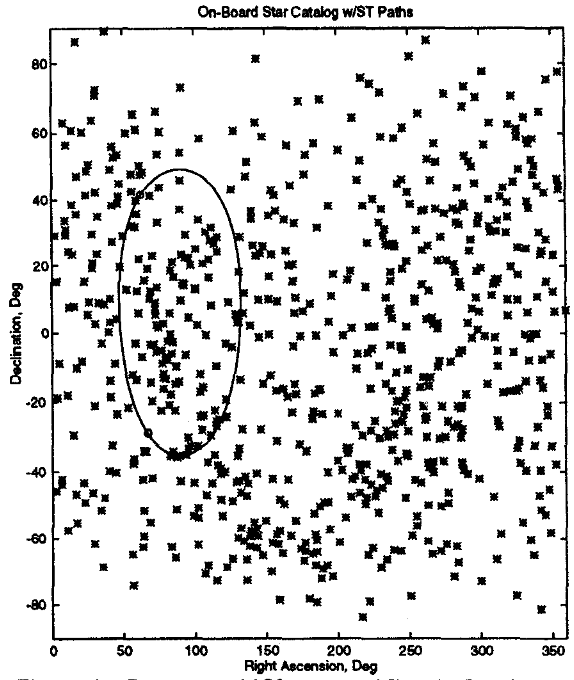

Figure 4. Catalog of 1183 stars of Visual Magnitude 2-5

At the end of every 8 seconds, the two star tracker's boresight vectors expressed in ECI frame are computed. Based on the computed boresight vectors, these stars present in the tracker 8^0x8^0 field of view are identified. Once the tracked star is identified, the corresponding star residuals are generated. The filter processes the star residuals one at a time from star tracker #1 and then followed by star tracker #2. At each update, the filter computes the filter gain matrix, updates the covariance matrix, the estimated state vector, and corrects the spacecraft attitude and gyro bias errors.

(6) Filter Performance

At every 2 seconds, the spacecraft attitude error quaternion, $\Delta Q_{ECI}^{S/C}$ is computed as follows:

$$\Delta Q_{ECI}^{S/C} = \hat{Q}_{S/C}^{ECI} * Q_{ECI}^{S/C} = \begin{bmatrix}\Delta q_1 & \Delta q_2 & \Delta q_3 & q_0\end{bmatrix}^T$$

where in pratical cases $q_0 \approx 1$.. The spacecraft's roll, pitch, and yaw errors are then given by :

Roll error = $2*\Delta q1$; pitch error = $2*\Delta q2$;
yaw error = $2*\Delta q3$

The 3σ bounds for the spacecraft attitude errors are given by : 3*sqrt (p_{ii}), for i=1, 2, and 3, where p_{ij} are the matrix elements of covariance matrix, P.

4. SUF Simulation Results

Typical two star tracker attitude determination results for one orbit of a single Monte Carlo run are shown in Figure 5, where the dotted curves represent the 3σ bound of the attitude errors based on the covariance analysis data. It can be seen that the attitude errors generally remain within the 3σ bound. The filter convergence time is less than 1000 seconds. Figure 6 shows the number of stars seen by each of the two star trackers at every 8 seconds period. As shown in the figure, the no star gaps are kept to minimum during this normal two star tracker operations. It is noted that pitch axis covariance peaks result whenever a gap in either tracker occurs, however, roll axis covariance tends to peak when tracker #2 gaps occur and yaw axis covariance peaks result when tracker #1 gaps occur. Similar observations were reported in [2].

Typical single star tracker attitude determination results for the last one orbit of a single Monte Carlo run are shown in Figure 7, where the dotted curves represent the 3σ bound of the attitude error. In this simulation case, the tracker #2 is only operated during the first 500 seconds, and then disabled for the rest of the mission time. Comparing the results of Figure 5 with Figure 7, it is clear that the two star tracker attitude errors are substantially less (more than 50 % less) than single star tracker errors.

