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Energy Management and Attitude Control Strategies using Flywheels[1]

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Abstract: This paper is devoted to the use of multiple flywheels that integrate the energy storage and attitude control functions in space vehicles. This concept, which we refer to as an Integrated Energy Management and Attitude Control (IEMAC) system, reduces the space vehicle bus mass, volume, cost, and maintenance requirements while maintaining or improving the space vehicle performance. To this end, we present two nonlinear IEMAC strategies (model-based and adaptive) that simultaneously track a desired attitude trajectory and desired energy/power profile. Both strategies ensure asymptotic tracking while the adaptive controller compensates for uncertain spacecraft inertia.

1 Introduction

Space vehicles, typically utilize separate devices to provide energy storage and attitude control. Conventionally, the energy collected from solar arrays is stored in chemical batteries for use when the spacecraft is in the earth's shadow. Attitude control is usually accomplished through an array of reaction wheels or control moment gyros. In contrast to the standard configuration, a suitable arrangement of four or more flywheels can integrate the energy storage and attitude control functions into a single system, and thereby, reduce the spacecraft's bus mass, volume, cost, and maintenance requirements while maintaining or improving the spacecraft's performance. We will refer to this system as an *Integrated Energy Management and Attitude Control* (IEMAC) system. Roughly speaking, the integration of these two functions is achieved by decomposing the model of the flywheel array into two separate control problems (*i.e.*, attitude tracking and energy/power tracking, respectively) [5, 6, 12]. As a result, the energy management function can be accomplished without affecting the attitude control function.

A comprehensive literature review of the IEMAC concept is presented in [6]. As noted in this work, the IEMAC concept has been investigated since the 1970s [2]; however, the enabling technologies have only recently reached a level of maturity that facilitates on-board evaluation. Also noted in [6] is the fact that most control designs for the IEMAC problem are based on linearization of the spacecraft dynamic equation. With this fact in mind, [12] used the nonlinear dynamic equation presented in [6] along with a Modified Rodrigues Parameters-based kinematic representation to design an attitude and power tracking control scheme using an array of reaction wheels in an arbitrary non-coplanar configuration. Recently, [5] extended the approach of [12] for flywheels operating in a variable-speed control moment gyro mode.

The goal of this paper is to develop nonlinear IEMAC strategies that simultaneously track a desired attitude trajectory and a desired energy/power profile. We consider a spacecraft model in which the flywheels are operated in a reaction wheel mode and the spacecraft attitude kinematics are parameterized by the unit quaternion. Using the backstepping [8] control design framework, we first develop a model-based IEMAC strategy that ensures asymptotic attitude, energy, and power tracking with no energy/power singularities (*i.e.*, when the controller loses the capability of tracking the desired energy or power profile [12]). We then present a second control strategy that actively compensates for parametric uncertainties associated with the spacecraft inertia matrix. This adaptive controller also ensures asymptotic attitude, energy, and power tracking; however, the controller mandates some conditions on the flywheel angular velocity in order to ensure that energy/power singularities are avoided. The proposed IEMAC strategies have the following advantages in comparison to the work of [5, 6, 12]: (i) both controllers ensure tracking of *both* the desired energy and the power profiles, and hence, allow the spacecraft load requirements to be specified either in terms of energy or power demands, (ii) the model-based controller does not contain energy/power singularities; hence, an additional singularity avoidance scheme [12] is not necessary, (iii) the adaptive controller does not require exact knowledge of the spacecraft inertia, and (iv) the control strategies are not characterized by attitude singularities since the spacecraft attitude kinematics are modeled using the unit quaternion representation as opposed to the Modified Rodrigues Parameters. The paper is organized as follows. In Section 2, we present the IEMAC system model while the control objective is stated in Section 3. The design of the model-based and adaptive controllers are presented in Sections 4 and 5, respectively. Section 6 contains the concluding remarks.

2 System Model

We consider a rigid spacecraft with actuators that provide body-fixed torques about a body-fixed reference frame $\mathcal{B}$ located at the center of mass of the spacecraft. The body-fixed torques are applied by an array of N (> 3) flywheels with fixed axis of rotation with respect to $\mathcal{B}$ (*i.e.*, reaction wheel-type mode of operation). The dynamic model for the

