

# Using Atmospheric Drag for Constellation Control of Low Earth Orbit Micro-satellites

Daniel N.J. du Toit<sup>1</sup>, J.J. du Plessis<sup>2</sup> and W.H. Steyn<sup>3</sup>

## Extended Abstract

In certain low Earth orbit (LEO) satellite missions it is required that two or more satellites must operate in a certain spacial configuration relative to each other. This paper introduces a simple concept of utilising aerodynamic drag to achieve this type of constellation control.

A necessary structural requirement for the satellites is that a change in projected area on a plane perpendicular to the velocity vector of the satellite can be brought about by means of an orientation adjustment. The aerodynamic force acting on the satellite can thus be controlled through a simple eigenaxis slew of a three-axis stabilised satellite. The slew can be done through conventional means, including thrusters, momentum exchange methods or magnetorquers. The presence of a GPS receiver on the satellite is necessary for accurate position information.

It is shown that certain critical parameters influence the bounds of control time and accuracy. These parameters include the physical properties of the satellites, the orbital configuration and the state of the atmosphere.

A control system to illustrate the concept is proposed and tested through detailed computer simulations. The simulations include the influence of other orbit perturbation forces acting on the satellite, like the effects of the Earth's oblateness, solar radiation pressure and lunisolar attractions. Cowell's method is used to integrate the equations of motion numerically.

Typical results for two satellites (mass: 10 kg; maximum cross-sectional area: 0.3947 m<sup>2</sup>; minimum cross-sectional area: 0.0875 m<sup>2</sup>) in a 450 km circular orbit are as follows: the two satellites can be moved 500 km apart within 98 orbits (~153 hours). The distance error is less than 1 %. The additional altitude loss due to the control effort is 1.53 km. The control time drops sharply for lower altitudes and higher areato-mass ratios of the satellites. For a 300 km altitude orbit and the same satellites, the control time is 30 orbits (~45 hours) with a distance error less than 1 %. The additional altitude loss in this case is 5.20 km.

The concept proposed in the paper introduces a very useful method of constellation control. The key feature is utilising aerodynamic drag, a natural phenomenon which is normally considered as an unwanted disturbance, especially for low Earth orbit missions. The structural and software requirements placed on the satellites are not stringent or restrictive and should easily be reconcilable with requirements arising from the primary mission objectives.

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## Abstract

**In certain low Earth orbit (LEO) satellite missions it is required that two or more satellites operate in a certain spacial configuration relative to each other. This paper introduces a simple concept of utilising atmospheric drag, a natural phenomenon which is normally considered as an unwanted disturbance, to achieve this type of constellation control. A control strategy to illustrate the concept is proposed and tested through detailed computer simulations, including the effects of a number of significant orbital perturbations. The control time to reach a specified separation distance between two satellites is determined as a function of certain critical parameters. These parameters include the physical properties of the satellite, the orbital configuration and the state of the atmosphere.**

## 1. INTRODUCTION

The theory of atmospheric drag and its influence on Earth orbiting satellites have been well analysed and studied (see [9, 10]). Drag is normally considered as an *unwanted* orbit perturbation force causing the satellite to deviate from the idealised Kepler orbit. This perturbation is especially significant at the altitudes of LEO satellites (200 km to 1000 km) and could present restrictions on the mission and structural design of a micro-satellite.

This paper will investigate the possibilities of benefiting from the presence of atmospheric drag, in other words *using* it. The requirements and conditions under which such benefits can be obtained will be outlined and the boundaries of the advantages will be investigated.

A necessary structural requirement for the satellites is that a change in projected area on a plane perpendicular to the velocity vector of the satellite can be brought about by means of an orientation adjustment. By controlling the satellite orientation, the magnitude of the drag acting on the satellite can thus be controlled. This control force is limited in magnitude and application direction, but when utilised correctly, it can influence the satellite motion to reach a desired effect.

The following example gives an application of the concept and will be used for the further study thereof.

### ***CHIPSAT mission***

The CHIPSAT mission is currently planned as part of the NASA MTPE project and will provide accurate position information for gravity modelling and signal occultation measurements for atmospheric profiling. For these measurements, a number of satellites in a constellation will each carry a specially configured GPS receiver. To increase

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sensitivity to the features of the Earth's gravity field, the satellites must be flown in circular orbits at the lowest possible altitude. At these low altitudes, however, the effect of atmospheric drag on the satellites becomes very significant. This has two major disadvantages for the mission: firstly the lifetime of the mission is limited and secondly the gravity field estimates coming from the precise position measurements will be contaminated with large errors due to atmospheric drag. The first constraint simply implies that the satellites must be higher than a certain minimum altitude to ensure enough lifetime. The second problem, however, may be overcome by means of a common mode cancellation scheme. Pairs of satellites orbiting in tandem and kept a certain distance apart could provide measurements from which the effects of drag can be cancelled and the effects of gravity variations can be deduced accurately. The distance between the satellites must be large enough to ensure sensitivity to gravity variations and small enough for the state of the atmosphere to be nearly identical at the two locations. The latter condition will ensure that the drag on the two satellites is nearly the same. A separation distance of 500 km to 1000 km is considered to be sufficient. The additional advantage of this scheme is that the effects of other non-conservative perturbing forces, such as solar radiation pressure, are also cancelled.

