Tests of Magnetometer/Sun-Sensor Orbit Determination Using Flight Data

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Abstract

A magnetometer-based orbit determination batch filter has been improved and tested with real flight data from Dynamics Explorer 2 (DE-2), the Magnetic Field Satellite (MAGSAT), and the Ørsted satellite. These tests have been conducted in order to determine the performance of a low-cost autonomous orbit determination system. The spacecraft's orbit, magnetometer biases, and (optionally) correction terms to the Earth's magnetic field are estimated by this filter. Sun-sensor data in addition to magnetometer data is used when field model correction terms are estimated. The filter improvement takes the form of a new dynamics model. The best performance that has been achieved with this filter is a maximum position error of 4.99 km for a 2-day batch of DE-2 data with 1st order/degree field model corrections, and 2.16 km for a 24-hour batch of MAGSAT data with 2nd order/degree field model corrections. The maximum position error for a 24-hour batch of Ørsted data is 59.50 km without field model corrections, but is only 2.47 km with 6th order/degree field model corrections.

I. Introduction

Orbit determination is an old topic in celestial mechanics and is an essential part of satellite navigation. Traditional ground-based tracking methods that use range and range-rate

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measurements can provide an orbit accuracy as good as a few centimeters[1]. Autonomous orbit determination using only onboard measurements can be a requirement of military satellites in order to guarantee independence from ground facilities[2]. The rapid increase in the number of satellites also increases the need for autonomous navigation because of bottlenecks in ground tracking facilities[3].

A filter that uses magnetometer measurements provides one possible means of doing autonomous orbit determination. This idea was first introduced by Psiaki and Martel[4] and has been tested by a number of researchers since then[3,5-8]. Magnetometers fly on most spacecraft for attitude determination and control purposes. Therefore, successful autonomous orbit determination using magnetometer measurements can make the integration of attitude and orbit determination possible and lead to reduced mission costs.

Ground-based tests of such systems that used real flight data reported position accuracies in the range from 8km to 125km[3,6-8]. Psiaki *et al*. conjectured that some of the inaccuracy was caused by uncertainty in the Earth's magnetic fields[6]. To combat this source of uncertainty, batch filters have been developed that estimate field model corrections in addition to the orbit of the spacecraft[9,10]. Either 3-axis star sensor data[9] or sun-sensor data[10], in addition to the magnetometer data, was added in these studies in order to make the orbit and the field model corrections simultaneously observable. These studies used truth-model simulation to test their designs, and they predicted improvements in the accuracy of magnetometer-based orbit determination. Their predicted position accuracies ranged down to better than 1 km.

The contributions of the present work are as follows. This is the first effort to test a field-model-correcting orbit determination filter with real flight data. The particular filter that has been tested is one that uses magnetometer and sun-sensor data, as in Ref. 10. Also, this work improves the orbit propagation dynamics model over what has been used in Refs 4-6 and 8-10 and shows that this refinement can improve the magnetometer-based orbit determination accuracy, especially for low altitude spacecraft (S/C).

The rest of this paper consists of 4 main sections plus conclusions. Section 2 explains how the batch filter of Ref. 10 has been improved for use in this study. Section 3 describes the spacecraft data that has been used in this study. The test results with real flight data are presented in Section 4.

II. Modifications of Batch Filter

The field-model-correcting orbit determination batch filter of Ref. 10 is extensively used in this study. In order to deal with real flight data, the orbital dynamics model had to be improved. This section explains the orbital dynamics model improvements and gives a review of the batch filter that has been used in this study.

A. Update of Orbital Dynamics Model

The orbital dynamics model that was used in the Ref. 10 filter considered only secular J_2 effects and the effects of drag on altitude and mean motion. It ignored the periodic effects of J_2, higher order gravity effects, and the drag effects on eccentricity. A more accurate dynamics model is required for dealing with real flight data.

The new dynamics model used in this study considers the full J_2 effects, and it uses a drag model that is based on the 1976 U.S. standard atmosphere. The orbit is calculated via direct numerical integration of the following equation of motion, which is expressed in an Earth-centered Earth-fixed (ECEF) reference frame.

$$\ddot{r} + 2? \times \dot{r} + ? \times (? \times r) = a_{inertial} = -\frac{m_{\oplus}}{\|r\|^3} r + a_{J2}(r) + a_d(r, \dot{r}) \tag{1}$$

where $?$ is the Earth's rotation rate vector, $m_{\oplus}$ is the geocentric gravitational constant, r is the position vector of the S/C in the ECEF frame, $a_{J2}(r)$ is the gravitational acceleration due to Earth Oblateness, and $a_d(r, \dot{r})$ is the acceleration due to drag. Note that the ECEF reference frame is defined with $+x$ pointing through the equator at the Greenwich meridian and $+z$ pointing through the north pole. The standard 4th order Runge-Kutta method with a fixed integration step size is used in the integration of eq.(1). $a_{J2}(r)$ is[11]

$$a_{J2}(r) = \frac{9}{2} \frac{\mu_{\oplus}}{\|r\|^5} J_2 R_{\oplus}^2 \left[\left(\frac{z}{\|r\|} \right)^2 - \left(\frac{1}{3} \right) \right] r + 3 \frac{\mu_{\oplus}}{\|r\|^7} J_2 R_{\oplus}^2 z \begin{bmatrix} zx \\ zy \\ -(x^2 + y^2) \end{bmatrix} \tag{2}$$

where $R_{\oplus}$ is the Earth's equatorial radius, J_2 is the gravity field's lowest zonal harmonic coefficient, and x, y, and z are the Cartesian coordinates of the S/C in the ECEF frame, $r = [x, y, z]^T$. $a_d(r, \dot{r})$ is

$$a_d(r, \dot{r}) = -\frac{\bar{b}}{2} r(r) \|\dot{r}\| \dot{r} \tag{3}$$

where r is the atmospheric mass density and $\bar{b} = C_D S / m$ is the inverse of the spacecraft's ballistic coefficient – C_D is the spacecraft's drag coefficient, S is its aerodynamic reference area, and m is its mass. $r(r)$ is calculated from a cubic spline interpolation of the natural log of tabulated data from the 1976 U.S. standard atmosphere[12].

The accuracy of the filter's new dynamics model has been checked and compared to that of the old dynamics model. This has been done by using another batch filter to estimate the 6 initial conditions for this dynamic model plus $\bar{b}$ by minimizing the sum of the square errors between the measured position time history of an actual S/C and the modeled position based on propagation of eq.(1). The closeness of this fit gives an indication of the model's propagation accuracy.

This dynamics evaluation filter has been run on real flight data from DE-2 and MAGSAT. The maximum position error of this test when using the old dynamics model is 2.16 km for a 24-hour MAGSAT data set and 4.35-4.90 km for a 48-hour DE-2 data set. The new dynamics model has a smaller maximum position error than the old one; 1.6 km for 24 hour MAGSAT data sets and 3.50-4.33 km for 48 hour DE-2 data sets. Thus, the new dynamics model has a slightly better orbital propagation accuracy than the old one and is accurate enough for use in the magnetometer-based navigation filter.

