Research on Attitude System of Active Magnetic Control Small Satellite

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Abstract. When enter orbit, small satellite often tumble as a result of disturbance. How to capture it promptly with finite magnetic torque is an important problem. Because of the coupling of dynamics and control, the small satellite control system is a nonlinear attitude control system with bounds. For high direction and steady precision, an effective method must be found. In this paper, combining with the bound conditions of magnetic torque, two methods are researched. The first is energy method. It use Lyapunov energy depletion theory, taking the Lyapunov function as a target function, determine the magnetic torque. Secondly the decoupling variable structure control theory is used to deal the magnetic torque. The results of computer simulation indicate that these methods have advantages of excellent robustness and easily designed and realized.

Introduce

Attitude control system is the pivotal subsystem of small satellite. At the beginning of satellite going into orbit, attitude is complicated and satellite usually tumble because of the disturbance of disassociation of satellite and rocket. So attitude control system must be able to use up the energy of satellite body immediately in order to stabilize the satellite in the state of proper earth-oriented equilibrium. Moreover, when satellite is disturbed by unlooked-for disturbance, it will also deviate the equilibrium position badly. So attitude control system ought to be able to capture gravity field again immediately. Making full use of the natural resources is a primary characteristic of small satellite, and gravity gradient with active magnetic control can make full use of the characteristic. In terms of it, there is a successful example overseas and many specialists studying on it in china. Active magnetic control system is an attitude control system with constraints. It is an important research task of the system to find an efficient control method.

Dynamics Model of Rigid Body Satellite

Small satellite viewed as rigid body, dynamics equation can be expressed as

$$\boldsymbol{I}\dot{\boldsymbol{w}} \quad {}^{\times}\boldsymbol{I} \quad \boldsymbol{M}_g \quad \boldsymbol{M}_c \quad \boldsymbol{M}_d \qquad (1)$$

where, $\boldsymbol{I}$ is inertia matrix of satellite which can be expressed as $\boldsymbol{I} = diag\{I_x, I_y, I_z\}$ when the satellite body coordinate is taken as inertia main axis coordinate; is a project in body coordinate of attitude rate vector in the inertia coordinate which can be expressed as $=\{\ {}_x\ {}_y\ {}_z\ \}^T$; $(\quad)^{\times}$ is a vector product operator; $\boldsymbol{M}_g$ is gravity gradient torque which is expressed as $\boldsymbol{M}_g=\{M_{gx} \quad M_{gy} \quad M_{gz}\}^T=3 \ {}^2_c\, \mathbf{c}_3^{\times} \boldsymbol{I}\, \mathbf{c}_3+o(\boldsymbol{e}^{\ 2})$, where $\mathbf{c}_3$ is a column matrix made up of the 3[rd] column of direction cosine matrix $\boldsymbol{C}_{BO}$ in the coordinate of body to orbit, $\boldsymbol{e}$ is the ratio of body dimension to distance between satellite and earth, which can be ignored, ${}_c$ is orbit rate in the circular orbit, $\boldsymbol{M}_c$ is magnetic torque, $\boldsymbol{M}_d$ is disturbance torque including residue magnetic torque, aerodynamic torque and etc.

Supposed $_{a}$ is a project in orbit coordinate of attitude rate vector in the inertia coordinate , it can be derived that :

$$_{a} = -\boldsymbol{C}_{BO}\{0\ \ -\ \ _{c}\ ,\ 0\}^{T} \qquad (2)$$

$$_{a} = \ + \ _{c}\ \mathbf{c}_2 \qquad (3)$$

where $\mathbf{c}_2$ is a column matrix made up of the 2nd column of $\boldsymbol{C}_{BO}$.

In order to avoid the calculation oddity, take the quaternion express the attitude of satellite. $\boldsymbol{C}_{BO}$ can be expressed as:

$$\boldsymbol{C}_{BO} = \begin{bmatrix} 2(q_0^2+q_1^2)-1 & 2(q_1q_2+q_0q_3) & 2(q_1q_3-q_0q_2) \\ 2(q_1q_2-q_0q_3) & 2(q_0^2+q_2^2)-1 & 2(q_2q_3+q_0q_1) \\ 2(q_1q_3+q_0q_2) & 2(q_2q_3-q_0q_1) & 2(q_0^2+q_3^2)-1 \end{bmatrix}$$

Suppose $\boldsymbol{q} = \{q_1\ \ q_2\ \ q_3\}^{T}$, because of $\dot{\boldsymbol{q}} = 0.5(\boldsymbol{q}^{\times} + q_0\mathbf{E})\ \ _{a}$

So $\dot{\boldsymbol{q}}$ can expressed that:

$$\dot{\boldsymbol{q}} = \mathrm{A}\ \ _{a} \qquad (4)$$

Where $\mathrm{A} = 0.5(\boldsymbol{q}^{\times} + q_0\mathbf{E})$, $\mathbf{E}$ express the unit array.

Lyapunov Energy Depletion Principle

For the satellite satisfying gravity gradient stability condition, taking (3) into dynamics equation (1), yield

$$\begin{aligned} &\boldsymbol{I}\dot{\boldsymbol{w}}_a + \boldsymbol{w}_a^{\times}\boldsymbol{I}\boldsymbol{w}_a + \grave{u}_c(\boldsymbol{I}\boldsymbol{w}_a^{\times}\boldsymbol{c}_2 - \boldsymbol{w}_a^{\times}\boldsymbol{I}\boldsymbol{c}_2 - \boldsymbol{c}_2^{\times}\boldsymbol{I}\boldsymbol{w}_a) \\ &\quad -3\grave{u}_c^{\,2}\boldsymbol{c}_3^{\times}\boldsymbol{I}\boldsymbol{c}_3 + \grave{u}_c^{\,2}\boldsymbol{c}_2^{\times}\boldsymbol{I}\boldsymbol{c}_2 = \boldsymbol{M}_c + \boldsymbol{M}_d \end{aligned} \qquad (5)$$