This paper will show that it is possible to design a control system, utilising atmospheric drag, to control the position of satellites relative to each other. The necessity of this control effort becomes obvious when considering the fact that a group of satellites will be launched together and will initially be close to each other in orbit. From this initial position the constellation can be set up to meet the special configuration requirements of the moving measurement grid.

### *Satellite properties*

The satellites considered for this study have the following nominal physical properties:

Shape: cylindrical disc  
Diameter (d): 0.7 m  
Height (h): 0.125 m  
Mass (m): 10 kg

The dimensions are chosen to meet the specifications of the available launch opportunity for CHIPSAT. With these dimensions, four satellites can be fitted in the available secondary payload envelope.

### *Orbital configuration*

The orbits considered are circular with a  $90^\circ$  inclination and an altitude of 450 km. The resulting lifetime of the satellite is approximately 7 years.

Before applying the concepts of atmospheric drag control to the above example, it is necessary to overview some theoretical aspects.

## 2. THEORETICAL PRELIMINARIES

### *2.1 Perturbation methods*

The nature of this investigation requires that the position of a satellite moving under the combined influence of the Earth's (non-ideal) gravitational field and a number of other perturbing forces must be calculated. Since this problem is highly non-linear and complicated in nature, it is necessary to make use of numerical methods.

There are two categories of methods to determine the motion of satellites moving under the influence of perturbative forces the methods of *special perturbations* and *general perturbations*. The methods of general perturbations are well studied (see [2, 4, 8, 12]) and are used to calculate the effect of perturbative forces on the orbital

parameters. Analytical integration of series expansions of the perturbative accelerations are carried out to calculate these changes over long periods of time.

The methods of special perturbations entail the step-by-step numerical integration of the equations of motion and provide the desired short-term solutions for the in-orbit position of the satellite (see [3, 4, 6, 12]). *Cowell's method* is simple and straightforward and falls into the class of special perturbations. It will be used for the purposes of this study.

### *Cowell's method*

The movement of a satellite under the influence of the Earth's gravity field as well as other perturbing forces is described by the following two first order differential equations:

$$\begin{aligned}\dot{\mathbf{r}} &= \mathbf{v} \\ \dot{\mathbf{v}} &= -\frac{\mu}{r^3} \mathbf{r} + \mathbf{a}_p\end{aligned}\quad (1)$$

where  $\mathbf{r}$  is the satellite's position vector (with magnitude  $r$ ),  $\mathbf{v}$  is the vector velocity,  $\mu$  is the Earth's gravitational constant and  $\mathbf{a}_p$  is the vector sum of all perturbing accelerations acting on the satellite. A Cartesian co-ordinate system is used, with origin at the centre of the Earth. The  $x$  axis points in the direction of the vernal equinox and the  $z$  axis coincides with the Earth's spin axis, pointing in the direction of the north pole. The numerical integration of (1) is known as Cowell's method.

### *2.2 Orbital perturbations*

It is necessary to model the perturbing forces acting on the satellite as vector accelerations in order to use Cowell's method. Detailed discussions on orbital perturbations can be found in most of the literature listed at the end of this article.

#### **2.2.1 Atmospheric drag**

The acceleration of the satellite due to atmospheric drag can be expressed as:

$$\mathbf{a}_{\text{drag}} = -\frac{1}{2} \rho \frac{C_D A}{m} v^2 \mathbf{i}_v \quad (2)$$

where  $\rho$  is the atmospheric density,  $C_D$  is the drag coefficient,  $A$  is the crosssectional area of the satellite perpendicular to the velocity vector,  $m$  is the mass of the  $v$  is the velocity of the satellite relative to the atmosphere and  $\mathbf{i}_v$  is a unit vector in the direction of the satellite's velocity. The negative sign indicates that the acceleration is in a direction opposite to this unit vector.

The density of the upper atmosphere can be modelled by a simple analytical equation (form [10]) if the following assumptions are made:

- the of the Earth is spherically symmetric;
- the scale height (see [9, 10]) is constant over the altitudes of interest; and
- there is no time variation in the density.

This results in the following model:

$$\rho = \rho_{p_0} \exp\left[-\frac{h - h_{p_0}}{H}\right] \quad (3)$$

where  $\rho_{p_0}$  is the atmospheric density at the initial perigee point,  $h$  and  $h_{p_0}$  are the altitudes of the evaluation point and the initial perigee point respectively and  $H$  is the constant scale height. The following mean values were taken from [14] for an altitude of 450 km:  $\rho_{p_0} = 1.585 \times 10^{-12} \text{ kg/m}^3$  and  $H = 62.2 \text{ km}$ .