B. Estimation Vector

The batch filter's estimation vector with the new dynamics model is

$$
\begin{aligned}
p = [&x_o, y_o, z_o, \dot{x}_o, \dot{y}_o, \dot{z}_o, \bar{b}, b_x, b_y, b_z, q_1^0, q_1^1, s_1^1, \dot{q}_1^0, \dot{q}_1^1, \dot{s}_1^1, \dot{g}_1^0, \dot{g}_1^1, \dot{h}_1^1, \\
&?g_1^0, ?g_1^1, ?h_1^1, ?g_2^0, ?g_2^1, ?h_2^1, ?g_2^2, ?h_2^2, ?g_3^0, \ldots\ldots\ldots, ?h_N^N]^{\mathrm{T}}
\end{aligned}
\tag{4}
$$

where the first 6 elements are the initial position and velocity of the S/C in the ECEF frame, the 7^{th} element is the inverse of the ballistic coefficient, the 8^{th} to 10^{th} elements constitute a magnetometer bias vector, and the remaining elements are correction terms to coefficients of a spherical harmonic expansion of the Earth's magnetic field[9,10]. The field correction elements include an external ring current field model and perturbation to the Earth's internal magnetic field model. Note that N is the maximum order and degree of the field model corrections.

C. Review of Batch Filter

Except for the improved dynamics model, the batch filter used in this study is the same as the one used in Ref. 10. It finds the p estimation vector that best approximates the measurement data based on the dynamics and measurement models.

Two pseudo measurements, the magnitude of the Earth's magnetic field and the cosine of the angle between the Earth's magnetic field vector and the sun direction vector, are used. These "measurements" are independent of the spacecraft's attitude. Let $B_{mes(k)}$ be the magnetic field vector that is measured by the magnetometer and let $\hat{s}_{mes(k)}$ be the sun unit direction vector that is measured by the sun-sensor, with both measurements occurring at sample time t_k. Both of these measurements are expressed in a common S/C-fixed coordinate system. Then the two pseudo measurements are given by

$$y_{1mes(k)}(p) = \sqrt{(B_{mes(k)} - b)^T (B_{mes(k)} - b)} \tag{5a}$$

$$y_{2mes(k)}(p) = \frac{\hat{s}^T_{mes(k)}(B_{mes(k)} - b)}{y_{1mes(k)}(p)} \tag{5b}$$

where $b = [b_x, b_y, b_z]^{\mathrm{T}}$ is the estimated magnetometer bias vector expressed in S/C coordinates.

The modeled values of the two pseudo measurements are

$$y_{1\,\mathrm{mod}}(t_k; p) = \sqrt{B^T_{sez} B_{sez}} \tag{6a}$$

$$y_{2\,\mathrm{mod}}(t_k; p) = \frac{\hat{s}^T_{ECIF}(t_k) \bullet (A_{ECIF/sez} \bullet B_{sez})}{y_{1\mathrm{mod}}(t_k; p)} \tag{6b}$$

where $B_{sez}[r(p,t), q^0_1, q^1_1, \ldots\ldots\ldots, \Delta h^N_N, t]$ is the modeled Earth's magnetic field vector in local south-east-zenith coordinates, $\hat{s}_{ECIF}(t_k)$ is the sun unit vector in an Earth-Centered Inertially

Fixed reference frame (ECIF) and $A_{ECIF\,/\,sez}[r(p,t),t]$ is the transformation matrix from local south-east-zenith coordinates to ECIF coordinates. B_{sez} is calculated using a spherical harmonic expansion and includes the effects of the estimated field model corrections[9].

The nonlinear least squares cost function that gets minimized by the batch filter is

$$J(p) = \frac{1}{2}\sum_{k=1}^{M}\left[\frac{y_{1\,\mathrm{mod}}(t_k;p) - y_{1mes(k)}(p)}{s_B}\right]^2 + \frac{1}{2}\sum_{k=1}^{M}\left[\frac{y_{2\,\mathrm{mod}}(t_k;p) - y_{2mes(k)}(p)}{s_{y2(k)}}\right]^2 \tag{7}$$

where M is the number of data samples, s_B is the standard deviation of the field magnitude measurement, and $s_{y2(k)}$ is the standard deviation of the cosine pseudo measurement, which are described in Ref. 10. The second term of the cost function is calculated only when sun-sensor data is available.

The Gauss-Newton iterative numerical method is used to solve this nonlinear least-squares problem[10]. It requires the calculation of the partial derivatives of the measurement errors with respect to the p vector, which it uses to find search directions. They are also used to compute the filter's predicted estimation error covariance.

D. An Alternate Filter that does not Estimate Field Model Corrections

An alternate batch filter has been developed in order to investigate the improvement in position accuracy that is due solely to the use of the new dynamics model. It does not estimate field model corrections nor does it use sun-sensor measurements. Otherwise, it is identical to the improved filter that has been described in this section. This filter is similar to the batch filter of Ref. 6, except that Ref. 6 used the poorer dynamics model mentioned above.

E. Additional Filters

Two additional batch filters have also been developed for test purposes. One is a field-model-correcting filter with an *a priori* drag coefficient estimate with a relatively small a *priori* drag variance. This filter keeps its estimated $\bar{b}$ nearly fixed at the *a priori* value 0.03 m^2/kg by adding an additional term to the cost function of eq. (7) that minimizes the weighted square error between $\bar{b}$ and its *a priori* value. This filter is useful for a higher altitude S/C (perigee altitude above the 500-600 km range) because $\bar{b}$ become nearly unobservable due to the minimal effects of drag at such altitudes.

The other filter is a field-model-correcting filter that does not estimate orbit. This filter uses tracked position instead of the estimated S/C orbit and only estimates field model correction terms. This filter can help to determine whether field model correction estimates are compromised by the simultaneous estimation of orbit.

III. Real Flight Data

Real flight data from three spacecraft, DE-2, MAGSAT, and Ørsted, has been used to test the batch filter. These spacecraft are described below. Data from a fourth spacecraft, the Tropical Rainfall Measuring Mission (TRMM), also has been tested. Its magnetometer data, however, was uncalibrated and the per-axis residual error standard deviations were over 300 nT. This poorly calibrated data lead to large maximum position errors for the orbit determination filter, on the order of 50 km. Therefore, the results for TRMM are not included in this study because an accurately calibrated magnetometer is necessary if one wants to apply this technique.

A. DE-2

The DE-2 spacecraft was launched in August 1981 to study the coupling between the magnetosphere, the ionosphere, and the upper atmosphere. DE-2 had roughly a 290 km perigee altitude, a 810 km apogee altitude, and a high inclination, 89.9°. It operated for about two years. Magnetometer data, 3-axis attitude data derived from sensors, and position derived from ground-based tracking data are all available for DE-2. The magnetometer data for DE-2 is known to have a 28 nT 1-σ accuracy. Sun-sensor data was not directly available. Therefore, it has been synthesized by transforming the known sun position in ECIF coordinates using DE-2's estimate of the attitude transformation matrix from ECIF coordinates to S/C coordinates. The attitude uncertainty of this transformation is about 0.3°. The position of the sun in the ECIF coordinate system is calculated using the algorithm in Ref. 13 which provides a 0.006° sun position accuracy. Eclipse by the Earth is also considered in synthesizing sun-sensor data.

