A PREDICTIVE ALGORITHM FOR ATTITUDE STABILIZATION AND SPIN CONTROL OF SMALL SATELLITES*

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Abstract

This paper presents a new algorithm for attitude stabilization and spin control of small satellites using only electromagnetic actuation. The approach takes advantage of the time-varying nature of the problem (the geomagnetic field changes through the orbit) by using the most appropriate control effort (according to an energy-based criterion) given the geomagnetic field and the satellite angular velocity at each actuation instant. The proposed controller is simulated and the results are discussed and compared with other approaches presented in the literature.

1 Introduction

Small satellites are nowadays an easy and cheap way to gain access to space and to all the advantages a satellite can provide (telecommunications, environment monitoring, etc). This class of LEO (Low Earth Orbit) satellites may be controlled by strict interaction with the geomagnetic field. A magnetic moment produced by coils placed on the satellite will produce a resultant torque by interaction with the geomagnetic field, which may be used for attitude control purposes. Nevertheless, this simple, low power consumption approach poses several interesting control difficulties, as the geomagnetic field viewed by a satellite changes along its orbit. Besides this time dependency, the mathematical description of this problem is highly non-linear, and new control strategies are needed to solve the attitude control demands of such a satellite.
This work has been carried out at the Intelligent Control Laboratory of ISR/IST under the ConSat project. A possible implementation on the PoSAT-1[1] satellite is envisaged.

2 Related Work

Several researchers have already begun to explore and solve the control problems posed by a LEO small satellite. Ong [4] proposes some intuitive control laws to tackle this problem, but the actuation is very restricted and does not take advantage of the time-varying nature of this problem. Steyn [5] approaches the control problem by using a Fuzzy Logic Controller that achieves better results than a Linear Quadratic Regulator (LQR), despite considering the constraint of actuating on a single coil at each actuation time. This approach suggests that non-linear and time-varying control methodologies should be further explored so that a better problem understanding and possible solutions may be found. Wisniewski [8] compares two non-linear solutions: sliding mode control and energy based control, achieving better results than LQRs based on linear periodic theory.

3 Problem Formulation

3.1 Coordinate systems

The following coordinate systems (C.S.) were used throughout this paper:

Control C.S.: This is a right orthogonal coordinate system built on the principal axes of the satellite with the origin placed in the center of mass. The x axis is the axis of the maximum moment of inertia, and the z axis is the minimum.

Orbital C.S.: This is a right orthogonal coordinate system fixed in the center of mass of the satellite. The z

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[1] PoSAT-1 is the first Portuguese Satellite in orbit, developed in a technology transfer program between University of Surrey and a Portuguese industrial and educational consortium lead by INETI.

axis points at the zenith (is aligned with the center of the Earth and points away of the Earth), the x axis points in the orbital plane normal direction and its sense coincides with the sense of the orbital angular velocity vector.

Inertial C.S.: This is a right orthogonal coordinate with origin in the center of mass of the satellite. The z axis is parallel to the rotation axis of the Earth and points towards the North Pole. The x axis is parallel to the line connecting the center of the Earth with the Vernal Equinox, and points towards the Vernal Equinox (the Vernal Equinox is the point where the ecliptic crosses the Earth equator going from South to North on the first day of spring).

3.2 Problem Description

The dynamics of a small satellite is well known and may be expressed in the Control CS as [7]:

$$\mathbf{I}\,{}^c\dot{\mathbf{\Omega}}_{ci} = -{}^c\mathbf{\Omega}_{ci} \times \mathbf{I}\,{}^c\mathbf{\Omega}_{ci} + {}^c\mathbf{N}_{ctrl} + {}^c\mathbf{N}_{gg} + {}^c\mathbf{N}_{dist} \quad (1)$$

where I is the inertia tensor, ${}^c\mathbf{N}_{ctrl}$ the control torque, ${}^c\mathbf{N}_{gg}$ the gravity gradient torque and ${}^c\mathbf{N}_{dist}$ a disturbance torque cause by aerodynamic drag and other effects. ${}^c\mathbf{\Omega}_{ci}$ is the angular velocity of the Control CS w.r.t. the Inertial CS written in the Control CS.
The control torque is obtained by electromagnetic interaction with the geomagnetic field [7],

$${}^c\mathbf{N}_{ctrl} = {}^c\mathbf{m} \times {}^c\mathbf{B} \quad (2)$$

where ${}^c\mathbf{m}$ is the control magnetic moment generated by the satellite coils and will be referred as the control variable throughout the paper. ${}^c\mathbf{B}$ is the geomagnetic field.
Equation 2 shows that the control torque is always perpendicular to the geomagnetic field, pointing out the non-controllability of electromagnetic actuation. The direction parallel to the geomagnetic field is not controllable, but the geomagnetic field changes along the orbit. This implies that, e.g., yaw, is not controllable over the poles but only a quarter of orbit later, approximately over the equator. Those characteristics must be adequately explored to appropriately regulate the satellite attitude. A time-varying predictive algorithm to determine the control moment, which takes advantage of the geomagnetic field changes, is proposed as a solution to this control problem.