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described IEMAC system is given by [7, 12]

$$\dot{h} = h^{\times} J^{-1} (h - Ah_f) \quad (1)$$

$$\dot{h}_f = \tau_f \quad (2)$$

where $h(t) \in \Re^3$ is the spacecraft angular momentum with respect to an inertial reference frame $\mathcal{I}$ expressed in $\mathcal{B}$, $h_f(t) \in \Re^N$ is the axial angular momentum of the flywheels, $J \in \Re^{3\times3}$ represents the constant, positive-definite, symmetric spacecraft inertia matrix, $A \in \Re^{3\times N}$ is a constant matrix whose columns contain the axial unit vectors of the N flywheels, $\tau_f(t) \in \Re^N$ is the control torque input, and the notation $a^{\times}$, $\forall a = [a_1, a_2, a_3]^T$, denotes the following skew-symmetric matrix

$$a^{\times} \triangleq \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}. \quad (3)$$

Note that $h(t)$ is related to $\omega(t) \in \Re^3$, the spacecraft angular velocity with respect to $\mathcal{I}$ expressed in $\mathcal{B}$, by the following equation

$$h = J\omega + Ah_f, \quad (4)$$

where $h_f(t)$ is related to $\omega_f(t) \in \Re^N$, the axial angular velocity of the flywheels, by[1]

$$h_f = J_f \omega_f. \quad (5)$$

The axial moments of inertia of the flywheels, $J_f \in \Re^{N\times N}$, is the constant, positive-definite, diagonal matrix defined as

$$J_f \triangleq \operatorname{diag}(j_{f1}, j_{f2}, ..., j_{fN}) \quad (6)$$

where the j_{fi}'s denote the inertia of each flywheel and $diag(\cdot)$ denotes a diagonal matrix with the enclosed diagonal terms.

The kinematics of the spacecraft attitude are represented by the unit quaternion [7] $\mathbf{q}(t) \triangleq \{q_o(t), q(t)\} \in \Re\times\Re^3$ which describes the orientation of $\mathcal{B}$ with respect to $\mathcal{I}$ expressed in $\mathcal{B}$. Note that the unit quaternion is subject to the constraint

$$q_o^2 + q^T q = 1. \quad (7)$$

The differential equation governing the attitude kinematics is given by

$$\dot{q} = \frac{1}{2}\left(q^{\times}\omega + q_o\omega\right) \quad (8)$$

$$\dot{q}_o = -\frac{1}{2} q^T \omega. \quad (9)$$

Remark 1 *The pseudo-inverse of the matrix A defined in (1), denoted by $A^+ \in \Re^{N\times 3}$, is given by*

$$A^+ \triangleq A^T\left(AA^T\right)^{-1} \quad \text{such that} \quad AA^+ = I_3. \quad (10)$$

where I_3 denotes the 3×3 identity matrix. As shown in [10], the above pseudo-inverse satisfies the so-called Moore-Penrose Conditions given below

$$\begin{array}{ll} AA^+A = A & A^+AA^+ = A^+ \\ \left(A^+A\right)^T = A^+A & \left(AA^+\right)^T = AA^+. \end{array} \quad (11)$$

In addition, the matrix $I_N - A^+A$, which projects vectors onto the null space of A, satisfies the following properties

$$\begin{array}{l} \left(I_N - A^+A\right)\left(I_N - A^+A\right) = I_N - A^+A \\ A\left(I_N - A^+A\right) = 0 \\ \left(I_N - A^+A\right)^T = I_N - A^+A \\ \left(I_N - A^+A\right)A^+ = 0. \end{array} \quad (12)$$

where I_N denotes the $N\times N$ identity matrix.

3 Control Objective

The control objective is to design the control input $\tau_f(t)$ in (2) to ensure that: (i) the flywheels track a desired energy/power profile, and (ii) the spacecraft tracks a desired attitude trajectory. We assume that the spacecraft attitude, spacecraft angular velocity, and flywheel angular velocities are measurable. Given that the total kinetic energy stored in the N flywheels due to their spin is given by $E(t) = \frac{1}{2}\omega_f^T(t)J_f\omega_f(t)$, the desired, stored kinetic energy will be defined as

$$E_d(t) \triangleq \frac{1}{2}\omega_{fd}^T(t)J_f\omega_{fd}(t) \quad (13)$$

where $\omega_{fd}(t) \in \Re^N$ represents the desired flywheel angular velocity yet to be specified. We can now quantify the energy storage tracking objective by the kinetic energy tracking error $\eta_E(t) \in \Re$ defined by

$$\eta_E \triangleq E_d - E. \quad (14)$$

Based on (14), the power tracking error is then given by

$$\dot{\eta}_E \triangleq \dot{E}_d - \dot{E}. \quad (15)$$

Thus, the energy/power tracking objective can be stated as $\lim_{t\to\infty} \eta_E(t), \dot{\eta}_E(t) = 0$.