Tests have been run using two sets of data extracted from a CD-ROM archive of the DE-2 data that is available from the National Space Science Data Center (NSSDC). One set was measured on November 1-2, 1981 (data set A), and the other set was measured on March 15-17, 1982 (data set B). Both data sets were measured when the activity index for solar magnetic storms was low. The sample period is 60 second, and both data sets are spread out over about 48 hours. There are, however, data gaps about every 2 or 3 hours, and some of these gaps are as long as 10 hours. Thus, the actual measurement coverage time is 13.9 hours and 6.7 hours for data sets A and B, respectively. Bad magnetometer data points have been removed before testing the batch filter. A bad point is considered to occur when the measured magnetometer data is off by over 1000 nT from the modeled magnetometer data based on the known S/C position from ground tracking. Two out of 835 data points in set A and 30 out of 433 data points in set B have been edited out.

The position tracking data of DE-2 is given in altitude, latitude, and longitude and is stated to be measured in geocentric coordinates assuming a spherical Earth. In the process of checking the accuracy of the filter's new dynamics model, however, it has been found that the position data must have been measured with respect to an ellipsoidal Earth, not with respect to a spherical Earth. Therefore, it is assumed that the tracking data of DE-2 has been measured with respect to the ellipsoidal World Geodetic Survey 1984 (WGS-84) model, which is used in describing the Earth as an ellipsoid throughout this study.

For data set B, star-sensor data for a single star has been synthesized in addition to sun-sensor data. This has been done because the sun is eclipsed by the Earth for most of the time when magnetometer and attitude data are available. It has been assumed that the star-sensor is measuring the unit direction vector to an imaginary star which is located 90° away from the sun in the equatorial plane of the ECIF coordinate system. This synthesized star-sensor data has been designed to mimic the beneficial effect that sun-sensor data would have if it were available.

B. MAGSAT

The MAGSAT spacecraft was launched in October 1979. The objective of this mission was to obtain accurate vector measurements of the near-Earth geomagnetic field for field modeling purposes. The orbital properties were : 97° inclination, 330 km perigee height, and 500 km apogee height. This S/C had a cesium scalar magnetometer for calibration purposes and a vector fluxgate magnetometer. The accuracy of the vector magnetometer was determined to be 3 nT 1-σ after inflight calibration using the scalar magnetometer. The 3-axis magnetic field components and the tracked position of the S/C are available in the CD-ROM archive of the

NSSDC. The magnetic field vector data is expressed in south, east, zenith local coordinates. Due to the imperfect calibration of the attitude determination system, frequent small jumps occur in the vector magnetometer data, but not in the magnitude of the measured magnetic field. Two MAGSAT data sets have been used. One set was measured on February 2, 1980 (data set A), and the other set was measured on March 12, 1980 (data set B). For the purpose of direct comparison with Ref. 6's results, the data set that was used in Ref. 6 also has been tested (this will be referred to as the old data). The old data set included position and magnetic field magnitude data and was obtained via direct contact with NASA personnel. Reference 6 stated that this data was measured on February 2, 1980, i.e. on the same day as data set A. Comparison of the old data and data set A, however, shows that the old data is quite different from data set A and may not have been measured on February 2, 1980. The sampling period of all three data sets was about 100 second without any gaps, and it was measured on magnetically quite days. Each data set lasts 24 hours.

Sun-sensor data has been synthesized for data sets A and B. This is similar to what has been done for DE-2, except that the y_{2mes} dot product is computed in south, east, zenith local level coordinates instead of in S/C coordinates.

C. Ørsted

The Danish satellite Ørsted was launched in Febuary 1999. The main objective of this mission is to provide a precise global mapping of the Earth's magnetic field. The orbit of the Ørsted has an apogee altitude of 865 km, a perigee altitude of 649 km, and an inclination of 96.48° as of Febuary 10, 2000. Ørsted has a scalar magnetometer and a fluxgate vector magnetometer. Vector magnetometer data in local south, east, zenith coordinates and absolute position data are available. Two data sets which were measured on December 21, 1999 (data set

A) and January 18, 2000 (data set B) have been tested. Both data sets were measured when the solar storm activity level was low. The data was sampled at about 30 second intervals and has a maximum data gap of 57 minutes. Each data set lasts 24 hours. Similar to MAGSAT, sun sensor measurements and the y_{2mes} dot product have been synthesized in local south, east, zenith coordinates.

IV. Results of Filter Tests Using Real Flight Data

Two batch filters, one without field model corrections and the other with field model corrections, have been tested with DE-2, MAGSAT, and Ørsted flight data. The magnetometer bias vector is estimated in both filters only for DE-2 because this is the only S/C whose magnetic field vector data is given in S/C coordinates. It would not make physical sense to define a bias in the local south, east, zenith coordinate system in which the MAGSAT or Ørsted data is available. The filter tests without field model corrections are useful because the effects of the improved dynamics model can be clarified through comparison with the results of Ref. 6 for DE-2 and MAGSAT. Its results also can be compared with the results of the field-model-correcting filter to check whether there is an advantage to using field model corrections.

The performance of these filters has been evaluated by comparing their S/C position estimates with those that have been derived from ground tracking data. The filters' predicted accuracies of their position estimates also have been calculated using the estimation vector's covariance and linearized covariance propagation techniques[9]. This predicted accuracy has been compared with the actual accuracy as a means of checking the validity of the filter's model.

A. DE-2 Results

a) Check of Raw Data Against the *a Priori* Magnetic Field Models

In the process of filtering DE-2 data, two different Earth's magnetic field models have been used as *a priori* starting points for the batch filter. One is the Langel and Estes's 80 model[14], and the other is an interpolated Definitive International Geomagnetic Reference Field (DGRF) model. Langel and Estes's 80 model was derived using the data from MAGSAT. The secular variation coefficients of this model are considered to be valid between 1978 and 1982, and the magnetic field in DE-2's 1981 – 1982 time frame is calculated by using this model of the secular variations. The interpolated DGRF model used DGRF 1980 and DGRF 1985 models that are provided by the National Geophysical Data Center (NGDC)[‡]. These two models have been interpolated to calculate a field model for the relevant parts of 1981 and 1982.

It is important to note that the interpolated DGRF model depends on future information that would not have been available in the 1981-1982 time frame, the 1985 DGRF model. The Langel and Estes's 80 model, on the other hand, only depends on measurements that were taken up to 1980. Therefore, Langel and Estes's 80 model would have been available during the operation period of DE-2 and thus is a reasonable model to be used in the test of an operational orbit determination filter. The reason for also considering the interpolated DGRF model is that it was thought that this model might be more accurate than Langel and Estes's 80 model for 1981. If so, then one could study the effects of the accuracy of the *a priori* magnetic field model on the performance of the batch filter.

The accuracy of these two field models has been assessed by considering their residual errors in comparison to the DE-2 magnetometer data. These calculations make use of the known

[‡] http://www.ngdc.noaa.gov/seg/potfld/tab1igrf.shtml#IGRF95.

S/C position and attitude to compute the modeled field in S/C coordinates. The per-axis rms value of the residual error of data set A with the Langel and Estes's 80 model ranges from 45 nT to 121 nT and the peak residual error component is 607 nT. The field magnitude has a residual error with an rms value of 24 nT and a peak value of 153 nT. The residual errors that are calculated using data set B and using the interpolated DGRF model are similar. The per-axis rms of the residual errors for both models is less than 0.7 % of the total magnitude of the measured field, which means that both models are quite accurate even before field model corrections are estimated.

b) Tests of the Filter that does not Estimate Field Model Corrections

Tests have been made for the filter that uses only magnetometer data and that does not estimate corrections to the field model. Recall that this is like the batch filter of Ref. 6, except that it has an improved dynamics model. These results isolate the effects of the improved dynamics model.