Multiplying (4) by $\boldsymbol{w}_a$ and simplifying, yield

$$\begin{aligned} &\frac{1}{2}\boldsymbol{w}_a^{T}\boldsymbol{I}\boldsymbol{w}_a + \frac{3}{2}\boldsymbol{w}_c^{2}\boldsymbol{c}_3^{T}\boldsymbol{I}\boldsymbol{c}_3 - \frac{1}{2}\boldsymbol{w}_c^{2}\boldsymbol{c}_2^{T}\boldsymbol{I}\boldsymbol{c}_2 \\ &= \int \boldsymbol{w}_a \cdot (\boldsymbol{M}_c + \boldsymbol{M}_d)dt \end{aligned} \qquad (6)$$

where the first term of the equation (6) is rotate energy of satellite to orbit coordinate, the second is defined as gravity gradient potential energy, and the third is defined as gravity eccentric potential energy. It is rewritten as

$$H = \frac{1}{2}\boldsymbol{w}_a^{T}\boldsymbol{I}\boldsymbol{w}_a + \frac{3}{2}\grave{u}_c^{\,2}\boldsymbol{c}_3^{T}\boldsymbol{I}\boldsymbol{c}_3 - \frac{1}{2}\grave{u}_c^{\,2}\boldsymbol{c}_2^{T}\boldsymbol{I}\boldsymbol{c}_2 \qquad (7)$$

H is total energy of satellite body, named as Hamilton function.

Apparently, taking the derivative of (7), it is yield

$$\frac{dH}{dt} = \boldsymbol{w}_a^{T} \cdot (\boldsymbol{M}_c + \boldsymbol{M}_d)$$

If satellite body coordinate is taken as main inertial axis coordinate, it is followed

$$\begin{aligned} H &= \frac{1}{2}\boldsymbol{w}_a^{T}\boldsymbol{I}\boldsymbol{w}_a \\ &+ \frac{3}{2}\grave{u}_c^{\,2}\left[(I_x - I_z)c_{13}^2 + (I_y - I_z)c_{23}^2\right] \\ &+ \frac{1}{2}\grave{u}_c^{\,2}\left[(I_y - I_x)c_{12}^2 + (I_y - I_z)c_{32}^2\right] \\ &+ \frac{1}{2}\grave{u}_c^{\,2}(3I_z - I_y) \end{aligned} \qquad (8)$$

If $I_y > I_x > I_z$, the sum of the above three terms is constantly nonnegative; and if the sum of the above three terms is equal to zero, correspondingly, it is derived

$$\left.\begin{aligned} &\boldsymbol{w}_a = 0 \\ &c_{13} = c_{23} = c_{12} = c_{32} = 0 \end{aligned}\right\} \qquad (9)$$

At the same time, satellite is just in the equilibrium state of earth-oriented and three axis stabilization. Therefor let

$$L = H - \frac{1}{2}\grave{u}_c^{\,2}(3I_z - I_y)$$

Where L is positive definite Lyapunov function, $L \to 0$ can be regarded as a control criteria of capture for gravity fields.

Because $\dfrac{dL}{dt} = \dfrac{dH}{dt} = \boldsymbol{w}_a^{T} \cdot (\boldsymbol{M}_c + \boldsymbol{M}_d)$, proper $\boldsymbol{M}_c$ can be selected in order to make $\boldsymbol{w}_a^{T} \cdot (\boldsymbol{M}_c + \boldsymbol{M}_d) < 0$, ensure L to decrease, and make satellite earth-oriented, that is Lypunov energy depletion theory.

Dividing District Control Theory

In response to $L \to 0$, there are four equilibrium states of satellite which correspond to the four values of $\boldsymbol{C}_{BO}$ which are *diag{1,1,1}*, *diag{-1,-1,1}*, *diag{1,-1,-1}*, and *diag{-1,1,-1}*. However, only one is suitable. Control decision is required to be exclusive strictly. Corresponding to the states of *diag{-1,1,-1}* and *diag{1,-1,-1}*, the gravity gradient pole is placed upside down which can be eliminated by using C_{33} as check-up criteria. Corresponding to the states of *diag{-1,-1,1}*, it can be corrected by attitude error feedback control.

According to lyapunov energy depletion theory, satellite may be stabilized in a unexpected equilibrium state. So in the stage of capture, the main mission is to stabilize satellite in a expected equilibrium state and be able to jump up from unexpected equilibrium regions.

When satellite is in the critical position of gravity gradient pole upright to orbit panel, it can be proved that satellite's gravity gradient potential energy added to eccentric potential energy is equal to $2\grave{u}_c^2(I_y - I_z)$. The value is the maximum of the sum of the above two in the state of satellite gravity gradient pole being in the panel upright to $\boldsymbol{z}_0$ axis, which is toward the nadir in orbit coordinate. When gravity gradient pole is in the orbit panel and horizontal, gravity gradient potential energy added to eccentric energy equals $\frac{3}{2}\grave{u}_c^2(I_x - I_z)$. The value is the minimum of the sum of the above two in the state of satellite gravity gradient pole being in the panel upright to $\boldsymbol{z}_0$ axis. Therefor, when the total energy of satellite is above $2\grave{u}_c^2(I_y - I_z)$, satellite can upside down itself; when it is below $\frac{3}{2}\grave{u}_c^2(I_x - I_z)$, satellite can't upside down itself. It is written as

$$\begin{aligned} E_1 &= \frac{3}{2}\grave{u}_c^2(I_x - I_z) \\ E_2 &= 2\grave{u}_c^2(I_y - I_z) \end{aligned} \quad (10)$$

In light of the principle of simplifying the control modes, the satellite movement states is divided according to energy (Seen figure 1). Region I: Rate of satellite is higher, and control mission is in fact a process of rate damping. Region II: $c_{33} < 0$, gravity gradient pole is upside- down, but satellite can make pole point to proper position depending on its energy. Region III: $c_{33} > 0$, satellite can't upside down, besides damping mission, there is a problem of adjusting satellite forward and backward in the region, and attitude error feedback control need to be adding. Region IV: $c_{33} < 0$, satellite can't upside down, energy need to be added in order to make gravity gradient pole rotate to a proper position, and its control logic is showed in figure 2.