We consider that the desired attitude of the spacecraft is described by a desired, body-fixed reference frame $\mathcal{D}$ whose orientation with respect to the inertial frame $\mathcal{I}$ is specified by the desired unit quaternion $\mathbf{q}_d(t) \triangleq \{q_{od}(t), q_d(t)\} \in \Re\times\Re^3$. The angular velocity of $\mathcal{D}$ with respect to $\mathcal{I}$ expressed in $\mathcal{D}$ is denoted by $\omega_d(t) \in \Re^3$. According to (8) and (9), the time derivative of $\mathbf{q}_d(t)$ is related to $\omega_d(t)$ by following dynamic equations

$$\dot{q}_d = \frac{1}{2}\left(q_d^{\times}\omega_d + q_{od}\omega_d\right) \quad (16)$$

$$\dot{q}_{od} = -\frac{1}{2}q_d^T\omega_d. \quad (17)$$

We make the assumption that the desired attitude motion of the spacecraft is specified such ω_d is bounded for all time.

To quantify the mismatch between the actual and desired spacecraft attitudes, we can compute the error quaternion $\mathbf{e}(t) \triangleq \{e_o(t), e(t)\} \in \Re\times\Re^3$ via the following quaternion product (see Theorem 5.3 of [9])

$$\mathbf{e} = \mathbf{q}_d^*\mathbf{q} \quad (18)$$

where $\mathbf{q}(t)$ was defined above, and $\mathbf{q}_d^*(t) \triangleq \{q_{od}(t), -q_d(t)\} \in \Re\times\Re^3$. The attitude error can also

[1] In formulating (5), one must make the modeling assumption that the flywheels spin at a much higher speed than the spacecraft[12] , *i.e.*, $\omega_f(t) \gg \omega(t)$.

be represented by the rotation matrix $\tilde{R}(e_o, e) \in SO(3)$ that brings $\mathcal{D}$ onto $\mathcal{F}$ which is defined as [7]

$$\tilde{R} = \left(e_o^2 - e^T e\right) I_3 + 2ee^T - 2e_o e^\times. \tag{19}$$

Based on (19), the attitude tracking objective can be stated as follows

$$\lim_{t\to\infty} \tilde{R}(e_o(t), e(t)) = I_3. \tag{20}$$

Now, since $\mathbf{e}(t)$ is a unit quaternion, its components satisfy the following constraint[7]

$$e_o^2 + e^T e = 1. \tag{21}$$

From (21), we can see that

$$0 \le |e_o(t)| \le 1 \qquad 0 \le \|e(t)\| \le 1 \tag{22}$$

for all time. It is also easy to see from (21) that

$$\text{if} \quad \lim_{t\to\infty} e(t) = 0, \quad \text{then} \quad \lim_{t\to\infty} e_o(t) = \pm 1; \tag{23}$$

hence, we know from (19) that if $\lim_{t\to\infty} e(t) = 0$, then the attitude tracking objective defined by (20) will be achieved. Finally, the differential equations that describe the attitude error kinematics are given by [1]

$$\begin{aligned} \dot{e} &= \frac{1}{2}e^\times\tilde{\omega} + \frac{1}{2}e_o\tilde{\omega} && (24) \\ \dot{e}_o &= -\frac{1}{2}e^T\tilde{\omega} && (25) \end{aligned}$$

where $\tilde{\omega}(t) \in \Re^3$, the angular velocity of $\mathcal{B}$ with respect to $\mathcal{D}$ expressed in $\mathcal{B}$, is defined as follows

$$\tilde{\omega} \triangleq \omega - \tilde{R}\omega_d. \tag{26}$$

To facilitate the subsequent control design, we will introduce the following notation. First, we can use (4), (5), and (26) to rewrite $\tilde{\omega}(t)$ as follows

$$\begin{aligned} \tilde{\omega} &= J^{-1}h - J^{-1}AJ_f\omega_f - \tilde{R}\omega_d \\ &= J^{-1}h - \tilde{R}\omega_d - J^{-1}A\bar{J}_f\bar{J}_f\omega_f \\ &= J^{-1}h - \tilde{R}\omega_d - J^{-1}\bar{A}\bar{\omega}_f && (27) \end{aligned}$$

where

$$\bar{J}_f \triangleq \operatorname{diag}\left(\sqrt{j_{f1}}, \sqrt{j_{f2}}, ..., \sqrt{j_{fN}}\right), \qquad \bar{A} \triangleq A\bar{J}_f, \quad \text{and} \quad \bar{\omega}_f \triangleq \bar{J}_f\omega_f. \tag{28}$$

Note that the pseudo-inverse of the matrix $\bar{A} \in \Re^{3\times N}$ defined in (28), denoted by $\bar{A}^+ \in \Re^{N\times 3}$, satisfies all the properties listed in Remark 1.