The above model will be used in the analysis and simulations to follow. The following three considerations help justify the approximation:

- there will be little altitude variation during the application of the model and the scale height can therefore be considered as approximately constant;
- the application period is small compared to most of the characteristic time scales of density variation; and
- short-term variations in atmospheric density (for example day-night) are ignored at this stage.

The drag coefficient ( $C_D$ ) is not as trivial to evaluate as it may seem. Since the density is very low at the altitudes of satellite orbits, even low Earth orbits, the ordinary continuum-flow theory of conventional aerodynamics has ceased to apply. According to [7] and [9], the appropriate regime is that of free-molecule flow. Various works have shown that a mean value for  $C_D$  of 2.2 can be taken with an error (standard deviation) which should not exceed 5 % (see [9, 10]).

In evaluating the velocity of the satellite relative to the atmosphere, the movement of the atmosphere relative to the Earth will be ignored.

### 2.2.2 Non-spherical gravitational field of the Earth

In deriving the ideal elliptical Keplerian orbit, it is assumed that the Earth can be modeled as a point mass with a spherical gravitational field. For the purposes of accurate orbit determination, this assumption is no longer valid. The mass of the Earth causes its gravitational field to deviate from the ideal spherical model. A convenient way to account for this variation is to model the Earth's gravitational potential by a spherical harmonics expansion (see example [2, 5, 8]). In this expansion, the value of the  $J_2$  zonal coefficient is three orders of magnitude larger than all the other coefficients and thus dominates the gravitational perturbative influences of Earth. It represents the equatorial bulge (oblateness) of the Earth. If all but this term is neglected and the gradient

of the scalar potential function is taken, the vector perturbative acceleration on satellite follows:

$$\mathbf{a}_{J_2} = \frac{-3\mu R_e^2 J_2}{2r^7} \begin{bmatrix} x(x^2 + y^2 - 4z^2) \\ y(x^2 + y^2 - 4z^2) \\ z(3x^2 + 3y^2 - 2z^2) \end{bmatrix} \quad (4)$$

where  $R_e$  is the radius of the Earth and  $x$ ,  $y$ , and  $z$  are the components of the satellite position ( $r$ ) resolved in the Cartesian coordinates mentioned earlier.

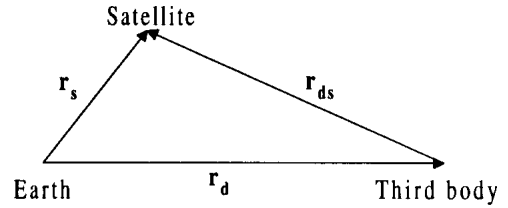
### 2.2.3 Third body attractions

The term *third body* refers to any other body in space besides the Earth which could have a gravitational influence on the satellite. The most significant influences for the LEO satellite come from the sun and the moon. Planetary gravitational influences are orders of magnitude smaller than these and will be ignored (see [5]).

The perturbing acceleration due to the gravitational attraction of a third body can be calculated as follows (from [2]):

$$\mathbf{a}_d = -\frac{\mu_d}{r_{ds}^3} (\mathbf{r}_s + f(q)\mathbf{r}_d) \quad (5)$$

where  $\mu_d$  is the gravitational parameter of the third body and the definitions of the vectors are given in the following diagram:



**Fig. 1: Vector definitions for third body attractions.**

The functions  $f$  and  $q$  are:

$$q = \frac{\mathbf{r}_s \cdot (\mathbf{r}_s - 2\mathbf{r}_d)}{\mathbf{r}_d \cdot \mathbf{r}_d}$$

$$f(q) = q \frac{3 + 3q + q^2}{1 + (1 + q)^2}$$

For simulations with third body perturbations included, it is necessary to have the position of the sun and the moon. The sun is modelled as an Earth satellite in a circular orbit with a radius of one astronomical unit and a period of 365.26 days. The inclination of the orbit is  $23.439^\circ$ . The moon is modelled as an Earth satellite in an orbit with an eccentricity of 0.055, semi-major axis of 384000 km and inclination of  $23^\circ$ .

## 2.2.4 Solar radiation pressure

Solar radiation pressure is a force on the satellite due to the momentum flux from the sun. For most satellites it acts in a direction radially away from the sun. The magnitude of the resulting acceleration on the satellite is given by:

$$a_{\text{Solar}} = KP \frac{A_s}{m} \quad (6)$$

where  $K$  is a dimensionless constant between 1 and 2 ( $K=1$ : surface perfectly absorbent;  $K=2$ : surface reflects all light),  $P$  is the momentum flux from the sun,  $A_s$  is the cross-sectional area of the satellite perpendicular to the sun-line and  $m$  is the mass of the satellite. The value of  $K$  is taken as 1.5 for the purpose of this study. The mean value of  $P$  is approximately  $4.4 \times 10^{-6}$  kg-m $^{-1}$  s $^{-2}$  at the distance of the Earth from the sun (from [14]).