Table 1 shows the maximum position error magnitude of this batch filter when operating on the DE-2 data sets. Two different field models have been tested and the results for the old dynamics model of Ref. 6, 9, and 10 are also presented for comparison purposes. The maximum position error magnitude is as low as 5.29 km with the new dynamics model and as low as 6.37 km with old dynamics model. The accuracy of the estimated position does not vary significantly with the choice of field model. Similarly, the effect of the change in the dynamics model is not big. The best performance is for the Langel & Estes's 80 field model and the new dynamics model.

Table 1. Maximum Position Error Magnitude of DE-2 Using the Filter that does not Estimate Field Model Corrections

Data set	Langel & Estes's 80 model		Interpolated DGRF model	
	New Dynamics Model	Old Dynamics Model	New Dynamics Model	Old Dynamics Model
A	5.29 km	7.70 km	6.65 km	6.95 km
B	5.87 km	6.37 km	8.13 km	7.34 km

The results for all of these cases, however, are far better than the results of Ref. 6, which reported 18 km maximum position error magnitude for DE-2 with the old dynamics model. This discrepancy in the Ref. 6 is probably due to a data processing error for the data of Ref. 6, which was obtained by direct contact with NASA personnel in 1990. This data was measured during the same time period as data set B from the NSSDC CD-ROM archive. The data set B magnetic field is quite different from that of the Ref. 6 data. The field components of the old data are different by up to 100 nT from the new data. The rms error between the old magnetic field magnitude data and the modeled field magnitude using known S/C position is over 40 nT, which is far bigger than the corresponding figure for the new data: 25 nT.

c) Tests of the Filter that Estimates Field Model Corrections

In order to use the batch filter that estimates field model corrections along with the orbit, sun-sensor or star-sensor data are needed in addition to magnetometer data. When using mixed data types in a filter, it is important to use reasonable predictions of their accuracies for the purpose of weighting their errors as in eq.(7). The 1-σ accuracy of the magnetometer data and the sun-sensor or star sensor data are assumed to be 15 nT and 0.15°, respectively. With σ values as inputs, the filter's optimal residual measurement errors show similar levels of accuracy.

The DE-2 results for this batch filter are summarized in Tables 2. Table 2 presents the results for data set A with simulated sun-sensor data, data set B with simulated sun-sensor data, and data set B with simulated star-sensor data. Langel & Estes' 80 magnetic field model is used as the *a priori* model for all cases. Note that the table has 10 rows and 7 columns. The first column gives the value of N, the highest order and degree of the field model corrections that the filter estimates – review eq. (4). Thus, each row corresponds to a different maximum order and degree of the filter's estimated field model corrections, and the complexity of the corrections (and of the filter) increases as one moves from the upper rows to the lower rows. (Note that in all cases considered in this paper the filter's field model uses a 10^{th}-order/10^{th}-degree field model. It uses *a priori* coefficients for terms that do not get corrected by the filter's estimation process.) The 2^{nd}, 4^{th}, and 6^{th} columns tabulate the filter's predictions of the maximum standard deviations of the position error magnitude for the three different cases. Unless otherwise noted, the term “maximum” refers to maximization of a given quantity over the entire batch interval. Columns 3, 5, and 7 give the actual maximum position error magnitudes for these three cases as computed by differencing the estimated positions with the true positions as derived from ground-based tracking data.

The results in Tables 2 reveal much about the operation of the filter. First, one can see that the maximum order and degree of the field model corrections should not be greater than 3. The position accuracy degrades markedly whenever field model corrections are included of 4^{th} order and degree or higher. The maximum position error magnitude ranges from 4.99 km to 9.82 km for different data sets if lower order corrections than 4^{th} order/degree are used, but these figures jump to 18 km for data set A and 63 km for data set B if the higher order correction terms are included.

Table 2. Maximum Position Error Magnitude with Field Model Corrections for Three Different DE-2 Data Sets

Order and Degree of Field Model Corrections	Data Set A with Synthesized Sun-sensor Data		Data Set B with Synthesized Sun-sensor Data		Data Set B with Synthesized Star-sensor Data	
	Predicted Maximum 1-σ Accuracy of Position (km)	Maximum Position Error Magnitude (km)	Predicted Maximum 1-σ Accuracy of Position (km)	Maximum Position Error Magnitude (km)	Predicted Maximum 1-σ Accuracy of Position (km)	Maximum Position Error Magnitude (km)
1st	1.00	4.99	1.69	7.54	1.19	7.48
2nd	1.09	5.05	2.24	9.45	1.42	9.82
3rd	1.37	5.99	4.32	8.01	1.61	8.92
4th	1.54	8.45	5.49	21.55	1.95	15.44
5th	1.84	12.25	7.36	23.71	3.24	22.07
6th	2.71	10.13	8.83	14.10	5.11	19.50
7th	3.56	13.47	13.17	19.90	7.87	13.61
8th	4.13	18.41	19.03	46.05	10.87	31.57
9th	4.56	13.58	32.42	55.14	16.08	43.23
10th	4.90	11.13	56.52	63.31	22.02	13.66

The position error magnitudes associated with data set A are smaller than the corresponding values for data set B. With 1st to 3rd order/degree field model corrections, data set A yields maximum position error magnitudes from 4.99 km to 5.99 km, but data set B's maximum position error magnitude ranges from 7.48 km to 9.82 km. This trend becomes more apparent for higher order/degree field model corrections.

The better position accuracy of data set A may come from the fact that the actual magnetometer data coverage time of data set A is two times longer than for data set B; 13.9 hours for data set A versus 6.7 hours for data set B. This effect is probably the reason why data set A yields lower predicted 1-σ position accuracies (compare columns 2, 4, and 6 of Table 2).

Recall that synthesized star-sensor data also has been tested for data set B because synthesized sun-sensor data is often unavailable due to eclipse. The results with synthesized star-sensor data, however, are not much different than those with synthesized sun-sensor data.

Overall, the results with field model corrections are not clearly better than those without field model corrections (review Table 1). The best case-A results with field model corrections (maximum position error of 4.99 km) are only slightly better than data A results without these corrections (maximum position error of 5.29 km or 6.65 km depending on the field model used). The best case-B results with corrections (maximum position error of 7.48 km) are not as good as the best results for the filter that does not estimate field model corrections (maximum position error of 5.87 km). These results may be caused by the low percentage of actual measurement coverage time for DE-2 over the 2-day span of the data.

Note that the dynamic model used in this study can cause a maximum position error of about 4 km over the 48 hour duration of a DE-2 data set: 3.50 km for data set A and 4.33 km for data set B. Therefore a maximum position error for magnetometer-based orbit determination as low as 4.99 km is very good. It is almost at the limit of the accuracy of the dynamics model.

The ratios of the maximum position error magnitudes to the filter's predicted position accuracies range from 2.27 to 6.66 for data set A and from 1.51 to 7.91 for data set B (except for the case of 10^{th} order/degree corrections). This implies that the filter's statistical model is roughly correct. This ratio would be about 3 if the statistical model were very accurate. This implies that the actual position error of the batch filter can be roughly estimated by using its predicted position accuracy.

