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Fully magnetic attitude control for spacecraft subject to gravity gradient[1]

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Interaction between control-current in coils and the geomagnetic field can be the means to stabilize a satellite about three axes. Nonlinear time-varying analysis shows how this new result is obtained.

Abstract

This paper presents stability and control analysis of a satellite on a near polar orbit subject to the gravity torque. The satellite is actuated by a set of mutually perpendicular coils. The concept is that interaction between the Earth's magnetic field and a magnetic field generated by the coils results in a mechanical torque used for attitude corrections. Magnetic actuation due to its simplicity and power efficiency is attractive on small, inexpensive satellites. This principle is however inherently nonlinear, and difficult to use since the control torque can only be generated perpendicular to the geomagnetic field vector. This paper shows that three-axis control can be achieved with magnetorquers as sole actuators in a low Earth near polar orbit. It considers the problem from a time-varying, nonlinear system point of view and suggests controllers for three-axis stabilization. A stability analysis is presented, and detailed simulation results show convincing performance over the entire envelope of operation of the Danish Ørsted satellite.

Keywords: Attitude control; Satellite control; Time-varying systems; Periodic motion; Lyapunov stability; Quaternion feedback

1. Introduction

Many valuable control methods have been developed over the past years since the first satellite was launched in 1957. Generally speaking all those techniques may be classified as active or passive. Active control means that an external source of energy is applied to drive an actuator. Typical actuators are: thrusters, reaction or momentum wheels and electromagnetic coils.

Magnetic torquing is attractive for small, cheap satellites in low Earth orbits. Magnetic control systems are relatively lightweight, require low power and are inexpensive. These were the main reasons to suggest this actuation principle for the Danish Ørsted satellite in an early phase when a spin-stabilized mission, i.e. two-axis control was foreseen. Later redefinition of the scientific objectives required an alternation of the control requirements to three-axis stabilization. The challenge was that three-axis control was not possible with an actuation principle that leaves the system controllable in only two degrees of freedom because the control torque can only be generated perpendicular to the local magnetic field of the Earth.

There is extensive literature covering satellite attitude control analysis and design. Most of the algorithms presented assume application of reaction wheels and/or thrusters. A geometric control approach to a satellite actuated by a set of two thruster jets was investigated by Byrnes and Isidori (1991). It was concluded that a rigid space-craft with actuators producing torque in only two directions independently cannot be locally asymptotically stabilized using continuously differentiable state feedback. A general framework for the an alysis of the attitude tracking control problem using Lyapunov theory was presented by Ting-Yung Wen and

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Krentz-Delgado (1991) and salient features of proportional-derivative controllers for attitude control were established. The approach by Ting-Yung Wen and Krentz-Delgado (1991) was extended by Egeland and Godhavn (1994), where an adaptive control scheme for satellite attitude control was derived using passivity theory.

The magnetic attitude control has the significant limitation that the control torque is always perpendicular to the local geomagnetic field vector, therefore, the magnetic torquing was mainly used for momentum desaturation. The number of internationally published papers on attitude control with use of magnetorquers is still rather small. A configuration with two magnetic coils and a reaction wheel was investigated by Cavallo et al. (1993). The proposed attitude control law was designed based on the sliding mode control. The problem of sole three-axis magnetic control was addressed by Musser and Ward (1989), where local stabilization of the satellite was achieved via implementation of an infinite-time-horizon linear qadratic regulator. An energy optimal solution with use of Riccati periodic equation was presented by Wisniewski (1995b). Another linear approach was given by Martel et al. (1988), where the linearized time-varying satellite motion model was approximated by a linear time-invariant counterpart.

Three-axis stabilization with use of magnetic torquing of a satellite without appendages was treated by Wisniewski (1994), where a sliding mode control law was provided and was shown to stabilize a tumbling satellite. Last but not least, a magnetic attitude control was treated by Steyn (1994). The author proposed a rule-based fuzzy controller and compared it with an adaptive LQR control. The references listed above did not consider global three-axis stability of a magnetic actuated satellite, which is covered in this paper.

Complete comprehension of the nature of the satellite control problem requires a new approach merging the control theory with physics of the rigid body motion. A linear analysis by Hughes (1986) showed that motion of a tri-inertial satellite in the gravitational field has four stable equilibria, all having the axis of minimum moments of inertia in the direction of the local vertical and maximum along orbit normal. In this paper the Lyapunov stability theory is employed to obtain global results. The Lyapunov function based on mechanical energy is formulated such that its minima coincide with the four equilibria described by Hughes (1986). One of these is chosen to be the reference for the control system. To obtain overall stability in all mission phases, several controllers are needed. A velocity controller is proposed first, that contributes to dissipation of both kinetic and potential energy. This controller is used for recovery from tumbling motion. Second, this controller is expanded to include attitude information for the local use in a normal science observation. Finally, an extension is made to obtain global stability by removing undesired equilibria including one with the boom upside down. This controller is used for boom upside down recovery.

The paper is organized as follows. Section 2 describes the Ørsted satellite configuration, Sections 3 and 4 gives mathematical model used for the attitude control design. Section 5 considers attitude stabilization at large. A controller with angular velocity feedback is introduced and proved to be asymptotically stable around four equilibria. This control law is shown to be especially effective for recovery of a satellite from tumbling motion with high value of the angular velocity. The next step taken is to modify the rate controller such that the desired reference attitude is the only stable equilibrium. The rate feedback is shown to be only locally asymptotically stable around the reference. To achieve global three-axis stabilization, attitude information is shown to be needed as a part of the control law. This extension is made in Sections 6 and 7. In Section 6 the focus is on local performance of the attirude controller for the normal operation. The satellite model is approximated by a linear periodic model, and the controller is designed employing Floquet stability analysis. A control algorithm for recovery from the inverted boom contingency operation is considered in Section 7. Simulation examples are provided to illustrate the theoretical findings.

The family of controllers presented in this paper with joint control action for detumbling, nominal operation and a boom upside-down condition provides an attitude control system which can achieve global three-axis attitude stabilization using solely magnetorquers for actuation. The Danish Ørsted satellite will be the first to fly three-axis stabilized using these new results.

2. Ørsted satellite configuration

The work has been motivated by the Danish Ørsted satellite mission. The purpose of the mission is to conduct a research program in the magnetic field of the Earth. The Ørsted satellite is a 60 kg auxiliary payload scheduled to be launched by a MD-Delta II launch vehicle in January 1999 into a 650×850 km orbit with a 96° inclination. When ground contact is established in orbit, an 8 m long boom is deployed by a ground command. The boom carries scientific instruments that must be displaced from the electro-magnetic disturbances present in the main body of the satellite. The Ørsted satellite configuration is depicted in Fig. 1. After boom deployment the normal operation phase controller is activated. The satellite will be three axis stabilized with its boom pointing outwards from the Earth. Formally, the control objective is that a certain coordinate system fixed in the satellite structure shall coincide with a reference coordinate

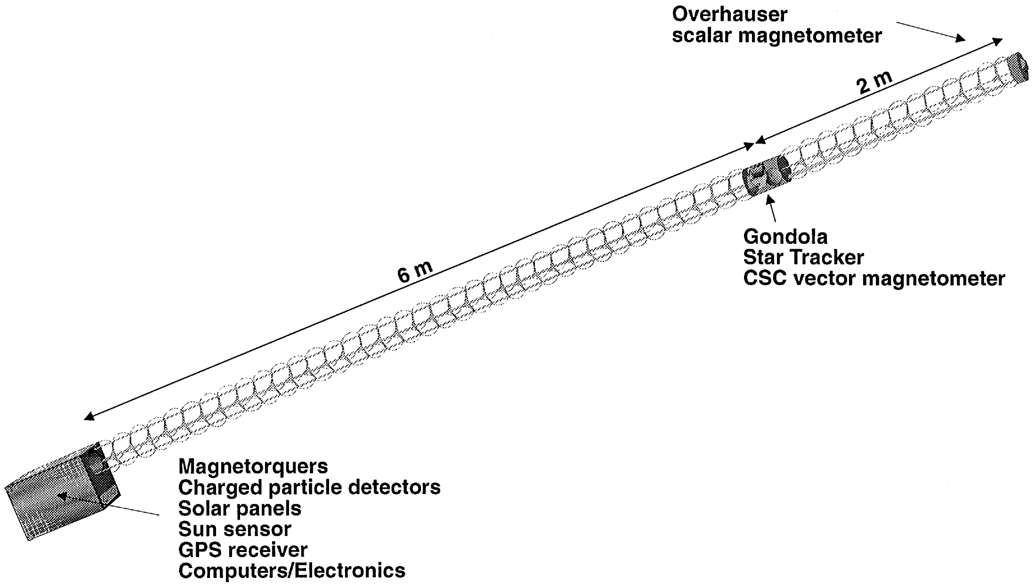

Fig. 1. The Ørsted satellite consists of a main body and an 8 m long scientific boom.

system fixed in the orbit. The pointing accuracy is required to be within 10° in pitch, roll, and yaw. The satellite is expected to be in the vicinity of the reference for most of the operational time, however, there are contingency phases due to some malfunctions in the subsystems causing the instrument boom to point upside down or the satellite to tumble. Requirements to the attitude control system include recovery from such contingency by turning the satellite back to the reference.

Stabilization of the Ørsted satellite is accomplished by active use of a set of mutually perpendicular magnetorquer coils. The coils are mounted in the x, y, and z facets of the satellite main body. A maximum producible magnetic moment is $20\ \mathrm{A\,m^2}$. The interaction between external magnetic field of the Earth and the magnetic field generated in the coils produces a mechanical torque, which is used to correct the attitude. The maximum mechanical torque produced by the coils is approximately 0.6×10^{-3} N m above the equator, and 1.2×10^{-3} N m above the Poles.

