MISSION PERFORMANCE MEASURES FOR SPACECRAFT FORMATION FLYING

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ABSTRACT

Clusters of low-performance spacecraft flying in formation may provide enhanced performance over single high-performance spacecraft. This is especially true for remote sensing missions where interferometry or stereographic imaging may provide higher resolution data. The configuration of such formations vary during an orbit due to orbital dynamics, and over larger time scales due to perturbations. Selection of a configuration should be based on overall performance of the formation. In this paper, performance measures are developed and evaluated based on integration over one orbit. The measures involve angular separation of spacecraft, instantaneous overlap access area, and an area-based measure of the separation of the spacecraft. An optimization scheme is used to determine the best configuration for a four-spacecraft formation.

INTRODUCTION

The formation flying concept has recently become popular due to its advantages in remote sensing missions and flexible long term mission capabilities. A multiple spacecraft platform allows increased flexibility in mission scenarios as new technology is developed. Distribution of components on a number of satellites allows the advantage that a single component failure results in the replacement of a small, cheap spacecraft and not mission failure. Areas of formation flying application in remote sensing include stereographic viewing, interferometry, and synthetic apertures. The advantage of formation flying for remote sensing missions is primarily the difference in perspective obtained from different satellites in a formation. Therefore it is necessary to design the orbits of spacecraft in a formation so that there is an appropriate separation between them to provide different viewing perspectives of targets of interest.

Formation flying is a relatively new concept. Preliminary investigations into formation flying were considered by Folta, Bordi, and Scolese.[1] Relative navigation control algorithms have been developed by Folta and Quinn[2] and DeCou.[3] The feasibility of using Goddard Space Flight Center's (GSFC) Onboard Navigation System for relative navigation was examined by Gramling *et al.*[4] Wickert *et al.*[5] investigated the feasibility of using a distributed architecture for a space-based radar detection system. However, performance measures that allow comparative formation orbit design have not received significant attention in the literature.

We develop general formation performance measures similar to those for single satellite missions. Orbit performance measures are developed that evaluate the separation between spacecraft in a formation. Algorithms for calculating the suggested measures are developed. Finally we postulate a hypothetical formation and apply the performance measures to demonstrate their application and nature.

SYMBOLS

A_o	overlap area, m^2
A_{refo}	reference overlap area, m^2
A_s	separation area, m^2
A_{refs}	reference separation area, m^2
A_{inner}	inner overlap area, m^2
A_{outer}	outer overlap area, m^2
α_{ij}	angle between the i^{th} and j^{th} satellites, rad
β	angle between sub-satellite points, rad
R_e	Earth radius, m
n	number of spacecraft in formation
n_i	number of points defining overlap region
ρ,ϵ	angular radii of horizon circles, rad
T	orbital period, s
θ	rotation angle of inner overlap area, rad

FORMATION MODELING

Spacecraft are subject to disturbing forces such as atmospheric drag, solar radiation pressure, and third body effects as well as the non-spherical central body force. The orbit performance measures to be developed subsequently require knowledge of the position of all spacecraft in the formation over a period. For a comparative study of candidate orbits the performance measure must be evaluated for many different initial states, resulting in a large number of orbit integrations. High fidelity force models slow the performance but are not expected to have a significant effect on preliminary formation design. Therefore, to simplify the preliminary analysis we neglect the disturbing forces and approximate the orbital dynamics with Keplerian motion.

For a formation composed of n satellites there are $6n$ degrees of freedom. A large number of state variables makes it difficult to examine the change in a performance measure due to changes in the initial state. We explore the design space of a formation by decreasing the number of state variables used to define a formation uniquely.

To ensure the formation's periodic motion over an orbit we assume all spacecraft have the same semi-major axis. This results in all of the spacecraft having the same period because the period depends only on the orbital parameter and the semi-major axis. If all the spacecraft in a formation do not have the same period then eventually it will separate and there will be no common coverage area. If we hold the semi-major axis constant for all satellites we decrease the number of state variables from $6n$ to $5n$.

To further reduce the number of state variables we assume all spacecraft to be in circular orbits. We obviously lose n state variables by fixing the eccentricity to zero. We lose another n variables because it is now simpler to locate a satellite in its orbital plane. For non-circular orbits we require two orbital elements to locate a satellite in the orbital plane, the argument of periapsis and the true anomaly. However for orbits with zero eccentricity the argument of periapsis is undefined. For a circular orbit, to locate the spacecraft's position in the orbital plane we require only a single variable, the true longitude at epoch. We have now reduced the number of state variables to define a formation to $3n$.

