ATTITUDE RECONSTRUCTION FOR FREJA, A MAGNETOMETER BASED APPROACH

Thomas Bak

Department of Control Engineering, Aalborg University, Fredrik Bajers Vej 7C, DK-9220 Aalborg, Denmark, Tel: +45 9635 8761, tb@control.auc.dk

Abstract: An attitude estimation algorithm has been developed and tested based on magnetometer data from the Freja spacecraft. The attitude has previously been determined once or twice per orbit using single point algorithms. Thermally induced inertia variations, however, prevent simple interpolation. The approach presented here, model thermal effects and preserve angular momentum, variations in the moments of inertia. Perturbations to the field measurements are avoided in the filter update. The algorithm allow attitude determination during eclipse, where no reliable attitude was previously available, thereby increasing the value of the science observations.

Keywords: Attitude algorithms, Extended Kalman filters, Smoothing, Satellite, Calibration, Magnetic fields.

1. INTRODUCTION

The analysis of data from low-budget space experiments is generally constrained by limited instrumentation packages. On the Freja magnetospheric research satellite no accurate attitude information is presently available. Determining the attitude may, however, greatly enhanced by the study of the magnetic and electric fields measured by Freja. Freja was launched into a 1763 by 597 km orbit at an 63 degree inclination on October 6, 1992.

The attitude has previously been determined once or twice per orbit based on data from an Earth horizon sensor and a Sun sensor. The attitude algorithm fail during eclipse when the Sun reference is unavailable. The passage of the spacecraft in and out of eclipse further complicates the problem. Presumably the coil booms shrink when they cool down during orbit night, thereby decreasing the moments of inertia. This prevents simple propagation of the spin frequency through the day/night transition. Many of the interesting phenomenon take place on the night side of the orbit, and a solution is therefore required, that allow the complete attitude time history during science observation to be determined.

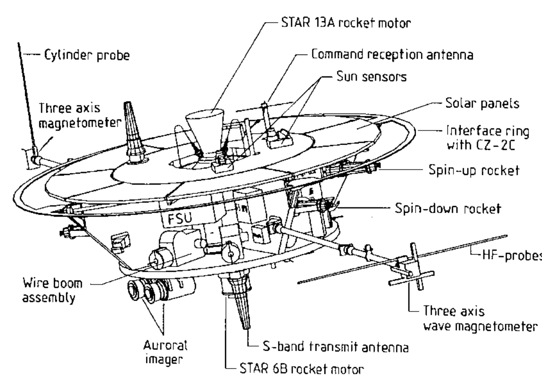

Fig. 1. The Freja satellite.

Freja is a magnetic mission and accurate magnetometer data is available during science observation periods which may provide us with information about the spacecraft attitude state. The accuracy of a magnetometer solution is, however, limited by field aligned currents which cause perturbations perpendicular to the main field as the spacecraft travels through the auroral oval.

For spacecraft attitude determination, the Kalman filter has proven useful for estimation using a combination of reference vectors and gyro measurements (Lefferts *et al.*, 1982). By maintaining an accurate model of the spacecraft dynamics the state and covariances may be propagated during periods with disturbed measurements without gyros. The Freja reconstruction may be carried out off-line in batches, and the attitude estimation can therefore be treated as a smoothing problem.

The present work makes two contributions. First, an estimator based on accurate dynamic models of the spacecraft is used to estimate the attitude attitude histories, during eclipse and during periods that lack accurate reference data. Second, it improves the estimator propagation model by estimating variations in the spin moment of inertia to account for thermally induced spin rate changes.

2. FREJA SMOOTHING PROBLEM

The problem here is to estimate the three-axis attitude time history $q(t)$ during science observation. The basic inputs are the magnetometer readings, the sun and spacecraft ephemeris. The attitude quaternion q parameterizes the transformation from inertial coordinates to spacecraft body coordinates.

The attitude reconstruction problem has been solved using an estimator. The estimator maintains an accurate dynamic model of the spacecraft which is used to propagate the attitude and angular velocity estimates through the data gap. The dynamic model is based on batch estimates of the on-orbit spacecraft inertias. The thermally induced changes in the moments of inertia is estimated as a constant in the filter, thereby conserving angular momentum. Updates of the model are performed whenever *reliable* magnetometer data are available. The data gaps are selected using an selection algorithm that may be based on empirical knowledge about the disturbances to the main field or an objective criteria as the satellite latitude.