4 Model-Based Control Design

A backstepping-type control design methodology [8] will be utilized to develop the control law. In the control design of this section, we will consider that the spacecraft and flywheel inertias are exactly known.

4.1 Attitude Tracking Objective

We begin by defining the following nonnegative function

$$V_1(e_o, e) \triangleq (e_o - 1)^2 + e^T e. \tag{29}$$

After taking the time derivative of (29), we obtain the following

$$\dot{V}_1 = 2(e_0 - 1)\dot{e}_0 + 2e^T\dot{e}. \tag{30}$$

After substituting (24) and (25) into (30), we obtain

$$\dot{V}_1 = -(e_0 - 1)e^T\tilde{\omega} + e^T e^\times\tilde{\omega} + e^T e_o\tilde{\omega} \;= e^T\tilde{\omega} \tag{31}$$

where we have used the fact that $e^T e^\times = 0$. After substituting (27) into (31) and then adding and subtracting the term $e^T J^{-1}\bar{A}\bar{\omega}_{fd}$ to the resulting expression, we obtain

$$\begin{aligned} \dot{V}_1 &= e^T\left(J^{-1}h - \tilde{R}\omega_d - J^{-1}\bar{A}\bar{\omega}_f + J^{-1}\bar{A}\bar{\omega}_{fd} - J^{-1}\bar{A}\bar{\omega}_{fd}\right) \\ &= e^T\left(J^{-1}h - \tilde{R}\omega_d - J^{-1}\bar{A}\bar{\omega}_{fd} + J^{-1}\bar{A}\eta\right) \end{aligned} \tag{32}$$

where the new auxiliary signals $\bar{\omega}_{fd}(t)$, $\eta(t) \in \Re^N$ are defined as follows

$$\bar{\omega}_{fd} \triangleq \bar{J}_f\omega_{fd} \qquad \eta \triangleq \bar{\omega}_{fd} - \bar{\omega}_f, \tag{33}$$

$\omega_{fd}(t)$ was introduced in (13), and $\bar{J}_f$, $\bar{\omega}_f(t)$ were defined in (28).

Based on the form of (32), we design the signal $\bar{\omega}_{fd}(t)$ as follows

$$\bar{\omega}_{fd} = \bar{A}^+ J\left(J^{-1}h - \tilde{R}\omega_d + K_1 e\right) + \left(I_N - \bar{A}^+\bar{A}\right)g \tag{34}$$

where $K_1 \in \Re^{3\times 3}$ is a constant, positive-definite, diagonal, control gain matrix, and $g(t) \in \Re^N$ is a differentiable, auxiliary function that will be later designed to facilitate the energy/power tracking objective. After substituting (34) into (32), we have

$$\dot{V}_1 = -e^T K_1 e + e^T J^{-1}\bar{A}\eta \tag{35}$$

where (12) was applied.

The structure of (35) motivates us to drive the signal $\eta(t)$ to zero. To this end, we develop the open-loop dynamics for $\eta(t)$ by taking the time derivative of the second equation of (33) and pre-multiplying the resulting expression by $\bar{J}_f$ to produce

$$\bar{J}_f\dot{\eta} = \bar{J}_f\,\dot{\bar{\omega}}_{fd} - \bar{J}_f\,\dot{\bar{\omega}}_f = \bar{J}_f\,\dot{\bar{\omega}}_{fd} - J_f\dot{\omega}_f \tag{36}$$

where (28) was used. After substituting the time derivative of (34) into (36), we obtain

$$\begin{aligned} \bar{J}_f\dot{\eta} = \; & \bar{J}_f\bar{A}^+ J\left(J^{-1}\dot{h} - \dot{\tilde{R}}\,\omega_d - \tilde{R}\dot{\omega}_d + K_1\dot{e}\right) \\ & + \bar{J}_f\left(I_N - \bar{A}^+\bar{A}\right)\dot{g} - J_f\dot{\omega}_f. \end{aligned} \tag{37}$$

After utilizing (1), (2), (4), (5), (24), and the fact that $\dot{\tilde{R}} = -\tilde{\omega}^\times\tilde{R}$, we can rewrite (37) as

$$\bar{J}_f\dot{\eta} = W - \tau_f \tag{38}$$

where the auxiliary term $W(e, e_o, \omega, h, \omega_d, \dot{\omega}_d, \dot{g}) \in \Re^N$ is defined as follows

$$\begin{aligned} W \triangleq \; & \bar{J}_f\bar{A}^+ h^\times\omega + \bar{J}_f\bar{A}^+ J\left(\tilde{\omega}^\times\tilde{R}\omega_d - \tilde{R}\dot{\omega}_d\right) \\ & + \tfrac{1}{2}\bar{J}_f\bar{A}^+ JK_1\left(e^\times\tilde{\omega} + e_0\tilde{\omega}\right) + \bar{J}_f\left(I_N - \bar{A}^+\bar{A}\right)\dot{g}. \end{aligned} \tag{39}$$