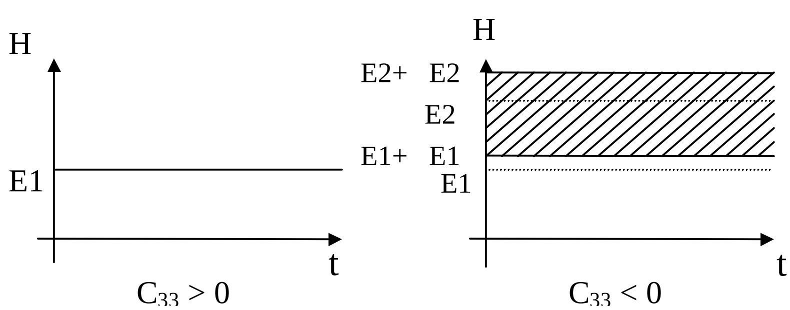

Figure 1 The schematic diagram of dividing district

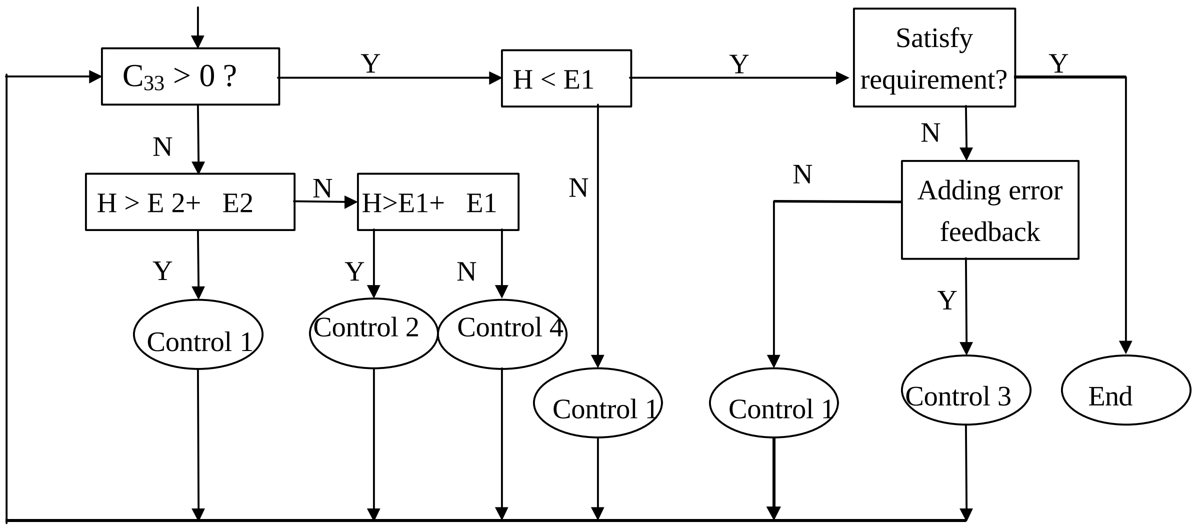

Figure 2 The logic diagram of dividing district control

The control modes of satellite is divided into four kinds:

Control 1: $\boldsymbol{w}_a^T \boldsymbol{M}_c < 0$, used in region I, in order to deplete satellite energy.

Control 2: $\boldsymbol{w}_a^T \boldsymbol{M}_c = 0$, it can be taken that $\boldsymbol{M}_c = 0$, used in region II, in order to maintain total energy of satellite body and expect gravity gradient pole upside down.

Control 3: $(\boldsymbol{w}_a + \boldsymbol{K}\boldsymbol{q})\boldsymbol{M}_c < 0$, used in region III, attitude error feedback control is added in order to ensure satellite stabilized in a expected equilibrium region. Because quaternion method is introduced to calculation, take $\boldsymbol{q}$ as the attitude error feedback control variables. Let $\boldsymbol{K}$ be a diagonal positive definitive matrix, whose value is determined by varied structure control theory. The value of $\boldsymbol{K}$ is decided by considering $\boldsymbol{w}_a$ and $\boldsymbol{q}$ comprehensively. When $\boldsymbol{w}_a$ is bigger, rate damping is primary and $\boldsymbol{K}$ may be valued as 0; when $\boldsymbol{w}_a$ is smaller, $\boldsymbol{K}$ is taken as a certain value to correct attitude angle, and to prevent energy from increasing quickly and satellite from jumping out of region and not stabilizing.

Control 4: $\boldsymbol{w}_a^T \boldsymbol{M}_c > 0$ used in region IV, in order to increase energy of satellite body and make it jump out of unexpected equilibrium region.

Magnetic control torque is affected by earth magnetic fields, and is constrained by magnetic dipole itself. The magnetic control torque created by earth magnetic field and magnetic dipole can be expressed as

$$\boldsymbol{M}_c = \boldsymbol{m}^{\times}\boldsymbol{B}$$

where $\boldsymbol{m}$ is magnetic dipole in the direction of three axes, and $\boldsymbol{B}$ is magnetic intensity in the position of satellite.

According to Lyapunov energy depletion theory, when $\boldsymbol{w}_a^T \boldsymbol{M}_c$ is negative, let $\boldsymbol{M}_c = -k_p \boldsymbol{w}_a$ is most efficient. That is, $\boldsymbol{M}_c$ is valued in the direction of $-\boldsymbol{w}_a$, k_p is proportional coefficient. But in fact, $\boldsymbol{M}_c$ can be valued only in the panel upright to **B.** So $-k_p \boldsymbol{w}_a$ is projected to the panel, then $\boldsymbol{M}_c'$ is obtained which satisfied earth magnetic field.