The general outline of the solution is given in Figure 2.

The filter is based on the following assumed model. Let $x(t)$ be a n dimensional state vector at time t. The continuous dynamics are then represented by

$$\frac{d}{dt}x(t) = f(x(t), t, u(t)) + w(t) \qquad (1)$$

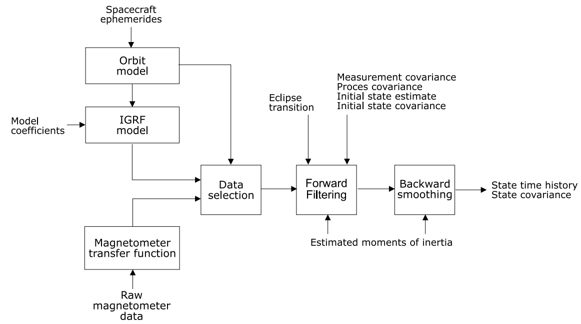

Fig. 2. Attitude determination solution.

where f is the dynamic model, and the process noise is a Gaussian white-noise process $w(t) \in \mathrm{NID}(0, Q(t))$.

The magnetometer measurements are discrete and given by

$$z_k = h_k(x_k, t_k) + v_k \qquad (2)$$

where $v_k \in \mathrm{NID}(0, R_k)$, the measurement noise, is a assumed to be a zero-mean discrete Gaussian white noise process.

Let an estimate of the initial state at t_0 be given and denote it by $\hat{x}_0$. The error in this initial estimate is assumed to be Gaussian with zero mean and covariance P_0. The filtered estimate at time t_j is designated by x_j^F. Given is a set of discrete measurements $\{z_k \mid k = 1, ...N\}$, we process the measurements sequentially forward in time and produce the sequence $\{\hat{x}_k^F, P_k^F, k = 1, \ldots, N\}$. The smoothing algorithm produces the sequence $\{\hat{x}_k^S, P_k^S, k = N, \ldots, 1\}$, with the initial smoothed estimate at N being provided by the filtered estimate at N.

In the Freja problem the dynamic equation Eq. (1) is non-linear and describe the rigid spinning spacecraft without momentum wheels. The measurement function Eq. (2) implicitly defines the measurements and varies from sample-to-sample depending on the orbit position.

The magnetic field observations are processed in a forward pass using an Extended Kalman Filter (EKF). A traditional discrete/continuous filter structure as found in (Jaswinski, 1970) was adopted. The measurement update is only performed when reliable magnetometer data are available. If not, the filter estimates are based on an accurate dynamics model for propagation.

The backward smoothing algorithm follows (Grewal and Andrews, 1993). The complete filter algorithm including smoothing is summarized in Figure 3.

2.1 *Filter State Vector*

The filter state vector is composed of the three vector elements of the error quaternion, three an-

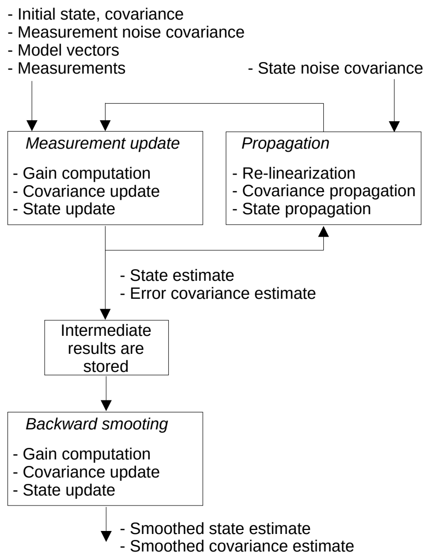

Fig. 3. Combined Smoothing and Filtering.

gular rate corrections, two spacecraft referenced torques, and a perturbation to the spin axis moment of inertia. The result is a nine dimensional state vector:

$$x = \left[q^T \;\; \omega_{bi}^T \;\; n_1,\, n_2,\, I_3\right]^T \tag{3}$$

where q $(= [q_{13}^T \;\; q_4]^T)$ is the inertial-to-body quaternion defined by

$$q_{13} \triangleq \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = n \sin(\rho/2) \tag{4}$$

$$q_4 \triangleq \cos(\rho/2) \tag{5}$$

where n is a unit vector corresponding to the axis of rotation and ρ is the angle of rotation.

