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THE EFFECT OF THE NON-CONSERVATIVE FORCES ON THE ATTITUDE DYNAMIC OF A VARIABLE GEOMETRY SATELLITE

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Abstract

This paper investigates which implication structural non-conservative forces may have on a variable geometry small satellite attitude control subsystem design. Also are presented analytic and numerical models in order to take in account such effects. Numerical tests using a common code were used to analyse the satellite during its operational life. A flexural satellite model was developed using the idea of dividing a connected bodies system, writing a set of equations for each subsystem and reassembling them in the connection points using Kane's method. A numerical model for the linearized satellite dynamics around the gravity gradient equilibrium position under the effect of the aerodynamic and solar torques was used. The numerical results confirm the deep influence of the structural damping on the satellite attitude dynamics. The effect of the measurement noise and of the control delay on the actuators were also considered. Thermo- elasticity and structural damage were modeled using a periodic disturbance and a noise generator on the state vector and a time varying value of the damping coefficient.

Keywords : ACS, Kane equations, damping, stochastic investigation.

Introduction

The design of a satellite ACS is deeply influenced by the dynamic response of the system, generally modelled by Euler's equations [1], finding good non-conservative forces models, in particular a good damping model, thus can be difficult but important as the response and attenuation of vibration are predominantly controlled by these effects [2].
However, even if in the analysis of the dynamic response of any system, non-conservative information is essential for satisfactory results. Our comprehension of this subject is considerably less encyclopedic than that of mass and stiffness systems; so, predicting vibration parameters with respect to non-conservative forces is difficult in practice. Therefore, using an analytical assumption about these effects in the analysis is a justified compromise also for space applications.
For a single degree of freedom system, damping theories are well established to include, for example,
(i) viscous damping, in which the damping force is assumed to be proportional to the relative velocity;
(ii) Coulomb damping, caused by the relative motion of surfaces sliding against each other;
(iii) structural damping introduced, together with others, to model energy dissipation of continuous elements.
Amongst these, viscous damping and structural (hysteretic) damping models are most commonly used [3]. It is important to note that the linear viscous damping ratio is independent of amplitude, whereas Coulomb and quadratic damping ratios are respectivelly inversely and directly proportional to the amplitude of motion.
The hysteretic damping involves a solution which is not in the real domain, the interpretation of which is that the damping force is proportional to displacement rather than velocity, and is 90° out of phase with displacement. This kind of model is introduced based on experimental observations that damping ratios decrease as

the frequency increases. It should be noted that hysteretic damping may only be used in the frequency domain; as an example, it leads to erroneous results for free vibration analysis in the time domain, since the eigenvalues will have both positive and negative real parts, and the response will diverge if there is no work done by external forces.

Due to the problems of obtaining a system damping information, there is no practical way of forming the physical damping matrix by using the finite element method even though damping exists in most mechanical systems or structures. Indeed, so common is damping that we tend to forget that is not in the natural order of things for oscillation to decay. The damping properties of a system vary according to the structural design or the material used. Numerous analytical models for prediction of damping at micromechanical, macromechanical and structural level are based on the assumption of linear viscoelasticity [4]. The different sources of energy dissipation in fiber-reinforced composites, which represent the most general material for aerospace application, can be classified in the following way: “Viscoelastic damping” due to the viscoelastic nature of matrix and fiber materials (the fiber damping must be included in the analysis for carbon and kevlar fibers having high damping as compared to other types of fiber), “Interphase damping” due to the interphase between fibers and matrix, “Damage damping” due to the damage, which can result in frictional damping due to slip in the unbound regions or in damping due to energy dissipation in the area of cracks, “Viscoplastic damping” due to the presence of high stress and strain concentration in local regions between fibers at large amplitudes of vibration or high stress levels even for applied stresses below the elastic limits, (especially in thermoplastic composites), “Thermoelastic damping” due to the cyclic heat flow from the region of compressive stress to the region of tensile stress due to the temperature rise of the composites under an applied load (expecially in metal composites). Due the approach adopted only some of these sources of non-conservativeness were modeled in this paper.

