Nonlinear Attitude Control for a Rigid Spacecraft by Feedback Linearization

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Abstract

Attitude control law design for spacecraft large angle maneuvers is investigated in this paper. The feedback linearization technique is applied to the design of a nonlinear tracking control law. The output function to be tracked is the quaternion attitude parameter. The designed control law turns out to be a combination of attitude and attitude rate tracking command. The attitude-only output function, therefore, leads to a stable closed-loop system following the given reference trajectory. The principal advantage of the proposed method is that it is relatively easy to produce reference trajectories and associated controller. The controller is also simplified with stable tracking performance.

Key Words : Spacecraft Attitude Control, Feedback Linearization, Quaternion Parameter, Output Tracking Control, Singularity Avoidance Strategy, Sliding Mode Control

1. Introduction

Significant research effort has been devoted to attitude controllers design for spacecraft large angle maneuvers(Wie, 1985,1989). Both closed-loop feedback and optimal open-loop designs are under extensive study. Recent trend is toward various robust feedback control approaches accounting for external disturbances, model uncertainty, and actuator saturation limits. Due to the nonlinear nature of the problem, control laws are dominated by nonlinear designs(Vadali,1986). The fixed gain feedback controller using body angular rate and quaternion error is regarded as one of the most distinct features of the controllers(Wie, 1989). Also variable structure control or sliding mode control has been investigated by Vadali(1986) resulting in robust maneuver performance. Nonlinear predictive control is based upon a pre-determined reference trajectory which is followed by the actual system under control(Crassidis,1986).

One potential nonlinear control design approach which has attracted significant attention recently is the feedback linearization(Lin, 1994 & Schumacher, 1998). The feedback linearization is generally able to handle nonlinearity more efficiently than conventional linearization techniques. A reference output function is defined *a priori*, and a controller is designed so that the actual output tracks the reference output. Therefore, the resultant control command consists of the reference and actual states of the system. The controller basically linearizes the system in the sense that stable closed-loop dynamics are achieved for the output function of the original nonlinear systems.

The principal objective of this study is to apply the feedback linearization technique to the large angle attitude maneuver of a rigid spacecraft model. The output function is a quaternion vector set which comprises three independent elements. The output function is then used to derive a feedback control law based upon the reference trajectory. The reference trajectory is generated in such a manner to satisfy terminal constraints at the initial time and final steady state. The resultant controller leads to smoothed output tracking performance. The key advantage derived from the proposed methodology is that the reference trajectory can be easily constructed using three quaternion elements only. In other words, the kinematics between angular velocity and quaternion do not have to be taken into account in the reference trajectory generation. The corresponding angular velocity profile is implicitly determined from the reference output history. Asymptotic output tracking and sliding mode controls are investigated to compare the performance.

2. Basic Dynamics and Control Theory

2.1 Attitude dynamics and kinematics

First, review on attitude dynamics and kinematics for a generic rigid spacecraft model is presented. The spacecraft model is shown in Fig. 1. For simplicity of analysis, the spacecraft is assumed to be a perfectly rigid body. The basic three-axis attitude dynamics of a rigid spacecraft are written in the form(Wie, 1995)

$$\mathbf{J}\dot{\omega} + \omega \times \mathbf{J}\omega = \mathbf{u} \tag{1}$$

where $\mathbf{J}$ is spacecraft inertia matrix consisting of the principal moment of inertia, ω is the body angular velocity vector, and $\mathbf{u}$ is the control input vector.

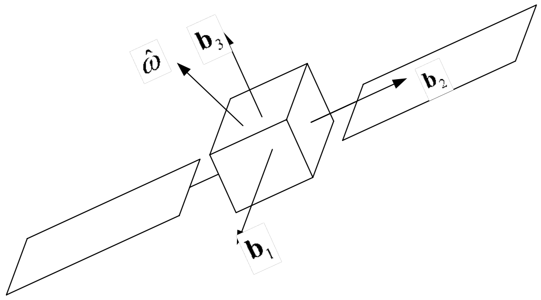


Fig. 1 Rigid spacecraft model

For attitude representation, the quaternion parameter is introduced as

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}_{13} \\ q_4 \end{bmatrix} \tag{2}$$

where each element satisfies(Wie,1989)

$$q_{13} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \mathbf{l}\sin\frac{\phi}{2}, \qquad q_4 = \cos\frac{\phi}{2} \tag{3}$$