The angular velocity ω_{bi} $(= [\omega_1, \omega_2, \omega_3]^T)$ are inertial body rates where the subscripts 1, 2, 3 denote the body axis vector components. n_1 and n_2 are spacecraft referenced random-walk torque. I_3 is a perturbation to the maximum moment of inertia. The torque components n_1 and n_2 allow the spin axis to precess in inertial space due to mismodeled torques. These components are defined to be perpendicular to the spin vector. I_3 is the maximum principal moment of inertia is used to model thermally induced inertia variations conserving angular momentum.

2.2 *Filter Propagation*

The dynamics are propagated using the well known Euler's equation (Wertz, 1978)

$$I \frac{d}{dt} \boldsymbol{\omega}_b = -\boldsymbol{\omega}_b \times (I \boldsymbol{\omega}_b) + N \tag{6}$$

where the term $N = [n_1 \;\; n_2 \;\; 0]^T$ is the estimated external torques. The z-direction moment of inertia I_{33} is based on the filter estimated deviations from the nominal. For the purpose of filter state estimation the external disturbance torques are approximated as random walk with

$$\dot{n}_1 = w_1 \qquad \dot{n}_2 = w_2, \tag{7}$$

and the change in the spin principal moment of inertia is modeled as a constant

$$\dot{I}_3 = 0 \tag{8}$$

The quaternion kinematic equations of motion are derived from the spacecraft's angular velocity and given by

$$\frac{d}{dt} q(t) = \frac{1}{2} S(\omega(t))\, q(t) \tag{9}$$

where $S(\boldsymbol{\omega})$ is defined as

$$S(\boldsymbol{\omega}) \triangleq \begin{bmatrix} -[\boldsymbol{\omega}\times] & \boldsymbol{\omega} \\ -\boldsymbol{\omega}^T & 0 \end{bmatrix}. \tag{10}$$

where the term $[\boldsymbol{\omega}\times]$ is a skew symmetric cross product equivalent matrix defined so that $[a\times]b = a \times b$.

2.3 *Measurement Formulation*

The direction of the magnetic field in the body frame b_b is related to the direction in the reference (inertial) frame b_i according to

$$b_b = TA(q_{bi})b_i \tag{11}$$

where T and $A \in SO(3)$ are a sensor alignment matrix and a attitude matrix respectively. b_i and $b_b \in R^3$. $A(q)$ is parameterized using the quaternion as

$$A(q) = (q_4^2 - q_{13}^T q_{13}) I_{3\times 3} + 2 q_{13} q_{13}^T - 2 q_4 [q_{13}\times] \tag{12}$$

2.4 *Linearized Equations*

The Kalman filter equations are based on a running linearization about the estimated trajectory. Given convergence of the filter we can assume that the angular distance to the estimated rotation is infinitesimal small which allows us to make some approximations. In that case we write the angle of rotation as $\Delta\theta$. Using that $\sin \Delta\theta \approx \Delta\theta$, and $\cos \Delta\theta \approx 1$, the infinitesimal quaternion δq becomes

$$\delta q = \begin{bmatrix} \Delta\boldsymbol{\theta}/2 \\ 1 \end{bmatrix} + O(|\Delta\boldsymbol{\theta}|^2) \tag{13}$$

where

$$\Delta\boldsymbol{\theta} \triangleq \Delta\theta n,$$

is the infinitesimal rotation vector, n is a unit vector corresponding to the axis of rotation, and $O(x)$ denotes a function which has the property that $O(x)/x$ is bounded as $x \to 0$. The components of $\Delta\boldsymbol{\theta}$ are termed the infinitesimal angles. Using the full quaternion directly in the filter results in covariance singularities (Lefferts *et al.*, 1982) and only the imaginary part of the error quaternion is therefore part of the filter error state vector which is hence reduced to an eight dimensional vector.

The attitude kinematics are linearized in terms of a perturbation quaternion times the nominal quaternion using quaternion multiplication.