The DeSat ACS

DeSat is a microsatellite for LEO orbits, studied and developed at the Aerospace Department and at the Electronic department of the University of Rome “La Sapienza” in collaboration with Alenia Spazio, in charge of testing a new mechanism for the deployment of space booms based on the ball re-cycling skews concept. It is 1 m long in the configuration with boom stowed and 3 m long in the configuration deployed with an octahedral shape, so it is strongly axialsymmetrical [5]. The ACS actuators consist in three magnetorques with a reaction wheel on the pitch axis (ϑ) [6, 7].

Considering its particular shape and its nominal attitude, the dynamics of DeSat is deeply influenced by the gravitational torque [8] so its dynamics near one of the equilibrium position of the gravitational torque under the effect of the aerodynamic and solar torques has been studied [9, 10]. Also the effect on the attitude control of the measurement noise and of the delay on the actuators were investigated using SIMULINK varying the values of the gains in the proportional-derivative (PD) law.

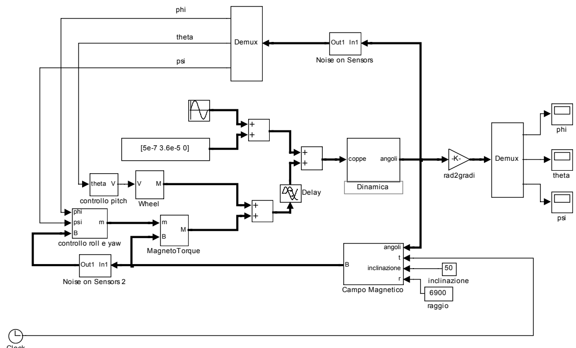

Figure 1. The Simulink model

The aerodynamic torque, due to the presence of atmosphere, for the conbsidered configuration has the direction of the pitch axis and has a constant value that varies from 2.37 10^{-6} Nm for the stowed configuration to 3.60 10^{-5} for the deployed one. The solar torque is due to the radiance pressure of the light emitted by the Sun. It has a pitch component (which varies with a period equal to the orbital period and an amplitude that varies along the year) and a roll component that varies with a period equal to one year. In this study has been considered the case of the maximum roll torque, taken constant, was considered. The values of the solar torque varies from 5.53 10^{-8} Nm for the stowed configuration to 8.00 10^{-7} Nm for the deployed one.

The following figures show that the coupling between roll (φ) and yaw (ψ) is “one-way”, meaning that the roll dynamics has a deep effect on the yaw, while the yaw does not influence the roll too much.

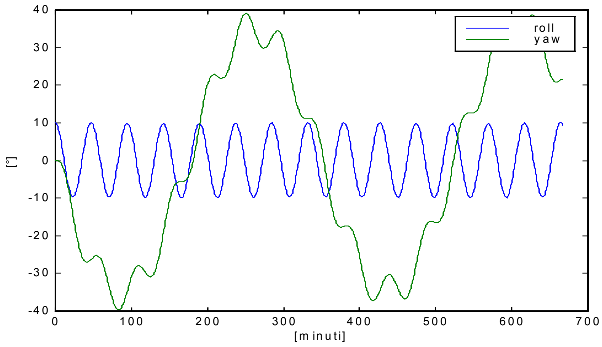

Figure 2. $\varphi(t), \psi(t)$ with initial conditions $\varphi(t) = 10°, \psi(t) = 0$ $\dot{\varphi}(t) = \dot{\psi}(t) = 0$ (Deployed Configuration)

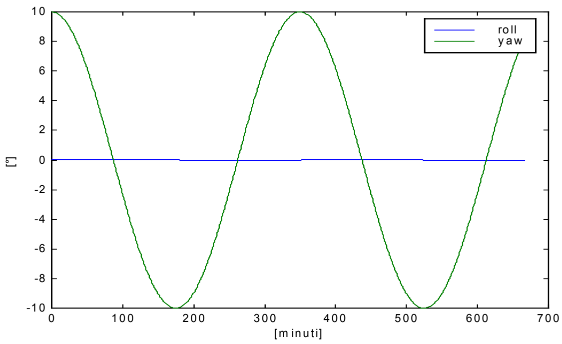

Figure 3. $\varphi(t), \psi(t)$ with initial conditions $\varphi(t) = 0, \psi(t) = 10°$ $\dot{\varphi}(t) = \dot{\psi}(t) = 0$ (Deployed Configuration)