Let $\boldsymbol{B}_0 = \boldsymbol{B} / |\boldsymbol{B}|$, $\boldsymbol{B}_0 = \{b_1, b_2, b_3\}^T$, $\boldsymbol{M}_c = \{M_{c1}, M_{c2}, M_{c3}\}^T$ then $\boldsymbol{M}_c' = \{M'_{c1}, M'_{c2}, M'_{c3}\}^T$ which can be expressed as

$$\left.\begin{aligned} M'_{c1} &= M_{c1} - b_1 \boldsymbol{D} M_{c1} \\ M'_{c2} &= M_{c2} - b_2 \boldsymbol{D} M_{c2} \\ M'_{c3} &= M_{c3} - b_3 \boldsymbol{D} M_{c3} \end{aligned}\right\} \tag{11}$$

where $\boldsymbol{D} = (M_{c1} b_1 + M_{c2} b_2 + M_{c3} b_3) / |\boldsymbol{M}_c|$

Given M_c', magnetic dipole in the direction of three axes of satellite body is calculated. The maximum named as m_m is selected among m_1, m_2 and m_3. The ratio of normal magnetic dipole to it can be calculated. Multiply the magnetic dipole in the direction of three axes by the ratio value, we can make magnetic dipole operate with full load. Multiplying magnetic control torque by the ratio value in the direction of three axes, we can create magnetic control torque satisfying the constraints of magnetic dipole. It can be seen that k_p is selected arbitrarily because it no connection with the final results.

Decoupling Variable Structure Control Theory

For the satellite not satisfying gravity gradient stability condition, the lyapunov energy depletion theory is inefficient. We must find another efficient method in order to stabilize the satellite in the state of proper earth-oriented equilibrium.

From (1) and (4), satellite attitude system can be expressed as:

$$\begin{cases} \dot{\boldsymbol{\theta}} = \boldsymbol{A}[\boldsymbol{\omega} - \boldsymbol{C}_{BO}\{0,-\omega_c,0\}^T] = \boldsymbol{f}_1(\boldsymbol{\omega},\boldsymbol{\theta}) \\ \dot{\boldsymbol{\omega}} = \boldsymbol{I}^{-1}\boldsymbol{M}_g + \boldsymbol{I}^{-1}\boldsymbol{M}_c - \boldsymbol{I}^{-1}\boldsymbol{\omega}^{\times}\boldsymbol{I}\boldsymbol{\omega} = \boldsymbol{f}_2(\boldsymbol{\omega},\boldsymbol{\theta}) + \boldsymbol{I}^{-1}\boldsymbol{M}_c \end{cases} \tag{12}$$

Introducing nonlinear alternation:

$$\begin{cases} \boldsymbol{y}_1 = \boldsymbol{\theta} \\ \dot{\boldsymbol{y}}_1 = \dot{\boldsymbol{\theta}} = \boldsymbol{f}_1(\boldsymbol{\omega},\boldsymbol{\theta}) = \boldsymbol{y}_2 \\ \dot{\boldsymbol{y}}_2 = \dfrac{\partial \boldsymbol{f}_1}{\partial \boldsymbol{\theta}} \boldsymbol{f}_1(\boldsymbol{\omega},\boldsymbol{\theta}) + \dfrac{\partial \boldsymbol{f}_1}{\partial \boldsymbol{\omega}} [\boldsymbol{f}_2(\boldsymbol{\omega},\boldsymbol{\theta}) + \boldsymbol{I}^{-1}\boldsymbol{M}_c] \end{cases} \tag{13}$$

Define the super plane equation is:

$$\boldsymbol{S} = \boldsymbol{K}(\boldsymbol{y}_1 - \boldsymbol{\theta}_1) + \dot{\boldsymbol{y}}_1 - \dot{\boldsymbol{\theta}}_r \tag{14}$$

Where $\boldsymbol{K}$ is diagonal positive definitive matrix, with the increasing of it's elements value convergent velocity increase. $\boldsymbol{\theta}_r$ and $\dot{\boldsymbol{\theta}}_r$ are expected parameter varied principle. To the three-axis stabilization and sun-oriented satellite, $\boldsymbol{\theta}_r$ can be given by the orbit and launching point. $\dot{\boldsymbol{\theta}}_r$=0. When $\boldsymbol{M}_c$ is selected to satisfy $\boldsymbol{S}\,\dot{\boldsymbol{S}} < 0$ $\boldsymbol{S}$ can be inclined to 0 as time inclined to infinitude. That is, satellite is sun-oriented and three-axis stabilized.

$$\dot{\boldsymbol{S}} = \boldsymbol{K}\boldsymbol{f}_1 + \frac{\partial \boldsymbol{f}_1}{\partial \boldsymbol{\theta}} \boldsymbol{f}_1 + \frac{\partial \boldsymbol{f}_1}{\partial \boldsymbol{\omega}} [\boldsymbol{f}_2 + \boldsymbol{I}^{-1}\boldsymbol{M}_c] \tag{15}$$

Define $\boldsymbol{S}_0$ is positive definitive diagonal matrix, yield:

$$\boldsymbol{M}_c = -[\frac{\partial \boldsymbol{f}_1}{\partial \boldsymbol{\omega}} \boldsymbol{I}^{-1}]^{-1} [\boldsymbol{K}\boldsymbol{f}_1 + \frac{\partial \boldsymbol{f}_1}{\partial \boldsymbol{\theta}} \boldsymbol{f}_1 + \frac{\partial \boldsymbol{f}_1}{\partial \boldsymbol{\omega}} \boldsymbol{f}_2 + \boldsymbol{S}_0 sgn\boldsymbol{S}] \tag{16}$$

Where $sgn\,\boldsymbol{S} = \{sgn\,S_2 \;\; sgn\,S_2 \;\; sgn\,S_3\}^T$

$$sgnS_i = \begin{cases} 1 & S_i > 0 \\ -1 & S_i < 0 \\ 0 & S_i = 0 \end{cases} \qquad (i = 1,2,3)$$

And: $\dot{\boldsymbol{S}} = -\boldsymbol{S}_0\, sgn\, \boldsymbol{S}$

Sliding mode equation is:

$$\dot{y}_2 = \;\; \boldsymbol{K}\, \boldsymbol{y}_2 \;\; \boldsymbol{S}_0 \,\mathrm{sgn}\, \boldsymbol{S}$$

As the above show, if $[\frac{\partial \boldsymbol{f}_1}{\partial \boldsymbol{\omega}} \boldsymbol{I}^{-1}]^{-1}$ exist, nonlinear system with the variable of $\boldsymbol{y}$ is the decoupling variable structure control.