PERFORMANCE MEASURES

Larson and Wertz[6] present several classical performance measures used to evaluate the effectiveness of orbit candidates. The instantaneous access area is commonly used to measure how much area a spacecraft is capable of viewing at a given instant. The area access rate is the rate at which land enters or leaves the access area. Multiple satellite missions are more complex and require more complicated performance measures.

Overlap Area

We define the overlap area as the area on the sphere that can be viewed simultaneously by all satellites in the formation as seen in Fig. 1. A measure of this type provides a conventional way of evaluating formation performance similar to the instantaneous access area for single satellite missions. The size of the overlap area is obviously dependent on the amount of separation between spacecraft in formation. For formations with small separations the overlap area is large and on the order of the instantaneous access area for a single satellite at the same radius. As formation separations increase the overlap area decreases.

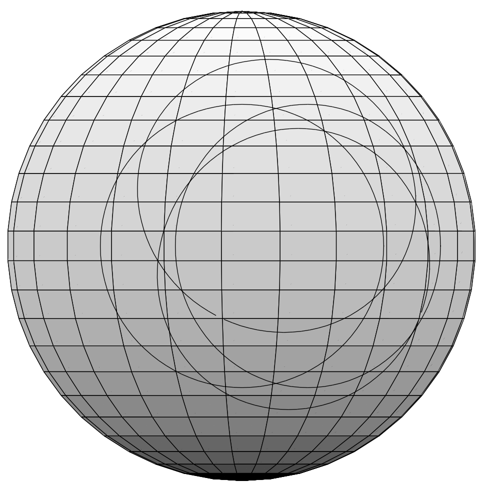

Figure 1: The Overlap Area

The algorithm to calculate the overlap area assumes a spherical Earth and uses spherical trigonometry[7] to calculate the area on the sphere. The overlap of small circles that define the horizons for each spacecraft defines the border of the region. For a point to lie in the overlap area it must either lie within the boundary of every horizon circle or on the borders. Using these criteria the intersection points defining the overlap region are determined. Knowing the defining intersection points the area is broken down into two sub-regions. The inner region is defined by connecting the intersection points with great circles. The outer region is defined by the lunes created by the horizon circles and the inner region. Calculation of the inner area is simple because its border is defined by great circles for which there is a simple spherical trigonometry relation:

$$A_{inner} = 2\pi R_e(1 - (n_i - 1))\theta \tag{1}$$

Because the outer region is bounded by small circles, there is a considerable increase in the complexity of the area calculation:

$$A_{outer} \quad = \quad 2\pi R_e - 2R_e \cos\rho \, \arccos\left(\frac{\cos\epsilon - \cos\rho\cos\beta}{\sin\rho\sin\beta}\right) \tag{2}$$

$$- 2R_e \cos\epsilon \ \arccos\left(\frac{\cos\rho - \cos\epsilon\cos\beta}{\sin\epsilon\sin\beta}\right)$$
$$- 2R_e \arccos\left(\frac{\cos\beta - \cos\epsilon\cos\rho}{\sin\epsilon\sin\rho}\right)$$

where ρ and ϵ are the angular radii of two small circles defining a lune and β is the center-to-center distance.

Separation Area

We define the separation area as the region enclosed by the outermost sub-satellite points as seen in Fig. 2. It is developed as a conflicting measure to the overlap area. For formations with small separations the separation area is small. As separations increase the separation area also increases. The border of the separation area is formed by connecting the outer sub-satellite points with great circles. Calculation of this area is simpler than the overlap area because its borders are defined by great circles. Once the border has been determined the rotation angle can be calculated using simple trigonometric relations and the area is calculated from Eq. (1).

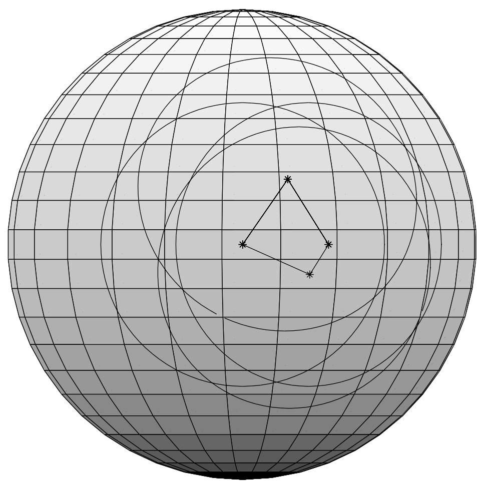

Figure 2: The Separation Area

Orbit Performance Measures

We desire measures that allow comparison of candidate orbits for satellites in formation. We develop a strategy that takes a measure of the instantaneous relative positions of a formation and integrate the measure over a period. The primary difference between the different orbit measures is the instantaneous relative position metric.