3. Notation

The notation used throughout the paper is listed in Table 1. Furthermore, the following terminology is introduced:

Control CS: right orthogonal coordinate system control spanned on principal axes of the satellite, the y axis along the axis of maximal moment of inertia, the z axis along the axis of the minimal, see Fig. 2,

Table 1
Notation

${}^{c}\mathbf{v}$, ${}^{o}\mathbf{v}$, ${}^{w}\mathbf{v}$	general notation for vectors: $\mathbf{v}$ resolved in Control CS, Orbit CS or World CS, respectively
$\boldsymbol{\Omega}_{cw}$	angular velocity of Control CS w.r.t. World CS
$\boldsymbol{\Omega}_{co}$	angular velocity of Control CS w.r.t. Orbit CS
$\boldsymbol{\Omega}_{ow}$	angular velocity of Orbit CS w.r.t. World CS
ω_o	orbital rate
$\mathbf{I}$	inertia tensor of the satellite
I_x, I_y, I_z	moments of inertia about x-, y- and z-principal axes
$\mathbf{N}_{ctrl}$	control torque
$\mathbf{N}_{gg}$	gravity gradient torque
$\mathbf{N}_{dist}$	disturbance torques
$\mathbf{m}$	magnetic moment generated by a set of the coils
$\mathbf{B}$	magnetic field of the Earth
$\tilde{\mathbf{B}}$	matrix representation of product $\mathbf{B}\times$
$\tilde{\tilde{\mathbf{B}}}$	matrix representation of double cross product $-\mathbf{B}\times(\mathbf{B}\times)$
${}^{c}_{o}\mathbf{q}$	quaternion representing rotation of Control CS w.r.t. Orbit CS
$\mathbf{q}$, q_4	vector part and scalar part of ${}^{c}_{o}\mathbf{q}$
$\mathbf{A}({}^{c}_{o}\mathbf{q})$	attitude matrix based on ${}^{c}_{o}\mathbf{q}$
$\mathbf{i}_o, \mathbf{j}_o, \mathbf{k}_o$	unit vectors along x-, y-, z-axis of Orbit CS
$\delta\mathbf{q}$	small perturbation of vector part of attitude quaternion, ${}^{c}_{o}\mathbf{q}$
$\delta\boldsymbol{\Omega}$	small perturbation of angular velocity ${}^{c}\boldsymbol{\Omega}_{co}$
n_{coil}	number of coil windings
i_{coil}	current in coil
A_{coil}	coil area
λ	quaternion gain in operational mission phase algorithm
$\underset{\sim}{\lambda}$	limit of stability of quaternion gain
$\bar{\lambda}$	quaternion gain in boom upside-down algorithm
$\mathbf{E}$	3×3 identity matrix

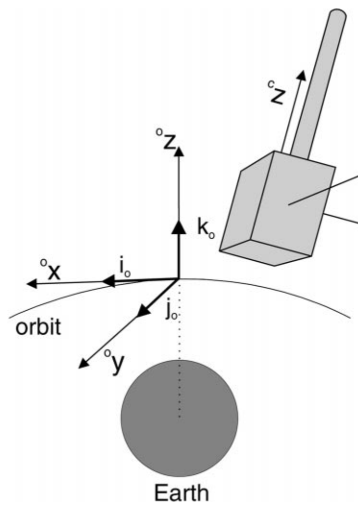

Fig. 2. Control and orbit coordinate systems.

Orbit CS: orbit (reference) right orthogonal coordinate system fixed in orbit, the y axis in the orbital plane normal direction, the z axis along zenith,

World CS: (inertial) right orthogonal coordinate system,

Pitch, *Roll*, *Yaw*: the angle describing satellite attitude, pitch referred to the rotation about the y axis of the Orbit CS, roll to the x axis, and yaw to the z axis,

Zenith: a unit vector in the Control CS along the line connecting the satellite centre of gravity and the Earth centre pointing away from the Earth. The vector with the opposite sense is *nadir*.

4. Modeling

The mathematical model of a satellite is described by well-known dynamic and the kinematic equations of motion, see, e.g. Hughes (1986) or Wertz (1990). The dynamics relates torques acting on the satellite to the satellite's angular velocity in the World CS. The kinematics provides integration of the angular velocity.

Satellite attitude is parameterized by four components of a quaternion describing rotation of the Control CS with respect to the Orbit CS. The use of quaternions is common for spacecraft's description motion, since it gives a singularity-free representation, see, e.g. Wertz (1990).

4.1. Equations of dynamics

The dynamic equation of motion for a satellite in low Earth orbit is

$$\mathbf{I}\,{}^{c}\dot{\boldsymbol{\Omega}}_{cw}(t) = -\,{}^{c}\boldsymbol{\Omega}_{cw}(t)\times\mathbf{I}\,{}^{c}\boldsymbol{\Omega}_{cw}(t) + {}^{c}\mathbf{N}_{ctrl}(t) + {}^{c}\mathbf{N}_{gg}(t) + {}^{c}\mathbf{N}_{dist}. \tag{1}$$

There are three torques represented in Eq. (1): the control torque, the gravity gradient torque and disturbance torques. The latter includes the aerodynamic drag, and a residual torque due to eccentricity of the orbit (it is assumed that the orbit is circular in the model used for control synthesis).

Using magnetorquer coils for control, a torque is generated by an interaction of the geomagnetic field with the magnetorquer current $i_{coil}(t)$ giving rise to a magnetic moment $m(t)$

$$m(t) = n_{coil}\, i_{coil}(t) A_{coil}. \tag{2}$$

With the coils placed perpendicular to the x, y and z-axes of the Control CS, the control torque acting on the satellite is

$${}^{c}\mathbf{N}_{ctrl}(t) = {}^{c}\mathbf{m}(t)\times{}^{c}\mathbf{B}(t). \tag{3}$$

The magnetic moment given in the Control CS, ${}^{c}\mathbf{m}$, is considered as the control signal throughout the paper. The torque originating from the gravity gradient (Wertz, 1990) is given by

$${}^{c}\mathbf{N}_{gg}(t) = 3\omega_o^2({}^{c}\mathbf{k}_o\times\mathbf{I}\,{}^{c}\mathbf{k}_o). \tag{4}$$

Disturbance torques are considered fairly small compared with the gravity gradient torque, and can be disregarded in the later analysis. This is a consequence of altitude of orbit chosen (Wisniewski, 1995a) and holds for most low earth polar orbit satellite missions.

4.2. Equation of kinematics

The attitude of the satellite is defined as an orientation of the Control CS relative to the Orbit CS. The attitude quaternion ${}^{c}_{o}\mathbf{q}$ consists of four parameters $[q_1\ \ q_2\ \ q_3\ \ q_4]^T$. Therefore, it does not provide a minimal attitude representation, which is locally 3 for SO(3), but offers global representation of kinematics without singularities. The first three components $[q_1\ \ q_2\ \ q_3]^T$ form the vector part of the quaternion and q_4 is regarded as the scalar part. The construction of the unit quaternion arises from the Euler theorem claiming that the rotation of coordinate systems can be uniquely described by a unit vector $\mathbf{e} = [e_1\ \ e_2\ \ e_3]^T$ giving an axis of rotation as well as its sense, and an angle of rotation ϕ. The four parameters of the quaternion ${}^{c}_{o}\mathbf{q}$ are then

$$[q_1\quad q_2\quad q_3\quad q_4]^T \equiv \left[e_1\sin\frac{\phi}{2}\quad e_2\sin\frac{\phi}{2}\quad e_3\sin\frac{\phi}{2}\quad \cos\frac{\phi}{2}\right]^T. \tag{5}$$

The kinematics is, when parameterized by the attitude quaternion

$$\dot{\mathbf{q}} = \tfrac{1}{2}\,{}^{c}\boldsymbol{\Omega}_{co}q_4 + \tfrac{1}{2}\,{}^{c}\boldsymbol{\Omega}_{co}\times\mathbf{q},$$
$$\dot{q}_4 = -\tfrac{1}{2}\,{}^{c}\boldsymbol{\Omega}_{co}\cdot\mathbf{q}. \tag{6}$$

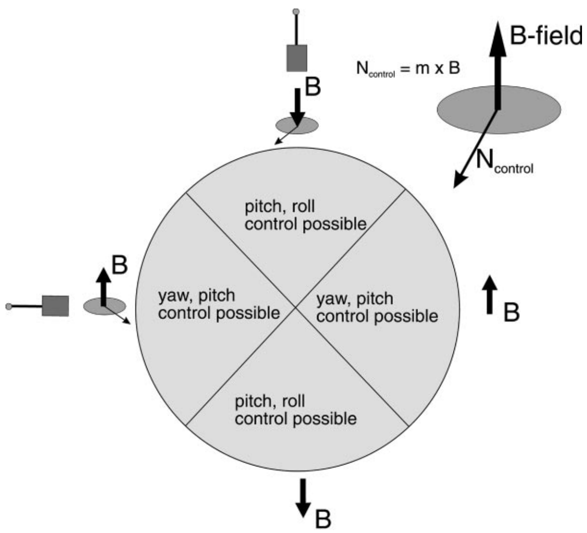

Fig. 3. Mechanical torque generated by magnetorquers is always perpendicular to the geomagnetic field vector. This implies that yaw is not controllable over poles, and roll is not controllable over equator.

The satellite's angular velocity relative to the World CS and the angular velocity with respect to the Orbit CS is given by the relation

$$^{c}\boldsymbol{\Omega}_{co} = {}^{c}\boldsymbol{\Omega}_{cw} - \omega_{o}\,{}^{c}\mathbf{j}_{o}. \tag{7}$$

Eqs. (1), (3), (4), (6) and (7) constitute the entire model of the satellite, where the magnetic moment $^{c}\mathbf{m}(t)$ is the control variable.

4.3. Control limitations

The satellite actuated by a set of magnetorquers has a serious limitation. The mechanical torque, produced by the interaction of the geomagnetic field and the magnetic field generated by the magnetorquers, is always perpendicular to the geomagnetic field vector. Thus, the direction parallel to the local geomagnetic field vector is not controllable. However, the geomagnetic field changes its orientation in the Orbit CS when the satellite moves in orbit. This implies that in a linear time-invariant setting yaw is not controllable over the poles but only a quarter of an orbit later over the equator, reaches a region with controllability. This is illustrated in Fig. 3. This problem has so far prevented three-axis control with only magnetorquer actuation. In the following, the control problem is approached as the nonlinear and time-varying problem. This will be shown to be a fruitful advance. The first step is to look at stabilization of angular velocity. The second to consider local stabilization of attitude and the last to attempt an extension to obtain global attitude stability.

5. Rate control

A tumbling satellite is one which rotates with relatively high value of angular velocity. In a satellite mission, this is a condition that can be experienced in various phases including contingencies where attitude sensors may be unavailable. It is vital to be able to stabilize such tumbling motion towards zero angular velocity. This requires dissipation of energy.

The aim of this section is to provide an algorithm for energy dissipation of the tumbling satellite. To achieve this, a rate detumbling controller is proposed which incorporates information about the angular velocity and time propagation of the geomagnetic field. When generating control currents, it is desirable that the produced magnetic moment is perpendicular to the local geomagnetic field vector, because this component is the only one that gives a nonzero control torque.