To active magnetic control small satellite, we can obtain $\frac{\partial \boldsymbol{f}_1}{\partial \boldsymbol{\omega}} = \boldsymbol{A}$ which satisfy the condition of variable structure control because of the existence of $[\boldsymbol{A}\boldsymbol{I}^{-1}]^{-1}$.

Expected control torque can be given as:

$$\boldsymbol{M}_c = -\boldsymbol{I}\boldsymbol{A}^{-1}[\boldsymbol{K}\boldsymbol{f}_1 + \frac{\partial \boldsymbol{f}_1}{\partial \boldsymbol{\theta}} \boldsymbol{f}_1 + \boldsymbol{A}\boldsymbol{f}_2 + \boldsymbol{S}_0 sgn\boldsymbol{S}] = \boldsymbol{f}_3(\boldsymbol{\omega},\boldsymbol{\theta},\boldsymbol{K},\boldsymbol{S}_0) \tag{17}$$

In fact, $\boldsymbol{M_c}$ obtained from the above expression is indirectly used to control attitude. Because $\boldsymbol{M_c}$ is also constrained by the local earth magnetic field intensity $\boldsymbol{B}$. Actual control torque is only caused in the plane $\boldsymbol{P}$ upright to $\boldsymbol{B}$. $\boldsymbol{M_{cp}}$ is obtained by projecting $\boldsymbol{M_c}$ into the plane $\boldsymbol{P}$ in order to satisfy the condition of $\boldsymbol{B}$ and $\boldsymbol{S}\dot{\boldsymbol{S}} < 0$ as possible as we can . Given $\boldsymbol{M_{cp}}$, the magnetic dipole along the three-axis can be calculated:

$$\boldsymbol{m} = \{m_x \ m_y \ m_z\}^{\boldsymbol{T}} = \boldsymbol{B}^{\times}\boldsymbol{M_{cp}} / \boldsymbol{B}^2 \quad (18)$$

When considering the external disturbance, yield:

$$\begin{aligned} \dot{\boldsymbol{\omega}} &= f_2(\boldsymbol{\omega}, \boldsymbol{q}) + \boldsymbol{I}^{-1}\boldsymbol{M_c} + \boldsymbol{I}^{-1}\boldsymbol{M_d} \\ \dot{\boldsymbol{S}} &= \frac{\partial \boldsymbol{f}_1}{\partial \boldsymbol{\omega}} \boldsymbol{I}^{-1}\boldsymbol{M_d} - \boldsymbol{S}_0 sgn\boldsymbol{S} \end{aligned} \quad (19)$$

It is obvious from (19) that $\boldsymbol{S}\dot{\boldsymbol{S}} < 0$ can be achieved only if $\boldsymbol{S}_0$ is big enough. The bigger the $\boldsymbol{S}_0$ is, the better the robustness is. When active magnetic control is adopted, $\boldsymbol{S}_0$ is selected according to the effect of local magnetic field and the constraint of magnetic dipole of satellite.

The Result of Computer Simulation

1. For the satellite satisfying gravity gradient stability condition, the Lyapunov energy depletion theory is applied to it.

The initial conditions:

$$\boldsymbol{\varphi}\,(\boldsymbol{roll}) = 10° \qquad \dot{\boldsymbol{\varphi}} = 3.5° / \boldsymbol{s}$$
$$\boldsymbol{\theta}\,(\boldsymbol{pitch}) = 10° \qquad \dot{\boldsymbol{\theta}} = 3.5° / \boldsymbol{s}$$
$$\boldsymbol{\psi}\,(\boldsymbol{yaw}) = 10° \qquad \dot{\boldsymbol{\psi}} = 3.5° / \boldsymbol{s}$$

Orbit parameters: altitude h=897km

Inclination i=98.98°

Normal magnetic dipole: $m = 15Am^2$

Disturbance torque:

$$M_{dx} = 5 \times 10^{-6}\ Nm,$$
$$M_{dy} = 5 \times 10^{-6}\ Nm,$$
$$M_{dz} = 5 \times 10^{-6}\ Nm$$

Satellite inertia: Before extending the pole,

$$\boldsymbol{I} = diag\{10.2, 12.4, 8.0\}$$

After extending the pole,

$$\boldsymbol{I} = diag\{162.2, 174.2, 9.6\}$$

Other parameters: $\boldsymbol{\Delta}E_1 = \boldsymbol{\Delta}E_2 = 0.05E_1$,

$$k_p = 1.0 \quad \boldsymbol{K} = diag\{10^{-3}, 10^{-3}, 10^{-3}\}$$

Using 7 orders earth magnetic field model, the results of digital simulation is showed in figure 3 and figure 4.

2. For the satellite not satisfying gravity gradient stability condition, variable structure control theory is applied to it.

The initial conditions:

$$\boldsymbol{\varphi}\,(\boldsymbol{roll}) = 10° \qquad \dot{\boldsymbol{\varphi}} = 2.3° / \boldsymbol{s}$$
$$\boldsymbol{\theta}\,(\boldsymbol{pitch}) = 10° \qquad \dot{\boldsymbol{\theta}} = 2.3° / \boldsymbol{s}$$
$$\boldsymbol{\psi}\,(\boldsymbol{yaw}) = 10° \qquad \dot{\boldsymbol{\psi}} = 2.3° / \boldsymbol{s}$$

Orbit parameters: altitude h=502km

Inclination i=97.4°

Normal magnetic dipole: $m = 15Am^2$

Disturbance torque:

$$M_{dx} = 1 \times 10^{-5}\ Nm,$$
$$M_{dy} = 1 \times 10^{-5}\ Nm,$$
$$M_{dz} = 1 \times 10^{-5}\ Nm,$$

Satellite inertia:

$$I = \begin{bmatrix} 13.61 & -0.085514 & -0.86155 \\ -0.085514 & 16.411 & 0.045257 \\ -0.86155 & 0.045257 & 18.613 \end{bmatrix}$$

Other parameters:

$$\boldsymbol{K} = diag\{0.005, 0.005, 0.005\},$$
$$\boldsymbol{S}_0 = diag\{0.001, 0.001, 0.001\}$$

Using 7 orders earth magnetic field model and giving Gauss coefficient according to DGRF model in 1995, the results of digital simulation is showed in from figure 5 to figure 7.