Based on (38) and (35), we now design the control input $\tau_f(t)$ as follows

$$\tau_f = W + K_2\eta + \bar{A}^T J^{-T} e. \tag{40}$$

where $K_2 \in \Re^{N\times N}$ is a constant, positive-definite, diagonal, control gain matrix. After substituting (40) into the open-loop dynamics of (38), we obtain the following closed-loop dynamics for $\eta(t)$

$$\bar{J}_f\dot{\eta} = -K_2\eta - \bar{A}^T J^{-T} e. \tag{41}$$

4.2 Energy/Power Tracking Objective

We now show how the function $g(t)$ in (34) is selected to facilitate the energy/power tracking objective. Note that we can use (13), (28), and (33) to write

$$\frac{1}{2}\bar{\omega}_{fd}^T\bar{\omega}_{fd} = E_d(t). \tag{42}$$

After substituting (34) into (42), we obtain

$$\left(\bar{A}^+u_c + \left(I_N - \bar{A}^+\bar{A}\right)g\right)^T\left(\bar{A}^+u_c + \left(I_N - \bar{A}^+\bar{A}\right)g\right) = 2E_d \tag{43}$$

where the auxiliary signal $u_c(e_o, e, h, \omega_d) \in \Re^3$ is defined as follows

$$u_c \triangleq J\left(J^{-1}h - \tilde{R}\omega_d + K_1 e\right). \tag{44}$$

After expanding (43) and making use of the properties in (12), we obtain

$$\left(\bar{A}^+u_c\right)^T\left(\bar{A}^+u_c\right) + g^T\left(I_N - \bar{A}^+\bar{A}\right)^T\left(I_N - \bar{A}^+\bar{A}\right)g = 2E_d \tag{45}$$

which can be rewritten as

$$\left\|\bar{A}^+u_c\right\|^2 + \left\|\left(I_N - \bar{A}^+\bar{A}\right)g\right\|^2 = 2E_d. \tag{46}$$

Now, given some desired kinetic energy function $E_d(t)$ or given some power requirement $P_d(t)$ where $E_d(t) = \int_0^t P_d(\sigma)d\sigma$, we can design $g(t)$ according to

$$\left\|\left(I_N - \bar{A}^+\bar{A}\right)g\right\| = \sqrt{2E_d - \left\|\bar{A}^+u_c\right\|^2} \tag{47}$$

to ensure that the kinetic energy tracks the desired kinetic energy. Note that there are many ways that $g(t)$ can be designed to satisfy (47).

Remark 2 *Note that (47) mandates that a condition be placed on $E_d(t)$. Specifically, we require that*

$$E_d(t) \geq \frac{\left\|\bar{A}^+u_c\right\|^2}{2} \quad \forall t \in [0,\infty), \tag{48}$$

to ensure that (47) is well-posed (See (44) for the definition of $u_c(t)$). This condition can be physically interpreted as requiring that the desired energy in the flywheels be large enough to supply the energy required for the attitude control function.

4.3 Composite Stability Analysis

We now use Lyapunov's stability theory to prove that the attitude, energy, and power tracking objectives have been met. The main result is summarized by the following theorem.

Theorem 1 *The torque control input composed of (40), (34), and (47) ensures asymptotic attitude, energy, and power tracking in the sense that*

$$\lim_{t\to\infty} e(t), \eta(t), \eta_E(t), \dot{\eta}_E(t) = 0. \tag{49}$$

Proof: We begin by defining the nonnegative function

$$V(e_o, e, \eta) \triangleq V_1(e_o, e) + \frac{1}{2}\eta^T\bar{J}_f\eta \tag{50}$$

where $V_1(t)$ was defined in (29). After taking the time derivative of $V(t)$ along (41) and substituting (35), we obtain

$$\begin{aligned}\dot{V} &= -e^T K_1 e + e^T J^{-1}\bar{A}\eta - \eta^T K_2\eta - \eta^T\bar{A}^T J^{-T} e \\ &\leq -\lambda_{\min}(K_1)\|e\|^2 - \lambda_{\min}(K_2)\|\eta\|^2\end{aligned} \tag{51}$$