and $\mathbf{l}$ is the Euler principal axis vector, and ϕ represents corresponding principal angle. The time derivative of the quaternion satisfies

$$\dot{\mathbf{q}} = \frac{1}{2}\Omega(\omega)\mathbf{q} = \frac{1}{2}\Xi(\mathbf{q})\omega \tag{4}$$

for which

$$\Omega(\omega) = \begin{bmatrix} -[\tilde{\omega}] & \omega \\ -\omega^T & \omega \end{bmatrix}, \quad \Xi(q) = \begin{bmatrix} q_4 I_{3\times 3} + [\tilde{\mathbf{q}}_{13}] \\ \text{-q}_{13} \end{bmatrix} \tag{5}$$

The notation $[\tilde{\omega}]$ represents

$$[\tilde{\omega}] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{6}$$

and similar definition is applied to $[\tilde{\mathbf{q}}_{13}]$. The quaternion is also subject to a constraint equation from Eq. (3) as

$$\mathbf{q}^T\mathbf{q} = \mathbf{q}_{13}^T\mathbf{q}_{13} + q_4^2 = 1 \tag{7}$$

Hence only three elements are independent, and other remaining element is determined automatically to satisfy the constraint. From a controller design view point, only three elements are needed for independent control action about three body axes of the spacecraft.

2.2 Feedback linearization

Principal idea of the feedback linerization is briefly presented herein. Majority portion of the material in this section is taken from Lin(1994). The feedback linearization technique is based upon linearized relationship between input and output. For a multi-input and multi-output nonlinear system of the form(Lin,1994)

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} \tag{8}$$

the output function is defined by

$$\mathbf{y}(t) = \mathbf{h}(\mathbf{x}(t)) \tag{9}$$

where $\mathbf{f}(\mathbf{x})$ and $\mathbf{g}(\mathbf{x})$ are functions of vector fields in R^n, and the control and output vector $\mathbf{u} = [u_1, \ldots, u_m]^T$, $\mathbf{y} = [y_1, \ldots, y_m]^T$ $\mathbf{h} = [h_1, \ldots, h_m]^T$ are corresponding multiple input and output functions.

The feedback linearization is to find integers $\rho_1, \rho_2, \ldots \rho_m$ and a feedback control of the form

$$\mathbf{u} = \alpha(\mathbf{x}) + \beta(\mathbf{x})\mathbf{v} \tag{10}$$

where α and β are smooth vector functions defined in a neighborhood of some point $\mathbf{x}_0 \in R^n$ and $\det\beta(\mathbf{x}_0) \neq 0$ such that the closed-loop system by the control input

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\alpha(\mathbf{x}) + \mathbf{g}(\mathbf{x})\beta(\mathbf{x})\mathbf{v} \\ \mathbf{y} &= \mathbf{h}(\mathbf{x}) \end{aligned} \tag{11}$$

has the property that the ρ th-order derivative of the output satisfies

$$\mathbf{Y}^{(\rho)}(t) = \begin{bmatrix} y_1^{(\rho_1)} \\ y_2^{(\rho_2)} \\ \vdots \\ y_1^{(\rho_1)} \end{bmatrix} = \begin{bmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_m(t) \end{bmatrix} = \mathbf{v}(t) \tag{12}$$

for which $\mathbf{v}(t)$ is an arbitrary function to be decided by a control objective. For each output((y_i) the *relative degree* ρ_i can be introduced as follows

$$\begin{aligned} y_i^{(1)} &= L_f h_i(x) \\ y_i^{(2)} &= L_f^{\rho_i - 1} h_i(x) \\ &\vdots \\ y_i^{(\rho_i)} &= L_f^{\rho_i} h_i(x) + \sum_{j=1}^{m} L_{g_j} L_f^{\rho_i - 1} h_i(x) u_j \end{aligned} \tag{13}$$

where $L_f^k h(x)$ is called the *Lie derivative* of $L_f^{k-1} h(x)$ along the vector field f, and it is assumed that for at least one $j, 1 \le j \le m, L_{g_j} L_f^{\rho_i - 1} h_i(x) \neq 0$ holds. Now by introducing the notations