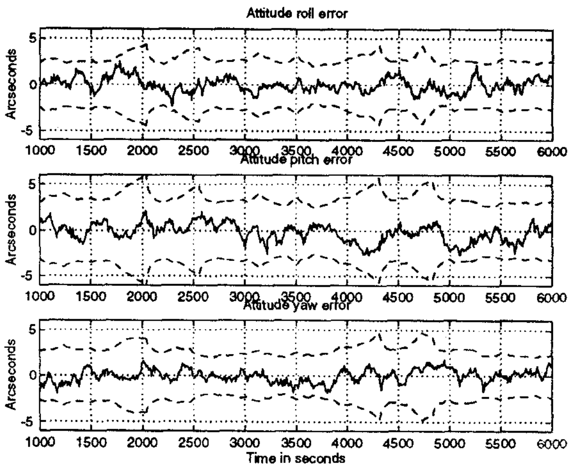

Figure 5. Attitude Determination Errors - Two Star Trackers Case

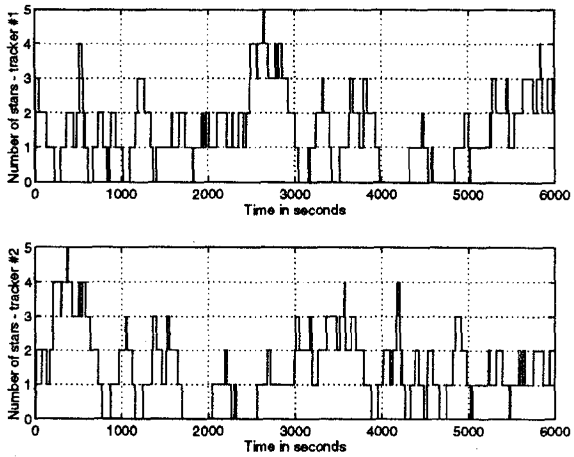

Figure 6. Stars Seen By Trackers - Two Star Trackers Case

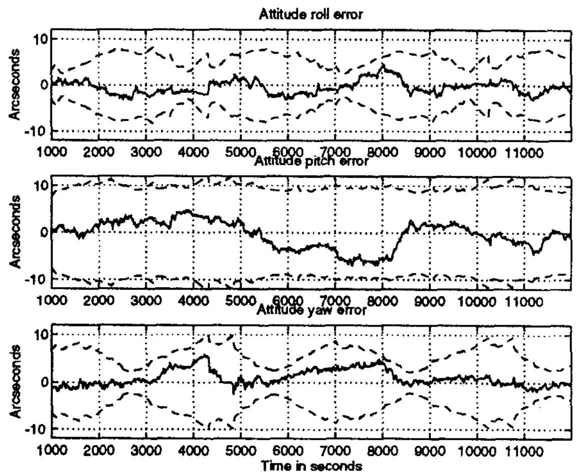

Figure 7. Attitude Determination Errors - Single Star Tracker Case

5. Conclusions

The overall attitude determination and corrections using celestial observations from star trackers for the LEO spacecraft has been described in this paper. A stellar update filter has been formulated and derived using a quaternion representation. The derived SUF is a 6-state Extended Kalman Filter which estimates 3 attitude errors and 3 gyro bias errors. A computer simulation model that simulates the basic functions of spacecraft Attitude Determination and Control System (ADCS), attitude determination sensors, and spacecraft motion has been developed and generated to evaluate the filter performance. Both covariance analysis and time-domain analysis have been performed using the simulation model to help us to allocate the subsystem flow down requirements. The simulation results indicate that the attitude knowledge requirement of 12 arc seconds ($3\ \sigma$) can be met with the proposed ADCS hardware components and attitude determination algorithms.

6. References

[1] : J. R. Wertz, "Spacecraft Attitude Determination and Control," D. Reidel Publishing Company, 1986.
[2] : P. Kudva and A. Throckmorton, "Attitude Determination Studies for EOS-AM1," AIAA GN&C Conference, 1-3 August 1994, pp. 188-195.