where $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue of the matrix. Based upon the structure of (50) and (51), we can show that $e(t), \eta(t) \in \mathcal{L}_\infty \cap \mathcal{L}_2$. From (1) and (4) it is clear that $h(t) \in \mathcal{L}_\infty$. We can utilize (44) and (47) to show that $u_c(t), g(t) \in \mathcal{L}_\infty$. Since $g(t) \in \mathcal{L}_\infty$, we can use (34) to show that $\bar{\omega}_{fd}(t) \in \mathcal{L}_\infty$. Since $\eta(t), \bar{\omega}_{fd}(t) \in \mathcal{L}_\infty$, we can use (28) and (33) to show that $\omega_f(t), \bar{\omega}_f(t) \in \mathcal{L}_\infty$; hence, we know $h_f(t) \in \mathcal{L}_\infty$ from (5). We can now use (4) to show $\omega(t) \in \mathcal{L}_\infty$ and then (26) and the fact that $\omega_d(t)$ is assumed bounded to show that $\tilde{\omega}(t) \in \mathcal{L}_\infty$. The previous boundedness statements along with (24) and (41) can be utilized to prove that $\dot{e}(t), \dot{\eta}(t) \in \mathcal{L}_\infty$. After taking the time derivative of (44) and (47), we are able to show that $\dot{u}_c(t), \dot{g}(t) \in \mathcal{L}_\infty$. Since $\dot{g}(t) \in \mathcal{L}_\infty$, we know from (39) and (40) that $\tau_f(t) \in \mathcal{L}_\infty$. Standard signal chasing arguments can now be employed to show that all other closed-loop signals remain bounded. Finally, we can use the above information along with Barbalat's Lemma [4, 11] to show that $\lim_{t\to\infty} e(t), \eta(t) = 0$.

In order to prove the third result of (49), we use (14), (42), and (28) to write

$$\eta_E = \frac{1}{2}\bar{\omega}_{fd}^T\bar{\omega}_{fd} - \frac{1}{2}\bar{\omega}_f^T\bar{\omega}_f. \tag{52}$$

After adding and subtracting the term $\frac{1}{2}\bar{\omega}_{fd}^T\bar{\omega}_f$ to the right-hand side of (52), we have

$$\eta_E = \frac{1}{2}\bar{\omega}_{fd}^T\bar{\omega}_{fd} - \frac{1}{2}\bar{\omega}_{fd}^T\bar{\omega}_f + \frac{1}{2}\bar{\omega}_{fd}^T\bar{\omega}_f - \frac{1}{2}\bar{\omega}_f^T\bar{\omega}_f. \tag{53}$$

After collecting terms in (53), we obtain

$$\begin{aligned}\eta_E &= \frac{1}{2}\bar{\omega}_{fd}^T\left(\bar{\omega}_{fd} - \bar{\omega}_f\right) + \frac{1}{2}\left(\bar{\omega}_{fd} - \bar{\omega}_f\right)^T\bar{\omega}_f \\ &= \frac{1}{2}\bar{\omega}_{fd}^T\eta + \frac{1}{2}\eta^T\bar{\omega}_f\end{aligned} \tag{54}$$

where (33) was used. Since we already know that $\lim_{t\to\infty}\eta(t) = 0$ and that the signals $\bar{\omega}_f(t)$ and $\bar{\omega}_{fd}(t)$ are bounded, it is clear from (54) that $\lim_{t\to\infty}\eta_E(t) = 0$. Finally, the fourth result of (49) can be proven by first showing that $\ddot{\eta}_E(t)$ is bounded (hence, $\dot{\eta}_E(t)$ is uniformly continuous) and then noting that $\eta_E(t)$ has a finite limit as $t \to \infty$. The integral form of Barbalat's Lemma [4, 11] can then be applied to show that $\lim_{t\to\infty}\dot{\eta}_E(t) = 0$. $\square$

5 Adaptive Control Design

In this section, we redesign the controller to compensate for the uncertainty associated with the spacecraft inertia matrix J. Since the system model is linear in the elements of J, we define a constant parameter vector, $\theta \in \Re^6$, as follows[2]

$$\theta \triangleq \left[\begin{array}{cccccc} J_{11} & J_{12} & J_{13} & J_{22} & J_{23} & J_{33} \end{array} \right]^T \qquad (55)$$

where J_{ij} are the elements of J. Since θ is unknown, the subsequent controller will contain an adaptation law that generates a dynamic parameter estimate, denoted by $\hat{\theta}(t) \in \Re^6$. The mismatch between the actual and estimated parameters is defined as

$$\tilde{\theta} \triangleq \theta - \hat{\theta} \qquad (56)$$

where $\tilde{\theta}(t) \in \Re^6$ denotes the parameter estimation error.