$$\mathbf{s} = [s_1, \ldots, s_m]^T$$

and

$$F(x) = \begin{bmatrix} L_f^{\rho_1} h_1(x) \\ L_f^{\rho_2} h_2(x) \\ \vdots \\ L_f^{\rho_m} h_m(x) \end{bmatrix}, \qquad Y^{\rho} = \begin{bmatrix} y_1^{(\rho_1)} \\ y_2^{(\rho_3)} \\ \vdots \\ y_m^{(\rho_m)} \end{bmatrix} \tag{14}$$

the following relationship in vector notations can be established

$$\mathbf{Y}^{(\rho)} = \mathbf{F}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\mathbf{u} \tag{15}$$

where

$$\mathbf{G}(\mathbf{x}) = \begin{bmatrix} L_{g_1} L_f^{\rho_1 - 1} h_1(x) & \cdots & L_{g_m} L_f^{\rho_1 - 1} h_1(x) \\ L_{g_1} L_f^{\rho_2 - 1} h_2(x) & \cdots & L_{g_m} L_f^{\rho_2 - 1} h_2(x) \\ \cdots & \cdots & \cdots \\ L_{g_1} L_f^{\rho_m - 1} h_m(x) & \cdots & L_{g_m} L_f^{\rho_m - 1} h_m(x) \end{bmatrix}$$

If $\mathbf{G}(\mathbf{x})$ is invertible for a give point $\mathbf{x}_0$, then the control input can be written as

$$\mathbf{u}(t) = \mathbf{G}^{-1}(\mathbf{x})(-\mathbf{F}(\mathbf{x}) + \mathbf{v}) \tag{16}$$

so that

$$\mathbf{Y}^{(\rho)}(t) = \mathbf{v}(t), \qquad t \in \Gamma \tag{17}$$

In the above derivation, $\rho_1, \rho_2, \cdots, \rho_m$ are *relative degrees* for the multi-input and multi-output systems. Now for the feedback control law, asymptotic tracking of the output signal is realized if the error signal is defined as

$$e_i(t) = y_i(t) - r_i(t), \quad i = 1, 2, \ldots m \tag{18}$$

where r_i is the *i*th reference trajectory for the corresponding output function. The control law is designed by selecting $\mathbf{v} = v_i (i = 1, 2, \ldots, m)$ as follows

$$v_i(t) = r_i^{(\rho i)} - c_{i(\rho_i - 1)} e_i^{(\rho_i - 1)} - \cdots - c_{i1} e_i^{(1)} - c_{i0} e_i \tag{19}$$

so that the closed-loop output dynamics result in

$$e_i^{(\rho_i)} + c_{i(\rho_i - 1)} e_i^{(\rho_i - 1)} + \cdots + c_{i1} e_i^{(1)} + c_{i0} e_i = 0 \tag{20}$$

where $i = 1, 2, \ldots, m$ and c_{ij} are constants for a stable closed-loop system. Obviously, the asymptotic tracking control law is more practical than the exact tracking law handling initial error between the reference and actual system in general cases.

In the similar context, the controller can be extended into a sliding mode control. For a system with vector *relative degrees* $\rho_1, \rho_2, \ldots, \rho_m$, the sliding surfaces are defined

$$s_i(t) = e_i^{(\rho_i - 1)} + c_{i(\rho_i - 1)} e_i^{(\rho_i - 2)} + \cdots + c_{i1} e_i + c_{i0} \int e_i dt = 0 \tag{21}$$

where $i = 1, 2, \ldots, m$. The stable sliding condition is dictated as follows

$$\frac{1}{2} \frac{ds_i^2}{dt^2} \le -\eta_i |s_i|, \qquad \eta_i > 0 \ \ (\text{i}=1,2,\ldots,\text{m}) \tag{22}$$

After some algebra, the final form of the sliding mode controller which enforces the sliding condition in Eq. (22) is presented in the form(Lin,1994)

$$\mathbf{u} = \mathbf{G}^{-1}(-\dot{\mathbf{s}} + Y^{(\rho)} - \mathbf{F} - \mathbf{k}\,\mathrm{sgn}(\mathbf{s})) \tag{23}$$

where $\mathbf{s} = [s_1, \ldots, s_m]^T$ is a vector of the sliding surfaces, $\mathbf{k} = [k_1, \ldots, k_m]^T$ is a gain matrix to be decided, and $\mathrm{sgn}(\mathbf{s}) = [\mathrm{sgn}(s_1), \ldots, \mathrm{sgn}(s_m)]^T$ represents the vector signum function.