A candidate for generation of the magnetic moment is an angular velocity feedback

$$^{c}\mathbf{m}(t) = (\mathbf{H}\,{}^{c}\boldsymbol{\Omega}_{co}(t)) \times {}^{c}\mathbf{B}(t), \tag{8}$$

where $\mathbf{H}$ is a positive-definite constant matrix. There are two main reasons to suggest this feedback:

(i) It contributes to dissipation of kinetic energy of the satellite.
(ii) It provides four stable equilibria depicted in Fig. 4. The equilibria are such that the z axis of the Control CS (the axis of the minimal moment of inertia) points in the direction of the z axis of the Orbit CS (zenith), and the unit vector of the y axis of the Control CS (the axis of the largest moment of inertia) is parallel to the y axis of the Orbit CS.

Global stability of the control law (8) can be expressed in the following theorem.

Theorem 1. *Consider the control law*

$$^{c}\mathbf{m}(t) = (\mathbf{H}\,{}^{c}\boldsymbol{\Omega}_{co}(t)) \times {}^{c}\mathbf{B}(t),$$

then the satellite, Eqs. (1)–(7), *in a nonequatorial orbit has* 4 *asymptotically stable equilibria*

$$\{(^{c}\boldsymbol{\Omega}_{co}, {}^{c}\mathbf{k}_{o}, {}^{c}\mathbf{j}_{o}) : (\mathbf{0}, \pm\,{}^{o}\mathbf{k}_{o}, \pm\,{}^{o}\mathbf{j}_{o})\}. \tag{9}$$

Magnetic torquing following Eq. (8) obviously introduces time dependency in the equations of the satellite motion. This time variation is periodic by nature, which arises from two superimposed periodic fluctuations of the geomagnetic field. One is due to revolution of the satellite about the Earth with the period corresponding to the orbital period, the second due to rotation of the Earth. The orientation of the orbit is fixed in an inertial coordinate system. Thus the rotation of the Earth is visible as fluctuation of the geomagnetic field vector's y component with frequency 1/24 h. The Earth's magnetic field seen from a satellite in a near polar orbit is illustrated in

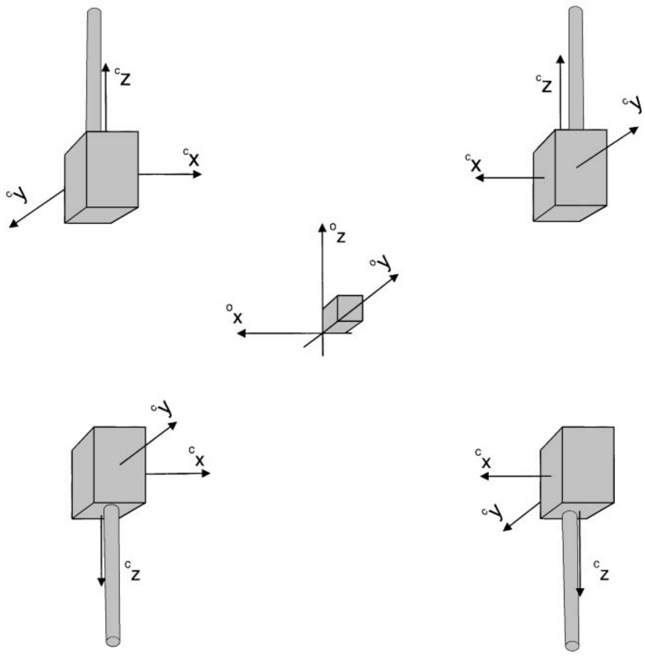

Fig. 4. Four locally stable equilibria of the angular velocity feedback (8): $\{({}^{c}\boldsymbol{\Omega}_{co}, {}^{c}\mathbf{k}_{o}, {}^{c}\mathbf{j}_{o}) : (\mathbf{0}, {}^{o}\mathbf{k}_{o}, {}^{o}\mathbf{j}_{o}), (\mathbf{0}, {}^{o}\mathbf{k}_{o}, -{}^{o}\mathbf{j}_{o}), (\mathbf{0}, -{}^{o}\mathbf{k}_{o}, {}^{o}\mathbf{j}_{o}), (\mathbf{0}, -{}^{o}\mathbf{k}_{o}, -{}^{o}\mathbf{j}_{o})\}$.

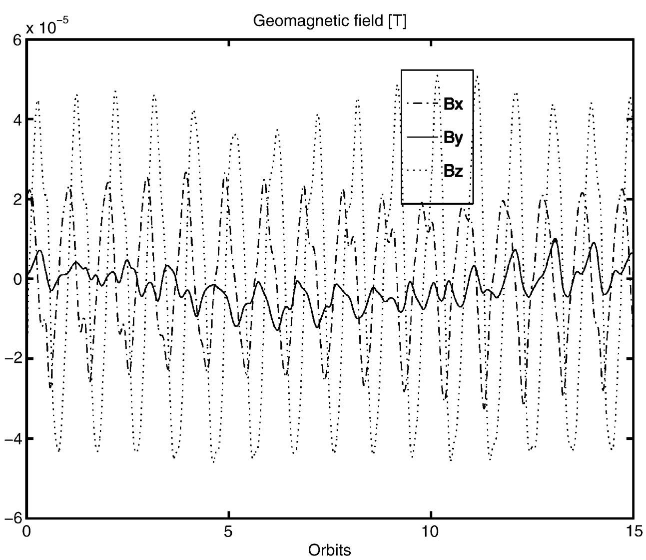

Fig. 5. The geomagnetic field vector seen from the Orbit CS calculated by the 10th order spherical harmonic model (24 h period in January 1999).

Fig. 5. The periodic character of the geomagnetic field is used in the proof of Theorem 1.

Proof. Consider the following scalar function expressing the total energy of the satellite. The total energy is the sum of kinetic energy of rotary motion, potential energy generated by the gravity gradient and the energy originating from the revolution of the satellite around the Earth (Hughes, 1986),

$$E_{tot} = E_{kin} + E_{gg} + E_{gyro}.$$

This leads to

$$E_{tot} = \tfrac{1}{2}\,{}^{c}\boldsymbol{\Omega}_{co}^{T}\,\mathbf{I}\,{}^{c}\boldsymbol{\Omega}_{co} + \tfrac{3}{2}\,\omega_0^2({}^{c}\mathbf{k}_o^{T}\,\mathbf{I}\,{}^{c}\mathbf{k}_o - I_z) + \tfrac{1}{2}\,\omega_o^2(I_y - {}^{c}\mathbf{j}_o^{T}\,\mathbf{I}\,{}^{c}\mathbf{j}_o). \tag{10}$$

The satellite's maximal moment of inertia is I_y, and I_z is the minimal. This implies that the function E_{tot} is positive definite. The potential energy due to the gravity gradient is zero if the satellite boom is pointing towards zenith or nadir, whereas the energy due to motion of the satellite in orbit is zero if the unit vectors $\mathbf{j}_o$ and $\mathbf{j}_c$ are aligned.

The time derivative of E_{tot} will now be shown to be negative semidefinite. With

$$\dot{E}_{tot} = {}^{c}\boldsymbol{\Omega}_{co}^{T}\,\mathbf{I}\,{}^{c}\dot{\boldsymbol{\Omega}}_{co} + 3\omega_o^2\,{}^{c}\mathbf{k}_o^{T}\,\mathbf{I}\,{}^{c}\dot{\mathbf{k}}_o + \omega_o^2\,{}^{c}\mathbf{j}_o^{T}\,\mathbf{I}\,{}^{c}\dot{\mathbf{j}}_o \tag{11}$$

Eqs. (1)–(7) are substituted into Eq. (11) yielding

$$\begin{aligned}\dot{E}_{tot} = {}&{}^{c}\boldsymbol{\Omega}_{co}^{T}(-\,{}^{c}\boldsymbol{\Omega}_{cw}\times\mathbf{I}\,{}^{c}\boldsymbol{\Omega}_{cw} + 3\omega_o^2\,{}^{c}\mathbf{k}_c^{T}\times\mathbf{I}\,{}^{c}\mathbf{k}_o + {}^{c}\mathbf{N}_{ctrl})\\ &- \omega_o\,{}^{c}\boldsymbol{\Omega}_{co}^{T}\,\mathbf{I}({}^{c}\mathbf{j}_o\times{}^{c}\boldsymbol{\Omega}_{co}) + 3\omega_o^2\,{}^{c}\mathbf{k}_o^{T}\,\mathbf{I}({}^{c}\mathbf{k}_o\times{}^{c}\boldsymbol{\Omega}_{co})\\ &- \omega_o^2\,{}^{c}\mathbf{j}_o^{T}\,\mathbf{I}({}^{c}\mathbf{j}_o\times{}^{c}\boldsymbol{\Omega}_{co}).\end{aligned} \tag{12}$$

Recognize that from Eq. (7) the following equality holds:

$$\begin{aligned}{}^{c}\boldsymbol{\Omega}_{co}^{T}({}^{c}\boldsymbol{\Omega}_{cw}\times\mathbf{I}\,{}^{c}\boldsymbol{\Omega}_{cw}) = {}&\omega_o\,{}^{c}\boldsymbol{\Omega}_{co}^{T}({}^{c}\mathbf{j}_o\times\mathbf{I}\,{}^{c}\boldsymbol{\Omega}_{co})\\ &+ \omega_o^2\,{}^{c}\boldsymbol{\Omega}_{co}^{T}({}^{c}\mathbf{j}_o\times\mathbf{I}\,{}^{c}\mathbf{j}_o).\end{aligned} \tag{13}$$

Hence Eq. (12) is reduced to the simple expression

$$\dot{E}_{tot} = {}^{c}\boldsymbol{\Omega}_{co}^{T}\,{}^{c}\mathbf{N}_{ctrl}. \tag{14}$$

If the proposed control law (8) is applied then Eq. (14) becomes

$$\dot{E}_{tot} = -\,({}^{c}\mathbf{B}\times{}^{c}\boldsymbol{\Omega}_{co})^{T}({}^{c}\mathbf{B}\times\mathbf{H}\,{}^{c}\boldsymbol{\Omega}_{co}), \tag{15}$$

or

$$\dot{E}_{tot} = -\,{}^{c}\boldsymbol{\Omega}_{co}^{T}\,\tilde{\mathbf{B}}^{T}\tilde{\mathbf{B}}\mathbf{H}\,{}^{c}\boldsymbol{\Omega}_{co}. \tag{16}$$

Here $\tilde{\mathbf{B}}$ is a skew symmetric matrix representing the cross product operator: ${}^{c}\mathbf{B}\times$. The matrix $\tilde{\mathbf{B}}^{T}\tilde{\mathbf{B}}$ is positive semidefinite and $\mathbf{H}$ is positive definite. The derivative of the total energy is thus negative semidefinite.