5.1 Attitude Tracking Objective

We begin with the same nonnegative function used in Section 4.1

$$V_{a1}(e_o, e) = (e_o - 1)^2 + e^T e. \qquad (57)$$

After taking the time derivative of (57) along (24) and (25) and simplifying the resulting expression, we obtain

$$\dot{V}_{a1} = e^T \tilde{\omega}. \qquad (58)$$

We now add and subtract the term $e^T K_1 e$ to the right-hand side of (58) to produce

$$\dot{V}_{a1} = -e^T K_1 e + e^T \eta_a \qquad (59)$$

where $K_1 \in \Re^{3\times 3}$ is a constant, positive-definite, diagonal, control gain matrix, and the auxiliary tracking error signal, denoted by $\eta_a(t) \in \Re^3$, is defined as follows

$$\eta_a \triangleq \tilde{\omega} + K_1 e. \qquad (60)$$

The backstepping design procedure along with the structure of (59) motivates us to drive the signal $\eta_a(t)$ to zero. After taking the time derivative of (60), multiplying through by J, and making use of (26), we obtain

$$J\dot{\eta}_a = J\dot{\omega} - J\dot{\tilde{R}}\,\omega_d - J\tilde{R}\dot{\omega}_d + JK_1\dot{e}. \qquad (61)$$

After utilizing (1), (2), (4), and (5), we can manipulate the dynamics for $\omega(t)$ into the following form

$$J\dot{\omega} = -\omega^\times J\omega - \omega^\times AJ_f\omega_f - A\tau_f. \qquad (62)$$

After substituting (62) along with (24) into (61) and then utilizing the fact that $\dot{\tilde{R}} = -\tilde{\omega}^\times \tilde{R}$, we have

$$J\dot{\eta}_a = Y\theta - \omega^\times AJ_f\omega_f - A\tau_f \qquad (63)$$

where the linear parametrization $Y(e, e_o, \omega, \omega_d, \dot{\omega}_d)\,\theta$ is defined as follows

$$Y(\cdot)\,\theta = -\omega^\times J\omega + J\left(\tilde{\omega}^\times \tilde{R}\omega_d - \tilde{R}\dot{\omega}_d\right) + \frac{JK_1}{2}\left(e^\times\tilde{\omega} + e_o\tilde{\omega}\right) \qquad (64)$$

and θ was defined in (55). Based on the structure of (63) and the subsequent stability analysis, we design the control torque input as follows

$$\tau_f = A^+\left(Y\hat{\theta} + K_2\eta_a + e - \omega^\times AJ_f\omega_f\right) + \left(I_N - A^+A\right)g_a \qquad (65)$$

where $K_2 \in \Re^{3\times 3}$ is a constant, positive-definite, diagonal, control gain matrix, $g_a(t) \in \Re^N$ is an auxiliary function that will be later designed to facilitate the energy/power tracking objective, and the parameter estimate $\hat{\theta}(t)$ is generated via the following dynamic update law

$$\dot{\hat{\theta}} = \Gamma Y^T \eta_a \qquad (66)$$

with $\Gamma \in \Re^{6\times 6}$ being a constant, positive-definite, diagonal, adaptation gain matrix. After substituting (65) into (63), we obtain the closed-loop dynamics for $\eta_a(t)$ as follows

$$J\dot{\eta}_a = -K_2\eta_a + Y\tilde{\theta} - e \qquad (67)$$

where (10), (12), and (56) have been utilized.

5.2 Energy/Power Tracking Objective

In order to ensure energy/power tracking, we take the time derivative of (14) and make use of (2) and (5) to obtain

$$\dot{\eta}_E = \dot{E}_d - \omega_f^T\tau_f. \qquad (68)$$

After substituting (65) into (68), we obtain the following expression

$$\dot{\eta}_E = \dot{E}_d - \omega_f^T A^+ u_{ca} - \omega_f^T\left(I_N - A^+A\right)g_a \qquad (69)$$

where the auxiliary signal $u_{ca}(e, e_o, \eta_a, \omega, \omega_d, \dot{\omega}_d, \omega_f) \in \Re^3$ is defined as follows

$$u_{ca} \triangleq Y\hat{\theta} + K_2\eta_a + e - \omega^\times AJ_f\omega_f. \qquad (70)$$