3. Output Tracking Controller using Quaternion Only

3.1 Output control function

As a choice for the output function in feedback linearziation, the attitude quaternion introduced in Eq. (3) is selected as($\mathbf{y} = [y_1, y_2, y_3]^T \in R^3$)

$$\mathbf{y} = \mathbf{q}_{13} \tag{24}$$

Note that another quaternion element q_4 is automatically determined from the constraint equation(Eq.(7)). Also, the angular velocity is implicitly prescribed from the kinematics in Eq. (4). Once the output function is defined, time derivative of the output is taken as

$$\dot{\mathbf{y}} = \dot{\mathbf{q}}_{13} = -\frac{1}{2} \omega \times \mathbf{q}_{13} + \frac{1}{2} q_4 \omega \tag{25}$$

Furthermore, by taking the derivative one more time, we arrive at

$$\begin{aligned} \ddot{\mathbf{y}} &= \frac{1}{4} \omega \times (\omega \times \mathbf{q}_{13}) - \frac{1}{4} \omega^T \mathbf{q}_{13} \omega + \frac{1}{2} ([\tilde{q}_{13}] + q_4 I_{3\times3})(J^{-1}\omega) \times (J\omega) \\ &\quad + \frac{1}{2} ([\tilde{q}_{13}] + q_4 I_{3\times3}) J^{-1} \mathbf{u} \\ &\equiv \alpha(\omega, \mathbf{q}) + \beta(\omega, \mathbf{q}) \mathbf{u} \end{aligned} \tag{26}$$

It turns out that the second derivative of the output shows explicit dependency on the control input. Therefore relative degrees are all equal to two, i.e., $\rho_i = 2 (i = 1, 2, 3)$, and input-output linearization is possible at this stage.

Meanwhile, the reference trajectory of the attitude quaternion should be constructed. For this purpose, it is assumed that initial($\mathbf{q}_{13}^0$) and steady state quaternion

($\mathbf{q}_{13}^f$) are prescribed as

$$q_{13} = q_{13}^0 \quad at \quad t = 0, \qquad q_{13}(t_f) = q_{13}^f \quad at \quad t = \infty$$

A candidate reference trajectory suitable for the above condition is generated as

$$\mathbf{r}(t) = \mathbf{q}_{13}^0 + (\mathbf{q}_{13}^f - \mathbf{q}_{13}^0)(1 - e^{-t/\tau}) \tag{27}$$

where $\mathbf{r} = [r_1, r_2, r_3]^T \in R^3$ and $\tau(>0)$ is a time constant to be selected. The reference trajectory satisfies boundary conditions, and it can be seen that high order derivatives of the reference trajectory exist. The time derivatives of the reference signal are

$$\dot{\mathbf{r}}(t) = (\mathbf{q}_{13}^f - \mathbf{q}_{13}^0)(1/\tau)e^{-t/\tau} \tag{28}$$

and

$$\ddot{\mathbf{r}}(t) = (\mathbf{q}_{13}^f - \mathbf{q}_{13}^0)(1/\tau^2)e^{-t/\tau} \tag{29}$$

The reference trajectory is ready for the tracking control law design.

3.2 Controller design

The time derivatives of the output and reference trajectories derived in the previous part can be combined together to build an asymptotic tracking control law. From the generalized form of the tracking control law in Eqs. (16) and (19), a tracking controller is proposed as

$$\mathbf{u} = \beta^{-1}[\ddot{\mathbf{r}}(t) - \alpha(t) - \mathbf{c}_1(\dot{\mathbf{y}}(t) - \dot{\mathbf{r}}(t)) - \mathbf{c}_0(\mathbf{y}(t) - \mathbf{r}(t))] \tag{30}$$

where α, β are parameters in Eq. (26), $\mathbf{c_1}, \mathbf{c_0}$ are constant diagonal matrices, and β^{-1} is assumed to exist. It can be easily shown that

$$\beta^{-1} = \frac{1}{q_4}\begin{bmatrix} q_4^2 + q_1^2 & q_3q_4 + q_2q_1 & q_3q_1 - q_2q_4 \\ -q_3q_4 + q_2q_1 & q_4^2 + q_2^2 & q_4q_1 + q_3q_2 \\ q_3q_1 + q_2q_4 & -q_4q_1 + q_3q_2 & q_4^2 + q_3^2 \end{bmatrix} \tag{31}$$

Note that the parameter β, from the definition in Eq. (26), is subject to singularity on the condition that

$$q_4 = \cos\frac{\phi}{2} = 0 \tag{32}$$

where q_4 turns out to be the determinant of the matrix $q_4 I_{3\times3} + [\tilde{q}_{13}]$. Namely, the principal angle rotation more than 180 degrees is subject to singularity problem. A singularity avoidance strategy needs to be implemented in such a case. For instance, a series of small maneuvers may be employed in lieu of a single large-angle maneuver.