	Deployed Configuration	Stowed Configuration
Roll (φ)	46.57 min	47.76 min
Pitch (ϑ)	53.78 min	55.37 min
Yaw (ψ)	348.01 min	348.01 min

Table 1. Free Oscillations Periods

The static response in pitch to the aerodynamic torque is 15° with the boom stowed and 11° with the boom deployed, equal to the mean value of the pitch oscillations.

The final values of the gains has been selected to keep the requested pointing precision during the deployment of the boom.

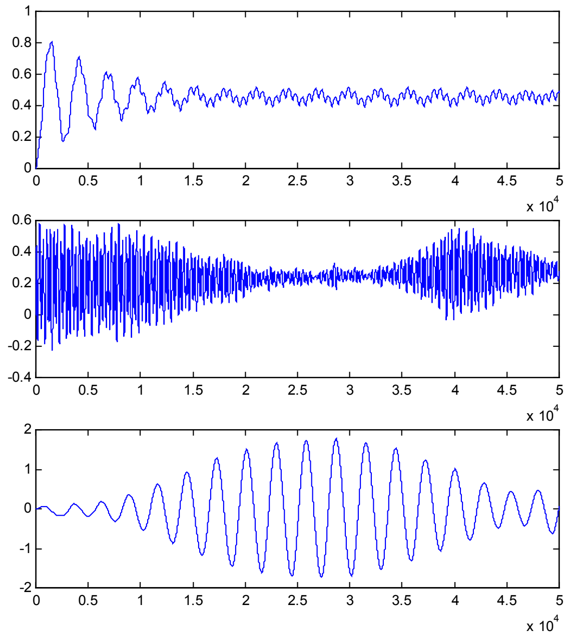

Figure 4. $\varphi(t), \vartheta(t), \psi(t)$ with c=0.05

	K_p	K_d
Pitch	9 V/rad	100 V/rad
Roll	10^{-6} Nm/rad	10^{-3} Nm/rad
Yaw	5 10^{-7} Nm/rad	10^{-8} Nm/rad

Table 2. PD gains

An linear-quadratic gaussian (LQ) state-feedback regulator for continuous plant using a stochastic approach is now object of study. Using a Kalman state-observer it permits to design an output feedback that minimize a cost function of the attitude error and of the strength of command needed for the control.

The DeSat attitude model was developed using Kane’s method which permits to write the equilibrium equations for a flexible articulated system [11]. It is possible to divide a complex system, formed by connected bodies, into subsystems and write a set of equations for each. The method consists in the following three steps:

(i) choose the dependent variables, called generalized or modal;
(ii) calculate the speeds and accelerations of the characteristic points of the system in terms of the generalized variables;
(iii) consider the inertial and external forces.
These sets can be reassembled summing, at the connection points, the generalized inertial and external forces.
Considering for example a flexible body fixed on a rigid one. The position of the generic point M of the flexible body can be expressed as

$$OM = R_{IC}\left[OC^* + C^*A + R_{CB}\left(AB^* + \sum_{i=1}^{v}\Phi_i(M)q_i\right)\right] \quad (1)$$

where R_{IC} is the rotation matrix from the rigid system to the inertial one, C^* the center of mass of the rigid body, A the point of junction of the two bodies, B^* the center of mass of the flexible body, R_{CB} the rotation matrix from the flexible system and the rigid one, Φ_i the i^{th} structural mode of the flexible body and q_i the modal variable.
The speed and the acceleration of every point of the system can be obtained simply deriving and then we can obtain the equation of motion in terms of the generalized variables (position and speed of the center of mass, attitude, angular speed, modal variables…).
It must be noted that, using this idea, the structural and the attitude dynamics become coupled both by the variation of the inertial properties and by the reaction force at the root of the flexible body.