In later simulations, the cross-sectional area perpendicular to the sun-line was always

taken as the maximum cross-sectional area to account for the worst possible case.

## 2.2.5 Other perturbation forces

There are other perturbation forces, but they are all significantly smaller than those mentioned in the previous sections and will be neglected during the subsequent analysis and simulations. These other perturbations include:

- aerodynamic lift;
- induced eddy currents in the satellite structure interacting with the Earth's magnetic field;
- Earth-reflected solar radiation pressure;
- drag due to solar wind;
- gravitational effects of Earth tides and ocean tides;
- relativity effects; and
- precession and nutation of the Earth's

## 3. USING ATMOSPHERIC DRAG

Consider the idealised situation of two identical Earth-orbiting satellites (S1 and S2) with the same initial position and velocity. Assume that the cross-sectional area perpendicular to the velocity vector (and hence the magnitude of the atmospheric drag) of each satellite can be changed arbitrarily between zero and some maximum value by changing the satellite's orientation. Assume further that the only other force acting on the satellites, besides atmospheric drag, is the gravitational attraction of an ideal point-mass Earth. As soon as the drag on a satellite is set to zero, that satellite would continue in an ideal Kepler orbit from the position and velocity at that moment.

The satellites will stay together in space as long as orientation manoeuvres on each take places simultaneously and identically - they would thus be accelerating through space together. Consider an orientation

manoeuvre, changing the cross-sectional area from zero through some arbitrary function back to zero. If this manoeuvre is done on both satellites, first on S1 and then, after a delay, on S2, they will both move to the same new Kepler orbit, but settle in different positions in that orbit, provided that the atmospheric density is a function of altitude only. If the new orbit is circular, the distance between them would remain constant. If the new orbit is not circular (non-zero eccentricity), the *average* distance between them would remain constant.

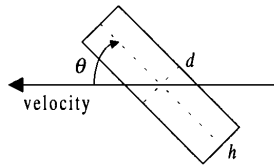
Two key simplifications in the above example are that the minimum cross-sectional area equals zero and that no other orbit perturbation forces are present. Both simplifications will now be considered.

### ***Non-zero minimum cross-sectional area***

The fact that the minimum cross-sectional area is not zero, combined with the exponential variation in density with altitude, will cause S 1 and S2 to lose different amounts of altitude during the manoeuvres and end up in different orbits. This would cause a continuous drift in the final distance between them. The following analysis will show that this drift will be very small.

For the given satellite, the cross-sectional area perpendicular to the velocity vector is a function of the incidence angle ( $\theta$ ) and is given by:

$$A(\theta) = \frac{\pi d^2}{4} \sin^2 \theta + dh \cos \theta \quad (7)$$



**Fig. 2: Definition of incidence angle ( $\theta$ ).**

Fig. 2 shows the definition of the incidence angle ( $\theta$ ), which, as a result of the cylindrical symmetry, is sufficient to specify the orientation.

The minimum area is 0.0875 m<sup>2</sup> at an angle of 0° and the maximum area is 0.3848 m<sup>2</sup> at an angle of 77.19°.

For circular orbits, the only secular effect of drag on the orbital elements is to reduce the semi-major axis, *i.e.* the altitude decreases. This change, approximated for a single revolution, is (from [11]):

$$\Delta a_{rev} = -2\pi \frac{C_D A}{m} \rho a^2 \quad (8)$$

with  $a$  the semi-major axis. For the given satellite and orbit configuration, the change is -39.2 m per orbit in the highdrag orientation and -8.9 m per orbit in the low-drag orientation. The value of the density was taken as 1.585x10<sup>-12</sup> kg/m<sup>3</sup>.

Consider S 1 and S2, both in the minimumdrag orientation. Assume that S1 is reorientated at some time to the maximumdrag orientation. After 50 orbits it is returned to the low-drag orientation and the manoeuvre is repeated on S2. The altitude loss would be approximately 2 km during the high-drag stage and approximately 0.45 km during the lowdrag stage to give a total altitude loss of 2.45 km for each satellite. For this small altitude variation, the corresponding variation in density is only 4 % (according to (3)). Differences in altitude loss during the two stages for the two satellites would tend to cancel each other since the exponential increase in density will be almost linear over this small altitude range. S 1 and S2 would thus still end up in nearly identical orbits and drift, due to the nonzero cross-sectional areas, would be minimal. Later sections will confirm control times in the order of 100 orbits for

a separation distance of 500 km. The above analysis would thus be applicable.