It is intuitive that for formations with little spacing the separation area is small while the overlap area is large. As separation of the formation increases the overlap area decreases and the separation area increases. The conflicting nature of the area measures can be utilized in an orbit performance measure such as,

$$W_1 = \frac{1}{T}\int_0^T \frac{1}{2}\left(\frac{A_s}{A_{refs}} + \frac{A_o}{A_{refo}}\right)\ dt \tag{3}$$

This metric integrates the sum of the normalized area measures over a period. However, the calculation of the overlap area is expensive. The overlap region is of interest for mission performance but is too expensive to be used as a formation design tool. We wish to capture the nature of the area measure technique without the computational expense.

Angular Separation Metric

The measure involving overlap and separation areas is computationally expensive due to the spherical trigonometry relations required to calculate the overlap area. We desire a metric that penalizes the formation if it is too close or too separated without the numerical complexity of the area method. We use the fact that formations with large separation areas also have large angular separations between spacecraft. Similarly, formations with large overlap areas have small angular separations. A measure utilizing separation angles is much less computationally demanding because the angle between satellites is much simpler to calculate than the area measures. We formulate a new measure

$$W_2 = \frac{1}{T}\int_0^T \frac{1}{n}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} w(\alpha_{ij})\ dt \tag{4}$$

The measure integrates over one period an instantaneous weight based on the angular separations of the satellites. The instantaneous weight w is a heuristic formula that measures the desirability of angular separation between the i^{th} and j^{th} spacecraft. Two forms for the function are considered as seen in Fig. 3. Both forms have a maximum of one at angular separations resulting in spatial separations of 3 km at a radius of 8000 km. The first function is a quartic polynomial which does not apply a negative weight for formations outside the allowed angular separation limit. The second function is parabolic and applies a negative weight for formations with unacceptable angular separations.

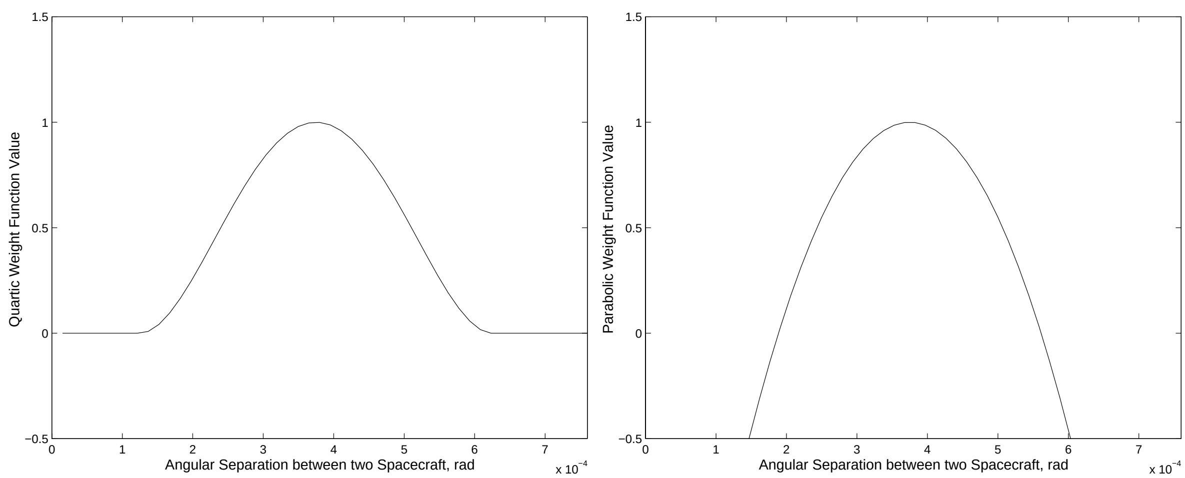

Figure 3: Heuristic Weight Functions, w

Weighted Separation Area Metric

We develop a third metric to capitalize on the relative simplicity of the separation area calculation in comparison to the overlap area. This measure is applicable to formations composed of more than 2 spacecraft because for 2 spacecraft scenarios the separation area is undefined. By applying a weight function to the separation area we penalize formations whose separation area is either too large or too small. This captures the conflicting nature of the overlap/separation measure with a significant decrease in computational complexity. By integrating the weight of the separation area over an orbit we can get a measure of orbit performance. Specifically

$$W_3 = \frac{1}{T}\int_0^T w(A_s)\ dt \tag{5}$$

For this case w is a heuristic formula of the form seen in Fig. 4. The ideal separation area occurs at $2 \times 10^6 m^2$. Allowable separations occur between $1 \times 10^6 m^2$ and $3 \times 10^6 m^2$. Configurations outside of this range receive a negative weight.