4 The predictive algorithm

4.1 Motivation

Using the satellite total energy as a Lyapunov candidate function [8] shows that its time derivative is given by:

$$\dot{E}_{tot} = {}^c\mathbf{\Omega}_{co}^T\,{}^c\mathbf{N}_{ctrl} \quad (3)$$

where ${}^c\mathbf{\Omega}_{co}$ is the angular velocity of the Control C.S. w.r.t. the Orbital C.S. expressed in the Control C.S. The equation $\dot{E}_{tot} = \mathbf{0}$ represents all the control torques that lie on a plane that is perpendicular to ${}^c\mathbf{\Omega}_{co}$. Therefore, imposing $\dot{E}_{tot} < \mathbf{0}$ is the same as constraining the control torque to lie 'behind' the plane perpendicular to ${}^c\mathbf{\Omega}_{co}$. Furthermore the control torque is obtained from (2), therefore the control torque must always be perpendicular to the geomagnetic field. As such, the solution of this control problem must satisfy two requirements:

$$\begin{cases} {}^c\mathbf{\Omega}_{co}^T\,{}^c\mathbf{N}_{ctrl} < \mathbf{0} \\ {}^c\mathbf{B}^T\,{}^c\mathbf{N}_{ctrl} = \mathbf{0} \end{cases} \quad (4)$$

It can be seen from (4) that although the solution to these constraints is not a linear space, it is nevertheless an unlimited subset of a plan embedded in a three-dimensional space, in the general case, or it doesn't exist if ${}^c\mathbf{\Omega}_{co}$ is parallel to ${}^c\mathbf{N}_{ctrl}$. This is equivalent to state that the solutions to this control problem are infinite in the general case, suggesting a control algorithm that should choose the optimum magnetic moment (or at least the best one given all the constraints) at each actuation instant to take advantage of the particular angular velocity and geomagnetic field. This approach differs from most of the others solutions available in the literature, which use a constant control law, independently of the current angular velocity and geomagnetic field.

4.2 Formulation

As in [5], the measurements of the current geomagnetic field and satellite angular velocity are used to determine the control magnetic moment. We start by defining a cost function based on the kinetic energy[2] :

$$J = \frac{1}{2}\,{}^c\mathbf{\Omega}_{co}^T\,\mathbf{\Lambda_\Omega}\,{}^c\mathbf{\Omega}_{co} \quad (5)$$

where $\mathbf{\Lambda_\Omega}$ is a positive definite gain matrix. More insight will be given regarding the choice of the cost function, when studying the algorithm stability in Section 4.3.
The dynamical model of the satellite is well known and understood so it can be used to check the influence of the magnetic moment on the angular velocity. The angular velocity of the Control CS w.r.t. the Inertial CS can be written as:

$$\begin{aligned} {}^c\mathbf{\Omega}_{ci} &= {}^c\mathbf{\Omega}_{co} + {}^c\mathbf{\Omega}_{oi} \\ &= {}^c\mathbf{\Omega}_{co} + {}^c\mathbf{A}_o\,{}^o\mathbf{\Omega}_{oi} \end{aligned} \quad (6)$$

where ${}^c\mathbf{A}_o = [{}^c\mathbf{i}_o\ {}^c\mathbf{j}_o\ {}^c\mathbf{k}_o]$ is the direct cosine matrix which transforms vectors expressed in the Orbit CS to the Control CS. Small satellites are usually launched into

[2]The use of $\mathbf{\Lambda_\Omega}$ instead of the inertia matrix was chosen due to the possibility of defining relative weights for the angular velocities

polar orbits with small eccentricities. Therefore, the angular velocity of the Orbital CS w.r.t. the Inertial CS is approximately given by:

$$ {}^{o}\boldsymbol{\Omega}_{oi} = [\omega_0 \ 0 \ 0]^T \tag{7} $$

where ω_0 is the angular velocity of the satellite revolution about Earth, Equation (6) becomes:

$$ {}^{o}\boldsymbol{\Omega}_{ci} = {}^{c}\boldsymbol{\Omega}_{co} + \omega_0 {}^{c}\mathbf{i}_o \tag{8} $$