### ***The presence of other perturbation forces***

The most significant other perturbation comes from the non-spherical gravitational potential of the Earth. The most critical influence is the periodic variation in altitude at a frequency of double the orbital rate. For the given satellite and orbital setup the peak to peak variation in altitude is approximately 5 km ( $\sim 1.1\%$ ).<sup>4</sup> Using equations (2) and (3), the resulting periodic variation in the magnitude of drag acceleration is  $\sim 8.4\%$ . If the high-drag time is an integer multiple of the orbital period, the average drag over the duration of the manoeuvre will be the same for both satellites and they will end up in approximately the same final orbit. The different positions in this final orbit will cause periodic oscillations in the separation distance, but the *average* distance should not drift significantly. If the high-drag time is not an integer multiple of the orbital period, the altitude losses for the satellites will be different and significant drift in the final distance will occur unless further control is applied.

Secular changes caused by the nonspherical Earth alters the orientation of the orbit, but should have very little effect on the relative velocity between the satellites.

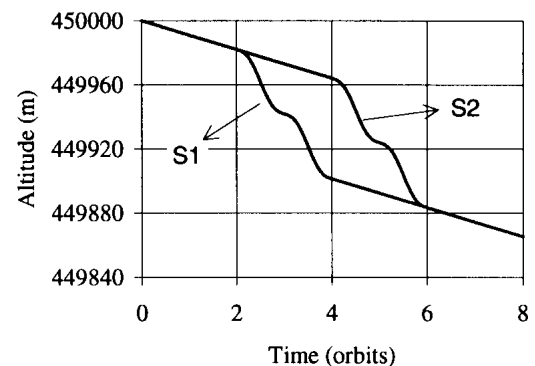
The effects of lunisolar attractions and solar radiation pressure are so small that their influence on the relative distance between the satellites can be assumed insignificant.

### ***Simulation results***

A PASCAL program was written to provide numerical solutions to the

equations of motion of the satellites with perturbations. The numerical integration is done with the fourth-order Runge-Kutta method and a fixed time step (see [6]). The smallest characteristic time scale comes from the Earth's oblateness effects periodic variations at 4 times the orbital rate. Since only short-term solutions are of interest, the time step can be chosen orders of magnitude smaller than this characteristic time scale to allow for great accuracy, while the simulation time should remain practical. Choosing a factor 50 leads to 200 steps per orbit. This time step (approximately 28 seconds for the 450 km circular orbit) has been used for all simulations.

A simulation has been carried out for two satellites with identical initial velocity and position vectors. After two orbits, the cross-sectional area of S 1 is changed within one time step from the minimum to the maximum and kept there for two full orbital periods after which it is changed back to the minimum value and the manoeuvre is repeated on S2. All perturbations, except atmospheric drag, were neglected in this example. The velocity vectors of the satellites were initiated to correspond with a uniform inward spiral in the low-drag orientation hence the slow, linear decrease in altitude before and after the high-drag manoeuvre on each satellite (fig. 3).

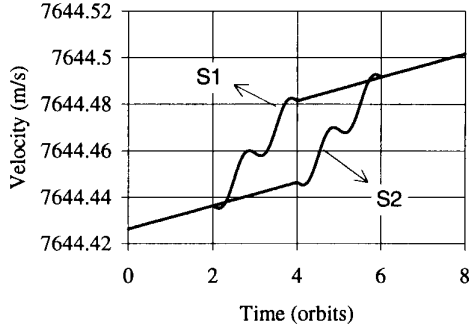


**Fig. 3: Altitude of S1 and S2.**

<sup>4</sup> For simulations with Earth oblateness perturbations included, the velocity of the satellite is initialised so that the *average* altitude is 450 km.



The velocity (fig. 4) increases as the altitude decreases<sup>5</sup> because Kepler's law must be satisfied ( $\mu = n^2 a^3 = \text{constant}$ ; with  $n$  the mean motion).



**Fig. 4: Velocity magnitude of S1 and S2.**

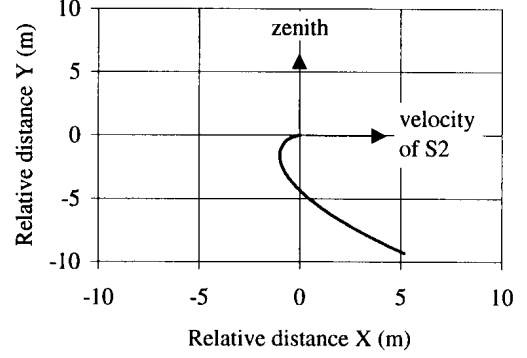
When the satellite is suddenly reorientated, its velocity vector still corresponds to the uniform inward spiral for the low-drag orientation and hence the oscillatory nature in the altitude and velocity. The velocity magnitude actually decreases at first after the re-orientation, but starts to increase as the satellite loses more and more altitude. If the high-drag time is not an integer multiple of the orbital period, the oscillations, with period equal to the orbital period, will remain after the manoeuvre.

Despite this complex dynamic behaviour, the *average* altitude decrease and velocity increase per orbit remain approximately linear.