B. MAGSAT Results

a) Check of Raw Data Against the *a priori* Magnetic Field Models

All MAGSAT data used in this test was measured in 1980. Therefore, Langel and Estes' 80 model has been used for the MAGSAT data sets. The accuracy of this field model has been tested, as in the DE-2 cases, by comparison of the measured field with the modeled field using known S/C positions. The residual magnitude errors of the magnetic field for the three different data sets of MAGSAT are 20.5 nT for the old Ref. 6 data, 24.5 nT for data set A, and 15.8 nT for data set B, and the component errors for data sets A and B have similar rms values. Data set A clearly has noisier magnetic field measurements than data set B. The old data used in Ref. 6 has an intermediate rms accuracy.

b) Tests of the Filter that does not Estimate Field Model Corrections

The batch filter without field model corrections has been tested for MAGSAT data sets A and B and for the old data used in Ref. 6. Table 3 shows this filter's maximum position error magnitude for the three different data sets and the two different dynamics models. As reported in Ref. 6, the maximum position error with the old dynamics model is around 8 km. The best maximum position error found with MAGSAT is 3.94 km with the new dynamics model and the old data. The performance on the old data and the performance on data set B are better than on data set A. This may be due to the noisier data of data set A.

Table 3. Maximum Position Error Magnitude for MAGSAT Using the Filter that does not Estimate Field Model Corrections

Dynamics Model	Maximum Position Error Magnitude for Old Ref. 6 Data	Maximum Position Error Magnitude for Data Set A	Maximum Position Error Magnitude for Data Set B
Old Dynamics Model	8.42 km	8.73 km	7.73 km
New Dynamics Model	3.94 km	7.02 km	4.56 km

Although the contribution of the improved dynamics model to the estimated orbit accuracy is not obvious for the DE-2 cases, there is an obvious improvement in the position estimation accuracy for MAGSAT. The difference in the contribution of the improved dynamics model for the two S/C cases may be related to the differences in orbital properties of DE-2 and MAGSAT. MAGSAT has a lower altitude orbit on average than that of DE-2. It is possible that the improved drag model of the new dynamics model contributes to the improvement of orbit propagation accuracy more for a lower altitude S/C and thus led to the improved position estimation accuracy for MAGSAT.

If this is so, then an improved position estimation accuracy can be expected with a refined dynamics model for a lower altitude S/C. It is important to note that there seems to be a "gain" factor from the orbital propagation accuracy to the overall position estimation accuracy. The orbital propagation accuracy for 1 day's worth of MAGSAT data is 1.28 km with the new dynamics model and 2.16 km with the old dynamics model. The magnetometer-based navigation filter experiences a larger decrease in the peak position error, 3.94 km with the new dynamics model versus 8.42 km with old dynamics model when operating on the old data set. These results suggest that further filter accuracy improvements might be achievable via further refinement of the orbital dynamics model.

c) Tests of the Filter that Estimates Field Model Corrections

The batch filter that estimates field model corrections in addition to the orbit has been tried for data sets A and B. The old data used in Ref. 6 only has the magnitude of the magnetic field and thus cannot be used in this filter. Table 4 summarizes the MAGSAT results for this filter. These results assume 1-σ accuracies of the magnetometer and sun-sensor data of 15 nT and 0.15°, respectively.

Table 4. Maximum Position Error Magnitudes with Field Model Corrections for Two Different MAGSAT Data Sets

Order and Degree of Field Model Corrections	Data Set A with Synthesized Sun-sensor Data		Data Set B with Synthesized Sun-sensor Data	
	Predicted Maximum 1-σ Accuracy of Position (km)	Maximum Position error Magnitude (km)	Predicted Maximum 1-σ Accuracy of Position (km)	Maximum Position Error Magnitude (km)
1st	0.86	3.89	0.82	3.23
2nd	0.89	4.09	0.91	2.16
3rd	1.07	3.58	1.04	2.64
4th	1.26	3.34	1.18	3.55
5th	1.51	3.56	1.50	3.39
6th	1.77	4.44	1.76	3.28
7th	2.10	4.87	2.09	5.52
8th	2.18	5.52	2.17	4.61
9th	2.24	7.05	2.23	3.81
10th	2.38	9.15	2.38	3.41

The best maximum position error magnitudes for the field-model-correcting filter are 3.34 km for data set A with 4th order/degree field model corrections and 2.16 km for data set B with 2nd order/degree field model corrections for data set B. The position error magnitude time histories for these best-case results are presented in Figure 1 along with the corresponding results from the filter that does not estimate field model corrections. There is a factor of two improvement in the position accuracy for best-case field model corrections in comparison to the accuracy without field model corrections for both data sets. Thus, unlike DE-2, field model corrections can improve the performance of magnetometer based orbit determination for MAGSAT. It is also true that the results with higher order/degree corrections for MAGSAT are not as bad as they are for DE-2. Recall from Table 2 that the position estimation error of the field-model- correcting filter becomes worse for DE-2 as the complexity of the order/degree of the field model corrections increases. For both data sets of MAGSAT, however, the results with higher order/degree corrections may not represent the best position estimate, but the estimate is still better than that of the filter without field model corrections - except for the 9th and 10th order/degree correction cases of data set A and the 7th and 8th order/degree correction cases of data set B. The relatively bad results in the 9th and 10th order/degree correction cases for data set A may be caused by the fact that data set A had noisier measurements.

The ratios of the maximum position error magnitudes to the filter's predicted position accuracies range from 2.32 to 4.60 for data set A and from 1.43 to 3.94 for data set B. This shows that the assumed accuracy of the measurement data in this filter is quite reasonable. Therefore, the maximum position error of the batch filter can be inferred very well from the predicted position accuracy.

C. Ørsted Results

There is a question of whether the DE-2 and MAGSAT results were helped by the fact that the data comes from a time period for which there is a very good *a priori* estimate of the Earth's magnetic field. This is true because the currently available field models for their flight time-frames are based on data that extends after the fact or are based on MAGSAT's accurate survey of the Earth. To clarify this issue, the orbit determination systems should be tested with recent magnetometer data that is measured when the only available Earth magnetic field model is one that is based only on past data of moderate accuracy. Ørsted data is useful for such a test because it carries an accurate magnetometer and is currently operational.

a) Check of Raw Data Against the *a priori* Magnetic Field Models

The *a priori* Earth magnetic field for the two Ørsted data sets have been calculated via propagation of the 1995 (for data set A) or 2000 (for data set B) International Geomagnetic

Reference Field (IGRF) model using secular terms. These models have been obtained from the NGDC[§]. Preliminary tests using these field models have been performed in order to check the accuracy of the raw Ørsted data. These tests involved comparisons of the measured Ørsted field with the modeled field in the local south, east, zenith coordinates. This comparison makes use of the Ørsted position data from ground tracking. The rms value of the magnetic field magnitude error is 72.22 nT for data set A and 22.52 nT for data set B. The peak absolute value of the residual error components ranges from 219 to 268 nT for data set A and from 102 nT to 579 nT for data set B.

The large rms of the residual error of data set A may be due mainly to the uncertainty in the magnetic field model. This uncertainty is caused by the propagation of the 1995 field model to the December 1999 time frame of data set A. As in MAGSAT, the residual errors of the magnetic field vector components in the local south, east, zenith coordinates have frequent small jumps.

b) Tests of the Filter that does not Estimate Field Model Corrections

The batch filter without field model corrections has been operated on Ørsted data. This filter uses only magnetometer data, and it does not estimate any field model corrections. Unlike DE-2 and MAGSAT cases which have a much lower perigee altitude, $\bar{b}$ is not practically observable for Ørsted because the effects of drag on its orbit are minimal. Therefore, unless otherwise noted, the filter that incorporates an a priori $\bar{b}$ estimate has been used on Ørsted data when orbit and, optionally, field model correction terms are estimated.