If the closed-loop system reaches steady state in such a way that

$$\lim_{t\to\infty}\mathbf{y}(t) = \mathbf{r}(t), \qquad \lim_{t\to\infty}\dot{\mathbf{y}}(t) = \dot{\mathbf{r}}(t) \tag{33}$$

then the relationship in Eq. (26) still leads to a meaningful solution with

$$\alpha(\omega, \mathbf{q}) + \beta(\omega, \mathbf{q})\mathbf{u} = \ddot{\mathbf{r}} \tag{34}$$

In general cases, once singularity condition takes place, which causes numerical difficulty in evaluating β^{-1}, then the state q_4 can be replaced with.

$$q_4^{'} = q_4 + \delta \tag{35}$$

where the parameter δ is a small number employed to prevent numerical singularity problem.

For further analysis of the control, the following Lyapunov function is introduced.

$$2U = (\mathbf{q}_{13} - \mathbf{r})^T\mathbf{Q}(\mathbf{q}_{13} - \mathbf{r}) + (\dot{\mathbf{q}}_{13} - \dot{\mathbf{r}})^T\mathbf{R}(\dot{\mathbf{q}}_{13} - \dot{\mathbf{r}}) \tag{36}$$

where $\mathbf{Q}, \mathbf{R}$ are positive definite weighting matrices. Time derivative of the Lyapunov function becomes

$$\dot{U} = (\mathbf{q}_{13} - \mathbf{r})^T\mathbf{Q}(\dot{\mathbf{q}}_{13} - \dot{\mathbf{r}}) + (\dot{\mathbf{q}}_{13} - \dot{\mathbf{r}})^T\mathbf{R}(\ddot{\mathbf{q}}_{13} - \ddot{\mathbf{r}}) \tag{37}$$

Re-arranging some terms and using the results in Eq. (26), it follows as

$$\begin{aligned}\dot{U} &= (\dot{\mathbf{q}}_{13} - \dot{\mathbf{r}})^T[\mathbf{Q}(\mathbf{q}_{13} - \mathbf{r}) + \mathbf{R}(\ddot{\mathbf{q}}_{13} - \ddot{\mathbf{r}})] \\ &= (\dot{\mathbf{q}}_{13} - \dot{\mathbf{r}})^T[\mathbf{Q}(\mathbf{q}_{13} - \mathbf{r}) + \mathbf{R}(\alpha(\omega, \mathbf{q}) + \beta(\omega, \mathbf{q})\mathbf{u} - \ddot{\mathbf{r}})]\end{aligned} \tag{38}$$

For stability in the Lyapunov sense, the following equation should hold

$$[\mathbf{Q}(\mathbf{q}_{13} - \mathbf{r}) + \mathbf{R}(\alpha(\omega, \mathbf{q}) + \beta(\omega, \mathbf{q})\mathbf{u} - \ddot{\mathbf{r}})] = -\mathrm{W}(\dot{\mathbf{q}}_{13} - \dot{\mathbf{r}}) \tag{39}$$

where W is a positive definite weighting matrix. Hence the stabilizing control law is derived as

$$\mathbf{u} = \beta^{-1}(\omega, \mathbf{q})[-\alpha(\omega, \mathbf{q}) + \ddot{\mathbf{r}} - \mathbf{G_1}(\dot{\mathbf{q}}_{13} - \dot{\mathbf{r}}) - \mathbf{G}_2(\mathbf{q}_{13} \text{-} \mathbf{r})] \tag{40}$$

where the gain matrices satisfy

$$\mathbf{G_1} = \mathbf{R^{-1}W}, \qquad \mathbf{G_2} = \mathbf{R^{-1}Q} \tag{41}$$

Hence, the control law in Eq.(30) is basically equivalent to that of Eq.(40). The control law by feedback linearization is therefore a stable tracking control law as proved by the Lyapunov function.