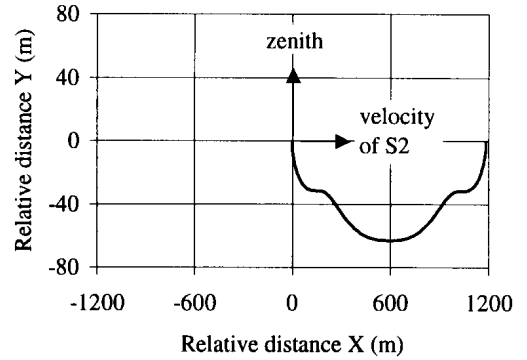
In fig. 5 and 6, the trajectories represent the motion of S1 relative to S2. Each graph's origin coincides with S2's position. The x-axes point in the direction of S2's velocity and the y-axes points radially away from the Earth. Fig. 5 is a close-up of the first half-orbit after S1 entered the high-drag orientation. S1 initially falls "behind" as it enters the high-drag stage and its velocity decreases, but then

<sup>5</sup> This is known as the "drag paradox".

"overtakes" S2 as it loses more altitude and the velocity starts increasing.



**Fig. 5: S1's motion relative to S2 (close-up).**



**Fig. 6: S1's motion relative to S2**

The final separation distance is ~1.2 km and stays approximately constant, since the two satellites end up in nearly identical orbits.

Let  $\delta$  be the difference in acceleration magnitude between the high-drag and lowdrag orientations. An expression for follows from (2) and is given by:

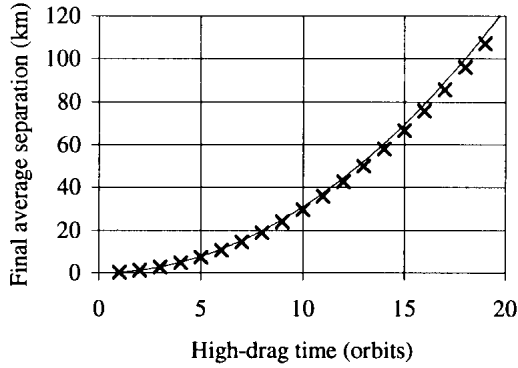
$$\delta = \frac{1}{2} \rho \frac{\mu C_D (A_{\max} - A_{\min})}{am} \quad (9)$$

with  $a$  the semi-major axis (the velocity of the satellite in a circular orbit is:  $v^2 = \mu/a$ ). If  $\delta$  is assumed constant and equal during the two stages of the control effort, the orbit-average velocity difference between the satellites will increase approximately linearly during each stage. The distance between them can be expected to increase

approximately quadratically. It follows that *the final* separation distance must be a quadratic function of the time each satellite spends in the high-drag orientation:

$$s \propto \delta(T_{hd})^2 \quad (10)$$

where  $s$  is the final separation distance and  $T_{hd}$  is the time each satellite spends in the high-drag orientation. The relationship has been verified by simulation for various high-drag periods and the results are shown in fig. 7. The thin line gives an analytical comparison.



**Fig. 7: Final average distance as a function of high-drag orientation time.**

All significant perturbation forces, as discussed earlier, were included.

#### 4. SPECIFYING A REFERENCE DISTANCE

##### 4.1 Feed-forward strategy

A simple strategy to reach any specified separation distance is to put S 1 in the highdrag orientation until half the distance is reached and then repeat the manoeuvre on S2, keeping it in the high-drag orientation for exactly the same length of time. This gives a time-optimal bang-bang strategy which is feed-forward in essence.

A requirement for success is that the average acceleration of the two satellites due to all perturbations, including drag, must be approximately equal during the

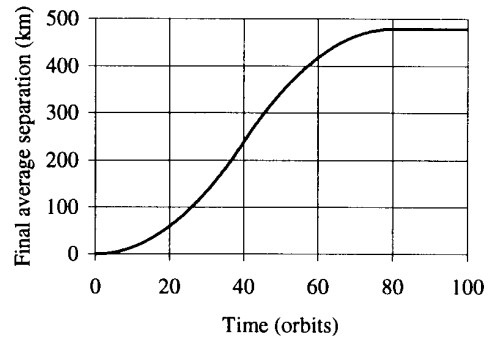
two stages. As discussed earlier, the highdrag time must be an integer multiple of the orbital period to meet this requirement. An error in the final distance is thus inevitable for this strategy. A strategy to correct this error will be discussed later.

Switching at an integer multiple of orbits, just before the halfway distance is reached, will always lead to a final distance smaller than the specified distance. This situation is more desirable than an overshoot because additional altitude is always lost during control efforts. At the end of each full orbit when S1 has been in the highdrag orientation, it is necessary to predict the separation distance one orbit ahead to determine whether to switch. This can easily be done, using the approximate quadratic relationship between distance and high-drag time (10):

$$s_{n+1} = s_n \left( \frac{n+1}{n} \right)^2 \quad (11)$$

where  $s_n$  is the separation distance after  $n$  orbits with  $n$  a positive integer. If  $s_{n+1}$  exceeds half the specified distance, the switch must occur after  $n$  orbits.