Define the perturbation quaternion δq as

$$\delta q = q \otimes \hat{q}^* \tag{14}$$

where $\otimes$ is the quaternion product (Shuster, 1993), q is the true quaternion and $\hat{q}$ is the estimated quaternion. We obtain the following linearized kinematics expressed in terms of the error quaternion (Bak, 1998),

$$\frac{d}{dt}\delta q_{13} = -\hat{\boldsymbol{\omega}} \times \delta q_{13} + \frac{1}{2}\Delta\boldsymbol{\omega} \tag{15}$$

Expressing the deviation from the estimated angular velocity by $\Delta\omega = \omega - \hat{\omega}$ and using a Taylor series expansion of the dynamics described by Eq. (6) we get the linearized dynamics

$$\frac{d}{dt}\Delta\boldsymbol{\omega} \approx I^{-1}\left([\,I\,\hat{\boldsymbol{\omega}}\times] - [\,\hat{\boldsymbol{\omega}}\times]\,I\right)\Delta\boldsymbol{\omega} + I^{-1}N. \tag{16}$$

2.5 *State Update*

When updating attitude knowledge error with the state estimates special care has to be taken to obey quaternion algebra. The body rates disturbance torques and spin inertia are updated by simple addition while the quaternion is updated using quaternion multiplication

$$\hat{q}_{k^+} = \begin{bmatrix} q_{13} \\ 1 \end{bmatrix}_k \otimes \hat{q}_{k^-}\,, \tag{17}$$

where $\hat{q}_{k^+}$ is the updated inertial-to-body attitude quaternion just after incorporation of the measurements and $\hat{q}_{k^-}$ is the propagated quaternion.

3. BATCH INTERVAL SELECTION, TUNING

An example of data available for processing is shown in Figure 4. The data is preprocessed as part of a nonlinear magnetometer calibration based on mean elements.

In the orbit described above the spacecraft attitude is nicely aligned with the spin axis perpendicular to the field lines. The field vector rotates in the spacecraft xy plane.

The measurements in Figure 4 are clearly disturbed in the part of the orbit from approximately 15 to 34 minutes. The phenomena is caused by field aligned currents that cause a disturbance

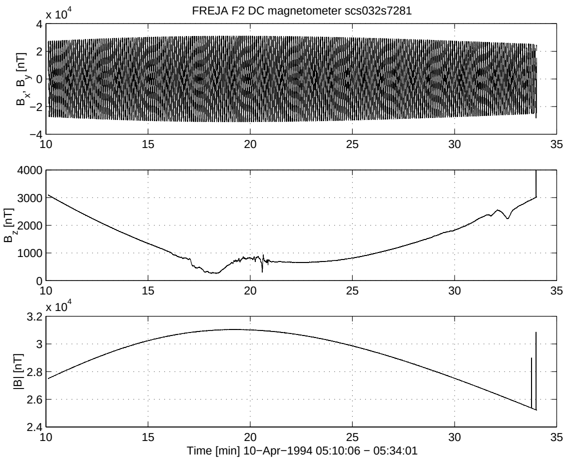

Fig. 4. Magnetic field measured by Freja science magnetometer.

perpendicular to the main field. Applying the filter directly the measured field with an error of 300 nT results in an error of approximately $\arctan(300/30000) = 0.6$ deg.

By visual inspection of the data, a data gap is selected leaving out data in the interval from approximately 16-33 minutes.

3.1 *Tuning and Initialization*

Selection of the initial covariance P_0, the measurement covariance R, process noise intensities Q, and the vector $\hat{x}_0$ tunes the filter. The initial covariance is selected as a unit diagonal matrix. The initial state estimates are derived from the coarse attitude data available in the present attitude history files. The measurement covariance is determined by the magnetometer noise whereas the process noise intensity has been hand-tuned to obtain filter/smoother measurement residuals.

The thermally induced variations in moments of inertia are know to be correlated with the day/night and night/day transitions. In order to allow the filter to track these inertia changes the process noise is adaptive and synchronized with the eclipse transitions.

4. RESULTS

The filter/smoother output includes several types of data that illustrate its performance. The measurement residual time histories indicate the filter/smoothers ability to fit the data. The attitude time histories illustrate the thermal motions and the general attitude state estimate.

Figure 5 shows the estimated and measured field measurements for the orbit 7281 which is entirely in eclipse and the attitude has therefore not previously been estimated.