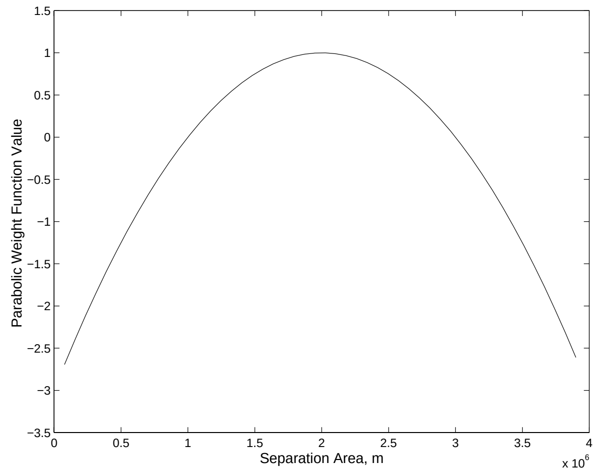

Figure 4: Parabolic Weight Function w for Angular Separation Metric

IMPLEMENTATION AND RESULTS

By fixing the semi-major axis for all spacecraft and assuming circular orbits we can represent a formation uniquely by $3n$ state variables. This is still a large number of state variables to examine the change in performance measures due to changes in the initial state. A further reduction in order is achieved by referencing all spacecraft to one of the spacecraft in the formation. Because the performance measures developed are not dependent on absolute position with respect to the Earth, only on the relative position of spacecraft, there is no loss of generality in doing so. We are simply picking one of a family of solutions that results in the same weight function value over an orbit. This is justified by realizing that there is an infinite number of state vectors such that the relative motion of the spacecraft is the same, only shifted in position with respect to the earth. Eliminating the three variables used to define the reference orbit we are left with $3(n-1)$ state variables to describe the formation.

As a preliminary example we define a formation to have four spacecraft with semi-major axes of 8000 km. We then have nine state variables to define the orbits uniquely. Although we have reduced the state variables from 24 to nine the resulting performance problem is still considerably complex. We now develop a representation that allows these nine variables to be represented by only two variables defined as the initial longitudinal and latitudinal separations. Since we are mainly concerned with the relative motion of the spacecraft, we have postulated a formation without concern to its path with respect to any specific points on the earth. We have fixed two spacecraft to be in circular, coplanar orbits. The separation between the coplanar spacecraft is defined as the longitudinal separation. The remaining two orbits are inclined at equal amounts with their ascending nodes placed $180°$ apart. At the initial epoch the satellites in the inclined orbits are at their maximum declination with respect to the coplanar orbits and lie midway between the two equatorial spacecraft. The initial separation between the inclined satellites is defined as the latitudinal separation.

Because we have only two state variables the effect of the initial state on the performance can be investigated using graphical techniques. The measures have been evaluated using MatLab over one period for a range of initial conditions. In all plots the independent variables have been converted from angular separation in radians, to spatial separation at the orbit radius to provide a more intuitive understanding.

Figure 5 shows the surface plot generated from the application of Eq. (4). To generate this plot the quartic angular weight function in Fig. 3 is used. The maximum value is seen to occur for a longitudinal separation of 4400 m and an initial latitudinal separation of 2800 m. The weight function is zero for all angles resulting in spatial separations of less than 1 km. As expected the performance measure is zero for those initial conditions that result in spatial separations always less than 1 km. Notice the longitudinal separation has a larger effect on the performance than the latitudinal separation. If the initial longitude is not acceptable according to the weight function it remains so over an orbit does and not contribute to the performance. The latitudinal separation has a different effect on performance due to the oscillatory nature of the inclined spacecraft with respect to the coplanar pair. If the initial separation is large and falls outside of the acceptable separation range there is still a contribution to the performance over the orbit. This is due to the fact that over an orbit the latitudinal separation starts at the initial value, goes to zero, and then increases back to original value. In doing so the angular separation between the inclined spacecraft must pass through the acceptable range and positively contribute to the performance.