[§] http://www.ngdc.noaa.gov/seg/potfld/tab1igrf.shtml#IGRF95.html and ftp://www.ngdc.noaa.gov/Solid_Earth/Mainfld_Mag/Models

The maximum position error magnitude is 59.50 km for data set A and 9.09 km for data set B when no field model corrections are estimated. Based on the large residual field model error of data set A in comparison to the *a priori* magnetic field model, it is no surprise that the estimated position without field model corrections has such a large error for that data set. The maximum position error of data set B is a lot smaller than that of data set A, but is still worse than any DE-2 or MAGSAT cases when the same filter is applied.

c) Tests of the Filter that Estimates Field Model Corrections.

The Ørsted data sets have also been filtered using the filter that estimates field model corrections along with orbit. Table 5 presents the results for these cases for different orders/degrees of the field model corrections for data sets A and B. The 1-σ accuracies of the magnetometer data and the sun-sensor data are assumed to be the same as for the DE-2 and MAGSAT cases.

For data set A, the maximum position error keeps decreasing from 58.72 km at 1st order/degree field model corrections down to 2.47 km at 6th order/degree field model corrections and stays around 3 to 4.36 km for higher-order corrections. For data set B, the error in the estimated position reaches a minimum for 5th order/degree field model corrections and remains around 5 to 6 km for all higher-order corrections. The best maximum position error with field model corrections is 2.47 km with 6th order/degree field model corrections for data set A which is very similar to the best estimate of 2.16 km for data set B of MAGSAT. The best maximum position error for data set B of Ørsted is 3.24 km with 5th order/degree field model corrections and is very much like the maximum position error of 3.34 km for data set A of MAGSAT. The slightly worse results of Ørsted data set B as compared to data set A may suggest that data set B is noisier than data set A.

Table 5. Maximum Position Error Magnitudes for Two Ørsted Data Sets with Estimation of Field Model Corrections

Order and Degree of Field Model Corrections	Data Set A with Synthesized Sun-sensor Data		Data Set B with Synthesized Sun-sensor Data	
	Predicted Maximum 1-σ Accuracy of Position (km)	Maximum Position error Magnitude (km)	Predicted Maximum 1-σ Accuracy of Position (km)	Maximum Position Error Magnitude (km)
1st	0.45	58.72	0.45	7.90
2nd	0.62	29.07	0.65	5.62
3rd	0.86	17.36	0.84	5.51
4th	0.99	8.25	0.95	5.94
5th	1.05	6.27	1.01	3.24
6th	1.07	2.47	1.09	5.98
7th	1.09	3.00	1.17	5.64
8th	1.11	4.36	1.27	4.90
9th	1.15	3.34	1.31	4.49
10th	1.21	3.21	1.37	4.68

Figure 2 shows the position error magnitude time histories for the best position estimate with field model corrections and for the one without field model corrections for each data set. The improvement of the accuracy of the estimated position due to field model corrections is more dramatic for data set A than for data set B. Recall that data set A was measured on December 1999. Therefore, the propagation of the 1995 IGRF model caused an *a priori* residual rms field error of over 70 nT, which is a lot bigger than the 23 nT residual error of data set B, which was measured on January 2000 and used the 2000 IGRF model. Therefore, the inaccuracy of the *a priori* Earth magnetic field model seems to be the driving factor in causing position estimation errors in the case of Ørsted data if one uses a filter that does not estimate corrections to the Earth's magnetic field. The Ørsted results confirm that the field-model-correcting batch filter can contribute to a substantial improvement in the orbit determination performance when the only available Earth magnetic field model is one from predictions and thus inaccurate.

This conclusion was further confirmed by a set of erroneous runs that was conducted in the course of this research. By mistake, Ørsted data set B was originally run using a propagation of the 2000 IGRF field model to 2010. This mistake caused very poor position estimation performance when no field model corrections were estimated or when only low-order corrections were estimated. When high order corrections were estimated, the position estimation errors become very small, comparable to those that have been obtained using the correct model. Thus, the field-model-correcting filter was able to correct the IGRF model dating bug.

Table 5 also shows that the maximum position error is far bigger than three times the filter's predicted position accuracy for lower order/degree corrections, but this ratio becomes reasonable for higher order/degree corrections. This is further confirmation that field model errors represent a significant source of systematic modeling error when no corrections, or only low-order corrections are made.

d) Tests of the Uniqueness and Reality of the Corrected Field Model

The following questions arise: Are the filter's field model corrections unique, and are they indicative of real variations in the Earth magnetic field? These questions are answered through three tests of the field-model-correcting filter with Ørsted data set B, which was measured in January 2000. These tests compare various corrected and uncorrected field models. Note that the field model correction terms from the field-model-correcting filter include corrections to the Earth's internal magnetic field and external ring current effects. Therefore, when a corrected field model is compared to an *a priori* propagated IGRF field model, then only internal terms are considered. If two different corrected field models are compared, then both internal and external terms are considered in these tests.

The first test is to check whether the corrected field is unique, which amounts to an observability test. Two different *a priori* field models, the 1995 IGRF model and the 2000 IGRF model, have been used in the filter, and 10^{th} order/degree corrections are estimated in each case. The rms value of the magnitude of the field vector error between the two resultant corrected field models has been computed along the orbit. It is 6.7×10^{-5} nT. The high level of agreement between the two different corrected field models implies that the corrected field model is unique, and therefore simultaneously observable with orbit.

The second test checks whether the filter's field model corrections cause the field model to become more accurate. This test compares the magnetic field vector based on the propagated 2000 IGRF field model without corrections with one based on the propagated 1995 IGRF field model. In one comparison the propagated 1995 IGRF model is uncorrected, and in the other case it is corrected by the filter based on data set B. The rms value of the magnitude of the field vector error between the propagated 1995 IGRF model and the 2000 IGRF model is 95.87 nT along the orbit before corrections get applied to the 1995 model. This rms error decreases as higher and higher orders and degrees of corrections are estimated and applied to the 1995 model. It decreases to 24.59 nT for 10^{th} order/degree corrections. This test shows that the field-model-correcting filter with higher order/degree corrections drives the inaccurately predicted January 2000 field of the propagated 1995 field model toward the more realistic IGRF 2000 field model. It also confirms that low order/degree corrections are not enough to predict the January 2000 field model from the 1995 model; for example, 1^{st} order/degree corrections left a 93.29 nT rms error.

The third test considers whether the simultaneous estimation of orbit has a negative effect on the estimation of field model corrections. In this test corrections to the 2000 IGRF field model are estimated for Ørsted data set B in two different ways. One way uses the field-model-correcting orbit determination filter. The other way uses the 2nd filter that is described in section II. E, the one that estimates only the corrections, not the orbit. Recall that this filter uses the tracking data to determine instantaneous spacecraft position. The corrected fields that have been estimated by these two techniques have been compared. The rms magnitude of their vector difference is 18.37 nT along the orbit. This small difference indicates that the simultaneous estimation of orbit does not have a large impact on the accuracy of the corrected field model.