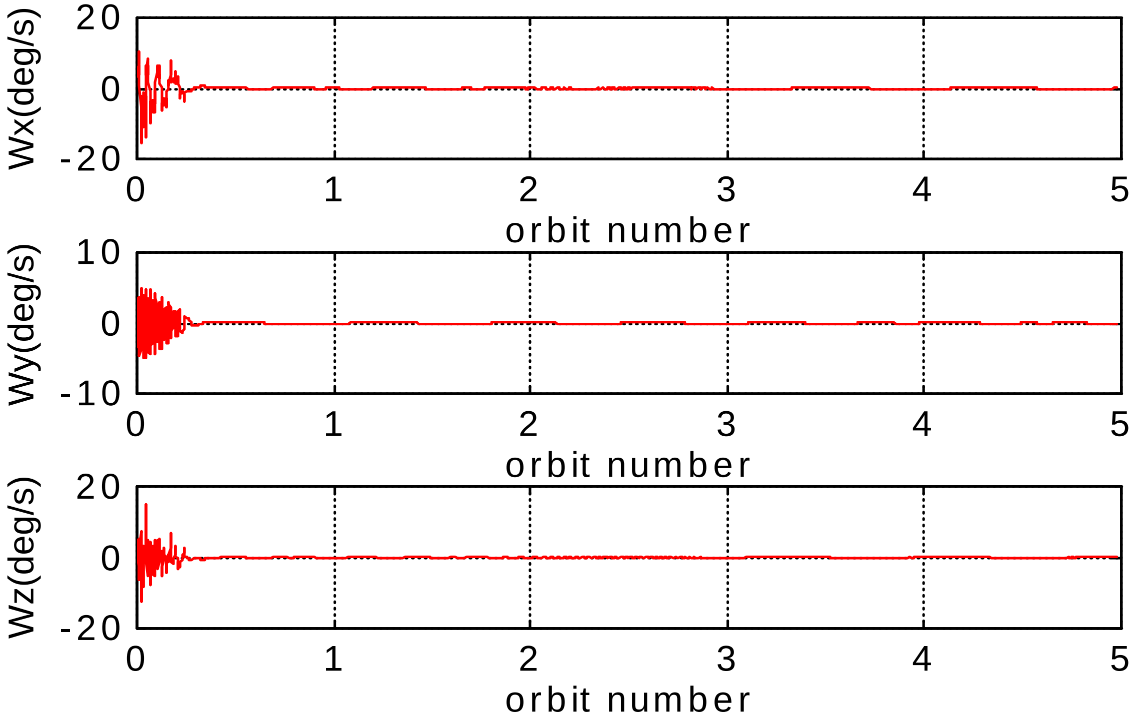

Figure 3 Active Magnetic Control of Gravity Gradient Stability Satellite
Attitude Rate (deg/s) Time Procedure(with Matlab 5.0)

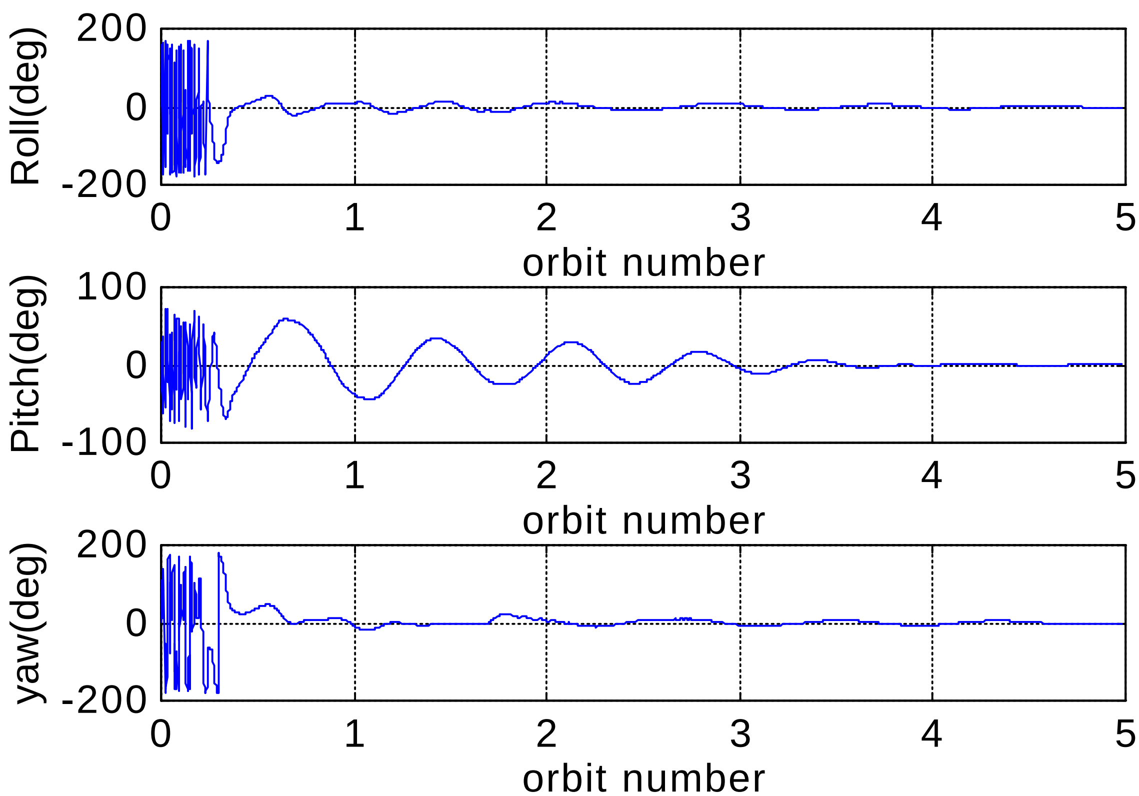

Figure 4 Active Magnetic Control of Gravity Gradient Stability Satellite
Attitude Error (deg) Time Procedure(with Matlab 5.0)