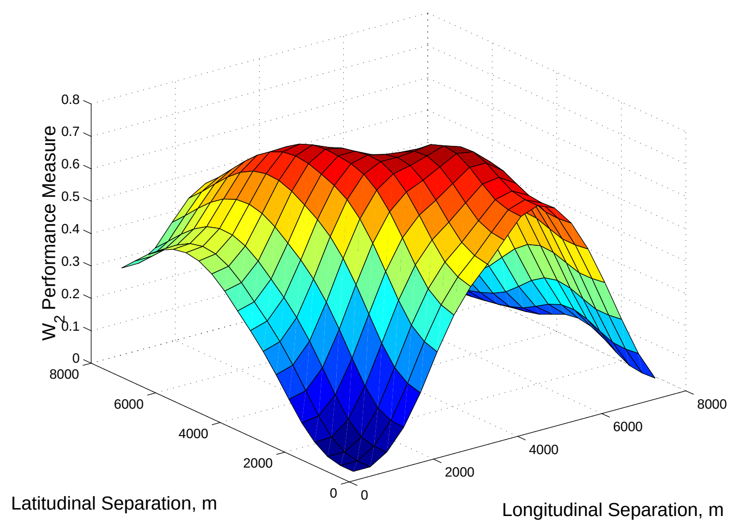

Figure 5: Angular Separation Measure with Quartic Weight Function

Application of the W_2 performance measure with the parabolic weight function is presented in Fig. 6. The optimum formation according to this performance measure is seen to occur for a longitudinal separation of 3190 m and an initial latitudinal separation of 3590 m. The effects of latitudinal and longitudinal separation on the parabolic case are similar to the quartic case. The longitudinal separation has a larger effect on the performance. The surface is smoother for the parabolic case than for the quartic weight function case. This smoothness is due primarily to the negative weight imposed by the parabolic function for separations that do not fall within the acceptable range.

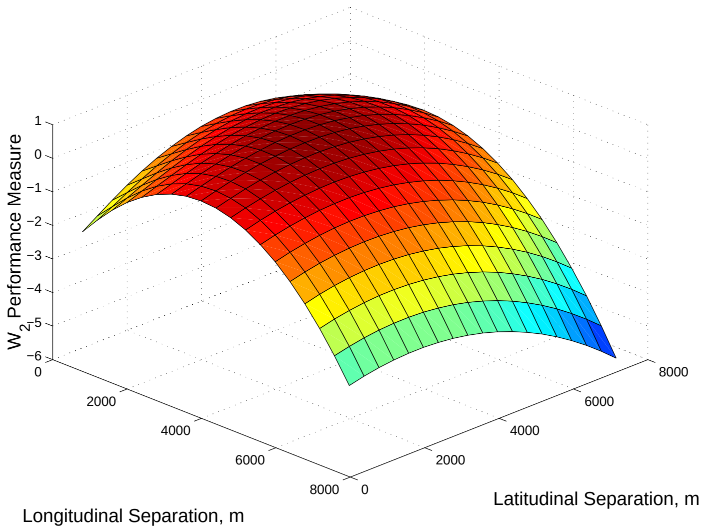

Figure 6: Angular Separation Measure with Parabolic Weight Function

The application of W_3 to the formation is seen in Fig. 7. The surface has a large region of near maximum weight as opposed to the surfaces in Fig. 5 and Fig. 6 that have distinct maxima. This demonstrates that there is a region within the design space that has near similar separation area integrals over an orbit. For scenarios with large longitudinal and latitudinal separations the performance is poor because the separation area is not acceptable over most of the orbit. If either of the separations goes to zero the separation area goes to zero and the performance measure tends toward the instantaneous weight function value at zero.