Application of the above tracking controller produces a stable output response governed by

$$\ddot{e}_i(t) + c_{i1}\dot{e}_i(t) + c_{i0}e_i(t) = 0 \tag{42}$$

where $e_i(t) = y_i(t) - r_i(t)(i = 1, 2, 3)$ represents error between the actual and reference outputs and $c_{ij} = diag(c_i)$. The design parameters $c_{ij}(i, j = 0,1)$ should be selected so that it produces a stable closed-loop system.

The feedback linearization tracking control law is based upon the condition that exact modeling data must be identified. In particular, the parameter $\alpha(\omega, \mathbf{q})$ is a function of the system inertia matrix J. If there is a modeling error in the system inertia matrix such as $J^{'} = (1 + \kappa)J$ where κ is a parameter representing the model uncertainty, then the performance of the control law shall be degraded. The time derivative of the Lyapunov function with the model error becomes

$$\dot{U} = -(\dot{\mathbf{q}}_{13} - \dot{\mathbf{r}})^T\mathbf{W}(\dot{\mathbf{q}}_{13} - \dot{\mathbf{r}}) - (\dot{\mathbf{q}}_{13} - \dot{\mathbf{r}})^T\mathbf{R}\tilde{\alpha}(\omega, \mathbf{q}) \tag{43}$$

where the parameter $\tilde{\alpha}(\omega, \mathbf{q})$ corresponds to a model error factor(κ) such as

$$\tilde{\alpha}(\omega, \mathbf{q}) = -\frac{1}{4}\kappa(\omega^T\omega)\mathbf{q}_{13} \tag{44}$$

Hence, the global stability is guaranteed on the condition that $\dot{U} < 0$ in Eq. (43), and the model uncertainty factor(κ) influences the stability condition directly.

Furthermore, the sliding mode controller can be designed from a vector sliding surface equation of the form

$$\mathbf{s} = \dot{\mathbf{e}} + \mathbf{c}_1\mathbf{e} + \mathbf{c}_0\int \mathbf{e}dt = 0 \tag{45}$$

Note that $\mathbf{e} = [e_1, e_2, e_3]^T$ denotes error vector, $\mathbf{s} = [s_1, s_2, s_3]^T$, and the sliding mode controller is designed on the condition that $\dot{\mathbf{s}} = 0$ which consequently leads to

$$\mathbf{u} = \beta^{-1}[\ddot{\mathbf{r}} - \alpha - \mathbf{c}_1(\dot{\mathbf{y}} - \dot{\mathbf{r}}) - \mathbf{c_0}(\mathbf{y} - \mathbf{r}) - \mathbf{k}\,\mathrm{sgn}(\mathbf{s})] \tag{46}$$

where $\mathbf{k}$ is a 3 by 3 diagonal gain matrix, and *sgn* represents the signum function. The signum function is replaced by a saturation function to minimize potential chattering phenomenon. In other words,

$$\mathrm{sgn}(f) = \begin{cases} \mathrm{sgn}(f) & \text{for } |\mathrm{f}| > \varepsilon \\ f/\varepsilon & \text{for } |\mathrm{f}| < \varepsilon \end{cases} \tag{47}$$

4. Simulation and Analysis

Simulation by the proposed control approach has been conducted. The mass moment of inertia of the model spacecraft are assumed to be *J*=*diag*[300, 320, 250](kg-m^2). Initial quaternion elements for simulation purpose are taken as $\mathbf{q}_{13}^0 = [0.2, 0.4, 0.5]^T$, $q_4^0 = 0.742$ while the target quaternion components are given by $\mathbf{q}_{13}^f = [0.6, -0.2, -0.4]^T$, $q_4^f = 0.663$ respectively. The controller gains $\mathbf{c}_1, \mathbf{c}_0$, and $\mathbf{k}$ are set to be diagonal matrices, for which each matrix consist of identical numbers of 0.2, 0.1, and 0.05. The time constant (τ) selected for the reference output is 10 seconds. Both asymptotic tracking control and sliding mode control scenarios are simulated separately. The simulation results are plotted in Fig. 2. Body angular velocity and quaternion responses are plotted. As it can be shown the quaternion parameter follows the final target $\mathbf{q}_{13}^f$ asymptotically. The parameter q_4 is not plotted here for simplicity. As a whole, satisfactory tracking performance is achieved. The angular velocity responses are dictated by the quaternion history, so that the convergence to zeros for the angular velocity is evident from Eq. (4).