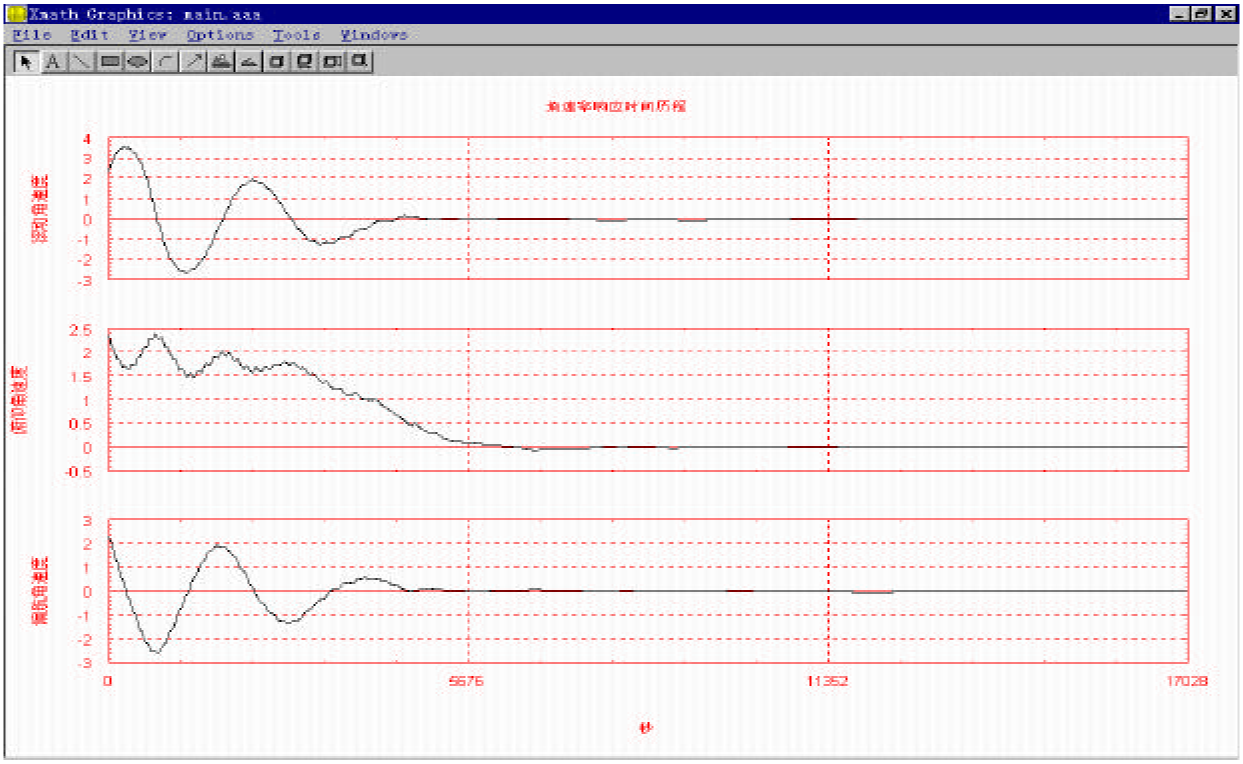

Figure 5 Active Magnetic Control of Non-gravity Gradient Stability Satellite
Attitude Rate (deg/s) Time Procedure (with Matrixx 5.0)