The Krasovskii–LaSalle lemma (see Lemma A.1 for details) is applicable in this proof since both the system and the control law (8) are periodic. The set S in Lemma 5

$$S = \{{}^{c}\boldsymbol{\Omega}_{co}, {}^{c}_{o}\mathbf{q} : \exists t \geq 0 \text{ such that } \dot{E}_{tot} = 0\}$$

contains elements ${}^{c}\boldsymbol{\Omega}_{co}, {}^{c}_{o}\mathbf{q}$ such that either of the vectors ${}^{c}\boldsymbol{\Omega}_{co}$ or $\mathbf{H}\,{}^{c}\boldsymbol{\Omega}_{co}$ is parallel to ${}^{c}\mathbf{B}({}^{c}\mathbf{B}\times{}^{c}\boldsymbol{\Omega}_{co} = \mathbf{0}$ or ${}^{c}\mathbf{B}\times\mathbf{H}\,{}^{c}\boldsymbol{\Omega}_{co} = \mathbf{0})$

$$S = \{{}^{c}\boldsymbol{\Omega}_{co}, {}^{c}_{o}\mathbf{q} : \exists t \geq 0\ {}^{c}\mathbf{B} \text{ parallel to } {}^{c}\mathbf{B} \text{ parallel to } \mathbf{H}\,{}^{c}\boldsymbol{\Omega}_{co}\}. \tag{17}$$

It will be shown that the only invariant set contained in the set S is the trivial trajectory $\mathbf{\Omega}_{co}(t) \equiv \mathbf{0}$, which corresponds to one of the equilibria (9).

Assume that the vector ${}^{c}\mathbf{B}(t)$ was parallel either to ${}^{c}\mathbf{\Omega}_{co}(t)$ or to $\mathbf{H}\,{}^{c}\mathbf{\Omega}_{co}(t)$ for each $t > t_0$ then from Eqs. (3) and (8) the control torque is equal to zero.

The angular velocity, ${}^{c}\mathbf{\Omega}_{co}$, is given by the dynamic equations (1), (3) and (7) for ${}^{c}\mathbf{N}_{ctrl} = \mathbf{0}$. The geomagnetic field determines the vector ${}^{o}\mathbf{B}$, the time propagation of which in the Orbit CS was depicted in Fig. 5. It is obvious that the vector ${}^{c}\mathbf{\Omega}_{co}$ (or $\mathbf{H}\,{}^{c}\mathbf{\Omega}_{co}$) can only be parallel to ${}^{o}\mathbf{B}$ instantaneously.

Notice that this is true for the geomagnetic field observed in all orbits with nonzero inclination. The only exception is the case of an equatorial orbit, where the geomagnetic field vector is always parallel to the orbit normal. Now, the angular velocity ${}^{c}\mathbf{\Omega}_{co}$ can always be parallel to the geomagnetic field since the satellite motion, such that the angular velocity ${}^{c}\mathbf{\Omega}_{co}$ and the principal axis $\mathbf{j}_c$ (coinciding with the maximum moment of inertia) are both perpendicular to the orbital plane, corresponds to a stable equilibrium (Hughes, 1986).

For all other orbits the largest invariant set contained in S is the trivial trajectory ${}^{c}\mathbf{\Omega}_{co} \equiv \mathbf{0}$. The angular velocity is thus zero for all $t \geq 0$ and the trajectory of the torque-free motion coincides with a stable equilibrium. There are four stable equilibria for the torque free motion (Hughes, 1986). $\{({}^{c}\mathbf{\Omega}_{co}, {}^{c}\mathbf{k}_o, {}^{c}\mathbf{j}_o)\colon (\mathbf{0}, \pm\,{}^{o}\mathbf{k}_o, \pm\,{}^{o}\mathbf{j}_o)\}$. Now, with the use of control law (8) these stable equilibria beome also asymptotically stable. □

Remark 2. The control law (8) can be used for three-axis magnetic stabilization of the satellite in a neighbourhood of one of the 4 equilibria stated in Theorem 1, thus also in the neighbourhood of the reference $\{({}^{c}\mathbf{\Omega}_{co}, {}^{c}\mathbf{k}_o, {}^{c}\mathbf{j}_o)\colon (\mathbf{0}, {}^{o}\mathbf{k}_o, {}^{o}\mathbf{j}_o)\}$, if $I_y > I_x > I_z$.

Remark 3. If the velocity controller (8) is applied for a limited time interval during an orbit, then the solution trajectory of the satellite motion still converges to one of the four equilibria in Theorem 1, since the total energy from Eq. (14) is constant if the control torque is zero, and is dissipated when the controller is active.

This control strategy is very useful if the attitude control can only take place via telecommand from a ground station, and time of radio contact is limited. The controller is also beneficial for a satellite with an attitude determination algorithm based on aa sun sensor, since the attitude information may not be available when the satellite is in eclipse. In both cases controller (8) is activated when the feedback signals are available and switched off otherwise. The control law is still stable.

It was proved that the satellite with the control law (8) is asymptotically stable around four equilibria (9). This simple control law was found well applicable for energy dissipation of a tumbling satellite. Simulation results of the detumbling phase controller based on Eq. (8) are presented in the next subsection.

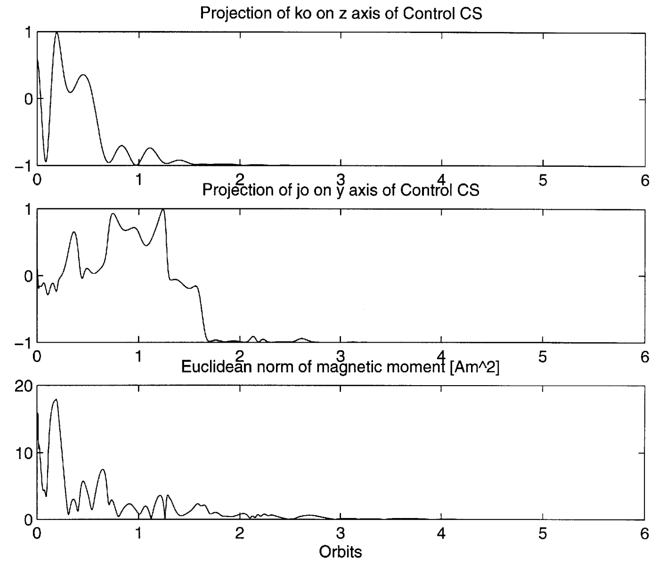

Fig. 6. Simulation using the angular velocity controller, ${}^{c}k_{oz}$ characterizes convergence of ${}^{c}\mathbf{k}_o$ towards ${}^{o}\mathbf{k}_o$, whereas ${}^{c}j_{oy}$ characterizes converges of ${}^{c}\mathbf{j}_c$ to ${}^{o}\mathbf{j}_o$. The satellite trajectory converges towards the equilibrium $\{({}^{c}\mathbf{\Omega}_{co}, {}^{c}\mathbf{k}_o, {}^{c}\mathbf{j}_o)\colon (\mathbf{0}, -{}^{o}\mathbf{k}_o, -{}^{o}\mathbf{j}_o)\}$. This is seen from ${}^{c}k_{oz}$ and ${}^{c}j_{oy}$ converging to -1.

5.1. Simulation results for detumbling controller

The performance of the detumbling phase controller is illustrated in Fig. 6. The initial angular velocity is ${}^{c}\mathbf{\Omega}_{co} = [0.003\ \ 0.005\ \ -0.003]^{\mathrm{T}}$ rad/s. Pitch, roll and yaw are 40°, −40° and 80°, respectively. The velocity gain applied in this simulation study is $\mathbf{H} = \mathrm{diag}([0.3\ \ 1.4\ \ 0.8]^{\mathrm{T}}) \times 10^{8}$ A m s/T. The satellite trajectory is seen to converge to the equilibrium $\{({}^{c}\mathbf{\Omega}_{co}, {}^{c}\mathbf{k}_o, {}^{c}\mathbf{j}_o)\colon (\mathbf{0}, -{}^{o}\mathbf{k}_o, -{}^{o}\mathbf{j}_o)\}$, with the boom upside-down. Other initial conditions could lead to a boom-up equilibrium.

6. Local attitude stability

The rate control achieved stabilization and energy dissipation as desired, but reaching a desired attitude with boom-up could not be guaranteed. In the operational mission phase of the Ørsted satellite the boom should be upright with the boom tip above the horizon. The attitude controller designed for this mission phase must thus be stable towards the reference

$$\{({}^{c}\mathbf{\Omega}_{co}, {}^{c}\mathbf{k}_o, {}^{c}\mathbf{j}_o)\colon (\mathbf{0}, {}^{o}\mathbf{k}_o, {}^{o}\mathbf{j}_o)\}. \tag{18}$$

This will be shown to be achievable by adding perturbations of the attitude to the control law (8). This is intuitively feasible since the maximum of the potential energy E_{gyro} in Eq. (10) is three times smaller than the maximum potential energy due to the gravity gradient. This implies that, a shift from the equilibria $\{({}^{\mathrm{c}}\boldsymbol{\Omega}_{\mathrm{co}}, {}^{\mathrm{c}}\mathbf{k}_{\mathrm{o}}, {}^{\mathrm{c}}\mathbf{i}_{\mathrm{o}})\colon (\mathbf{0}, \pm {}^{\mathrm{o}}\mathbf{k}_{\mathrm{o}}, -{}^{\mathrm{o}}\mathbf{j}_{\mathrm{o}})\}$ to $(\mathbf{0}, \pm {}^{\mathrm{o}}\mathbf{k}_{\mathrm{o}}, {}^{\mathrm{o}}\mathbf{j}_{\mathrm{o}})$ requires less energy than the move from $(\mathbf{0}, -{}^{\mathrm{o}}\mathbf{k}_{\mathrm{o}}, \pm {}^{\mathrm{o}}\mathbf{j}_{\mathrm{o}})$ to $(\mathbf{0}, {}^{\mathrm{o}}\mathbf{k}_{\mathrm{o}}, \pm {}^{\mathrm{o}}\mathbf{j}_{\mathrm{o}})$.