Damping models

The following equation describes all of the commonly used damping models:

$$f_d(x,\dot{x}) = c\dot{x}\left|\dot{x}\right|^{i-1} \quad (2)$$

where c is the damping coefficient. The value of i determines the specific damping model, i.e. $i=0$ represents the Coulomb damping, $i=1$ represents the linear viscous damping, $i=2$ the quadratic damping [12].
For a ligthly damped system, where all three preceding damping mechanisms may be present, a first-order approximation for the total damping in the system can be obtained by fitting the data to the previous equation. Other models such as "stick-slip" type models, elasto-plastic and bilinear models can be invoked to describe the variation of the system damping with amplitude but are not suitable for a satellite structure description, in the case of metals, such models can describe the dissipation of energy in frictional modes, e.g. in-filled panels and partitions and between structural framing and cladding.
Typically in engineering pratice a viscous damping model, assigned on the basis of the material, or experimentally evaluated, is used for the sake of simplicity as it lends itself to a linear equation of motion. Any source of non linearity is ignored in this approach. This concept has been extended to represent equivalent viscous damping, whereby the energy dissipated by a nonlinear system in a steady state vibration is equated to the energy dissipated by an equivalent viscous system.
The results of the simulation for the pitch angle with these damping models are shown in figure 5. Even if there are some differences, these are small enough to be neglected, so in the following analyses only the linear viscous damping will be considered, and the aim is to find an expression for the damping constant c which involves the real effects the satellite meets with in its operational life.

Damping coefficient models

While the main source of dissipative effects is the structural damping due to the flexibility of the boom, and since the satellite is axialsymmetric, we can suppose a rotation about the yaw axis change not to the damping coefficients, i.e. they will have the form $\zeta_\varphi, \zeta_\vartheta = f(\varphi,\vartheta)$, $\zeta_\psi = f(\varphi,\vartheta,\psi)$.
The damping coefficient is a π periodic function of the angles and, in the nominal attitude when the satellite is in a stable equilibrium and the gravitational forces do not bend the boom, it will have its minimum value. Furthermore we can suppose, for the deployed configuration, a mean value of the damping coefficient of 0.05, typical of the

energy dissipation of the complex structures, a 10% variation while the cross angle varies up to 90° and a 5% variation for the direct one.

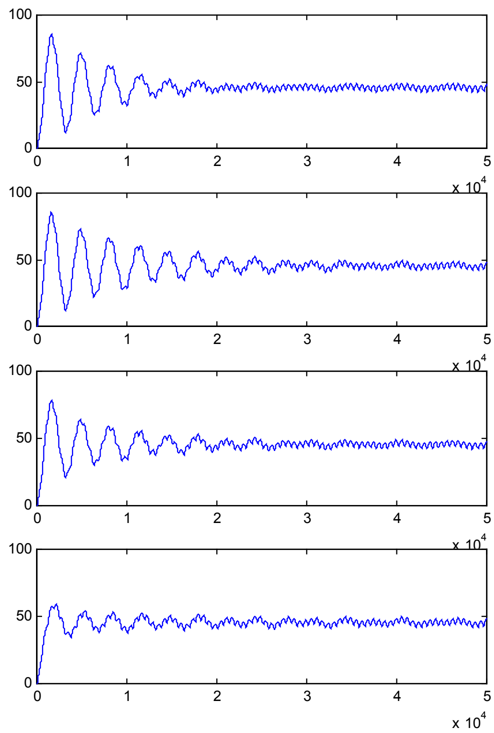

Figure 5. Controlled $\vartheta(t)$ considering i=0,1,2,6

From these values the following expressions could be found

$$\zeta_{\psi} = A + \left(B\cos 2\varphi + C\cos 2\vartheta + D\cos 2\psi\right) \quad (3)$$

	A	B	C	D
ζ_{φ}	0.05	-0.0025	-0.005	0
ζ_{ϑ}	0.05	-0.005	-0.0025	0
ζ_{ψ}	0.05	-0.005	-0.005	-0.0025

Table 3. ζ coefficients.