The derivative of Equation (8) now becomes:

$$ {}^{c}\dot{\boldsymbol{\Omega}}_{ci} = {}^{c}\dot{\boldsymbol{\Omega}}_{co} + \omega_0 {}^{c}\mathbf{i}_o \times {}^{c}\boldsymbol{\Omega}_{co} \tag{9} $$

substituting in the dynamics equation (1) and neglecting the disturbance torque we get:

$$ \begin{aligned} \mathbf{I}^{c}\dot{\boldsymbol{\Omega}}_{co} &= \mathbf{I}^{c}\boldsymbol{\Omega}_{ci} \times {}^{c}\boldsymbol{\Omega}_{ci} \\ &+ \mathbf{I}({}^{c}\boldsymbol{\Omega}_{co} \times \omega_0 {}^{c}\mathbf{i}_o) \\ &+ {}^{c}\mathbf{N}_{gg} + {}^{c}\mathbf{N}_{ctrl} \end{aligned} \tag{10} $$

Equation (10) is used to predict the evolution of the angular velocity produced by a given control torque by discretising it, considering a small time step Δt:

$$ \begin{aligned} \frac{{}^{c}\boldsymbol{\Omega}_{co}(t+\Delta t) - {}^{c}\boldsymbol{\Omega}_{co}(t)}{\Delta t} &\approx \mathbf{I}^{-1}(\mathbf{I}^{c}\boldsymbol{\Omega}_{ci}(t) \times {}^{c}\boldsymbol{\Omega}_{ci}(t)) \\ &+ {}^{c}\boldsymbol{\Omega}_{co}(t) \times \omega_0 {}^{c}\mathbf{i}_o(t) \\ &+ \mathbf{I}^{-1c}\mathbf{N}_{gg}(t) + \mathbf{I}^{-1c}\mathbf{N}_{ctrl}(t) \end{aligned} \tag{11} $$

which may be written as:

$$ \begin{aligned} {}^{c}\boldsymbol{\Omega}_{co}(t+\Delta t) &= {}^{c}\boldsymbol{\Omega}_{co}(t) + \Delta t \mathbf{f}(t) + O((\Delta t)^2) \\ \mathbf{f}(t) &= \mathbf{I}^{-1}\Big(\mathbf{I}^{c}\boldsymbol{\Omega}_{ci}(t) \times {}^{c}\boldsymbol{\Omega}_{ci}(t)\Big) \\ &+ {}^{c}\boldsymbol{\Omega}_{co}(t) \times \omega_0 {}^{c}\mathbf{i}_o(t) \\ &+ \mathbf{I}^{-1c}\mathbf{N}_{gg}(t) + \mathbf{I}^{-1c}\mathbf{N}_{ctrl}(t) \end{aligned} \tag{12} $$

and the prediction equation is obtained by discarding the higher order terms[3]:

$$ {}^{c}\hat{\boldsymbol{\Omega}}_{co}(t+\Delta t) = {}^{c}\boldsymbol{\Omega}_{co}(t) + \Delta t \mathbf{f}(t) \tag{13} $$

where the ${}^{c}\hat{\boldsymbol{\Omega}}_{co}$ is the predicted angular velocity. It can be seen from (13) that it is possible to predict the effect that a given control torque will produce on the angular velocity. This prediction requires only the knowledge of the current angular velocities and attitude, readily available from the attitude determination system. Using the prediction equation (13) and (2) it is possible to choose from the available magnetic moments the one that minimizes the cost function (5), once the geomagnetic field value is available from the magnetometers.

[3]Recall that eq. (12) corresponds to the Euler method for numerically solving first order differential equations.