The rms errors between the various field models show similar trends in the entire altitude range from 300 km to 900 km. The propagated 1995 IGRF model with estimated 10th order/degree corrections is 3 to 4 times closer to the uncorrected 2000 IGRF model than is the propagated, uncorrected 1995 IGRF model. Thus, the field model corrections from the Ørsted orbit determination filter are applicable to a broad range of altitude beyond those of Ørsted trajectory.

These tests confirm that the field model corrections from the orbit determination filter are unique and that they provide reasonable estimates of the true magnetic field.

D. Summary Results

Figure 3 compares the maximum position errors of the batch filter with field model corrections for different order/degree corrections for all three S/C. As discussed earlier, it can be seen that the performance of the field-model-correcting filter for different order/degree corrections is different for the three S/C. The maximum position error decreases a lot for data set A of Ørsted as the order/degree of the field model corrections increases. The results of Ørsted data set B and of both MAGSAT data sets improve modestly for lower order/degree corrections, but then do not vary much as the order/degree of the corrections increases further. Fortunately, they maintain low position estimation errors for most types of field model corrections. DE-2 performance varies dramatically from Ørsted and MAGSAT and degrades as the size of the order/degree of the corrections increases.

The trends of the maximum position error of the field-model-correcting filter with different order/degree corrections may imply the following. Very high order/degree field model corrections improve the performance of filter significantly for an inaccurate magnetic field model like the one used for data set A of Ørsted (propagation of the 1995 IGRF model to December, 1999). If, however, the *a priori* model is more accurate (like the 2000 IGRF model used in data set B of Ørsted and the 1980 IGRF model used for MAGSAT), then lower order/degree corrections are enough to achieve good accuracy. In this latter case, higher order/degree corrections will produce similar performance as with lower order/degree corrections because the higher order correction terms will not be very significant. In the DE-2 cases, however, the position estimation becomes worse with higher order/degree corrections. This may be due to the small measurement coverage time of magnetometer data with frequent and long data gaps.

V. Conclusions

Several magnetometer-based orbit determination batch filters have been developed and tested using real flight data. The basic filter estimates a S/C's orbit along with magnetometer biases, and, optionally, correction terms to a spherical harmonic model of Earth's magnetic field. If field model corrections are estimated, then the filter requires sun-sensor data as well as magnetometer data. The only enhancement to these filters as compared to previous work has been to use an improved orbital dynamics model and to allow the use of an *a priori* drag parameter estimate.

The filter performs well for flight data from the DE-2, MAGSAT, and Ørsted spacecraft. Using only magnetometer data without field model corrections, the maximum position error is 5.29 km and 5.87 km for two DE-2 data sets and 3.94 km to 7.02 km for three MAGSAT data sets. This filter's maximum position error for one Ørsted data set is 59.50 km. This large error is mainly due to error in the *a priori* Earth magnetic field model. The other Ørsted data set yields a 9.09 km maximum position error largely because it was given a better *a priori* field model.

The performance of the batch filter with Earth magnetic field model corrections varies among the different S/C. This filter augments the magnetometer data with sun-sensor data in order to make the field model corrections observable. The addition of field model corrections does not make significant improvements in the case of DE-2, and the addition of high-order corrections dramatically degrades the DE-2 position estimation accuracy. The performance with MAGSAT data improves by as much as a factor of 2 if field model corrections with order/degree of about 2 to 4 are estimated. Higher-order corrections do not improve MAGSAT, but neither do they degrade its performance by much. Ørsted data shows the most benefit from the estimation of field model corrections. In one case, the peak position estimation errors are decreased from 59.50 km when no field model corrections are estimated down to 2.47 km when corrections up to 6^{th} degree/order are estimated. In a case with a better *a priori* Earth magnetic field model, the improvements due to the estimation of field model corrections are less dramatic. The general trends of the MAGSAT and Ørsted results suggest that it would be wise to include field model corrections up to order and degree 6 if there is any significant uncertainty in the *a priori* Earth's magnetic field. The DE-2 results imply that the estimation of field model corrections will work poorly if there are long gaps in the magnetometer or sun sensor data.

The uniqueness and realism of the corrected magnetic field model has been tested using an Ørsted data set measured on January 2000. The corrected field model is unique for 10th order/degree field model corrections regardless of the *a priori* field model. Correction of a propagated 1995 IGRF field model using data from January 2000 reduces the rms value of magnitude of field vector differences from the 2000 IGRF field model. The rms difference is 95.87 nT before the field corrections, but it is only 24.59 nT after one applies the filter's estimated field model corrections. This shows that the field-model-correcting orbit determination filter produces realistic field model perturbations. It has also been confirmed that simultaneous estimation of orbit and field model corrections does not degrade the estimation of field model corrections.

Two other implications of this research have to do with the importance of measurement and model accuracy. The DE-2 results imply that a poor magnetometer calibration will greatly degrade the position estimation accuracy. MAGSAT results imply that an accurate orbital dynamics model is needed in order to get the best possible estimation accuracy from the filter. This is especially true at low altitudes, where the effects of atmospheric drag are most significant.

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Figure Captions

Fig. 1. Position error magnitude time histories for the batch filter without field model corrections and for the ones with the optimal order/degree of field model corrections. Top plot : MAGSAT data set A; Bottom plot : MAGSAT data set B.

Fig. 2. Position error magnitude time histories for the batch filter without field model corrections and for the ones with the optimal order/degree of field model corrections. Top plot : Ørsted data set A; Bottom plot : Ørsted data set B.

Fig. 3. Maximum position errors of the batch filter with field model corrections for different order/degree corrections for DE-2, MAGSAT, and Ørsted.

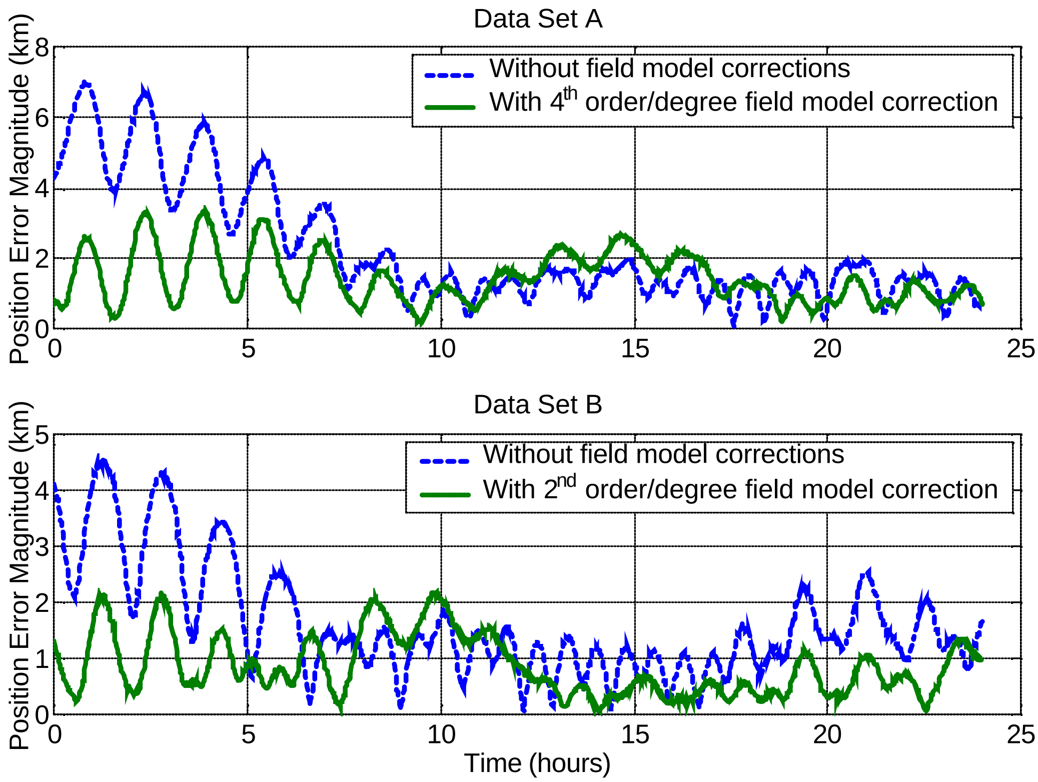

Fig. 1. Position error magnitude time histories for the batch filter without field model corrections and for the ones with the optimal order/degree of field model corrections. Top plot : MAGSAT data set A; Bottom plot : MAGSAT data set B.