A proposed control law for the three-axis attitude stabilization is

$$ {}^{\mathrm{c}}\mathbf{m}(t) = \mathbf{H}\, {}^{\mathrm{c}}\boldsymbol{\Omega}_{\mathrm{co}}(t) \times {}^{\mathrm{c}}\mathbf{B}(t) - \boldsymbol{\Lambda}\mathbf{q}(t) \times {}^{\mathrm{c}}\mathbf{B}(t), \tag{19} $$

where attitude information is added to the rate control law. Here $\mathbf{H}$ and $\boldsymbol{\Lambda}$ are positive-definite matrices. The properties of control law (19) will be analyzed using linear control theory.

In Eq. (19) a small perturbation of the vector part of the attitude quaternion is added to the controller in Eq. (8). For small $\boldsymbol{\Lambda}$ the satellite is stable in the neighbourhood of the reference $\{({}^{\mathrm{c}}\boldsymbol{\Omega}_{\mathrm{co}}, {}^{\mathrm{c}}\mathbf{k}_{\mathrm{o}}, {}^{\mathrm{c}}\mathbf{j}_{\mathrm{o}})\colon (\mathbf{0}, {}^{\mathrm{o}}\mathbf{k}_{\mathrm{o}}, {}^{\mathrm{o}}\mathbf{j}_{\mathrm{o}})\}$, since the differential equations describing the motion of the satellite are well posed. At this point of the analysis the gain $\mathbf{H}$ is assumed to be constant. The next step of the design is to compute the gain $\boldsymbol{\Lambda}$ for a given $\mathbf{H}$ such that the system is still locally stable and the domain of local stability can be extended.

The system is first linearized. The satellite motion is considered in a neighbourhood of the following reference: the angular velocity of the satellite rotation w.r.t. the Orbit CS is zero (${}^{\mathrm{c}}\boldsymbol{\Omega}_{\mathrm{co}} = \mathbf{0}$), and the attitude is such that the Control CS coincides with the Orbit (${}^{\mathrm{c}}_{\mathrm{o}}\mathbf{q} = [0\ \ 0\ \ 0\ \ 1]^{\mathrm{T}}$).

The linearization of the angular velocity is additive. From Eq. (7) the angular velocity ${}^{\mathrm{c}}\boldsymbol{\Omega}_{\mathrm{cw}}$ is

$$ {}^{\mathrm{c}}\boldsymbol{\Omega}_{\mathrm{cw}} = \mathbf{A}({}^{\mathrm{c}}_{\mathrm{o}}\mathbf{q})[0\ \ \omega_0\ \ 0]^{\mathrm{T}} + \delta\boldsymbol{\Omega}, \tag{20} $$

where $\delta\boldsymbol{\Omega}$ is a small perturbation.

Linearization of the attitude quaternion is multiplicative. Two successive rotations are needed. The first is the transformation from the Orbit CS to a reference coordinate system, the second from the reference coordinate system to the Control CS. The reference coordinate system is the Orbit CS in the paper, thus the rotation from the Orbit CS to the reference coordinate system is trivially given by the identity operation.

$$ {}^{\mathrm{c}}_{\mathrm{o}}\mathbf{q} = \widetilde{\delta\mathbf{q}} \cdot [0\quad 0\quad 0\quad 1]^{\mathrm{T}} = \widetilde{\delta\mathbf{q}}, \tag{21} $$

where $\widetilde{\delta\mathbf{q}}$ is a small perturbation of the attitude quaternion, ${}^{\mathrm{c}}_{\mathrm{o}}\mathbf{q}$, and "$\cdot$" is an operation of quaternion multiplication, for details see Wertz (1990). According to the physical interpretation of a quaternion given in Eq. (5)

$$ \widetilde{\delta\mathbf{q}} = \left[e_1 \sin\frac{\delta\phi}{2} \quad e_2 \sin\frac{\delta\phi}{2} \quad e_3 \sin\frac{\delta\phi}{2} \quad \cos\frac{\delta\phi}{2} \right]^{\mathrm{T}}, \tag{22} $$

and for a small angle ϕ

$$ \widetilde{\delta\mathbf{q}} \approx [\delta q_1 \quad \delta q_2 \quad \delta q_3 \quad 1]^{\mathrm{T}} \equiv \begin{bmatrix} \delta\mathbf{q} \\ 1 \end{bmatrix}. \tag{23} $$

The linearized equations of motion (1)–(7) are then (for more details see Wisniewski, 1996)

$$ \frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} \delta\boldsymbol{\Omega} \\ \delta\mathbf{q} \end{bmatrix} = \mathbf{A}\begin{bmatrix} \delta\boldsymbol{\Omega} \\ \delta\mathbf{q} \end{bmatrix} + \tilde{\tilde{\mathbf{B}}}(t)(\mathbf{H}\,\delta\boldsymbol{\Omega} + \boldsymbol{\Lambda}\,\delta\mathbf{q}), \tag{24} $$

where

$$ \mathbf{A} = \begin{bmatrix} 0 & 0 & \omega_{\mathrm{o}}\sigma_x & -6\omega_{\mathrm{o}}^2\sigma_x & 0 & 0 \\ 0 & 0 & 0 & 0 & 6\omega_{\mathrm{o}}^2\sigma_y & 0 \\ \omega_{\mathrm{o}}\sigma_z & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & -\omega_{\mathrm{o}} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \\ 0 & 0 & \frac{1}{2} & \omega_{\mathrm{o}} & 0 & 0 \end{bmatrix}, $$

$$ \sigma_x = \frac{I_y - I_z}{I_x}, \qquad \sigma_y = \frac{I_z - I_x}{I_y}, \qquad \sigma_z = \frac{I_x - I_y}{I_z}, $$

$$ \tilde{\tilde{\mathbf{B}}} = \begin{bmatrix} \mathbf{I}^{-1} \begin{matrix} -{}^{\mathrm{o}}B_y^2 - {}^{\mathrm{o}}B_z^2 & {}^{\mathrm{o}}B_x\, {}^{\mathrm{o}}B_y & {}^{\mathrm{o}}B_x\, {}^{\mathrm{o}}B_z \\ {}^{\mathrm{o}}B_x\, {}^{\mathrm{o}}B_y & -{}^{\mathrm{o}}B_x^2 - {}^{\mathrm{o}}B_z^2 & {}^{\mathrm{o}}B_y^2\, {}^{\mathrm{o}}B_z \\ {}^{\mathrm{o}}B_x\, {}^{\mathrm{o}}B_z & {}^{\mathrm{o}}B_y\, {}^{\mathrm{o}}B_z & -{}^{\mathrm{o}}B_x^2 - {}^{\mathrm{o}}B_y^2 \end{matrix} \\ \mathbf{0} \end{bmatrix}. $$

As discussed in Section 5, variation of the local geomagnetic field observed in the Orbit CS reflects both the orbital motion and the rotation of the Earth. However, for the near polar orbits the main contribution to the geomagnetic field propagation is caused by the orbital motion as illustrated in Fig. 5. In order to simplify the analysis, an approximation of the geomagnetic field is calculated with a period equivalent to one orbit. This is done by averaging the geomagnetic field. The influence of the difference between the real geomagnetic field and this approximation is evaluated in a subsequent simulation study.

In the sequel local stability of the satellite is analysed using Floquet theory (Mohler, 1991). Consider the following form of Eq. (24)

$$ \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \hat{\mathbf{A}}(\lambda, t)\mathbf{x}, \tag{25} $$

where

$$ \hat{\mathbf{A}}(\boldsymbol{\Lambda}, t) = \mathbf{A} + \mathbf{B}(t)[\mathbf{H}\quad \boldsymbol{\Lambda}], $$

$\mathbf{H}$ is considered to be a constant matrix, whereas $\boldsymbol{\Lambda}$ is a matrix of parameters. Furthermore,

$$ \tilde{\mathbf{A}}(\boldsymbol{\Lambda}, t) = \hat{\mathbf{A}}(\boldsymbol{\Lambda}, t + T), \quad \text{for } T = 2\pi/\omega_{\mathrm{o}}. $$

According to Floquet theory system (25) is asymptotically stable if all characteristic multipliers, i.e. the eigenvalues of the monodromy matrix $\boldsymbol{\Psi}_{\hat{A}}(t_0, \boldsymbol{\Lambda})$ for a certain value of $\boldsymbol{\Lambda}$ lie in the open unit circle.

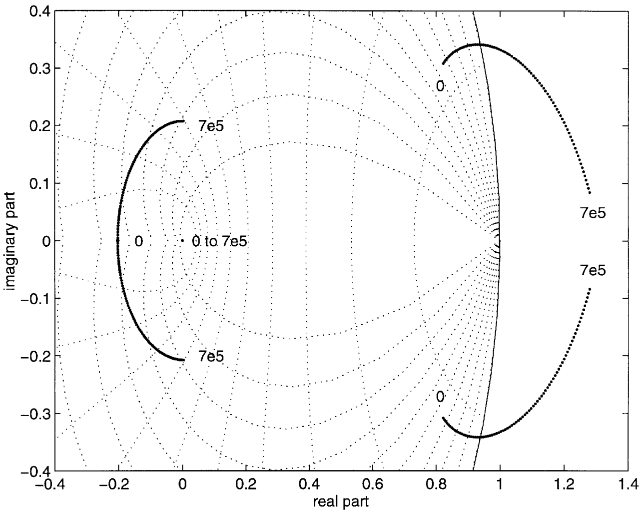

Fig. 7. The locus for the characteristic multiplier for $\lambda \in [0,\ 7\times10^5]$ A m^2/T, $h = 1\times10^8$ A m s/T using Eq. (19) as the control law.

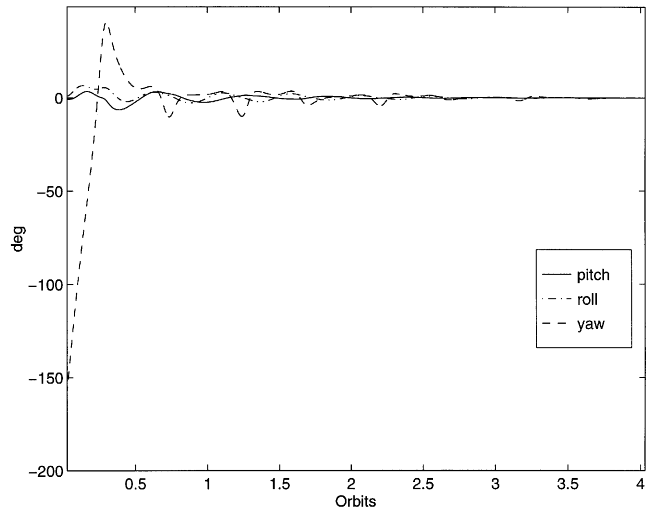

Fig. 8. The satellite trajectory converges from the equilibrium $\{({}^{c}\mathbf{\Omega}_{co},\ {}^{c}\mathbf{k}_{o},\ {}^{c}\mathbf{j}_{o}) : (\mathbf{0},\ {}^{o}\mathbf{k}_{o})\}$ to the reference.