Fig. 8 shows a simulation result for the above strategy and a specified separation distance of 500 km. All significant perturbation forces were included for this simulation.



**Fig. 8: Separation distance.**

The final average separation distance is  $\sim 480$  km and the high-drag time for each satellite is 40 orbits.

The oblateness perturbation causes a small oscillation ( $\sim 0.5$  km amplitude) in the final distance, as explained earlier. This oscillation is not visible in the graph, but will be shown later.

The altitude losses during the two stages of the control effort are summarised in table 1 for a similar simulation with only the atmospheric drag perturbation included. The values correspond very well with the predicted values from equation (8): i.e. 1569 m and 357 m over 40 orbits for the high- and low-drag stages respectively.

	<i>High drag</i>	<i>Low drag</i>	<i>Total</i>
<i>S1</i>	1632	368	2000
<i>S2</i>	1641	359	2000
<i>S1-S2</i>	-9	9	$\approx 0$

Table 1: Comparison of altitude drops (m).

It is clear that the differences in the respective control stages cancel each other and the total altitude loss is nearly identical for both satellites. If no control was applied and the satellites were kept in the low-drag orientation all the time, the altitude loss would have been approximately 720 m in 80 orbits, thus the additional altitude loss due to the control effort is approximately equal to 1280 m. This is negligible compared to the 450 km initial altitude.

Following the observation that the total altitude losses for the two satellites are nearly identical, it is expected that drift in the separation distance after the control manoeuvres should be very slow. Fig. 9 confirms this for the simulation where all perturbations were included. The abscissa represents the time since the end of the control manoeuvre. The dotted line represents

an ideal linear drift of  $-1.02$  m per orbit.

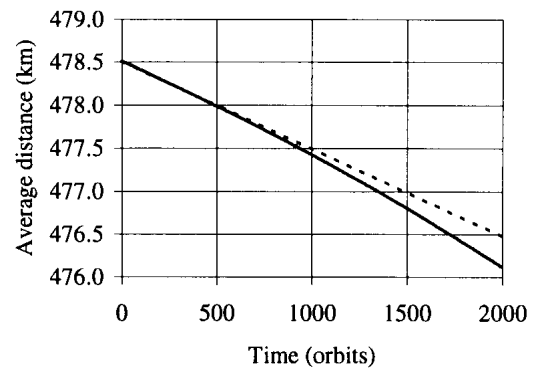


Fig. 9: Distance drift after the control effort.

In applying the above strategy, it was assumed that the satellites could be reorientated in a single time step. This would require a  $77^\circ$  slew in 28 seconds. For slower slew rates, the strategy should remain successful as long as the typical reorientation time is small in comparison to the orbital period. The feasibility of controlling the incidence angle of such a small micro-satellite by means of cheap reaction wheels and magnetorquers is demonstrated by the development of SUNSAT6, a 60 kg imaging micro-satellite (see [13]).

#### 4.2 Adding feedback

In order to correct the finite distance error after application of the bang-bang manoeuvre of the previous section, a further bang-bang manoeuvre can be done. At the end of the first complete bang-bang manoeuvre, the final distance ( $s_{\sim}$ ) and the time each satellite spent in the high-drag orientation ( $nl$  orbits) are known quantities. Using this knowledge, together with the quadratic relationship of (10), it is possible to scale the cross-sectional area for the high-drag orientation to reach the desired distance almost exactly with second bang-bang manoeuvre. The scaling

6 SUNSAT is developed by the University of Stellenbosch and will be launched by NASA as a secondary payload with the ARGOS mission in March 1997.

is necessary since the second manoeuvre must also span an integer number of orbits, for the same reasons cited earlier. The scaling law is:

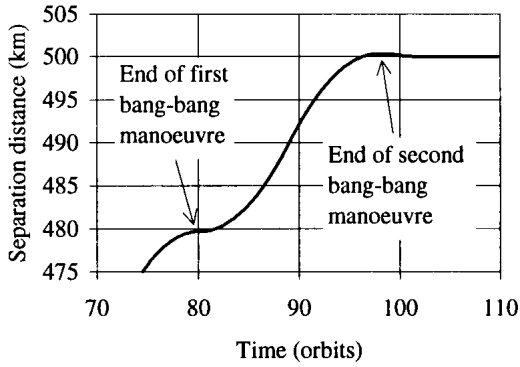
$$A_{high} = A_{min} + \alpha(A_{max} - A_{min}) \quad (12)$$

with the scaling factor

$$\alpha = \frac{s_e}{s_1} \left( \frac{n_1}{n_e} \right)^2$$

where  $s_e$  is the distance error and  $n_e$  is the number of high-drag orbits for the second bang-bang manoeuvre. The integer  $n_e$  must be computed so that  $\alpha$  is smaller than unity.