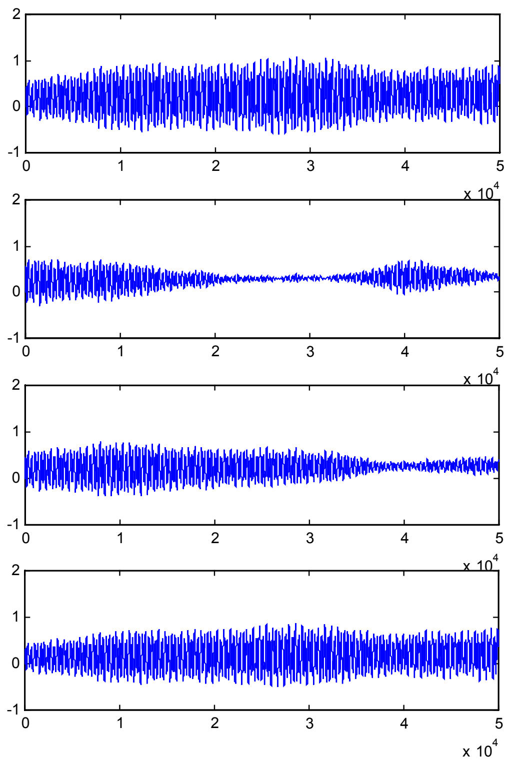

Figure 6. Uncontrolled $\vartheta(t)$ considering i=0,1,2,6

The attitude damping will be smaller in the stowed configuration because the structure is stiffer so there will be a smaller influence of the structural dynamics on the attitude one.
It is important to note that our functions are based on the consideration that on a damping coefficient the effect of the "cross" angles is greater than the "direct" one; for example, considering the effect of a pitch misalignment on the roll damping, a roll oscillation added up to a pitch misalignment causes, under the effect of the "tide" forces, a boom torsion which increases the load on the joints.

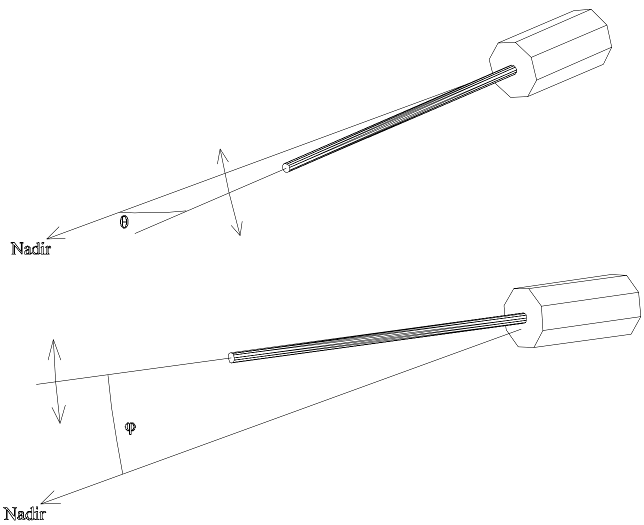

Figure 7. The “cross” influence.

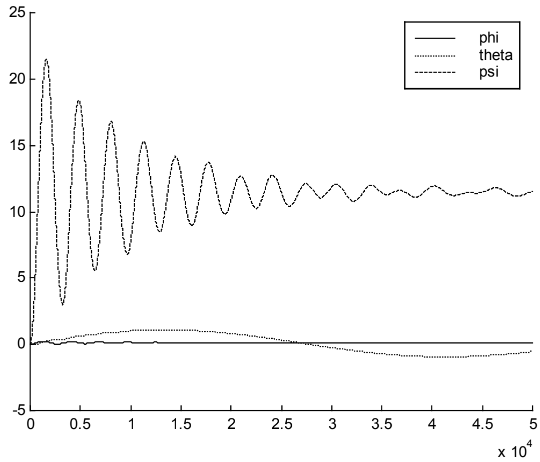

Figure 8. Uncontrolled $\varphi(t), \vartheta(t), \psi(t)$ with i=1

Another way to model such effects is to use an exponential function.
Considering the angle range limited between 0 and π/2, a mean value of 0.05 and a variation of ±10% for the cross angle and ±5% for the direct angle, the expression of the pitch damping coefficient is

$$\zeta_{\vartheta} = 0.0419\, e^{0.04557\, \vartheta^2 + 0.08672\, \varphi^2} \quad (4)$$