Now, such $\mathbf{\Lambda}$ are found for which the linearized satellite system (24), with a certain fixed value of the positive definite matrix $\mathbf{H}$, is stable. This can be done by plotting a locus for the characteristic multipliers as a function of $\mathbf{\Lambda}$. An example of a scalar case $\mathbf{H} = h\mathbf{E}$ and $\mathbf{\Lambda} = \lambda\mathbf{E}$ is shown in Fig. 7. The value of the velocity gain, h, was chosen equal 1×10^8 A m s/T. Then the locus of the characteristic multiplier for $\lambda \in [0, 7\times10^5]$ A m^2/T was plotted. The gain $\tilde{\lambda} = 5\times10^5$ A m^2/T is the limit of stability. Hence, a certain $\hat{\lambda} < \tilde{\lambda}$ shall be chosen for which the satellite motion is guaranteed to be asymptotically stable. For $\lambda = 0$ controller (19) is equivalent to the velocity controller, which is stable.

6.1. Simulation results for normal operation phase controller

A Monte Carlo simulation was made to investigate stability towards the reference (18). The initial state is random with the nominal condition as the mean value. The envelope for the Monte Carlo simulation includes values of the attitude above the horizon, i.e. ${}^{c}k_{oz} > 0$ and $\mathbf{\Omega}_{co} = 0$. The chosen attitude controller has velocity gain, $\mathbf{H} = 1\times10^8\,\mathbf{E}$ A m s/T, and the quaternion gain is $\mathbf{\Lambda} = \hat{\lambda}\mathbf{E} = 3\times10^5\,\mathbf{E}$ A m^2/T.

An example of the simulation test for the satellite on a circular orbit is depicted in Fig. 8. The simulation starts at the equilibrium with the yaw axis turned 180° away from the desired direction, $\{({}^{c}\mathbf{\Omega}_{co},\ {}^{c}\mathbf{k}_{o},\ {}^{c}\mathbf{j}_{o}) : (\mathbf{0},\ {}^{o}\mathbf{k}_{o},\ -{}^{o}\mathbf{j}_{o})\}$. The trajectory is within the margin of 10° from the reference already after one orbit. The effect of the attitude term in the controller is seen to make the equilibrium $\{(\mathbf{0},\ {}^{o}\mathbf{k}_{o},\ -{}^{o}\mathbf{j}_{o})\}$ unstable, thus extending the region of convergence to the initial values of the attitude for which the boom is above the horizon and the initial satellite angular relative to the Orbit CS is zero.

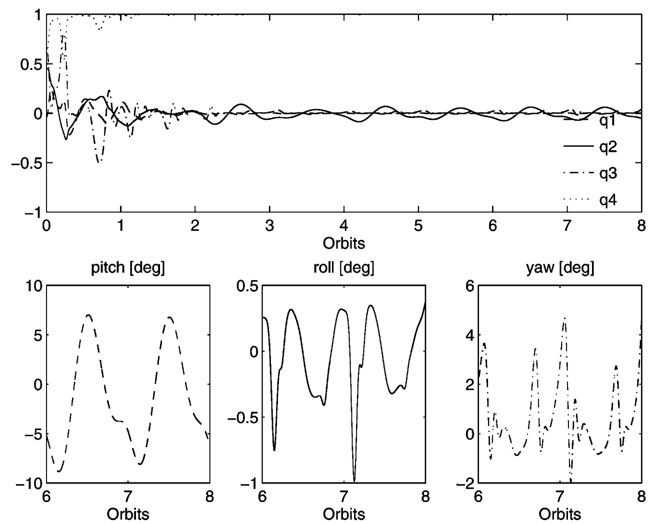

Fig. 9. Performance of the constant gain controller for the Ørsted satellite on the elliptic orbit influenced by the aerodynamic drag. The initial conditions are 40° pitch, − 40° roll and 80° yaw. The resultant attitude is within 8°.

The time constants of the closed-loop system are on the order of one orbit, therefore if the satellite is subject to cyclic disturbances with fundamental frequency corresponding to the orbital period, this disturbance will only be marginally damped. Aerodynamic drag in an elliptic orbit gives exactly this type of disturbances. The simulation results for the Ørsted satellite on impact of the aerodynamic drag and the torque due to eccentricity of the orbit is illustrated in Fig. 9. The attitude error is within 8°, which is within the required limit of $\pm$ 10°.

The linear attitude controller performs satisfactory for all initial value of attitude ${}^{c}k_{oz} > 0$ (boom is upright). A global stabilizing attitude controller will be investigated in the next section.

7. Contingency operation controller for inverted boom

A condition with the boom upside-down, i.e. pointing towards the centre of the Earth, may likely occur due to considerable influence of the aerodynamic torque. Such undesired attitude must be corrected by the attitude control system. Hence, the attitude control problem is to turn the satellite from the equilibria $\{(\mathbf{0}, -{}^{o}\mathbf{k}_o, \pm{}^{o}\mathbf{j}_o)\}$ to the upright attitude, ${}^{c}k_{oz} > 0$. This issue will be addressed in this section and a control law will be suggested to cope with the boom-down condition.

The following control law is considered

$$ {}^{c}\mathbf{m}(t) = \mathbf{H}\,{}^{c}\boldsymbol{\Omega}_{co}(t) \times {}^{c}\mathbf{B}(t) + \eta(t)\mathbf{q}(t) \times {}^{c}\mathbf{B}(t), \tag{26} $$

where $\mathbf{H}$ is a positive constant matrix. $\eta(t)$ is a piecewise continuous positive scalar function satisfying

$$ \eta(t) = \text{const} > 0, \quad t \in (kT_s, (k+1)T_s), \quad k = 1, 2, \ldots, $$
$$ \eta(kT_s) > \eta((k+1)T_s) > 0, \tag{27} $$

where T_s is a positive constant. It is noted that control law (26) has a time-varying attitude gain $\eta(t)$ compared with control law (19).

Before the features of control law (26) are discussed, the following theorem is presented. Its proof is incorporated in Appendix B.

Theorem 4. *Consider control law* (26) *then the satellite, given by Eqs.* (1)–(7), *has* 4 *asymptotically stable local equilibria*

$$ \{({}^{c}\boldsymbol{\Omega}_{co}, {}^{c}\mathbf{k}_o, {}^{c}\mathbf{j}_o) : (\mathbf{0}, \pm{}^{o}\mathbf{k}_o, \pm{}^{o}\mathbf{j}_o)\}. $$

The stability analysis shows that the equilibrium $(\mathbf{0}, {}^{o}\mathbf{k}_o, {}^{o}\mathbf{i}_o)$ is locally stable if $\eta(t) = \tilde{\eta} < \tilde{\eta}$, where $\tilde{\eta}$ is the limit of stability for control law (19). On the other hand, if $\eta(t) = \bar{\eta}$ is large such that the quaternion feedback is the most significant component on the r.h.s. of Eq. (1)

$$ \mathbf{I}\,{}^{c}\dot{\boldsymbol{\Omega}}_{co} \approx \tilde{\eta}(\mathbf{q} \times {}^{c}\mathbf{B}) \times {}^{c}\mathbf{B} \tag{28} $$

then the vectors $\mathbf{I}\,{}^{c}\boldsymbol{\Omega}_{co}$ and $(\mathbf{q} \times {}^{c}\mathbf{B}) \times {}^{c}\mathbf{B}$ become parallel, and

$$ \mathbf{I}\,{}^{c}\boldsymbol{\Omega}_{co}^{T}((\mathbf{q}(t) = {}^{c}\mathbf{B}) \times {}^{c}\mathbf{B}) > 0. \tag{29} $$

It was assumed in Eq. (29) that the vectors ${}^{c}\mathbf{B}$ and $\mathbf{q}$ are not parallel. This is viable, since the controller can be activated when the most favourable conditions in orbital motion for the boom upside-down algorithm occur, as illustrated in Fig. 10. It follows from Eq. (29) that

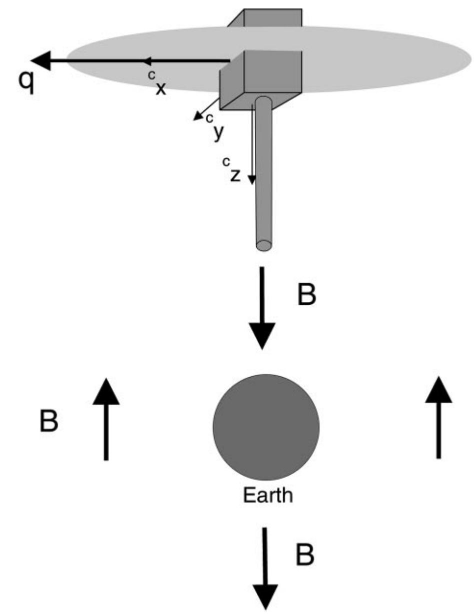

Fig. 10. Boom upside-down algorithm is activated in the regions of North or South Poles.

$$ {}^{c}\boldsymbol{\Omega}_{co}^{T}((\mathbf{q} \times {}^{c}\mathbf{B}) \times {}^{c}\mathbf{B}) > 0, \tag{30} $$

since $\mathbf{I}$ is positive definite. If Eq. (29) is always satisfied, then the satellite is asymptotically stable about the reference $\{(\mathbf{0}, {}^{o}\mathbf{k}_o, {}^{o}\mathbf{j}_o)\}$. To prove this statement, the following Lyapunov candidate function is proposed:

$$ \begin{aligned} E_{tot} &= \tfrac{1}{2}\,{}^{c}\boldsymbol{\Omega}_{co}\mathbf{I}\,{}^{c}\boldsymbol{\Omega}_{co} + \tfrac{3}{2}\,\omega_o^2({}^{c}\mathbf{k}_o^{T}\mathbf{I}\,{}^{c}\mathbf{k}_o - I_z) \\ &\quad + \tfrac{1}{2}\,\omega_o^2(I_y - {}^{c}\mathbf{j}_o^{T}\mathbf{I}\,{}^{c}\mathbf{j}_o) + \eta(q_1^2 + q_2^2 + q_3^2 \\ &\quad + (1 - q_4)^2), \end{aligned} \tag{31} $$

where η is a positive constant. The Lyapunov candidate function (31) resembles the total energy in Eq. (10) (Proof of Theorem 1). An extra attitude quaternion term has been added, however.