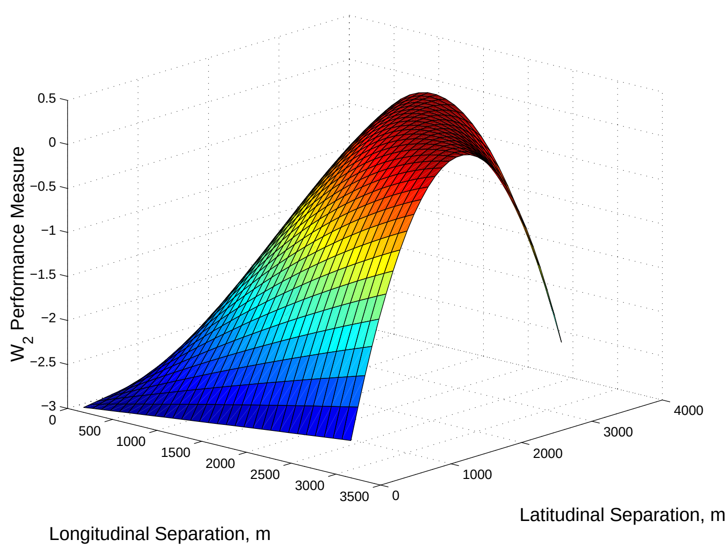

Figure 7: Weighted Separation Area Measure

We now investigate a higher order formation model using the W_2 performance measure with the parabolic instantaneous weight function seen in Fig. 3. Previously we justified representing the state of a formation of four spacecraft with nine variables. We now reduce the order by two to seven. Specifically we allow the longitude of the ascending node, inclination, and true longitude at epoch for the two inclined orbits to vary. We also allow the longitudinal separation to vary by allowing the true longitude at epoch of the leading coplanar spacecraft to vary.

For seven state variables the solution can no longer be found using graphical techniques. Instead we use Newton's method[8] to determine a maximum in the performance. To begin the iterative scheme we use the seven-variable representation of the optimal two-variable solution as an initial guess. The state vector is updated according to

$$\mathbf{x}_{i+1} = \mathbf{x}_i - \left[\frac{\partial \mathbf{F}}{\partial \mathbf{x}}(\mathbf{x}_i)\right]^{-1} \mathbf{F}(\mathbf{x}_i) \tag{6}$$

where $\mathbf{F}(\mathbf{x}_n)$ is the gradient of W_2 and $\partial\mathbf{F}(\mathbf{x}_n)/\partial\mathbf{x}$ is the Jacobian of W_2. Because analytical derivatives of W_2 are not available approximations for the gradient and the Jacobian are found using one-sided finite differencing.

A maximum in the performance measure W_2 using the parabolic weight function is found. To illustrate the results the relative motion of the spacecraft has been exaggerated and illustrated in Fig. 8. There is no relative motion between the coplanar spacecraft so they are seen as fixed points. The inclined orbits produce figure eight patterns over one orbit. For clarity we label the left figure eight as orbit one and the right as orbit two. At the initial epoch the spacecraft in orbit one is at the top of the figure eight. We define clockwise motion for a figure eight to be the type of motion occurring in the upper half of the figure. Spacecraft one moves in clockwise motion according to this definition. The spacecraft in orbit two begins at the bottom of the figure eight and also moves in clockwise motion. For the actual example formation, these figure eight's appear as nearly straight vertical lines with a slight separation between them.

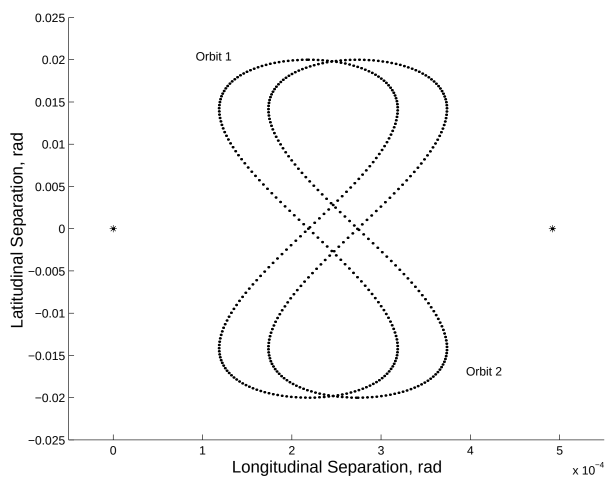

Figure 8: Exaggerated Relative Motion for Seven-Variable Solution

CONCLUSIONS

The goal of this paper is to contribute to the area of performance measures for formation flying satellites. We developed general measures similar to those for single spacecraft missions. The overlap area and the separation area represent conflicting measures of the amount of separation in a formation. Orbit performance measures were developed to ascertain if the separation in a formation is meeting necessary requirements over an orbit. The metrics are based on angular separation and area based measures of separation. Two formation models were postulated. A simple two degree of freedom formation was investigated and optimum configurations determined according to the performance measures using graphical techniques. A more complex seven-variable model was examined for one of the orbit performance metrics and an optimum solution found using Newton's method.

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