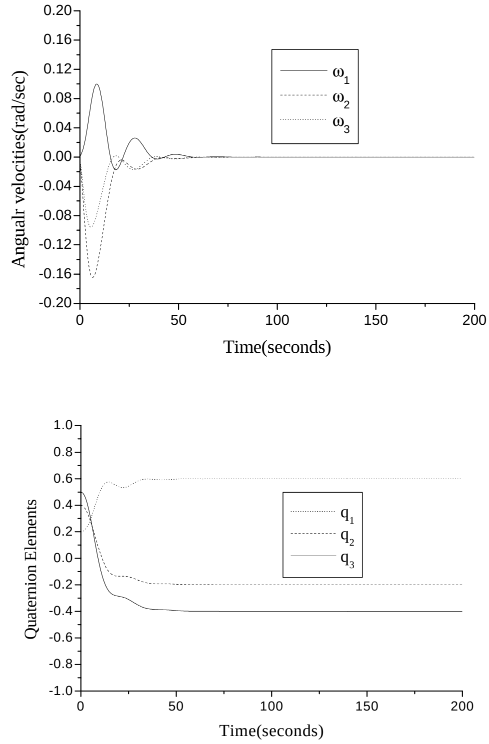


Fig. 2 Simulation results with the tracking control law

Next additional set of initial and final attitude are investigated with $\mathbf{q}_{13}^0 = [0.2, 0.4, 0.5]^T$, $q_4^0 = -0.742$ and $\mathbf{q}_{13}^f = [0.6, -0.2, -0.632]^T$, $q_4^f = 0.447$. In this case the state q_4 crosses the singular point. As discussed early, there may exist a singular condition. Time response of q_4 is displayed in Fig. 3. As one can see, q_4 reaches the singular point less than 10 seconds. The control gain parameter(β) has been modified by the singularity avoidance strategy as Eq. (35). From Fig. 3, it is hard to see visual difference between the two cases: one with original gain and the other with modified gain. Also, other quaternion parameters(q_1, q_2, q_3) are plotted in Fig. 4. They still show satisfactory tracking performance despite the singularity problem. It turns out that the singularity did not raise significant difficulty in the process of numerical simulation because a finite time step was taken for numerical integration. The exact singularity condition was not easy to realize since it is also sensitive to the closed-loop dynamics.

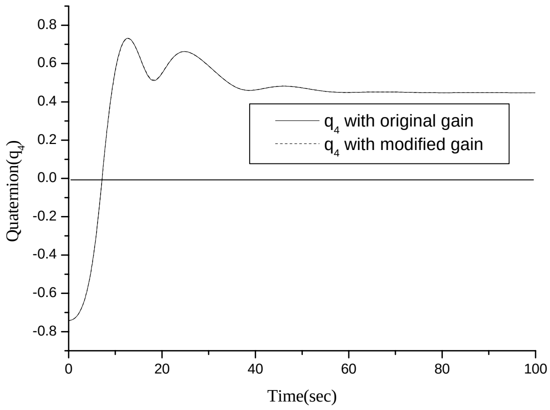


Fig. 3 Time response of the parameter q_4

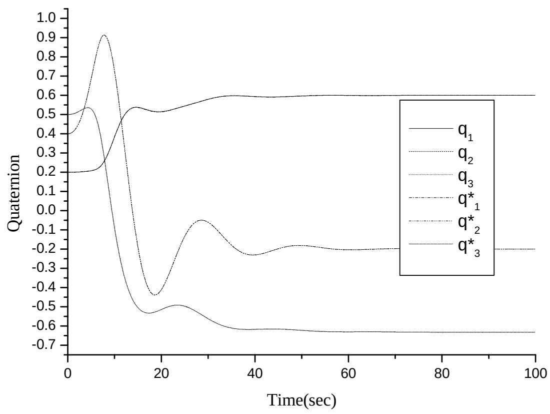


Fig. 4 Time responses of the parameters q_1, q_2, q_3

The control command trends are provided in Fig. 5. Some sharp rise in the control command, in particular, around the singularity point is observed. The near-singular condition causes the control input to change with large values in derivative.