This process can now be repeated until the separation distance is within a specified error margin of the specified distance.



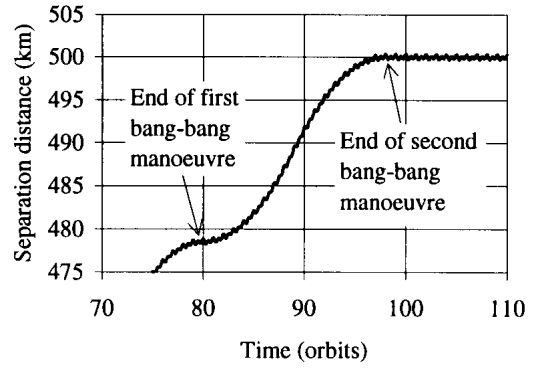
**Fig. 10: Feedback control (only drag).**

Successive bang-bang manoeuvres will take place with feedback occurring after every complete manoeuvre.

Fig. 10 shows the result of a simulation with drag as the only perturbation force. The error margin has been set to zero and the system will continuously execute bangbang manoeuvres to get closer and closer to the specified distance.

In the next simulation result, the other perturbations were included and the error margin has been set to 1 % (5 km). The separation distance falls within this error margin after two bang-bang manoeuvres. The first manoeuvre lasts 80 orbits and the second manoeuvre lasts 18 orbits for a total

control time of 98 orbits (+ 6 days, 8 hours and 36 minutes).



**Fig. 11: Feedback control (all perturbations).**

The small oscillation due to the Earth's oblateness effects is now visible.

#### 4.3 Minimum control time

The minimum control time necessary to reach a specified separation distance within a specified error margin is a good measure of the performance of the control strategy. Using (9) and (10), the control time has the following dependency on the satellite properties and orbital parameters:

$$T \propto \sqrt{\frac{ams}{\rho C_D (A_{max} - A_{min})}} \quad (13)$$

Consider a variation in altitude. The relationship between altitude ( $h$ ) and density is (from (3)):

$$\rho \propto \exp\left[\frac{-h}{H}\right] \quad (14)$$

Using this relationship with (13), the relationship between control time and altitude follows:

$$T \propto \sqrt{R_E + h} \exp\left[\frac{h}{2H}\right] \quad (15)$$

where  $R_E$  is the radius of the Earth and  $a=R_E+h$ . Fig. 12 shows simulation results for a varying altitude, confirming the predicted relationship. The error margin has been set to 1 %. This has been reached by a maximum of two complete bang-bang manoeuvres for every altitude simulated.

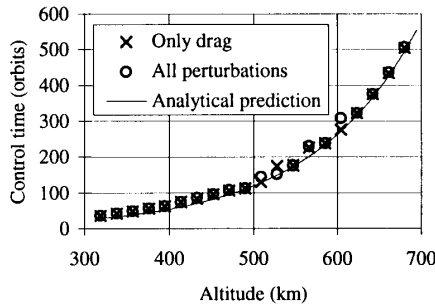


Fig. 12: Control time as a function of altitude.

If the satellite mass is taken as the independent variable in (13) and is assumed constant, then the proportionality can be expressed as:

$$T \propto \sqrt{m} \quad (16)$$

Fig. 13 shows simulation results of total control time as a function of the satellite mass. The specified error margin of 1 % has again been reached by a maximum of 2 complete bang-bang manoeuvres.

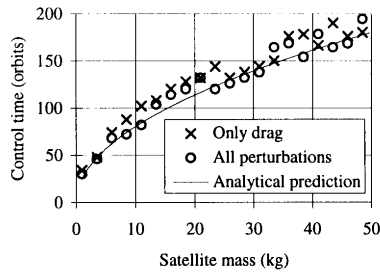


Fig. 13: Control time as a function of the satellite mass.

The relationship between control time and density (13) is also very useful when

considering uncertainties in the atmospheric density. If, for example, during high solar activity, the average density increases by a factor 2, the minimum control time could be expected to decrease by a factor  $1/\sqrt{2}$ . A simulation with all perturbations included confirmed this, with a total control time of 70 orbits (for 1 % error) under these conditions.

## 5. CONCLUSION

It has been shown that it is possible to use atmospheric drag to control LEO satellites relative to each other. The potential of the proposed scheme in terms of control time and accuracy has been determined and yielded satisfactory results in the presence of all significant orbit perturbations.

Key assumptions have been that variations in atmospheric conditions take place on time scales which are large compared to the typical control times and that reorientation times for the satellites are short compared to the orbital period. The authors are currently busy investigating control strategies, including adaptive strategies, for the case where these assumptions are not valid.

The possibility of utilising atmospheric drag for tight control of the Kepler orbit parameters is also the topic of further study by the authors.

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