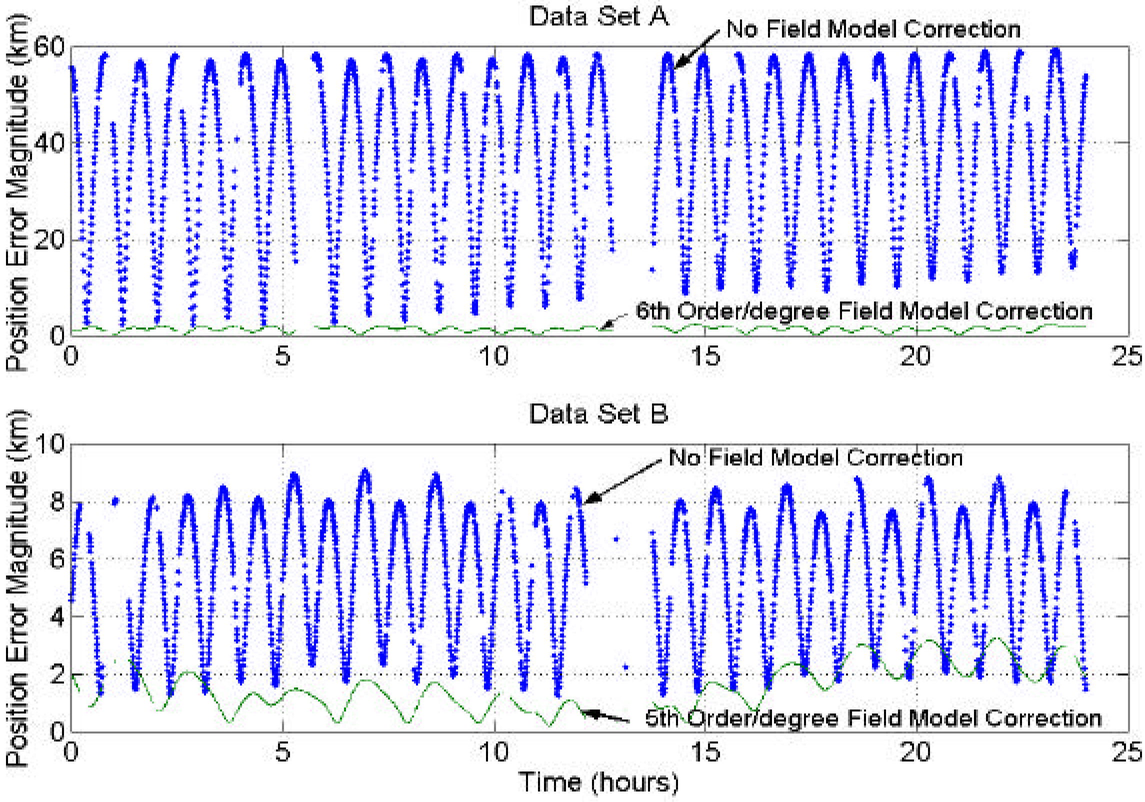

Fig. 2. Position error magnitude time histories for the batch filter without field model corrections and for the ones with the optimal order/degree of field model corrections. Top plot : Ørsted data set A; Bottom plot : Ørsted data set B.

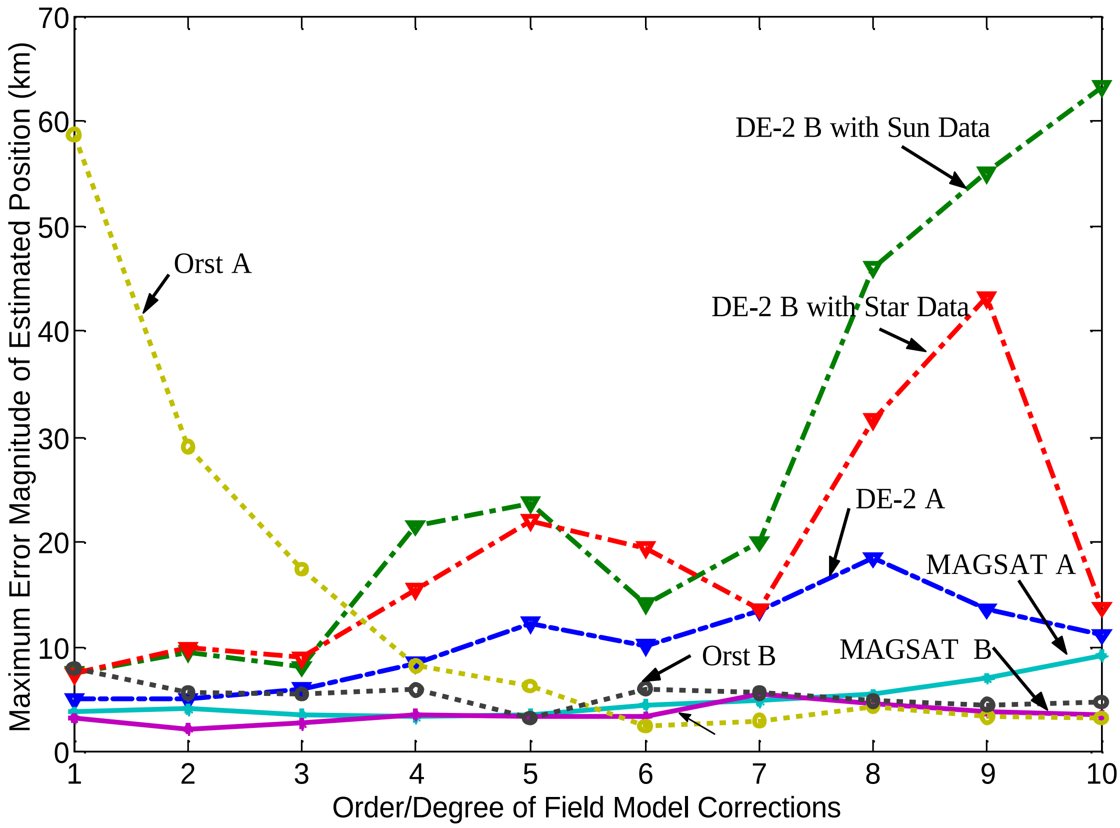

Fig. 3. Maximum position errors of the batch filter with field model corrections for different order/degree corrections for DE-2, MAGSAT, and Ørsted.

Hee and Psiaki, Fig. 3.