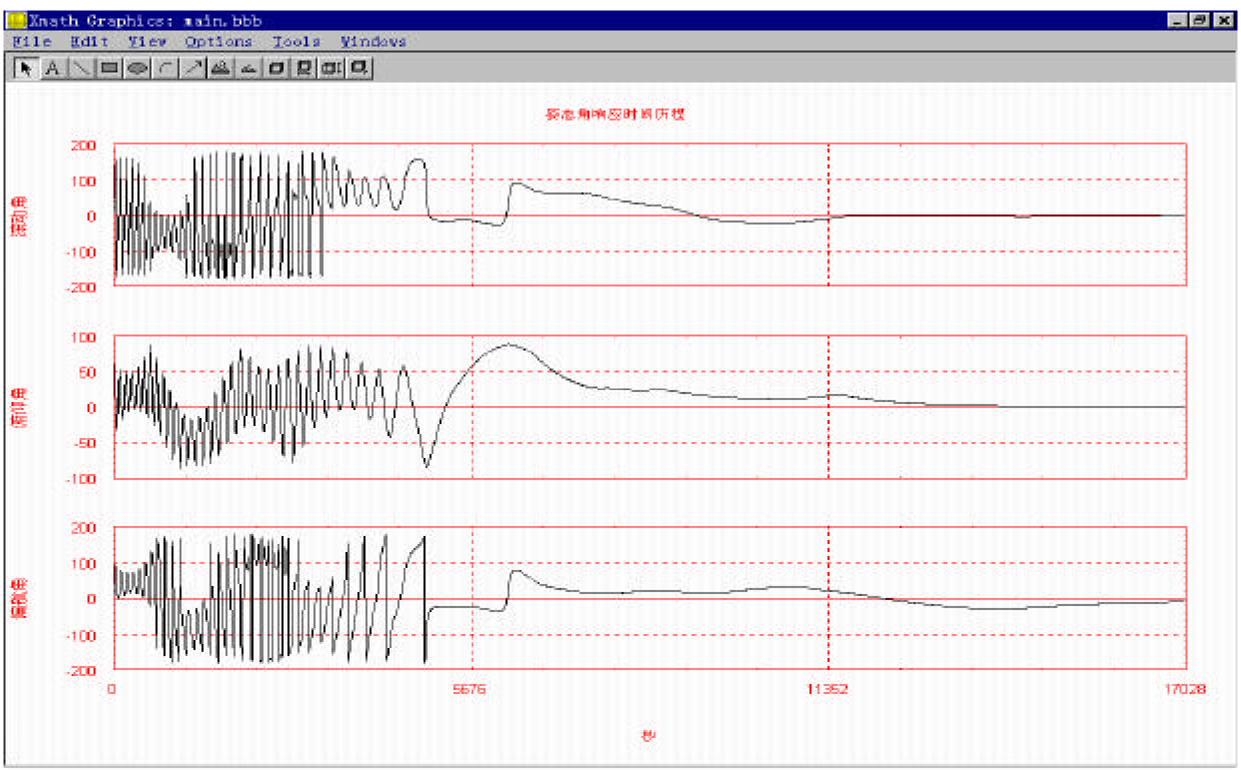

Figure 6 Active Magnetic Control of Non-gravity Gradient Stability Satellite
Attitude Error (deg)Time Procedure (with Matrixx 5.0)

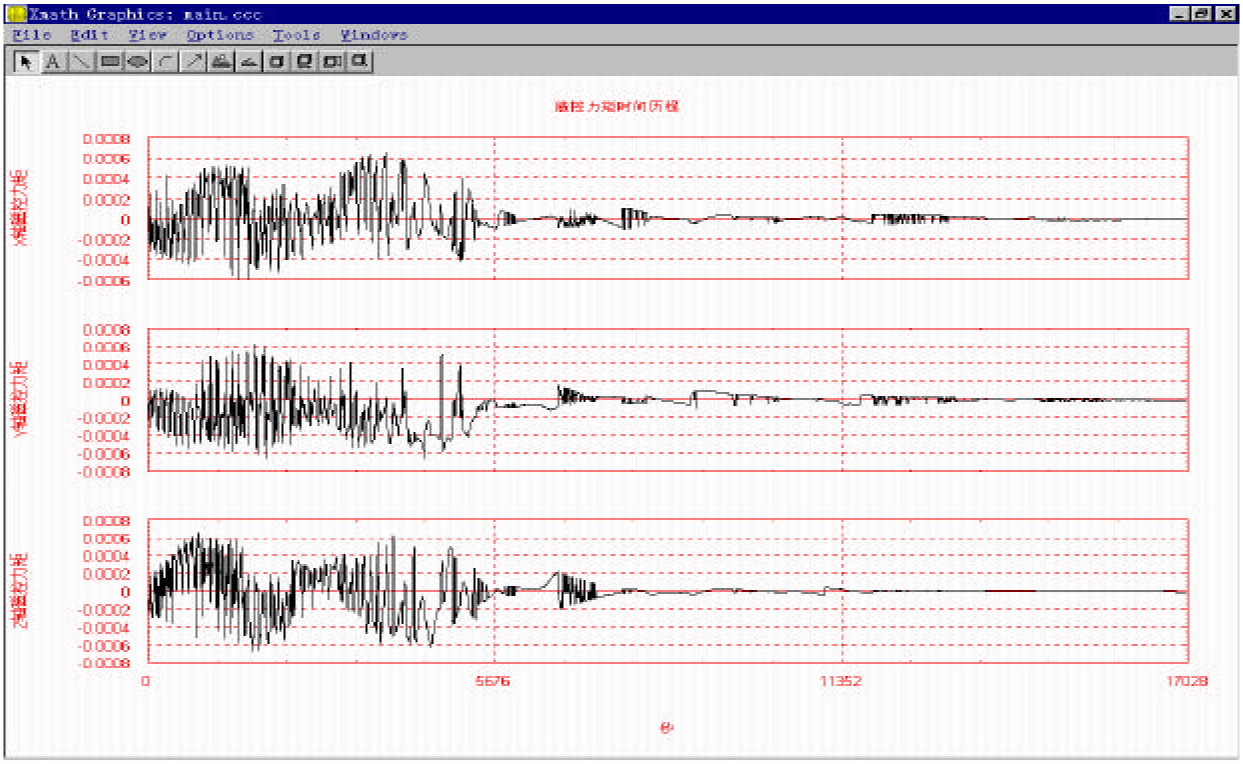

Figure 7 Active Magnetic Control of Non-gravity Gradient Stability Satellite Control Torque (Nm)Time Procedure(with Matrixx 5.0)

Summary

For gravity gradient stability satellite, we can conclude as follow:

1. Stability in global region can be ensured because of finding the system Lyapunov function.
2. The scheme is simple and efficient enough to satisfy the requirement of initial capture.
3. The control accuracy: (pointing accuracy: $<1^o$ in pitch and roll axes , $<3^o$ in yaw axis).

For non-gravity gradient stability satellite, we can conclude as follow:

1. Nonlinear theory is used to avoid the question caused by linearity design, which is of good robust characteristic.
2. The scheme is simple and easy to be achieved.
3. The control accuracy (pointing accuracy: $<1^o$ in pitch and roll axes, $<3^o$ in yaw axis; stability(3σ): 0.005^o/s) satisfied the requirement of system.

References

1. Hughes P C. Spacecraft Attitude Dynamics. John wiley and sons, Us, 1986.
2. Stickler A C, Alfriend K T. An Elemental Magnetic Attitude control System. AIAA Mechanics and Control of Flight Conference, 1974.
3. Zimbelman, D. J. Wilmot, and S.Evangelista, The Attitude Control System Design for the Transition Region and coronal Explorer Mission, Processings of the 10th Annual AIAA/USU Conference on small satellites, Logan, Utah, September 1996.
4. Wertz, J.R., editor, Spacecraft Attitude Determination and Control, Chapter 12.2, Kluwer Academic Publishers, 1978.