4.3 Stability study

The total energy of satellite is composed of a kinetic term and a potential term,

$$ E_{total} = E_{kin} + E_{pot}. \tag{14} $$

Their sum, the total energy, can be considered constant, since the dissipative forces and torques actuating on a satellite are very weak. By dissipating the kinetic energy, the total energy is also decreased. Since the system is not fully controllable it is not possible to place the satellite in a zero kinetic energy configuration and keep it there because gravity-gradient torques will impose a libration movement converting potential energy to kinetic energy. All potential energy is converted to kinetic energy during the libration movement. Should all kinetic energy be dissipated by the predictive algorithm, the only stable configuration for the satellite would be a minimum total energy one (${}^{c}\mathbf{k}_o = \pm {}^{o}\mathbf{k}_o$).
There is, however, a situation under which the predictive algorithm is not capable of dissipating energy, when using on-off actuation and if the actuation instants are coincident with zero kinetic energy configurations. This situation can be avoided by guaranteeing that the libration movement period, which is a function of the inertia moments and the satellite angular velocity around earth [1], is different from the actuation period.
To show that the proposed algorithm is indeed global uniform asymptotical stable we start by considering a Lyapunov candidate function as defined in (14). The kinetic energy based on (5) will effectively be dissipated, since it can be expressed as:

$$ \begin{aligned} E_{kin}(t+\Delta t) &= \hat{E}_{kin}(t+\Delta t) \\ &+ O((\Delta t)^2) + O((\Delta t)^4) \end{aligned} \tag{15} $$

where $\hat{E}_{kin}$ is the kinetic energy computed using the predicted angular velocity (13). Assuming that the minimization algorithm is working correctly, we will have:

$$ \hat{E}_{kin}(t+\Delta t) < E_{kin}(t) \tag{16} $$

substituting (14) in (15) we get:

$$ E_{kin}(t+\Delta t) - E_{kin} < O((\Delta t)^2) + O((\Delta t)^4) \tag{17} $$

dividing by Δt and assuming Δt as small as wanted, we can write:

$$ \begin{aligned} &\lim_{\Delta t \to \infty} \frac{E_{kin}(t+\Delta t) - E_{kin}(t)}{\Delta t} \\ &< \lim_{\Delta t \to \infty} \frac{O((\Delta t)^2) + O((\Delta t)^4)}{\Delta t} \\ &\Leftrightarrow \\ &\dot{E}_{kin} < 0 \end{aligned} \tag{18} $$

Using the fact that $\dot{E}_{kin} < 0$ and the fact that ${}^c\boldsymbol{N_{ctrl}}$ is chosen by minimizing (5) we will show that the total energy is dissipated. Rewriting (14) we have:

$$\dot{E}_{kin} = \dot{E}_{total} - \dot{E}_{pot} < 0 \tag{19}$$

from this we see that when $\dot{E}_{pot} < 0$ the total energy verifies $\dot{E}_{total} < 0$. When $\dot{E}_{pot} > 0$ we can only state that $\dot{E}_{total} < \dot{E}_{pot}$. But $\dot{E}_{total} = {}^c\mathbf{\Omega}_{co}^T\, {}^c\mathbf{N}_{ctrl}$ and ${}^c\mathbf{N}_{ctrl}$ is chosen as to minimize (5), the chosen value for ${}^c\mathbf{N}_{ctrl}$ will also guarantee $\dot{E}_{total} < 0$, since the lowest value for $\dot{E}_{total}$ implies the lowest value for the cost function (5) at the next time step. Unfortunately there does not always exist a ${}^c\mathbf{N}_{ctrl}$ the ensures $\dot{E}_{total} < 0$ as discussed in section 4.1. In the rare occasions when ${}^c\mathbf{\Omega}_{co}$ is parallel to ${}^c\mathbf{N}_{ctrl}$ the algorithm is forced to choose ${}^c\mathbf{m} = [0\ 0\ 0]^T = {}^c\mathbf{N}_{ctrl}$ which is the best available solution.

In summary, when the potential energy decreases, the total energy also decreases, since the kinetic energy is always decreasing. When the potencial energy increases, all the solutions that decrease the total energy also decrease the kinetic energy, and these solutions are found (when they exist) by minimizing (5).

So far we have only shown that the system is globally stable, but not asymptotically stable. To show asymptotical stability we realize that ${}^c\mathbf{m}$ is computed based on the curent angular velocity and geomagnetic field, so ${}^c\mathbf{m} = g({}^c\Omega_{co}, {}^c\mathbf{B}(t))$ and therefore the dynamics (1) equation is periodic with the same period of the satellite's orbit. Using a periodic extension to Lyapunov stability theory due to Krasovskii-LaSalle [3] we can show that the system is indeed globally uniformly asymptotically stable towards the reference ${}^c\mathbf{\Omega}_{co} = [0\ 0\ 0]^T$, ${}^c\mathbf{k}_o = \pm{}^o\mathbf{k}_o$. The proof is similar to the one in [8] and will not be repeated here.