The attitude quaternion satisfies the constraint equation $q_1^2 + \cdots + q_4^2 = 1$, thus

$$ \begin{aligned} E_{tot} &= \tfrac{1}{2}\,{}^{c}\boldsymbol{\Omega}_{co}^{T}\mathbf{I}\,{}^{c}\boldsymbol{\Omega}_{co} + \tfrac{3}{2}\,\omega_o^2({}^{c}\mathbf{k}_o^{T}\mathbf{I}\,{}^{c}\mathbf{k}_o - I_z) \\ &\quad + \tfrac{1}{2}\,\omega_o^2(I_y - {}^{c}\mathbf{j}_o^{T}\mathbf{I}\,{}^{c}\mathbf{j}_o) + 2\eta(1 - q_4). \end{aligned} \tag{32} $$

The time derivative of the Lyapunov candidate function is

$$ \dot{E}_{tot} = {}^{c}\boldsymbol{\Omega}_{co}^{T}\,{}^{c}\mathbf{N}_{ctrl} + \eta\,{}^{c}\boldsymbol{\Omega}_{co}^{T}\mathbf{q}. \tag{33} $$

In Eq. (33) the following was used:

$$ \frac{d}{dt}2\eta(1 - q_4) = \eta\,{}^{c}\boldsymbol{\Omega}_{co}\mathbf{q}. \tag{34} $$

Finally, $\dot{E}_{tot}$ is

$$\dot{E}_{tot} = -\mathbf{H}\,{}^c\boldsymbol{\Omega}_{co}^T\tilde{\mathbf{B}}^T\tilde{\mathbf{B}}\,{}^c\boldsymbol{\Omega}_{co} - \tilde{\eta}\,{}^c\boldsymbol{\Omega}_{co}^T\tilde{\mathbf{B}}^T\tilde{\mathbf{B}}\mathbf{q} + \eta\,{}^c\boldsymbol{\Omega}_{co}^T\mathbf{q}, \quad (35)$$

which is negative definite for sufficiently large $\tilde{\eta}$. Notice that Eq. (29) is satisfied only until the angular velocity term in Eq. (1) becomes dominant. The objective of this investigation, however, is not to derive a complete globally stabilizing controller in one step, but rather to get a control law providing the necessary acceleration to turn the satellite from the upside down to upright attitude.

It was assumed that the vectors ${}^c\mathbf{B}$ and $\mathbf{q}$ should not be parallel. This implies that the boom upside-down algorithm must be triggered in the zones near the North or South Poles. As mentioned in Section 4.2, the vector part of the attitude quaternion, $\mathbf{q}$, determines the axis of rotation from the Orbit CS to the Control CS. If the boom is upside-down, then $\mathbf{q}$ is perpendicular to the z-axis of the Orbit CS (the zenith). The zenith, though, is parallel to the geomagnetic field vector over the poles, and ${}^c\mathbf{B}$ and $\mathbf{q}$ are perpendicular. The boom upside-down situation is illustrated in Fig. 10.

From the analysis carried out so far it follows that for $\eta(t) = \bar{\eta}$ large enough the satellite trajectory is turned from the boom upside-down towards the boom upright attitude. For practical implementation, $\bar{\eta}$ shall be chosen larger than

$$\frac{\max(\|\mathbf{N}_{gg}\|)}{\min(\|{}^o\mathbf{B}\|^2)} \approx 9\times 10^5\ \mathrm{N\,m/T}$$

such that the control torque is larger than the gravity gradient. Recapitulating, if $\eta(t) = \hat{\eta}$ the system is asymptotically stable for all values of attitude such that the boom tip is above the horizon. If $\eta(t) = \bar{\eta}$ the satellite boom axis is turned from upside-down to upright. Furthermore, if $\eta(t)$ satisfies Eq. (27) then the satellite is locally asymptotically stable towards the four equilibria $\{({}^c\boldsymbol{\Omega}_{co}, {}^c\mathbf{k}_o, {}^c\mathbf{j}_o):(\mathbf{0}, \pm{}^o\mathbf{k}_o, \pm{}^o\mathbf{j}_o)\}$. The following algorithm is now straightforward.

If the boom tip is below the horizon, ${}^ck_{oz} \le 0$, implement control law (26) with $\eta(t) = \bar{\eta}$. The satellite reaches a boom upright attitude, ${}^ck_{oz} > 0$, and $\eta(t)$ shall gradually decrease from $\bar{\eta}$ to $\hat{\eta}$. Nevertheless, due to possibly large angular velocity involved the satellite may again arrive at a boom upside-down attitude. However, if ${}^ck_{oz} \le 0$ the magnetic moment, ${}^c\mathbf{m}$, is set to $\mathbf{0}$, until the boom is upright once again.

This procedure uses the fact that the total energy of the satellite for torque-free motion, ${}^c\mathbf{m} = \mathbf{0}$, is preserved as proved in Theorem 1. The above algorithm makes the equilibria $\{(\mathbf{0}, -{}^o\mathbf{k}_o, \pm{}^o\mathbf{j}_o)\}$ unstable. The equilibrium $(\mathbf{0}, {}^o\mathbf{k}_o, -{}^o\mathbf{j}_o)$ is also unstable since $\eta(t)$ converges to a constant nonzero value $\hat{\eta}$. Finally, only one equilibrium $(\mathbf{0}, {}^o\mathbf{k}_o, {}^o\mathbf{j}_o)$ remains asymptotically stable, thus it is globally asymptotically stable.

7.1. Simulation results for contingency operation controller

A simulation study has confirmed our hypothesis that the boom upside-down control algorithm provides globally asymptotically stable satellite motion. Simulation results are shown in Figs. 11–13. The initial conditions are such that the satellite has the upside-down attitude corresponding to the equilibrium $\{({}^c\boldsymbol{\Omega}_{co}, {}^c\mathbf{k}^o, {}^c\mathbf{j}_o):(\mathbf{0}, -{}^o\mathbf{k}_o, {}^o\mathbf{j}_o)\}$. The velocity gain is

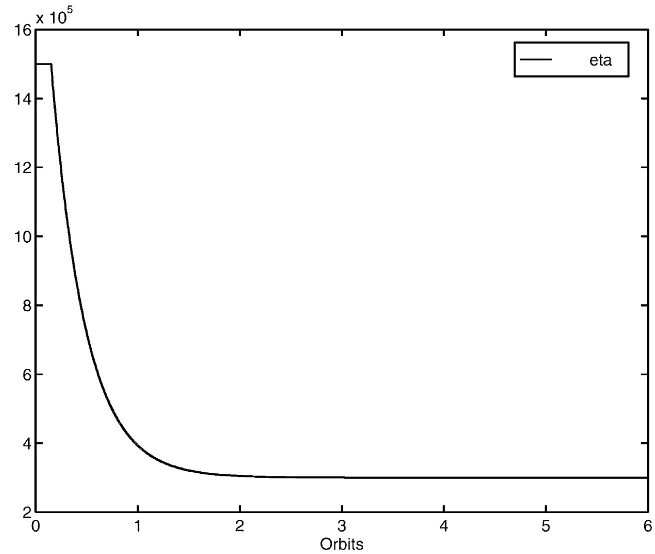

Fig. 11. The velocity gain $\mathbf{H}$ is 1×10^8 $\mathbf{E}$ A m s/T. The attitude gain $\eta(t)$ is time varying with an initial value $15\cdot 10^5$ A m^2/T. It converges to $\hat{\eta} = 3\times 10^5$ A m^2/T.

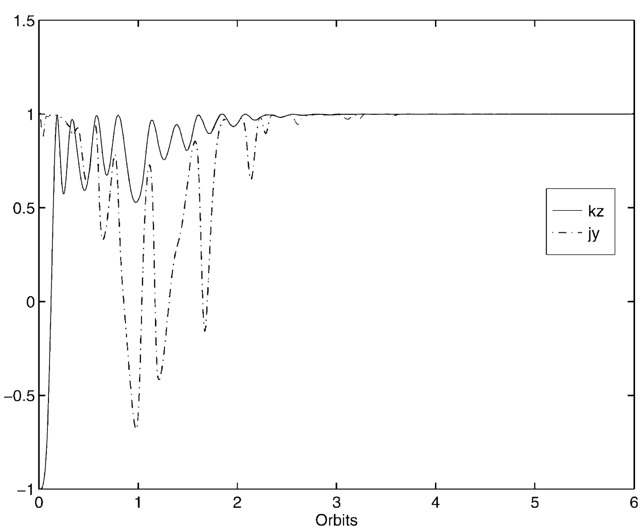

Fig. 12. ${}^ck_{oz}$ characterizes convergence of ${}^c\mathbf{k}_o$ towards ${}^o\mathbf{k}_o$ (if ${}^ck_{oz} < 0$ satellite is upside-down), whereas ${}^cj_{oy}$ characterizes convergence of ${}^c\mathbf{j}_o$ towards ${}^o\mathbf{j}_o$.

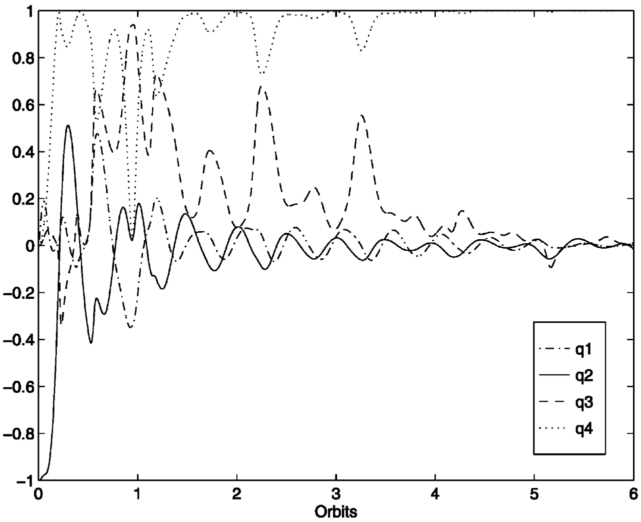

Fig. 13. The attitude quaternion, ${}^c_o\mathbf{q}$ converges to $[0, , 0, 1]^T$ from an upside-down attitude.