From the form of (69), we design the signal $g_a(t)$ such that the following equation is satisfied

$$\omega_f^T\left(I_N - A^+A\right)g_a = \dot{E}_d - \omega_f^T A^+ u_{ca} + k_E\eta_E \qquad (71)$$

where k_E is a constant, positive control gain. The minimum norm solution of (71) is given by

$$g_a = \begin{array}{l} \left[\left(I_N - A^+A\right)\omega_f\left[\omega_f^T\left(I_N - A^+A\right)\omega_f\right]^{-1}\right. \\ \left.\left(\dot{E}_d - \omega_f^T A^+ u_{ca} + k_E\eta_E\right)\right] \end{array} \qquad (72)$$

where (12) was used. Note that this solution exists if $\left(I_N - A^+A\right)\omega_f \neq 0$, which implies that $\omega_f \neq 0$ or that $\omega_f(t)$ be in the column space of $I_N - A^+A$. Thus, if the solution exists, substitution of (72) into (69) gives

$$\dot{\eta}_E = -k_E\eta_E. \qquad (73)$$

5.3 Composite Stability Analysis

Theorem 2 *The torque control input composed of (65), (66), and (72) ensures asymptotic attitude, energy, and power tracking in the sense that*

$$\lim_{t\to\infty} e(t), \eta_a(t), \eta_E(t), \dot{\eta}_E(t) = 0, \qquad (74)$$

provided that $\omega_f(t) \neq 0$ or that $\omega_f(t)$ be in the column space of $I_N - A^+A$ for all time.

[2] Since J is known to be symmetric, θ is only required to have 6 elements instead of the 9 which the inertia matrix actually contains.

Proof: We define the following nonnegative function

$$V_a(e_o, e, \eta_a, \eta_E, \tilde{\theta}) = V_{a1}(e_o, e) + \frac{1}{2}\eta_a^T J \eta_a + \frac{1}{2}\eta_E^2 + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \tag{75}$$

where $V_{a1}(t)$ was defined in (57). After taking the time derivative of $V_a(t)$ along (67), (73), and (66) (note from (56) that $\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$) and making use of (59), we obtain

$$\dot{V}_a = -e^T K_1 e - \eta_a^T K_2 \eta_a - k_E \eta_E^2. \tag{76}$$

Arguments similar to those used in the proof of Theorem 1 can now be followed to arrive at the results of (74) and show the boundedness of all signals.

5.4 Elimination of Flywheel Velocity Assumption

While formulating (5) we originally made the assumption that the flywheels spin at a much higher speed than the spacecraft. In this section, we provide an extension for how this limitation can be eliminated for the adaptive controller. To this end, we now rewrite the axial angular momentum of the flywheels, which was originally defined in (5), as follows

$$h_f = J_f\left(\omega_f + A^T\omega\right). \tag{77}$$

After taking the time derivative of (4) and making use of (1) and (2), we can manipulate the dynamics for $\omega(t)$ into the following form

$$J\dot{\omega} = -\omega^\times J\omega - \omega^\times A h_f - A\tau_f. \tag{78}$$

If we now substitute (77) back into (78), we obtain

$$J\dot{\omega} = -\omega^\times J\omega - \omega^\times A J_f \omega_f - \omega^\times A J_f A^T \omega - A\tau_f \tag{79}$$

After substituting (79) along with (24) into (61) and then utilizing the fact that $\dot{\tilde{R}} = -\tilde{\omega}^\times \tilde{R}$, we have

$$J\dot{\eta}_a = Y\theta - \omega^\times A J_f \omega_f - \omega^\times A J_f A^T \omega - A\tau_f \tag{80}$$

where $Y(e, e_o, \omega, \omega_d, \dot{\omega}_d)\theta$ was previously defined in (64). Based upon the structure of (80) and the stability analysis provided above, we design the new control torque input as follows

$$\begin{aligned}\tau_f = \; & A^+\left(Y\hat{\theta} + K_2\eta_a + e - \omega^\times A J_f \omega_f - \omega^\times A J_f A^T \omega\right) \\ & + \left(I_N - A^+ A\right) g_a\end{aligned} \tag{81}$$

where $\hat{\theta}(t)$ is updated according to (66) and $g_a(t)$ was designed in (72).

It is important to mention that the elimination of the flywheel velocity assumption cannot be done in the context of the model-based control strategy of Section 4. Specifically, the stability analysis proof fails for the model-based technique since we are no longer able to show boundedness of the spacecraft angular velocity ω due to the new definition of h_f given in (77).

6 Conclusions

In this paper, we have developed two nonlinear controllers for a spacecraft/flywheel system which integrate the energy storage and attitude control functions into a single strategy. The system model assumed that the flywheels are operating in a reaction wheel-type mode. A model-based controller was initially designed that guaranteed asymptotic attitude, energy, and power tracking with no energy/power singularities. To overcome the need for exact knowledge of the spacecraft inertia, an adaptive controller was then designed that ensured the same asymptotic tracking results; however, the controller required some conditions on the flywheel angular velocity to ensure that energy/power singularities are avoided. Future work will incorporate the control of the discharge cycle of the flywheel during times of eclipse when no power is being generated by the solar arrays. The problem of unloading the wheels introduces the complexity of decelerating the wheels at a given rate to maintain the necessary power requirements for the spacecraft.

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