4.4 Implementation

4.4.1 Unrestricted actuators

For ideal actuators the minimization of the cost function is done on a continuous unlimited subset of a plan. An iterative method for the cost function minimization was required, so a Genetic Algorithm (GA) [2] was implemented. Any other iterative algorithm could be used but the GA was chosen because of its fast convergence characteristics, since in this problem the geomagnetic field is constantly changing. The implemented GA uses the standard techniques and two special operators: elitism, under which the best solution is always preserved and transmitted to the next generation, and cloning, by which we insert into the population the solution ${}^c\mathbf{m} = [0\ 0\ 0]^T$. Cloning is justified because it has been found through simulation that sometimes the algorithm would converge to magnetic moments parallel to the geomagnetic field after the stabilization had been completed. The solution ${}^c\mathbf{m} = [0\ 0\ 0]^T$ performs the same action (do nothing), but preserves power, as it does not use the magnetorquers for that purpose.

4.4.2 Restricted actuators

PoSAT-1 as other satellites of the UoSAT class has reduced control capabilities due to the restricted nature of its actuators.

Satellite design factors have restricted the values of the control magnetic moment to only three different values of positive/negative polarity. Combining this restriction with the single-coil-actuation the available set of magnetic moments is reduced to only 18 different values (6 for the x coils, 6 for the y coils and 6 for the z coils).

Power consumption is another serious restriction, which reflects on PoSAT actuation capabilities. For each actuation on a coil there must be at least a back-off time of 100 seconds to recharge the power supplies. This means that the actuators have at most a duty cycle of 3%, since the maximum actuation time is only 3 seconds. Considering these constraints, there are only 19 available magnetic moments: the 18 already referred and the do nothing solution ${}^c\mathbf{m} = [0\ 0\ 0]^T$. With such a restricted search space it is not necessary to use an iterative minimization algorithm, because all solutions may be evaluated and the best one (the one that minimizes (5)) is chosen.

5 Simulation results

Several simulations were performed using ConSat simulator [6] where perfect attitude determination is assumed and no disturbance torques (ex: aerodynamic drag or other effects) are considered.

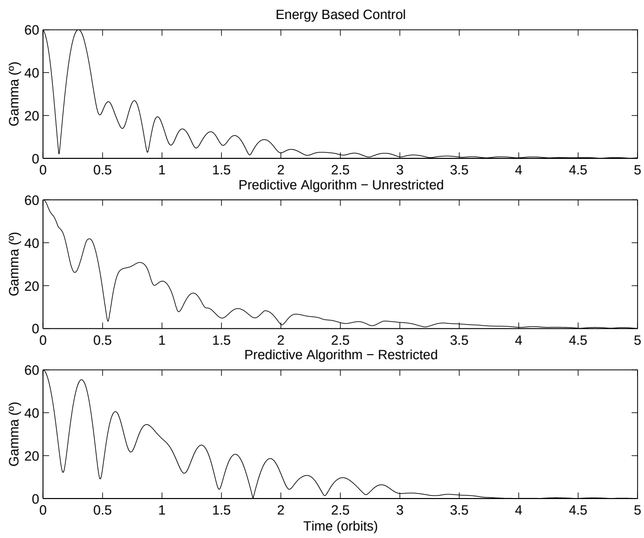

Figure 1: γ evolution for Energy Based Control and the Predictive Algorithm. Initial condition $\gamma = 60°$, ${}^c\mathbf{\Omega}_{co} = [0\ 0\ 0.0625]^T$. Desired reference $\gamma = 0°$, ${}^c\mathbf{\Omega}_{co} = [0\ 0\ 0]^T$.

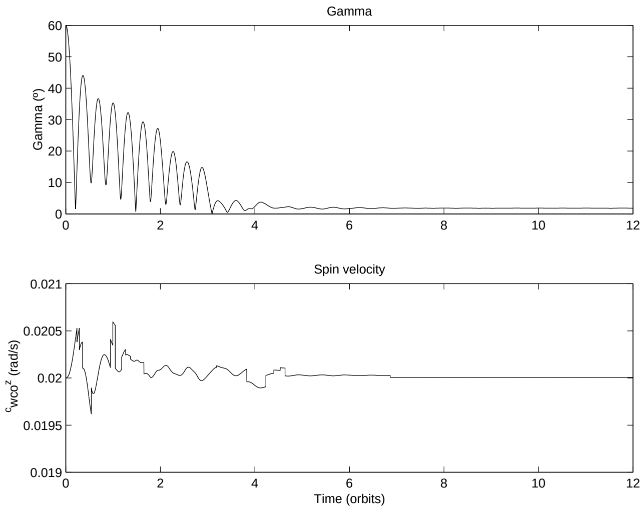

Figure 2: γ and spin velocity evolution for the predictive algorithm. Initial condition $\gamma = 60°, {}^c\Omega_{co} = [0\ 0\ 0.02]^T$. Desired reference $\gamma = 0°, {}^c\Omega_{co} = [0\ 0\ 0.02]^T$.