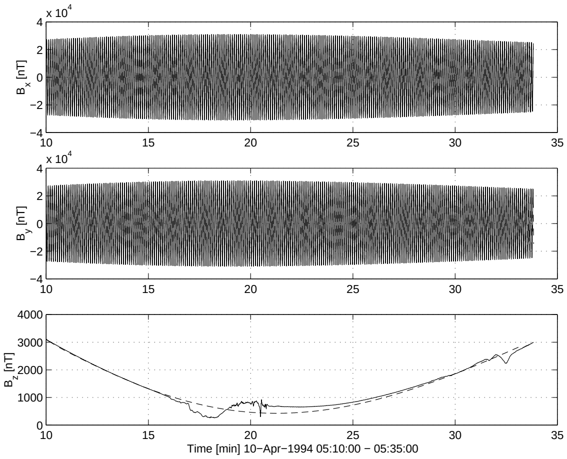

Fig. 5. Estimated and estimated magnetic field measurements.

4.1 *Measurement Residual Time Histories*

The measurement residual vector is $z_k - h(\hat{x}^s_{k+}, t_k)$. The result should be a noise sequences with a RMS values less than or equal to the diagonal elements of the measurement noise covariance, R. If the process noise intensity were very small, then the measurement residual sequences would be almost white noise with RMS values approaching the diagonal elements of R.

Figure 6 shows the estimated F2 measurement residual time history.

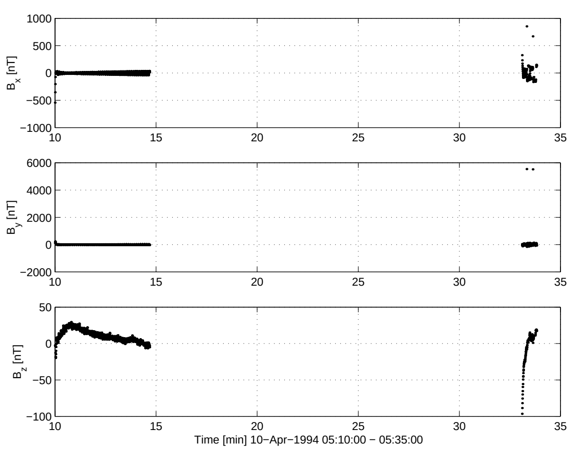

Fig. 6. Measurement residual time histories.

The residual time histories look as should be expected. The data gaps are clear. The convergence in the z-component of the measurement was expected due to the low dynamics. The RMS values are around 15 nT. Reasonable RMS noise levels indicates that the process noise is probably tuned correctly. Figure 7 presents the measurement residuals in terms of the angular separation between the estimated magnetic field and the measured field. The RMS value of angular error in the interval where the magnetic measurements are not disturbed is 0.02 deg, which is in good agreement with the 15 nT RMS error.

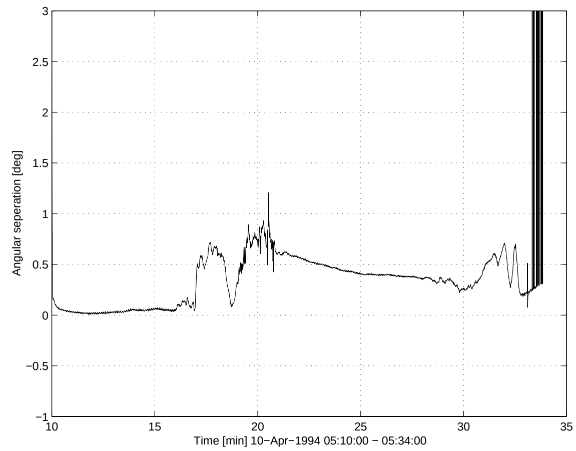

Fig. 7. Angular separation between measured and estimated magnetic fields.

While the measurements are not used in the filter during the data gap, the discrepancy between the estimated field (based on the smoothed attitude) and the actually measured field has scientific interest and is therefore shown in Figure 8.

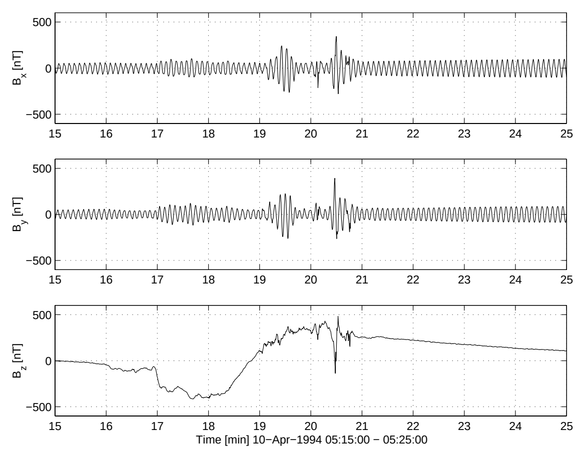

Fig. 8. Measurement residuals during data gap.