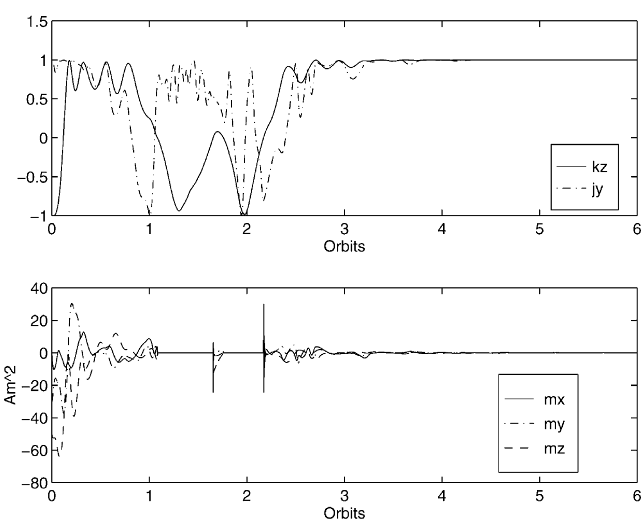

Fig. 14. The velocity and attitude gains are $\mathbf{H} = 1 \times 10^8\, \mathbf{E}$ A m s/T and $\bar{\eta} = 27 \times 10^5$ A m^2/T. The attitude controller is activated when ${}^c k_{oz} > 0$ (if ${}^c k_{kz} \leq 0$ then ${}^c\mathbf{m} = \mathbf{0}$).

$\mathbf{H} = 1 \times 10^8 \mathbf{E}$ A m s/T, and the quaternion gain is chosen to gradually decrease from $\bar{\eta} = 15 \times 10^5$ A m^2/T to $\hat{\eta} = 3 \times 10^5$ A m^2/T. The controller is seen to have a satisfactory performance. It takes less than half an orbit to turn the satellite up, and it is stabilized to the operational region within 6 orbits. This is rather convincing considering the available mechanical torque which is less than 1.2×10^{-3} N m, only three times more than the magnitude of the maximum gravity gradient torque for this satellite. A simulation for the same initial conditions but with an initial attitude gain $\hat{\eta} = 27 \times 10^5$ A m^2/T is depicted in Fig. 14. The acceleration imposed by the attitude controller is high enough to turn the boom upright, but the controller is not able to decelerate the motion when ${}^c k_{oz} > 0$, and the satellite turns upside-down again. The attitude controller is seen to be active only when ${}^c k_{oz} > 0$. The combined control action is seen to be globally asymptotically stable.

8. Conclusions

This paper has shown that it is feasible to obtain a three-axis stable platform in low Earth orbits using magnetorquers as the only actuators for the attitude control system.

More detailed, this work contributes to the development of proportional-derivative feedback control based only on magnetic torquing for low Earth orbit satellites. A stepwise refinement from locally to globally stabilizing controllers were proposed, and rigorous stability analysis was carried out. A velocity controller, using a cross product between the angular velocity and the local geomagnetic field, was shown to give four stable equilibria, one of which was the reference. Methods for perturbing the satellite motion from three undesired equilibria were then investigated. It was shown using results from the theory of periodic systems that a locally stable solution could be found. This controller had a proportional term with constant gain as part of the nonlinear algorithm. Full global asymptotic stability was required to cope with an extreme condition like a boom upside-down contingency. To handle this, a time-varying gain was introdued in the proportional term and switching used to avoid generation of de-stabilizing torques. This control algorithm was shown to make the satellite globally asymptotically stable to the reference. Achieving three-axis stabilization is quite remarkable result for a system that is only controllable in two axes in a linear time invariant setting.

Simulation results showed the proficiency of the new control algorithms for three different mission phases: the detumbling phase, the normal operation phase, and a contingency operation phase with inverted boom. These controllers are used in the Danish Ørsted satellite. This mission is believed to be the first to fly three-axis stabilized with magnetorquers as sole actuators for attitude control.

Acknowledgements

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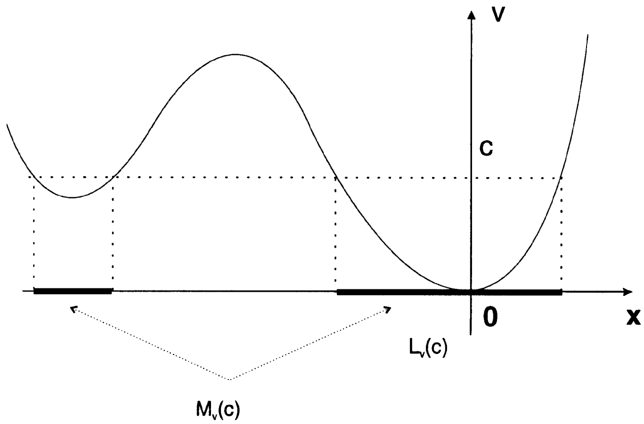

Fig. 15. An example of level set $L_v(c)$. The set $M_y(c)$ consists of two subsets, whereas the level set, $L_v(c)$ is the subset of $M_v(c)$, which is containing the equilibrium.

Appendix A. Periodic extension of Lyapunov stability

The aim of this appendix is to present an important lemma by Krasovskii–LaSalle lemma (Vidyasagar, 1993). The focus is on the stability of the class of periodic nonlinear systems

$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}(t)) \quad \text{where } \mathbf{x} \in \mathbb{R}^n \text{ and } t \in \mathbb{R}_+ \tag{A.1}$$

satisfying

$$\forall t \geq 0 \quad \mathbf{x}\mathbf{f}(t + T, \mathbf{x}) = \mathbf{f}(t, \mathbf{x}).$$

As an introduction to Krasovskii–LaSalle lemma definitions of two auxiliary sets is provided. Firstly, the definition of a level set is introduced.

To begin with consider a C^1 locally positive definite function $v : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}$ such that v is periodic with the same period T as the system. Now, a level set $L_v(c)$ is the connected subset of a set

$$M_v(c) = \{\mathbf{x} \in \mathbb{R}^n : \exists t \geq 0 \text{ such that } v(t, \mathbf{x}) \leq c\} \tag{A.2}$$

containing the equilibrium $\mathbf{0}$. An example of the level set, $L_v(c)$ is depicted in Fig. 15. The set $M_v(c)$ consists of two subsets. The level set, $L_v(c)$ is the subset of $M_v(c)$, which contains the equilibrium. The second auxiliary set is $A_v(c)$

$$A_v(c) = \{\mathbf{x} \in L_v(c) : \forall t \geq 0\, v(t, \mathbf{x}) \leq c\}. \tag{A.3}$$

Notice that the difference between $L_v(c)\,(M_v(c))$ and $A_v(c)$ is in the quantifiers. Now, the stability of a periodic nonlinear system is established in Krasovski–LaSalle lemma.

Lemma A.1 (Krasovskii–LaSalle). *Consider the periodic system (A.1). Suppose there exists a C^1 function $v : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}$ such that v is periodic with the same period as the system, v is locally positive definite. Moreover, there exists an open neighbourhood N of 0 such that*

$$\forall t \geq 0, \quad \forall \mathbf{x} \in N \qquad \dot{v}(t, \mathbf{x}) \leq 0. \tag{A.4}$$

Choose a constant $c > 0$ such that the level set $L_v(c)$ is bounded and contained in N. Finally let

$$S = \{\mathbf{x} \in L_v(c) : \exists t \geq 0 \text{ such that } \dot{v}(t, \mathbf{x}) = 0\}, \tag{A.5}$$

and let M denote the largest invariant set of the system in hand contained in S. Then

$$\mathbf{x}_0 \in A_v(c),\ t_0 \geq 0 \Rightarrow \lim_{t \to \infty} d(\mathbf{x}(t, t_0, \mathbf{x}_0), M) = 0, \tag{6}$$

where $\mathbf{x}(t, t_0, \mathbf{x}_0)$ is the solution of Eq. (A.1) which at initial time t_0 passes through the initial point $\mathbf{x}_0$. $d(\mathbf{y}, M)$ denotes the distance from the point $\mathbf{y}$ to the set M.

Appendix B

Proof of Theorem 4. For simplicity of notation the equation of satellite Eqs. (1)–(7) with controller (8) are represented by

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), t), \tag{B.1}$$

and the equations of satellite motion with controller (26), for constant $\varepsilon(t) = \varepsilon(kT)$, are denoted as

$$\dot{\mathbf{x}}(t) = \mathbf{f}_k(\mathbf{x}(t), t). \tag{B.2}$$

Furthermore, differential equation (B.1) for the initial condition $\mathbf{x}(t_0) = \mathbf{x}_0$ has the solution $\mathbf{x}(t, t_0, \mathbf{x}_0)$, and differential equation (B.2) for the initial condition $\mathbf{x}(t_0) = \mathbf{x}_0$ has the solution $\mathbf{x}_k(t, t_0, \mathbf{x}_0)$.

The kinematic and dynamic differential equations are Lipschitz, and the following is true

if $\lim_{k \to \infty} \mathbf{f}_k(\mathbf{x}(t), t) = \mathbf{f}(\mathbf{x}(t), t)$ then

$$\lim_{k \to \infty} \mathbf{x}_k(t, t_0, \mathbf{x}_0) = \mathbf{x}(t, t_0, \mathbf{x}_0),$$

thus

if $\lim_{t \to \infty} \mathbf{x}(t, t_0, \mathbf{x}_0) = \mathbf{y}_f$ then

$$\lim_{t \to \infty} \mathbf{x}_k(t, t_0, \mathbf{x}_0) = \mathbf{y}_f.$$

This means, if $\lim_{t \to \infty} \varepsilon(t) = 0$, each trajectory of the satellite actuated by control law (26) converges to one of the asymptotically stable equilibria of system (B.1): $\{({}^{c}\mathbf{\Omega}_{co}, {}^{c}\mathbf{k}_o, {}^{c}\mathbf{j}_o) : (\mathbf{0}, \pm {}^{o}\mathbf{k}_o, \pm {}^{o}\mathbf{j}_o)\}$. $\square$

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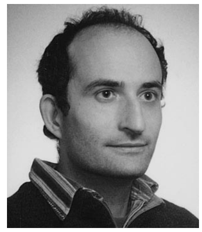

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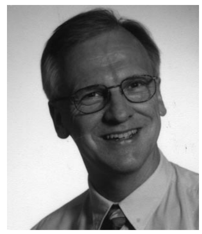

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