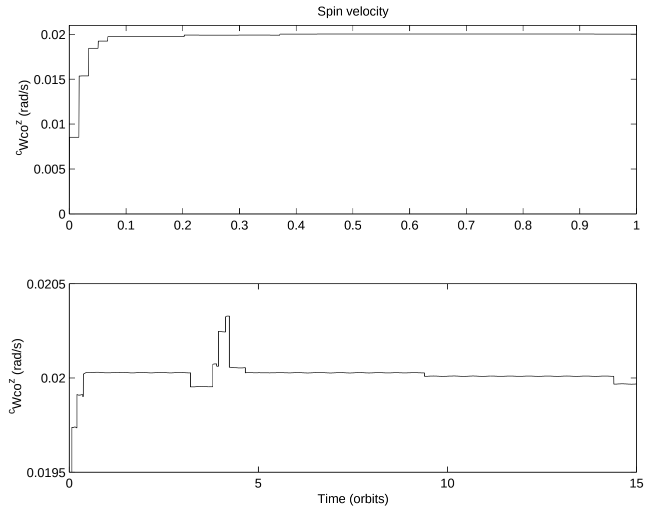

Figure 3: Spin velocity evolution for the predictive algorithm. Initial condition $\gamma = 5°, {}^c\Omega_{co} = [0\ 0\ 0]^T$. Desired reference $\gamma = 0°, {}^c\Omega_{co} = [0\ 0\ 0.02]^T$. The top plot is a detailed view of the results during the first orbit.

Simulations were performed for attitude stabilization only and attitude stabilization with spin control, where the used cost function was a variation from (5),

$$\begin{array}{c} J = \frac{1}{2} {}^c\Omega_{ref}^T \Lambda_\Omega {}^c\Omega_{ref} \\ {}^c\Omega_{ref} = {}^c\,\Omega_{co} - [0\ 0\ \omega_{spin}]^T \end{array} \qquad (20)$$

Figure 1 shows that the performance attained with the predictive algorithm, for attitude stabilization only, is similar to energy based control proposed by [8]. γ, the angle between the local vertical and the boom axis, is reduced from 60° to less then 5° in only 3 orbits.

It is interesting to note that the results attained with restricted actuators are similar to unrestricted actuators, and since the computational effort involved is considerably smaller, this algorithm is a valid solution for the available on board computer resources.
For attitude stabilization and spin control the algorithm's libration damping performance is slightly reduced since 4 orbits are now necessary to reduce γ to 5° and steady state error of 2° is attained, while maintaining the spin velocity at a reference of 0.02 rad/s. Figure 2 shows that spin velocity oscillates around the reference while libration is being damped but the set point is attained again as soon as the perturbation is rejected and the oscillation amplitudes reduced, being inferior to 0.0006 rad/s.

To test the algorithm spin control performance the satellite was spinned-up from 0 to 0.02 rad/s with an initial γ value of 5°. Simulation results plotted in Figure 3 show that the predictive algorithm takes less then 19.2 minutes or 12 actuations to set the spin velocity within a neighborhood of 0.001 rad/s (top figure) and achieves a final accuracy of less then 0.0005 rad/s (bottom figure). These are encouraging results since the actuators restrictions are quite severe. However, the dissipated energy is superior to energy based control since it is impossible to generate a magnetic moment perpendicular to the geomagnetic field at all actuation instants.

6 Conclusions and future work

A new algorithm for attitude stabilization and spin control was proposed and was shown to be asymptotically stable. Simulation results showed good performance, when compared to the algorithms proposed in the literature. For restricted actuators the low computational demands allow the implementation in the on board available computer resources. The reduced computational needs of the algorithm when used with restricted actuators suggest its use also with unrestricted actuators. Guaranteeing that the available set of control magnetic moments is perpendicular to the geomagnetic field can reduce the power requirements, which is a critical factor for small satellites. A possible algorithm improvement is to use a time varying set of magnetic moments, e.g. proportional to the angular velocity. This way, large moments are available to quickly reduce the satellite velocity, and small moments are available when the satellite will be near the desired set point allowing for a precise control.

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