Furthermore, to take into account directly the properties of the material, it is possible to add a function multiplying the expression above. Considering the dependence from the elastic modulus E, the damping coefficient get the form

$$\zeta'(\varphi\vartheta\psi, E) \propto \left(\frac{1}{E}\right) \cdot \zeta(\varphi\vartheta\psi) \quad (5)$$

The results are summarized in the following figure in which it can be noted that these damping equations model the dynamic effect of a slow tendency to the equilibrium position.
The cyclic load due to the thermal environment, causes a fatigue degradation of the mechanical properties of the DeSat, in particular of the elastic modulus, causing an increase in the material damping. It was used the Palmgren-Miner’s cumulative damage model as shown in the following expression:

$$\zeta(E) = \frac{\zeta_0}{1 - \sum_i \frac{n_i}{N_i}}. \quad (6)$$

where ζ_0 is the damping coefficient at the beginning of the service life, n_i is the number of cycles the i-th external load is applied for and N_i is the fatigue life of the structure at the i-th load amplitude.
To have an idea of the effect of the fatigue damage on the attitude dynamics, the pitch evolution at beginning of the life, after 1 year and after 2 years are shown. Pitch oscillations tend to decay during the service life due to thermal fatigue loads but this effect is mainly negative as the structure rapidly tends to failure.

Stochastic investigations

To take in account the other effects of the structural dynamics on the attitude dynamic, such as thermo- elasticity and structural damage, a periodic disturbance and a noise generator on the state vector and a time varying value of the damping coefficient has been introduced.

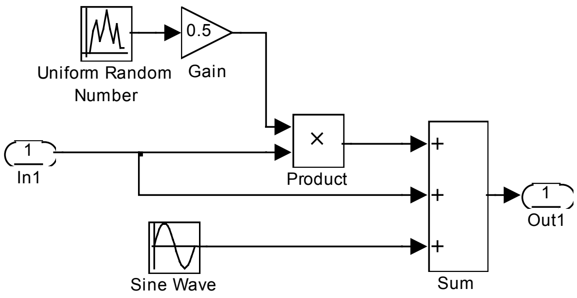

Figure 9. The stochastic investigation.

The noise on the state has been introduced into the SIMULINK model "feed-back" branch. This block has a uniform random numbers generator, which introduces a 50% variance on the state vector, and sine wave source, with an amplitude of $1\cdot10^{-5}$ rad/s on the roll and pitch angular speed and of 0.017 rad on the roll and pitch angle, which reproduce the effect of the thermal environment on the boom. No effect on the yaw DOF.