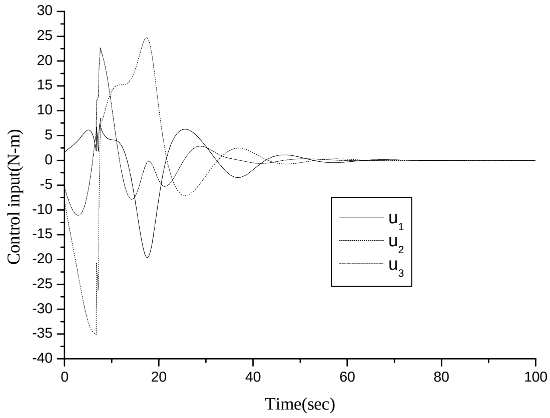


Fig. 5 Time responses of the control command

In order to verify the effect of the singularity avoidance technique, the time derivate of the control command is plotted in Fig. 6. There is a sharp peak around the singular point. As it can be shown the maximum peak in the time rate of change of the control command is significantly reduced by the singularity avoidance method. Such a sharp change in the control command may cause a saturation problem in the actual hardware systems. Fig. 6 therefore verifies the useful advantage of the avoidance scheme.

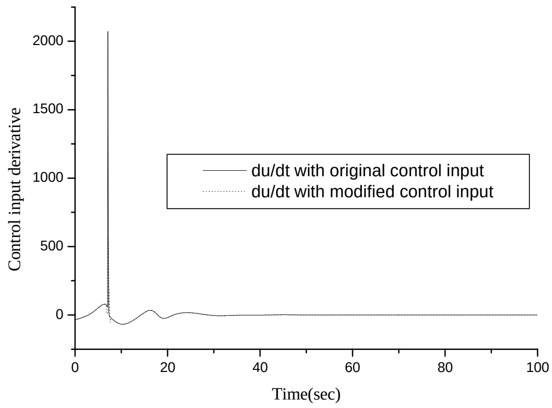
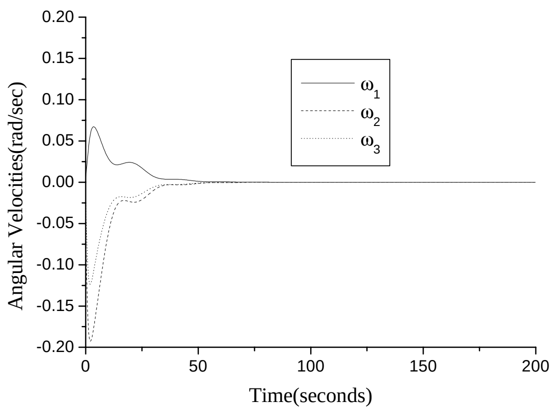


Fig. 6 Time rate of change of the control command

Next simulation with the sliding mode control law in Eq. (46) is conducted with the results presented in Fig. 7. Identical initial condition and control parameters are adopted as those in Fig. 2. It is not easy to see the difference between the results in Fig. 1 and Fig. 7. The sliding mode control needs to be analyzed from other aspects such as disturbance rejection and robustness with respect to modeling error, which is not pursued in this study.



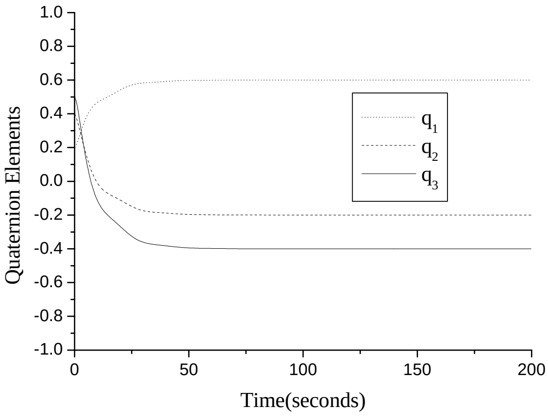


Fig. 3 Simulation results with the sliding mode control

5. Conclusions

It was shown that the feedback linearization technique could be applied to large angle attitude maneuver of a rigid spacecraft model. Three quaternion attitude parameters are used to generate a set of reference motion. The simulation results demonstrate the useful merit of the proposed control law. Reference angular velocities are implicitly determined from the reference attitude. Stable output tracking performance was successfully achieved. The simple reference trajectory synthesis methodology proposed in this study could be used to construct corresponding output control command easily. Robustness issues for the proposed controller still leave room for further investigation.

Acknowledgment

The present work was supported by National Research Lab.(NRL) Program(M1-0203-00-0006) by the Ministry of Science and Technology, Korea. Authors fully appreciate the financial support.

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