The discrepancy in B_x and B_y in Figure 8 has two different causes: 1) in the data gap the integration of small spin-rate uncertainties over a large portions of the orbit, leads to significant uncertainty in the spin angle, 2) in the update interval a close inspection of the phase of the error shows that it is 90 degrees out of phase with the field and it may, therefore, be attributed to field magnitude errors. The first problem could be addressed by including another measurement that would allow us to fix the spin rate. A possible solution to the latter problem would be to estimate an magnitude correction to the IGRF model. Both modifications should reduce the periodic error and might yield accuracy gains.

4.2 *Attitude Time Histories*

Plots of the attitude quaternion estimates are not very useful due to the rapid motion of the satellite.

More useful are plots of the spin vector direction versus time as seen in Figure 9.

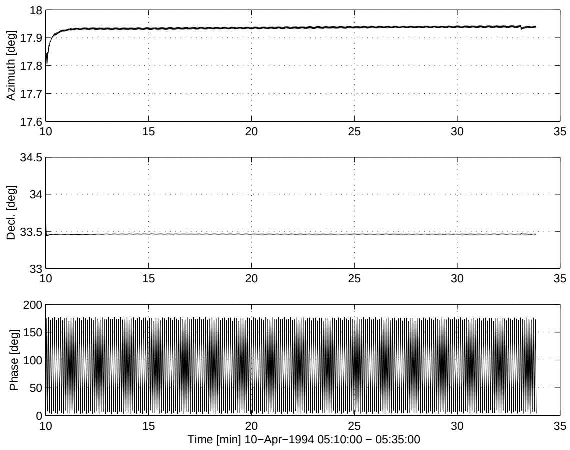

Fig. 9. Spin vector azimuth, declination and phase estimates versus time.

At the present time no reference data is available that would allow an attitude accuracy check of the data.

The covariance however, provide an estimate of the accuracy of the algorithm. The covariance results are shown in Figure 10.

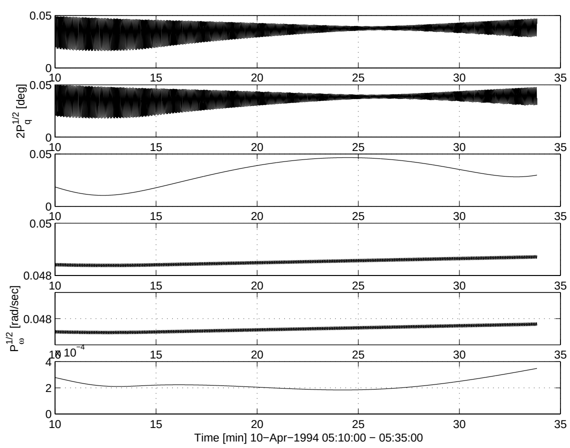

Fig. 10. Estimated square of the covariances for attitude and rate.

The covariances clearly show the smoothing of the results. The uncertainty increases as we moves into the data gap. The figure indicates sigma values of 0.05 deg.

5. CONCLUSION

The full use of Freja data require spacecraft attitude state during the complete science observation phases. A filter/smoother was developed and tested on Freja data. The filter avoided areas with perturbations to the main geomagnetic field that would otherwise influence the attitude solution. Using a smoother allowed data before and after the data gap to influence the whole interval resulting in reduced attitude uncertainty during the gap.

The propagation was based on an accurate dynamics model of the spacecraft. Good in-flight parameter identification of inertia elements, and sensor calibration helped improve filter performance significantly.

In summary, the Freja attitude reconstruction algorithm has solved a the initial science problem, getting attitude during eclipse. The filter is based on magnetometer data only combined with an accurate dynamics model. The estimated error covariance indicate a 1σ accuracy of 0.05 deg over long periods that lack direct sensor data. The measurement residual plots, however, indicate an error of approximately 0.09 deg, and to obtain an absolute accuracy we do require a reference attitude.

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