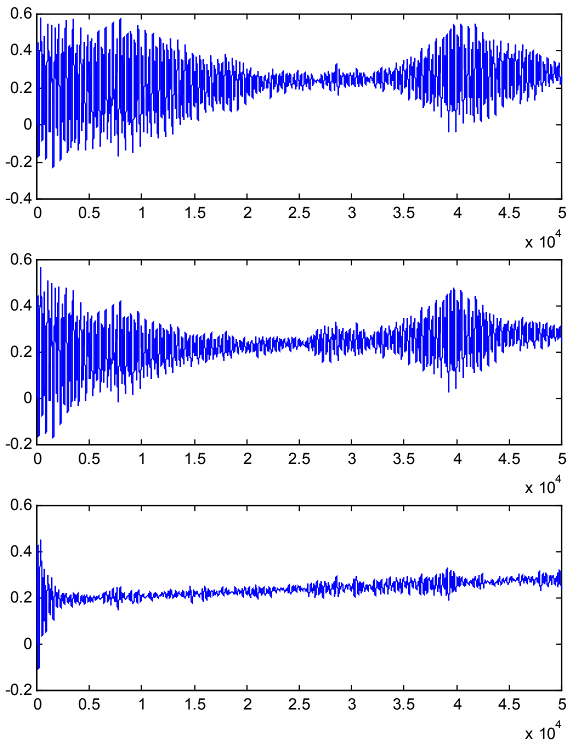

Figure 10. $\vartheta(t)$ at beginning of the service life, after 1 year and after 2 years

The responses of the system are shown in these figure, without and with the noise; with the noise the response is greater.

This fact has a real effect on the attitude control system. The new control law gains are deeply influenced as an increase derivative gains due to the damping actions is needed.

	K_p	K_d
Pitch	9 V/rad	100 V/rad
Roll	$3\ 10^{-6}$ Nm/rad	10^{-2} Nm/rad
Yaw	$5\ 10^{-7}$ Nm/rad	10^{-7} Nm/rad

Table 4. PD gains

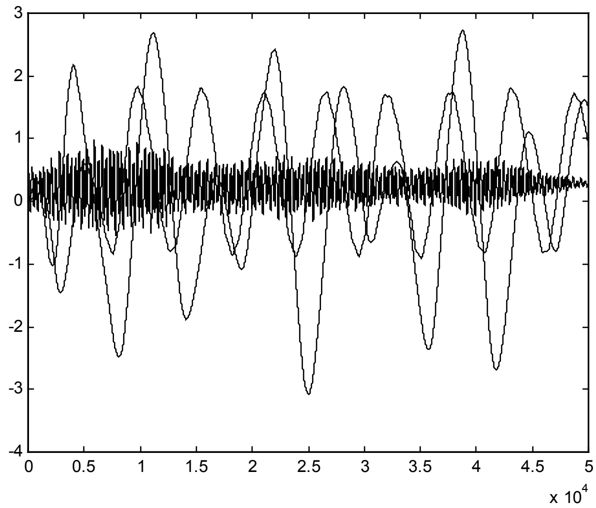

Figure 11. $\varphi(t), \vartheta(t), \psi(t)$ with a state noise

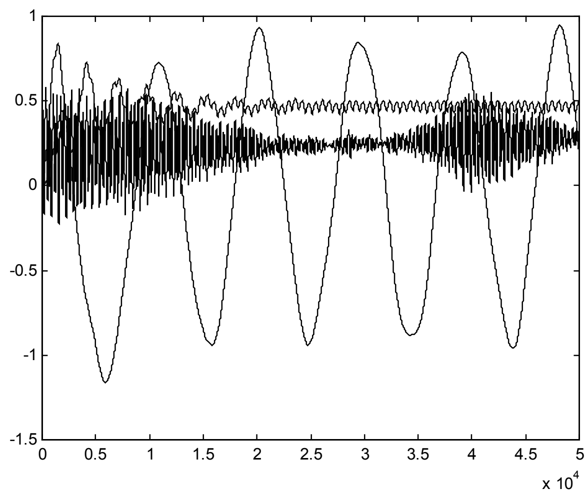

Figure 12. $\varphi(t), \vartheta(t), \psi(t)$ without state noise

Summary and Conclusions

We have presented a time varying stochastic analysis of the attitude dynamics of a variable geometry small satellite. Several environmental effects were investigated in order to find models which can describe the most important effects the space environment can have on the satellite attitude dynamics. The numerical investigation, referring to the framework of non linear discrete structural mechanics, allows to synthesize the controller without learning any special language. The simulation result allows the use of a linear viscous damping model with a periodic coefficient function of the attitude angles and the structural stiffness. The fatigue damage effect is that pitch oscillations tend to decay during the service life due to thermal loads but this effect is mainly negative as the structure rapidly tends to failure. The state noise effect is mainly an increase on the control law derivative gains.

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