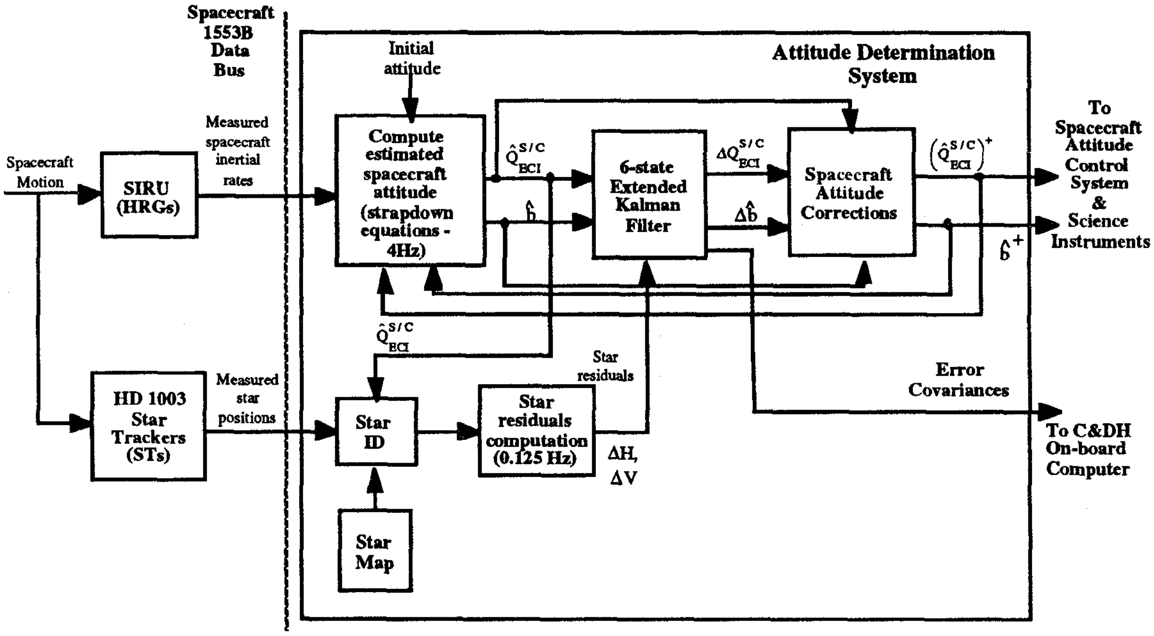

Figure 1. Functional Block Diagram of Attitude Determination Algorithm

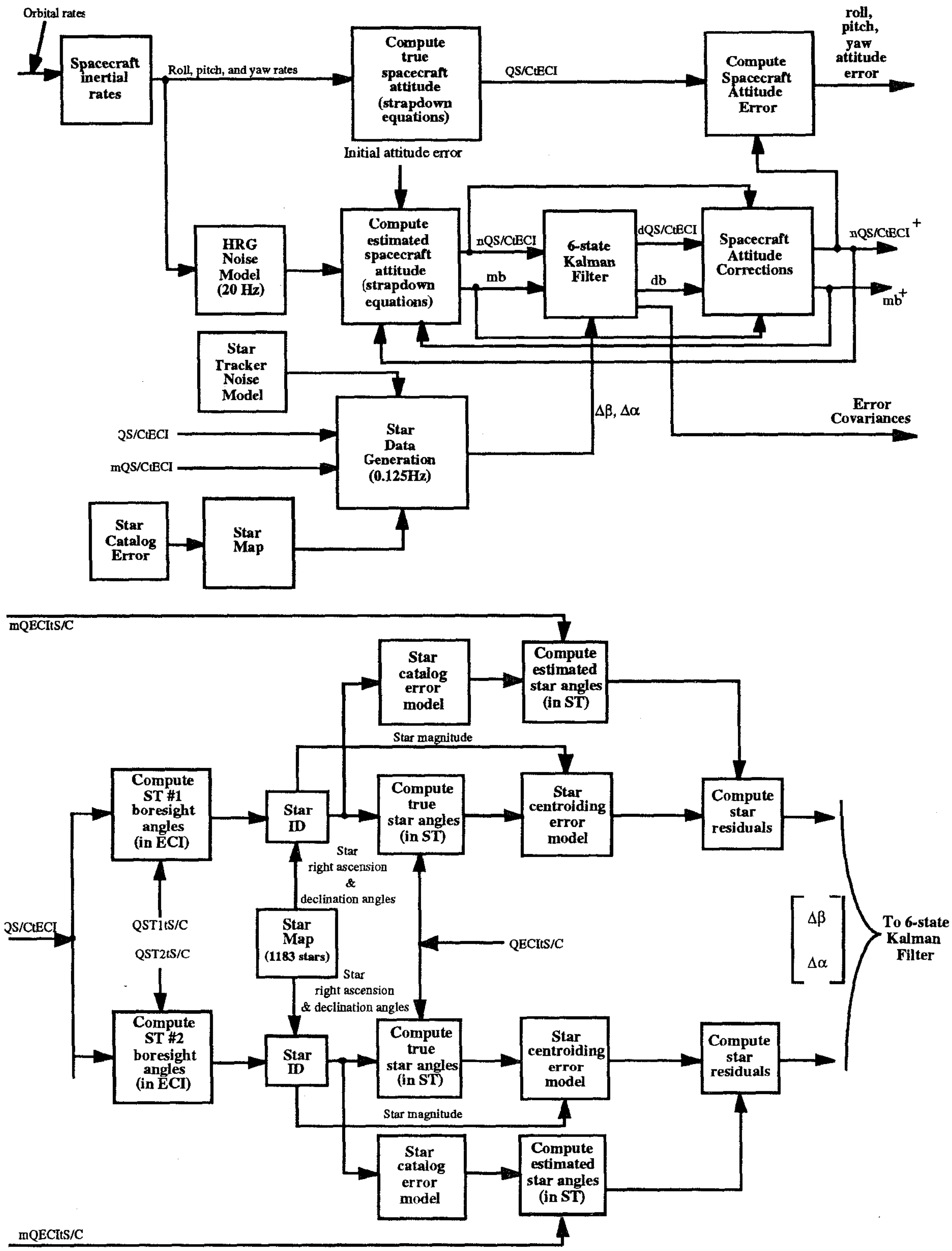

Figure 2. Spacecraft Attitude Determination Simulation Model - Data Flow